Optimal Design of Delay Reduction Contract in EU Air Traffic Management∗

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April 11, 2014

Abstract

In the context of SESAR, Air Navigation Service Provider (regulator) can provide a delay reduction service to airlines. Therefore, this paper studies the optimal design of delay reduction contract signed between a regulator and a monopoly airline. This paper first builds a model which fits characteristics of EU air transport sector. Specifically, a delay function only including exceptional event delay captures slot controls and no congestion of flights at major European airports. Moreover, the fixed number of flights captures grandfather right and “use it or lose it” rule in EU slot allocation mechanism. Then, this paper derives optimal contracts analytically mainly under two scenarios, that is, the regulator provides the service with or without using public funds. Furthermore, this paper conducts comparative-static analysis to study the effects of some relevant exogenous variables, for instance, safety standard and the passenger’s value of time, on optimal contracts, in which information rent is found to increase the possibility that contracting variables move in opposite directions. Besides, this paper also uses numerical examples to gain insights.

Keywords: delay reduction service; optimal contract; comparative statics

JEL classification: D82; D86; L93

∗I acknowledge Estelle Malavolti and Yassine Lefouili for their insightful comments and unfailing support. I also thank Patrick Fève, Pierre Dubois, Daniel Garrett, Miaomiao Dong, and the participants of 10th EBES Conference, 2013 ECORE Summer School, China Meeting of Econometric Society 2013, seminar at Shandong University, Journée de la Recherche ENAC 2014, and RES Annual Conference 2014 for helpful discussions. All remaining errors are mine.
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1 Introduction

Aviation in Europe is expected to experience a rapid growth\(^1\) and endure severe air traffic delay in the future. To satisfy the development of EU air transport sector, in 2004, European Union and EUROCONTROL founded the Single European Sky ATM\(^2\) Research (SESAR) programme, where satisfying future safety needs and reducing delays are the important targets\(^3\). In the context of SESAR, to reduce delays, Air Navigation Service Provider (ANSP), that is, the regulator in this paper, can provide a delay reduction service to airlines\(^4\).

By describing air traffic management in a two-dimensional space, we can see how delay reduction service works. When facing potential delays, an airline will contact the regulator to find a solution to reduce delay. After receiving the airline’s request, the

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\(^1\) According to the SESAR Annual Report 2012 (European Commission and EUROCONTROL, 2013), for the number of flights, in 2012, there are 9.6 million flights. However, in 2030, there will be 16.9 million flights. For the number of passengers, in 2012, there are 1.6 billion passengers. However, in 2030, there will be 2.7 billion passengers.

\(^2\) ATM is the abbreviation of Air Traffic Management.

\(^3\) The European ATM Master Plan - Edition 2 (European Commission and EUROCONTROL, 2012).

\(^4\) Some technology changes in SESAR make the provision of the service possible, for instance, air/ground datalink for System Wide Information Management, Flight Data Processing & tools to handle four-dimensional trajectories, multi constellation Global Navigation Satellite System, support dynamic flow management, and support flexible use of airspace.
regulator can find out several equilibria (for instance, E1 to E10 in Figure 1) satisfying all regulation constraints. Then, by costly calculation, evaluation, and coordination, the regulator can determine the equilibrium (for instance, E7) which can reduce delay most and then implement it. In the short run, the service can be provided for free because of generous funds of SESAR. In the long run, however, the regulator will face financial constraints. In fact, authority is thinking about asking airlines to pay for funding the service, through a contract, the so-called delay reduction contract in this paper. Therefore, this paper is to study the optimal design of delay reduction contract.

This paper first builds a model which fits for characteristics of EU air transport sector. At all major European airports, take-off and landing slots are controlled by aviation authority and thus there is no congestion of flights\(^5\). This implies that the only source of air traffic delay in EU is exceptional events, for instance, adverse weather conditions and technical problems of aircraft. To capture this characteristic, this paper uses a delay function only including exceptional event delay, which consists of the delay due to exceptional events in own slots and the delay induced by the delayed flights in previous slots. Moreover, because of grandfather right and “use it or lose it” rule in EU slot allocation mechanism, in this paper, the number of flights is fixed. Then, this paper derives optimal contracts analytically mainly under two scenarios, that is, the regulator provides the service with or without using public funds. Furthermore, because contracts should be adjusted over time according to the evolution of variables, this paper studies the effects of some relevant exogenous variables, for instance, safety standard, the number of flights, and the passenger’s value of time, on optimal contracts. In particular, this paper also finds that, under incomplete information, the airline’s information rent not only decreases social welfare as we expect but also increases the possibility that contracting variables move in opposite directions. Besides, by choosing proper values of parameters and proper specifications of functions, this paper also uses numerical examples to gain insights, for instance, illustrating when the regulator will provide the service with using public

\(^5\)In this paper, congestion of flights is defined as the situation such that airlines schedule more flights than that capacity can support, and consequently, flights may be delayed even when no exceptional event at airports happens. Moreover, congestion delay is defined as the one due to congestion of flights. According to these definitions, congestion delay at European airports in some literature is in fact the delay induced by the delayed flights in previous slots which we will see. The differentiation between congestion delay and the delay induced by the delayed flights in previous slots makes sense because, by differentiation, we can clearly see the cause of delay.
funds and how a pooling contract affects the airline’s net benefit from the service.

Because airlines may have private information about their value of time\textsuperscript{6}, this paper considers the adverse selection problem in the design of optimal contract. Thus, this paper is closely related to incentives theory and regulation literature, for instance, Baron and Myerson (1982), Baron and Besanko (1984), Laffont and Tirole (1986), and Caillaud et al. (1988). Caillaud et al. (1988) summarizes two types of regulator’s objective function, that is, distributional objectives and cost of public funds. Baron and Myerson (1982) and Baron and Besanko (1984) use the distributional-objectives objective function while Laffont and Tirole (1986) uses the objective function with cost of public funds. This paper considers the cost of public funds. However, the roles of players (passengers, the airline, and the regulator) in this paper are different from those in the papers above. In this paper, the regulator provides the service at a cost and thus is in fact the “producer” while the airline pays the service and thus is the “consumer”. Moreover, passengers can also benefit from the service after the airline signs delay reduction contract. Thus, the regulator as a social planner will consider consumer (passenger) surplus, the airline’s profit, and the regulator’s cost. Because passengers benefit from the service but in the contract do not have to directly pay to the regulator, it is possible that the airline’s benefit from the service is not enough to cover the total cost of providing the service. In this case, the regulator has to use public funds to subsidy the service and thus will also consider the cost of public funds in the objective function.

This paper also directly relates to air transport economics literature. Generally speaking, flight delay\textsuperscript{7} includes exceptional event delay and congestion delay. If not considering airport capacity change, a commonly used delay function, for instance, the one in Brueckner (2002), Pels and Verhoef (2004), Brueckner (2005), Basso (2008), and Yang and Zhang (2011), is only an increasing function of total number of flights, which captures congestion delay. Such delay function fits the characteristic of US air transport sector, that is, congestion of flights is the main source of delay\textsuperscript{8}. However, in

\textsuperscript{6}Airlines always know better about their values of time than the regulator.

\textsuperscript{7}Flight delay can be measured by the gap between flight actual and scheduled arrival time.

\textsuperscript{8}In US, only four airports, that is, Ronald Reagan Washington National Airport (Reagan National), John F. Kennedy International (JFK), LaGuardia International (LaGuardia), and Newark Liberty International (Newark), are subject to slot controls of US Federal Aviation Administration (United States Government Accountability Office, 2012). At other major US airports, airlines schedule more flights than that capacity can support. Consequently, more and more flights are delayed, even under normal weather conditions (Whalen \textit{et al.}, 2007).
Europe, as we have seen, exceptional events are the only source of delay. Thus, a delay function only including exceptional event delay is used for analyzing EU air transport sector. Moreover, this paper benefits from the framework for modeling passenger utility in Brueckner (2004), Brueckner and Flores-Fillol (2007), and Flores-Fillol (2010). In both Brueckner (2004) and Brueckner and Flores-Fillol (2007), passenger utility function includes passenger’s income, airline ticket price, passenger’s travel benefit, and schedule delay\textsuperscript{9} cost. In Flores-Fillol (2010), passenger utility function also includes congestion damage. However, this paper focuses on exceptional event delay and safety standard, instead of schedule delay and congestion delay. Thus, in this paper, besides passenger’s income, airline ticket price, and passenger’s travel benefit, passenger utility function only includes exceptional event delay cost and passenger’s utility gain from a safety standard.

The remainder of the paper is organized as follows. Section 2 introduces the model and discusses the optimal design of delay reduction contract in four scenarios. Section 3 conducts comparative-static analysis to study the effects of some relevant exogenous variables on optimal contracts. Section 4 uses numerical examples to gain insights. Section 5 concludes.

\section{The model}

The model considers an air transport market connecting two European airports. Passengers in the market have mass $N$ and the market is served by a monopoly airline.

Conditional on use of the airline, passenger utility is given by

$$v = y - p + b + a(s) - \alpha D(s).$$

Specifically, $y$ is the passenger’s income. $p$ is the airline ticket price. $b$ is the passenger’s travel benefit which is uniformly distributed on the support $[\zeta, \xi]$. $a(s)$ is the passenger’s utility gain from a safety standard $s$ where $a'(s) \geq 0$. Note that safety standard $s$ is exogenous and can vary within $[\underline{s}, \overline{s}]$ where $\underline{s}$ is far larger than the minimum safety requirement. $\alpha$ is the passenger’s value of time. $D(s)$ is the expected delay per flight (in time units).

\textsuperscript{9}According to Basso (2008), schedule delay is defined as the expected gap between passengers’ actual and desired departure time.
The expected delay per flight, that is, exceptional event delay, is given by

\[
D(s) = 2 \left[ \sum_{k=0}^{+\infty} \frac{\left( \frac{\beta T}{T} \right)^k}{k!} \frac{e^{-\left( \frac{\beta T}{T} \right)}}{k!} kg(s) + \gamma\beta \left( \frac{T}{f} \right)^{-1} g(s) \right],
\]

where i: delay due to exceptional events in own slot; ii: delay induced by the delayed flights in previous slots.\(^{10}\)

This delay function reflects that exceptional events are the only source of delay at major European airports. The first term in the square brackets is the delay due to exceptional events in own slot. We assume that the number of exceptional events in a slot follows a Poisson distribution with parameter \(\frac{\beta T}{f}\), where \(\beta\) is the exceptional event arriving rate, \(T\) is the number of available hours, and \(f\) is the number of flights. We assume that flights are evenly spaced during available hours, which implies that slot for each flight is equal and the duration of a slot is given by \(\frac{T}{f}\). Therefore, the parameter \(\frac{\beta T}{f}\) is in fact the expected number of exceptional events in a slot. \(k\) is the number of exceptional events. \(g(s)\) is the amount of delay caused by an exceptional event. The amount of delay increases with safety standard, that is, \(g'(s) \geq 0\), which captures the fact that the higher safety standard the regulator maintains is, the longer the delay will be. We assume that the total amount of delay caused by \(k\) exceptional events is \(kg(s)\). The second term in the square brackets is the delay induced by the delayed flights in previous slots. This term is inversely related to the duration of a slot \(\frac{T}{f}\) because one flight may be more severely affected by the delayed flights in previous slots if the duration of a slot becomes shorter. This term is also related to the exceptional event arriving rate \(\beta\) and the amount of delay \(g(s)\) because one flight may be more severely affected by the delayed flights in previous slots if exceptional events arrive at previous slots more frequently and the delay caused by an exceptional event in previous slots becomes longer. Moreover, \(\gamma\) is a positive parameter, which is called the delay externality parameter in the following parts. A larger \(\gamma\) implies a severer effect of the delayed flights in previous slots. The square brackets times two because there are two airports.

