The cost recoverability of transport infrastructure under the risk of natural disasters

Ryo Itoh *

April 28, 2017

Abstract
This study examines cost recoverability, or whether expected revenue of the optimal congestion toll exceeds the cost of the optimal investment in transportation infrastructure, when its capacity is uncertain because of natural disasters. The government control degree of reliability of entire transportation system by combining two types of infrastructure; unstable infrastructure whose capacity decreases when a disaster occurs; and a stable one whose capacity is constant regardless of the disaster. Under the assumption of risk-averse preferences of households, the theorem of strict cost-recovery presented by Mohring and Harwitz (1962) does not holds despite the congestion toll is controlled in a completely flexible manner given any incidents, and the optimally designed unstable (stable) infrastructure is (not) cost recoverable because benefit of investment mitigating disutility of risk, called risk premium, is not covered by the revenue of congestion toll. We also show entire transportation infrastructure is not cost recoverable if (and only if) transport cost is specified by a linear function, and if price elasticity of transport demand is less than one as many empirical studies has shown.

Keywords: natural disaster, cost recovery, (un)stable infrastructure, risk aversion.

JEL classification: R42, D81.

*University of Tohoku, Aramaki aza aoba 6-3-09, Aoba-ku, Sendai, Miyagi, Japan 467-8579. E-mail: itoh@se.is.tohoku.ac.jp Tel: +81-022-795-4420
1 Introduction

Transportation infrastructure always faces a risk that it is seriously destroyed by natural disasters such as earthquakes and following tsunami-waves, as seen in Japan’s earthquake in 2011. After such earthquakes, recovering entire facility of infrastructure need significantly long period, from several months to several years, and the disaster area suffers from a significant loss in production and income during the recovery ¹. Although such risk or volatility caused by natural disasters is considered unfavorable, it is too significant for each people to hedge by insurance or deposit, even if we only consider monetary loss. Therefore, considering these risks and developing sufficiently robust transportation system towards disasters is an important task of government.

One typical and effective policy measure to mitigate such uncertainty is developing another stable infrastructure for backup for the unstable one; one example is developing bypass a route for a trunk road and another example is to develop multiple hub ports in nearby regions. For example, after the Japan’s earthquake, inland roads were promptly recovered and used instead of the coastal roads seriously damaged by the tsunami. Similarly, the development of a multi-modal transportation network is another effective measure. For instance, when water transportation on the Mississippi River in the United States was hampered in early 2013 because of dry weather, railways were instead used to transport freight such as cereal and coal.

However, because developing reliable infrastructure is often costly, the government must both evaluate the associated costs and benefits as well as design planning institutions for financing and managing such infrastructure. Bearing these issues in mind, this study considers the issue of optimal reliability of transportation system focusing on the “cost recoverability”, or whether cost of optimal investment in transportation infrastructure is equal to or less than revenue of optimal congestion toll of it. Note that our cost-recoverability is used in broader meaning than the original cost recovery theorem stated by Mohring and Harwitz (1962) because our definition includes the case when congestion toll revenue exceeds the cost of investment.

¹For instance, the Joban railway line, which is an arterial railway connecting Tokyo and coastal area of Fukushima prefecture, has been destroyed in the East Japan Great earthquake in 2011 and is still unavailable in May of 2017.
We then call the original theorem of Mohring and Harwitz as "strict" cost recovery theorem for distinguishing our definition from the original one.

We consider a situation that the economy faces a risk to have some natural disaster which destroy part of infrastructure and then increase transportation cost. Because firms input transportation services as well as labor for their production, decrease in transportation facility decreases firms' productivity and then wages paid to labor. We also assume that households have risk-averse preference and hence dislike risk in income in the future; and then the government should consider to mitigate the risk when it develops a transportation system. We consider a government which constructs two congestible infrastructures; one is unstable infrastructure whose capacity decrease when a disaster occurs; and another is stable one whose capacity is constant regardless of the incidents. Stability of entire transportation system is a policy variable which is controlled by combination of these two infrastructures. We also assume flexible congestion tolls which differ by infrastructure and observed incidents; so government can change congestion toll observing disaster.

