Joint Provision of International Transport Infrastructure*

Se-il Mun
Graduate School of Economics, Kyoto University,
Yoshida Hon-machi, Sakyo-ku, Kyoto 606-8501, Japan
mun@econ.kyoto-u.ac.jp

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Abstract

We consider the following scheme for the development of international transport infrastructure: two countries jointly establish an operator of the infrastructure that are responsible of collecting the user charge, maintenance, etc.; the costs for infrastructure investment are covered by financial contributions from two countries; and the revenue from infrastructure charge is distributed according to the share of contributions. The governments of two countries choose the amount of contribution so as to maximize the national welfare. Assuming that the infrastructure use is non-rival, we show that financing infrastructure by the revenue from user charges is better than financing by tax revenue. We extend the analysis by incorporating congestion in infrastructure use. It is shown that independent decisions on contributions by two governments attains the first-best optimum when the operator sets the user charge so that the toll revenue just covers the cost for the investment. We further examine the condition that joint provision is realized in Nash equilibrium.

Keywords: international transport infrastructure, joint provision, congestion, self-financing

JEL Classification: L91, L98, R48

1 Introduction

There are many bridges or tunnels in the world crossing the border between two countries. These facilities touch the territories of both countries, and thereby decisions to construct them should be jointly made by the governments on both sides of the border. This paper deals with the problem concerning the joint provision of cross-border transport infrastructure. We consider the following scheme for the infrastructure development: two countries jointly establish an operator of the infrastructure that are responsible of collecting the user charge, maintenance, etc.; the costs for infrastructure investment are covered by financial contributions from two countries, and the revenue from infrastructure charge is distributed according to the share of contributions. Similar practices can be found in the real world. For example, United States and Canada jointly established the Niagara Falls Bridge Commission to finance, construct and operate the Rainbow Bridge.

We develop a simple two-country model in which the transportation cost between countries depends on the capacity and user charge (e.g., road toll) of infrastructure. The governments of two countries choose the amount of contribution so as to maximize the national welfare. The sum of contributions is spent for investment, and thereby determines the capacity of the infrastructure. We consider two cases: the infrastructure use is non-rival; and congestible. In the non-rival case, financing the cost for infrastructure investment by revenue from user charges encourages the contributions from two governments, and thereby attains the higher welfare than financing by tax revenue. We next compare the following two situations: (i) optimal capacity choice for the two countries as a whole; (ii) equilibrium capacity choice by independent decisions by two governments. It turns out that capacity in equilibrium (case (ii)) is smaller (larger) than the optimum (case (i)) if user charge is smaller (larger) than the level at which the revenue cover the cost for capacity investment. In the case of congestible infrastructure, it is shown that independent decisions on contributions by two governments attains the first-best optimum when the operator sets the user charge so that the toll revenue just covers the cost for the investment. This is an extension of the well-known self-financing theorem by Mohring-Harwitz (1962). Unlike the original setting where a single government chooses the capacity based on benefit-cost criterion, we obtained the result in the situation that the capacity is determined by non-cooperative contributions by multiple governments.

There is a large body of literature on pricing and capacity choice of transport infrastructure in the system of multiple governments (see e.g. review by De Borger and Proost (2012)). Mun and Nakagawa (2010) consider the cross-border transport infrastructure that consists of two links, each of which is constructed and operated by the government of its territory. They evaluate the effects of alternative pricing and investment policies for the infrastructure on economic welfare of two countries. Recently, Brueckner (2014) investigates pricing and capacity choice of a congestible bridge between jurisdictions in a monocentric metropolitan area. He assumes that the capacity of the bridge is determined solely by the government of the jurisdiction on the outer side. This assumption is reasonable in the context of a monocentric metropolitan area since bridge is used only by the residents in outer locations. In this
setting, Brueckner shows that decentralized capacity choice with budget-balancing user fee attains the efficient allocation. This paper can be regarded as an extension of Brueckner’s analysis to the case that there are users on both side of the bridge and multiple governments share the cost for capacity investment. Verhoef (2012) also obtains the self-financing result in the setting that users of the infrastructure have market powers. His result is strong in that self-financing holds in broader situations where capacity cost does not exhibit constant returns. Note that the subsidy from the government is required to attain the efficiency and self-financing in Verhoef’s model. In contrast, the scheme proposed in this paper attains the efficiency through voluntary contributions from two governments imposing cost recovery to the operator.

