Airport Congestion and Endogenous Slot Allocation

[ PRELIMINARY ]

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Abstract

This paper analyzes the optimal slot allocation in the presence of airport congestion. We model peak and offpeak slots as vertically differentiated products, and congestion limits the number of peak slots that the airport can allocate. Inefficiency emerges when the airport does not exploit all its slots. We show that for a private airport, inefficiency may arise if the airport is not too congested and the per-passenger fee is small enough, while with a public airport it does not emerge. Furthermore, a private airport tends to give different slots to flights with same destination, while a public airport more likely favors identical slots to flights with same destination. We conclude that a private airport may be less efficient in terms of use of slots, and its allocation choices satisfy a lower number of passangers than a public airport.

JEL classification: R41; H23; H21.

Keywords: Slot allocation; Airport congestion; Vertical differentiation.
1 Introduction

Airline traffic growth has outpaced capacities at many of the world’s major airports in the past decades, leading them to operate in congestion conditions. For instance, over half of Europe’s 50 largest airports have already reached or are close to their saturation points in terms of declared ground capacity (Madas and Zografos, 2008). Airport congestion has been managed through control and management of slots, including slot sales, slot trading and slot auctions. According to the European Council Regulation EEC95/93, a “slot” is the permission to use airport infrastructure for landing or take-off on a specific time. Under a slot system, the airport authority determines the total number of slots to make available, and slots are distributed among the airlines according to some allocation rule. Congestion calls for management of slots. For example, FAA (Federal Aviation Administration) capped peak hour flight movements at New York La Guardia, J.F.Kennedy, and Newark airports. As for Chicago’s O’Hare airport, FAA persuaded two major airlines United and American Airlines to reduce peak flight activities while prohibiting smaller airlines from increasing flights to fill the gap.

In the analysis of airport congestion, the economic literature focused mainly on “congestion pricing” (for which carriers pay a toll according to their contribution to congestion) as a regulatory tool to deal with congestion\textsuperscript{1}. However, despite its theoretical feasibility, congestion pricing has not been practised in the real world. By contrast, slot allocation is the usual business in the management of airports. Thus, a theoretical analysis investigating the interaction between slot allocation and congestion seems highly policy relevant.

In this paper we analyse slot allocation in the presence of airport congestion. We examine a setting where an airport sorts slots according to different departure flights, and airlines compete in the flights market. As in Brueckner (2002), we model peak and off-peak slots as products of different qualities in a model of vertical differentiation. Peak slots are congested, mirroring the situations of capacity shortages at peak hours faced by many airports. We analyze

\textsuperscript{1}For an early contribution on congestion pricing see Levine (1969). Recent representative studies include Brueckner (2002, 2005). Under congestion pricing, carriers could place as many flights as they wish provided they pay the toll, thus the overall level of congestion is determined by airline decisions.
both a private and a public airport being restricted to levy a uniform per-passenger fee for flight activities, this being pre-determined by administrative bodies. The analysis of a private airport also seems relevant. Although airports have long been owned by governments, there has been a significant worldwide trend towards government facilities privatization beginning from the middle of the 1980s. We consider separately the case where two flights towards a same destination (denoted as “pairwise flights” along the paper) are served by two airlines, and where they are owned by a monopoly. In this complete information setting, airlines know the total provision of slots and that each participating airline would a single slot. We define as “allocation inefficiency” the situation in which not all the slots available are exploited.

Our approach differs from Brueckner (2002) as follows. In Brueckner (2002)’s framework, a monopoly airport chooses the critical points on the continuum that respectively define whether to fly or not and whether to fly in peak slots or off-peak slots. Focusing on finding out the optimal congestion pricing, he implicitly assumes that airport capacity is sufficient to meet peak hour demands. Unlike Brueckner (2002), our interest stems from the scarcity of peak hour slots. Thus we focus on the allocation instead of using the pricing tool.

The airport’s ownership and the airline market’s structure affect the results. For a private airport and duopoly airline markets, the airport tends to assign different slots, unless the per-passenger fee is high enough, and the airport is “not too busy”, i.e., if the number of peak slots is higher than the number of destinations. In this case, allocation inefficiency arises when the airport is not too busy and the per-passenger fee is sufficiently low. The results are driven by two effects. First, an off-peak flight in one market would attract additional lower-valuation passengers, yielding a wider range of passengers to travel in total. Second, consumers benefit from harsher competition. Thus when airlines obtain peak/peak slots, they compete head-to-head and more passengers would be served. While on the contrary, in the presence of quality differentiation, competition is less intense, and airlines provide passengers with different taste. When the per-passenger fee is small the first effect outweighs the second effect.

Following UK many major airports in Europe, Australia and Asia have followed suit and have undergone privatization or are in the process of being privatized. In principle, privatization is characterized by the transfer of ownership structure from state-owned to private enterprises.
In the case of monopoly, the airport prefers to assign identical time slots to pairwise flights. This is due to the fact that, without differences in terms of type of slots, it is possible to extract a higher mark-up. When the airport is public, allocation inefficiency does not emerge. Also, in a public airport it is more likely that pairwise flights are assigned to identical time slots. The intuition is simple. Airline markets with same slot quality are more favorable to passengers, given the higher intensity of competition. Since a public airport is interested in consumer surplus also, this type of slot configuration is usually preferred.

The emergence of allocation inefficiency in our results corresponds to the common practice in airport management of declaring a number of slots being lower than an airport’s full capacity (Mac Donald, 2007, and De Wit and Burghouwt, 2008). Indeed, as De Wit and Burghouwt (2008) point out, “an efficient use of the slots at least requires a neutral and transparent determination of the declared capacity”.

So far, slot allocation has drawn relatively little interest in the economic literature, with few but noteworthy contributions. Barbot (2004) models slots for airline activities as products of either high or low quality, and carriers choose the number of flights they serve. She shows that slot allocation improves efficiency according to the criteria for assessment, and welfare in fact decreases after re-allocation. Unlike the present paper, in Barbot (2004)’s model carriers could place as many flights as they want. Our paper limits the number of peak slots, in order to address congestion. Verhoef (2008) and Brueckner (2009) compare the pricing policy and slot policy regimes. They show that the first best congestion pricing and slot trading/auctioning generate the same amount of passenger volume and total surplus. They investigate a single congested period. Their contributions do not distinguish between peak and off-peak hours, and allow the airport to allocate slots without charges. Although this seems a plausible description of some public airports, a non-profit behavior does not seem likely in the presence of a private airport. Departing from Verhoef (2008) and Brueckner (2009), we assume that each airline operates a single flight. Our approach models certain time intervals that are most desired by all passengers as peak period. In particular, the total number of slots that an airport could place in peak period does not meet the demand of passengers.
The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the baseline results, in which the airport is assumed to be private and an airline duopoly serves each destination. Section 2 and 5 show the changes in the results when either a monopoly airline is operating for each destination or the airport is publicly owned, respectively. Section 6 discuss some extensions. In particular, we show the changes in the results when (i) collusion emerges between the airport and one of the airlines in the duopoly case, (ii) the per-passenger fee is endogenously determined by the airport and (iii) density is heterogenous among different flights. Section 7 concludes.

2 A model of slot quality

We consider an economy with one private airport, where trips to \( N \) destinations \( d \in D = \{1, 2, \ldots, N\} \) are served by \( 2N \) airlines. Each destination, also called a market, is served by two flights being run by separate airlines. Denote a flight as \( f: f \in \mathcal{F} = \{1, 2, \ldots, 2N\} \). Formally this can be depicted as a mapping \( I \) from flights \( f \) to destinations \( d : I(f) = d \). In turn, we write the inverse mapping function from flights to destinations as \( A(d) = \{f : I(f) = d\} \). We assume that airports at destinations are uncongested, so that the allocation decisions do not affect the flight scheduling of destination airports. Furthermore, we focus on the case of solely single trip departing flights. Return trip flights can be dealt with by either an identical analysis with two runways, or simply by discounting a scale factor if there is a single runway. Destinations are independent in the sense that they are neither substitutes nor complements, therefore the demand for one destination is irrelevant to demands for other destinations.

We assume that the quality differential is characterized only by the departing time. Although the quality of an airline depends on many factors, this approach would allow us to concentrate on the congestion issue. There are two travel periods, denoted as peak and off-peak. A peak period represents the time window that consists of the most desirable travel times in a day, for instance early morning and late afternoon. The peak period may contain a collection of disjoint time intervals like 7:00-9:00 and 17:00-19:00. An off-peak period, by

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3Section 5 analyses the case with a public airport.
4Section 4 extends the analysis to the case where a private airport interacts with airline monopolies.
contrast, contains all other time intervals that do not belong to the collection of peak period. In order to address the problem of peak slots congestion, the off-peak period is assumed to be uncongested, i.e., airport capacity can serve all flights within off-peak period time intervals. Conversely, the peak period is congested in the sense that airport capacity cannot serve all flights within peak period time intervals. This assumption captures traffic patterns at many airports such as Paris Charles de Gaulle, Amsterdam Schiphol, Munich and Brussels airport (NERA, 2004). All potential passengers perceive and agree with the slot associated with time slot, and agree over peak load hours (denoted as subscripts $h$) are more preferable than the off-peak load hours (denoted as subscripts $l$) at equal price. At peak hours the demand to use airport runways is much higher than off-peak hours. Thus slot qualities $s_l$ and $s_h$ are exogenously perceived: $s_h > s_l > 0$. An airline offering mid-day schedules are more favorable to all passengers than another airline that offers late evening airlines. Finally, a slot allocation is defined as the mapping $m$ from airline $f$ to a slot type $i$, $m: F \rightarrow \{l, h\}$, so that $m(f) = i$. For instance, $m(f) = h$ reads as airline $f$ is allocated peak time slot.

We assume in each market the airlines engage in quantity competition. We denote $p_{ii'}^f$ as the price charged by an airline $f$ flying to destination $d = I(f)$ that takes off at slot $i$ while its competitor on the same destination takes off at slot $i', i, i' \in \{l, h\}$. Similarly, $q_{ii'}^f$ denotes the number of passenger served by this flight. Following the general framework of vertical differentiation (Gabszewicz and Thisse, 1979), in each market the demand is generated from a unit mass of passengers indexed by a type parameter $v$. Passengers differ in tastes, the taste parameter is described by $v \in [0, 1]$, $v$ being uniformly distributed with unit density. Demand addressed to flight $f$ is defined by the set of passengers who maximize their utility when flying with airline $f$, rather than flight $f'$ to the same destination or refraining from flying. Accordingly, each passenger flying to destination $d$ purchases a flight ticket of airline $f$ if her utility $vs_i - p_{ii'}^f$ is higher than the utility $vs_i - p_{ii'}^{f'}$ of flying with the competitor and higher than the utility of not traveling. We assume each passenger flies at most once, and, if a passenger does not fly, her reservation utility is zero. It follows that the more convenient

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5The assumption of quantity competition is common in the airline economics literature. See Brueckner (2002), Pels and Verhoef (2004). Brander and Zhang (1990) find empirical evidence that the rivalry between duopoly airlines is consistent with Cournot behavior.
the slot of departing time slot, the higher the utility reached by passengers for a given price. Formally, a potential passenger in the destination market \( d \) with the airlines \( f \) and \( f' \), \( d = I(f) = I(f') \), has the following preferences:

\[
U^d = \begin{cases} 
vs_i - p_{ii}' & \text{if she flies with airline } f \text{ in slot } s_i \text{ at price } p_{ii}' \\
vs_{i'} - p_{i'i}' & \text{if she flies with airline } f' \text{ in slot } s_{i'} \text{ at price } p_{i'i}' \\
0 & \text{if she does not fly.}
\end{cases}
\]

The airlines choose the number of seats in order to maximize profits. Airline costs include airport per-passenger charge \( \phi \), while marginal operating costs are normalized to zero for the moment. We do not consider the entry of airlines in the airport and assume that fixed costs are sunk. Thus the profit of an airline \( f \) competing in the destination market \( d \) with another airline \( f' \), \( d = I(f) = I(f') \), is given by:

\[
\pi_i^f = \left( p_{ih}(q_{ih}, q_{hi}) - \phi \right) q_{ih} \text{ if } m(f) = l \text{ and } m(f') = h
\]

\[
\pi_{hi}' = \left( p_{hi}'(q_{hi}' , q_{hi}') - \phi \right) q_{hi}' \text{ if } m(f) = h \text{ and } m(f') = l
\]

for an off-peak/on peak slot configuration and

\[
\pi_{ii}^f = \left( p_{ii}(q_{ii}, q_{ii}') - \phi \right) q_{ii} \text{ if } m(f) = m'(f') = i \in \{l, h\}
\]

\[
\pi_{ii}' = \left( p_{ii}'(q_{ii}', q_{ii}) - \phi \right) q_{ii}' \text{ if } m(f) = m'(f') = i \in \{l, h\}
\]

for the on peak/on peak and off-peak/off-peak slot configuration.