\(^{10}\)In reality, there is also En-Route delay, which, however, only accounts for a small proportion of all delays (Central Office for Delay Analysis (CODA) of EUROCONTROL, 2012; 2013). Thus, we only consider the delay due to exceptional events in slots at airports.
Figure 2 can help us understand the allocation of slots at airports.

![Diagram of slot allocation at airports]

Figure 2: Allocation of slots at airports

Besides traveling by plane, passengers also have an outside option, for instance, traveling by train, which gives a utility

\[ v_0 = y + z, \]

where \( z \) is the net benefit of outside option.

Therefore, a passenger chooses to travel by plane when

\[ y - p + b + a(s) - \alpha D(s) \geq y + z, \]

or when travel benefit satisfies \( b \geq p - a(s) + \alpha D(s) + z \). The number of passengers traveling by plane then equals

\[
q = \int_{p-a(s)+\alpha D(s)+z}^{\xi} \frac{N}{\xi - \zeta} db
\]
\[ \eta = \left[ \xi - p + a(s) - \alpha D(s) - z \right] \frac{N}{\xi - \zeta}. \]

In this model, the number of flights \( f \) is fixed. The reason of this specification is due to the EU slot allocation mechanism. Specifically, an air carrier having operated its particular slots for at least 80% during the summer/winter scheduling period is entitled to the same slots in the equivalent scheduling period of the following year (so-called grandfather rights). Consequently, slots which are not sufficiently used by air carriers are reallocated (so-called “use it or lose it” rule).\(^{11}\) Given these regulation rules, most slots at airports are sufficiently used. In this way, it is hard for the airline to increase the number of flights unless through secondary trading. The airline also does not arbitrarily decrease the number of flights because of the possibility of losing slots and then facing competition from entrants. Given the fixed number of flights, load factor changes when the number of passengers changes.

The airline’s profit is

\[
\pi = pq - C_{air}(\theta, s, q) = pq - (\tau q + cf + \theta fD(s)).
\]

Airline’s cost \( C_{air}(\theta, s, q) \) consists of variable cost \( (\tau q) \) and fixed cost \( (cf + \theta fD(s)) \). For the variable cost, \( \tau \) is the marginal cost per seat. For the fixed cost, \( cf \) is the total fixed operating cost where \( c \) is the fixed operating cost of a flight and \( \theta fD(s) \) is the supply side delay cost where \( \theta \) is the airline’s value of time\(^{12}\). \( \theta \) may be unobservable to regulator. However, it is common knowledge that \( \theta \) belongs to the set \( \Theta = \{\overline{\theta}, \underline{\theta} \} \) where \( \overline{\theta}, \underline{\theta} > 0 \) and \( \Delta \theta = \overline{\theta} - \underline{\theta} > 0 \). If \( \theta \) is the airline’s private knowledge, the airline can be the one with \( \overline{\theta} \) or \( \underline{\theta} \) with probabilities \( \mu \) and \( 1 - \mu \) respectively.

The airline maximizes its profit by setting a ticket price \( p \). Taking the first-order condition of \( \pi \) with respect to \( p \) and letting \( \eta = \frac{N}{\xi - \zeta} \), we can obtain

\[
p^*(s) = \frac{1}{2} \left( \xi + \tau + a(s) - z - \alpha D(s) \right),
\]

\[
q^*(s) = \frac{1}{2} \eta \left( \xi - \tau + a(s) - z - \alpha D(s) \right),
\]

\(^{11}\)See http://ec.europa.eu/transport/modes/air-airports/slots_en.htm for more details.

\(^{12}\)According to European airline delay cost reference values (University of Westminster, 2011), the airline’s value of time consists of a variety of costs, for instance, fuel burn costs, aircraft maintenance costs, and pilot and cabin crew salaries and expenses.
\[
\pi^*(\theta, s) = -\frac{1}{4} \eta \left[ 2(\xi - \tau + a(s) - z) - \alpha D(s) \right] \alpha D(s) - \theta f D(s)
\]

\[
+ \frac{1}{4} \eta (\xi - \tau + a(s) - z)^2 - cf.
\]

Then, we can calculate passenger utility and consumer surplus.

\[
v^*(s) = y - p^*(s) + b + a(s) - \alpha D(s).
\]

\[
cs^*(s) = \int_{p^*(s) - a(s) + \alpha D(s) + z}^{\xi} \left\{ y - p^*(s) + b + a(s) - \alpha D(s) \right\} \eta db + \int_{\xi}^{p^*(s) - a(s) + \alpha D(s) + z} (y + z) \eta db
\]

\[
= -\frac{1}{8} \eta \left[ 2(\xi - \tau + a(s) - z) - \alpha D(s) \right] \alpha D(s) + \frac{1}{8} \eta (\xi + \tau - a(s) + z)^2
\]

The regulator signs a contract with the airline. The contracting variables are \( r \) and \( t \). \( r \) is the degree of delay reduction service the regulator provides to the airline, where \( r \) belongs to \([0, 1]\). \( t \) is the transfer the airline pays to the regulator. We can understand \( r \) in the following way. \( r \) is determined by the ratio \( \frac{i}{n} \), where \( i \) is the number of equilibria the regulator will work on and \( n \) is the total number of equilibria the regulator can find out. Specifically, the regulator will pick \( i \) equilibria which are very likely the most effective ones by experience. Note that the regulator does not know exactly whether they are indeed the most effective equilibria before further evaluation. Then, the regulator will calculate, evaluate, and coordinate for these \( i \) equilibria and thus find one which can reduce delay most. \( \sigma \ln (1 + r) \) is the fraction of delay reduction the airline can enjoy if the degree of the service it purchases is \( r \). \( \sigma \) belongs to \([0, \frac{1}{\ln 2}]\) measuring the effectiveness of the service and a larger \( \sigma \) implies a higher degree of the effectiveness of the service. In fact, when the number of equilibria which the regulator will work on increases, the regulator can find a very effective equilibrium in a higher possibility. Moreover, because the regulator always tries best to pick equilibria which are very likely the most effective ones by experience, the additional equilibrium is less possibly a very effective one. Therefore, the marginal value of the service is positive but decreasing with the degree, which is captured by the logarithmic function \( \ln (1 + r) \) as we have seen. After obtaining the
service, the expected delay per flight reduces from \( D(s) \) to \( D(s) \left[ 1 - \sigma \ln (1 + r) \right] \).

Then, passenger utility, ticket price, the number of passengers, the airline’s profit, and consumer surplus become

\[
V^*(s, r) = y - P + b + a(s) - \alpha D(s) \left[ 1 - \sigma \ln (1 + r) \right],
\]

\[
P^*(s, r) = \frac{1}{2} \{ \xi + \tau + a(s) - z - \alpha D(s) \left[ 1 - \sigma \ln (1 + r) \right] \},
\]

\[
Q^*(s, r) = \frac{1}{2} \eta \{ \xi - \tau + a(s) - z - \alpha D(s) \left[ 1 - \sigma \ln (1 + r) \right] \},
\]

\[
\Pi^*(\theta, s, r) = -\frac{1}{4} \theta f D(s) \left[ 1 - \sigma \ln (1 + r) \right] + \frac{1}{4} \eta \{ \xi - \tau + a(s) - z \}^2 - cf,
\]

\[
CS^*(s, r) = -\frac{1}{8} \eta \{ \xi + \tau + a(s) + z \}^2 - \frac{1}{2} \eta \xi \{ \xi + \tau - a(s) - z \} + \frac{1}{2} \eta \xi^2 - \eta \zeta (y + z) + \eta \xi y.
\]

In fact, we can rewrite \( V^*(s, r) \), \( P^*(s, r) \), \( Q^*(s, r) \), \( \Pi^*(\theta, s, r) \), and \( CS^*(s, r) \) as follows.

\[
V^*(s, r) = v^*(s) + \frac{1}{2} \alpha D(s) \sigma \ln (1 + r),
\]

\[
P^*(s, r) = p^*(s) + \frac{1}{2} \alpha D(s) \sigma \ln (1 + r),
\]

\[
Q^*(s, r) = q^*(s) + \frac{1}{2} \eta \alpha D(s) \sigma \ln (1 + r),
\]

\[
\Pi^*(\theta, s, r) = \pi^*(\theta, s) + \frac{1}{4} \eta \alpha^2 D(s)^2 \sigma^2 \left[ \ln (1 + r) \right]^2 + q^*(s) \alpha D(s) \sigma \ln (1 + r)
\]

\[
\begin{align*}
&\text{initial profit} &\text{demand side delay reduction benefit} \\
&\Theta f D(s) \sigma \ln (1 + r), &\text{supply side delay reduction benefit}
\end{align*}
\]

\[
CS^*(s, r) = cs^*(s) + \frac{1}{2} \left\{ \frac{1}{4} \eta \alpha^2 D(s)^2 \sigma^2 \left[ \ln (1 + r) \right]^2 + q^*(s) \alpha D(s) \sigma \ln (1 + r) \right\}.
\]

We can find that all of passenger utility, ticket price, the number of passengers, the airline’s profit, and consumer surplus become larger. The reason why passengers
become better, that is, passenger utility and consumer surplus increase, even when
ticket price increases is that the utility gain from delay reduction is larger than the
utility loss from the increase of ticket price. For convenience, in the following parts,
I will call \( q^* (s) \), instead of \( Q^* (s, r) \), as the number of passengers.

The regulator’s cost of providing the delay reduction service \( C_{reg} (s, r) \) is

\[
C_{reg} (s, r) = m (s) r,
\]

where \( m (s) \) is the marginal cost of providing the service, which increases with safety
standard, that is, \( m' (s) \geq 0 \). In fact, when providing the service, the regulator
has to spend more time on evaluation and coordination for satisfying a higher safety
standard, which will inevitably result in a higher cost.

Moreover, in this model, the regulator is a social planner whose objective is to
maximize social welfare\(^{13}\), that is,

\[
W = CS^* (s, r) + \Pi^* (\theta, s, r) - C_{reg} (s, r) - \lambda (C_{reg} (s, r) - t) \\
\text{s.t. } t \leq C_{reg} (s, r),
\]

where \( \lambda \) is the shadow cost of public funds.

The timeline is shown in Figure 3.

![Figure 3: Timeline](image)

In this model, the passenger’s value of time \( \alpha \) and the airline’s value of time \( \theta \)
are important parameters because delay affects passengers through \( \alpha \) and affects the

\[^{13}\text{In this social welfare function, the weights of passengers and the airline are equal. In fact, there is}
\text{no evidence to show that the regulator will place greater emphasis on either one of passengers and}
\text{the airline. Therefore, putting equal weights on them is a natural and fair specification.}\]
airline through $\alpha$ and $\theta$. Throughout the paper, we design contracts according to the values of $\theta$. Then, according to the value of $\alpha$, we discuss in four scenarios.

