Pigouvian taxation under uncertain demand: Are demand-responsive congestion pricing worth it?

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Short abstract
Pigouvian taxes, such as congestion charges, often need to be determined well in advance, before the exact demand is known. Moreover, demand often varies across seasons, days and times of the day, while the tax usually needs to be fixed in most of these dimensions. This paper derives the optimal pigouvian tax in such situations, and the welfare that is lost by having a fixed tax compared to a tax responding perfectly to varying demand. The magnitude of these effects is illustrated with a numerical analysis of a congestion charge. They turn out to be relatively small for typical variations in demand, which means that the welfare gain from having perfectly demand-responsive congestion charges, rather than the same charge every weekday, is likely to be small.

Introduction
An optimal pigouvian tax depends on the demand: all else equal, higher demand implies a higher tax. However, demand often varies across days and seasons, while the tax usually needs to be fixed across most of these dimensions. The purpose of this paper is to derive the optimal pigouvian tax in such situations, and the welfare loss resulting from having to set a fix pigouvian tax rather than one which varies dynamically with demand.

Consider for example congestion charges. Road traffic varies systematically according to season, time of day and day of the week. It is in principle possible to let the congestion charges vary across of all these dimensions, but there is a limit to how much they can vary before they become incomprehensible to the driver. On top of systematic variations in demand, there is random demand variation across days and times of the day. In practice, it is all but infeasible to vary the charges according to stochastic variation, since this would make them unpredictable to drivers. Finally, congestion charges usually need to be fixed well in advance, usually months or even years, for technical, legal and political reasons.

Hence, there are many sources of demand variation. Some of them are random, while some are in principle predictable, but from the point of view of setting a pigouvian tax they can be treated as “random variations”, since there may be cognitive or legal constraints on how much the pigouvian tax can respond to these demand variations.

The design of an optimal pigouvian tax in such a situation needs to take this variation in demand into account. As will be shown, the optimal fixed tax under varying demand is in general not equal to the optimal tax at average demand. The only case when they coincide is when the externality is proportional to demand with a constant proportionality factor. However, this is often not the case. For example, in the case of road congestion, the externality (i.e. the delay) will increase faster than linearly with demand.

The second question of the paper is how much benefits are lost when the pigouvian tax needs to be fixed, rather than respond dynamically to varying demand. Returning to the example of congestion charges, the idea of perfectly demand-responsive, i.e. dynamically determined, congestion charges has attracted quite a lot of interest and development efforts in the transport planning sector, from urban planners, economists and engineers alike. Moreover, operational congestion charges usually do not vary a lot. For example, the Stockholm congestion charges are equal for all charging stations, equal in both directions, equal in the morning and afternoon rush hours, and equal the entire year except that there is no charge in July. The question is how much additional benefits it would bring to allow charges to vary along more dimensions, or in the more extreme case, respond dynamically to random demand variations across days or even times of day. This paper provides a general formula for this, and illustrates the magnitudes with a numerical example of a congestion charge.
Main results

Assume that the travel time \( t \) depends on the road traffic volume \( D \), so we have \( t = t(D) \). A road congestion function is characterized by \( t'(D) > 0 \) and \( t''(D) > 0 \). In a general setting, this is the average private cost function. For road traffic, \( t(D) \) is usually called a volume-delay function (VDF). The generalized travel cost \( c \) is the sum of the travel time and a toll \( \tau \), so we have \( c = t(D) + \tau \). Let \( p(D) \) be the inverse demand function, i.e. \( p(D) = c \). Now, assume that demand is stochastic: it depends on a stochastic term \( \varepsilon \) such that for a given \( \varepsilon \) we have the inverse demand function \( p(D + \varepsilon) = c \). \( \varepsilon \) has a density function \( f(\varepsilon) \), and we assume that \( E(\varepsilon) = 0 \) (if not stated otherwise) and that \( \text{var}(\varepsilon) \) exists. The problem is now to find the fixed toll that maximizes expected social benefits, \( E(B(\tau)) \), i.e. the change in consumer surplus plus toll revenues integrated over \( \varepsilon \).