This paper is also related to the literature on voluntary provision of public good (Bergstrom, Blume and Varian (1986), Cornes and Sandler(1996), Batina and Ihori(2005)). In the absence of congestion, the service provided by the infrastructure we consider is excludable but non-rival. In this case, optimal infrastructure charge should be zero, and thereby the formula to determine the capacity of the infrastructure is reduced to that corresponding to the voluntary provision of public good, leading to under provision. In other words, optimal pricing (or financing by taxation) suffers from too small capacity. There have been several proposals to induce the efficient voluntary provision of public good (e.g., Falkinger (1996)). We show that imposing user charge gives an incentive to increase the amount of voluntary contribution and larger welfare in the case of non-rivalry (i.e., without congestion). Furthermore, if the infrastructure is congestible, pricing attain the optimal level of capacity by voluntary contributions.

This paper is organized as follows. In Section 2, we examine the outcome of the proposed scheme assuming that infrastructure use is non-rival (i.e., no congestion). Section 3 extends the analysis to the case allowing congestion. Section 4 considers the possibility of single provision where one of two countries builds and operates the infrastructure. We investigate the condition that joint provision is realized in Nash equilibrium. Section 5 concludes the paper.

2 User Charge and Capacity Investment for Non-rival Infrastructure

2.1 Setting

Consider an economy with two countries, indexed by $i$ ($i = 1, 2$). In each country, there is transport demand to another country, which crosses the border using the international transport infrastructure. The transportation cost depends on the capacity and user charge (e.g., road toll) of infrastructure. The demand function is given by $D_i(f + t(k))$, where $f$ is the infrastructure charge, and $t(k)$ is the user cost that depend on the capacity of
the infrastructure, \( k \). \( f + t(k) \) is the full cost of transportation per trip\(^1\). The demand function is strictly decreasing and differentiable. We assume that an investment in transport infrastructure increases capacity, thereby saves the user cost and that the investment is decreasing return to scale: \( t' \equiv dt/dk < 0, t'' \equiv d^2t/dk^2 > 0 \). We also assume that the transport infrastructure is produced with constant returns to scale technology: the cost of infrastructure investment is linearly increasing in capacity.

There is an operator of the infrastructure jointly established by two countries, which constructs the facility and collects the user charge. The costs for infrastructure investment are covered by financial contributions from two countries. We assume that the revenue from the infrastructure charge is distributed according to the share of contributions. The national welfare in country \( i \) is defined as the sum of users’ welfare and the dividend of the revenue minus the expenditure for financial contribution, as follows

\[
W_i = \int_{f+t(k)}^{\infty} D_i (p) \, dp + \frac{k_i}{k} f (x_1 + x_2) - p^k k_i
\]

where \( k_i \) is the amount of financial contribution from Country \( i \). \( k_1 + k_2 = k \) should hold; \( x_i = D_i (f + t(k)) \) is the number of trips from Country \( i \) and \( p^k \) is unit cost of infrastructure investment. It is convenient to rewrite the national welfare as

\[
W_i = \int_{f+t(k)}^{\infty} D_i (p) \, dp + \frac{k_i}{k} \Pi
\]

where \( \Pi \) is the profit of the infrastructure project, \( \Pi = f (x_1 + x_2) - p^k k \). The second term on the RHS, \( \frac{k_i}{k} \Pi \) can be interpreted as the dividend of profit.

### 2.2 Social Optimum

In this paper, the social optimum is characterized as the solution to a global welfare maximization problem. The global welfare is defined as the sum of the two countries’ national welfares, as follows

\[
W(f, k) = \int_{f+t(k)}^{\infty} D_1 (p) \, dp + \int_{f+t(k)}^{\infty} D_2 (p) \, dp + \Pi
\]

Let us suppose that the infrastructure charge, \( f \), is fixed. The optimality condition with respect to the capacity is

\[
-(x_1 + x_2) t' + f(x_{1k} + x_{2k}) = p^k
\]

where \( x_{ik} \equiv \frac{\partial D_i}{\partial k} > 0 \). Let the solution of (3) be \( K^O(f) \). Differentiating the global welfare function with respect to \( f \) at \( k = K^O(f) \), we have

\[
\frac{dW(f, K^O(f))}{df} = f(x_{1f} + x_{2f}) < 0
\]

\(^{1}\text{Measuring by the number of trips is naturally applicable to passenger transportation, such as Tourism, shopping. In the case of freight transportation, the quantity (e.g., weight of goods) is the unit of measurement, but hereafter we use trips as the unit of measurement.}\)
where \( x_{if} \equiv \frac{\partial D_i}{\partial f} < 0 \). The above inequality states that the global welfare is maximized at \( f = 0 \) while the capacity is determined by (3). In other words, the optimal pricing policy is that infrastructure use should be free of charge. This is natural since marginal cost of usage is zero for the non-rival infrastructure. Under this optimal pricing, (3) is reduced to

\[
-(x_1 + x_2) t' = p^k
\]  

(5)

\( f = 0 \) together with (5) is the condition for the first-best optimum. The LHS of (5) is the number of users, \( (x_1 + x_2) \) multiplied by the marginal benefit of a user (i.e., saving of the transport cost) by increasing the capacity, \( -t' \). The RHS is the marginal cost of increasing the capacity. (5) is the social benefit-cost rule for transport project. It also has the same formal structure as the Samuelson’s condition for optimal public good provision.