2.1 Scenario 1: $\alpha = 0$

When the passenger’s value of time is zero, consumers cannot enjoy any benefit from delay reduction service. In this scenario, because only the airline can enjoy the benefit of the service and the airline has to pay all the relevant costs, the airline in fact delegates the provision of delay reduction service to the regulator. Thus, the regulator’s (the airline’s) objective is to maximize

$$W = \Pi^* (\theta, s, r) - C_{reg} (s, r).$$

Note that information structure (complete and incomplete information) plays no role here. Taking the first-order condition of $W$ with respect to $r$ and denoting the optimal degrees for the airline with $\bar{\theta}$ and $\theta$ by $\bar{r}^*$ and $r^*$ respectively, we can obtain

$$\bar{r}^* = \frac{\bar{\theta} f_D (s) \sigma}{m (s)} - 1,$$
$$r^* = \frac{\theta f_D (s) \sigma}{m (s)} - 1.$$

Moreover, the second-order conditions can also be satisfied.

$$\frac{\partial^2 W}{\partial r^2} \bigg|_{r = \bar{r}^*} = -\frac{\bar{\theta} f_D (s) \sigma}{(1 + \bar{r}^*)^2} < 0,$$
$$\frac{\partial^2 W}{\partial r^2} \bigg|_{r = r^*} = -\frac{\theta f_D (s) \sigma}{(1 + r^*)^2} < 0.$$

Then, we have the following result.

**Proposition 1.** Delay reduction service will be provided to the airline with $\bar{\theta}$ and $\theta$ if and only if it is effective enough, that is,

$$\sigma > \frac{m (s)}{\bar{\theta} f_D (s)}.$$
**Proof.** To ensure positive degrees, we immediately have

\[ r^* > 0, \ z^* > 0 \iff \sigma > \frac{m(s)}{\theta f D(s)} > \frac{m(s)}{\theta f D(s)}. \]

\[ \square \]

2.2 **Scenario 2:** \( \alpha > \frac{\xi - \tau + a(s) - z}{D(s)} \)

According to passenger utility, we have

\[ \xi \geq b \geq p - a(s) + \alpha D(s) + z \geq \tau - a(s) + \alpha D(s) + z \]
\[ \Rightarrow \xi - \tau + a(s) - z \geq \alpha D(s). \]

Thus, when \( \alpha > \frac{\xi - \tau + a(s) - z}{D(s)} \), no passenger will choose to travel by plane. Then, the air transport market will close down and there will be no such contract.

2.3 **Scenario 3:** \( 0 < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)} \) and providing service without using public funds

When \( 0 < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)} \), both passengers and the airline can benefit from the service. Moreover, when \( 0 < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)} \), there are two scenarios. The first one is that the airline’s benefit from the service is larger than the cost of providing the service. In this scenario, because there is cost associating with public funds, it is optimal for the regulator to ask the airline to pay the total cost of providing the service, that is, \( t = C_{reg}(s, r) \).

Note that we focus our discussion on interior solutions.

2.3.1 **Complete information**

Under complete information, the regulator maximizes

\[ W = CS^*(s, r) + \Pi^*(\theta, s, r) - C_{reg}(s, r). \]

Taking the first-order condition of \( W \) with respect to \( r \) and denoting the optimal
degrees for the airline with \( \theta \) and \( \theta \) by \( r_{FB} \) and \( r_{FB} \) respectively, we can obtain

\[
\Omega = \frac{3\eta\alpha^2 D(s)^2 \sigma^2 \ln (1 + r_{FB})}{\text{LOGR}} - \left[4m(s)(1 + r_{FB}) - 3\eta(\xi - \tau + a(s) - z - \alpha D(s))\alpha D(s)\sigma - 4\theta fD(s)\sigma\right] = 0,
\]

\[
\Omega = \frac{3\eta\alpha^2 D(s)^2 \sigma^2 \ln (1 + r_{FB})}{\text{LOGR}} - \left[4m(s)(1 + r_{FB}) - 3\eta(\xi - \tau + a(s) - z - \alpha D(s))\alpha D(s)\sigma - 4\theta fD(s)\sigma\right] = 0,
\]

where \( \text{LOGR} \) (LOGR) is a logarithmic function of \( r_{FB} \) (\( r_{FB} \)) and \( \text{LR} \) (LR) is a linear function of \( r_{FB} \) (\( r_{FB} \)). Thus, the solution is determined by the intersection of logarithmic and linear function. Moreover, we assume that the solutions can satisfy the airline’s participation constraints. A detailed discussion about solutions is in Appendix A.1.

### 2.3.2 Incomplete information

Under incomplete information, the regulator maximizes the expected social welfare, that is,

\[
W = \mu(\text{CS}^*(s, \overline{r}) + \Pi^*(\overline{\theta}, s, \overline{r}) - \text{C}_{\text{reg}}(s, \overline{r})) + (1 - \mu)(\text{CS}^*(s, \underline{r}) + \Pi^*(\underline{\theta}, s, \underline{r}) - \text{C}_{\text{reg}}(s, \underline{r})).
\]

Taking the first-order condition of \( W \) with respect to \( \tau \) and \( \underline{r} \), we can obtain exactly the same solution as under complete information, that is, \( \tau_{FB} \) and \( \underline{r}_{FB} \). However, the contract may not be incentive compatible. Because we have assumed that \( \tau_{FB} \) and \( \underline{r}_{FB} \) can satisfy the airline’s participation constraints, we only need to check whether the airline’s incentive compatibility constraints can be satisfied. If the contract is incentive compatible, we must have

\[
\Pi^*(\overline{\theta}, s, \sigma_{FB}) - \text{C}_{\text{reg}}(s, \overline{r}_{FB}) \geq \Pi^*(\overline{\theta}, s, \underline{r}_{FB}) - \text{C}_{\text{reg}}(s, \underline{r}_{FB}),
\]

\[
\Pi^*(\overline{\theta}, s, \sigma_{FB}) - \text{C}_{\text{reg}}(s, \overline{r}_{FB}) \geq \Pi^*(\overline{\theta}, s, \sigma_{FB}) - \text{C}_{\text{reg}}(s, \sigma_{FB}).
\]
Denoting the airline’s net benefit from the service by $u$, that is,
$$u = \Pi^* (\theta, s, r) - \pi^* (\theta, s) - C_{reg} (s, r)$$
and further denoting the net benefits of the airline with $\bar{\theta}$ and $\hat{\theta}$ by $\bar{u}$ and $\hat{u}$ respectively, that is,
$$\bar{u} = \Pi^* (\bar{\theta}, s, \pi^{FB}) - \pi^* (\bar{\theta}, s) - C_{reg} (s, \pi^{FB}),$$
$$\hat{u} = \Pi^* (\hat{\theta}, s, \pi^{FB}) - \pi^* (\hat{\theta}, s) - C_{reg} (s, \pi^{FB}),$$
we can rewrite incentive compatibility constraints as
$$\bar{u} \geq \hat{u} + \Delta \theta f D (s) \sigma \ln (1 + \pi^{FB}),$$
$$\hat{u} \geq \bar{u} - \Delta \theta f D (s) \sigma \ln (1 + \pi^{FB}).$$

We can confirm the second incentive compatibility constraint. From
$$CS^* (s, \pi^{FB}B) + \Pi^* (\theta, s, \pi^{FB}) - C_{reg} (s, \pi^{FB}) \geq CS^* (s, \pi^{FB}) + \Pi^* (\theta, s, \pi^{FB}) - C_{reg} (s, \pi^{FB}),$$
we can obtain
$$u \geq \bar{u} - \Delta \theta f D (s) \sigma \ln (1 + \pi^{FB}) + (CS^* (s, \pi^{FB}) - CS^* (s, \pi^{FB}B)).$$

Because consumer surplus increases with the degree of the service, that is, $CS^* (s, \pi^{FB}) \geq CS^* (s, \pi^{FB}B)$, we can confirm the second constraint.

For the first constraint, we have
$$\Pi^* (\bar{\theta}, s, \pi^{FB}) - C_{reg} (s, \pi^{FB}) - \Pi^* (\bar{\theta}, s, \pi^{FB}) + C_{reg} (s, \pi^{FB}) = -\left[ \Pi^* (\theta, s, \pi^{FB}) - C_{reg} (s, \pi^{FB}) - \Pi^* (\theta, s, \pi^{FB}) + C_{reg} (s, \pi^{FB}) \right] + \Delta \theta f D (s) \sigma \left[ \ln (1 + \pi^{FB}) - \ln (1 + \pi^{FB}) \right],$$
the sign of which is not clear. If the sign is positive, the contract with $\pi^{FB}$ and $\pi^{FB}B$ is incentive compatible. Otherwise, the contract with $\pi^{FB}$ and $\pi^{FB}B$ is not incentive compatible, that is, the airline with $\bar{\theta}$ will choose $\pi^{FB}B$, instead of $\pi^{FB}$. In this case,
a pooling contract dominates the separating contract (with \( r^{FB} \) and \( r^{FB} \)). Denoting the optimal degree in the pooling contract by \( \tilde{r} \), because \( \tilde{r} \) maximizes the regulator’s objective function, we have

\[
\mu \left( CS^* (s, \tilde{r}) + \Pi^* (\tilde{\theta}, s, \tilde{r}) - C_{reg} (s, \tilde{r}) \right) + (1 - \mu) \left( CS^* (s, \tilde{r}) + \Pi^* (\bar{\theta}, s, \tilde{r}) - C_{reg} (s, \tilde{r}) \right) \\
\geq \mu \left( CS^* (s, r^{FB}) + \Pi^* (\tilde{\theta}, s, r^{FB}) - C_{reg} (s, r^{FB}) \right) \\
+ (1 - \mu) \left( CS^* (s, r^{FB}) + \Pi^* (\bar{\theta}, s, r^{FB}) - C_{reg} (s, r^{FB}) \right),
\]

which implies that it is optimal for the regulator to choose a pooling contract, instead of a separating contract.

In fact, \( \tilde{r} \) is determined by

\[
\tilde{\Omega} = \frac{3\eta \alpha^2 D(s)^2 \sigma^2 \ln (1 + \tilde{r})}{\log \frac{\tilde{\Omega}}{LR}} - \left[ 4m(s)(1 + \tilde{r}) - 3\eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma - 4\tilde{\theta} f D(s) \sigma \right] = 0,
\]

where \( \tilde{\theta} = \mu \tilde{\theta} + (1 - \mu) \bar{\theta} \) and \( \tilde{\theta} \leq \tilde{\theta} \leq \bar{\theta} \).

Incentive compatibility constraints can be satisfied by the pooling contract but at the cost of loss of flexibility in allocations. We then consider participation constraints as follows.

\[
\Pi^* (\tilde{\theta}, s, \tilde{r}) - \pi^* (\tilde{\theta}, s) - C_{reg} (s, \tilde{r}) \geq 0, \\
\Pi^* (\bar{\theta}, s, \tilde{r}) - \pi^* (\bar{\theta}, s) - C_{reg} (s, \tilde{r}) \geq 0.
\]

Because of

\[
\Pi^* (\tilde{\theta}, s, \tilde{r}) - \pi^* (\tilde{\theta}, s) > \Pi^* (\bar{\theta}, s, \tilde{r}) - \pi^* (\bar{\theta}, s),
\]

we can obtain

\[
\Pi^* (\tilde{\theta}, s, \tilde{r}) - \pi^* (\tilde{\theta}, s) - C_{reg} (s, \tilde{r}) > \Pi^* (\bar{\theta}, s, \tilde{r}) - \pi^* (\bar{\theta}, s) - C_{reg} (s, \tilde{r}),
\]

which implies that the pooling contract is feasible as long as we assume that the participation constraint of the airline with \( \bar{\theta} \) can be satisfied.

