Modelling long distance travel when intermodal schedule delay matters.

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Abstract
This paper focuses on the differences between a discrete choice approach and an assignment approach to mode choice for long distance travel. If travellers compare schedule delay across both lines and modes when they chose an itinerary for a trip the will be a self-selection mechanism at work. This can lead to severe biases in a traditional discrete choice model for mode choice both with respect to demand predictions and evaluation of changes. An alternative is to let an assignment model also handle mode choice by including all public transport modes in the same assignment. The assignment model should be based on the assumption that travellers use timetable information and face a "reliable" system. While this handles the self-selection aspect and also gives a proper treatment of multimodal trips, it does away with the major advantage of discrete choice modelling: The ability to deal with unobserved aspects that affects the choices. Depending on the importance of unobserved aspects, a pure assignment approach will also introduce biases. Simple examples are used to demonstrate the issues involved and we also show that the discrete choice approach and the assignment approach in principle can be combined.

Introduction
In travel demand models for short distance (daily) trips, public transport is usually treated as one mode even though the system may consist of a mixture of bus-, tram-, rail- and subway lines. A public transport assignment model handles the distribution between these modes and transfers between them. A common assumption in these assignment models is that frequencies are so high that travel losses don't use timetable information. Without use of timetable information, door-to-door travel time is perceived as stochastic, but the variance is too small to warrant the effort of timetable use. In this situation it is reasonable to assume that they will tend to base the choice of boarding points, lines and transfer points on a strategy that minimizes expected (generalized) travel time (Spiess and Florian, 1989).

For long distance travel, the most common approach is to treat public transport modes like bus, train, air and boat separately and model the choice of mode with discrete choice models. The basic assumption of a discrete choice model is that travellers have mode specific and unobservable preferences and that also unobservable attributes of different modes may affect the choices. The discrete choice approach is - for example - used both in Sweden (Sampers) and in Norway (NTM). Assignment is then made separately for each mode. A consequence is that one main mode must be defined for all trips between each origin destination pair. This main mode is typically defined as the

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mode used the longest distance. Multimodal trips will only occur in the models if an access/egress model for the main modes handles this aspect adequately, something that can be a tricky task.

A characteristic of public transport services for long distance travel is that frequencies are much lower than in urban transit (often an hour or more between departures of a line) and travellers use timetable information when they plan their trips. They will tend to choose an itinerary before the trip starts and may also book in advance. Waiting time at stops consequently becomes relatively less important, with the possible exception of necessary waiting time at transfer points, while the deviance between preferred departure and/or arrival times and the times offered by different services ("schedule delay") can be substantial.

The topic of this paper is some consequences for the modelling of travel demand and for the evaluation of changes in consumers' surplus that arise when travellers compare schedule delay across modes and this also affects mode choice. To illustrate the issues involved we use some simple examples and compare the extreme cases of "pure" assignment and discrete choice and also a model that is a mixture of the extreme cases.

**Assignment vs. discrete choice**

A set of timetables for the lines in a model that maintains the fixed headway of each line might be termed a timetable instance. A set of published timetables is an example of a timetable instance. The long distance trips we observe, whether in travel surveys or traffic statistics are conditional on the actual timetable instance that the travellers have experienced. An assignment model, however, purports to give results that can be interpreted as the mean values of travel time, waiting time etc. over all feasible timetable instances that maintain the fixed headway of each line, and will thus rarely be able to exactly predict the observed travel times and passenger volumes.

There are also different assignment principles or algorithms for public transport that make different assumptions, in particular regarding the information possessed and used by travellers and the distribution of interarrival times for the lines in the model. For long distance travel the most appropriate assumptions are that travellers have full information about departure time, travel time and travel cost for each line and that the system is considered as reliable in the sense that the headway of each line is taken as fixed. An assignment model based on these assumptions, plus the assumption that travellers' preferred departure and/or arrival times are uniformly distributed on the time line (Hasselström 1981), we may term FIRSy (Full Information and Reliable System).

