ABSTRACT

Travelcards are used in many parts of the world as a form of payment for public transport that is convenient for frequent users. In essence it involves a one-time payment $T$ at the beginning of a period that covers all trips within that period. Carbajo (1988) applies the two-part tariff approach to find the optimal (welfare maximizing) value for $T$ assuming a nil effect of $T$ on the demand schedule of each and every individual (no income effect). Here we deal with an urban area where individual trips increase with income, but where car ownership – correlated with income – makes the public transport share diminish towards high income segments. A theoretical model is developed to find the optimal values (maximum social welfare with a budget constraint) for $T$ and, simultaneously, for a single ticket $P$, considering the effect of $T$ on available income as well as differences across individuals regarding car ownership. The model is applied using parameters associated with monthly travel in Santiago, Chile, where both income and car ownership are highly concentrated and correlated. We obtain that the two richest segments choose to pay for the single ticket and the other eight choose to buy the travelcard.

Keywords: travelcard, income, pricing
1 INTRODUCTION

Travelcards are used in many parts of the world as a method to facilitate and induce the use of public transport (PT). They are offered as an alternative to the single ticket that consists of a fixed fee that allows unlimited trips within its validity period. Generally, this ticket type is convenient for the city residents (and even for visitors if they stay long enough), because users compares the travelcard value ($T$) with the mandatory expenditure that they should pay for the expected trips using the single ticket ($P$). Table 1 contains the monthly travelcard values (there are also daily, weekly and annual) and the single ticket price for five European cities; the bottom row shows the number of equivalent trips that users can make if they spent the value of the travelcard in single tickets. Note that in all five cities the travelcard is worth buying even if individuals travel less than twice a day.

<table>
<thead>
<tr>
<th>City</th>
<th>Madrid</th>
<th>Paris</th>
<th>Rome</th>
<th>Berlin</th>
<th>London</th>
</tr>
</thead>
<tbody>
<tr>
<td>Travelcard [€]</td>
<td>54.6</td>
<td>65.1</td>
<td>35</td>
<td>78</td>
<td>139</td>
</tr>
<tr>
<td>Single ticket [€]</td>
<td>1.5 – 2</td>
<td>1.7</td>
<td>1.5</td>
<td>2.6</td>
<td>5.4</td>
</tr>
<tr>
<td>Equivalent trips</td>
<td>36 – 27</td>
<td>38</td>
<td>23</td>
<td>30</td>
<td>26</td>
</tr>
</tbody>
</table>

Experiences around the world have been studied by White (1981), FitzRoy and Smith (1998, 1999) and Gschwender (2007), among others. The travelcard literature follows the two-part tariff theory (Oi, 1971; Feldstein, 1972; Brown and Sibley, 1986; Wilson, 1997), dismissing income effects. This theory is applied by Carbajo (1988) to the PT market in order to find the optimal $P$ and $T$ using a taste parameter to differentiate across users. In this paper we develop and apply a theoretical model to find the optimal values (maximum social welfare with a budget constraint) for a travelcard and, simultaneously, for a single ticket, considering the effect of $T$ on the available income as well as differences across individuals regarding car ownership. As these effects work in opposite direction, the single taste parameter approach is not applicable.

In the next section the approach developed by Carbajo (1988) is summarized, showing its limitations regarding income. In section 3 an analytical model that captures both effects mentioned above is developed. Next the model is applied to Santiago, Chile, a city where both income and car ownership are highly concentrated and correlated. The final section contains a synthesis, conclusions and directions for further research.

2 TASTE OR INCOME

2.1 Carbajo (1988) model

A two part tariff $(P,T)$ consists in separating the charge for a product into an entry fee $T$ (to be allowed into the market) and a payment $P$ for each unit consumed, as used in water, electricity and telecommunication markets. This structure allows gains in efficiency due the possibility of lowering the price to get $P$ near the marginal cost, but it also can induce consumers to exit the market because they refuse to pay the entry fee. The lower the price $P$ the higher $T$ should be in order to cover the producers’ costs, causing different reactions in consumers (assuming they are not all equal).
The differences among consumers are mainly treated in two forms: by means of their income, as in Oi (1971) and Feldstein (1972), or by means of a taste parameter that is distributed in the population and that represents consumption intensity, as done by Brown and Sibley (1986) and Wilson (1997). In both treatments, however, the effect of \( T \) on the individual income (and, therefore, on the demand curve) is considered negligible; Oi assumes that demands are invariant to changes in income or in the entry fee and Feldstein considers that the effect is negligible because \( T \) is only a little fraction of the consumers income. No author considers the possibility that there could be markets (like public transport) or segments of the population where this assumption might not be reasonable.