### 2.3 Capacity investment under joint provision

The government of each country takes the infrastructure charge as given, and chooses the amount of financial contribution \( k_i \) so as to maximize the national welfare defined by (1). The optimality condition for the government of country \( i \) is

\[
-x_i t' + \frac{k_j}{k^2} f(x_1 + x_2) + \frac{k_i}{k} f(x_{1k} + x_{2k}) = p^k, \quad j \neq i
\]  

(6)

The first term on the LHS of (6) is the users’ marginal benefit in the home country, the second and third terms are the effects on the dividend through changes in the share of contribution and in capacity, respectively. For the special case, \( f = 0 \), (6) is reduced to

\[
-x_i t' = p^k
\]  

(7)

Comparing (7) with (5), we see that the national government ignores the benefit of users in other country, which leads to too small capacity. This discrepancy is essentially the same as that between voluntary provision and optimal provision of public good (Cornes and Sandler (1996), Batina and Ihori (2005)).

Recall that \( f = 0 \) is the optimal pricing policy. This implies that the first-best optimum is never achieved under the decisions by the national government.

Let us examine the effects of increasing the level of infrastructure charge on the amount of contributions and the level of economic welfare. Summing up the investment rule (6) for two countries yields

\[
-(x_1 + x_2) t' + \frac{1}{k} f(x_1 + x_2) + f(x_{1k} + x_{2k}) = 2p^k
\]  

(8)

Let the solution of (8) be \( K^J(f) \). Totally differentiating (8) with respect to \( k \) and \( f \), evaluated at \( f = 0 \), we have

\[
\left. \frac{dk}{df} \right|_{f=0} = \frac{dK^J(0)}{df} = \frac{1}{k} \frac{(x_1 + x_2)}{(x_1 + x_2)t'' + (x_{1k} + x_{2k})t'}
\]  

(9)
The denominator of the RHS of (9) is positive from the second-order condition for (6). Thus we have \( \frac{dK^J(0)}{df} > 0 \) : \( k \) is increased by increasing \( f \) from zero. Differentiating the global welfare function with respect to \( f \) while \( k \) is determined by the national governments: \( k = K^J(f) \), we have

\[
\frac{dW(0, K^J(0))}{df} = p^k \frac{dK^J(0)}{df} > 0
\]

The above analysis is summarized as follows.

**Proposition 1** Increasing the infrastructure charge from zero improves the global welfare through expanding the capacity of infrastructure.

If the infrastructure charge is zero, the national government should use the tax revenue to finance the contribution to the infrastructure project. Also note that optimal infrastructure charge is zero in the non-rival case, so increasing the infrastructure charge from zero means deviation from optimal pricing. The above proposition implies that shifting the revenue source from taxes to user charging, i.e., deviation from optimal pricing, is welfare-improving.

We address the next question: what the efficient infrastructure charge looks like under the joint provision based on the voluntary contributions by the national governments. We assume that the infrastructure project is profitable if the operator exercises the market power\(^2\).

**Proposition 2** Assume that \( \Pi \) is increasing with \( f \) for \( 0 \leq f < \tilde{f} \), where \( \tilde{f} = \arg \max \Pi \), and \( \Pi > 0 \) at \( \tilde{f} \). And let \( \hat{f} < \tilde{f} \), at which \( \hat{f} (x_1 + x_2) - p^k K^J(\hat{f}) = 0 \).

(i) Capacity determined by contributions by two national governments is equal to the optimal capacity at \( \hat{f} \), i.e., \( K^J(\hat{f}) = K^O(\hat{f}) \), and thereby \( W(\hat{f}, K^J(\hat{f})) = W(\hat{f}, K^O(\hat{f})) \);

(ii) \( K^J(f) < K^O(f) \), if \( f < \hat{f} \) and vice versa;

(iii) There exists an infrastructure charge \( f^* \) at which the global welfare is maximized under joint provision, i.e., \( f^* = \arg \max \frac{dW(f, K^J(f))}{df} \);

(iv) \( f^* \) is smaller than \( \hat{f} \).

**Proof.** (i) (8) is rewritten as

\[
-(x_1 + x_2)t' + \frac{1}{k} \Pi + f(x_{1k} + x_{2k}) = p^k
\]

(10)

The above equation is reduced to (3) at \( f = \hat{f} \) where \( \Pi = 0 \).