The theorem of "strict" cost recovery was originally proposed by Mohring and Harwitz (1962), who showed that the optimal investment in a congestible infrastructure is "just" equal to the optimal congestion toll revenue, when an infrastructure investment is assumed to have constant returns to scale (CRS). The CRS of an investment (i.e., only when the function of trip cost is homogeneous of degree zero in regards to traffic volume and infrastructure capacity) is necessary so that the theorem holds (Mohring and Harwitz 1962; Small 1999). Further, in the case of heterogeneous demand such as peak and off-peak passengers, the theorem does not hold with an inflexible congestion toll (Arnott and Kraus 1995; Bichsel 2001).

Some studies also consider uncertainty in transport demand and facility. Kraus (1982) investigated the effect of uncertain demand for planners who must choose the level of congestion toll as well as of investment only using uncertain information on passengers’ utility function. He found that demand uncertainty distorts the strict cost recovery theorem in expected value terms, suggesting that the optimal investment to maximize expected welfare exceeds the ex-

\footnote{However, Arnott et al. (1993) showed that the theorem holds for the bottleneck congestion model in which users choose a departure time even when flexible (i.e., time-dependent) peak-road pricing is unavailable.}
pected revenue of the congestion toll. On the contrary, Lindsey (2009) and Lindsey and De Palma (2014) showed that even when a planner faces uncertain shocks in demand and infrastructure facilities when designing infrastructure, strict cost recovery holds in expected value terms if the toll is fully flexible (i.e., allowing the first-best pricing for each shock) 3. Furthermore, Arnott and Kraus (1998) considered the accumulation of transportation capital for the long-run growth (i.e., volatile) in transportation demand. They conjectured that the strict cost recovery holds in present value terms if information on future demand is complete and the toll is flexible over time; this conjecture is proved by Lindsey and de Palma (2014).

On the contrary, this paper focuses on uncertainty in transportation facility combined with risk-averse preference of households toward the uncertainty in income. We show that the strict cost-recovery does not hold owing to the risk-averse preference even when government can flexibly change congestion toll observing the state. Our setting is justified by a different situation from Lindsey (2009); we consider a sufficiently long-lasting incidents such as destruction of infrastructures by a big natural disaster, and such risk is too significant for each household to hedge. Furthermore, considering combination of stable and unstable infrastructures for controlling the stability of the entire transportation system and investigating cost-recoverability of each of them is completely new. 4. This setting allows our model to investigate how the role of each infrastructure affects its cost recoverability. Such investigation provides us with implications for multi-modal planning and for how to transfer congestion toll revenues between stable and unstable infrastructures.

This study firstly examines cost recoverability in terms of expected revenue, called ex-ante cost recoverability, as well as the previous studies. We shows that completely unstable infrastructure is ex-ante cost-recoverable while the stable one is not. Investment in the back-up infrastructure not only increases expected product but also mitigates risk or volatility of households’ income. Although the latter benefit should be taken into account in the optimal investment although its value is not covered by congestion toll. However, not only the ex-ante viewpoint, we also also consider ex-post cost recoverability, or whether the revenue yielded in

3According to Lindsey (2009), a flexible toll cannot achieve cost recovery if planners receive only a probabilistic signal of demand shocks.
4Planners can develop completely reliable infrastructures by investing only in stable infrastructures.
each state can cover the cost. Our analysis shows that the same results also hold in case of ex-
post cost recoverability in the ordinal state in which no natural disaster appears. The results
of these two analysis suggest that the unreliable infrastructure needing a back-up facility is
profitable, while an infrastructure which is considered as a back-up facility needs subsidy.

The remainder of this paper is structured as follows. Section 2 describes the model. Section
3 examines the cost recoverability of each individual infrastructure under general settings. Sec-
tion 4 examines the model under given several specification of the model. Section 5 concludes.

2 The model

We suppose competitive firms inputting transportation service for production, and households
provide labor to firms force for receiving wage. The central government develops two trans-
portation infrastructures, and imposes a congestion toll on each of them.