Aiming at finding the optimal value of travelcards, Carbajo (1988) extends the Brown and Sibley model to the PT market, representing both the travelcard and the single ticket as special cases of a two part tariff \((P, T)\): the travelcard as \((0, T)\) and the single ticket as \((P, 0)\). Under this consideration, each user will choose the alternative that yields the largest consumer’s surplus. Following the demand scheme represented in Figure 1a, the surplus \(CS_P\) gained by a user who chooses single ticket is \(A\), with \(Q_P\) trips per period. With the travelcard the surplus \(CS_T\) will be \(A + B - T\), making \(Q_T\) trips regardless the value of \(T\); the election will depend on the sign of \(B - T\).

![Figure 1. Single ticket and travelcard surplus according to Carbajo (1988)](image)

In Figure 1b we represent three individuals (Active, Middle and Passive) with different PT demands schemes. For any single ticket price, their trips will fulfill the condition \(Q_{act}(P) > Q_{mid}(P) > Q_{pas}(P)\). This implies that if \(T = d + e\) each user will choose as shown in Table 2.

<table>
<thead>
<tr>
<th>User</th>
<th>(CS_P)</th>
<th>(CS_T)</th>
<th>Election</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active</td>
<td>(a + b + c)</td>
<td>(a + b + c + f)</td>
<td>Choose (T)</td>
</tr>
<tr>
<td>Middle</td>
<td>(a + b)</td>
<td>(a + b)</td>
<td>Indifferent</td>
</tr>
<tr>
<td>Passive</td>
<td>(a)</td>
<td>(a - e)</td>
<td>Choose (P)</td>
</tr>
</tbody>
</table>

Carbajo assumes that users differ in terms of a taste parameter \(\theta\) that represents the intensity of use of PT. For a given pair \((P, T)\), there will be an indifferent user \(\bar{\theta}\), characterized by \(CS_P = CS_T\), that will establish the limit between users who choose \(P\) or \(T\). Specifically, users with \(\theta > \bar{\theta}\)
choose buying the travelcard and those with $\theta < \bar{\theta}$ choose the single ticket. Carbajo assumes that we know the taste parameter density $h(\theta)$, its cumulative distribution $H(\theta)$ and the range where it belongs, $\theta \in [\underline{\theta}, \overline{\theta}]$. With this information, the trips made by a user $\theta$ are $q(P, \theta)$ for a single ticket user and $q(0, \theta)$ for a travelcard user, for any value of $T$. The indifferent user is implicitly determined by the expression that equals the surpluses obtained from $T$ and $P$ as shown in equation (1), which yields a function $\bar{\theta}(P, T)$.

$$\int_{\underline{\theta}}^{\overline{\theta}} q(p', \theta)dp' - T = \int_{\underline{\theta}}^{\overline{\theta}} q(p', \theta)dp'$$  

(1)

Considering a linear cost function $C = F + m \cdot Q$, where $F$ is the fixed cost and $m$ the trip marginal cost, the social welfare for given $P$ and $T$ is the sum of the users’ consumer surplus and the operators’ profits, that is,

$$SW = \bar{\theta}(P, T) \int_{\underline{\theta}}^{\overline{\theta}} \left[ \int_{P}^{\infty} q(p', \theta)dp' \right] \cdot h(\theta') \cdot d\theta' + \int_{\bar{\theta}(P, T)}^{\overline{\theta}} \left[ \int_{0}^{\infty} q(p', \theta)dp' - T \right] \cdot h(\theta') \cdot d\theta' +$$

$$+ \int_{\underline{\theta}}^{\bar{\theta}(P, T)} (P - m) \cdot q(p, \theta') \cdot h(\theta') \cdot d\theta' + \int_{\bar{\theta}(P, T)}^{\overline{\theta}} [T - m \cdot q(0, \theta')] \cdot h(\theta') \cdot d\theta' - F$$  

(2)

The first and the second terms represent benefits of the single ticket and travelcard users, respectively; the other three are the profits of the operators. Carbajo maximizes this $SW$ function under a budget constraint to obtain the optimal values, $P^*$ and $T^*$. Then the taste parameter for the indifferent user $\bar{\theta} = \bar{\theta}(P^*, T^*)$ and the corresponding trips for each user are obtained.