The airport earns the charge \( \phi \) for each passenger. It chooses the slot allocation mapping \( m(\cdot) \) that maximizes it profits. We get the following program:

\[
\max_{m(\cdot)} \Pi = \sum_{d=1}^{N} \phi \left( q_{m(f),m(f')}^f + q_{m(f'),m(f)}^f \right) \text{ where } I(f) = I(f') = d.
\]

subject to the peak slot capacity constraint

\[
\# \{ f : m(f) = h \} \leq M,
\]

\footnote{In Section 3.3 we also consider positive operating costs. These have been assumed in some contributions, such as Pels and Verhoef (2004), Brueckner and Van Dender (2008), and Basso (2008).}
where $M$ is the total number of peak slots. To avoid a cumbersome discussion of ties, we assume that $M$ is even. Constraint (4) implies that the overall allocated peak slots cannot exceed the total number of available peak slots. Peak capacity could not accommodate all flights $M < 2N$. By contrast, the off-peak capacity could accommodate all flights (there is no constraint for the off-peak slots).

We then define allocation inefficiency as follows.

**Definition 1** Allocation inefficiency emerges when at least one peak slot is not used.

This assumption seems natural. In the presence of airport congestion, leaving some peak slots unused represents some degree of inefficiency.

Figure 1 shows the timing of the game. In the first stage the airport allocates peak and off-peak slots for a given fee $\phi$. In the second stage airlines choose number of seats to supply $q_{it}^f$ based on slot allocation. In the third stage passengers in each destination decide whether to fly with a peak period airline, an off-peak period airline, or not fly at all. The equilibrium concept is the subgame perfect equilibrium by backward induction.

### 3 Baseline results

In this section we show the baseline results of the model. In particular, we consider the scenario where per-passenger fee $\phi$ is uniform. For the sake of simplicity we omit superscript $d$ in the ensuing analysis, as the demand in one market is not influenced by the demand in other markets.
3.1 Passengers

In the third stage, a passenger decides whether to fly and, if so, the time of flying. Let us consider the destination market \( d \) with two airline competitors \( f \) and \( f' \), i.e. \( d = I(f) = I(f') \). If the two flights in this market obtain a peak and an off-peak slot, the passenger with a taste parameter \( \bar{v} \) such that \( \bar{v}s_l - p_{lh}^f = \bar{v}s_h - p_{hl}^f \) is indifferent between flying at peak load time and at off-peak load time. Likewise, the passenger with a taste parameter \( v \) such that \( vs_l - p_{lh}^f = 0 \) is indifferent between flying at off-peak time and not flying at all. Thus airline markets are partially covered.

In this case, if two flights in a single market obtain slots of same quality, the passenger with a taste parameter \( v_0 \) such that \( v_0s_l - p_{lh}^f = 0 \) with \( p_{lh}^f = p_{hl}^{f'} \), or \( v_0s_h - p_{hh}^f = 0 \) with \( p_{hh}^f = p_{hh}^{f'} \) is indifferent between flying and not flying. Three market configurations may arise at equilibrium: (i) the two airlines obtain peak/off-peak slots; and both obtain either (ii) off-peak slots, or (iii) peak slots. They are characterized by the following demand function, respectively:

\[
\begin{align*}
(i) \quad & q_{lh}^f(p_{lh}, p_{hl}^{f'}) = \frac{p_{hl}^{f'} - p_{lh}^f}{s_h - s_l}
q_{hl}^{f'}(p_{lh}, p_{hl}^{f'}) = 1 - \frac{p_{hl}^{f'} - p_{lh}^f}{s_h - s_l}, \quad (5) \\
(ii) \quad & q_{lh}^f(p_l) + q_{hl}^{f'}(p_l) = 1 - \frac{p_l}{s_l}, \quad (6) \\
(iii) \quad & q_{hh}^f(p_{hh}) + q_{hh}^{f'}(p_{hh}) = 1 - \frac{p_{hh}}{s_h}. \quad (7)
\end{align*}
\]

In turn, the inverse demand functions corresponding to the three possible market structures are given respectively as follows:

\[
\begin{align*}
(i) \quad & p_{lh}^f = s_l \left( 1 - q_{lh}^f - q_{hl}^{f'} \right), \quad (8) \\
& p_{hl}^{f'} = s_h \left( 1 - \frac{s_l}{s_h} q_{lh}^f - q_{hl}^{f'} \right).
\end{align*}
\]

\[
\begin{align*}
(ii) \quad & p_l = s_l \left( 1 - q_{ll}^f - q_{ll}^{f'} \right), \quad (9) \\
(iii) \quad & p_{hh} = s_h \left( 1 - q_{hh}^f - q_{hh}^{f'} \right). \quad (10)
\end{align*}
\]

\footnote{Motta (1993) shows that Cournot competition can be studied only with partial market coverage, since the demand function can not be inverted with full market coverage.}
3.2 Airlines

In the second stage, airlines set their optimal supply of seats. We analyze each possible configuration (peak/off-peak; peak/peak; off-peak/off-peak) separately. Consider first a market where pairwise flights obtain different slots, then according to (1), airlines’ profits are expressed by:

\[
\pi_{lh}^f = \left[ s_l(1 - q_{lh}^f - q_{hl}^f) - \phi \right] q_{lh}^f,
\]

\[
\pi_{hl}^{f'} = (s_h - s_lq_{lh}^f - s_hq_{hl}^f - \phi)q_{hl}^{f'}.
\]

Airlines choose the number of seats to maximise profits, for any given \( \phi \). The first-order conditions are:

\[
\frac{\partial \pi_{lh}^f}{\partial q_{lh}^f} = -\phi + (1 - q_{lh}^f - q_{hl}^f)s_l - q_{lh}^f s_l = 0,
\]

\[
(11)
\]

\[
\frac{\partial \pi_{hl}^{f'}}{\partial q_{hl}^{f'}} = -\phi + s_h - 2q_{hl}^f s_h - q_{hl}^{f'} s_l = 0.
\]

\[
(12)
\]

Solving (11) and (12) simultaneously with respect to \( q_{lh}^f \) and \( q_{hl}^{f'} \) yields:

\[
q_{lh}^f = \frac{s_h s_l - \phi (2s_h - s_l)}{(4s_h - s_l)s_l},
\]

\[
(13)
\]

\[
q_{hl}^{f'} = \frac{2s_h - \phi - s_l}{4s_h - s_l}.
\]

\[
(14)
\]

To ensure interior solutions, we assume the condition \( 0 < \phi < \phi_1 \equiv \frac{s_h s_l}{2s_h - s_l} \).

Note that \( q_{lh}^f < q_{hl}^{f'} \) for all \( 0 < \phi < \phi_1 \). If a market obtains different slots, then the airline with peak slot serve more passengers in equilibrium than its off-peak competitor. Plugging (13) and (14) into (8) yields

\[
p_{lh}^f = \frac{s_h(2\phi - s_l)}{4s_h - s_l},
\]

\[
(15)
\]

\[
p_{hl}^{f'} = \frac{2s_h^2 + s_h(3\phi - s_l) - \phi s_l}{4s_h - s_l},
\]

\[
(16)
\]
being always positive, and where

\[ p_{hl}' - p_{lh}' = \frac{(s_h - s_l)(s_h + 2\phi)}{4s_h - s_l} > 0. \]

Since \( q_{lh}^f < q_{hl}' \), \( p_{lh}' < p_{hl}' \), then the airline flying during the peak slot yields higher profit than its off-peak slot counterpart.

Consider next the optimal number of seats provided in markets (ii) and (iii) where both airlines obtain the same slots \( i = m(f) = m(f') \). The airlines face the demand

\[ q_{ii}^f + q_{ii}' = 1 - \frac{p_i}{s_i}. \]  (17)

Plugging (17) and (19) into airline profits (2) yields:

\[
\begin{align*}
\pi_{ii}^f &= \left(1 - q_{ii}^f - q_{ii}'\right)s_i - \phi \right) q_{ii}^f, \\
\pi_{ii}' &= \left(1 - q_{ii}^f - q_{ii}'\right)s_i - \phi \right) q_{ii}'. 
\end{align*} \]  (18)

The first order conditions of \( \pi_{ii}^f \) and \( \pi_{ii}' \) with respect to \( q_{ii}^f \) and \( q_{ii}' \), respectively, yield:

\[
\begin{align*}
-\phi - q_{ii}^f s_i + s_i(1 - q_{ii}^f - q_{ii}') &= 0 \\
-\phi - q_{ii}' s_i + s_i(1 - q_{ii}^f - q_{ii}') &= 0.
\end{align*} \]

By solving the above two equations for \( q_{ii}^f \) and \( q_{ii}' \) we obtain the optimal number of seats served by two airlines, which are identical due to symmetry:

\[ q_{ii}^f = q_{ii}' = \frac{s_i - \phi}{3s_i}. \]  (19)

To ensure interior solutions, we make the assumption \( 0 < \phi < s_i \) with \( s_i > \phi_1 \), \( i \in \{h, l\} \). Hence \( 0 < \phi < \phi_1 \) is sufficient condition for \( q_{ih}^f \), \( q_{hl}' \), \( q_{ii}' \) to be positive.

For all \( 0 < \phi < \phi_1 \), we have \( q_{ih}^f > q_{hh}^f > q_{ii}^f > q_{lh}^f \). Plugging (19) into (18) and (10) yields:

\[ p_{il} = \frac{s_l + 2\phi}{3}, \quad p_{hh} = \frac{s_h + 2\phi}{3}. \]  (20)
3.3 Airport

In the first stage, the airport maximizes its profit by allocating peak slots subject to congestion. In this setting, the presence of congestion implies that the number of peak slots is lower than the total number of flights. Hence, allocating one peak slot to a market implies to take that peak slot away from another market. This in turn implies that, since the type of market allocation influences another market, then the airport needs to consider the differences in the allocation of two peak slots, in the configuration of two markets together. For example, putting two peak slots to the same market might leave another market with two off-peak slots. If configuration peak/peak is preferred to peak/off-peak, but the combination peak/peak + off-peak/off-peak gets less passengers than two configurations peak/off-peak, then the best allocation is one peak slot in each market. This despite the fact that, in a single market, the configuration peak/peak reaches the best outcome.

Begin the analysis by comparing the number of seats in each configuration for a single market. When two flights in a market obtain peak/off-peak slots, according to (13) and (14), the number of passengers in a market is given by

\[
q_{lh}^f + q_{0lh}^f = \frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)}. \tag{21}
\]

whereas with configuration peak/peak, the number of passenger in a market is

\[
q_{ii}^f + q_{0ii}^f = \frac{2(s_i - \phi)}{3s_i}, i \in \{h, l\}. \tag{22}
\]

Note that for all \(s_h > s_l > 0\), \(q_{lh}^f + q_{0lh}^f < \min \{q_{ih}^f + q_{0ih}^f, q_{hh}^f + q_{0hh}^f\}\). In words, off-peak/off-peak is strictly dominated by peak/peak and peak/off-peak in terms of number of passengers, which implies that in a market having either one or two peak slots is better than having only off-peak slots. Conversely, by comparing the values of \(q_{lh}^f + q_{0lh}^f\) in (21) with \(q_{hh}^f + q_{0hh}^f\) given by (22):

\[
\frac{s_h(3s_l - 2\phi) - s_l^2}{s_l(4s_h - s_l)} \geq \frac{2(s_h - \phi)}{3s_h}.
\]
if and only if $\phi \leq \phi_2 \equiv \frac{s_l s_h}{6 s_h - 2 s_l}$, where

$$\phi_1 - \phi_2 = \frac{s_l (2 s_h - s_l)}{2 (3 s_h - s_l)} > 0.$$  

Consider next the differences in the allocation of two peak slots. Since two peak slots can be allocated either in one market (thus leaving another market with two off-peak allocation) or in two market, we must also compare $Q_1 = q_{lh}^f + q_{hl}^f$ to $Q_2 + Q_3 = q_{hh}^f + q_{ll}^f$, yielding

$$q_{lh}^f + q_{hl}^f - q_{hh}^f - q_{ll}^f = -\frac{1}{3} (s_h - s_l) \frac{(2 s_h - s_l) \phi - s_h s_l}{s_h s_l (4 s_h - s_l)} > 0 \quad \text{for} \quad \phi < \phi_1.$$  

Figure 2 compares the configurations in a single market according to airport fees, by illustrating how $Q_1 = q_{lh}^f + q_{hl}^f$, $2Q_2 = q_{hh}^f + q_{ll}^f$ and $2Q_3 = q_{ll}^f + q_{hh}^f$ are linear decreasing functions in $\phi$ with $\phi \in (0, \phi_1)$. Two effects emerge from Figure 2. The first effect entails that having an off-peak flight in one market, depicted by $Q_1$, would attract additional passengers with low $v$, yielding a wider range of passengers to travel. The second effect entails that consumers benefit from harsher competition. This is to say, when airlines obtain peak/peak
slots, depicted by \( 2Q_2 \), they compete head-to-head and thus the outcome is more beneficial to passengers, since more passengers would be served. On the contrary, in the presence of slot differentiation, competition is less intense, and airlines provide passengers with different taste. When \( \phi \) is small the first effect outweighs the second effect, so that \( Q_1 \) is greater than \( 2Q_2 \). On the other hand, configuration \( Q_2 + Q_3 \) is always dominated by the alternatives \( 2Q_2 \) and \( Q_1 \).

