Duality and second-best congestion pricing: compensated or uncompensated demand∗

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Abstract

We explore the characteristics of first- and second-best congestion pricing rules under different measures of total (societal) benefit. In particular, we study whether and under which conditions the well-known pricing rules based on the uncompensated Marshallian demand function as measure of total benefit are equal to pricing rules based on compensated Hicksian demand functions. Our theoretical analysis will be supported by numerical simulation models.

Both normative and positive economic analysis need to address issues concerning the correct measurement of the well-being of individuals and the public policy measures to be taken to improve this well-being. The contribution of Harberger (1971) has spurred a wave of research into measuring welfare and its application in public policy appraisal. He argued that Marshallian consumer surplus should form the core of applied welfare economics. In a reaction, Chipman and Moore (1980) show that consumer surplus can only be an exact measure of the change in welfare under the assumption that the demand function is integrable and consistent with a well-behaved utility function on the one hand, and a constant marginal utility of income on the other hand. Although shown to be theoretically incorrect, consumer surplus has the advantage to rely on the observable Marshallian demand function.

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resulting in modest data requirements. This is presumable the main reason for its remained popularity in applied economics ever since (Slesnick, 1998).

There is a clear consensus amongst scholars that the compensated Hicksonian demand function represents the correct measurement. In this article, however, we show that for a large set of public policy questions that can be characterized as first- and second-best problems the choice between compensated or uncompensated demand functions does not alter the policy rules. We use the classic two-route problem as our leading example. We assume two parallel congestible routes and a constraint preventing the government from levying a fee on both roads (Lévy-Lambert, 1968; Verhoef et al., 1996; Small and Verhoef, 2007).

For example, we analyse the appropriate Lagrangian which the regulator needs to solve in the simplest case where both roads are perfect substitutes. The total (consumer) benefits that are included in this optimization problem are usually defined as the area under the uncompensated Marshallian demand function, however we use the more appropriate Hicksian demand functions instead. A close inspection of the first-order conditions in both cases, reveals that the only difference between them is that the second-best pricing rule would be evaluated at Hicksian instead of Marshallian inverse demand. The expressions for the Wardrop equilibrium constraints are the same (Wardrop, 1952), just as for the marginal (external) cost levels. Hence, if the inverse Hicksian demand equals the inverse Marshallian demand, both pricing rules will yield equal outcomes.

We show that if evaluated at optimal output (road use) levels, the inverse of the Hicksian demand function that is based on the original utility level before the price change will be equal to the inverse of the Marshallian demand function. Hence, applying the Hicksian demand function based on compensating variation will yield same pricing rules as applying Marshallian demand function. If one would evaluate the Hicksian demand function at the utility level users would attain after the price change, hence equivalent variation, the Hicksian and Marshallian price rules diverge. This divergence in positively related to the magnitude of the implied price change by the second-best policy.

We extend this particular analysis in several directions. First, we allow for more roads in our network. As long as the roads are perfect substitutes, or there are no demand interdependencies, the insights are the same as for the standard two-route problem. Second, we allow for demand and cost interactions between routes, hence for imperfect substitutes and for multiple
price changes. For multiple price changes we find that the pricing rules are not path dependent, but the actual equilibrium outcomes are.

We use a numerical analysis to show clearly how duality can be used to analyse whether the theoretical difference between Markhalian and Hicksian demand functions plays a role in determining first- and second-best pricing rules. The numerical analysis shows that the results hold for well-known specification of the utility functions (and implied demand systems), such as Cobb-Douglas, CES and quadratic utility function.

Our analysis shows that for a large set of constrained optimization problems, evaluation at the optimum causes the Hicksian inverse demand function to be equal to its Marshallian counterpart, resulting into consistent pricing rules. Although the message is subtle, it is by no means trivial: it provides the theoretical justification for using simple Marshallian inverse demand functions as measure for total benefits when considering first- and second-best policy rules using instruments that can be adjusted at the margin.

Key references