Carbajo says that $\theta$ may be thought of as a vector of characteristics or as the individual’s income, but the consequences of including income are not discussed, particularly the fact that the expenditure in PT is not always negligible with respect to income and, in consequence, the two part tariff assumptions of no income effect and invariance of $Q_T$ with $T$ could not be directly applicable to this model.

### 2.2 The role of income

If users are identified by their income, the role of this variable in PT demand has to be understood. Let us look at a case that represents the reality in most Latin American capital cities. In 2002 an origin-destination survey (EODH) was made in Santiago, Chile, where 15 thousand families were asked for their transports habits. People were classified into ten income segments and the car ownership (CO, cars per thousand individuals) of every segment was obtained as shown in Figure 2a: CO increases sharply with income, such that the richest segment exhibits nine times the CO of the poorest segment; this explains the yellow dots in Figure 2b that shows the monthly car trips by income segment. The red dots are the total trips. Both curves present a similar shape (logistic), with trips growing faster at the beginning until a constant is reached asymptotically. The difference between these two curves (blue dots) represents the PT trips,
which decrease with income: poor people use PT more intensively. In other words, urban trips increase with income but a car ownership effect makes the demand for PT trips in higher income segments smaller than the demand of individuals in lower income segments, as represented in Figure 3b.

![Figure 2. Car ownership and trips variation with income](image)

Let us consider now an individual within a given income segment $I_l$, whose PT trips demand is $X(P, I_l)$. If he chooses to travel paying with the single ticket $P_0$, he will make $X(P_0, I_l)$ trips, but, if he chooses to pay $T_0$ for the travelcard, demand in the $(P, X)$ space will move towards the origin, as shown in Figure 3a. In this case, the user will make $X(0, I_l - T_0)$ trips, less than $X(0, I_l)$ as assumed in the literature. The effect of paying for a travelcard on the PT demand diminishes as $I_l$ increases, as represented in Figure 3c.

![Figure 3. Demand movements with income](image)

Therefore, two differences with Carbajo (1988) appear when introducing income in the travelcard analysis: i) travelcard trips depend on the travelcard value – they are independent for Carbajo - because of the available income effect that can be relevant in some segments; and ii) the effect at an individual level goes in opposite direction with respect to what happens across individuals belonging to different segments because of CO. The double effect of income makes it necessary to reformulate the travelcard problem.
3 ANALITICAL MODEL

3.1 General formulation and the role of the indifferent income.

Let us consider an individual in an income segment $I$ and whose trips in PT are represented by a function $X(P,T,I)$, that has associated an inverse $P = X^{-1} = P(X,T,I)$. Following Figure 4 the user obtains a surplus $A$ if he chooses to pay with single ticket (making $X_P$ trips) or $A + B - T$ if he buys the travelcard, making $X_T = X(0,T,I)$ trips that depend on the value of $T$ and are less than $X(0,0,I)$. The analytical expressions for the surpluses are shown in equations (3).

![Figure 4. Single ticket and travelcard surplus considering income as purchasing power.](image)

$$CS_P = \int_{P}^{\infty} X(\phi,0,I) d\phi \quad \quad \quad \quad CS_T = \int_{0}^{X(0,T,I)} P(\phi,0,I) d\phi - T \quad (3)$$