The discussion about the solution is in Appendix A.2.
Then, we discuss the effects of the pooling contract on passengers and the airline. Because both the airline with \( \overline{\theta} \) and \( \overline{\theta} \) will choose \( r^{FB} \), we just need to study how passengers and the airline are affected when the degree changes from \( r^{FB} \) to \( \tilde{r} \). Because of \( r^{FB} \leq \tilde{r} \), all of passenger utility \( V^*(s,r) \), ticket price \( P^*(s,r) \), the number of passengers \( Q^*(s,r) \), and consumer surplus \( CS^*(s,r) \) will increase. The reason why passengers become better when ticket price \( P^*(s,r) \) increases is that the utility gain from a higher fraction of delay reduction is larger than the utility loss from the increase of ticket price. Moreover, how the airline’s net benefit will change is not clear. Therefore, in the Section 4, we will give an numerical example to see how the pooling contract affects the airline’s net benefit.

2.4 Scenario 4: \( 0 < \alpha \leq \frac{\xi-\tau+a(s)-z}{D(s)} \) and providing service with public funds

When \( 0 < \alpha \leq \frac{\xi-\tau+a(s)-z}{D(s)} \), the second scenario is that the airline’s benefit from the service is smaller than the cost of providing the service. In this scenario, the airline’s transfer is unable to cover the total cost of providing the service, that is, \( t < C_{reg}(s,r) \). However, as long as the social benefit of the service outweighs the social cost, it is optimal for the regulator to use public funds to cover the part of cost which cannot be covered by the airline’s transfer.

Note that we focus our discussion on interior solutions.

2.4.1 Complete information

Under complete information, the equation determining \( \pi^{FB} \) is exactly the same as the one determining \( \pi^{SB} \). Moreover, except the value of \( \theta \), the equation determining \( \pi^{FB} \) is also the same as the one determining \( \pi^{SB} \). Thus, the analysis for \( \pi^{FB} \) and \( \pi^{FB} \) is omitted henceforth.

2.4.2 Incomplete information

Under incomplete information, the regulator maximizes the expected social welfare, that is,

\[
W = \mu \left[ CS^*(s,\overline{r}) + \Pi^* (\overline{\theta}, s, \overline{r}) - C_{reg} (s, \overline{r}) - \lambda \left( C_{reg} (s, \overline{r}) - \overline{t} \right) \right] \\
+ (1 - \mu) \left[ CS^*(s, \tilde{r}) + \Pi^* (\tilde{\theta}, s, \tilde{r}) - C_{reg} (s, \tilde{r}) - \lambda \left( C_{reg} (s, \tilde{r}) - \tilde{t} \right) \right].
\]
Under incomplete information, to have the airline self-selecting properly within the menu, incentive compatibility constraints must be satisfied, that is,

\[ \Pi^* (\bar{\theta}, s, \bar{r}) - \bar{t} \geq \Pi^* (\bar{\theta}, s, r) - \bar{t}, \]
\[ \Pi^* (\bar{\theta}, s, r) - \bar{t} \geq \Pi^* (\bar{\theta}, s, \bar{r}) - \bar{t}. \]

Moreover, participation constraints must also be satisfied, that is,

\[ \Pi^* (\bar{\theta}, s, \bar{r}) - \bar{t} \geq \pi^* (\bar{\theta}, \bar{s}), \]
\[ \Pi^* (\bar{\theta}, s, r) - \bar{t} \geq \pi^* (\bar{\theta}, s). \]

Denoting the airline’s information rent by \( u \), that is,

\[ u = \Pi^* (\theta, s, r) - \pi^* (\theta, s) - t \]

and further denoting the information rent of the airline with \( \bar{\theta} \) and \( \theta \) by \( \bar{u} \) and \( u \) respectively, that is,

\[ \bar{u} = \Pi^* (\bar{\theta}, s, \bar{r}) - \pi^* (\bar{\theta}, s) - \bar{t}, \]
\[ u = \Pi^* (\theta, s, r) - \pi^* (\theta, s) - \bar{t}, \]

we can write incentive compatibility and participation constraints as

\[ \bar{u} \geq u + \Delta \theta f D (s) \sigma \ln (1 + r), \]
\[ u \geq \bar{u} - \Delta \theta f D (s) \sigma \ln (1 + r), \]
\[ \bar{u} \geq 0, \]
\[ u \geq 0. \]

At the optimum of the regulator’s problem, the first and the fourth constraints must be binding. Thus, we have \( \bar{u} = \Delta \theta f D (s) \sigma \ln (1 + r) \) and \( u = 0 \). Then, plugging \( \bar{t} = \Pi^* (\bar{\theta}, s, \bar{r}) - \pi^* (\bar{\theta}, s) - \Delta \theta f D (s) \sigma \ln (1 + r) \) and \( \bar{t} = \Pi^* (\bar{\theta}, s, r) - \pi^* (\bar{\theta}, s) \) into the regulator’s objective function \( W \), taking the first-order condition of \( W \) with respect to \( r \) and \( r \), and denoting the optimal degrees for the airline with \( \bar{\theta} \) and \( \theta \) by \( \tau^{SB} \) and
Let $\Omega$ be a variable, we can obtain

$$\Omega = \frac{(3 + 2\lambda) \eta^2 D(s)^2 \sigma^2 \ln (1 + r_{SB})}{4(1 + \lambda) m(s)(1 + r_{SB}) - (3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma - 4(1 + \lambda) \bar{\theta} D(s) \sigma}$$

(1)

$$\Omega = \frac{(3 + 2\lambda) \eta^2 D(s)^2 \sigma^2 \ln (1 + r_{SB})}{4(1 + \lambda) m(s)(1 + r_{SB}) - (3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma - 4(1 + \lambda) \bar{\theta} D(s) \sigma + 4 \frac{\mu}{1 - \mu} \lambda \Delta \theta D(s) \sigma} = 0,$$

(2)

where $LOGR$ is a logarithmic function of $r_{SB}$ and $LR$ is a linear function of $r_{SB}$. Thus, the solution is determined by the intersection of logarithmic and linear function. A detailed discussion about solutions is in Appendix A.3.

### 3 Comparative-static analysis

We can say that the solution in Scenario 4 is a general form of all the solutions because by setting $\alpha = 0$ or $\lambda = 0$, we can obtain the equations determining the solutions in Scenario 1 and 3. Therefore, in this section, we will only do comparative-static analysis for the solutions in Scenario 4.

When $0 < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)}$, we do not have explicit solutions. Thus, we consider derivatives of implicit functions.

**Lemma.** For a variable $x$,

$$\frac{\partial r}{\partial x} = -\frac{\partial \Omega / \partial x}{\partial \Omega / \partial r}.$$

**Proof.** The proof is in Appendix B.1.

We know $\frac{\partial \Omega}{\partial r} = \text{slope} (LOGR) - \text{slope} (LR)$ and for any solution, $\text{slope} (LOGR) <
slope \((LR)\). Thus, we have \(\frac{\partial \Omega}{\partial r} < 0\). Then, we can obtain \(\text{sign} \left( \frac{\partial r}{\partial x} \right) = \text{sign} \left( \frac{\partial \Omega}{\partial x} \right)\), which implies that, to explore the effect of the increase of a variable on the optimal degree of delay reduction service, we just need to study its effect on the first-order condition.

3.1 The effect of safety standard on degree

Undoubtedly, safety is the highest priority in air transport sector and safety standard always becomes higher unceasingly. Then, how the improvement of safety standard affects the optimal degree of delay reduction service? Before answering this question, we first give three definitions, the safety elasticity of delay, the safety elasticity of cost, and the safety elasticity of passenger safety benefit.

Definition. The safety elasticity of delay, cost, and passenger safety benefit is defined as, respectively,

\[
\varepsilon_{gs} = \frac{dg(s)}{g(s)} \frac{s}{ds},
\]

\[
\varepsilon_{ms} = \frac{dm(s)}{m(s)} \frac{s}{ds},
\]

\[
\varepsilon_{as} = \frac{da(s)}{a(s)} \frac{s}{ds}.
\]

The safety elasticity of delay (cost, passenger safety benefit) measures the percentage change in air traffic delay caused by an exceptional event (marginal cost of providing delay reduction service, passenger’s utility gain from a safety standard) in response to a one percent change in safety standard.

Then, we have the following result.

Proposition 2.

1. For each \(r^{SB}\), there exists a threshold \(\varepsilon^{SB}\) such that \(r^{SB}\) increases with \(s\) if and only if

\[
\varepsilon_{gs} - \varepsilon_{ms} \geq \varepsilon^{SB} \equiv - \frac{(3 + 2\lambda) \eta \alpha D(s) \sigma s}{2 (1 + \lambda) (1 + r^{SB}) m(s)} \frac{\partial V^*}{\partial s} (s, r^{SB}).
\] (3)

2. For each \(r^{SB}\), there exists a threshold \(\varepsilon^{SB}\) such that \(r^{SB}\) increases with \(s\) if and only if

\[
\varepsilon_{gs} - \varepsilon_{ms} \geq \varepsilon^{SB} \equiv - \frac{(3 + 2\lambda) \eta \alpha D(s) \sigma s}{2 (1 + \lambda) (1 + r^{SB}) m(s)} \frac{\partial V^*}{\partial s} (s, r^{SB}).
\] (4)
Proof. We first consider the effect of $s$ on $r^{SB}$. Taking the derivative of $\Omega$ in (1) with respect to $s$, we can obtain

\[
\frac{\partial \Omega}{\partial s} = \left\{ \frac{2 (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma^2 \ln (1 + r^{SB}) - (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma}{\hat{\beta} T} \right. \\
\left. + (3 + 2 \lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha \sigma + 4 (1 + \lambda) \bar{\theta} f \sigma \right\} \\
\frac{\partial \Omega}{\partial s} = \left\{ \frac{\partial D(s)}{\partial s} \frac{\partial \Omega}{\partial \Omega} \right\} \\
+ (3 + 2 \lambda) \eta \alpha D(s) \sigma a'(s) - 4 (1 + \lambda) \left( 1 + \bar{\pi}^{SB} \right) m'(s). \tag{5}
\]

By using Definition and (1), we can obtain

\[
\frac{\partial \Omega}{\partial s} = \frac{1}{s} \left\{ 4 (1 + \lambda) \left( 1 + \bar{\pi}^{SB} \right) m(s) (\varepsilon_{gs} - \varepsilon_{ms}) \\
+ (3 + 2 \lambda) \eta \alpha D(s) \sigma \left\{ a(s) \varepsilon_{as} - \alpha D(s) \left[ 1 - \sigma \ln \left( 1 + \bar{\pi}^{SB} \right) \right] \varepsilon_{gs} \right\} \right\}.
\]

Then, by introducing the direct effect of the improvement of safety standard on passenger utility, that is,

\[
\frac{\partial \hat{V}^*}{\partial s} (s, \bar{\pi}^{SB}) = \frac{1}{2} \left\{ a'(s) - 2 \alpha \left[ \sum_{k=1}^{+\infty} \frac{\beta T k}{(k-1)!} e^{-\frac{\beta T}{T}} \right] \left[ 1 - \sigma \ln \left( 1 + \bar{\pi}^{SB} \right) \right] g'(s) \right\} \\
= \frac{1}{2s} \left\{ a(s) \varepsilon_{as} - \alpha D(s) \left[ 1 - \sigma \ln \left( 1 + \bar{\pi}^{SB} \right) \right] \varepsilon_{gs} \right\},
\]

and plugging $\frac{\partial \hat{V}^*}{\partial s} (s, \bar{\pi}^{SB})$ into $\frac{\partial \Omega}{\partial s}$, we can obtain (3).