If all that matters for the travellers’ choices of mode and lines are the times and costs that can be included in an assignment model, mode choice will in principle be part of an assignment problem that can be handled by an appropriate assignment model (i.e., FIRSy). The model will simultaneously assign travellers to modes and lines and multimodal trips will automatically be included. A probability distribution over modes and lines for a given OD-pair is primarily generated because travellers have different preferences with respect to departure and/or arrival time. The assignment will also give us mean values for travel time, cost and schedule delay that depend on the total supply of services. A noteworthy consequence of the "pure" assignment model used simultaneously for all modes, is that the mean value of travel times and schedule delay for the users of one mode in general will depend on the level of service offered by other modes. Or to put it differently: The mode and line shares will not have the independence of irrelevant alternatives (IIA) property. The reason is that there will be a self-selection process at work. In the FIRSy-model one can specify ride time, fare and operator for each line of each mode, which can be many. This means for example that competition between operators due to varying fares, frequencies and ride times can easily be analysed.
On the other hand, a discrete choice model for mode choice will work in 2 steps. First an assignment (skim) is made for each mode to get the level of service for this mode. Level of service data (LoS, i.e., travel times and costs) are then used as inputs to the discrete choice model that gives the modal split. The level of service data assigned to the users of one mode is in this case independent of the level of service for other modes. One advantage of the discrete choice approach is that it explicitly takes into consideration that choices made by individual travellers may not be fully explained by the variables included in a model. There is also an extensive theory for estimation of discrete choice models and for extension of discrete choice to more than one dimension, for example destination choice. However, if schedule delay is of importance in the choice of mode, the discrete choice models have a problem that has yet to solved, while a pure assignment approach has a problem when there are (random) mode specific preferences.

In a sense we can say that a pure assignment model and a discrete choice model represent two extreme cases. If travellers consider intermodal schedule delay, but we still have heterogeneous preferences and unobserved attributes of different modes, the “true” model may be somewhere in between. For the modelling of long distance travel this raises two questions:

1) To what extent do travellers compare itineraries across modes when they plan for long distance trips? They will obviously do so for trips that necessarily will involve transfer between modes and long headways are involved, but this only affects a certain share of long distance trips.
2) To what extent will unobserved trip and mode specific attributes and preferences affect mode choice?

By unobserved mode and trip specific attributes we mean that:
- Travellers may have different preferences with respect to travel modes.
- There may be trip-specific circumstances that favour a particular mode for a specific trip.
- Travellers may have different perceptions of the reliability of different modes, i.e., the frequency and magnitude of delays as compared to published timetables.

To this we may also add the measurement errors that will be involved for individual trips when we use zonal-based models. The measures used for access to and egress from a public transport system will for each pair of zones - in this case - at best - be a reasonably accurate estimate of the mean value for the trips between the two zones. The problem related to this access//egress measurement error is, however, the same for both kinds of models.

A simple formal formulation of the "extreme" models and a mixed model
To avoid the complications posed by a complex network and multiple transfers we shall in this section consider one OD-pair served only by one or more direct lines of different modes. One of the modes may also be car, which will have zero headway.

Notation:

\[ M = \text{the set of available modes} \]
\[ l_{m,i} = \text{line } "i" \text{ of mode } M \]
\[ l_{m,i} \in L_m = \text{the set of lines available for mode } M \]
\[ Pr(l_{m,i} | m) = \text{the probability that line } i \text{ is chosen conditional on mode } m \text{ being chosen} \]
\[ c_{m,i} = \text{generalized travel cost for line } i \text{ of } m, \text{exclusive of the cost of schedule delay} \]
\[ c_{m,i}^{\text{cond}} = \sum_{l_{m,i} \in L_m} Pr(l_{m,i} | m) \cdot c_{m,i} = \text{mean value for mode } m \text{ conditional on assignment } L_m \]
\[ h_{m,i} = \text{headway of line } i \text{ of mode } m \]
\( h^{\text{Cond}}_m = \text{mean schedule delay for mode } m \text{ conditional on assignment only on } L_m \)
\( v_{m,i} = \text{a random variable for line } i \text{ of mode } m \text{ uniformly distributed on the unit interval} \)
\( e_m = \text{a random variable on the interval } < -\infty, +\infty > \text{ for mode } m \)
\( DC = \text{superscript for discrete choice model} \)
\( AS = \text{superscript for assignment model} \)

We assume that the all other variables are scaled to the same measurement unit as the headways.

**Discrete choice.**

The DC model consists of three steps.