(ii) Since \( \Pi \) is increasing with \( f \), \( \Pi \leq 0 \iff f \leq \hat{f} \). If \( \Pi > 0 \), the LHS of (10) is larger than the LHS of (3) thereby \( K^J(f) < K^O(f) \) and vice versa.

(iii)(iv) We know \( \frac{dW(0, K^J(0))}{df} > 0 \) from Proposition 1, and \( \frac{dW(\hat{f}, K^J(\hat{f}))}{df} = \frac{dW(\hat{f}, K^O(\hat{f}))}{df} < 0 \) from (4). Thus there must be \( f^*, 0 < f^* < \hat{f}, \) where \( \frac{dW(f^*, K^J(f^*))}{df} = 0 \).

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\(^2\)This assumption is applicable if the demand and user cost functions are not too convex.
Figure 1 is drawn based on the Propositions 1 and 2. $W(f, K^O(f))$ and $W(f, K^J(f))$ are loci of global welfare when capacity is determined optimally and by joint provision, respectively. As Proposition 1 states, the global welfare is increased by increasing $f$ from zero. From (4), the optimal value function $W(f, K^O(f))$ is decreasing with $f$, so the first-best is attained at $f = 0$. These two curves touch at $f = \hat{f}$ where the revenue just covers the cost for investment. We also see that there exists a point $f^*$ where the global welfare under joint provision is maximized ((iii) of Proposition 2). This point can be regarded as the second-best.

Figure 1

From (iv) of the Proposition 2, the revenue from the infrastructure charge at $f^*$ is not sufficient to cover the cost for investment. Thus (i) together with (4) implies that the break-even pricing is most efficient among the schemes in which capacity investment is financed solely by the revenue from the infrastructure. (ii) states that over investment of capacity may arise if the infrastructure charge is larger than $\hat{f}$. This result never arises in Mun and Nagagawa (2010), which examines a number of alternative pricing schemes for the cross-border transport infrastructure consisting of two links, but they all result in under investment.

2.4 Equilibrium with break-even pricing

Equilibrium is described as a game by three players, i.e., two governments and the operator. Two governments choose the investment level, as described in the previous section. The operator sets the level of infrastructure charge according to the pricing policy. Recall that the operator is established by two governments. So the pricing policy is determined by the agreement of two governments. Once the pricing policy is fixed, the operator behaves as an independent player of the game. We assume that two governments agree to adopt break-even pricing. We focus on this case because this pricing rule is commonly adopted in practice of regulation. It is also a good reason that break-even pricing leads to efficient outcome, as shown in the Proposition 2\(^3\).

The operator sets the level of infrastructure charge such that the revenue equal to the cost for investment, taking the contributions from two governments as given. Let us denote by $F^b(k)$ the response function of the operator, which is obtained by solving the following equation for $f$
\[ f (x_1 + x_2) - p^k k = 0 \]

\(^3\)Note that Proposition 2 shows that the break-even pricing is the third-best: there is the second-best infrastructure charge, $f^*$. However the profit of the infrastructure project is negative under the second-best pricing. It is also the advantage of break-even pricing that implementation is much easier. On the other hand, finding the second-best charge would be difficult in practice.
The response of the governments is described by $K^J(f)$, as discussed earlier. Nash equilibrium is characterized by the solution $(f^b, k^b)$ of the following system of equations.

$$
\begin{align*}
  f^b &= F^b(k^b) \\
  k^b &= K^J(f^b)
\end{align*}
$$

Equilibrium is stable when the response functions are positioned as in Figure 2, where the curve of $K^J(f)$ crosses $F^b(k)$ from above. Other than the break-even, we can also consider various pricing policies, for which positions of the operator’s response function are changed.

3 Congestible Infrastructure

We extend the analysis to the case that the infrastructure is congestible. Congestion is described by the user cost function $c\left(\frac{x_1 + x_2}{k}\right)$, where we assume $c'>0$. Accordingly the national welfare is written as $W_i = \int_{f+c(\frac{x_1+x_2}{k})}^{\infty} D_i(p) \, dp + \frac{k_i}{k} f (x_1 + x_2) - p^k i$, and the global welfare is the sum of the two countries’ national welfares, $W(f, k) = W_1 + W_2$.

The conditions for the global welfare maximization (first-best) are

$$
\begin{align*}
  f &= \langle x_1 + x_2 \rangle_{k} c' \\
  \left(\frac{x_1 + x_2}{k}\right)^2 c' &= p^k
\end{align*}
$$

These two conditions are standard formulas for congestion problem: (11) states that the infrastructure charge is equal to the congestion externality; (12) states that the social marginal benefit (reduction of cegestion) from capacity expansion should be equalized to marginal cost of investment.