In this set-up, destination markets can have only three types of slot allocations. There are \( n_1 \) destination markets with peak/off-peak allocations such that \( n_1 = \# \{ d : m(f) \neq n(f') \text{ and } d = I(f) = I(f') \} \). The peak/peak allocations includes \( n_2 \) destinations markets where \( n_2 = \# \{ d : m(f) = m(f') = h \text{ and } d = I(f) = I(f') \} \). Finally the number of destination markets that receive off-peak/off-peak allocations is given by \( n_3 = \# \{ d : m(f) = m(f') = l \text{ and } d = I(f) = I(f') \} \). As a consequence the airport allocation problem (3) simplifies to the following linear programming problem:

\[
\begin{align*}
\max_{n_1, n_2, n_3} & \quad n_1 \left( q_{lh}^f + q_{lh}^{f'} \right) + n_2 \left( q_{hh}^f + q_{hh}^{f'} \right) + n_3 \left( q_{ll}^f + q_{ll}^{f'} \right) \\
\text{subject to} & \quad n_1 + n_2 + n_3 = N \\
& \quad n_1 + 2n_2 \leq M \\
& \quad 0 \leq n_1, n_2, n_3 \leq N
\end{align*}
\]

The solution of this problem is found in Appendix A. This is

(i) \( n_1 = \min \{ M, N \} \), \( n_2 = 0 \), \( n_3 = N - n_1 \) if \( \phi < \phi_2 \) - case 1;

(ii) \( n_1 = M \), \( n_2 = 0 \) and \( n_3 = N - M \) if \( \phi_2 < \phi < \phi_1 \) and \( N \geq M \) - case 3;

(iii) \( n_1 = 2N - M \), \( n_2 = M - N \), \( n_3 = 0 \) if \( \phi_2 < \phi < \phi_1 \) and \( M > N \) - case 4.

The description of single cases is in Appendix A. In solution (i) and (ii), the airport never allocates two flights to the same destination in the peak slots. It rather separates the airlines in different slots. Note that solution (i) and (ii) occur always when the airport is busy (destinations are more than peak slots, \( N > M \)). In this case, the relative scarcity of peak slots implies that allocating one slot in one market always subtracts it to another market. Then the
best strategy for the airport is to compare the configurations of two markets: “peak/peak+off-peak/off-peak” against “peak/off-peak”. Since $Q_1 > Q_2 + Q_3$, the airport favors the peak/off-peak configuration. When the airport is not busy ($M > N$) and configuration peak/off-peak yields a higher profit than configuration peak/peak ($\phi < \phi_2$), then solution (i) occurs. In this case, the airport under-uses the peak slot capacity, leading to allocation inefficiency. Indeed the number of peak slots exploited is $N$, whereas $M - N$ slots would be left unused.

In solution (iii), the airport is not busy ($M > N$) and configuration peak/peak yields a higher profit than configuration peak/off-peak ($\phi > \phi_2$). Given the relative abundance of peak slots, allocating one slot in one market not always subtracts it to another market, thus for some markets the best strategy is to compare single market configurations. The trade-off between the constraint due to the number of peak slots (for which the condition $Q_1 > Q_2 + Q_3$ matters) and the fact that $2Q_2 > Q_1$ in the single market results in a combination of peak/peak ($M - N$) and peak/off-peak ($2N - M$) configurations in equilibrium. The foregoing discussion can be summarized in the following proposition.

**Proposition 1** Suppose all markets are served by duopoly airlines, and the airport is private. For $M \leq N$, the airport uses all available peak slots, and favors “peak/off-peak” configuration. For $M > N$ and

- $\phi \in (0, \phi_2]$, the airport does not use all available peak slots (inefficiency), and the configuration is “peak/off-peak” in each market;
- $\phi \in [\phi_2, \phi_1)$, the airport uses all available peak slots, and implements $(M - N)$ “peak/peak” over $(2N - M)$ “peak/off-peak” configuration.

Figure 3.a describes the equilibria in the cases of private airport with duopoly markets. The numbers in brackets refer to the notation adopted in Appendix A. Proposition 1 highlights an important issue. When peak slots are not scarce relative to the number of markets ($M > N$), a private airport would leave a number of peak slots unused when the pre-determined fee is small, thereby resulting in allocative inefficiencies. In reality, such behavior may be expressed by misreporting true airport handling capacity. This is in line with De Wit and Burghouwt (2008), who find that efficient slot use can be affected by capping available slots through capacity declaration. To obtain some intuition as to why
airport has a propensity to leave some peak slots unused rather than using all, recall in Figure 2 the ranking of $Q_1$ and $Q_2$ is driven by two effects in a market. More specifically, effect 1 outweighs effect 2 when $\phi \in (0, \phi_2]$. In this case the airport favors peak/off-peak and thus determining allocation inefficiency. When $\phi \in [\phi_2, \phi_1)$, the airport favors peak/peak and as a consequence no peak slots would be optimally left unused. On the contrary, in a very busy airport where available peak slots are scarce relative to total demand ($M \leq N$), it is optimal to allocate all available peak slots.

**Positive airline operating costs** For the sake of completeness, we end the section by discussing the case where airlines have identical operating costs $c > 0$. The analysis can be developed in a similar vein as before, where the airline marginal cost is now $c + \phi$ rather than $\phi$ only. Naturally, in both configurations the volume of passengers is larger without operating cost. The conclusion drawn from the comparison under peak-off peak configuration also applies here. It follows that, with positive operating costs airport profit is also smaller in each configuration. The condition required to guarantee positive passenger
volumes in equilibrium is
\[ 0 < \phi < \phi'_1 \equiv \frac{s_h s_t}{2s_h - s_t} - c, \]
while the threshold determining the preference between peak/peak and peak/off-peak is
\[ \phi'_2 \equiv \frac{s_h s_t}{6s_h - 2s_t} - c. \]
Therefore, the above proposition now reads with \( \phi'_1 \) and \( \phi'_2 \) substituting for \( \phi_1 \) and \( \phi_2 \). If the cost \( c \) is small enough so that \( \phi'_2 > 0 \), the proposition presents the same configurations and the same issue of allocation inefficiency. The configuration peak-off/peak induces more passenger volume than configuration peak/peak so that the airport does not distribute all available peak slots and inefficiency arises. However, if \( c \) is large enough so that \( \phi'_2 \) becomes negative, all available peak slots are distributed and allocation inefficiency never arises.

4 Airline monopolies

Having examined competition between duopolists in each market, we shall now investigate the case in which each market is served by a single airline that acts as a monopolist operating two flights in the same destination markets. As with the baseline model, we analyze the second stage in each possible configuration separately, while the analysis of the third stage remains unchanged. Beginning with the peak/off-peak configuration, the airline profit is:

\[ \pi^M = (s_h - s_t q_{lt}^M - s_h q_{lh}^M - \phi) q_{lh}^M + [s_t (1 - q_{lt}^M - q_{hl}^M) - \phi] q_{hl}^M, \]

where the superscript \( M \) stands for “monopoly”. The first-order conditions are:

\[ \frac{\partial \pi^M}{\partial q_{lt}^M} = (1 - 2q_{lt}^M - 2q_{hl}^M) s_t - q_{hl}^M s_h - \phi = 0, \]  
\[ \frac{\partial \pi^M}{\partial q_{hl}^M} = s_h - q_{hl}^M (s_h + 2s_t) - \phi = 0. \]
Solving the first-order conditions for passenger volumes, one obtains:

\[ q_{lh}^M = \frac{s_h - \phi}{s_h + 2s_l}, \]  
(29)

\[ q_{hl}^M = s_l \frac{(2s_l - s_h) - s_h \phi}{(2s_l + s_h)^2}. \]  
(30)

The condition \( 0 < \phi < \phi^M \equiv \frac{s_l(2s_l - s_h)}{s_h} \) is sufficient to ensure interior solutions, implying also \( 2s_l > s_h \). In contrast to the duopoly case, now \( q_{lh}^M > q_{hl}^M \) for all \( 0 < \phi < \phi^M \). Next, compare the monopoly outcome (29) and (30) and the duopoly outcome (13) and (14).

\[ q_{lh}^M - q_{hl}^M < 0 \quad \text{and} \quad q_{lh}^M - q_{hl}^M > 0, \]

for \( 0 < \phi < \phi^M \) (see computation in Appendix B). Comparing total passenger volume in two cases shows that:

\[ q_{hl}^M + q_{lh}^M - \left( q_{hl}^l + q_{lh}^l \right) < 0, \]

for \( 0 < \phi < \phi^M \). As expected, the total number of seats provided by the monopolist is smaller than the duopolists. Replacing \( q_{lh}^l \) and \( q_{hl}^l \) in (8) with (29) and (30) yields:

\[ p_{lh}^M = \frac{s_l [2\phi (s_h + s_l) + s_l (3s_h + 2s_l)]}{(s_h + 2s_l)^2}, \]  
(31)

\[ p_{hl}^M = \frac{s_l^2 (\phi + s_h) + s_h s_l (4s_h + \phi) + 2\phi s_l^2}{(s_h + 2s_l)^2}. \]  
(32)

Comparing prices with the duopoly case, one gets:

\[ p_{lh}^M - p_{lh}^l = \frac{s_h s_l^2 (s_l - 2\phi - 2s_l^3 (\phi + s_l) + 8s_h^2 s_l^3 - s_h^3 (2\phi + s_l))}{(4s_h - s_l) (s_h + 2s_l)^2} > 0, \]

\[ p_{hl}^M - p_{hl}^l = \frac{\phi s_h^3 - 8\phi s_h^3 s_l - 2s_h s_l^2 + \phi s_h s_l^2 + 2s_h^4 + 8s_h^2 s_l - 8s_h^2 s_l^2 + 4s_h s_l^3}{(4s_h - s_l)(s_h + 2s_l)^2} > 0. \]

Of course, prices are higher in monopoly than in duopoly. These preliminary results can be summarized in the following lemma.
**Lemma 1** In a market of peak/off-peak slots, the airline monopolist (i) serves more passengers in the off-peak period than in the peak period, and (ii) provides less peak period seats, more off-peak period seats, and less total seats than in the duopoly case. The price of travelling is higher under monopoly at any time.

Consider next the numbers of seat provided in markets (ii) and (iii) where a monopoly airline obtains identical slots. In this case, we denote the slot type \((h\ or\ l)\) the monopolist obtains by \(i\), it follows that the monopolist’s profit is given by:

\[ \pi_i^M = [(1 - q_i^M) s_i - \phi] q_i^M, \quad \text{with } i \in \{h, l\}. \]

Taking the first-order condition for number of seats we get the equilibrium number of seats the monopoly would provide:

\[ q_i^M = \frac{s_i - \phi}{2s_i}, \quad q_h^M = \frac{s_h - \phi}{2s_h}. \] (33)

To ensure interior solutions, suppose condition \(0 < \phi < s_l\) is satisfied. It can be verified that \(\phi^M < s_l\), hence \(\phi < \phi^M\) is sufficient condition for interior solutions in all configurations. Plugging (33) into (18) and (10) yields:

\[ p_l^M = \frac{s_l + \phi}{2}, \quad p_h^M = \frac{s_h + \phi}{2}. \] (34)

We can compare the outcomes of different configurations of a single market. From (29), (30) and (33), one obtains:

\[ q_{hl}^M + q_{lh}^M - q_h^M < 0, \quad q_{hl}^M + q_{lh}^M - q_l^M < 0, \quad \text{and} \quad q_h^M - q_l^M > 0 \]

for all \(2s_l > s_h > s_l > 0\) and \(0 < \phi < \phi^M\) (see computation in Appendix B). Taken together, a ranking of configuration is straightforward in terms of passenger volume: peak/peak is superior to off-peak/off-peak, which is better than peak/off-peak.

