Modeling the relation between income and commuting distance

Giulia Carra
Institut de Physique Théorique, CEA, CNRS-URA 2306, F-91191, Gif-sur-Yvette, France

Ismir Mulalic
DTU Denmark

Mogens Fosgerau
DTU Denmark and
Royal Institute of Technology, Sweden

Marc Barthelemy
Institut de Physique Théorique, CEA, CNRS-URA 2306, F-91191, Gif-sur-Yvette, France and
Centre d’Analyse et de Mathématique Sociales, (CNRS/EHESS) 190-198,
avenue de France, 75244 Paris Cedex 13, France

We discuss the distribution of commuting distances and its relation to income. Using data from Great Britain, US and Denmark, we show that the commuting distance is (i) broadly distributed with a tail decaying typically as $1/r^\gamma$ with $\gamma \approx 3$ and (ii) an average growing slowly as a power law with an exponent less than one that depends on the country considered. The classical theory for job search is based on the idea that workers evaluate potential jobs on the wage as they arrive sequentially through time. Extending this model with space, we obtain predictions that are strongly contradicted by our empirical findings. We then propose an alternative model that is based on the idea that workers evaluate potential jobs based on a quality aspect and that workers search for jobs sequentially across space. We assume that the density of potential jobs depends on the skills of the worker and decreases with the wage. The predicted distribution of commuting distances decays as $1/r^3$ and is independent of the distribution of the quality of jobs. We find our alternative model to be in agreement with our data. This type of approach opens new perspectives for the modeling of urban phenomena.

Keywords: Statistical Physics — Urban economics — Mobility — Job-search — Modeling

INTRODUCTION

The recent availability of datasets on almost all aspects of life opens the exciting perspective of a scientific approach to social and urban systems validated by data. In particular, many interesting models were developed in urban economics [1–3] and despite the strength of their predictions, there is often a gap between these models and real data. A fundamental problem in economics is about the decision process of individuals for choosing their job, their residence location, etc. In particular, the labor market is a main topic of interest in economics, which leads to an interest in commuting and the choice of work and residential locations. We focus here on the job search process, which has a direct impact on the spatial distribution of commuting trips. The seminal contributions on search theory in economics [4–6] rely on the central assumption that individuals choose among different job offers that arrive sequentially in time, by maximizing their expected discounted net wage, while waiting to accept a job is costly. This is a very strong assumption that should be tested against empirical data.

Surprisingly, the standard model of job search [5] does not integrate space (space is however taken into account in some labour market studies, see for example [7]). We introduce the spatial component and derive the consequences of the job search model for the distribution of the commuting distance. In particular, we show that the basic McCall model [5] cannot explain some fundamental statistical features observed in empirical data. Therefore we propose a new stochastic model that does not rely on the important assumption of optimal control, but on the idea that an offer can be accepted if it has a certain level of ‘quality’ that integrates a large number of parameters specific to each individual. We find excellent agreement between this new model and data for Denmark, the UK, and the US. Going beyond the particular problem of the relation between income and commuting distances, our model provides a search-based microfoundation for models of spatial patterns that can be found in the mobility literature [8].

In the first part of this paper, we present an empirical analysis of the commuting distance distribution for Denmark, the UK, and the US, and analyze how the average commuting distance scales with income. In a second theoretical section, we derive the probability distribution for the commuting distance from the spatial extension of the standard job search model and compare it with our empirical results, showing that the standard theoretical framework is not in agreement with data.
pose a new stochastic model which does not rely on the optimal strategy assumption, and where workers evaluate potential jobs sequentially across space and based on a quality aspect. We then show that this new model is in excellent agreement with our data.

**EMPIRICAL RESULTS**

In this section, we investigate the distribution of commuting distances and its relation to individual income using three different datasets: Denmark, the UK [9], and the US [10]. These datasets are produced by national agencies and national household surveys (see Materials and Methods for details) and record at an individual level the commuting distance and the income range. These datasets cover the whole country and take into account all transportation modes. For the UK the data is for the years 2002-2012, for the US three different years are available (1995, 2001, 2009), and for Denmark we have access to 10 years (2001-2010).

The average commuting distance

We first focus on the simplest quantity, the average commuting distance, and how it varies with income. The results for the three countries studied here are shown in Fig. 1 (left column). In agreement with basic equilibrium models of urban economics [11–13] workers with higher incomes will have longer commuting distances, we observe that the average distance essentially grows with income, with the exception of the US where no particular trend can be detected.