Under the scheme of joint provision by two governments, each government chooses the amount of contribution to maximize the national welfare. The optimality condition for the government of country $i$ is

$$
\begin{align*}
  x_i \left[ \frac{(x_{1k} + x_{2k})}{k} - \frac{(x_1 + x_2)}{k^2} \right] c' + \frac{k_i}{k^2} f (x_1 + x_2) + \frac{k_i}{k} f(x_{1k} + x_{2k}) = p^k, \quad j \neq i
\end{align*}
$$

Summing up the investment rule (13) for two countries and rearranging, we have

$$
\begin{align*}
  \left(\frac{x_1 + x_2}{k}\right)^2 c' + \left[ f - \frac{\langle x_1 + x_2 \rangle_{k} c'}{k} \right] (x_{1k} + x_{2k}) + \frac{f}{k} (x_1 + x_2) = 2p^k
\end{align*}
$$

This specification implies that the user cost function is homogeneity of degree zero in volume and capacity.
As in the case of non-rivalry, we examine the consequence of break-even pricing. Let us substitute the zero-profit condition, \( f(x_1 + x_2) - p^k k = 0 \) into the above equation, we have the following

\[
\left(1 - \frac{k(x_{1k} + x_{2k})}{x_1 + x_2}\right) \left[\left(\frac{x_1 + x_2}{k}\right)^2 c' - p^k\right] = 0
\]

The above equality holds when the condition for optimal capacity, (12) holds. And zero profit together with optimal capacity leads to (11), the optimal pricing rule. Thus we have

**Proposition 3** Under the break-even pricing, the first-best charge and capacity is attained by contributions from two governments, each of which seeks to maximize the national welfare.

The above proposition shows that the self-financing theorem by Mohring-Harwits (1962) is applicable to the case that the capacity is determined in a decentralized way. It is obvious that the first-best is attained under the zero profit and optimal capacity rule. New finding here is that the optimal capacity condition is derived from non-cooperative choices of contribution by the national governments.

### 4 Joint Provision vs Single Provision

We have not discussed whether joint provision is actually realized: two governments have incentive to join the infrastructure project. There are alternative ways to provide international infrastructure. One common alternative is that one of two countries builds and operates the transport infrastructure\(^5\). We call this case as "single provision" hereafter. Brueckner (2013) considers exactly this situation: the bridge between jurisdictions of mid city and suburb is controled by the government of suburb. Brueckner shows that, when the bridge capacity cost is financed by budget-balancing tolls, decentralized capacity choices by local government generate the social optimum.

This section examines whether joint provision is realized by the decisions of two governments seeking to maximize the national welfare. Each government chooses whether to join the infrastructure project by providing the contribution or not\(^6\). If one government makes a positive amount of contribution while another does not, the outcome is the single provision. There are four possible combinations of choices by two national governments: case \(Y Y\) (joint provision) in which both countries provide positive amount of contribution; case \(NN\) in which no country provides contribution; case \(YN\) (single provision by country 1) in which country 1 provides a contribution while country 2 does not; and vice versa for the case \(NY\). Let us denote the national welfare of country \(i\) for the four cases by \(W_{iY Y}, W_{iN N}, W_{iY N}, W_{iN Y}\), respectively.

\(^5\)Even in this case, two governments should agree on the infrastructure project, since it touches the territories of both countries.

\(^6\)The problem discussed in this section is similar to the voluntary participation of public good provision (Saijo and Yamato (1999), Furusawa and Konishi (2011)). The difference is that the infrastructure use is excludable. In this sense, our problem might be easier than the case of public good provision.
The conditions under which joint provision is Nash equilibrium are written as follows
\[ W_{YY} > W_{NY}^1 \text{ and } W_{YY} > W_{NY}^2 \]

### 4.1 Non-rival case

When the infrastructure use is non-rival, \( W_{YY}^i \) are obtained by substituting to (1) the capacity obtained in Section 2. In cases \( YN \) or \( NY \), the infrastructure charge and capacity is determined by the decision of the government implementing the infrastructure project. Without loss of generality, we consider the case \( YN \) in which country 1 builds and operates the infrastructure. The problem to be solved by the government of country 1 is

\[
\max_{f,k} \int_{f+t(k)}^{\infty} D_1(p) \, dp + f(x_1 + x_2) - p^k k
\]  

(15)

The optimality conditions with respect to the user charge and the capacity of infrastructure are respectively

\[
x_2 + f(x_1f + x_2f) = 0
\]

(16)

\[-x_1t' + f(x_1k + x_2k) = p^k
\]

(17)

Let us denote the solution of the above equation by \((f^{YN}, k^{YN})\). From (16) and (17), we have \( f^{YN} > 0 \) and \(- (x_1 + x_2)t' = p^k \). In words, user charge in the single provision is higher than the efficient level (i.e., zero), and the investment rule is efficient. Substituting \((f^{YN}, k^{YN})\) to the objective function in (15), we have \( W_{YN}^1 \). And we obtain \( W_{YN}^2 \).