We then consider the effect of $s$ on $r^{SB}$. Taking the derivative of $\Omega$ in (2) with
respect to $s$, we can obtain

$$
\frac{\partial \Omega}{\partial s} = \left[ 2 (3 + 2 \lambda) \eta \alpha^2 D (s) \sigma^2 \ln \left( 1 + \tau^{SB} \right) - (3 + 2 \lambda) \eta \alpha^2 D (s) \sigma \right. \\
+ (3 + 2 \lambda) \eta (\xi - \tau + a (s) - z - \alpha D (s)) \alpha \sigma + 4 (1 + \lambda) \theta f \sigma - 4 \frac{\mu}{1 - \mu} \lambda \Delta f \sigma \\
\left. \right] + \frac{\partial \Omega}{\partial D (s)} \\
\cdot 2 \sum_{k=1}^{+\infty} \left( \frac{\beta T}{T} \right)^k e^{-\left( \frac{\beta T}{T} \right)} \left\{ \begin{array}{c}
\frac{\partial D (s)}{\partial s} \\
\frac{\partial \Omega}{\partial D (s)} \frac{da (s)}{ds} \\
\frac{\partial \Omega}{\partial m (s)} \frac{dm (s)}{ds}
\end{array} \right\} \\
+ (3 + 2 \lambda) \eta \alpha D (s) \sigma \alpha'^{'} (s) - 4 (1 + \lambda) \left( 1 + \tau^{SB} \right) m' (s). 
$$

By using Definition and (2), we can obtain

$$
\frac{\partial \Omega}{\partial s} = \frac{1}{s} \left\{ 4 \left( 1 + \lambda \right) \left( 1 + \tau^{SB} \right) m (s) (\varepsilon_{gs} - \varepsilon_{ms}) \\
+ (3 + 2 \lambda) \eta \alpha D (s) \sigma \left\{ a (s) \varepsilon_{as} - \alpha D (s) \left[ 1 - \sigma \ln \left( 1 + \tau^{SB} \right) \right] \varepsilon_{gs} \right\} \right\} .
$$

Then, by plugging $\frac{\partial V^* (s, \tau^{SB})}{\partial s}$ into $\frac{\partial \Omega}{\partial s}$, we can obtain (4).

Proposition 2 tells us that, for each degree, there exists a threshold such that the degree increases with safety standard if and only if, the difference between the safety elasticity of delay, which is conducive to the increase of the degree, and the safety elasticity of cost, which is not conducive to the increase of the degree, is larger than the threshold, which is a function of the direct effect of the improvement of safety standard on passenger utility. Moreover, if passengers can directly benefit from a higher safety standard, the condition ensuring that the degree increases with safety standard can be easier to achieve.

To illustrate (3) in Proposition 2, we should first analyze the effects of safety standard on the degree in (5).

In fact, the first term in (5) measures the effects of the longer delay resulting from the improvement of safety standard on the degree. Specifically, we can find a direct and an indirect effect. On the one hand, the longer the delay is, the more
delay reduction the airline and passengers will enjoy. In other words, the longer delay will increase the marginal benefit of society from the service. Thus, the longer delay will directly motivate the regulator to increase the degree\textsuperscript{14}. On the other hand, the longer delay will decrease the number of passengers and a smaller number of passengers implies a lower marginal benefit of society. Thus, the longer delay will indirectly motivate the regulator to decrease the degree.

The second term in (5) measures the effect of the higher passenger’s utility gain resulting from the improvement of safety standard on the degree. The higher passenger’s utility gain will increase the number of passengers and a larger number of passengers implies a higher marginal benefit of society. Thus, the higher passenger’s utility gain will motivate the regulator to increase the degree.

The third term in (5) measures the effect of the higher marginal cost of providing the service resulting from the improvement of safety standard on the degree. Undoubtedly, the higher marginal cost will motivate the regulator to decrease the degree.

In fact, (3) is the synthesis of the effects above. More precisely, it is the condition about whether the effects which are conducive to the increase of the degree can dominate the ones which are not.

Then, we consider (4) in Proposition 2. From (6), besides the four effects in (5), we can find another effect. Specifically, the longer delay resulting from the improvement of safety standard will increase the information rent of the airline with \( \overline{\theta} \), which is a function of \( \overline{\ell}^{SB} \), and thus motivate the regulator to decrease \( \overline{\ell}^{SB} \). In fact, except that the effect of information rent has been incorporated into the value of \( \overline{\ell}^{SB} \), we can explain (4) in the same way as for (3).

### 3.2 The effect of exceptional event arriving rate on degree

In this part, we analyze the effect of the exceptional event arriving rate \( \beta \) on the optimal degree of delay reduction service. Specifically, we have the following result.

**Proposition 3.** The sign of \( \frac{\partial r}{\partial \beta} \) is the same as that of \( \frac{\partial \Omega}{\partial D(s)} \), which measures the effect of delay on the optimal degree. Then, we consider \( \frac{\partial \Omega}{\partial D(s)} \).

1. For each \( \overline{\ell}^{SB} \), there exists a threshold \( \overline{\alpha}^{SB} \) such that \( \overline{\ell}^{SB} \) increases with \( D(s) \)

\textsuperscript{14} \( \varepsilon_{gs} \) in (3) represents this direct effect.
if and only if
\[
\alpha \leq \alpha^{SB} \equiv \frac{2}{D(s)} \sqrt{\frac{(1 + \lambda) m(s) (1 + \tau^{SB})}{(3 + 2\lambda) \eta \sigma [1 - \sigma \ln (1 + \tau^{SB})]}}.
\] (7)

2. For each \( \tau^{SB} \), there exists a threshold \( \alpha^{SB} \) such that \( \tau^{SB} \) increases with \( D(s) \) if and only if
\[
\alpha \leq \alpha^{SB} \equiv \frac{2}{D(s)} \sqrt{\frac{(1 + \lambda) m(s) (1 + \tau^{SB})}{(3 + 2\lambda) \eta \sigma [1 - \sigma \ln (1 + \tau^{SB})]}}.
\] (8)

Proof. We first consider the effect of \( \beta \) on \( \tau^{SB} \). Taking the derivative of \( \Omega \) in (1) with respect to \( \beta \), we can obtain
\[
\frac{\partial \Omega}{\partial \beta} = \left[ 2 (3 + 2\lambda) \eta \alpha^2 D(s) \sigma^2 \ln (1 + \tau^{SB}) - (3 + 2\lambda) \eta \alpha^2 D(s) \sigma \right]
\]
\[
+ (3 + 2\lambda) \eta \left( \xi - \tau + a(s) - z - \alpha D(s) \right) \alpha \sigma + 4 (1 + \lambda) \overline{\theta} f \sigma
\]
\[
\cdot 2 \sum_{k=1}^{+\infty} \left( \frac{\beta T}{T} \right)^{k-1} e^{-\left( \frac{\beta T}{T} \right)} \left( k - \frac{\beta T}{T} \right) g(s) + \gamma f g(s) \right].
\] (9)

By using (1), we can obtain
\[
\frac{\partial \Omega}{\partial \beta} = \frac{1}{D(s)} \left\{ 4 (1 + \lambda) m(s) (1 + \tau^{SB}) - (3 + 2\lambda) \eta \alpha^2 D(s) \sigma [1 - \sigma \ln (1 + \tau^{SB})] \right\}
\]
\[
\cdot 2 \sum_{k=1}^{+\infty} \left( \frac{\beta T}{T} \right)^{k-1} e^{-\left( \frac{\beta T}{T} \right)} \left( k - \frac{\beta T}{T} \right) g(s) + \gamma f g(s) \right].
\] (10)
Because of $\frac{\partial D(s)}{\partial \beta} > 0$, we can obtain $\text{sign} \left( \frac{\partial \Omega}{\partial \beta} \right) = \text{sign} \left( \frac{\partial \Omega}{\partial D(s)} \right)$. Therefore, what we should do is to determine the sign of $\frac{\partial \Omega}{\partial D(s)}$. Then, we can obtain (7) according to $\frac{\partial \Omega}{\partial D(s)}$ in (10).

We then consider the effect of $\beta$ on $\Omega^{SB}$. Taking the derivative of $\Omega$ in (2) with respect to $\beta$, we can obtain

$$
\frac{\partial \Omega}{\partial \beta} = \left[ 2 (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma^2 \ln (1 + \ell^{SB}) - (3 + 2 \lambda) \eta \alpha D(s) \right]
$$

$$
+ (3 + 2 \lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha \sigma + 4 (1 + \lambda) \theta f \sigma - 4 \frac{\mu}{1 - \mu} \lambda \Delta \theta f \sigma
$$

$$
\cdot 2 \left[ \sum_{k=1}^{+\infty} \left( \frac{\beta T}{T} \right)^{k-1} e^{-\left( \frac{\beta T}{T} \right)} \frac{\left( k - \frac{\beta T}{T} \right) T f}{(k - 1)!} g(s) + \gamma \frac{f}{T} g(s) \right].
$$

(11)

By using (2), we can obtain

$$
\frac{\partial \Omega}{\partial \beta} = \frac{1}{D(s)} \left\{ 4 (1 + \lambda) m(s) (1 + \ell^{SB}) - (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma \left[ 1 - \sigma \ln (1 + \ell^{SB}) \right] \right\}
$$

$$
\cdot 2 \left[ \sum_{k=1}^{+\infty} \left( \frac{\beta T}{T} \right)^{k-1} e^{-\left( \frac{\beta T}{T} \right)} \frac{\left( k - \frac{\beta T}{T} \right) T f}{(k - 1)!} g(s) + \gamma \frac{f}{T} g(s) \right].
$$

(12)

Then, we can obtain (8) according to $\frac{\partial \Omega}{\partial D(s)}$ in (12).

Proposition 3 tells us that, for each degree, there exist a threshold such that the degree increases with the exceptional event arriving rate if and only if the passenger’s value of time is smaller than a threshold.

To illustrate (7) in Proposition 3, we should first analyze the effects of the exceptional event arriving rate on the degree in (9).

From (9), we can find two effects. Note that the increase of the rate will increase
the expected delay per flight. Then, we have a direct and an indirect effect of the longer delay on the degree. On the one hand, the longer delay will increase the marginal benefit of society from the service and thus directly motivate the regulator to increase the degree. On the other hand, the longer delay will decrease the number of passengers and a smaller number of passengers implies a lower marginal benefit of society. Thus, the longer delay will indirectly motivate the regulator to decrease the degree.

Then, for (7), the case that $\alpha$ is smaller (larger) than a threshold implies that the effect of the increase of delay on the number of passengers, that is, a part of the indirect effect, will be relatively insignificant (significant) and thus the direct effect will dominate (be dominated by) the indirect effect, that is, the increase of the rate, or delay, will motivate the regulator to increase (decrease) the degree. Intuitively, the increase of delay can always motivate the regulator to increase the degree. In fact, (7) shows that this happens only when the passenger’s value of time is relatively low. If passengers value their time relatively much, a significant proportion of them will tend to choose outside option when delay becomes longer. In this way, the delay reduction service will be less valuable for a shrinking market and thus the degree of the service will decrease.