The two most important questions are how the tax should be set when demand varies randomly, and how much welfare is lost when the tax cannot vary perfectly with demand but needs to be constant. The first question is answered by Theorem 1, and the second by Theorem 2.

**Theorem 1.** The optimal fixed pigouvian tax (e.g. congestion charge) under demand variation is

\[
\tau^* = \int t'(D + \varepsilon)(D + \varepsilon)f(\varepsilon)d\varepsilon
\]

If the average cost function \( t(D) \) (the volume-delay function) can be approximated by a second-degree polynomial over the relevant range of \( \varepsilon \), this expression becomes

\[
\tau^* = \hat{\tau}^* + t''(D) \cdot \text{var}(\varepsilon)
\]

where \( \hat{\tau}^* \) is the optimal pigouvian tax under deterministic demand (\( \varepsilon = 0 \)).

**Theorem 2.** Compared to an optimal demand-responsive pigouvian tax, the fixed tax \( \tau^* \) yields less benefits. If the marginal social cost (MSC) and the inverse demand function can be approximated by linear functions in the relevant interval, the loss of benefits is approximately

\[
\Delta B = E(\Delta B(\varepsilon)) = \frac{1}{2} \left( \frac{\partial \tau^*}{\partial D} \right)^2 \frac{1}{-p'(D^*) + \text{MSC}' - p'(D^*) + t'(D^*)} \cdot \text{var}(\varepsilon)
\]

where \( D^* \) is the demand corresponding to the tax \( \tau^* \), \( \text{MSC}' = 2t'(D^*) + t''(D^*) \) is the derivative of the marginal social cost and \( \frac{\partial \tau^*}{\partial D} = t'(D^*) + t''(D^*)D^* \) is the derivative of the optimal tax, all evaluated at \( D^* \).

Theorem 3 says that under varying demand, the standard pigouvian tax will generate more benefits the larger the variation is, as long as the marginal externality is increasing (upwards-bending volume-delay function). Intuitively, this is because the added benefits of the toll when demand is higher than average are larger than the lost benefits when the demand is lower than average.

**Theorem 3.** Compared to a situation with no variation in demand, a pigouvian tax will yield more benefits if \( t''(D) > 0 \). The added benefits are proportional to \( \text{var}(\varepsilon) \).

Finally, taking demand variation into account when setting the toll adds benefits which are proportional to the variance in demand.

**Theorem 4.** The added benefits of taking demand variation into account when setting the pigouvian tax, i.e. the difference in benefits between \( \tau \) and \( \hat{\tau}^* \), is proportional to \( \text{var}(\varepsilon) \).

**Numerical simulation**
To illustrate the numerical magnitudes of the effects of demand uncertainty on the pigouvian tax and its benefits, we present a stylized but realistic example of a congestion charge. Consider a road where the travel time $t$ depends on the traffic volume $D$ according to a standard BPR function, and where traffic demand during a given hour varies randomly between days. Standard deviations are typically in the order of 5%. The seasonal variation is slightly larger, and comes on top of the random variation between days. The difference between the weeks with highest demand and the weeks with lowest demand (disregarding vacation periods) are typically in the order of 15%. In the simulations, we use two uniform distributions: $\pm 10\%$ of demand and $\pm 25\%$ of demand.

The numerical simulations show two main conclusions:

1) The addition to the optimal toll by taking demand variation into account is rather small, but the addition to total expected welfare is non-negligible. Hence, it is worth setting the toll optimally (i.e. taking demand variation into account).

2) The added benefits by having a completely demand-responsive toll (that adjusts perfectly to demand variations) are small. Hence, it does not appear worthwhile to implement perfectly demand-responsive tolls, considering the technical and cognitive complexity.