We investigate the Nash equilibrium based on the specific forms of the demand function and user cost function as

\[
x_i = A_i \exp[-\alpha (f(t(k))]
\]

(18)

\[ t(k) = -\beta \ln k,
\]

(19)

where \(A_i, \alpha, \beta\) are parameters.

We have the following result in the case that break-even pricing is adopted in joint provision.

**Proposition 4** Suppose that break-even pricing is adopted in the case of joint provision. Under the demand and user cost functions given by (18) and (19), joint provision is Nash equilibrium if the following holds

\[
(1 - \alpha \beta) \frac{\alpha \beta}{1 - \alpha \beta} e^{\frac{\alpha \beta - \alpha (1 - \alpha \beta)}{(1 - \alpha \beta)^{1/2}}} < 1
\]

(20)

\[7\] In the case of single provision, the national government can control the operation of the infrastructure. So we assume that the government chooses the user charge and the capacity of the infrastructure. On the other hand, in the case of joint provision, no single government can choose the level of infrastructure charge by oneself. There are various alternative ways to determine the pricing policy, so we allow flexibility in pricing under joint provision.
where \( s \equiv \min\{\frac{A_1}{A_1 + A_2}, \frac{A_2}{A_1 + A_2}\} \), i.e., the share of demand from the smaller country.

**Proof.** See Appendix A.

In the special case that two countries are symmetric, \( s = 0.5 \), the inequality (20) is approximately equivalent to \( \alpha \beta < 0.4227 \). Joint provision is unlikely when two countries are asymmetric. In the very asymmetric case, \( s = 0.1 \), joint provision is realized only if \( \alpha \beta < 0.0994 \). Calibrated values in Mun and Nakagawa (2010) satisfy the inequality (20) even in this very asymmetric case.\(^8\)

### 4.2 Congestible Case

We follow the formulation in Section 3, to describe capacity choice in joint provision, case YY. For the case YN (single provision by country 1), the national government solves the following problem:

\[
\max_{f,k} \int_{f+c(x_1+x_2)}^{\infty} D_1(p) \, dp + f(x_1 + x_2) - p^k \tag{21}
\]

The optimality conditions with respect to the user charge and the capacity of infrastructure are respectively

\[
-x_1 \left( 1 + c' \left( \frac{x_1 f + x_2 f}{k} \right) \right) + (x_1 + x_2) + f(x_1 f + x_2 f) = 0 \tag{22}
\]

\[
-x_1 c' \left( \frac{x_1 + x_2}{k^2} \right) + \frac{(x_1 k + x_2 k)}{k} + f(x_1 k + x_2 k) = p^k \tag{23}
\]

(22) is rewritten as follows

\[
f = (x_1 + x_2) \frac{c'}{k} - \frac{x_2}{(x_{1p} + x_{2p})} \tag{24}
\]

where \( x_{ip} \equiv \frac{\partial D_1}{\partial (f+c(x_1+x_2))} < 0 \). The first term on the RHS of (24) is the congestion externality, and the second term is the mark-up, so the user charge in the single provision is higher than the optimal level.\(^9\) The investment rule (23) is reduced to the same as in the social optimum, (12). However, due to the excessively high user charge, the capacity under single provision is smaller than that in the social optimum.

We have the following result.

**Proposition 5** *Joint provision with break-even pricing is Nash equilibrium.*

\(^8\)Calibrated value is \( \alpha \beta = 0.0499 \). The details of the calibration are provided in the working paper version that is downloadable at http://www.econ.kyoto-u.ac.jp/~mun/papers/Pricing_and_investment091006.pdf

\(^9\)As shown by Proposition 3, joint provision with break-even pricing attains the first-best, in which infrastructure charge is equal to the congestion externality.
Proof. See Appendix B. ■

Consider the choice of country 1 between cases YY and NY. In both cases, the national welfare of country 1 is equal to the users’ benefit, so depend solely on the full price of transportation, \( f + c \left( \frac{x_1 + x_2}{k} \right) \). In the case NY, users in country 1 incur the higher full price than the case YY since the user charge is higher and the capacity is smaller. So the country is better off by choosing the joint provision.

So far, we assume that, under the single provision, the national government providing the infrastructure rationally maximizes the national welfare by choosing the high user charge and smaller capacity. However, as discussed in Section 2.4, it might be also the case that the break even pricing is adopted in the single provision. So we examine this case and obtain the following result.

**Proposition 6** If the break-even pricing is adopted in both joint provision and single provision of the congestible infrastructure, two cases yield the same outcome, and they attain the first-best optimum.