For Denmark, we observe an increasing range and a saturation at large income values, while for the UK we observe a plateau at low income values. In the range where the increase is observed we can fit the data by a power law of the form

\[ p(Y) \sim Y^\beta \]  

where \( Y \) is the individual income and where the exponent \( \beta \) depends on the country considered. For the US, the fit gives an exponent \( \beta \approx 0 \) indicating that there is no clear trend. For the UK, the plateau occurs in the low income range \([10^2, 10^4]\) (GBP/year) around the commuting distance value \( r \approx 5 \) miles. The fit on UK data for incomes higher than 5,000 £ (for all modes and all years), gives an exponent value \( \beta \approx 0.5 \) (in the range \([0.53, 0.66]\) when considering different years). For the Danish data, in contrast we observe a strong dependence with a large exponent of order 0.8 for yearly incomes larger than 250,000 DKK and smaller than 500,000 DKK (for lower incomes we observe a small plateau). Depending on the year considered, the exponent \( \beta \) varies in the range \([0.61, 0.88]\).

![Fig. 1.](image)

**FIG. 1.** Left column: Average commuting distance versus income for different years. In dark blue, the commuting distance is averaged over all years. (Top) UK data. This loglog plot displays a plateau for small values of income followed by a regime, when fitted by a power law (see inset), gives an exponent \( \beta \approx 0.5 \) \([0.53, 0.66]\). In the inset the average commuting distance is averaged over all years and the power law fit gives an exponent \( \beta = 0.58 \). (Middle) US data. In this loglog plot we do not observe an income dependence. Indeed, a power law fit gives an exponent \( \beta \approx 0 \). (Bottom) Danish data. The power law fit on the commuting distance averaged over all years (in the inset) gives an exponent \( \beta = 0.77 \).

Right column: Commuting distance distribution for different income classes. The probability distribution is shown for different income classes. In dark blue we show the distribution for a particular value of the income for which the fit on the tail is shown in the inset. (Top) UK data (averaged over all available years). The distribution displays a slow decaying tail and a power law fit on the tail \((r > 10 \text{ miles})\) give exponents in the range \([2.67, 3.16]\) (in the inset \(\gamma = 3.16\)). (Middle) US data (averaged over all available years). The power law fit on the tail \((r > 20 \text{ miles})\) gives values for the exponent in the range \([2.98, 5.02]\) (in the inset \(\gamma = 3.9\)). (Bottom) Danish data (all years give the same result and we choose here to show the year 2008). The power law fit on the tail \((r > 20 \text{ km})\) gives values for \(\beta\) in the range \([2.11, 2.53]\) (in the inset \(\gamma = 2.28\)).

The distribution of commuting distance

We now consider the full distribution of the commuting distance. We show in Fig. 1 (right column) the distribution of commuting distance for different incomes for
Denmark, the UK, and the US. All these plots display a slowly decaying tail that can be fitted for large values of $r$ by a power law of the form $P(r) \sim r^{-\gamma}$ with $\gamma \in [2.11, 2.53]$ for Danish data, $\gamma \in [2.67, 3.16]$ for the UK data, $\gamma \in [2.98, 5.02]$ for the US, depending on the income class considered.

There are two important facts that we can extract from these empirical observations. First, for all datasets studied here, the distribution is broad. This means that the variation range of commuting distances is extremely large. Indeed, we observe that with a non-negligible probability, individuals in Denmark, UK, and US are commuting on distances of the order of a few hundred kilometers. Second, the shape of the distribution and the large distance behavior are remarkably similar among the different countries we have studied here, with a power law decay with exponent of order $\gamma \approx 3$. These non-trivial features are very important as they provide a test for any models that aims to describe spatial commuting patterns.

The three datasets observed here display a slow increase of the average commuting distance with income and, more importantly, a slowly decaying tail for large distances. These are the two characteristics that we would like to understand theoretically. We will begin with a discussion of the standard job search model proposed in economics [3][6], and compare their prediction to our empirical observations. This will lead us to propose another model, the ‘closest opportunity’ model with predictions in much better agreement with data.