The airport has the same problem as (23) with the new values for \(q_{i,i'}^M\). The solution is given by (see Appendix A) \(n_1 = 0, n_2 = \min\{M/2, N\}, n_3 = N - n_2\) - case 6. Therefore, the airport concentrates the competitors in the same slots and extensively uses the peak slot capacity.
Proposition 2 Suppose all markets are served by monopoly airlines. Then a private airport uses all available peak slots for peak/peak market configurations.

Figure 3.b describes the equilibria in the case of private airport with monopoly markets. The equilibrium type is the same in the entire parameter range. Here it is worth stressing that, unlike the duopoly airline case, the allocation of slots is not influenced by whether the number of slots is relatively abundant or not compared to the number of markets. Specifically, since off-peak/off-peak dominates peak/off-peak, the airport would either allocate two peak slots to a market, or no peak slots to a market at all.

5 Public airport

We now investigate the case of a public, welfare-maximizing airport. This allows us to obtain some insights on how the airport’s ownership influences slot allocation. In this regard, social welfare $W$ is represented by the sum of airport’s profits $\Pi$, passenger surplus (denoted as $CS$) and airlines’ profits:

$$W = \Pi + CS + n_1 (\pi_{hl} + \pi_{lh}) + 2 (n_2 \pi_{hh} + n_3 \pi_{ll}) .$$

Consider an airline duopoly in each market. Since airport and airlines operating costs are normalized to zero, then airport profits come from total per-passenger fees, whereas airline profits are the ticket income less total per-passenger fees paid to the airport. In turn, passenger surplus is represented by the total gross utility generated from flying minus all ticket payments. Since monetary transfers between airlines and airport cancel out, and so do transfers between passengers and airlines, then social welfare equals the sum of passengers’ gross utility in all $2N$ markets. Thus $W$ can be rewritten as:

$$W = n_1 \left( \int_{\underline{v}}^{\bar{v}} vs_ldv + \int_{\underline{v}}^{1} vs_hdv \right) + n_2 \int_{\bar{v}}^{1} vs_hdv + n_3 \int_{\bar{v}}^{1} vs_l dv ,$$

(35)

where

$$\underline{v} = \frac{p_{fh}}{s_l}, \bar{v} = \frac{p_{hl} - p_{fh}}{s_h - s_l} - \frac{p_{lh}}{s_l}, \bar{v}' = \frac{p_{hh}}{s_h}, \bar{v}'' = \frac{p_{ll}}{s_l} .$$

(36)
The analysis of the second and third stage remains the same as in the baseline model. For notational simplicity we define $B_{hl} = \int_0^1 vs_h dv + \int_0^1 vs_l dv$, $B_{hh} = \int_0^1 vs_h dv$ and $B_{ll} = \int_0^1 vs_l dv$, so that (35) becomes

$$W = n_1B_{hl} + n_2B_{hh} + n_3B_{ll},$$

We will consider first the case with duopoly airlines and then the case with monopoly airlines.

### 5.1 Airline duopolies

Putting (15), (16) and (20) together with (36) yields:

$$v = \frac{s_h(s_l + 2\phi)}{s_l(4s_h - s_l)}, \quad \bar{v} = \frac{2s_h + \phi}{4s_h - s_l}, \quad \bar{v}' = \frac{s_h - 2\phi}{3s_h}, \quad v' = \frac{s_l - 2\phi}{3s_l}. \quad (37)$$

Assuming $\phi < \phi^P = \frac{s_l}{2}$ ensures that the market is partially covered. Substituting (37) into (35) and solving the integrals yields:

$$B_{hl} = \frac{4\phi s_h s_l(s_l - 2s_h) + s_h s_l(12s_i^2 - 5s_h s_l + s_l^2) - \phi^2(4s_h^2 + s_h s_l - s_i^2)}{2s_l(4s_h - s_l)^2},$$

and

$$B_{ii} = \frac{2(\phi + s_i)(2s_i - \phi)}{9s_i}, \quad i \in \{h, l\}.$$

By comparing the number of seats obtained in each configuration, we get (see Appendix B) $B_{hh} > B_{hl} > B_{ll} > 0$ and $(B_{hh} + B_{ll})/2 < B_{hl}$. Therefore we now arrive at a situation similar to the private airport case with $\phi \in [\phi_2, \phi_1)$. The following proposition summarizes the features of the equilibrium.

**Proposition 3** Suppose all markets are served by duopoly airlines, and the airport is public. For $M \leq N$, the airport uses all available peak slots, and favors “peak/off-peak” configuration. For $M > N$ the airport implements $(M - N)$ “peak/peak” and $(2N - M)$ “peak/off-peak” configurations.

The results are qualitatively similar to the case with private airport for $\phi_2 < \phi < \phi_1$. However, now the public airport would use all available peak slots in any case, hence, inefficiency does not emerge when the airport is public.
5.2 Airline monopolies

We now turn to the interplay between a public airport and airline monopolies. Putting (31), (32) and (34) together with (36) yields:

\[ v = \frac{s_h(5s_l + \phi) + 2s_l^2 + s_h^2}{(s_h + 2s_l)^2}, \bar{v} = \frac{s_h^2 + s_h(5s_l + \phi) + 2s_l^2}{(s_h + 2s_l)^2}, \bar{v}' = \frac{s_h + \phi}{2s_h}, \bar{v}' = \frac{s_l + \phi}{2s_l}. \]

Substituting (38) into (35) and solving the integrals yields:

\[ B_{hl+lh} = \frac{s_h s_l (20s_l^3 + 5s_h^2 s_l + 32s_h s_l^2 - s_h^3) - 2s_l (s_h^4 + 4s_h^3 s_l + 3s_h^2 s_l^2 + 8s_h s_l^3 + 4s_l^4)}{2(s_h + 2s_l)^4} - \frac{s_l (3s_l + \phi)}{8s_l}, \]

and

\[ B_{ii} = \frac{(s_i - \phi)(3s_i + \phi)}{8s_i}, i \in \{h, l\}. \]

Comparing the number of seats obtained in each configuration, we get (see Appendix B) \( B_{hh} > \max\{B_{ll}, B_{hl+lh}\} \). Now we compare \( B_{hl+lh} \) and \( B_{ll} \). We show that \( B_{hl+lh} - B_{ll} > 0 \) for \( s_l < s_h < 1.5s_l \) and \( \phi < \phi_p^M \); or \( 1.5s_l < s_h < 2s_l \) and \( 0 < \phi < \phi_p^M \), where

\[ \phi_p^M \equiv \frac{s_l (7s_l^3 + 18s_h^2 s_l - 20s_h s_l^2 - 24s_l^3)}{s_h (s_h^2 + 4s_h s_l + 12s_l^2)}. \]

As before, we then evaluate the differences in the allocation of two peak slots. In this case (see Appendix B) \( B_{hh}^M + B_{ll}^M > 2B_{hl+lh}^M \). We get the following solution (Appendix A): \( n_1 = 0, n_2 = \min\{M/2, N\}, n_3 = N - n_2 \). - case 6 (\( \phi_{PM} > \phi > \phi_1^P \)) and case 2 (for \( \phi < \phi_{PM} \)), yielding

**Proposition 4** Suppose all markets are served by monopoly airlines. Then a public airport uses all available peak slots for peak/peak market configurations.

Figure 4.b describes the equilibria in the case of public airport with monopoly markets. The results are qualitatively similar to the case with private airport and airline monopolies. The allocation of slots is not affected by the relationship
between the number of destinations and the number of peak slots, the airport would either allocate two peak slots to a market, or no peak slots to a market at all.

6 Extensions

6.1 Airport-Airline Collusion

This section investigates the consequences of collusion between the airport and one airline. Since airports and airlines are vertically connected, the targets of both on passenger volumes are coincident, despite the divergent interests in per-passenger fees. Hence it is not surprising that an airport establishes vertical agreements similar to vertical mergers with one or multiple airlines that are not in a same city pair. This implies that the airport and the airline in the agreement maximize joint profits.

The third stage does not change compared to the baseline case. In the second stage, airlines set their optimal supply of seats. Note that, when a peak/off-peak configuration emerges, the airport might collude with the airline obtaining
either the peak or the off-peak slot. We analyze each possible configuration separately.

**Peak/off-peak market with peak slot airline colluding**

Consider a market where pairwise flights obtain different slots and the peak slot airline colludes with the airport. Denote the joint profits of airport-airline as $\Pi_p : \Pi_p \equiv \Pi + \pi_{hl}^f$. Then according to (1), profits are expressed by:

\[
\begin{align*}
\pi_{hl}^f &= s_t (1 - q_{lh}^f - q_{hl}^f) - \phi q_{lh}^f, \\
\Pi_p &= (s_h - s_l q_{lh}^f - s_h q_{hl}^f) q_{hl}^f + \phi (q_{lh}^f + q_{hl}^f).
\end{align*}
\]

The former expression shows the profit of the non colluding airline, while the later shows the joint profit of the peak slot airline and the airport.

We now characterize the airlines’ profit-maximizing numbers of seats for any given $\phi$. The procedure is the same as in the baseline case. The airlines choose $q_{lh}^f$ and $q_{hl}^f$ to maximize their profits, and the first-order conditions are:

\[
\begin{align*}
\frac{\partial \pi_{lh}^f}{\partial q_{lh}^f} &= -\phi + (1 - q_{lh}^f - q_{hl}^f) s_t - q_{lh}^f s_t = 0, \\
\frac{\partial \Pi_p}{\partial q_{hl}^f} &= s_h - 2q_{hl}^f s_h - q_{lh}^f s_t = 0.
\end{align*}
\]

Solving (39) and (40) simultaneously with respect to $q_{lh}^f$ and $q_{hl}^f$ yields:

\[
\begin{align*}
q_{lh}^f &= \frac{s_h (s_t - 2\phi)}{s_t (4s_h - s_t)}, \\
q_{hl}^f &= \frac{2s_h + \phi - s_t}{4s_h - s_t}.
\end{align*}
\]

Condition $0 < \phi < \frac{s_t}{2}$ ensures interior solutions. Note that $q_{lh}^f < q_{hl}^f$ for all $\phi > 0$. In this case the peak slot airline serves more passengers in equilibrium than its off-peak counterpart. We compare next $q_{lh}^f$ and $q_{hl}^f$ in the case of collusion with those in the basic model (13) and (14). The colluding airline:

\[
\frac{2s_h + \phi - s_t}{4s_h - s_t} - \frac{\phi - 2s_h + s_t}{s_l - 4s_h} = \frac{2\phi}{4s_h - s_t} > 0,
\]
while its counterpart:

\[
\frac{s_h (s_l - 2\phi)}{s_l (4s_h - s_l)} - \frac{s_h s_l - \phi (2s_h - s_l)}{(4s_h - s_l)s_l} = \frac{\phi}{s_l - 4s_h} < 0.
\]

It’s hardly surprising to see that the colluding airline serves more passengers than it would do if there were no collusion, and its non-colluding counterpart serves fewer passengers than it would do if there were no collusion.

Comparing profits yields

\[
\Pi_p - \pi_{lh} = \frac{s_h [(s_h - s_l) s_l + 3\phi(s_l - \phi)]}{(4s_h - s_l)s_l} > 0,
\]

for \(0 < \phi < \frac{s_l}{2}\), implying that the joint profit of colluding airport-airline is larger than that of the non-colluding airline.