While both infrastructure provide homogeneous transportation service, cost of the service
depends on infrastructure capacity and usage, namely the congestion level and the congestion
toll imposed by the government. The economy faces the risk of natural disasters by which part
of infrastructure is destroyed and unavailable. Hence, disasters influence firms’ production and
then wage income of households, but people dislike such uncertain income; and then government
care volatility of income as well as its expected value when developing infrastructure.

2.1 Transportation technology

The economy faces two future states $r = \{A, B\}$, where state A faces no big disaster while a
big disaster occurs in state B; hence we call state A and B as non-disaster state and disaster
state, respectively. The probability that states $A$ and $B$ appear are $\theta$ and $1 - \theta$, respectively.
There are two types of infrastructures, $i = \{S, U\}$: the facility of infrastructure $S$ is constant
under both states, while only part of full capacity of infrastructure $U$ is unavailable in state
$r = B$. Hence, we call infrastructures $S$ and $U$ stable and unstable, respectively.

We suppose that the government changes the congestion toll after observing the prevailing
state. Moreover, the two infrastructures are assumed to be congestible and separated. When
households consume transportation services by using each infrastructure, per trip cost increases
with total traffic, while it decreases in capacity.

Let us denote the traffic volume of infrastructure $i$ in state $r$ as $X_{ir} \geq 0$. Also, effective capacity and full capacity of infrastructure $i$ are denoted by $K_{ir}$ and $K_i$, respectively. Because we assume that full capacity is available when the infrastructure has no trouble, $K_{SA} = K_{SB} = K_S$ holds for infrastructure $S$, while $K_{UA} = K_U$ and $K_{UB} = \delta K_U$ holds for infrastructure $U$, where $\delta \in [0,1]$ describes fraction of unstable infrastructure available in the disaster state, hence $1 - \delta$ is called destruction rate.

We assume that function of trip cost is homogeneity of degree zero in regards to traffic volume and infrastructure capacity; and then describe it as $C_i(x)$, where $x_{ir} \equiv X_{ir}/K_{ir}$ denotes the load factor. We also assume that $C_i(x_{ir})$ is twice differentiable, $C'_i(x_{ir}) > 0$ holds, and infrastructure $i$ is unavailable if $K_{ir} = 0$. Because we assume that cost of the infrastructure per unit of capacity is constant and normalized to one, capacity is equivalent to amount of money spent for investment. This means that twice the monetary budget to an infrastructure would be able to accept twice the amount of transportation at the same cost. Such constant returns to scale in investment of transportation infrastructure is well known as a necessary condition for the strict cost-recovery theorem of Mohring and Harwitz (1962).

Finally, central government can levy toll $t_{ir}$ from each usage of the infrastructure whose value is dependent on the state.

### 2.2 Production and income of households

We assume that firms produce homogeneous output by inputting labor and transportation service. For neglecting economy of scale in each firm, we assume that technology of production is homogeneous in degree one in regards of labor and transportation service. Since the total labor supply in the economy is fixed to $L$ unit, we can simply describe value of total output as $F(X_r)$, where $X_r = X_{rL} + X_{rW}$ is total input of transportation service and composite good, respectively.

Given state $r$, each representative firm employing one unit of labor chooses $X_r$ so that maximize their profit $F(X_r) - p_r X_r - w_r$, where $w_r$ is wage, and $p_r = C_i(x_{ir}) + t_{ir}$ is full price of

---

5We assume price of output is exogenously given and fixed to one regardless of state.
transportation service which should be equalized between infrastructures available in each state in the equilibrium of mode choice because each infrastructure provides homogeneous service. Therefore, given the equilibrium level of \( p_r \), the representative firm maximizes \( F(X_r) - p_r X_r \); hence the first order condition for the profit maximization is described as

\[
F'(X_r) - p_r = 0. \tag{1}
\]

Finally, because of zero profit condition in the competitive market, equilibrium wage is determined as follows.

\[
w_r = F(X_r) - p_r X_r. \tag{2}
\]

### 2.3 Social welfare and risk-averse preference

The role of the central government is to decide on the degree of investment in each infrastructure, \( K_S \geq 0 \) and \( K_U \geq 0 \), as well as their congestion tolls in each state, \( t_{Sr} \) and \( t_{Ur} \). It is assumed that all the cost for constructing infrastructure is levied by lamp sum tax from the current household, and all the revenue of the toll is redistributed to households in each state.