Then, we consider (8) in Proposition 3. From (11), besides the two effects in (9), we can find another effect. Specifically, the longer delay resulting from the increase of the rate will increase the information rent of the airline with $\theta$, which is a function of $r_{SB}$, and thus motivate the regulator to decrease $r_{SB}$. In fact, except that the effect of information rent has been incorporated into the value of $r_{SB}$, we can explain (8) in the same way as for (7).

Moreover, we can also find a sufficient condition for $\frac{\partial \Omega}{\partial D(s)} \geq 0$ and $\frac{\partial \Omega}{\partial D(s)} \geq 0$.

**Corollary 1.** $r_{SB}$ and $\bar{r}_{SB}$ increase with $D(s)$ if

$$\alpha \leq \frac{2}{D(s)} \sqrt{\frac{(1 + \lambda) m(s)}{(3 + 2\lambda) \eta \sigma}}. \quad (13)$$

**Proof.** The proof is in Appendix B.2. \qed

For (13) in Corollary 1, if $\alpha$ is smaller than a threshold, the effect of the increase of delay on the number of passengers, that is, a part of the indirect effect, will be so insignificant that the direct effect will always dominate the indirect effect, that is,
the increase of the rate, or delay, will always motivate the regulator to increase the degree.

Then, if we also consider the relevant thresholds $\bar{\alpha}^{FB}$ and $\underline{\alpha}^{FB}$ under complete information and combining the ones in Proposition 3, we will have Corollary 2.

**Corollary 2.** When $\beta$ increases, there is a possibility that $\bar{r}^{SB}$ and $\underline{r}^{SB}$ move in opposite directions. Moreover, the existence of information rent increases that possibility.

**Proof.** Comparing the thresholds, we can obtain $\bar{\alpha}^{SB} \geq \underline{\alpha}^{SB}$. Thus, the difference between the thresholds creates the possibility that $\bar{r}^{SB}$ and $\underline{r}^{SB}$ move in opposite direction. Moreover, if we also consider complete information, we can find $\underline{\alpha}^{SB} \leq \bar{\alpha}^{FB} \leq \bar{\alpha}^{SB} = \bar{\alpha}^{FB}$. For the values of $\alpha$ such that $\underline{\alpha}^{FB} \leq \alpha \leq \bar{\alpha}^{FB}$, $\bar{r}^{FB}$ and $\underline{r}^{FB}$ move in opposite directions. In the same way, for the values of $\alpha$ such that $\underline{\alpha}^{SB} \leq \alpha \leq \bar{\alpha}^{SB}$, $\bar{r}^{SB}$ and $\underline{r}^{SB}$ move in opposite directions. However, we have $\bar{\alpha}^{SB} - \underline{\alpha}^{SB} \geq \bar{\alpha}^{FB} - \underline{\alpha}^{FB}$. This implies that, because of information rent, the possibility that $\bar{r}^{SB}$ and $\underline{r}^{SB}$ move in opposite directions becomes larger. We can better see this in Figure 4.

![Diagram](image)

**Figure 4:** Information rent and movements of degrees

In fact, the analysis for the effect of the delay externality parameter $\gamma$ on the optimal degree is essentially the same as this part and is omitted henceforth.

For convenience, in the following parts, let

$$
\Phi \equiv 4 (1 + \lambda) m(s) (1 + \bar{r}^{SB}) - (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma \left[ 1 - \sigma \ln (1 + \bar{r}^{SB}) \right], \\
\Psi \equiv 4 (1 + \lambda) m(s) (1 + \underline{r}^{SB}) - (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma \left[ 1 - \sigma \ln (1 + \underline{r}^{SB}) \right].
$$
3.3 The effect of number of available hours on degree

In this part, we analyze the effect of the number of available hours $T$ on the optimal degree of delay reduction service. Specifically, we have the following result.

**Proposition 4.** When $\bar{r}^{SB}$ ($T^{SB}$) increases with $D(s)$, $\bar{r}^{SB}$ ($T^{SB}$) increases with $T$ if and only if $\gamma \leq \Gamma$. Otherwise, $\bar{r}^{SB}$ ($T^{SB}$) increases with $T$ if and only if $\gamma \geq \Gamma$, where

$$\Gamma \equiv \sum_{k=1}^{+\infty} \frac{(\frac{\beta T}{T})^{k-1} e^{-\left ( \frac{\beta T}{T} \right )} (k-\frac{\beta T}{T})^{2} (k-1)!}{T^{2} g(s)}.$$ 

*Proof.* We first consider the effect of $T$ on $r^{SB}$. Taking the derivative of $\Omega$ in (1) with respect to $T$, we can obtain

$$\frac{\partial \Omega}{\partial T} = \left[ \frac{2 (3 + 2\lambda) \eta \alpha^{2} D(s) \sigma^{2} \ln \left ( 1 + \bar{r}^{SB} \right ) - (3 + 2\lambda) \eta \alpha^{2} D(s) \sigma}{\frac{\partial D(s)}{\partial T}} \right ] + (3 + 2\lambda) \eta \left ( \xi - \tau + a(s) - z - \alpha D(s) \right ) \alpha \sigma + 4 (1 + \lambda) \bar{f} f \sigma$$

$$+ 2 \sum_{k=1}^{+\infty} \frac{(\frac{\beta T}{T})^{k-1} e^{-\left ( \frac{\beta T}{T} \right )} (k-\frac{\beta T}{T})^{2} (k-1)!}{T^{2} g(s)} \left ( \left ( k-\frac{\beta T}{T} \right ) \beta \bar{f} g(s) - \gamma \beta \frac{T^{2} g(s)}{T^{2}} \right ).$$

We have discussed $\frac{\partial \Omega}{\partial D(s)}$ in the last part. Thus, what really matters here is when the number of available hours increases how the expected delay per flight will change, that is, the sign of $\frac{\partial D(s)}{\partial T}$. In fact, letting $\Gamma \equiv \sum_{k=1}^{+\infty} \frac{(\frac{\beta T}{T})^{k-1} e^{-\left ( \frac{\beta T}{T} \right )} (k-\frac{\beta T}{T})^{2} (k-1)!}{T^{2} g(s)}$, we can obtain

$$\frac{\partial D(s)}{\partial T} \geq 0 \Leftrightarrow \gamma \leq \Gamma,$$

which shows that the expected delay per flight increases with the number of available hours if and only if the delay externality parameter is small enough.

Because of $\frac{\partial \Omega}{\partial D(s)} = \frac{1}{D(s)} \Phi$, we have $\text{sign} \left ( \frac{\partial \Omega}{\partial D(s)} \right ) = \text{sign} (\Phi)$ and then we can discuss cases by the sign of $\Phi$. Therefore, when $\Phi \geq 0$, that is, $\frac{\partial r^{SB}}{\partial D(s)} \geq 0$,

$$\frac{\partial \bar{r}^{SB}}{\partial T} \geq 0 \Leftrightarrow \gamma \leq \Gamma.$$
and when $\Phi < 0$, that is, $\frac{\partial r^{SB}}{\partial D(s)} < 0$,

$$\frac{\partial r^{SB}}{\partial T} \geq 0 \iff \gamma \geq \Gamma.$$ 

We then consider the effect of $T$ on $r^{SB}$. Taking the derivative of $\Omega$ in (2) with respect to $T$, we can obtain

$$\frac{\partial \Omega}{\partial T} = \left[ \frac{2(3 + 2\lambda) \eta \alpha^2 D(s) \sigma^2 \ln(1 + r^{SB}) - (3 + 2\lambda) \eta \alpha^2 D(s) \sigma}{\frac{\partial \Omega}{\partial D(s)}} + (3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha \sigma + 4(1 + \lambda) \theta f \sigma - 4 \frac{\mu}{1 - \mu} \lambda \Delta \theta f \sigma \right]$$

$$\cdot \sum_{k=1}^{+\infty} \frac{\left( \frac{3T}{T} \right)^{k-1} e^{-\left( \frac{3T}{T} \right) k \frac{3}{T}} g(s) - \gamma \beta f \frac{1}{T^2} g(s)}{(k - 1)!} \right].$$

From $\frac{\partial \Omega}{\partial T}$, we can find that the only difference between $\frac{\partial \Omega}{\partial T}$ and $\frac{\partial \Omega}{\partial D(s)}$ is the derivative of $\Omega$ and $\Omega$ with respect to $D(s)$. Thus, we can directly obtain the conclusion. Again, because of $\frac{\partial \Omega}{\partial D(s)} = \frac{1}{D(s)} \Psi$, we have $\text{sign} \left( \frac{\partial \Omega}{\partial D(s)} \right) = \text{sign} (\Psi)$ and then we can discuss cases by the sign of $\Psi$. Therefore, when $\Psi \geq 0$, that is, $\frac{\partial r^{SB}}{\partial D(s)} \geq 0$,

$$\frac{\partial r^{SB}}{\partial T} \geq 0 \iff \gamma \leq \Gamma$$

and when $\Psi < 0$, that is, $\frac{\partial r^{SB}}{\partial D(s)} < 0$,

$$\frac{\partial r^{SB}}{\partial T} \geq 0 \iff \gamma \geq \Gamma.$$ 

Proposition 4 tells us that, when the degree increases (decreases) with the expected delay per flight, the degree increases with the number of available hours if and only if the delay externality parameter is smaller (larger) than a threshold.
To illustrate the change of $\tau^{SB}$ in Proposition 4, we should first analyze the effects of the number of available hours on the degree in (14).

From (14), we can find four effects. On the one hand, the increase of the number of available hours will increase delay due to exceptional events in own slot. Then, we have a direct and an indirect effect of the longer delay on the degree. First, the longer delay will increase the marginal benefit of society from the service and thus directly motivate the regulator to increase the degree. Second, the longer delay will decrease the number of passengers and a smaller number of passengers implies a lower marginal benefit of society. Thus, the longer delay will indirectly motivate the regulator to decrease the degree. On the other hand, the increase of the number of available hours will decrease delay induced by the delayed flights in previous slots. Then, we have a direct and an indirect effect of the shorter delay on the degree. Third, the shorter delay will decrease the marginal benefit of society from the service and thus directly motivate the regulator to decrease the degree. Fourth, the shorter delay will increase the number of passengers and a larger number of passengers implies a higher marginal benefit of society. Thus, the shorter delay will indirectly motivate the regulator to increase the degree.

Then, for the change of $\tau^{SB}$, we can discuss in two cases.

**Case 1**: when $\Phi \geq 0$,

$$\frac{\partial \tau^{SB}}{\partial T} \geq 0 \Leftrightarrow \gamma \leq \Gamma.$$  

In this case, the degree increases with delay. This implies that the net effect of the first and second effect is to increase the degree and the net effect of the third and fourth effect is to decrease the degree. Then, what matters for the comparison between two net effects is that, when the number of available hours increases, which one will dominate, the change of delay due to exceptional events in own slot or the change of delay induced by the delayed flights in previous slots? The result shows that a small (large) delay externality parameter favours the change of delay due to exceptional events in own slot (delay induced by the delayed flights in previous slots). Then, the net effect of the first and second effect (third and fourth effect) dominates the other and thus the degree will increase (decrease) with the number of available hours.

**Case 2**: when $\Phi < 0$,

$$\frac{\partial \tau^{SB}}{\partial T} \geq 0 \Leftrightarrow \gamma \geq \Gamma.$$
In this case, the degree decreases with delay. This implies that the net effect of the first and second effect is to decrease the degree and the net effect of the third and fourth effect is to increase the degree. Then, what matters for the comparison between two net effects is that, when the number of available hours increases, which one will dominate, the change of delay due to exceptional events in own slot or the change of delay induced by the delayed flights in previous slots? The result shows that a large (small) delay externality parameter favours the change of delay induced by the delayed flights in previous slots (delay due to exceptional events in own slot). Then, the net effect of the third and fourth effect (first and second effect) dominates the other and thus the degree will increase (decrease) with the number of available hours.