**Step 1:** Solve the assignment problem for mode “m” \((m=1,\ldots,M)\)

If we use the FIRSy assignment principle, the probability distribution over lines is defined by

\[
Pr(l_{m,i}|m) = Pr(c_{m,i} + v_{m,i}h_{m,i} \leq \min_{k\neq i} \left[c_{m,k} + v_{m,k}h_{m,k}\right])
\]  

(1)

To calculate the probabilities we must integrate over a series of polynomials. The underlying assumption is that the travellers preferred departure and/or arrival times are uniformly distributed on the time line and the probabilities must be interpreted as mean the values over the set of all feasible timetables that maintain the fixed headway of each line.

**Step 2:** Calculate \(\bar{c}^{\text{Cond}}_m\) and \(\bar{h}^{\text{Cond}}_m\) where the "Cond" superscripts indicate that the values are based on a conditional assignment that only includes the lines of mode \(m\).

**Step 3:** Calculate the mode choice probabilities by solving:

\[
Pr^{DC}(m) = Pr\left(-\left(\bar{c}^{\text{Cond}}_m + \bar{h}^{\text{Cond}}_m\right) + e_m > \max_{n\neq m} \left[-\left(\bar{c}^{\text{Cond}}_n + \bar{h}^{\text{Cond}}_n\right) + e_n\right]\right)
\]  

(2)

If \(m\) \((m=1,\ldots,M)\) is distributed IID-Gumbel, (2) give us a very simple version of the familiar multinomial logit model with the scale parameter \((\mu)\) determined by the variance of the random terms. Thus (2) can on this assumption also be written as:

\[
Pr^{DC}(m) = \frac{e^{-\mu\left(\bar{c}^{\text{Cond}}_m + \bar{h}^{\text{Cond}}_m\right)}}{\sum_{j=1}^{M} e^{-\mu\left(\bar{c}^{\text{Cond}}_j + \bar{h}^{\text{Cond}}_j\right)}}
\]  

(3)

Any logit model estimated on observed trips will be more complicated than (3) and will have separate parameters for generalized cost and schedule delay (if not constrained to be the same) and also alternative specific constants and different variables that allows for segmentation. The cost and headway variables may also be subject to transformations. (3) is therefore an extremely "naïve" version compared to an estimated model for mode choice.

By assumption the assignment step and the mode choice step are independent and the unconditional probability \(Pr( l_{m,i} )\) is given by:

\[
Pr(l_{m,i}) = Pr(l_{m,i}|m) \cdot Pr^{DC}(m)
\]  

(4)

Note that it is \(c_m\) and \(h_m\) \((m=1,\ldots,M)\) that enter the "(dis)utility-functions". These variables are not the variables that the individual travellers base their choice of mode on. From the point of view of
estimation of a logit model based on observations of individual trips, this means that the deviations from the mean values for individual trips will be included in the random terms and may sometimes destroy the initial IID assumption for the actual random terms and in particular the implied assumption of constant variance of random terms across modes.

**Assignment for modes and lines simultaneously.**

Assignment for modes and lines simultaneously simply means that we make an unconditional assignment by the same principle as (1), i.e., by use of the FIRSy algorithm, and include all combinations of stops, lines and modes (hyper path) in the same assignment, i.e.,

\[
Pr(l_{m,i}) = Pr(c_{m,i} + v_{m,i} + h_{m,i} \leq \min_{n \in M, j \in L_n} [c_{n,j} + v_{n,j} + h_{n,j}])
\]  

(5)

From the assignment results we can subsequently calculate the mode probabilities as:

\[
Pr^{AS}(m) = \sum_i Pr(l_{m,i}) \quad \text{for} \quad m = 1, ..., M
\]  

(6)

The assignment results also allow us to calculate mean cost over all lines and modes, \(\bar{c}\), and mean schedule delay, \(\bar{h}\), but also the means for each mode separately, i.e., \(\bar{c}_m^{UnC}\) and \(\bar{h}_m^{UnC}\) for \(m = 1, ..., M\) (where the superscript UnC indicate that the values are from an unconditional assignment). In this case \(\bar{c}_m^{Cond}\) and \(\bar{h}_m^{UnC}\) will depend on the characteristics of the lines belonging to the other modes.