**Peak/off-peak market with off-peak slot airline colluding**

We turn now to the case where pairwise flights still obtain different slots, with the off-peak slot airline now colludes with the airport. Denote the joint profit of airport-airline as \(\Pi_o : \Pi_o \equiv \Pi + \pi_{lh}^f\). Using this notation, from (1) it follows that airlines’ profits are:

\[
(i) \quad \begin{cases}
\Pi_o = s_l(1 - q_{lh}^f - q_{hl}^f) - \phi q_{lh}^f + \phi (q_{lh}^f + q_{hl}^f) \\
\pi_{hl}^f = (s_h - s_l q_{lh}^f - s_h q_{hl}^f - \phi) q_{hl}^f
\end{cases}
\]

Given \(\phi\), airlines choose the number of seats \(q_{lh}^f\) and \(q_{hl}^f\) to maximize profits. The first-order conditions are:

\[
\frac{\partial \Pi_o}{\partial q_{lh}^f} = (1 - q_{lh}^f - q_{hl}^f) s_l - q_{lh}^f s_l = 0,
\]

\[
\frac{\partial \pi_{hl}^f}{\partial q_{hl}^f} = -\phi + s_h - 2q_{hl}^f s_h - q_{lh}^f s_l = 0.
\]

The numbers of passengers in equilibrium are:

\[
q_{lh}^f = \frac{s_h + \phi}{4s_h - s_l},
\]

\[
q_{hl}^f = \frac{2(s_h - \phi) - s_l}{4s_h - s_l}.
\]
As before, we compare \( q^f_{lh} \) and \( q^f_{hl} \) with the numbers of passengers in the case without collusion. For the colluding airline:

\[
\frac{s_h + \phi}{4s_h - s_l} - \frac{s_h s_l - \phi \left(2s_h - s_l\right)}{(4s_h - s_l)s_l} = \frac{2\phi s_h}{(4s_h - s_l)s_l} > 0,
\]

and for its counterpart:

\[
\frac{2 \left(s_h - \phi\right) - s_l}{4s_h - s_l} - \frac{\phi - 2s_h + s_l}{s_l - 4s_h} = \frac{\phi}{s_l - 4s_h} < 0.
\]

Hence, the colluding airline now serves more passengers than the previous situation where collusion is absent, and its non-colluding counterpart serves fewer passengers than before.

One can show that \( \Pi_o > \pi^f_{hl} \) for either \( s_h > 2.5s_l \), and \( 0 < \phi < \frac{s_l}{2} \) or \( s_h \leq 2.5s_l \) and \( \phi < \frac{5s_h - s_l}{3} \). Conversely, \( \Pi_o < \pi^f_{hl} \) for \( s_h < 2.5s_l \) and \( \frac{5s_h - s_l}{3} < \phi < \frac{s_l}{2} \). Therefore, the colluding off-peak airline does not necessarily yield higher profits than its non-colluding counterpart.

**Identical slots market with one airline colluding**

Consider next a market where both airlines obtain slots type \((i, i') \in \{(h, h), (l, l)\}\). Define the joint profit of airport-airline in a market with slot type \(i\) as \( \Pi_i : \Pi_i \equiv \Pi + \pi^f_{i'i} \). From (2), it follows that airlines profits are:

\[
\begin{align*}
\Pi_i &= \left[\left(1 - q^f_{ii'v} - q^f_{i'iv}\right)s_i - \phi\right] q^f_{i'i'v} + \phi \left(q^f_{ii'v} - q^f_{i'iv}\right), \\
\pi^f_{i'i} &= \left[\left(1 - q^f_{ii'v} - q^f_{i'iv}\right)s_{i'i} - \phi\right] q^f_{i'iv},
\end{align*}
\]

with \((i, i') \in \{(h, h), (l, l)\}\). The first-order conditions of \( \Pi_i \) and \( \pi^f_{i'i} \) with respect to \( q^f_{ii'} \) and \( q^f_{i'iv} \) respectively, yields:

\[
\begin{align*}
-q^f_{ii'} s_i + s_i \left(1 - q^f_{ii'} - q^f_{i'iv}\right) &= 0, \\
-\phi - q^f_{i'iv} s_i + s_i \left(1 - q^f_{i'iv} - q^f_{i'iv}\right) &= 0.
\end{align*}
\]
Solving the two equations for $q_{ii}$ and $q_{ii}'$, we obtain:

$$q_{ii} = \frac{s_i + \phi}{3s_i},$$
$$q_{ii}' = \frac{s_i - 2\phi}{3s_i}.$$

Note that $\Pi_i - \pi_{ii}' = \frac{\phi(s_i - \phi)}{s_i} > 0$, that is, the colluding airline has a higher profit than its non-colluding counterpart.

We are now in a position to compare joint colluding profits in these four configurations: $\Pi_p, \Pi_o, \Pi_h$ and $\Pi_l$. One can check that (i) $\Pi_p - \Pi_o > 0$, (ii) $\Pi_p - \Pi_l > 0$, (iii) $\Pi_p - \Pi_h > 0$, implying that $\Pi_1$ dominates all other three joint profits. The discussion is summarized in the next proposition.

**Proposition 5** Suppose all markets are served by duopoly airlines, and the airport is private. Also, suppose that the airport collude with one airline in a market. Then the optimal airport allocation is peak/off-peak, with the colluding airline being assigned to a peak slot.

### 6.2 Endogenous Per-Passenger Fee

In the above analysis, the per-passenger fee $\phi$ was taken as given by the airport, reflecting the fact that an airport is generally subject to government regulation. We now relax this assumption by allowing the airport to choose $\phi$. In particular, the timing of the game is now as follows. There are four stages of the game. In the first and second stage, the airport sets $\phi$ and allocates slots so as to maximize its profits, respectively. The last two stages of the game do not change compared to the previous analysis: in the third stage, airlines compete in quantities, in the fourth stage, passengers buy (or not) one ticket for their destinations. In what follows we focus on introducing endogenous per-passenger fees in the baseline case. For the sake of completeness, the other cases (private airport and airline monopolies, public airport and airline duopolies/monopolies) with the same procedure can be found in the appendix.

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8 Similarly, the airport can perform the two actions simultaneously. Indeed, in both cases the number of destination markets in equilibrium $n_1, n_2$ and $n_3$ are functions of the per-passenger fee, so that no issues of time inconsistency emerge.
Begin by noting that the airport’s allocation choice does not change compared to the baseline case, so that Proposition 1 still holds. For each possible allocation choice emerging in Proposition 1, the airport (i) maximizes its objective function with respect to per-passenger fees \( \phi \) and (ii) verifies if the optimal level of per passenger fees is consistent with the allocation choice. Let us consider the three cases of Proposition 1:

\[
\Pi(\phi) = \begin{cases} 
\Pi^a(\phi) & \text{for } (\phi, N) \in [0, \phi_2] \times R^+ \\
\Pi^b(\phi) & \text{for } (\phi, N) \in [\phi_2, \phi_1] \times [M, \infty] \\
\Pi^c(\phi) & \text{for } (\phi, N) \in [\phi_2, \phi_1] \times [0, M]
\end{cases}
\]

where

\[
\Pi^a(\phi) = \left[ \min\{M, N\} \left( q''_{th} + q''_{hl} \right) + (N - \min\{M, N\}) \left( q''_{lt} + q''_{lt} \right) \right] \phi,
\]

\[
\Pi^b(\phi) = M \left( q''_{th} + q''_{hl} \right) + (M - N) \left( q''_{lt} + q''_{lt} \right),
\]

\[
\Pi^c(\phi) = (2N - M) \left( q''_{th} + q''_{hl} \right) + (M - N) \left( q''_{hh} + q''_{hh} \right).
\]

In the first case, there are two possible situations, according to whether \( N > M \) or \( N < M \). For \( N > M \), the airport’s problem is:

\[
\max_{\phi} \Pi_a(\phi) = \max_{\phi} \left[ M \frac{s_h (3s_l - 2\phi) - s_l^2}{s_l (4s_h - s_l)} + (N - M) \frac{2(s_l - \phi)}{3s_l} \right] \phi. \quad (41)
\]

The first order condition with respect to \( \phi \) is:

\[
\frac{\partial \Pi_a(\phi)}{\partial \phi} = s_l \left[ s_l (M + 2N) + 4\phi (M - N) \right] - s_h \left[ s_l (M + 8N) + 4\phi (M - 4N) \right] = 0,
\]

while the second order condition is

\[
\frac{\partial^2 \Pi_a(\phi)}{\partial \phi^2} = 4 \left[ s_h (M - 4N) + s_l (N - M) \right] > 0 \quad \text{for } N > M.
\]

Solving \((42)\) for \( \phi \) yields

\[
\phi_{a1}^* = \frac{s_l \left[ s_l (M + 2N) - s_h (M + 8N) \right]}{4 \left[ s_h (M - 4N) + s_l (N - M) \right]}.
\]

The next step is to verify if the optimal per-passenger fee is consistent with the
allocation considered. In other words, we need to check whether \(0 < \phi^{**} < \phi_2\),
this being the necessary condition in order to obtain this equilibrium allocation.
We get
\[
\phi^{**}_{a1} - \phi_2 = \]
\[
- \frac{s_l [s_h^2(5M + 16N) - 6s_h s_t(M + 2N) + s_t^2(M + 2N)]}{4 (3s_h - s_t) (s_h(M - 4N) + s_t(N - M))} > 0, \quad \text{for } N > M. \quad (44)
\]
Since \(\phi^{**}_{a1} > \phi_2\), the optimal per-passenger fee is out of the parameters’ range
allowing this slot allocation. Hence in the range \([0, \phi_2]\), the best choice for the
airport is to set \(\phi^{**} = \phi_2\).

For \(M > N\), the airport’s problem in the first case is
\[
\max_{\phi} \Pi_a(\phi) = \max_{\phi} N \frac{s_h (3s_l - 2\phi) - s_t^2}{s_t (4s_h - s_l)} \phi.
\]
The first order condition with respect to \(\phi\) is:
\[
\frac{\partial \Pi_a(\phi)}{\partial \phi} = - \frac{N [s_h (4\phi - 3s_l) + s_t^2]}{s_t (4s_h - s_l)} = 0, \quad (45)
\]
while the second order condition is
\[
\frac{\partial^2 \Pi_a(\phi)}{\partial \phi^2} = - \frac{4Ns_h}{4s_h s_t - s_t^2} < 0.
\]
Solving (45) for \(\phi\) yields
\[
\phi^{**}_{a2} = - \frac{s_l (s_t - 3s_h)}{4s_h}.
\]
Again, we verify if the optimal per-passenger fee is consistent with the allocation
considered. We get
\[
\phi^{**}_{a2} - \phi_2 = \frac{s_l (7s_h^2 - 6s_h s_t + s_t^2)}{4s_h (3s_h - s_t)} > 0.
\]
As before, the optimal per-passenger fee is out of the parameters’ range allowing
this slot allocation. The ongoing discussion can be summarized as follows.

**Proposition 6** Suppose all markets are served by duopoly airlines, and the
airport is private. Then the airport never sets the per-passenger fees below $\phi_2$.

Therefore, the level of per-passenger fees set by a private airport is never too low (never lower than $\phi_2$).

We now turn on the second case, that requires $\phi_2 < \phi < \phi_1$ and $N > M$. Again, the airport problem and its solution are (41) and (43), respectively. Also, equation (44) ensures that the optimal choice of $\phi$ is consistent with the optimal allocation in the second stage. To complete the analysis, we have an interior solution in the market (third) stage for

$$\phi^{\alpha} - \phi_1 = -\frac{s_l}{4} \left[ \frac{6M s_h^2 - s_h s_l (7M + 8N) + s_l^2 (M + 2N)}{2s_h - s_l} \right] < 0,$$

for

$$M < M_b \equiv \frac{8ns_h s_l - 2ns_l^2}{-7s_h s_l + 6s_h^2 + s_l^2} \text{ and } s_l < s_b \equiv \frac{1}{2} \left( 5 - \sqrt{17} \right) s_h,$$

or

$$s_l > s_b.$$

As in the rest of the paper, we limit the analysis to the range in which interior solutions emerge in the market stage. These findings show

**Proposition 7** Suppose all markets are served by duopoly airlines, and the airport is private. For $N > M_b > M$ and $s_l < s_b$ or $N > M$ and $s_l > s_b$, then the airport favors “peak/off-peak” configuration and sets $\phi^* = \phi_1^*$.