The purpose of the government is to maximize the social welfare, or value of future income of the representative household measured by the present safe income. First, income in each state is equivalent to the consumable income of households, which is described as

\[
I_r = w_r + \sum_{i=(S,U)} X_{ir} t_{ir} = F(X_r) - \sum_{i=(S,U)} X_{ir} C(x_{ir}) \tag{3}
\]

However, the income differs by states and we assume that households dislike the uncertainty. Therefore, considering the risk-averse preference of households, social welfare or present value of the income as follows.

\[
V = E(I_r) - \alpha \text{Var}(I_r) - K_S - K_U \tag{4}
\]

where \( E(I_r) \) and \( \text{Var}(I_r) \) are expected value and variance of \( I_s \), and parameter \( \alpha \geq 0 \) represents degree of risk averse. This formula means that a combination of future uncertain
incomes \((I_A, I_B)\) is indifferent to \(E(I_r) - \alpha \text{Var}(I_r)\) unit of safe income received today, hence the risky income is discounted by its volatility and \(\alpha\) \(^6\). Although we employ such simplified form of risk-averse utility here, the robustness of the result under the utility function presented by von-Neuman and Morgenstern is examined in Appendix. Because construction cost of infrastructures must be paid in the present period, they are equivalent to the same amount of the safe income. Government maximizes the social welfare \(V\) by controlling \(t_{ir}\) and \(K_i\). \(^7\).

3 Optimal policy and cost recovery theorem

3.1 Optimal toll

We first consider the optimal usage of infrastructure. We can consider that the government can choose \(X_{ir} > 0\) instead of \(t_{ir}\) because the government can control \(X_{ir} > 0\) via \(t_{ir}\) for any \(i\) and \(r\). Since \(X_{ir}\) only influences \(I_r\) but does not \(I_r'\) in the different state, just considering maximization of \(I_r\) instead of \(I_r'\) is enough when we determine the optimal \(X_{ir}\). Because \(\text{Var}(I_r) = \theta(1 - \theta)(I_A - I_B)^2\) holds, the first order conditions are described as follows.

\[
\frac{\partial I_r}{\partial X_{ir}} = F'(X_r) - C_i(x_{ir}) - C'_i(x_{ir})x_{ir} = 0
\]  

(5)

From equations (1) and (5), it is obvious that the Pigouvian congestion toll \(t_{ir} = C'(x_{ir})x_{ir}\) is the socially optimal charge for usage of infrastructure because the conditions for private optimization and social optimization described by those two equations are equivalent when giving the congestion toll.

3.2 Optimal investment and ex-ante cost recoverability

In turn, we consider whether each infrastructure is cost recoverable in regards of the expected revenue; that is defined as \(E(R_{ir}) \geq K_i\). \(^8\)

From Mohring and Harwitz (1962), now remember that \(dI_r/dK_i\bigg|_{t_{ir} = c'(x_{ir})x_{ir}} = C'_i(x_{ir})x_{ir}X_{ir}/K_i = R_{ir}/K_i\) holds from the envelop theorem, where \(R_{ir} = t_{ir}X_{ir}\) is revenue of congestion toll from

\(^6\)Note that \(I_r\) is the present value which is already discounted by some interest rate, which is exogenous and independent of state.