Also: While the ratio \(Pr^{DC}(m)/Pr^{DC}(n)\) are independent of the characteristics of other modes than \(m\) and \(n\), this will not be the case for the ratio \(Pr^{AS}(m)/Pr^{AS}(n)\).

**The mixed model**

The basics of a mixed model only involve a small extension of (5). In (7) the mode-specific random terms are included. For all lines of a mode the same random terms are deducted. With a symmetric distribution it does not matter whether we add or deduct the mode-specific random terms, but with a skew distribution like – for instance – the Gumbel, we should deduct the random terms in the case of a minimization problem.

\[
Pr^{Mix}(l_{m,i}) = Pr(c_{m,i} + v_{m,i} + h_{m,i} - \varepsilon_m \leq \min_{n \in M, j \in L_n} [c_{n,j} + v_{n,j} + h_{n,j} - \varepsilon_n])
\]  

(7)

Unfortunately there is at present no assignment algorithm that solves this problem, but for small and simple problems it can be solved by simulation. Essentially this means that we have to make a sufficient number of draws of the random terms \(\varepsilon_i\) \((i = 1, ..., M)\), make an assignment for each draw and take the average over all assignments. In the example below we will – instead of using thousands random draws – utilize the fact that the Gumbel distribution can be approximated very accurately by 5 data points with appropriate weights. This is shown by Larsen et al. (2010) and is exemplified with the numerical example used below.

On the other hand, if \(\varepsilon_i\) is from a Gumbel-distribution, the mode choice probabilities generated for this simple case, will also be from a special type of mixed logit where the "mixing" is generated by the random variables on the unit interval, \(v_{m,i}\). As opposed to a "standard" mixed-logit where we need to make assumptions regarding the distributions of one or a few random variables, we will in this case have a number of random variables equal to the total number of lines, but the distribution is known and is the same for all random variables.
Consumer surplus when headways are changed

When passengers employ time schedules in order to choose modes and lines, only the FIRSy-model can calculate consumer surplus correctly, neither “Rule of a Half” nor the logit model by use of the logsum. The reason is that only the FIRSy-model takes into account the self-selection mechanism that is involved when travellers compare itineraries across lines and modes. This section highlights the importance of this.

To fix the ideas, assume that we have two lines – A and B – belonging to different modes and connecting the same origin and destination, each with the same ride time, but with differing headways, \( h_A \) and \( h_B \) respectively, where \( h_A \leq h_B \).

We first describe the estimation by use of the FIRSy-model. The time spells between departures are \((T_A, T_A+h_A)\) and \((TB, T_B+h_B)\) where \(T_A\) and \(T_B\) are random and independent. A traveller chooses the line whose departure time is nearest earlier the “ideal” departure time (we can think of a situation where the traveller needs to reach the destination at latest at a certain time, but does not want to arrive unnecessarily early.) Ideal departure times are assumed being uniformly distributed among travellers.

The probability that mode B will be chosen conditional that the delay time (time between actual and ideal departure, denoted \( x \)) for mode A is

\[
Pr(B | x) = \frac{x}{h_A h_B} \tag{8}
\]

Hence, the unconditional probability that mode B be chosen is

\[
Pr(B) = \int_0^{h_A} \frac{x}{h_A h_B} dx = \frac{h_A}{2h_B} \tag{9}
\]

Consequently the probability that line A is chosen is:

\[
Pr(A) = 1 - Pr(B) = 1 - \frac{h_A}{2h_B} \tag{10}
\]

The expected delay times (time between actual and ideal departure) for the respective lines are

\[
E(t_A) = \frac{1}{2} h_A \frac{1-(2h_A/3h_B)}{1-(h_A/2h_B)}, \quad E(t_B) = \frac{1}{3} h_A \tag{11}
\]

(Note that the expected delay time for mode B does not depend on that mode’s headway!) Combining these results, it follows that the expected delay time for the journey is

\[
E(t) = \frac{1}{2} h_A - \frac{1}{6} \frac{(h_A)^2}{h_B} \tag{12}
\]

Assume now that a researcher ex post tries to calculate the mean generalised cost (time delay), i.e., he observes the demands \( Pr(A)\) and \( Pr(B)\), but ignores the fact that travellers employ time schedules. He then calculates the expected delay times to be half the headway. He will then estimate the expected delay time, denoted \( E(t^*) \), by:

\[
E(t^*) = Pr(A) \frac{h_A}{2} + Pr(B) \frac{h_B}{2} = \frac{3}{4} h_A - \frac{1}{4} \frac{(h_A)^2}{h_B} \tag{13}
\]
This is a 50 per cent over-estimate of the time delay. The change in time delay due to any change in headways will thus also be over-estimated by 50 per cent.