Proof. See Appendix C. ■

The optimality of the single provision with break-even pricing is already known: Brueckner (2013) obtains essentially the same result based on the model of locational equilibrium in a monocentric city with multiple jurisdictions. Proposition 5 is obtained by combining this result with Proposition 3. Now we know that the joint provision is indifferent from the single provision. This result suggests that the countries may not undertake the joint provision. Even if two cases attain the same outcome by adopting break-even pricing, the joint provision would require the transaction cost in the process to reach agreement on design of the facility, pricing policy, organization, etc.

5 Conclusion

This paper examines the scheme for joint provision of international transport infrastructure, in which two countries jointly establish an operator of the infrastructure that are responsible of collecting the user charge, maintenance, etc.; the costs for infrastructure investment are covered by financial contributions from two countries; and the revenue from infrastructure charge is distributed according to the share of contributions. Assuming that the infrastructure use is non-rival, we show that financing infrastructure by the revenue from user charges is better than financing by tax revenue. We extend the analysis by incorporating congestion in infrastructure use. It is shown that independent decisions on contributions by two governments attains the first-best optimum when the operator sets the user charge so that the toll revenue just covers the cost for the investment. We further investigate the governments’ choice between joint provision and single provision. For reasonable values of parameters, joint provision is realized in Nash equilibrium.
Appendix A: Proof of Proposition 4

For the specified functions (18) and (19), the equation to determine the contribution from the country \( i \) in the case of joint provision (case YY), (6), is written as follows

\[
\alpha \exp[-\alpha f]k^{\alpha\beta-2} \{ fk_j(A_1 + A_2) + \beta kA_1 + \alpha \beta f k_i (A_1 + A_2) \} = p^k, \quad j \neq i \quad (25)
\]

The second order condition is

\[
1 - \alpha \beta > 0
\]

Aggregating (25) for two countries and solving the resulting equation for \( k \), we have

\[
K^J(f) = \left( \frac{\exp[-\alpha f](A_1 + A_2)(f + \beta + \alpha \beta f)}{2 p^k} \right)^{\frac{1}{1-\alpha\beta}}
\]

The operator’s response under break-even pricing, \( F^b(k) \), is derived from \( f(x_1 + x_2) - p^k k = 0 \). Then we have the solution as follows,

\[
f^{YY} = \frac{\beta}{1 - \alpha \beta}
\]

\[
k^{YY} = \left( \frac{\exp[-\frac{\alpha \beta}{1-\alpha\beta}](A_1 + A_2)\beta}{p^k (1 - \alpha \beta)} \right)^{\frac{1}{1-\alpha\beta}}
\]

The formula to calculate the national welfare (1) becomes \( \frac{x_i}{\alpha} + \frac{k_i}{K} \Pi \). Putting the above solution, we have

\[
W^{YY}_i = \frac{A_1}{\alpha} \left( \frac{(A_1 + A_2)\beta}{p^k (1 - \alpha \beta)} \right)^{\frac{1}{1-\alpha\beta}} \exp \left[ -\frac{A_1}{(A_1 + A_2)(1 - \alpha \beta)} \right]
\]

In the single provision, the user charge and the capacity of the infrastructure are determined by (16) and (17). In the case NY where country 2 provides the infrastructure, the solution is

\[
f^{NY} = \frac{A_1}{\alpha(A_1 + A_2)}
\]

\[
k^{NY} = \left( \frac{(A_1 + A_2)\beta}{p^k} \right)^{\frac{1}{1-\alpha\beta}} \exp \left[ -\frac{A_1}{(A_1 + A_2)(1 - \alpha \beta)} \right]
\]

\[
W^{NY}_i = \frac{A_1}{\alpha} \left( \frac{(A_1 + A_2)\beta}{p^k} \right)^{\frac{1}{1-\alpha\beta}} \exp \left[ -\frac{A_1}{(A_1 + A_2)(1 - \alpha \beta)} \right]
\]

The expressions for the case YN are obtained likewise.

Substituting the above results to the conditions for the joint provision to be Nash equilibrium, \( W^{YY}_1 > W^{NY}_1 \) and \( W^{YY}_2 > W^{YN}_2 \), we have

\[
(1 - \alpha \beta)\frac{\alpha \beta}{1-\alpha\beta} e^{\frac{\alpha \beta}{(1-\alpha\beta)^2}} - \frac{A_1}{(A_1 + A_2)(1 - \alpha \beta)} < 1
\]

\[
(1 - \alpha \beta)\frac{\alpha \beta}{1-\alpha\beta} e^{\frac{\alpha \beta}{(1-\alpha\beta)^2}} - \frac{A_2}{(A_1 + A_2)(1 - \alpha \beta)} < 1
\]
It is seen that the inequality for the smaller country is critical. Thus (20) is the condition for Nash equilibrium.