Then, we consider the change of $\tau^{SB}$ in Proposition 4. From (15), besides the four effects in (14), we can find another two effects. Specifically, fifth, the increase of the number of available hours will increase delay due to exceptional events in own slot and thus increase the information rent of the airline with $\bar{\theta}$, which is a function of $\tau^{SB}$. This will motivate the regulator to decrease $\tau^{SB}$. Sixth, the increase of the number of available hours will decrease delay induced by the delayed flights in previous slots and thus decrease the information rent. This will motivate the regulator to increase $\tau^{SB}$. In fact, for the change of $\tau^{SB}$, except that the effects of information rent have been incorporated into the value of $\tau^{SB}$, we can also discuss in two cases and develop the same explanations as for $\frac{\partial \tau^{SB}}{\partial T}$.

3.4 The effect of number of flights on degree

In this part, we analyze the effect of the number of flights $f$ on the optimal degree of delay reduction service. Specifically, we have the following result. Note that, to avoid confusion, in this part, we denote partial derivatives by $\frac{\partial \Omega}{\partial f}$ and $\frac{\partial \Omega}{\partial T}$.

**Proposition 5.** Let

\[
\Gamma_f \equiv \Gamma - 2(1 + \lambda) \overline{\theta} D(s)^2 \sigma \frac{1}{\psi(1 - \mu)} T \frac{1}{\beta g(s)},
\]

\[
\Gamma_f \equiv \Gamma - 2 \left[ (1 + \lambda) \overline{\theta} - \frac{\mu}{1 - \mu} \lambda \Delta \theta \right] D(s)^2 \sigma \frac{1}{\psi} T \frac{1}{\beta g(s)}.
\]

1. When $\tau^{SB}$ increases (decreases) with $D(s)$, $\tau^{SB}$ increases with $f$ if and only if
\[
\gamma \geq (\leq) \Gamma_f, \text{ where } \Gamma_f \leq (>) \Gamma.
\]

2(i). When \( r^{SB} \) increases with \( D(s) \) and \( \theta \geq (\geq) \frac{\mu}{1-\mu} \Delta \theta \), \( r^{SB} \) increases with \( f \) if and only if \( \gamma \geq \Gamma_f \), where \( \Gamma_f \leq (>) \Gamma \).

2(ii) When \( r^{SB} \) decreases with \( D(s) \) and \( \theta \geq (\geq) \frac{\mu}{1-\mu} \Delta \theta \), \( r^{SB} \) increases with \( f \) if and only if \( \gamma \leq \Gamma_f \), where \( \Gamma_f \geq (\leq) \Gamma \).

Proof. We first consider the effect of \( f \) on \( r_{SB} \). Taking the derivative of \( \Omega \) in (1) with respect to \( f \), we can obtain

\[
\frac{\partial \Omega}{\partial f} = \frac{2 (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma^2 \ln (1 + r^{SB}) - (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma}{\partial \Omega_{D(s)}} + (3 + 2 \lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha \sigma + 4 (1 + \lambda) \bar{\theta} f \sigma.
\]

By using (1), we can obtain

\[
\frac{\partial r_{SB}}{\partial f} = \frac{2 (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma^2 \ln (1 + r^{SB}) - (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma}{\partial \Omega_{D(s)}} + (3 + 2 \lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha \sigma + 4 (1 + \lambda) \bar{\theta} D(s) \sigma.
\]

Then, letting \( \Gamma_f \equiv \Gamma - 2 (1 + \lambda) \bar{\theta} D(s)^2 \sigma \frac{T}{\Phi} g(s) \), we can obtain that when \( \Phi > (\leq) 0 \),

\[
\frac{\partial r^{SB}}{\partial f} \geq 0 \iff \gamma \geq (\leq) \Gamma_f,
\]

where \( \Gamma_f \leq (\geq) \Gamma \).

We then consider the effect of \( f \) on \( r^{SB} \). Taking the derivative of \( \Omega \) in (2) with respect to \( f \), we can obtain

\[
\frac{\partial \Omega}{\partial f} = \frac{2 (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma^2 \ln (1 + r^{SB}) - (3 + 2 \lambda) \eta \alpha^2 D(s) \sigma}{\partial \Omega_{D(s)}}
\]
\[
+ (3 + 2\lambda) \eta (\xi - \tau + a (s) - z - \alpha D (s)) \alpha \sigma + 4 (1 + \lambda) \theta f \sigma - 4 \frac{\mu}{1 - \mu} \lambda \Delta f \sigma
\]

\[
\cdot 2 \left[ - \sum_{k=1}^{\infty} \left( \frac{\beta T}{T} \right)^k e^{-\left( \frac{\beta T}{T} \right)} \left( k - \frac{\beta T}{T} \right) \frac{1}{T} \frac{1}{T} g (s) + \gamma \beta \frac{1}{T} g (s) \right]
\]

\[
+ 4 (1 + \lambda) \theta D (s) \sigma - 4 \frac{\mu}{1 - \mu} \lambda \Delta D (s) \sigma.
\]  

(17)

By using (2), we can obtain

\[
\frac{\partial \Omega}{\partial f} = \frac{2}{D (s)} \left\{ - \sum_{k=1}^{\infty} \left( \frac{\beta T}{T} \right)^k e^{-\left( \frac{\beta T}{T} \right)} \left( k - \frac{\beta T}{T} \right) \frac{1}{T} \frac{1}{T} g (s) + \gamma \beta \frac{1}{T} g (s) \right\} \Psi
\]

\[
+ 2 \left[ (1 + \lambda) \theta - \frac{\mu}{1 - \mu} \lambda \Delta \theta \right] D (s) \sigma.
\]

Then, letting \( \Gamma_f \equiv \Gamma - 2 \left[ (1 + \lambda) \theta - \frac{\mu}{1 - \mu} \lambda \Delta \theta \right] D (s) \sigma \), we can obtain that

(1) when \( \Psi > 0 \) and \( \theta > \left( \frac{\mu}{1 - \mu} \lambda \Delta \theta \right) \),

\[
\frac{\partial \Omega}{\partial f} \geqslant 0 \iff \gamma \geqslant \Gamma_f,
\]

where \( \Gamma_f \leqslant \left( \frac{\mu}{1 - \mu} \right) \); 

(2) when \( \Psi < 0 \) and \( \theta > \left( \frac{\mu}{1 - \mu} \lambda \Delta \theta \right) \),

\[
\frac{\partial \Omega}{\partial f} \geqslant 0 \iff \gamma \leqslant \Gamma_f,
\]

where \( \Gamma_f \geqslant \left( \frac{\mu}{1 - \mu} \right) \).

In fact, for completeness, we also have to discuss the cases where \( \Phi = 0, \Psi = 0 \). However, because such cases are trivial, the discussion for them is omitted henceforce.

Proposition 5 tells us that, when the degree for the airline with a higher value
of time increases (decreases) with the expected delay per flight, the degree increases with the number of flights if and only if the delay externality parameter is larger (smaller) than a threshold; when the degree for the airline with a lower value of time increases with the expected delay per flight and the distortion of information rent is insignificant (significant), the degree increases with the number of flights if and only if the delay externality parameter is larger than a threshold; when the degree for the airline with a lower value of time decreases with the expected delay per flight and the distortion of information rent is insignificant (significant), the degree increases with the number of flights if and only if the delay externality parameter is smaller than a threshold.

From $\frac{\partial D(s)}{\partial f}$ in (16) and (17), we can obtain

$$\frac{\partial D(s)}{\partial f} \geq 0 \Leftrightarrow \gamma \geq \Gamma,$$

which shows that the expected delay per flight increases with the number of flights if and only if the delay externality parameter is large enough. We will use this condition in the following analysis.

To illustrate the change of $\tau^{SB}$ in Proposition 5, we should first analyze the effects of the number of flights on the degree in (16).

From (16), we can find five effects. Note that the increase of the number of flights will decrease delay due to exceptional events in own slot. Then, we have a direct and an indirect effect of the shorter delay on the degree. First, the shorter delay will decrease the marginal benefit of society from the service and thus directly motivate the regulator to decrease the degree. Second, the shorter delay will increase the number of passengers and a larger number of passengers implies a higher marginal benefit of society. Thus, the shorter delay will indirectly motivate the regulator to increase the degree. Moreover, the increase of the number of flights will increase delay induced by the delayed flights in previous slots. Then, we have a direct and an indirect effect of the longer delay on the degree. Third, the longer delay will increase the marginal benefit of society from the service and thus directly motivate the regulator to increase the degree. Fourth, the longer delay will decrease the number of passengers and a smaller number of passengers implies a lower marginal benefit of society. Thus, the longer delay will indirectly motivate the regulator to decrease the degree. Besides, fifth, the increase of the number of flights will also directly increase the airline’s supply
side delay cost, which implies a higher marginal benefit of society, and thus motivate the regulator to increase the degree.

Then, for the change of $\tau^{SB}$, we can discuss in two cases.

Case 1: when $\Phi > 0$, 
\[
\frac{\partial \tau^{SB}}{\partial f} \geq 0 \iff \gamma \geq \Gamma_f.
\]

In this case, the degree increases with delay. This implies that the net effect of the first and second effect is to decrease the degree and the net effect of the third and fourth effect is to increase the degree. Then, what matters for the comparison between two net effects is that, when the number of flights increases, which one will dominate, the change of delay due to exceptional events in own slot or the change of delay induced by the delayed flights in previous slots?

We have shown that, if the delay externality parameter is large, that is, $\gamma \geq \Gamma$, the change of delay induced by the delayed flights in previous slots will dominate. Then, the net effect of the third and fourth effect dominates the other and thus the net effect of the first four effects is to increase the degree. Combining the fifth effect, which is also conducive to the increase of the degree, the degree will increase with the number of flights.

However, if the delay externality parameter is small, that is, $\gamma < \Gamma$, the change of delay due to exceptional events in own slot will dominate. Then, the net effect of the first and second effect dominates the other and thus the net effect of the first four effects is to decrease the degree. However, the fifth effect is to increase the degree. In fact, if delay externality parameter is not very small (very small), that is, $\gamma \geq (<) \Gamma_f$, where $\Gamma_f \leq \Gamma$, the net effect of the first four effects will be less (more) significant than the fifth effect and thus the degree will increase (decrease) with the number of flights.

Case 2: when $\Phi < 0$, 
\[
\frac{\partial \tau^{SB}}{\partial f} \geq 0 \iff \gamma \leq \Gamma_f.
\]

In this case, the degree decreases with delay. This implies that the net effect of the first and second effect is to increase the degree and the net effect of the third and fourth effect is to decrease the degree. Then, what matters for the comparison between two net effects is that, when the number of flights increases, which one will dominate, the change of delay due to exceptional events in own slot or the change of delay induced by the delayed flights in previous slots?
We have shown that, if the delay externality parameter is large, that is, $\gamma \geq \Gamma$, the change of delay induced by the delayed flights in previous slots will dominate. Then, the net effect of the third and fourth effect dominates the other and thus the net effect of the first four effects is to decrease the degree. However, the fifth effect is to increase the degree. In fact, if the delay externality parameter is not very large (very large), that is, $\gamma \leq (>) \Gamma_f$, where $\Gamma_f > \Gamma$, the net effect of the first four effects will be less (more) significant than the fifth effect and thus the degree will increase (decrease) with the number of flights.