Assume now instead that the researcher observes the modes A and B in isolation, (non-dependent) and tries to assess the change in expected time delay employing “Rule of a Half.” To fix the ideas, assume that first \( h_A = h_B = h \). He will then notice that the demand is

\[
\Pr(B) = \frac{1}{2} \text{ and believes that } E(t_B) = 0.5h
\]

(14)

Next, assume that \( h_B \) is increased by 50 per cent: \( h_B = 1.5h \). He will notice (ex post) that the demand now is

\[
\Pr(B) = \frac{1}{3} \text{ and believes that } E(t_B) = 0.75h
\]

(15)

“Rule of a Half” will now calculate the increase in total delay time to \((5h/48)\). But according to (12) the increase is only \( h/18 \), so Rule of a Half over-estimates the time loss with 87.5 per cent.

Finally, assume that the researcher employs the logsum from a logit model to calculate time delays in the previous example. He assumes that the deterministic part of individual’s utilities in terms of time loss are measured as

\[
u = a - 0.5h
\]

(16)

where \( a \) is a constant and \( h \) is headway. When \( h_A = h_B = h \) he observes the demands

\[
\Pr(A) = \Pr(B) = \frac{1}{2}
\]

(17)

Which is in accordance with the logit probabilities. Next, when \( h_B = 1.5h \) he observes the probabilities

\[
\Pr(A) = \frac{2}{3} \text{ and } \Pr(B) = \frac{1}{3}
\]

so he calibrates his model accordingly:

\[
\frac{e^{a-0.5h}}{e^{a-0.5h} + e^{a-0.75h}} = \frac{2}{3}, \text{ i.e. } h = 4\ln(2)
\]

(18)

(We must thus measure time in a unit such that \( h = 4\ln(2) \) in order that the assumed Gumbel random terms are normalised \((0,1)\).)

The loss in consumer surplus is now calculated as

\[
\ln(e^{a-0.5h} + e^{a-0.5h}) - \ln(e^{a-0.5h} + e^{a-0.75h}) = 0.2877 = 0.10376h
\]

(20)

This is an overestimate of the true value \( h/18 \) by 87 per cent.

**Conclusions**

When travellers are assumed to consult timetables, computing consumer surplus is considerably more demanding than when they make their decision based only on headways (and other characteristics unrelated to departure / arrival times). The often-overlooked feature is that when the headway is changed for one line, the average delay time will be affected also for other lines. A problem is that erroneous estimates of consumer surplus may pass undetected. A plain vanilla logit model cannot
handle the situation, nor can a simple “rule of a half”. The FIRSy-model seems to be the best suited for the task. However, as mentioned, in its simplest form, the FIRSy-model does not fully take individual preferences with respect to modes into account, but can partly by use of segmentation.

Note that the difference between the estimation of consumers surplus according to the FIRSy-model on the one the hand and Rule of a half and logsum on the other, may be both larger and smaller than what the analysis above shows, dependent on the assumed headways and change of headway. Note also that the difference between the results of consumers’ surplus according to FIRSy-model on the one hand and logsum or “rule of a half” on the other, does not affect the main message: that only the FIRSy-model can calculate consumer surplus correctly when travellers only consider travel times, prices and schedule delays when they make their choices.

**Comparison of the "extreme" models and a mixed model by use of an example**

It is difficult to compare the consequences of a pure assignment model, a DC model and a mixture of the two analytically. Despite the fact that the consequences will be case dependent, we will use a simple example to demonstrate some of the implications related to choice of concept for modelling long distance travel. The example is taken from Larsen et al. (2010). The supply of services for journeys on a hypothetical OD-pair is shown in Table 1.