\(^7\)one may consider that the government should deposit fund for compensating for the loss of income caused by disaster, providing complete risk-hedge for big natural disasters is actually impossible even for the central government.
infrastructure $i$ in state $s$. This means that contribution of investment on income is evaluated by the average revenue of congestion toll per investment under the assumption of constant return to scale in transportation investment; hence the strict cost recovery theorem of Mohring and Harwitz (1962) holds if there is no uncertainty. Therefore, assuming that the optima $K_i$ is strictly positive, the first order condition for investment in infrastructure $i$ given the optimal congestion toll is described as

$$
\frac{\partial V}{\partial K_i} \bigg|_{t_{ir}=C'(x_{ir})x_{ir}} = \frac{\partial E(I_r)}{\partial K_i} - \alpha \sum_{l \in \{A,B\}} \frac{\partial \text{Var}(I_r)}{\partial I_l} \frac{dI_l}{dK_i} - 1
$$

$$
= \frac{E(R_{ir})}{K_i} - \alpha \sum_{l \in \{A,B\}} \frac{\partial \text{Var}(I_r)}{\partial I_l} \frac{dI_l}{dK_i} - 1
$$

$$
= 0. \tag{6}
$$

From equation (6), we can easily confirm that the strict cost recovery theorem, $E(R_{ir}) = K_i$, holds without the second term. This is why the benefit of investment on expected income, described by $\frac{\partial E(I_r)}{\partial K_i}$, is completely covered by revenue of congestion fee, $\frac{E(R_{ir})}{K_i}$. However, in our model a benefit of investment described by the second term distorts the theorem.

The second term of (6) describes value of marginal volatility of income yielded by the investment and then called the risk premium of investment. Now we intuitively assume $I_A > I_B$, and then $\frac{\partial \text{Var}(I_r)}{\partial I_A} > 0 > \frac{\partial \text{Var}(I_r)}{\partial I_B}$ holds. Therefore, if contribution of the investment is larger on income of ordinary state than the state of disaster, whose risk premium is negative because it extends volatility of income.

Equation (6) shows that none of the risk premium is not covered by the revenue of congestion toll. If the second term is negative and the investment can mitigate the income risk, $E(R_{ir}) < K_i$ holds and then such infrastructure is not cost recoverable. Contrary, if the investment on an infrastructure extend the income risk, it is cost recoverable. These characteristics are confirmed in the following equation yielded from (6).

$$
K_i^* = E(R_{ir}) - 2\alpha \theta (1 - \theta)(I_A - I_B)(R_{iA} - R_{iB}), \tag{7}
$$

where note that $\frac{\partial \text{Var}(I_r)}{\partial I} = \theta (1 - \theta)(I_A - I_B)^2$. Further, by adding (7) for two infrastructures,
where \( R_r \equiv R_{Sr} + R_{Ur} \) and \( K^* \equiv K^*_S + K^*_U \) describes total revenue of congestion fee and cost of investment in entire transportation system. Therefore, the following proposition readily holds.

**Proposition 1.** Each individual transportation infrastructure is (not) cost recoverable if revenue of its congestion toll is larger (or smaller) in state B than in state A. This also holds for the entire transportation network. That is, i) \( K_i^* > (<) E(R_r) \) holds if \( R_{iA} < (>) R_{iB} \) holds, ii) \( K^* > (<) E(R_r) \) holds if \( R_{A} < (>) R_{B} \) holds.

From this proposition, contribution to the income risk, or the risk premium, is measured by comparing monetary revenue of the infrastructure in each state because of \( dI_r/dK_i|_{t_r = C_i(x_{ir})x_{ir}} = R_{ir}/K_i \) again. If \( R_{iA} < R_{iB} \) holds, such infrastructure largely contributing to the situation facing the disaster has positive risk-premium. This is also true for the entire transportation system, as well as each individual infrastructure.

Now it is intuitively obvious that \( R_{SA} < R_{SB} \) holds because \( X_{SA} < X_{SB} \) and \( C''_S(x_{Sr}) > 0^8 \). Further, \( R_{UA} > R_{UB} = 0 \) is also obvious in case of \( \delta = 0 \). Therefore, the following proposition holds;

**Proposition 2.** i) The stable infrastructure is not cost recoverable; ii) The completely unstable infrastructure with \( \delta = 0 \) is cost recoverable.