For given $P$ and $T$, the equality $CS_P = CS_T$ defines an individual with income $\bar{I}(P,T)$ who is indifferent between both payment alternatives, that is when $T = B$ in Figure 4. The indifferent user establishes the limit between users who choose $P$ or $T$. Let us analyze which alternative is chosen by individuals with $\bar{I}_I$. Following Figure 5, consider an individual with income $\bar{I}_I$ that is indifferent between $P_0$ and $T_0$, i.e. $T_0 = B + \delta_2$ such that $CS_{P_0} = CS_{T_0}$. If the single ticket price is raised to $P_1 > P_0$, keeping $T_0$ constant, the user will obtain a surplus of $A$ if he chooses $P_1$. With this, $CS_{P_1} = A < A + \delta_1 = CS_{P_0} = CS_{T_0}$ such that he will choose the travelcard over $P_1$. To keep the indifference of the individual it is necessary to raise the travelcard value in order to reduce its surplus and match it with the single ticket surplus. So, there is $T_1 > T_0$ that makes the individual with income $\bar{I}_I$ indifferent between $T_1$ and $P_1$. Analogously, when the single ticket price is lowered to $P_2 < P_0$ the individual chooses $P_2$ over $T_0$; it would be necessary to lower $T$ to keep the indifference (point 2: $T_2$ equilibrates $P_2$). The three indifferent pairs are shown in the $(P,T)$ plane in Figure 5: $(P_0,T_0)$, $(P_1,T_1)$ and $(P_2,T_2)$ represented by points 0, 1 and 2 respectively. Finally, it is easy to show that any individual is indifferent between the alternatives in the pair $(P,T) = (0,0)$, because both surpluses are represented by the whole area behind the demand function. We conclude that there is an indifference curve $IC$ in the $(P,T)$ plane that increases from the origin where the individual with income $\bar{I}_I$ is indifferent between $P$ and $T$. 
The same exercise of raising or lowering the single ticket price can be made at any point of the IC, which shows that the IC divides the \((P,T)\) plane in two regions: the individual of income \(I_i\) will choose the travelcard if it is offered any pair located above his IC, and the single ticket if offered a pair located below his IC, as shown in Figure 6.

Individuals belonging to different income segments will have different IC. Following Figure 7 consider two individuals from different income segments such that \(I_{\text{Poor}} < I_{\text{Rich}}\).

Based on the analysis of Figure 4, the travelcard value that equilibrates \(P_0\) is represented by \(B_{\text{Rich}} = T_{\text{Rich}}^0\) for \(I_{\text{Rich}}\) and by \(B_{\text{Poor}} = T_{\text{Poor}}^0\) for \(I_{\text{Poor}}\). As \(B_{\text{Poor}} > B_{\text{Rich}}, T_{\text{Poor}}^0 > T_{\text{Rich}}^0\), which implies that the IC of the rich individual is to the left of (or above) the IC of the poor.
Finally, when a pair \((P^*, T^*)\) is offered to the population there will be an indifferent user represented by the income \(\tilde{I}(P^*, T^*)\), that will have the pair \((P^*, T^*)\) on his \(IC\), as is shown in Figure 8. As explained through Figure 7, segments with \(I > \tilde{I}\) have their \(IC\) above that of \(\tilde{I}\). As the offered pair falls below their \(IC\), they will choose to travel paying \(P^*\) for each trip, as obtained in Figure 6. Analogously, segments with \(I < \tilde{I}\) will buy the travelcard.

![Figure 8. Election based on \(\tilde{I}(P^*, T^*)\)](image)

If the indifferent income was known, the choices would be known as well and an expression for the social welfare \(SW\) could be written, i.e.

\[
SW = \sum_{I > \tilde{I}} N_i \cdot CS_p + \sum_{I < \tilde{I}} N_i \cdot CS_T + (P - m) \cdot \sum_{I > \tilde{I}} N_i \cdot X(P, 0, I) + \sum_{I < \tilde{I}} N_i \cdot T - m \cdot \sum_{I < \tilde{I}} N_i \cdot X(0, T, I) - F
\]  

(4)

where the surpluses are specified in equation (3), \(N_i\) is the number of individuals in the segment of income \(I_i\), \(m\) is the marginal cost for trips and \(F\) is the operators’ fixed cost. The first two terms are the consumers’ surpluses and the last four, the operators’ profits. Note that the income from the single tickets is associated to trips, while the income from the travelcards is associated to the number of individuals who buy it.