**THEORETICAL MODELING**

The spatial optimal job search model

The stopping problem [14] is a classical problem in optimal control and has been applied in many different areas [15][19]. In particular, in economics it has been applied to the job search problem [3][6] and we will consider as a starting point the McCall model [5]. Here we study the consequences of this model for the spatial distribution of distances between residences and jobs depending on the income. We begin by describing this model in its simplest version. The job search process is sequential in time, and a worker who is unemployed at time 0, reviews at every time step a random wage offer $w$ drawn from a distribution with density $f$ (and cumulative $F$). At each time step, the worker can either accept the current job offer and keep it forever, or she can pay a waiting cost $c$ to discard the offer and wait for the next offer. In this model, with an offer $w$ at hand, the worker maximizes her expected value $v(w)$

$$v(w) = \langle \sum_{t=0}^{\infty} \mu^t y_t \rangle,$$  

where the brackets denote the average over the offer distribution and where $y_t$ denotes the income of the worker at time $t$ (the discount factor is $\mu < 1$). The worker’s income is either $y_t = w$ if she accepts the offer or $y_t = -c$ if she refuses it. The classical way to solve this problem is to write the Bellman equation for this stopping process which reads [20]

$$v(w) = \max \left\{ \frac{w}{1-\mu}, -c + \mu \int v(w') f(w') dw' \right\}.$$  

Here, $v(w)$ is the value of a current wage offer $w$. It is equal to the maximum of two terms: the first term is the discounted wage and occurs of the current offer is accepted, and the second term occurs if the current offer is rejected, in which case the worker pays the waiting cost and obtains the value of the next period offer. The optimal strategy that solves this equation is to accept the current offer if it is larger than a reservation wage $\tau$ and to refuse it if it is lower. The reservation wage satisfies the equation

$$\frac{\tau}{1-\mu} = -c + \frac{\mu}{1-\mu} \left[ \tau F(\tau) + \int_{\tau}^{\infty} w' f(w') dw' \right].$$  

By solving this equation, we obtain a function $\tau$ that depends on the offer distribution. The number of trials $N$ before accepting a job offer thus follows a geometric distribution

$$P(N) = (1 - p)^{N-1} p,$$  

where $p$ is the probability of accepting an offer

$$p = \int_{\tau}^{\infty} f(w) dw.$$  

A skill level $I$ governs the distribution of wage offers and the reservation wage and hence $p$ and the income will depend on this level $I$.

Space is absent at this point and we will now extend the McCall model with space in the simplest possible way. We assume that the individual now reviews the job offers sequentially in the order of increasing distance from the home. This means that the worker, starting from home, will examine the offer and will choose the first one that is above her reservation wage in order to stay optimal. We will also assume that the jobs are uniformly distributed in space with density $p$ and the probability that the individual accepts an offer is still given by Eq. (5). If a worker has accepted the $N^{th}$ offer, the probability that she has moved a distance $r$ from its residence is given by
a classical result for the $N$th nearest neighbors in dimension $d = 2$ for uniformly distributed points

$$P(R = r|N) = \frac{2}{(N-1)!} \frac{1}{r} (\rho \pi r^2)^N e^{-\rho \pi r^2}. \quad (6)$$

The distribution of the commuting distance $R$ is then given by

$$P(R = r) = \sum_{N \geq 1} P(r|N)P(N) \quad (7)$$

and since the distribution of $N$ is geometric (Eq. [4]), we obtain

$$P(R = r) = 2p \rho \pi r e^{-p \rho \pi r^2}. \quad (8)$$

This distribution is not a power law and decreases exponentially over a scale of order $\sim 1/\sqrt{\rho}$ where $1/\sqrt{\rho}$ corresponds to a typical interdistance between different offers (tau and therefore $p$ depend on the income $Y$ and so does this distance too). We also note that the average commuting distance decreases if the spatial density of opportunities $\rho$ increases. A decrease in the number of job openings during economic downturns then leads to increasing commuting distances.

The result Eq. [8] is thus not consistent with the empirical evidence. In addition we also considered another generalization of the McCall model with transport costs, and we showed that it also cannot be consistent with empirical observations (see the general argument presented in the Materials and Methods section). It thus seems at this point that the optimal strategy is not consistent with data. At first glance this is however not unexpected. Indeed the empirical observations reveal that the commuting distances vary over a very broad range which suggests that the individuals behavior might depend on a large number of uncontrolled factors. We thus have to find an alternative model for explaining the empirical findings. In the next section, we will propose such a model and compare its predictions with data.

**The closest opportunity model**

In this new model, we revisit two important assumptions of the McCall model: i) workers search through space and not through time, and ii) jobs are chosen based on some ‘quality’ aspect and not on the wage (see for instance [22, 23]).