The important point from the above propositions is that cost-recoverability depends on stability of infrastructure. Since the contribution of the stable infrastructure mitigating the income risk cannot be covered by by its congestion toll revenue, and then subsidy from the

---

\( K^* \equiv K^*_S + K^*_U = E(R_r) - 2\alpha \theta(1 - \theta)|I_A - I_B|(R_A - R_B) \)

---

\(^8\text{The reason for} X_{SA} < X_{SB} \text{ is as follows. Now we temporally assume} X_{SA} > X_{SB}. \text{At this time} C_S(x_{SA}) > C_S(x_{SB}) \text{ and} t_{SA} > t_{SB} \text{ hold; hence user cost is lower in state B (i.e.,} p_A > p_B). \text{At this time, demand for infrastructure should satisfy} X_A = D(p_A) < D(p_B) = X_B \text{ and then} X_{SA} < X_A < X_B = X_{SB} \text{ should holds. However, this is contradiction to the temporally assumption temporally assume} x_{SA} > x_{SB}.\)
general account of the government is necessary to compensate the risk-premium. Inversely, despite the unstable infrastructure has negative risk-premium, that cost is not displayed in its revenue; and then the unstable infrastructure yields profit only by itself. These two results also suggest transfer of the profit from unstable infrastructure to the stable one for covering its financial deficit. However, it is unclear whether the transfer completely solves the problem or not since cost recoverability in entire transportation system is still unclear in our generalized setting.

3.3 Cost recoverability in regards of the ex-post revenue

Examining the cost recoverability in regards of the expected revenue gives us an important information in considering how to fund the cost of construction before the construction. However, the government actually receives some revenue from the toll only after some state is definitely given. The government must prepare for balancing its fiscal budget in each state, hence considering the ex-post cost recoverability is also important for policy making. From proposition 1, the cost recoverability in the ordinary state (i.e., $r = A$) is easily understood as follows;

Lemma 1. When infrastructure $i$ is (not) cost-recoverable in regards of expected revenue, such infrastructure is (not) cost recoverable in state $A$.

Proof: $K^*_i > E(R_{ir})$ is equivalent to $R_{iA} < R_{iB}$ holds. At this time, $R_{iA} < E(R_{ir}) < R_{iB}$ holds and then $K^*_i > E(R_{ir}) > R_{iA}$ holds. This proof can be analogously extended to the case when $K^*_i < E(R_{ir})$.

From Lemma 1 and Propositions 2, the following result is readily shown.

Proposition 3. When state $A$ realize, i) the stable infrastructure is not cost recoverable, and ii) the completely unstable infrastructure with $\delta = 0$ is cost recoverable.

Although these results tells us the ex-post cost recoverability only in the ordinary state,
that in the state of disaster (i.e., \( r = B \)) is still unclear. However, cost recoverability in the
ordinary state will be specifically important because such state is more likely to occur, and
because the balance sheet of the government is strictly checked comparing with the state of
disaster. Therefore, in the ordinary state the government must explain why the deficit from
the well designed stable infrastructure is inevitable; and then covering them by some subsidy
is reasonable At this time, the present paper will provide the backbone for the policy. On
the contrary, in the state of disaster, special account for reconstruction is settled since quick
recovering people’s lives and various social facilities from the disaster is the first priority rather
than budget balancing unlike the ordinary state.

4 Model specifications

This section offers some model specifications to support the results of the analysis using the
generalized model.

4.1 Linear transportation costs and the cost recoverability of the entire
transportation network

We suppose that the transportation costs of both infrastructures are given as the following
linear function of traffic volume:

\[ c_S(x) = cx, \quad c_U(x) = \beta x, \quad c > 0, \quad \beta \in (0, 1) \]  

(8)

Because such linear transportation costs are well known to be derived from the simple
bottleneck congestion model (c.f. Arnott et al. 1993), the two infrastructures are interpreted
as two separate highways. If Pigouvian congestion fee, \( t_{Sr} = cx_{Sr} \) and \( t_{Ur} = \beta x_{Ur} \), is charged
to each infrastructure, their full price is described as \( p_{Sr} = 2cx_{Sr} \) and \( p_{Ur} = 2c\beta x_{Ur} \). Because
the full price is equalized between two infrastructures in equilibrium, load factor must hold the
following:

\[ x_{Sr} = \beta x_{Ur} = x_r \equiv \frac{X_{Sr} + X_{Ur}}{K_{Sr} + K_{Ur}/\beta} \]