Maximizing \(SW\) subject to a budget constraint leads to

\[
\frac{P - m}{P} = \frac{\theta}{\sum_{I > \tilde{I}} \alpha_i \cdot |\eta_{P, I_i}|}
\]

(5)

\[
T = \sum_{I < \tilde{I}} \beta_i \cdot |\eta_{T, I_i}| \cdot X(0, T, I_i) \cdot \lambda^{-1} \cdot \{P[X(0, T, I_i), 0, I_i] - m \cdot (1 + \lambda)\}
\]

(6)

where \(\theta = \frac{\lambda}{1 + \lambda}\) with \(\lambda\) the multiplier of the restriction, \(\alpha_{I_i} = N_i \cdot X(P, T, I_i) / \sum_{I > \tilde{I}} N_i \cdot X(P, T, I_i)\) and \(\beta_{I_i} = N_i / \sum_{I < \tilde{I}} N_i\).

Expression (5) shows that the optimal single ticket value makes the mark up ratio proportional to the inverse of the sum of the price elasticity of the demands (\(\eta_{P, I}\)), each weighted by a coefficient \(\alpha_{I_i}\) that represents each segment PT trips proportion with respect to the total. Expression (6) indicate that the optimal value of the travelcard is the sum, over all segments that choose \(T\), of the
product of the proportion of individuals per segment (β_\text{i}), the elasticity of the demand with respect to \( T (\eta_{T,i}) \), and a term that depends on the PT demand with travelcard and the difference between its associated willingness to pay and the marginal cost amplified by the multiplier of the budget restriction; this suggests that \( T \) gets positive (exists) due to the presence of groups with strong propensity to use PT (low car ownership, strong income effect).

### 3.2 Solution to the problem with many income segments and unknown indifferent income

So far the indifferent income has been assumed known and, therefore, the payment alternative chosen by every income segment is known. However, the indifferent income depends on the solution \( (P^*, T^*) \). What to do? Assume that there are \( k \) income segments, with \( I_1 \) the poorest and \( I_k \) the richest, and let \( I_j < \bar{I} < I_{j+1} \). This means that the IC associated to \( \bar{I} \) is below the IC of the segment of income \( I_{j+1} \) and above the IC associated to \( I_j \). This implies that income segments with \( I \geq I_{j+1} \) will choose to travel paying \( P \) per trip, while segments with \( I \leq I_j \) will buy the travelcard. As curves intersect only at the origin, the \( k \) segments define \( k + 1 \) zones in the \( (P, T) \) space. In each of those cases an optimal pair \( (P_j^*, T_j^*) \) can be found by solving

\[
\begin{align*}
\text{Max} \quad & SW \quad \text{s.t.} \quad \pi > 0 \quad (\lambda) \\
& P > P(T, I_j) \\
& P(T, I_{j+1}) > P
\end{align*}
\]

where \( P(T, I_j) \) is the IC of the segment with income \( I_j \). This way the second and third constraint imposes that the solution belongs to the space between ICs that is compatible with the choices assumed in the SW expression. Solving the \( k + 1 \) cases, the one with the largest \( SW \) among those that are feasible will be the optimal solution of the problem.

### 4 APPLICATION

#### 4.1 Demand function, welfare measure and indifference curves.

The monthly PT trips made by a user with income \( I_i \) will be represented by

\[
X(P, T, I_i) = A_i - B_i \cdot \frac{P + \Delta_i}{I_i - T} \label{eq:8}
\]

where the parameter \( A_i \) is imposed as the total trips made (including private and public transport) by any individual with income \( I_i \). \( B_i \) is related to the elasticities with respect to the single ticket and to the travelcard, while \( \Delta_i \) is associated with those variables different from price that influence the choice between car and PT, which depends on car ownership. This comes from the idea that if public transport was free, there will still be people who would use the car anyway.

