Economics of Crowding in Public Transport

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Abstract

Many public transport systems are crowding at peak times. This paper proposes a structural model in which public transport users face a trade-off between crowding and schedule delay costs and choose when to travel. We derive the individual cost functions for a uniform fare regime. We then consider several alternative regimes and cases including the social optimum, a step-function fare, heterogeneous users, and bi-modal competition. We focus on the economic insights of crowding in public transport and differences from the bottleneck model of automobile travel.

Extended Abstract for the ITEA Conference

1 Research questions

Many transport policies are designed to shift travel from private road vehicles to public transport and other cleaner modes. Yet most of these policies do not consider crowding on public transport which degrades in-vehicle comfort and limits the effectiveness of mode-shifting policies. For instance, in a study of the West Midlands (UK) rail network, Baker et al. (2007) have shown that the most important service attribute for a rail journey is overcrowding, followed by reliability and frequency.

To analyze the welfare effects of public transit crowding, and policies to alleviate it, in a conceptually consistent way it is necessary to use a structural model that incorporates trip scheduling decisions, an empirically plausible crowding cost function, and alternative pricing (i.e., fare) regimes. Several papers in the transport economics literature have laid much of the groundwork for such a model. Vickrey’s (1969) bottleneck congestion model is the seminal work on scheduling of automobile trips. Arnott et al. (1990, 1993) extended it to time-varying tolling schemes and capacity investment decisions. Tabuchi (1993) added public transit by considering a setting in which travelers can choose between driving and taking a rail service with scale economies and no crowding.
Danielis and Marcucci (2002) and Mirabel and Reymond (2011) have studied various pricing regimes in the two-mode-bottleneck model. Huang (2000) introduced a linear crowding cost function, but his model excludes scheduling decisions because only one train or bus is assumed to be available. Henderson (1981) developed a model with both scheduling decisions and a general congestion cost function, but he applies it to private rather than public transportation. Recently, de Palma et al. (2014) proposed an analytical formulation of crowding cost in public transportation and analyzed optimal scheduling and pricing.

We build on this corpus of work by adapting the Henderson (1981) model to public transportation with crowding. We derive equilibrium for various pricing/fare regimes and alternative assumptions about users’ trip-timing and crowding preferences.

2 Methodology

We develop a structural model of a public transit system featuring a fixed number of users, $N$, and a continuum of trains. In-vehicle travel time is assumed to be independent of both vehicle occupancy and departure time, and without loss of generality it is normalized to zero. Commuters therefore arrive at the destination as soon as they depart, and departure time and arrival time can be considered synonymous. In the base-case regime the fare is assumed to be independent of time of day, and it is normalized to zero. Users incur two types of cost: a piecewise-linear schedule delay cost that is incurred if they arrive before or after their preferred arrival time, and an increasing crowding cost that depends on the number of users entering the train at the same time.

Each commuter chooses when to arrive at the train station by trading off schedule delay costs and crowding costs. Equilibrium obtains when no commuter can decrease his journey cost by changing his departure time, taking all other commuters’ departure times as fixed. Thus, as in the Vickrey (1969) model the equilibrium is a pure-strategy Nash equilibrium with departure times as the decision variables. The social optimum is reached when the marginal social cost of a trip is the same in any train during the peak hour.

3 Results

We derive the individual cost functions for a uniform (flat) fare regime and we prove that if the crowding cost function is strictly increasing, the individual cost at equilibrium is a concave function of $N$. This property of the model differs from the bottleneck model where the equilibrium cost is a linear function of $N$. The reason is that as $N$ increases, and the equilibrium cost rises, more users take each train so that the duration of the travel period increases less than proportionally with $N$. The first and last users, who only incur schedule delay costs, therefore experience costs that rise less than in proportion to $N$ and in equilibrium the same is true of all users.
Next, we derive the socially optimal departure rate that minimizes total user costs. We show that the social optimum can be decentralized using a continuously time-varying toll. Tolling reduces the deadweight loss of crowding, and generates fare revenue that could be used to expand public transit service. However, unlike in the bottleneck model the social optimum lasts longer than the flat-fare equilibrium and users incur a higher cost than they do in the flat-fare equilibrium. Optimal transit pricing is less effective than tolling in the bottleneck model because the crowding cost function is assumed to be smooth whereas the bottleneck queuing delay function features a strict capacity constraint such that queuing can be completely eliminated with no reduction in throughput. Some crowding of transit is therefore efficient even in the social optimum.

The final section of the paper explores the implications of several dimensions of heterogeneity in users’ trip-timing preferences. In one variant we assume that users differ in their unit costs of schedule delay, but have the same preferred arrival times and the same crowding cost functions. We show that users of different types sort themselves systematically by departure time. Those least averse to schedule delay travel at the beginning and the end of the travel period while those most averse to schedule delay choose to make their trips at the peak. Equilibrium costs for each user type can be derived in closed-form. The solution procedure is similar to that of the bottleneck model with heterogeneous users, but the equilibrium costs are nonlinear rather than linear functions of the number of users of each type.

References