Table 1. Example for one OD-pair

<table>
<thead>
<tr>
<th>Alternative:</th>
<th>On board (min)</th>
<th>Headway (min)</th>
<th>Acc+Egress (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train1</td>
<td>120</td>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>Train2</td>
<td>130</td>
<td>90</td>
<td>20</td>
</tr>
<tr>
<td>Train3</td>
<td>140</td>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>Bus</td>
<td>120</td>
<td>120</td>
<td>25</td>
</tr>
<tr>
<td>Car</td>
<td>160</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

For simplicity we assume that the cost is the same for all alternatives and time has the same weight irrespective of mode and time component. A model shall distribute travellers between the three modes (train, bus and car), but also between three train lines. The assignment algorithm referred to in this section is the FIRSy-algorithm.

A model of type DC will make an assignment for train giving a distribution on the train lines and mean values for the level of service variables as for “Train combined” in Table 2. Bus will have mean schedule cost of 60 minutes, i.e., half the headway. If a DC-model uses PT as mode along with car, an assignment for train and bus combined gives the level of service for the PT-mode as "PT-combined" in Table 2.
Table 2. Assignments with FIRSy-algorithm

<table>
<thead>
<tr>
<th>Assignments:</th>
<th>Mean on-board, min</th>
<th>Mean schedule cost, min</th>
<th>Mean acc+egg, min</th>
<th>Mode probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train combined</td>
<td>125.278</td>
<td>18.889</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>PT combined</td>
<td>124.215</td>
<td>16.812</td>
<td>20.807</td>
<td></td>
</tr>
<tr>
<td>Of this: Train users</td>
<td>125.027</td>
<td>17.267</td>
<td>20</td>
<td>0.8385</td>
</tr>
<tr>
<td>Of this: Bus users</td>
<td>120.000</td>
<td>14.447</td>
<td>25</td>
<td>0.1615</td>
</tr>
<tr>
<td>All combined (incl. car)</td>
<td>132.883</td>
<td>8.3858</td>
<td>18.101</td>
<td></td>
</tr>
<tr>
<td>Of this: PT alts.</td>
<td>123.530</td>
<td>11.277</td>
<td>20.895</td>
<td>0.7435</td>
</tr>
<tr>
<td>Train alts.</td>
<td>124.299</td>
<td>11.951</td>
<td>20</td>
<td>0.6104</td>
</tr>
<tr>
<td>Bus</td>
<td>120.000</td>
<td>10.487</td>
<td>25</td>
<td>0.1331</td>
</tr>
<tr>
<td>Car</td>
<td>160.000</td>
<td>0</td>
<td>10</td>
<td>0.2565</td>
</tr>
</tbody>
</table>

A mode choice model with three modes will then get the following LoS-variables that can be added up or given different weights and possibly subject to transformations in the mode choice model:

Table 3. LoS-variables for mode choice

<table>
<thead>
<tr>
<th>Assignments:</th>
<th>Mean on-board, min</th>
<th>Mean schedule cost, min</th>
<th>Mean acc+egg, min</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train</td>
<td>125.278</td>
<td>18.889</td>
<td>20</td>
<td>164.167</td>
</tr>
<tr>
<td>Bus</td>
<td>120.000</td>
<td>60.000</td>
<td>25</td>
<td>205.000</td>
</tr>
<tr>
<td>Car</td>
<td>160.000</td>
<td>0</td>
<td>10</td>
<td>170.000</td>
</tr>
<tr>
<td>With PT mode:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PT</td>
<td>124.215</td>
<td>16.812</td>
<td>20.807</td>
<td>161.834</td>
</tr>
</tbody>
</table>

Notice that schedule cost for the bus alternative becomes quite high compared to train and in a mode choice model the schedule cost for bus is unaffected by what happens to the headways of trains and vice versa.

Now, if the choice between bus and train lines is strongly affected by the deviation between preferred and actual arrival times while mode specific unobserved variables and preferences are of minor importance, it would be natural to treat train and bus together as a public transport mode and let the assignment determine mode choice between train and bus. This gives us the LoS-variables for PT in Table 3 and among the PT modes; bus gets a share of 16.15 per cent. However, the main thing to notice here is what happens to "Mean schedule cost”. The bus alternative now gets a mean schedule cost of 14.447 instead of 60 and for train the mean schedule cost becomes 17.267 instead of 18.889 minutes. Inclusion of bus in the assignment also affects the mean on-board time for train.