We finally examine the third case, under which $\phi_2 < \phi < \phi_1$ and $N < M$. the airport’s problem is now:

$$\max_{\phi} \Pi_c(\phi) = \max_{\phi} \left[ (2N - M) \frac{s_h (3s_l - 2\phi) - s_l^2}{s_l (4s_h - s_l)} + (N - M) \frac{2(s_h - \phi)}{3s_h} \right] \phi.$$

The first order condition with respect to $\phi$ is:

$$\frac{\partial \Pi_c(\phi)}{\partial \phi} = 0,$$

$$\frac{s_h^2 [12\phi(M - 2N) - s_l(M - 10N)] + s_h s_l [s_l(M - 4N) + 16\phi(N - M)] + 4\phi s_l^2 (M - N)}{3s_h s_l (4s_h - s_l)} = 0,$$

30
while the second order condition is

\[
\frac{\partial^2 \Pi_c(\phi)}{\partial \phi^2} = \frac{4 \left[ 4s_h s_l(N-M) + 3s_h^2(M-2N) + s_l^2(M-N) \right]}{3s_h s_l(4s_h - s_l)} < 0,
\]

for

\[
M < M_c \equiv \frac{N \left( 6s_h^2 - 4s_h s_l + s_l^2 \right)}{3s_h^2 - 4s_h s_l + s_l^2}.
\]

Solving (46) for \( \phi \) yields

\[
\phi^*_c = \frac{s_h s_l \left( s_h(M-10N) - s_l(M-4N) \right)}{4 \left[ 4s_h s_l(N-M) + 3s_h^2(M-2N) + s_l^2(M-N) \right]}.
\]

As before, we verify if the optimal per-passenger fee is consistent with the allocation considered. We get

\[
\phi^*_c - \phi_2 =
\]

\[
- \frac{s_h s_l \left[ 3s_h^2(M+6N) - 2s_h s_l(2M+7N) + s_l^2(M+2N) \right]}{4 (3s_h - s_l) \left[ 4s_h s_l(N-M) + 3s_h^2(M-2N) + s_l^2(M-N) \right]} > 0, \text{ for } M < M_c.
\]

(47)

Therefore, for \( M < M_c \), then optimal allocation and optimal per-passenger fees are consistent. When \( M > M_c \), \( \phi^*_c \) is (i) out of the parameter range \([\phi_2, \phi_1]\), and (ii) is a minimum. Thus the best choice for the airport is to set \( \phi^* = \phi_2 \).

These results can be summarized as follows.

**Proposition 8** Suppose all markets are served by duopoly airlines, and the airport is private. For \( M > N \), the airport implements \((M-N) \) “peak/peak” over \((2N-M) \) “peak/off-peak” configuration. Moreover, for \( M_c > M > N \), the airport sets \( \phi^* = \phi^*_c \). For \( M > M_c > N \), it sets \( \phi^* = \phi_2 \).

### 6.3 Heterogeneous Density

In this section, we assume heterogeneous density across markets. For simplicity we study an economy with only two markets that exhibit different densities: the small market 1 has a lower density \( \delta_1 \), the large market 2 has a higher density \( \delta_2 > \delta_1 \). We discuss three examples: the airport allocates either, one, two or
three peak slots in the two markets.

To begin with, we first look at the case where peak slots are highly scarce, where one peak slot is available to four slot demands, $M = 1$, $N = 2$. Since, for every market, off-peak/off-peak is strictly dominated by peak/peak and peak/off-peak (see section 3.3), there are two ways to allocate the peak slot: (1) one of the two airlines from the small market, and (2) one of the two airlines from the large market.

The demand functions for each airline in the two possible configurations are:

1. **Configuration 1.** Market 1:

   $$q_{lh}^{1f} = \delta_1 \left( \frac{p_{hl}^{1f} - p_{lh}^{1f}}{s_h - s_l} - \frac{p_{hl}^{1f}}{s_l} \right), \quad q_{hl}^{1f'} = \delta_1 \left( 1 - \frac{p_{hl}^{1f} - p_{lh}^{1f}}{s_h - s_l} \right);$$

market 2:

   $$q_{ll}^{2f} + q_{ll}^{2f'} = \delta_2 \left( 1 - \frac{p_{ll}^{2f}}{s_l} \right).$$

2. **Configuration 2.** Market 1:

   $$q_{ll}^{1f} + q_{ll}^{1f'} = \delta_1 \left( 1 - \frac{p_{ll}^{1f}}{s_l} \right);$$

market 2:

   $$q_{lh}^{2f} = \delta_2 \left( \frac{p_{hl}^{2f'} - p_{lh}^{2f}}{s_h - s_l} - \frac{p_{lh}^{2f}}{s_l} \right), \quad q_{hl}^{2f'} = \delta_2 \left( 1 - \frac{p_{hl}^{2f'} - p_{lh}^{2f}}{s_h - s_l} \right).$$

The number in the superscript denotes the market considered. In the similar manner with (13), (14) and (19), we could derive the optimal passenger volumes served by each airline and consequently each market in equilibrium. A comparison of the equilibrium passenger volumes in the two configurations yields:

$$q_{lh}^{1f} + q_{hl}^{1f'} + q_{ll}^{2f} + q_{ll}^{2f'} - \left( q_{ll}^{1f} + q_{ll}^{1f'} + q_{lh}^{2f} + q_{hl}^{2f'} \right)$$

$$= (\delta_1 - \delta_2) \left[ s_l (9 + s_h + 2s_l) + 2\phi (s_h - s_l) \right] < 0.$$  

implying configuration 2 yields a higher number of passenger than configuration 1. The ranking in this simple framework suggests
Proposition 9 Suppose an economy with two duopolies with different density levels, a private airport and a single peak slot. Then the airport allocates the peak slot to one of the airlines in the large market, and inefficiency would not arise.

We investigate next the case where peak slots have a moderate scarcity at the airport. In this setting there are two peak slots available for two markets, \( M = N = 2 \). It is straightforward to see that allocation “two peak slots to market 2” dominates allocation “two peak slots to market 1”. Indeed, the large market has a bigger multiplier for density \( \delta_2 > \delta_1 \). This, together to the fact that off-peak/off-peak is strictly dominated by peak/peak and peak/off-peak, implies that there are two possible allocations: (1) two peak slots to the large market, and (2) one peak slot to each market. The demand functions for each airline in these two configurations are:

1. Configuration 1. Market 1:

\[
q_{1l}^{1f} + q_{1l}^{1f'} = \delta_1 \left( 1 - \frac{p_{1l}}{s_l} \right);
\]

market 2:

\[
q_{1h}^{2f} + q_{1h}^{2f'} = \delta_2 \left( 1 - \frac{p_{2h}}{s_h} \right).
\]

2. Configuration 2. Market 1:

\[
q_{1h}^{1f} = \delta_1 \left( \frac{p_{1h}^{1f'} - p_{1l}^{1f}}{s_h - s_l} - \frac{p_{1h}^{1f}}{s_l} \right), \quad q_{1h}^{1f'} = \delta_1 \left( 1 - \frac{p_{1h}^{1f'} - p_{1l}^{1f}}{s_h - s_l} \right);
\]

market 2:

\[
q_{1h}^{2f} = \delta_2 \left( \frac{p_{2h}^{2f'} - p_{2l}^{2f}}{s_h - s_l} - \frac{p_{2h}^{2f}}{s_l} \right), \quad q_{1h}^{2f'} = \delta_2 \left( 1 - \frac{p_{2h}^{2f'} - p_{2l}^{2f}}{s_h - s_l} \right).
\]

Comparing the two configurations we obtain

\[
q_{1l}^{1f} + q_{1l}^{1f'} + q_{lh}^{2f} + q_{lh}^{2f'} - \left( q_{1h}^{1f} + q_{1h}^{1f'} + q_{lh}^{2f} + q_{lh}^{2f'} \right) > 0
\]
for
\[ \phi > \phi_3 \equiv \frac{s_h s_l (\delta_1 + \delta_2) (s_h + 2s_l + 9)}{2 (s_h - s_l) [\delta_2 (3s_h - s_l) - \delta_1 s_h]}. \]

This result can be summarized as follows.

**Proposition 10** Suppose an economy with two duopolies with different density levels, a private airport and two peak slots. Then, for either \( \phi_3 > \phi_1 > \phi \) or \( \phi_1 > \phi_3 > \phi \), the airport allocates one peak slot to each market; for \( \phi_1 > \phi > \phi_3 \), the airport allocates two peak slots to the large market, and inefficiency would not arise.

When the per-passenger fee is sufficiently small, the allocation is fair and does not favor any market so that each destination is equally served. For high per-passenger fees, the denser market obtains all available slots. As a consequence, passengers in the small market have no chance to fly at peak hours, while passengers in the big market can not fly at off-peak hours.

Finally, we examine the example where peak slots are relatively abundant. In particular, there are three peak slots to be allocated to two markets, \( M = 3 \), \( N = 2 \). Given that configuration off-peak/off-peak is strictly dominated, we can set aside the situation where the airport leaves one slot unused and gives two slots to the big market. Indeed, the airport could be better off by giving the unused one to the small market. It follows that there are three plausible configurations: (1) two peak slots to market 2- one peak slot to market 1, (2) two peak slots to market 1- one peak slot to market 2, and (3) one peak slot to each market.

1. **Configuration 1.** Market 1:

   \[ q_{1h}^f = \delta_1 \left( \frac{p_{1h}^{1f} - p_{1h}^{1f}}{s_h - s_l} - \frac{p_{1h}^{1f}}{s_l} \right), \quad q_{1h}^{1f'} = \delta_1 \left( 1 - \frac{p_{1h}^{1f}}{s_h} \right); \]

   market 2:

   \[ q_{2h}^f + q_{2h}^{1f'} = \delta_2 \left( 1 - \frac{p_{2h}^2}{s_h} \right). \]

2. **Configuration 2.** Market 1:

   \[ q_{hh}^f + q_{hh}^{1f'} = \delta_1 \left( 1 - \frac{p_{hh}^1}{s_h} \right); \]
market 2:
\[ q_{lh}^{2f} = \delta_2 \left( \frac{p_{hl}^{2f'} - p_{lh}^{2f}}{s_h - s_l} \right), \quad q_{hi}^{2f'} = \delta_2 \left( 1 - \frac{p_{hi}^{2f'} - p_{lh}^{2f}}{s_h - s_l} \right); \]

3. Configuration 3. market 1:
\[ q_{lh}^{1f} = \delta_1 \left( \frac{p_{hi}^{1f} - p_{hi}^{1f}}{s_h - s_l} \right), \quad q_{hi}^{1f'} = \delta_1 \left( 1 - \frac{p_{hi}^{1f} - p_{hi}^{1f}}{s_h - s_l} \right); \]

market 2:
\[ q_{lh}^{2f} = \delta_2 \left( \frac{p_{hi}^{2f'} - p_{hi}^{2f}}{s_h - s_l} \right), \quad q_{hi}^{2f'} = \delta_2 \left( 1 - \frac{p_{hi}^{2f'} - p_{hi}^{2f}}{s_h - s_l} \right). \]

Comparing the three configurations we obtain

configuration 1 > 2 > 3 when \( \phi > \phi_4 \);
configuration 1 < 2 < 3 when \( \phi < \phi_4 \).

with
\[ \phi_4 \equiv \frac{s_h s_l (s_h + 2s_l + 9)}{2 (s_h - s_l) (3s_h - s_l)}. \]

Therefore

**Proposition 11** Suppose an economy with two duopolies with different density levels, a private airport and three peak slots. Then, for \( \min (\phi_1, \phi_4) > \phi \), the airport allocates one peak slot to each market, and leaves one peak slot unused (inefficiency); for \( \phi_1 > \phi > \phi_4 \), the airport allocates two peak slots to the large market and one peak slot to the small market.

Proposition 11 implies that when market has different consumer densities, allocation inefficiency would arise if the per-passenger fee is sufficiently small. On the other hand, if the per-passenger fee lies in a certain range, allocation outcome is efficient, with the denser market obtaining both peak slots and smaller market obtaining one peak slot. Such allocation favors the denser market, which is a result of airport’s profit maximizing behavior.
7 Concluding remarks

We have shown the optimal slot allocation in the presence of airport congestion in a model where peak and off-peak slots are modelled as products of different qualities in a vertically differentiated setting. Allocation inefficiency emerges when the airport does not exploit all its slots. The airport’s ownership matters in terms of slot allocation. In particular in a private airport, under-utilization of slots may emerge if the airport is not too congested and the per-passenger fee is small enough. We have also shown that a private airport tends to give different slots to flights with same destination, while a public airport favors identical slots. Hence the results support a public ownership of the airport. A private airport appears less efficient in terms of use of slots, and its allocation choices lead to satisfy a lower number of passengers than a public airport.
References


8 Appendix A

The airport has the following linear programming problem to solve

\[
\max_{n_1, n_2, n_3} \Pi = n_1 Q_1 + 2n_2 Q_2 + 2n_3 Q_3, \\
\text{s.t.} \\
\quad n_1 + n_2 + n_3 = N, \\
\quad n_1 + 2n_2 \leq M, \\
\quad 0 \leq n_1, n_2, n_3 \leq N.
\]

Using \(n_3 = N - n_1 - n_2\) we can re-write

\[
\mathcal{P} \equiv \max_{n_1, n_2, n_3} \Pi = n_1 (Q_1 - 2Q_3) + n_2 (2Q_2 - 2Q_3) + 2N Q_3,
\]

\text{s.t.} \\
\quad n_1 + 2n_2 \leq M, \\
\quad 0 \leq n_1 + n_2 \leq N.