Nextly, we describe demand function for transportation as \( X_r = D(p_r) \), where \( p_r = 2x_r \)
and \( X_r = X_{Sr} + X_{Ur} \). We assume \( D'(p_r) < 0 \). At this time, because of \( t_r = p_r/2 \), total revenue
of the entire transportation system is described as \( R_r = p_r D(p_r)/2 \); hence the following holds.

\[
\frac{\partial R_r}{\partial K_{Ur}} = \frac{dp_r}{dK_{Ur}} \left( p_r D'(p_r) + D(p_r) \right) 2
= \frac{dp_r}{dK_{Ur}} X_r(1 - \epsilon)
\]

Where \( \epsilon \equiv -p_r D'(p_r)/D(p_r) \) is price elasticity of transportation demand. From \( D'(p_r) < 0 \) and \( c'(x) > 0 \) it is readily seen \( \frac{dp_r}{dK_{Ur}} < 0 \). Therefore, if price elasticity \( \epsilon \) is larger (or smaller) than one in any \( p_r \), \( \frac{\partial R_r}{\partial K_{Ur}} > 0 \) holds and then \( R_A \) is larger (or smaller) than \( R_B \); hence the following holds from proposition 1.

**Proposition 4.** When supposing linear transportation costs, as described in equation (8), entire transportation is (not) cost recoverable when \( \epsilon \geq 1 \) (or \( \epsilon \leq 1 \)) holds for any \( p_r \).

The result for the cost recovery of the entire transportation system depends on the price elasticity of transportation demand, in line with the numerous empirical studies of this issue. The survey by Oum et al. (1992), for example, showed that most studies of transportation in the United States have reported that elasticity of demand is far below one, particularly for intra-city and road transportation (see also Goodwin (1992), which drew a similar conclusion).

In such a relatively inelastic demand, entire transportation system is not cost recoverable and then subsidy is required to construct the optimal infrastructure. When transportation is vital input for the production process, it is difficult to save inputting it even facing a rise in transportation cost owing to disaster. At this time, firms and then households suffer from a significant losses caused by the disaster. Therefore, for mitigating such big risk we need sufficiently large investment in the stable infrastructure which is not cost-recoverable. Therefore, share of investment in the stable infrastructure of the entire investment becomes high, and then entire transportation system also becomes not cost-recoverable.

### 4.2 Numerical analysis

We examine a simple numerical analysis for answering several questions which are still remained, such as cost-recoverability of unstable infrastructure in case of in the case of positive \( \delta > 0 \).
Firstly, we suppose a production as the following CES function:

\[ F(X_r, L) = -(X_r^{\rho} + L^{\rho})^{-1/\rho} \]

We know that price elasticity of transportation demand is higher (or lower) than one in case of \( \rho > 0 (\rho < 0) \). Also, we give transportation cost by (8).

Figure 1 shows optimal investment and revenues from each infrastructure which is yielded from the results of numerical simulation of the model with the above function. In this simulation, we give \( \rho = -4 \) to focus on the case that transportation demand is relatively inelastic as seen in the most empirical studies \(^9\).

Firstly, when \( 1 - \delta \), degree of destruction rate of infrastructure \( U \), is small, investment is concentrated in the unstable infrastructure because it is relatively efficient in regards of expected facility. As the destruction rate increases and risk of disaster rises, government firstly mitigate the risk by increasing capacity of the unstable infrastructure when the degree of risk, \( 1 - \delta \), is relatively small. However, when the degree of risk increases, government reduces \( K_U \) while increases \( K_S \) because increasing unstable infrastructure is insufficient for hedging the big risk.

Although the unstable infrastructure is cost-recoverable when the destruction rate is sufficiently large as mentioned in proposition 2 mentioned, it is not cost recoverable when the degree of the risk is relatively small. This is because entire transportation system owes to the unstable infrastructure when the risk is small, and it is not cost-recoverable because we suppose low elasticity of transportation demand by assuming negative \( \rho \). As mentioned above, investment \( K_U \) to mitigate the small risk when \( 1 - \delta \) is small; and hence that investment plays a role of mitigating the risk premium and then part of that benefit is not covered by the revenue of congestion toll.