We still consider the problem of a worker who looks for a job starting from her residence (that we assume to be located at $r = 0$). We also assume that job offers are uniformly distributed across space with density $\rho$. In order to take income into account, we assume that each worker is characterized by a skill level $I$ that we will identify with income. The density of jobs $\rho = \rho(I)$ relevant for the worker depends on the skill level and we assume that it is simply $\rho = \rho_0/I^{\alpha}$, such that higher skill jobs are less dense than lower skill jobs. The exponent $\alpha$ depends on the country under consideration and reflects many exogenous factors concerning job offers at a certain skill level [22, 23].

The McCall model assumes that jobs are primarily characterized by the wage they offer. We depart from this and assume instead that each job is characterized by a random ‘quality’ $X$ that encodes many factors. The job quality is distributed according to $f$ (with corresponding cumulative distribution $F$) and job qualities are independent.

We assume that a given worker has a reservation quality value $\tau$ (in the same spirit as the reservation wage), and she will keep expanding her search radius until this threshold is met. We denote by $R$ the commuting distance and its cumulative thus reads

$$P(R \leq r|\tau) = P(X[0, r] \geq \tau) = 1 - F(\tau)\rho \pi r^2. \quad (9)$$

We now take into account that workers have different search costs and different expectations, which leads them to have different reservation qualities. We consider the reservation quality as random, distributed according to a density $g(\tau)$, and obtain the cumulative distribution of commute distances

$$P(R \leq r) = \int g(\tau)P(R \leq r|\tau)d\tau, \quad (10)$$

with corresponding density

$$P(R = r) = -2\rho \pi r \int g(\tau)F(\tau)^\rho \pi r^2 \log F(\tau) d\tau. \quad (11)$$

The first term in this integral is the probability that a worker has reservation quality $\tau$, the second term is the probability that all offers are below $\tau$ in the disk of radius $r$, and the last term (the log) corresponds to the probability that at least one offer is above $\tau$ in the circular band $[r, r + d\tau]$ (see Fig. 2(a) for a simple illustration of this process).

A simple and natural assumption for the distribution of the reservation quality $\tau$ is that it is the same as the distribution of job quality $F$. Then Eq. (11) simplifies in a remarkable way as follows

$$P(R = r) = -2\rho \pi r \int f(\tau) F(\tau)^\rho \pi r^2 \log F(\tau) d\tau$$

$$= -2\rho \pi r \int_0^1 x^{\rho \pi r^2} \log x dx$$

$$= \frac{2\rho \pi r}{(1 + \rho \pi r^2)^2}. \quad (12)$$

Under these assumptions, the distribution of commuting distances does not depend on the distribution of job quality, an effect that was already observed in the specific case discussed in [8], and the model proposed here...
The optimal control model predicts a behavior for the average commuting distance that depends on the offer distribution, while in the closest opportunity model, this behavior is independent from it, as long as we assume that the offer and reservation quality distributions are the same. We observe a behavior \( \tau \sim I^{\alpha/2} \) for the closest opportunity model where \( \alpha \) depends on the country considered, and we can interpret the empirical results in our framework. For the US, we observe an exponent \( \beta_{US} \approx 0 \) indicating that we should consider the density of jobs seems to be independent from the skill level in this country. For the UK and Danish dataset, we observe a non-zero exponent with \( \beta_{UK} \approx 1/2 \) for the UK and a larger value for Denmark \( \beta_{DK} \approx 0.8 \). These results indicate that the density of jobs decreases with the skill level, more in Denmark than in the UK. This argument suggests that in the US the spatial distribution of jobs is independent from the skill level while in Europe, there seems to be a stronger dependence on space of the job market, imposing a larger commute for more specialized jobs (for a discussion in equilibrium theory about the spatial distribution of workers and skill levels, see for example [21]).

The behavior of the average commuting distance is however not the crucial prediction for distinguishing between models. Indeed, for the simple spatial extension of the McCall model presented here, the distribution of \( r \) decreases very quickly (Eq. (8)) and is not a broad distribution (the extension with transport costs can lead to a power law but with a fine tuning of parameters, see the material and methods section). In sharp contrast, in the closest opportunity model, we have a broad distribution of the form given by Eq. (14) which decays as \( P(r) \sim r^{-3} \) for large \( r \) and this theoretical result implies a data collapse on the unique curve given by

\[
\tau(I) = \frac{1}{2} \sqrt{\frac{\pi}{\rho}} I^{\alpha/2},
\]

which is a power law with exponent \( \beta = \alpha/2 \).