We get the following solution:

1. If \((Q_1 - 2Q_3) > 0\) and \((2Q_2 - 2Q_3) > 0\), and \(Q_1 > 2Q_2\) then \(n_1 = \min\{M, N\}, n_2 = 0, n_3 = N - n_1\);

2. If \((Q_1 - 2Q_3) > 0\) and \((2Q_2 - 2Q_3) > 0\), and \(Q_2 + Q_3 > Q_1\) then \(n_2 = \min\{M/2, N\}, n_1 = 0, n_3 = N - n_2\);

3. If \((Q_1 - 2Q_3) > 0\) and \((2Q_2 - 2Q_3) > 0\), \(2Q_2 > Q_1 > Q_2 + Q_3\) and \(N \geq M\), then \(n_1 = M, n_2 = 0\) and \(n_3 = N - M\);

4. If \((Q_1 - 2Q_3) > 0\) and \((2Q_2 - 2Q_3) > 0\), \(2Q_2 > Q_1 > Q_2 + Q_3\) and \(M > N > M/2\), then \(n_1 = 2N - M, n_2 = M - N\) and \(n_3 = 0\);

5. If \((Q_1 - 2Q_3) > 0\) and \((2Q_2 - 2Q_3) > 0\), \(2Q_2 > Q_1 > Q_2 + Q_3\) and \(M/2 > N\), then \(n_1 = 0, n_2 = M/2\) and \(n_3 = N - M/2\).

6. If \((Q_1 - 2Q_3) < 0\) and \((2Q_2 - 2Q_3) > 0\), then \(n_2 = \min\{M/2, N\}, n_1 = 0, n_3 = N - n_2\).
7. If \((Q_1 - 2Q_3) < 0\) and \((2Q_2 - 2Q_3) < 0\), then \(n_1 = n_2 = 0\) and \(n_3 = N\).

8. If \((Q_1 - 2Q_3) > 0\) and \((2Q_2 - 2Q_3) < 0\), then \(n_1 = \min\{M, N\}\), \(n_2 = 0\), \(n_3 = N - n_1\).

Note that Case 5 is not applicable because we assumed \(M < 2N\).

### 8.1 Applications of the solution

#### 8.1.1 Duopolies and private airport

For the duopoly case, let \(Q_1 = q_{hl} + q_{lh}\), \(Q_2 = q_{hh}\) and \(Q_3 = q_{ll}\). We know that \(Q_1 > 2Q_2 > 2Q_3\) if \(\phi < \phi_2\) and \(2Q_2 > Q_1 > 2Q_3\) if \(\phi_2 < \phi < \phi_1\). Also, \(Q_1 > Q_2 + Q_3\). If \(\phi < \phi_2\) we get \(Q_1 > 2Q_2\) so that case 1 applies: \(n_1 = \min\{M, N\}\), \(n_2 = 0\), \(n_3 = N - n_1\);

If \(\phi_2 < \phi < \phi_1\), we get:

i. if \(N > M\), then \(n_1 = M\), \(n_2 = 0\) and \(n_3 = N - M\) (case 3);

ii. if \(M > N > M/2\), then \(n_1 = 2N - M\), \(n_2 = M - N\) and \(n_3 = 0\) (case 4);

This yields the solution:

- \(n_1 = \min\{M, N\}\), \(n_2 = 0\), \(n_3 = N - n_1\) if \(\phi < \phi_2\) or if \(\phi_2 < \phi < \phi_1\) and \(N > M\);

- \(n_1 = 2N - M\), \(n_2 = M - N\), \(n_3 = 0\) if \(\phi_2 < \phi < \phi_1\) and \(M > N\).

#### 8.1.2 Monopolies and private airport

We have \(Q_1 = q_{lh} + q_{hl}\), \(Q_2 = q_{hh}/2\) and \(Q_3 = q_{ll}/2\). We know that \(2Q_2 > 2Q_3 > Q_1\). So, we are in case 6 where \(n_2 = \min\{M/2, N\}\), \(n_1 = 0\), \(n_3 = N - n_2\).

#### 8.1.3 Duopolies and public airport

For the duopoly case, let \(Q_1 = B_{hlh}\), \(Q_2 = B_{hh}/2\) and \(Q_3 = B_{ll}/2\). Let \(M = n_f\).

We know \(2Q_2 > Q_1 > 2Q_3\) if \(\phi < \phi_P^1\) and \(2Q_2 > 2Q_3 > Q_1\) if \(\phi > \phi_P^1\). Also, \(Q_2 + Q_3 > Q_1\) for \(\phi > \phi_P^1\).

Therefore, if \(\phi > \phi_P^1\), we have case 6 where \(n_2 = \min\{M/2, N\}\), \(n_1 = 0\), \(n_3 = N - n_2\). For \(\phi_P^1 > \phi > \phi_P^4\), we are in case 2 where \(n_2 = \min\{M/2, N\}\),

39
\( n_1 = 0, n_3 = N - n_2. \) For \( \phi < \phi^P_1 \), then the relationship between destinations and peak slots matters. For \( M > N \), case 4 emerges where \( n_1 = 2N - M, n_2 = M - N \) and \( n_3 = 0. \) For \( N > M \), case 3 occurs, according to which \( n_1 = M, n_2 = 0 \) and \( n_3 = N - n_2. \)

### 8.1.4 Monopolies and public airport

For the monopoly case, let \( Q_1 = B_{hllh}, Q_2 = B_{hh}/2 \) and \( Q_3 = B_{ll}/2. \) Let \( M = n_f. \) We know \( 2Q_2 > Q_1 > 2Q_3 \) if \( \phi < \phi^{PM} \) and \( 2Q_2 > 2Q_3 > Q_1 \) if \( \phi > \phi^{PM}. \) Also, \( Q_2 + Q_3 > Q_1. \)

Therefore, the resulting equilibrium is always \( n_2 = \min\{M/2, N\}, n_1 = 0, n_3 = N - n_2. \)

### 9 Appendix B

#### 9.1 Monopoly airlines

We compare the monopoly outcome (29) and (30) and the duopoly outcome (13) and (14).

\[
q^M_{hl} - q^f_{hl} = \frac{2s_l^3 + s_l^2 (5s_h + 4\phi) - s_h s_l (11s_h - 5\phi) - s_h^2 (2s_h + 3\phi)}{(4s_h - s_l) (s_h + 2s_l)^2} < 0,
\]

\[
q^M_{lh} - q^f_{lh} = \frac{(s_h - s_l) [3s_h s_l + \phi (2s_h + s_l)]}{s_l (4s_h - s_l) (s_h + 2s_l)} > 0,
\]

for \( 0 < \phi < \phi^M_1. \) Comparing total passenger volume in two cases shows that:

\[
q^M_{hl} + q^M_{lh} - (q^f_{hl} + q^f_{lh}) = \frac{s_l (s_h^3 - 8s_h^2 s_l - s_h s_l^2 + 2s_l^3) + 2\phi (s_h^3 + s_h^2 s_l + s_l^3)}{s_l (4s_h - s_l) (s_h + 2s_l)^2} < 0,
\]

for \( 0 < \phi < \phi^M_1. \) As expected, the total number of seats provided by the monopolist is smaller than the duopolists. Replacing \( q^f_{hl} \) and \( q^f_{hl} \) in (8) with
(29) and (30) yields:

\[ p^M_{hl} = \frac{s_l [2\phi (s_h + s_l) + s_l (3s_h + 2s_l)]}{(s_h + 2s_l)^2}, \]

\[ p^M_{lh} = \frac{s_l s_h + \phi (s_h + s_l)}{(s_h + 2s_l)}. \]

We also compare the outcomes of different configurations of a single market. From (29), (30) and (33), one obtains:

\[ q^M_{hl} + q^M_{lh} - q^M_h = \frac{4\phi s^2_l - 3\phi s^2_h + s^3_h - 2s^2_h s_l}{2s_h (s_h + 2s_l)^2} < 0, \]

\[ q^M_{hl} + q^M_{lh} - q^M_l = \frac{\phi s^2_h + s_h s_l (s_h - 2s_l)}{2s_h (s_h + 2s_l)^2} < 0, \]

\[ q^M_h - q^M_l = \frac{\phi (s_h - s_l)}{2s_h s_l} > 0, \]

for all \( 2s_l > s_h > s_l > 0 \) and \( 0 < \phi < \phi^M_1 \).

### 9.2 Public Airport

#### 9.2.1 Duopoly

By comparing the number of seats obtained in each configuration, we get:

\[ B_{hh} - B_{lt} = \frac{2(s_h - s_l)(\phi^2 + 2s_h s_l)}{9s_h s_l} > 0, \]

\[ B_{hh} - B_{hl+th} = \frac{(s_h - s_l)(s^2_h s_l (20s_h + s_l) + 4s_h s_l \phi (2s_h + s_l) + \phi^2 (36s^2_h - 19s_h s_l + 4s^2_l))}{18s_h s_l (s_l - 4s_h)^2} > 0 \]

\[ B_{hl+th} - B_{lt} = \frac{(s_h - s_l)(108s^2_h s_l + s_l (8s^2_l - 4s_l \phi - 13\phi^2) + s_h (28\phi^2 - 65s^2_l - 8s_l \phi))}{18s_l (s_l - 4s_l)^2} > 0, \]

Hence we obtain \( B_{hh} > B_{hl+th} > B_{lt} \) for all \( 0 < \phi < \frac{s_l}{2} \).

Next, we evaluate the differences in the allocation of two peak slots:

\[ B_{hh} + B_{lt} - 2B_{hl+th} = \]
depending on $<$ imply that, for $B_{hh} + B_{ll} < 2B_{hl+lh}$.

9.2.2 Monopoly

By comparing the number of seats obtained in each configuration, we get:

$$B^M_{hh} - B^M_{ll} = \frac{(s_h - s_l)(\phi^2 + 3s_h s_l)}{8s_h s_l} > 0,$$

for all $0 < \phi < \frac{s_l}{s_h}$. Hence, $B_{hh} + B_{ll} < 2B_{hl+lh}$.