From equation (8) the consumers’ surpluses and the social welfare can be obtained as

\[
SW = \sum_{i > I} \frac{N_i I_i}{2B_i} \left( A_i - B_i \frac{P + \Delta_i}{I_i} \right)^2 + \sum_{i < I} \frac{N_i I_i}{2B_i} \left( A_i - B_i \frac{\Delta_i}{I_i} \left( 1 - \frac{T}{I_i - T} \right) \right) \left( A_i - B_i \frac{\Delta_i}{I_i - T} \right) - \sum_{i < I} N_i \cdot T
\]

\[
+ (P - m) \cdot \sum_{i > I} N_i \left( A_i - B_i \frac{P + \Delta_i}{I_i} \right) + \sum_{i < I} N_i \cdot T - m \cdot \sum_{i < I} N_i \left( A_i - B_i \frac{\Delta_i}{I_i - T} \right) - F \label{eq:9}
\]
The first three terms are the consumers’ surpluses and the other three are the operators’ profits. The terms associated to money spent on travelcards cancel out because it is a welfare transfer. Finally, from $CS_p = CS_T$ an expression for the IC of the segment with income $I_i$ is obtained:

$$P(T, I_i) = \frac{A_i \cdot I_i}{B_i} - \Delta_i - \frac{I_i}{B_i} \sqrt{\left( A_i - \frac{B_i \cdot \Delta_i}{I_i} \left( 1 - \frac{T}{I_i - T} \right) \right)} \left( A_i - \frac{B_i \Delta_i}{I_i - T} \right) - \frac{2T B_i}{I_i}$$  \hspace{1cm} (10)

Knowing the social welfare and the IC of every segment problems (7) can be solved.

### 4.2 Case parameters and solution for $P$ and $T$

The method was applied to the case of Santiago, Chile. In order to obtain representative parameters for the demand of each income segment we used data from the OD Survey (EODH) previously described, where 15 thousand families were randomly selected to study their travel patterns during weekdays, saturdays and sundays. Using a matching procedure we were able to construct weekly and monthly trip patterns representative of each of the 10 income segments. As explained earlier, $A_i$ is obtained as the total trips made. Next we used the observed single ticket price and the number of trips made by each income segment as one point of the demand curve in equation (8). A second point was obtained by estimating the maximum price that the users were willing to pay for PT. Data showed that the 78.4% of all the taxi trips cost less than 2000 Chilean pesos; we imposed this figure as the maximum willingness to pay for PT, assuming that a user would travel by taxi if the fare was higher. For short, the point $(0, 2000)$ belongs to the demand of each income segment. From these points the parameters $B_i$ and $\Delta_i$ were obtained; they are shown in Table 3 along with the size of each segment. The corresponding demands and IC are shown in Figure 9.

<table>
<thead>
<tr>
<th>Seg</th>
<th>Income</th>
<th>$A_i$</th>
<th>$B_i$</th>
<th>$\Delta_i$</th>
<th>$N_i$</th>
<th>Seg</th>
<th>Income</th>
<th>$A_i$</th>
<th>$B_i$</th>
<th>$\Delta_i$</th>
<th>$N_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74333</td>
<td>69.9</td>
<td>2612.5</td>
<td>390.8</td>
<td>67593</td>
<td>6</td>
<td>203535</td>
<td>75.7</td>
<td>5162.7</td>
<td>983.5</td>
<td>159398</td>
</tr>
<tr>
<td>2</td>
<td>119650</td>
<td>71.3</td>
<td>3346.5</td>
<td>549.6</td>
<td>234593</td>
<td>7</td>
<td>267811</td>
<td>80.5</td>
<td>6268.0</td>
<td>1440.5</td>
<td>218319</td>
</tr>
<tr>
<td>3</td>
<td>145970</td>
<td>72.4</td>
<td>3962.0</td>
<td>665.8</td>
<td>210603</td>
<td>8</td>
<td>363701</td>
<td>87.2</td>
<td>7363.3</td>
<td>2306.8</td>
<td>85567</td>
</tr>
<tr>
<td>4</td>
<td>164288</td>
<td>73.3</td>
<td>4370.3</td>
<td>756.5</td>
<td>205612</td>
<td>9</td>
<td>574484</td>
<td>91.1</td>
<td>7809.6</td>
<td>4698.6</td>
<td>65917</td>
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<tr>
<td>5</td>
<td>182966</td>
<td>74.3</td>
<td>4755.7</td>
<td>859.9</td>
<td>214250</td>
<td>10</td>
<td>1112151</td>
<td>90.8</td>
<td>11835.1</td>
<td>6528.1</td>
<td>54760</td>
</tr>
</tbody>
</table>

Data to estimate a representative cost function was obtained from the transport authority in Santiago, Transantiago, as every operator has reported its cost and trips every three months since the implementation of a new system in February 2007. With information until March 2013 we estimated that $F$ was $23,011$ million per month and $m = 132.6$ per trip. With this parameters we have all the information required to obtain the optimal values for $P$ and $T$.