Appendix B: Proof of Proposition 5

The conditions for the joint provision to be Nash equilibrium are $W_{1}^{YY} > W_{1}^{NY}$ and $W_{2}^{YY} > W_{2}^{YN}$.

We examine $W_{1}^{YY} > W_{1}^{NY}$ first. Let us simplify the notation as $x \equiv x_{1} + x_{2}$, $x_{f} \equiv x_{1f} + x_{2f}$, $x_{k} \equiv x_{1k} + x_{2k}$. The national welfare of country 1 in two cases are

\[
W_{1}^{YY} = \int_{f_{YY} + c \left( \frac{x}{k_{YY}} \right)}^{\infty} D_{1} (p) \, dp
\]

\[
W_{1}^{NY} = \int_{f_{NY} + c \left( \frac{x}{k_{NY}} \right)}^{\infty} D_{1} (p) \, dp
\]

Note that the profit from the infrastructure project disappears in the case YY, since break-even pricing is adopted. Therefore, $W_{1}^{YY} > W_{1}^{NY}$ is equivalent to $f_{YY} + c \left( \frac{x}{k_{YY}} \right) < f_{NY} + c \left( \frac{x}{k_{NY}} \right)$. As shown in the Section 3, under the joint provision with break-even pricing, the infrastructure charge is equal to the congestion externality, i.e., $f_{YY} = x_{f}^{c} \frac{c}{k}$. On the other hand, the infrastructure charge under single provision (case NY) is $f_{NY} = \frac{x_{1}^{c}}{k} - \frac{x_{1}}{x_{p}}$ from (24). Thus for given $k$, $f_{YY} < f_{NY}$.

The investment rule in both cases is (12). Totally differentiating (12) yields the following.

\[
\left[ c' \left( \frac{-2x^{2}}{k^{3}} + \frac{2x \cdot x_{k}}{k^{2}} \right) + c'' \left( \frac{x^{2}}{k^{2}} \right) \left( \frac{-x}{k^{2}} + \frac{x_{k}}{k} \right) \right] \, dk + \left[ c' \left( \frac{2x \cdot x_{f}}{k^{2}} \right) + c'' \left( \frac{x^{2}}{k^{2}} \right) \left( \frac{x_{f}}{k} \right) \right] \, df = 0
\]

The first bracket is negative from the second-order condition for optimality. And the second bracket is negative since $x_{f} < 0$. Thus $\frac{dk}{df} < 0$ should hold on the locus of (12). Synthesizing the above results, we have $f_{YY} < f_{NY}$ and $k_{YY} > k_{NY}$. Thus $f_{YY} + c \left( \frac{x}{k_{YY}} \right) < f_{NY} + c \left( \frac{x}{k_{NY}} \right)$. In words, in the case NY, users in country 1 incur the higher full price than the case YY since the user charge is higher and the capacity is smaller. So the country 1 is better off by choosing the joint provision. $W_{2}^{YY} > W_{2}^{YN}$ is shown in a similar manner.

Appendix C: Proof of Proposition 6

The government providing the infrastructure chooses the user charge and the capacity, subject to the break-even condition. The problem to be solved is

\[
\max_{f,k} \int_{f + c \left( \frac{2x_{1} + x_{2}}{k} \right)}^{\infty} D_{1} (p) \, dp + f \left( x_{1} + x_{2} \right) - p^{k} k
\]

s.t. $f \left( x_{1} + x_{2} \right) - p^{k} k = 0$
The optimality conditions with respect to $f$ and $k$ are respectively

$$
-x_1 \left( 1 + c' \frac{(x_1 f + x_2 f)}{k} \right) + (1 + \lambda) \left[ (x_1 + x_2) + f(x_1 f + x_2 f) \right] = 0
$$

$$
-x_1 c' \left( -\frac{(x_1 + x_2)}{k^2} + \frac{(x_1k + x_2k)}{k} \right) + (1 + \lambda) \left[ f(x_1k + x_2k) - p^k \right] = 0
$$

where $\lambda$ is the Lagrange multiplier of the break-even constraint. Combining the two optimality conditions to eliminate the Lagrange multiplier yields the following

$$
\left( 1 - \frac{(x_1k + x_2k)}{(x_1 + x_2)k} \right) \left( c' \frac{(x_1 + x_2)^2}{k^2} - p^k \right) = 0
$$

The above equality holds when the condition for optimal capacity, (12) holds. And this optimal capacity together with break-even condition leads to (11), the optimal pricing rule.

References


Figure 1  Infrastructure charge and global welfare

Figure 2  Response functions of governments and operator