On the contrary, the stable infrastructure is not cost recoverable as proposition 2 stated. However, its financial deficit peaks out in the intermediate \( 1 - \delta \) although investment in \( K_S \) monotonically rises with the destruction rate. This is because the volatility of transportation demand is lower than 1 if and only if \( \rho \) is negative.

---

\(^9\) If \( X_r \) is negligibly small comparing with \( L \), price elasticity of transportation demand is lower than 1 if and only if \( \rho \) is negative.
cost is rather small in sufficiently large destruction rate because almost all transportation investment concentrates in the completely stable infrastructure, $S$. Therefore, value of risk premium decreases and then almost all cost is covered by the expected revenue in such high $1 - \delta$. The same property is also observed as for $E(R) - K_S - K_U$, or the total financial deficit of the entire transportation system.

[Please insert Figure 1 around here]

Figure 1. Optimal investment and revenue of congestion toll

$\rho = -4, \alpha = 5, \theta = 0.95, \beta = 0.6, L = 20$

5 Conclusion

This study considers the issue of optimal reliability of transportation system focusing on the “cost recoverability”, when households dislike risks caused by uncertain transportation condition. Because risk-premium of transportation investment is not covered congestion toll revenue, stable infrastructure is not cost recoverable while the unstable one is cost recoverable, in regards of ex-ante or expected congestion toll revenue. Further, the same results hold in regards of the ex-post congestion toll revenue in the non-disaster state. These results suggest a subsidy or transfer for developing the stable infrastructure which enhances reliability of the entire transportation system.

Finally, there are so many remained issues on development and management of transportation infrastructure facing disaster and risk-averse preference of households, and we mention only several of them. First, we should consider how the market is distorted if each infrastructure is developed and managed by risk-averse private companies. Specifically, if stable infrastructure is privatized, how to reward its back-up facility is an issue. Second, uncertainty in transportation cost may influence location of households and firms despite we neglect the problem. Third,
Acknowledgements

The author deeply thanks for Oliver Feng Yeu SHYR, Se-il Mun, Masamitsu Onishi and participants of Urban Economic Workshop in Kyoto University, 4th Asian seminar in Regional Science, 61st North American Regional Science council for their helpful comments. The author also acknowledges the financial support from Grant-in-Aid for JSPS Fellows No. 24730216.

Appendix. Robustness under the risk aversion of von-Neumann and Morgenstern

Although one may consider that assuming welfare as equation (4) is over specified, it is still possible to derive the same result from more generalized form of risk-aversion. Following the expected utility function of Von-Neuman and Morgenstern, we assume the welfare function as follows.

\[ V = E[v(I_r - K_S - K_U)] \] (9)

where \( v'(\cdot) > 0, v''(\cdot) < 0 \)

where \( v \) describes long-run indirect utility when \( s \) appears given a total present value income by \( I_r - K_S - K_U \). Because \( v \) is concave function, expected value of it represents risk averse preference of households.

First, it is obvious that the Pigouvian congestion toll that maximizes \( I_s \) for given investment is the optimal \( t_{ir} \), as in the original setting. Second, from the first order condition for \( K_i \), following equation is derived.

\[ K_i^* = E(R_{ir}) - \theta \frac{E(v'_A) - v'_A (R_{iA} - R_{iB})}{E(v'_r)} \] (10)

where \( v'_r \equiv \frac{\partial v(I_r - K_S - K_U)}{\partial I_r} \)

This equation is equivalent to equation (7) when replacing \( \frac{\theta [E(v'_A) - v'_A]}{E(v'_r)} \) with \( 2 \alpha \theta (1 - \theta) |I_A - I_B| \); both of them represent positive sign since \( I_A > I_B \) and then \( v'_A < v'_B \) holds because
of concavity of \( v \). Therefore, from the equation we can easily derive propositions 1, and hence all the preceding propositions.

References


Figure 1. Optimal investment and revenue of congestion toll