In the next section, we will discuss how all these theoretical results compare with data and what are the theoretical consequences of our model.
we plot the rescaled commuting distance distribution for different income categories and we observe a very good collapse, except for the lower income category in the UK for which the square root behavior is not applicable.

We note that the empirical fit (Fig. 2b,c,d) gives an exponent smaller than the value 3 for large \( r \) predicted by this theoretical distribution and the agreement between data and the model is better demonstrated with this data collapse. We note that there are might be different reasons for a smaller value of the exponent \( \gamma \). First, there is a limited range of \( r \) and a direct fit on the tail of a function given by Eq. \((14)\) would lead to an exponent value different than 3. Second, we show in the Materials and Methods section, that transport cost could explain a smaller value of this exponent.

We also show the fit by the form given by Eq. \((16)\) for a given income category (in the dataset the income is divided in different categories that are different from one year to another and it is then difficult to average over all of them). The agreement with data is very good for the UK and the US, but there are some discrepancies in the DK case. It seems that for this Danish case there are other heterogeneities that are not taken into account in our model. Clearly, further studies on other countries are needed in order to elucidate this point.

**DISCUSSION AND PERSPECTIVES**

With the availability of always more precise data we can test a number of predictions of models for the urban structure and its processes. In this work we predict the distribution of commuting distances and how it is related to income.

We showed that the empirical data cannot support the standard McCall model (based on optimal strategy) for the job searching process. Instead, we showed that a simple model, based on the closest opportunity that meets the expectation of the individual is able to predict correctly the behavior of the average commuting distance with income in terms of the density of jobs offers. More importantly, this model is able to provide the correct form of the distance distribution, its broad tail, and the data collapse predicted by its form.

In other words, previous models relied on the idea that workers wait for a job that pays enough, while in the new model, workers search space for a job that is good enough.

Although further studies on more countries is certainly needed, this stochastic model provides a microscopic foundation for a large class of mobility models and opens many interesting directions in modeling mobility and leading to testable predictions.

**MATERIALS AND METHODS**

**Data description**

**UK data**

We use data from the UK National Travel Survey (NTS) for the years 2002 – 2012 \[9\]. Each year’s sample has a size of 15,048 addresses and was designed to provide a representative sample of households in Great Britain. A weighting methodology was developed to adjust for non-responses and drop-offs in the travel recording. Data collection is obtained from face-to-face interviews and a seven day travel record of individual daily travel activity.

We specifically exploit the individual and the trip files of this dataset. The individual file is used to determine the income category of each individual (data provides 23 income bands). The trip file allows us to link individuals with their weekly commuting trips for which we know the actual distance. In order to compute the average commuting distance as a function of the income class, we first average the commuting distance of each individual, no matter the transportation mode used, over the number of commuting trips undertaken during the week. We then average these quantities over all individuals for a given income category. When we consider average values from these data, we do not distinguish between different transportation modes or the geographical locations of the origin and destination of the trip.

**US data**

We use data from the 1995, 2001 and 2009 national household travel survey (NHTS) \[10\], a survey of the civilian, non-institutionalized population of the United States. The NHTS datasets contain data for respectively 42,033, 26,032 (with approximately 40,000 add-on interviews for the latest version ) and 150,147 households. Weighting factors are used in order to take into account nonresponses, undercoverage, and multiple telephones in a household.

These datasets, allow us to associate to each worker the income category she belongs to (we have for this dataset 18 different income bands) and the one-way distance to workplace. For the 2009 NHTS, the personal income is not provided, in this case we define the income category of each individual as the household income divided by the household size.

**Danish data**

The Danish data are derived from annual administrative register data from Statistics Denmark for the years
2001 – 2010. We observe the full population of workers, and for each year, we have information on the workers annual income and their commuting distance. We use the post-tax income. The commuting distances have been calculated using information on exact residence and workplace addresses using the shortest route. Note that for these data, no weighting methodology has been developed as we observe the full population of workers in the country.