In the "extreme" case that car also is treated as a route choice option we get the LOS-variables for "All combined" in Table 2 and car will get a share of 0.2565 (1-0.7435) and the schedule cost of the PT alternatives are further reduced.

The point to be made here is that as long as travellers compare schedule delay across modes and this to some extent affects mode choice, the effects we see above will be present, although not as strongly as revealed by pure assignments based on the inputs in Table 1. The consequence is that an assignment
for each mode separately will produce LoS-variables that are more or less biased and the amount of bias will vary with the circumstances. It may also affect the (implicit) random terms in the utility-functions introducing both correlation and non-constant variance across modes and OD-pairs. Modelling mode choice in long distance travel by DC models can thus involve some potentially very severe problems both in estimation and applications of the models.

On the other hand, the basic assumption of DC models: that some of the aspects relevant for mode choice are unobservable and can be subsumed to a term that can be treated as stochastic, may still be a realistic assumption. As long as mode choice is influenced unobservable aspect specific to each mode and varying between travellers, a combined assignment on all public transport modes is also problematic as a modelling approach.

**Assignment with mode-specific random terms (mixed model)**

As mentioned it is shown by Larsen et al. (2010), that it is actually possible to combine assignment and a discrete choice model in one step, at least for simple cases. This is facilitated by the fact that – for example – 5 data points with appropriate weights can approximate the Gumbel distribution. The results from a combined model based on the example in Table 1 are shown below in Table 4.

<table>
<thead>
<tr>
<th>Variance/scale parameter for mode-specific random terms</th>
<th>0=pure RDT</th>
<th>15/0.086</th>
<th>20/0.064</th>
<th>40/0.032</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Train1</td>
<td>0.416</td>
<td>0.361</td>
<td>0.341</td>
<td>0.311</td>
</tr>
<tr>
<td>Train2</td>
<td>0.127</td>
<td>0.105</td>
<td>0.101</td>
<td>0.096</td>
</tr>
<tr>
<td>Train3</td>
<td>0.068</td>
<td>0.067</td>
<td>0.068</td>
<td>0.066</td>
</tr>
<tr>
<td>Train total</td>
<td>0.610</td>
<td>0.533</td>
<td>0.509</td>
<td>0.472</td>
</tr>
<tr>
<td>Bus</td>
<td>0.133</td>
<td>0.132</td>
<td>0.140</td>
<td>0.178</td>
</tr>
<tr>
<td>Car</td>
<td>0.257</td>
<td>0.335</td>
<td>0.351</td>
<td>0.350</td>
</tr>
<tr>
<td>Mean in-vehicle</td>
<td>132.9</td>
<td>135.8</td>
<td>136.4</td>
<td>136.2</td>
</tr>
<tr>
<td>Mean acc/eggr</td>
<td>18.1</td>
<td>17.3</td>
<td>17.2</td>
<td>17.4</td>
</tr>
<tr>
<td>Mean SCD</td>
<td>8.4</td>
<td>9.6</td>
<td>10.3</td>
<td>13.1</td>
</tr>
<tr>
<td>Mean ε chosen alt.</td>
<td>0.0</td>
<td>-14.9</td>
<td>-21.2</td>
<td>-46.5</td>
</tr>
<tr>
<td>Mean total</td>
<td>159.4</td>
<td>147.8</td>
<td>142.7</td>
<td>120.2</td>
</tr>
</tbody>
</table>

Source: Larsen et al. (2010)

In the simulation it is assumed that the same value of the stochastic term applies to all trains. It is also assumed that the stochastic term has the same standard deviation for all modes. This assumption can be relaxed. The mean total time includes the value of the Gumbel term for the chosen alternatives and thus has the same interpretation as a logsum from a discrete choice model.

As revealed by Table 4, increasing the standard deviation of the random term makes the market shares of the three modes more equal, and they will converge to 1/3 as the standard deviation increase towards infinity and the random terms completely dominate the mode choice.
In the example used above the main mode approach is used in the sense that the same random term is applied to all train alternatives, while bus and car have separate random terms. Access/egress time (or cost) is assumed to be walk links or to be input from an access/egress model. The FIRSy-algorithm used for assignment presupposes use of timetables and the assignments must be interpreted as the mean value over all feasible instances of timetables that maintain the fixed headway of each line. It is further assumed that preferred arrival/departure times are uniformly distributed on the time axis.