The ten IC determinate eleven zones where problem (7) must be solved. Table 4 shows the results obtained for each of the eleven cases solved and the variation of social welfare with respect to the base case, where the only payment alternative is the single ticket.
In case 10 there is no value for $T$ that fulfills the conditions of the problem. The optimal solution with the largest associated $SW$ is case 8, where only the two richest segments choose the single ticket. Table 5 shows the relative change of PT trips in each case with respect to the base. Users who buy the travelcard always increase their PT trips as expected, while single ticket users either maintain or reduce them because the new single ticket price is always larger (equal in one case) than in the base case.

Table 5: Trip variation [%]

<table>
<thead>
<tr>
<th>Income segments</th>
<th>Cases</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case: every one chooses P</td>
<td>1230</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>10th case: everyone chooses T</td>
<td>27075</td>
<td>21705</td>
<td>21705</td>
<td>21705</td>
<td>21705</td>
<td>21705</td>
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Figure 10 shows graphically how the solution looks like. The green and blue curves are the IC of the 8th and 9th income segments, respectively. The red point correspond to the optimal pair \((P^*,T^*)\) and is on the IC of the 8th income segment, which means that individuals who belongs to it will be indifferent between \(P^*\) and \(T^*\).

The slashed horizontal line represents the marginal cost and the nearly vertical one is the locus of all pairs that make the operators’ profit zero, such that every pair located to the right side fulfills the budget constraint. The continuous red curves are the SW levels corresponding to the 8th case; note that the unconstrained maximum SW is outside the feasible region delimited by the ICs. The optimal point is the closest to the unconstrained optimum that fulfills the budget constraint within the region determined by the 8th case.

In conclusion, the optimal single ticket–travelcard pair for Santiago, for the parameters and functions that were used in this application, is \((P^*,T^*) = (663; 22,176)\). This pair implies that the two richest segments of the city will travel paying \(P\) for each trip made and the other eight segments, the most intense in PT trips, will buy the travelcard.

5 CONCLUSIONS

We have developed and applied a model to obtain, simultaneously, the optimal values for a monthly travelcard \((T)\) for PT trips and the price of a single ticket \((P)\). Individuals are characterized by their car ownership (correlated with income) and income as purchasing power. This induces a double effect of income on PT trips demand, as lower income groups use PT more intensively than richer groups for the same price, but the income effect is stronger. So the demand for PT trips not only depends on the price, but also on the travelcard value and the income segment of the individual, something that has been ignored in the literature. For any given pair \((P,T)\) the user will choose the alternative with the largest associated surplus. An indifferent user (with income \(I\)) is defined as the one that receives the same surplus from both alternatives. This user is shown to be such that individuals with \(I_t > \bar{I}\) will choose the single
ticket and those with \( I_j < \bar{I} \) will buy the travelcard for a given pair \((P,T)\). If \( \bar{I} \) was known the election of each user would be known and both \( P^* \) and \( T^* \) that maximizes the social welfare under a budget constraint could be found. But \( \bar{I} \) depends on the offered pair \((P^*,T^*)\) and, therefore, the choice is unknown. Solving \( k + 1 \) cases (where \( k \) is the number of income segments) assuming \( \bar{I} \) known within a restricted region of the \((P,T)\) plane, the overall optimum can be found by comparison.

An application of the model to Santiago, Chile, was presented. Ten income segments were considered and data from the origin-destination survey and from the operators’ reports was used to estimate the PT demand parameters, and the marginal and fixed costs of the system. An optimal pair \((P^*,T^*) = (663; 22,176)\) was found; only the two richest income segments would choose the single ticket and the other eight would buy the travelcard.

There are many possible directions for further research. For example, the incorporation of a continuous distribution of income capturing both effects. Also, differences among tastes can be considered for individuals within the same income segment, coupling our general approach with Carbajo’s (1988). A third direction is the incorporation of mode choice, giving the model the possibility to capture behavior of users in a better way and to include travel time as a decision factor.

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