Economic modeling of traffic congestion that captures both demand side behavior of travelers and supply side of congestion technology in a realistic and mathematically tractable way is a challenging task. Nevertheless, potentially large benefits from the use of such model motivate the endeavor. In part, these benefits stem from a more precise welfare assessment of various transport policy measures.

Our paper proposes a new model of recurrent traffic congestion which arises due to a daily commuting of workers from suburban residential areas to a central business district. We employ a well-known Vickrey’s (1969) dynamic bottleneck model in combination with the recently developed models of bathtub urban congestion (Arnott, 2013; Fosgerau and Small, 2013; Fosgerau, 2014). Our model is able to account for several observed empirical regularities in road transportation, such as (i) hypercongestion, i.e., an increase of travel time with an increase of the cars density above certain threshold level, (ii) traffic jam, i.e., a complete stop of a traffic flow for a certain period of time and (iii) urban-wide diffusion of congestion (see, among others, Geroliminis and Daganzo, 2008, and Daganzo et al., 2011). While previous economic literature has certain difficulty in accounting for these empirical regularities, our model is able to produce these features in user equilibrium. Moreover, we aim to test welfare implications of optimal road pricing, road capacity improvements and other policy measures in a more realistic and richer setup, as compared to the previous studies.

The supply side of the model is as follows. We study travel dynamics in a hypothetical central business district of a city (hereafter, the CBD) which is a workplace location for homogeneous, atomistic daily commuters from outside of the city. Drivers travel from home to work on a congestion-free road up to the entrance to the CBD. A traffic bottleneck is located at the entrance to the CBD. One might think of a bridge or a junction that the drivers have to pass to get to the CBD. If the incoming traffic flow of drivers exceeds the bottleneck capacity level, a queue emerges in front of the bottleneck, just as in the standard bottleneck model. After a driver passes the bottleneck, she enters the CBD. To reach her final destination within the CBD, a driver incurs travel time which is a function of the total number of drivers travelling within the CBD at each time instance. Specifically, this travel time is longer if there are more drivers travelling in the CBD. One might think of this travel time as a required time to
get through an intrinsic traffic flow within the city centre, which public transport, pedestrians and cyclists create. The more drivers there are on streets of the city, the slower the flow is, and the more time it takes to reach the workplace.

Important feature of our model is the spillback effect that the drivers within the CBD impose on a capacity of the bottleneck at the entrance to the CBD. Large density of drivers within the city causes the bottleneck capacity to decline. Bottleneck capacity drops to zero in the limiting case, and this brings the incoming traffic flow at the entrance to the CBD to a standstill. Traffic jam is thus occurs at the entrance to the CBD. To put it differently, an increasing density of cars within the CBD assumes a role of a cork for a bottleneck which becomes less capacious. A cork function governs the bottleneck capacity level, as cars density changes within the CBD. Specifically, cork function relates contemporaneous bottleneck capacity and accumulated over time cars density. Such lagged time dependency of the bottleneck capacity and the cars density generates convenient way of modeling traffic jam. A parking function, which defines arrival times of cars to their final destinations within the CBD, also depends on cars density. Dynamic discrepancy of the intensities of parking and car inflow produces a rich set of results, including hypercongestion and traffic jam.

The demand side of the model is driven by the utility function used in the dynamic bottleneck model (Arnott et al., 1993), under which homogeneous drivers have a common preference for arriving at a workplace at certain moment in time. We follow convections of the standard bottleneck model and assume “––” utility specification. That is, each driver derives time-invariant marginal utilities from each additional time unit spent at home and in a vehicle. A piece-wise linear marginal utility of being at a work is such that before (after) preferred arrival time it is below (above) marginal utility of being at home. User equilibrium in the cork model satisfies dynamic equilibrium condition which postulates that no individual driver has incentives to unilaterally change her behavior. The only margin of behavior in our model is departure time from home.