How would a multinomial DC model perform on the same example, assuming that all time components have the same weight? In that case the modes get the sum of travel time components in Table 3. The mode shares will depend on the standard deviation of the random terms, and using the same alternatives as in Table 4 we get:

<table>
<thead>
<tr>
<th>Scale parameter</th>
<th>0.086</th>
<th>0.064</th>
<th>0.032</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train</td>
<td>0.611</td>
<td>0.568</td>
<td>0.476</td>
</tr>
<tr>
<td>Bus</td>
<td>0.018</td>
<td>0.042</td>
<td>0.129</td>
</tr>
<tr>
<td>Car</td>
<td>0.370</td>
<td>0.391</td>
<td>0.395</td>
</tr>
</tbody>
</table>

Let us, for the sake of the argument, assume that the "correct" model is a combined assignment and DC model with scale parameter 0.064 and we observe the market shares in Table 4 for this value. If we start out with the idea that the "correct" model is a pure assignment model, the model will initially produce the results in the first column of Table 4. The FIRSy assignment model will thus need some calibration in order to reproduce the observed distribution on modes, even though the parameters used in the assignment initially were "correct".

If we believe that a multinomial logit model is the "correct" model for choice of mode we will use the LoS-variables in Table 3 and try to find a set of model parameters that reproduce the observed results. Again we might end up with different parameters from the ones used for the combined model in Table 4 and might even end up with transformation of variables and a nested or cross-nested logit model even though a simple multinomial logit model is the "correct" DC model according to our assumption.

What this simple example shows us is:

- If travellers compare itineraries (and schedule delay) across different modes when they decide on how to travel, a mode choice model based on the main mode concept and LoS-data produced by separate assignments for each main mode will have some problems both related to estimation and application.
- Handling mode choice by a FIRSy-assignment model has the advantage that multimodal trips automatically are taken care of and estimated schedule delay is based on the total supply of services. However, when unobserved aspects and attributes of the different alternatives available also influence travellers’ decisions, an assignment model will also have problems.
- In both cases we will calibrate or estimate a model that did not generate the data we observe with respect to distribution on modes (and lines) and with unknown consequences when model is used for prediction and evaluation of policy measures.
A somewhat disturbing fact is that the simple example we have used here is based on mode specific random terms that are IID-Gumbel, but the fact that people compares itineraries across modes may produce observable data that seemingly indicates a much more complicated error-structure.

An additional complication is that the data we observe is generated by a situation where a specific set of timetables (one timetable instance) are at work, while the assignment model produces results that must be taken as the mean value over all timetable instances.

There is much to say for long distance models that can combine the basic ideas of assignment models and discrete choice models. There are, however, several problems that must be solved before such combined models can be applied on a large scale. Among these are:

✓ How to handle multimodal trips?
✓ The computational effort needed on large-scale networks?
✓ How to estimate/calibrate the parameters of combined models?

Some conclusions
Both the use of pure assignment models and discrete choice models has some merits in modelling long distance travel. If long distance travellers base their choices on timetable information and behave as if the public transport alternatives are reliable, this should also be reflected in the assignment principle. To the extent we can verify that mode-specific (and unobservable) preferences are relatively unimportant in the choice between different public transport modes, there are major advantages in letting an assignment model handle both the distribution on lines and modes for public transport and reserve only the choice between car and public transport for a discrete choice model. In particular, this will "solve" the problem of multimodal trips, which, in the case of a DC-model, needs rather elaborate models for access and egress to the main modes in order to be handled properly. In this case the FIRSy-model will also provide the proper inputs for assessment of changes to the system.

On the other hand, if mode specific preferences are important also for the choice among public transport modes, the crucial question is the extent to which travellers also consider and compare the itineraries offered by different modes. If these types of considerations are important, the proper model will be a mixture of assignment and discrete choice and both a pure assignment and a "traditional" discrete choice model may severely bias both demand forecasts and evaluation of policy measures.

References