By comparing the number of seats obtained in each configuration, we get:

$$B^M_{hh} - B^M_{hl+lh} =$$

$$3s_h^6 + s_h^5(28s_l + 6\phi) + s_h^4(52s_l^2 + 16s_l + 12\phi) + 8s_h^3s_l(-8s_l^2 - 6s_l + \phi + \phi^2) +$$

$$8s_h(s_h + 2s_l)^4 \left(\phi^2 - 4s_l^2 - 16s_h^3s_l^3\phi^2 - 16s_l^4\phi^2\right) > 0,$$

$$B^M_{hl+lh} - B^M_{ll} =$$

$$-\left(2s_l^2 - s_h(s_l + \phi)\right)(s_h^2(\phi - 7s_l) + 2s_h^3s_l(2\phi - 9s_l) + 4s_h^3s_l^2(5s_l + 3\phi) + 24s_l^4) \geq 0,$$

depending on $s_l$ and $s_h$. When $s_l < s_h < 1.5s_l$ and $\phi < \phi^M_p$; or $1.5s_l < s_h < 2s_l$ and $\phi < \phi^M_1$: $B^M_{hl+lh} - B^M_{ll} > 0$, where

$$\phi^M_p \equiv \frac{s_l(7s_h^3 + 18s_h^2s_l - 20s_h s_l^2 - 24s_l^3)}{s_h(s_h^2 + 4s_h s_l + 12s_l^2)}.$$

Following the procedure already adopted in the baseline case, these results imply that, for $M > N$, the highest number of seats in a market are provided by adopting the configuration “peak/peak”. We then evaluate the differences in the allocation of two peak slots:

$$\frac{\partial Q(\phi)}{\partial n_2} = B^M_{hh} + B^M_{ll} - 2B^M_{hl+lh} =$$

$$\frac{4s_h^3s_l(\phi)(3s_h^2 + 8s_h s_l - 12s_l^2) - \phi^2(s_h^5 + s_h^4 s_l + 8s_h^3 s_l^2 - 8s_h^2 s_l^3 + 16s_h s_l^4 + 16s_l^5)}{16s_h s_l(s_h + 2s_l)^4}$$

$$+ \frac{s_h s_l(3s_h^5 + 35s_h^4 s_l + 56s_h^3 s_l^2 - 88s_h^2 s_l^3 - 16s_h s_l^4 + 48s_l^5)}{16s_h s_l(s_h + 2s_l)^4} > 0.$$
9.3 Endogenous per passenger fee

9.3.1 Private airport and airline monopolies

We next examine the case of private airport with monopoly airlines. According to Proposition (2), this case exhibits one allocation type. From Proposition (2), the solution is

\[ n_1 = 0, \quad n_2 = M = 2, \quad n_3 = N - M = 2, \]

so that airport profits are

\[
\max_{\phi} \Pi(\phi) = \phi \left[ \frac{M}{2} \left( q_{hh}^f + q_{hh}^r \right) + \left( N - \frac{M}{2} \right) \left( q_{ll}^f + q_{ll}^r \right) \right] \\
= \phi \left[ \frac{M}{2} \cdot \frac{s_h - \phi}{2s_h} + \left( N - \frac{M}{2} \right) \frac{s_l - \phi}{2s_l} \right]
\]

The first order condition with respect to \( \phi \) yields

\[ \phi^M = \frac{Ns_h s_l}{2Ns_h - M(s_h - s_l)}. \]

The second-order condition is

\[
\frac{\partial^2 \Pi(\phi)}{\partial \phi^2} = -2 \left( \frac{M}{4s_h} + \frac{M + 2N}{4s_l} \right) < 0.
\]

Next we need to check if the optimal per-passenger fee lies in the admissible parameter range of \( \phi \). In particular, the necessary condition for interior solution in every possible configuration for a monopolist is \( \phi < \phi^M \). For convenience, we denote

\[ \widehat{M} \equiv \frac{Ns_h (3s_h - 4s_l)}{s_h^2 - 3s_l s_h + 2s_l^2}, \]

Comparing \( \phi^*_1 \) and \( \phi^M \), yields

\[
\phi^M - \phi^{M*} = s_l \left( \frac{2s_l}{s_h} - \frac{N s_h}{(2N - M) s_h + Ms_l} - 1 \right),
\]

where

- \( \phi^M - \phi^{M*} > 0 \) for \( 0.75 s_h < s_l \leq 0.81 s_h \) and \( M < \widehat{M} \), or \( 0.81 s_h < s_l \leq s_h \);
- \( \phi^M - \phi^{M*} < 0 \) for \( 0.5 s_h < s_l \leq 0.75 s_h \) or \( 0.75 s_h < s_l \leq 0.81 s_h \) and \( \widehat{M} < M < 2N \).
Hence the optimal fee depends on a combination between the relationship among \( s_h \) and \( s_l \), and the relative scarcity of peak slots. When the quality difference is small, the optimal fee is \( \phi^M \), when the difference is large, the optimal fee is \( \phi^M^* \). When it lies in between, the optimal fee depends on \( M \). The ongong analysis can be summarized as:

**Proposition 12** Suppose all markets are served by monopoly airlines, and the airport is private. Then the airport favors “peak/peak” configuration and sets \( \phi^* = \min \{ \phi^M, \phi^M^* \} \).

### 9.3.2 Public airport and airline duopolies

In this setting, there are two cases according to whether \( N > M \). For \( N > M \), the solution is \( n_1 = M, n_2 = 0, n_3 = N - M \), and the condition \( \phi < \phi^P = \frac{s_l}{2} \) must hold to ensure interior solution. We could write profit as

\[
\max_{\phi} \Pi(\phi) = \phi \left[ M (q_{hl}^f + q_{lh}^f) + (N - M) (q_{ll}^f + q_{ll}^f) \right]
\]

\[
= \phi \left( \frac{2(N - M) (s_l - \phi)(2s_l - \phi)}{s_l} + \frac{M (12s_h^3 s_l - 5s_h^2 s_l^2 + s_l^2 s_h - 8s_h s_l^2 \phi + 4s_h^2 \phi^2 - s_h s_l \phi^2 + s_l^2 \phi^2)}{2(4s_h - s_l)^2 s_l} \right)
\]

The second-order condition is

\[
\frac{\partial^2 \Pi(\phi)}{\partial \phi^2} = -\frac{M (s_h - s_l)(8s_h s_l - 84s_h \phi + 4s_l^2 + 39s_l \phi) + 4N (4s_h - s_l)^2(s_l + 3\phi)}{9s_l(4s_h - s_l)^2} < 0.
\]

Solve from first order condition we obtain \( \phi_1^{P*} \) and \( \phi_2^{P*} \)

\[
\phi_1^{P*} = \frac{16M s_h^2 s_l - 8M s_h s_t^2 - 64N s_h s_t^2 + 128N s_h^2 s_t - 8M s_t^3 + 8N s_t^3 - \sqrt{\Delta}}{2(-123M s_h s_t + 96N s_h s_t + 84M s_t^2 - 192N s_t^2 + 39M s_t^2 - 12N s_t^2)} > \phi^P
\]

\[
\phi_2^{P*} = \frac{16M s_h^2 s_l - 8M s_h s_t^2 - 64N s_h s_t^2 + 128N s_h^2 s_t - 8M s_t^3 + 8N s_t^3 + \sqrt{\Delta}}{2(-123M s_h s_t + 96N s_h s_t + 84M s_t^2 - 192N s_t^2 + 39M s_t^2 - 12N s_t^2)} < 0
\]
where $\phi_1^* > \phi_2^*$ and

\[
\Delta \equiv \left(8Ms_h s_l^2 - 16 Ms_h^2 s_l + 64N s_h s_l^2 - 128 N s_h^2 s_l + 8M s_l^3 - 8 N s_l^3\right)^2 - \\
4 \left(-123M s_h s_l + 96 N s_h s_l + 84M s_h^2 - 192 N s_h^2 + 39M s_l^2 - 12 N s_l^2\right) \times \\
\left(73M s_h s_l^3 - 173Ms_h^2 s_l^2 + 108Ms_h^3 s_l - 64 N s_h s_l^3 + 128 N s_h^2 s_l^2 - 8M s_l^4 + 8N s_l^4\right).
\]

Hence the optimal fee is $\phi^* = \phi^P = \frac{s_l^4}{2}$.

Consider next $N < M$, for which the airport profits are

\[
\max_\phi \Pi(\phi) = \left[(2N - M) \left(q_{hl}^l + q_{lh}^l\right) + (N - M) \left(q_{hh}^l + q_{hh}^l\right)\right] \phi.
\]

The first-order with respect to $\phi$ yields:

\[
\phi_3^{P*} = \frac{4s_h s_l \left[s_h (M - 10N) + 2s_l^2 (M - 10N) - s_l^2 (M - N)\right] - \sqrt{s_l^2 s_l \Delta}}{3 \left[s_l^2 s_l (46N - 55M) + s_h s_l^2 (23M - 14N) + 36 s_h^2 (M - 2N) + 4 s_l^2 (N - M)\right]} > \phi^P
\]

\[
\phi_4^{P*} = \frac{4s_h s_l \left[s_h (M - 10N) - 2s_l^2 (M - 10N) + s_l^2 (M - N)\right] + \sqrt{s_l^2 s_l \Delta}}{3 \left[s_l^2 s_l (46N - 55M) + s_h s_l^2 (23M - 14N) + 36 s_h^2 (M - 2N) + 4 s_l^2 (N - M)\right]} < 0,
\]

where

\[
\hat{\Delta} \equiv 8s_h^4 s_l \left(677M^2 + 1148MN - 1420N^2\right) + s_l^3 s_l^2 \left(-4471M^2 - 6916MN + 3044N^2\right) \\
+ 6s_l^3 s_l^2 \left(223M^2 + 664MN - 428N^2\right) + s_h s_l^4 \left(-127M^2 - 1168MN + 1052N^2\right) \\
- 432s_l^5 \left(5M + 22N\right)\left(M - 2N\right) + 4s_l^5 \left(M - N\right)\left(M + 26N\right).
\]

The second-order condition yields

\[
\frac{\partial^2 \Pi(\phi)}{\partial \phi^2} = \frac{s_l^2 s_l \left(8s_h (M - 10N) + 3\phi (46N - 55M)\right) + s_h s_l^2 \left(-4s_h (M - 10N) + 69M \phi - 42N \phi\right)}{9s_h s_l \left(s_l - 4s_h\right)^2} + \\
\frac{-4s_l^3 (M - N) \left(s_h + 3\phi\right) + 108\phi s_l^3 (M - 2N)}{9s_h s_l \left(s_l - 4s_h\right)^2} < 0 \text{ for } \phi > \overline{\phi}^P,
\]

where

\[
\overline{\phi}^P \equiv \frac{8M s_h^3 s_l - 4M s_h^2 s_l^2 - 4Ms_h s_l^3 - 80Ns_h s_l^3 + 40Ns_h^2 s_l^2 + 4Ns_h^3 s_l^3}{165Ms_h s_l^2 - 69Ms_h s_l^2 - 138Ns_h^2 s_l^2 + 42Ns_h s_l^2 - 108Ms_h^3 + 216Ns_h^3 + 12Ms_h^4 - 12Ns_h^4}.
\]

45
Comparing $\tilde{\phi}^P$ and $\phi^P$:

$$\tilde{\phi}^P - \phi^P = \frac{s_I (s_h^2 s_I (218N - 173M) + s_h s_{I}^2 (61M - 34N) + 4s_{h}^3 (31M - 94N) + 12s_{h}^3 (N - M))}{6 (s_h^2 s_{I} (55M - 46N) + s_{h} s_{I}^2 (14N - 23M) - 36s_{h}^3 (M - 2N) + 4s_{h}^3 (M - N))} < 0.$$ 

Thus the optimal per-passenger fee is, again, $\phi^* = \phi^P = \frac{s_h}{2}$. Combining the two cases, we summarize:

**Proposition 13** Suppose all markets are served by duopoly airlines, and the airport is public. For $M \leq N$, the airport uses all available peak slots, and favors “peak/off-peak” configuration. For $M > N$ the airport implements $(M - N)$ “peak/peak” and $(2N - M)$ “peak/off-peak” configurations. In both cases, the airport sets $\phi^* = \frac{s_h}{2}$.

**9.3.3 Public airport and airline monopolies**

According to (??), the equilibrium allocation is $n_1 = 0$, $n_2 = M/2$, $n_3 = N - M/2$. And the interior solution condition $\phi < \phi^M = \frac{s_I (2s_I - s_h)}{s_h} > 0$ must hold. The airport profits are

$$\max_{\phi} \Pi(\phi) = \phi \left[ \frac{M}{2} (q''_{hh} + q''_{lh}) + \left( N - \frac{M}{2} \right) \left( q''_{ll} + q''_{lI} \right) \right]
= \phi \left[ \frac{M (s_h - \phi) (3s_h + \phi)}{16 s_h} + \frac{(s_I - \phi) (3s_I + \phi) (N - M/2)}{8 s_I} \right].$$

The first-order condition yields:

$$\phi^{PM*}_{1} = \frac{4Ns_h s_I - \sqrt{16N^2 s_h^2 s_{I}^2 - 3(3M (s_I - s_h) - 6Ns_h) (Ms_h s_I (s_h - s_I) + 2Ns_h s_{I}^2)}}{3(M (s_h - s_I) - 2N s_h)} > 0$$

$$\phi^{PM*}_{2} = \frac{4Ns_h s_I + \sqrt{16N^2 s_h^2 s_{I}^2 - 3(3M (s_I - s_h) - 6Ns_h) (Ms_h s_I (s_h - s_I) + 2Ns_h s_{I}^2)}}{3(M (s_h - s_I) - 2N s_h)} < 0,$$

while the second-order condition is:

$$\frac{\partial^2 \Pi(\phi)}{\partial \phi^2} = \frac{3M \phi (s_h - s_I) -Ns_h (4s_I + 6\phi)}{8s_I s_h} < 0,$$

for $\phi > 0$. Therefore

**Proposition 14** Suppose all markets are served by monopoly airlines. Then a
public airport uses all available peak slots for peak/peak market configurations, and charges a fee $\phi^* = \min \{ \phi^*_1, \phi^*_{PM^*} \}$. 