Including transport cost in the McCall model

We discuss here the general case for the McCall model where there is a transport cost associated with distance. The distance from the home of worker to a job offer is then a random variable $R$ having density $2\pi pr$, which is independent of the wage $W$ associated with the job. In order to link the probability of accepting a job to space, we assume a linear transport cost $\delta R$ that is paid by the worker if she accepts a job. Ultimately, she cares about the net wage $W - \delta R$. The optimal strategy of the worker involves a reservation wage $\tau$ and the worker accepts the first offer that offers a net wage $W - \delta R > \tau$. These assumptions already imply that the commuting distance for the accepted job satisfies $R < \frac{W - \delta}{\delta}$. Then the tail behavior of the commuting distance cannot follow a power law if $W$ has a bounded distribution and we therefore allow $W$ to have an unbounded distribution. The density of commuting distances is

$$P(R = r|W - \delta R > \tau) = \frac{P(R = r) P(W - \delta r > \tau)}{P(W - \delta R > \tau)}$$

$$= \frac{2\pi pr (1 - F(\tau + \delta r))}{\int_0^\infty 2\pi rs (1 - F(\tau + \delta s)) ds}.$$  \hfill (17)

From this, we can observe that

$$\frac{\partial \ln P}{\partial \ln r} = 1 - \frac{\frac{f(\tau + \delta r)}{1 - F(\tau + \delta r)} \delta r.}$$ \hfill (18)

which shows that in general $P$ does not decay as a power law, unless $\frac{f(\tau + \delta r)}{1 - F(\tau + \delta r)} = \frac{\zeta}{r}$ for some $\zeta > 1$. In the specific case where $W$ follows a power law with $F(W) = 1 - W^{-\nu}, W > 1$, we obtain

$$\frac{\partial \ln P}{\partial \ln r} = 1 - \frac{\zeta r}{\tau + \delta r},$$ \hfill (19)

which tends to $1 - \frac{\zeta}{\tau}$ as $r \to \infty$. This model thus leads to a power law for the distribution of commute distances, if the distribution of wage offers follows a power law. If we consider all wages, the Pareto law tells us that they can be broadly distributed, but this is not the quantity needed here. Indeed we are considering here the offer distribution for a given set of skills and it is very unlikely that a given individual will sample offers that range over the whole income distribution.

We can then compute the relationship between the average commuting distance and income in this model. For $w - \delta r > \tau$, we have

$$P(R = r|W = w, W - \delta R > \tau) = \frac{P(R = r, W = w, w - \delta r > \tau)}{P(W = w, \delta R < w - \tau)}$$

$$= \frac{2r}{(w - \tau)^2};$$ \hfill (20)

which leads to the conditional expectation

$$E(R|W = w, W - \delta R > \tau) = \int_0^{\frac{w - \tau}{\delta}} \frac{2r^2}{(w - \tau)^2} dr$$

$$= \frac{2w - \tau}{3\delta}.$$

This model thus predicts that for a linear transport cost, the expected commute distance is always linear in income which does not fit the empirical findings.

In any case, it seems that in order to predict results consistent with empirical observations (a power law with exponent 3 for the distribution, and a power law behavior for the average distance), this model needs fine-tuning of the parameters, in sharp contrast with the closest opportunity model.

Including transport costs in the closest opportunity model

Workers base their decisions on transport costs that depend not only on distance but also on monetary costs and travel time. We shall see how transport costs can be accommodated by the closest opportunity model proposed in this paper. This is useful as we get exact predictions regarding how the observables of the model are modified by transport costs. The model can then also be used for prediction in cases when transport costs change.

Letting the variable $r$ represent the transport cost, the closest opportunity model predicts $d \log P(R = r)/d \log(r) = -3$. In general we may expect that the transport cost is an increasing and concave function of distance, since travelers switch to faster modes for longer trips. Denoting the physical distance by $\ell$, we assume that $r \sim \ell^\nu$, where $0 < \nu < 1$. In terms of distance we then find that

$$\frac{d \log P(R = r)}{d \log \ell} = \frac{d \log P(R = r)}{d \log r} \frac{d \log r}{d \log \ell} = -3\nu. \hfill (21)$$

For the income elasticity, the model predicts a relationship between transport cost and income that is $\beta = d \log r(t)/d \log I = 1/2$. The elasticity of commuting distance with respect to income is then larger:

$$\frac{d \log r^{\delta r}(Y)}{d \log Y} = 1/2 + \frac{1}{2\nu}. $$
Observing the commuting distance rather than transport cost, we thus expect an exponent in the tail of the distribution smaller than 3 in absolute value and an income elasticity of the average commuting distance that is greater than 1/2. It is thus possible to back out the exponent $\nu$ from both observed exponents.

**DATA ACCESSIBILITY**


USA data [10] are provided by the U.S. Department of Transportation, Federal Highway Administration, National Household Travel Survey. These can be downloaded from URL address: http://nhts.ornl.gov.

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