Trip Timing and Crowding on Rail Transit Lines: Theory and Application

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INTRODUCTION

Crowding and travel delays are commonplace on many transit systems in developed and developing countries.¹ A recent roundtable report by the International Transport Forum (OECD, 2014) identifies crowding as a major source of inconvenience that increases the generalized cost of travel. Crowding occurs not only while riding buses and trains, but also when boarding and alighting from them, while waiting on platforms or at bus stops, and while accessing stations by escalator, elevator, or on foot (King et al., 2014). Several recent studies have documented the aggregate cost of crowding on transit networks. For example, Prud'homme et al. (2012) estimate that the 8% increase in densities in the Paris subway between 2002 and 2007 imposed a welfare loss of at least €75 million per year. Veitch et al. (2013) estimate the annual total cost of crowding in Melbourne metropolitan trains in 2011 at AUS $280 million.

Similar to road traffic congestion, congestion on transit networks varies widely by time of day, and it cannot be studied effectively without taking into account users' time-of-use decisions. Users face a choice between traveling in a crowded train and arriving when they want, and traveling earlier or later to avoid crowding but arriving at an inconvenient time (i.e., incurring schedule delay costs).

LITERATURE

A number of theoretical studies have analyzed the trade-off between crowding costs and schedule delay costs (e.g., Huang et al., 2005; Tian et al., 2007, 2009a; de Palma, Kilani and Proost, 2015; Kaddoura et al., 2015). Empirical studies have also estimated how disutility from in-vehicle time increases with the number of users (e.g., Douglas and Karpouzis, 2005; Tian et al., 2009b; Haywood and Koning, 2013; Kroes et al., 2014; de Lapparent and Koning, 2015); see Li and Hensher (2011) and Wardman and Whelan (2011) for literature reviews.

A recent paper by the authors (de Palma, Lindsey and Monchambert, 2015) draws on this literature to develop a structural model of crowding on a single rail transit line. Train service is assumed to operate between two points following a timetable with a uniform headway between trains. Riders are ex ante identical. They know the timetable, and choose which train to take. The paper derives the user equilibrium and socially optimal distribution of passengers across trains, shows how the optimum can be decentralized using train-specific fares, and characterizes the welfare gains from optimal pricing. Given price-elastic total demand, it compares total usage, private costs, and welfare in the user equilibrium, the social optimum, and an optimal uniform-fare regime. It also derives the optimal timetable, number of trains, and train capacity for each fare regime. Finally, it compares the results with those obtained from the Vickrey (1969) bottleneck model as well as the flow congestion models of Henderson (1974, 1981) and Chu (1995).

THE MODEL AND APPLICATIONS

In this paper we build on de Palma, Lindsey and Monchambert (2015) in four directions. First, we allow users to differ in their preferred arrival times. Second, we extend the transit line from a single origin-destination pair to a many-to-one network while returning to the assumption that users have identical desired arrival times. All users are still assumed to travel to the same destination, but they board trains at different stations.

Third, we investigate different functional forms for the schedule delay cost and crowding cost functions. As de Palma, Lindsey and Monchambert (2015) show, the welfare gains from differentiated fares are sensitive to the shape of the schedule delay cost function as well as total ridership since it affects average train loads. That paper focuses on deriving closed-form analytical solutions for the user equilibrium and social optimum. Here, we consider alternative functional forms including a quadratic schedule delay cost function with asymmetric early and late arrival costs, and a hyperbolic crowding cost function that features an absolute capacity limit on the number of riders that can be accommodated on each train.

Fourth, we apply the model to two distinctly different mass transit systems. One is the RER A line in Paris, France, which served about 1.14 million travelers per working day in 2011. The other is the Capital Line in Edmonton, Alberta, which carried just over 100,000 travelers per working day in 2013.2 The two lines each have a station that is mainly a "destination" or "off" station during the morning peak: "La Défense" on the Paris RER A line, and "University" on the Edmonton Capital Line. We treat these stations as the destination of the many-to-one network of the model, and ignore stations further downstream in either direction.

The following data are available for the two lines:

Paris RER A line:

- The train timetable

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2 Since the Capital Line is the only light rail transit line in Edmonton, the rail transit network is linear with no connecting stations. A new line called the Metro Line is scheduled to open in early 2015.
- Train capacities, measured in number of seats and square meters of floor area
- An O-D matrix in 15-minute time slices during the morning peak for the West branch of the RER A line

Edmonton Capital Line:
- The train timetable for northbound and southbound runs throughout the day
- Train design capacities for the rolling stock (Siemens-Duewag U2 and Siemens SD-160 cars).
- Passengers boarding, passengers alighting, and passengers departing from each station on each train throughout the day

Data on trip-timing preferences are not available. For the costs of early and late arrival we draw on various estimates in the literature. For preferred arrival times we assume that, in the case of homogeneous preferences, the time at which the largest number of passengers alights (and crowding is correspondingly the highest) is the preferred arrival time. To handle heterogeneous preferences, we assume that the most heavily loaded train arrives at the fractal of the distribution predicted in the user equilibrium of the model.\(^3\)

Given these data and assumptions we can compute the equilibrium distribution of departure and arrival times for all users originating upstream of the destination stations. We can then also derive the socially optimal departure distribution, the optimal differentiated fare scheme, and the welfare gains from implementing differentiated fares.

**PRELIMINARY RESULTS**

Preliminary results have been derived for two of the extensions.

**Heterogeneous users:** When user groups differ in their desired arrival times the equilibrium departure pattern can take one of several forms. If the distribution is very spread out, users self-select into groups that travel in disjoint time intervals and do not interact. If the distribution is somewhat narrower, user groups take different trains, but the crowding costs imposed by each group deter users from other groups in taking the same trains so that the existence of each group raises the equilibrium travel costs of other groups. If the distribution is relatively narrow, different groups can occupy the same trains. Moreover, if schedule delay and crowding cost functions are linear, the equilibrium aggregate train load pattern is the same as if all users have the same preferred arrival time although the total costs of schedule delay are smaller.

**Differentiated fares:** The degree of variation in optimal train fares is sensitive to the shape of the crowding cost function. Under plausible assumptions, fares on the most popular trains can be quite high and may be opposed on acceptability grounds. As Tian et al. (2007) show, with inelastic demand it is possible to reduce fares a certain amount for users boarding at stations

\(^3\) As noted below, if schedule delay and crowding cost functions are both linear, the user equilibrium is — within limits — independent of the distribution of preferred arrival times, so we do not expect the results to be too sensitive to this assumption.
downstream of the first (i.e., the most remote) station while still supporting a social optimum. However, this fare adjustment creates a loss of efficiency if demand is elastic. We will experiment with modified fare regimes to explore the trade-off between efficiency and acceptability.

REFERENCES