However, if the delay externality parameter is small, that is, $\gamma < \Gamma$, the change of delay due to exceptional events in own slot will dominate. Then, the net effect of the first and second effect dominates the other and thus the net effect of the first four effects is to increase the degree. Combining the fifth effect, which is also conducive to the increase of the degree, the degree will increase with the number of flights.

Then, we consider the change of $r_{SB}$ in Proposition 5.

From (17), besides the five effects in (16), we can find another three effects. Sixth, the increase of the number of flights will decrease delay due to exceptional events in own slot and thus decrease the information rent of the airline with $\bar{\theta}$, which is a function of $r_{SB}$. This will motivate the regulator to increase $r_{SB}$. Seventh, the increase of the number of flights will increase delay induced by the delayed flights in previous slots and thus increase the information rent. This will motivate the regulator to decrease $r_{SB}$. Eighth, the increase of the number of flights will also directly increase the information rent and thus motivate the regulator to decrease $r_{SB}$.

In fact, for the change of $r_{SB}$, we can discuss in four cases.  

Case 1: when $\Psi > 0$ and $\bar{\theta} \geq \frac{\mu}{1-\mu} \frac{\lambda}{1+\lambda} \Delta \theta$,

$$\frac{\partial r_{SB}}{\partial f} \geq 0 \iff \gamma \geq \Gamma_f.$$  

This case is essentially the same as Case 1 of $\frac{\partial r_{SB}}{\partial f}$.

Case 2: when $\Psi > 0$ and $\bar{\theta} < \frac{\mu}{1-\mu} \frac{\lambda}{1+\lambda} \Delta \theta$,

$$\frac{\partial r_{SB}}{\partial f} \geq 0 \iff \gamma \geq \Gamma_f.$$  

The difference between this case and Case 1 is that, in this case, the net effect of the fifth and eighth effect is to decrease the degree.
We have shown that, if the delay externality parameter is large, that is, \( \gamma \geq \Gamma \), the change of delay induced by the delayed flights in previous slots will dominate. Then, the net effect of the third and fourth effect dominates the net effect of the first and second effect and thus the net effect of the first four effects is to increase the degree. However, the net effect of the fifth and eighth effect is to decrease the degree. In fact, if the delay externality parameter is (not) very large, that is, \( \gamma \geq (\leq) \Gamma_f \), where \( \Gamma_f > \Gamma \), the net effect of the first four effects will be more (less) significant than the net effect of the fifth and eighth effect and thus the degree will increase (decrease) with the number of flights.

However, if the delay externality parameter is small, that is, \( \gamma < \Gamma \), the change of delay due to exceptional events in own slot will dominate. Then, the net effect of the first and second effect dominates the net effect of the third and fourth effect and thus the net effect of the first four effects is to decrease the degree. Combining the net effect of the fifth and eighth effect, which is also conducive to the decrease of the degree, the degree will decrease with the number of flights.

Case 3: when \( \Psi < 0 \) and \( \theta \geq \frac{\mu}{1-\mu} \frac{\lambda}{1+\lambda} \Delta \theta \),

\[
\frac{\partial r^{SB}}{\partial f} \geq 0 \iff \gamma \leq \Gamma_f.
\]

This case is essentially the same as Case 2 of \( \frac{\partial r^{SB}}{\partial f} \).

Case 4: when \( \Psi < 0 \) and \( \theta < \frac{\mu}{1-\mu} \frac{\lambda}{1+\lambda} \Delta \theta \),

\[
\frac{\partial r^{SB}}{\partial f} \geq 0 \iff \gamma \leq \Gamma_f.
\]

The difference between this case and Case 3 is that, in this case, the net effect of the fifth and eighth effect is to decrease the degree.

We have shown that, if the delay externality parameter is large, that is, \( \gamma \geq \Gamma \), the change of delay induced by the delayed flights in previous slots will dominate. Then, the net effect of the third and fourth effect dominates the net effect of the first and second effect and thus the net effect of the first four effects is to decrease the degree. Combining the net effect of the fifth and eighth effect, which is also conducive to the decrease of the degree, the degree will decrease with the number of flights.

However, if the delay externality parameter is small, that is, \( \gamma < \Gamma \), the change of delay due to exceptional events in own slot will dominate. Then, the net effect of the
first and second effect dominates the net effect of the third and fourth effect and thus
the net effect of the first four effects is to increase the degree. However, the net effect
of the fifth and eighth effect is to decrease the degree. In fact, if the delay externality
parameter is (not) very small, that is, \( \gamma \leq (>) \Gamma_f \), where \( \Gamma_f < \Gamma \), the net effect
of the first four effects will be more (less) significant than the net effect of the fifth and
seventh effect and thus the degree will increase (decrease) with the number of flights.

3.5 The effect of passenger’s value of time on degree

In this part, we analyze the effect of the passenger’s value of time \( \alpha \) on the optimal
degree of delay reduction service. Specifically, we have the following result.

Proposition 6.

1. For each \( r_{SB} \), there exists a threshold \( \bar{\alpha}_{SB} \) such that \( r_{SB} \) increases with \( \alpha \) if

\[
\alpha \leq \bar{\alpha}_{SB} \equiv \frac{\xi - \tau + a(s) - z}{2D(s) \left[ 1 - \sigma \ln (1 + \tau_{SB}) \right]}. \tag{18}
\]

and thus

\[
\frac{\partial r_{SB}}{\partial \alpha} \geq 0 \iff \alpha \leq \bar{\alpha}_{SB} \equiv \frac{\xi - \tau + a(s) - z}{2D(s) \left[ 1 - \sigma \ln (1 + \tau_{SB}) \right]}. \tag{20}
\]

2. For each \( \bar{r}_{SB} \), there exists a threshold \( \underline{\alpha}_{SB} \) such that \( r_{SB} \) increases with \( \alpha \) if

\[
\alpha \leq \underline{\alpha}_{SB} \equiv \frac{\xi - \tau + a(s) - z}{2D(s) \left[ 1 - \sigma \ln (1 + \tau_{SB}) \right]}. \tag{19}
\]

Proof. We first consider the effect of \( \alpha \) on \( r_{SB} \). Taking the derivative of \( \Omega \) in (1) with
respect to \( \alpha \), we can obtain

\[
\frac{\partial \Omega}{\partial \alpha} = (3 + 2\lambda) \eta D(s) \sigma \left[ 2\alpha D(s) \sigma \ln (1 + \tau_{SB}) - \alpha D(s) + (\xi - \tau + a(s) - z - \alpha D(s)) \right]
\]

and thus

\[
\frac{\partial r_{SB}}{\partial \alpha} \geq 0 \iff \alpha \leq \bar{\alpha}_{SB} \equiv \frac{\xi - \tau + a(s) - z}{2D(s) \left[ 1 - \sigma \ln (1 + \tau_{SB}) \right]}. \tag{20}
\]

Following the same way, we can obtain

\[
\frac{\partial \bar{r}_{SB}}{\partial \alpha} \geq 0 \iff \alpha \leq \underline{\alpha}_{SB} \equiv \frac{\xi - \tau + a(s) - z}{2D(s) \left[ 1 - \sigma \ln (1 + \tau_{SB}) \right]};
\]

Proposition 6 tells us that, for each degree, there exists a threshold such that the
degree increases with the passenger’s value of time if and only if the passenger’s value of time is smaller than a threshold.

To illustrate (18) in Proposition 6, we should first analyze the effects of the passenger’s value of time on the degree in (20).

From (20), we can find two effects, specifically, a direct and an indirect effect. On the one hand, the increase of the passenger’s value of time will increase the marginal benefit of society from the service and thus directly motivate the regulator to increase the degree. On the other hand, the increase of the passenger’s value of time will decrease the number of passengers and a smaller number of passengers implies a lower marginal benefit of society. Thus, the increase of the passenger’s value of time will indirectly motivate the regulator to decrease the degree.

Then, for (18), the case that \( \alpha \) is smaller (larger) than a threshold implies that the effect of the number of passengers on the degree, that is, a part of the indirect effect, will be relatively insignificant (significant) and thus the direct effect will dominate (be dominated by) the indirect effect, that is, the increase of the passenger’s value of time will motivate the regulator to increase (decrease) the degree.

In fact, we can explain (19) in the exactly same way as for (18).

Moreover, we can also find a sufficient condition for \( \frac{\partial r_{SB}}{\partial \alpha} \geq 0 \) and \( \frac{\partial r_{SB}}{\partial \alpha} \geq 0 \).

**Corollary 3.** \( r_{SB} \) and \( r_{SB}^{\bar{\alpha}} \) increase with \( \alpha \) if

\[
\alpha \leq \frac{\xi - \tau + a(s) - z}{2D(s)}. \tag{21}
\]

**Proof.** The proof is in Appendix B.3.

For (21) in Corollary 3, if \( \alpha \) is smaller than a threshold, the effect of the number of passengers on the degree, that is, a part of the indirect effect, will be so insignificant that the direct effect will always dominate the indirect effect, that is, the increase of the passenger’s value of time will always motivate the regulator to increase the degree.

Then, if we also consider the relevant thresholds \( \alpha^{FB} \) and \( \alpha^{FB} \) under complete information and combining the ones in Proposition 6, we will have Corollary 4.

**Corollary 4.** When \( \alpha \) increases, there is a possibility that \( r_{SB} \) and \( r_{SB}^{\bar{\alpha}} \) move in opposite directions. Moreover, the existence of information rent increases that possibility.

**Proof.** The proof follows the same way as for Corollary 2. \( \square \)
3.6 The effect of cost per seat on degree

In this part, we analyze the effect of the cost per seat \( \tau \) on the optimal degree of delay reduction service. Specifically, we have the following result.

**Proposition 7.** \( \bar{r}^{SB} \) and \( L^{SB} \) decrease with \( \tau \).

**Proof.** Taking the derivative of \( \bar{\Omega} \) in (1) and \( \Omega \) in (2) with respect to \( \tau \), we can obtain

\[
\frac{\partial \bar{\Omega}}{\partial \tau} = \frac{\partial \Omega}{\partial \tau} = -(3 + 2\lambda) \eta \alpha D(s) \sigma < 0.
\]

(22)

Proposition 7 tells us that the degree always decreases with the cost per seat. From (22), we can find a indirect effect. Specifically, the increase of the cost per seat will decrease the number of passengers and a smaller number of passengers implies a lower marginal benefit of society from the service. Thus, the increase of the cost per seat will indirectly motivate the regulator to decrease the degree.

In fact, by using Proposition 7, we can make useful predictions. For instance, the escalations of fuel prices and pilot and cabin crew salaries will raise the cost per seat. Thus, the regulator should optimally respond by decreasing the degree of delay reduction service.

Moreover, the analysis for the effect of the net benefit of outside option \( z \) on the optimal degree is essentially the same as this part and is omitted henceforth.

4 Numerical examples

In this section, we use numerical examples to gain some insights, which are not obvious in theoretical analysis.

4.1 When regulator uses public funds?

In this model, whether the regulator uses public funds plays a significnat role in the design of optimal contracts but we cannot theoretically explain when the regulator will use public funds. Therefore, we will do computer simulations and develop a theory illustrate this problem. Note that all the details of simulations are in Appendix C.1.
We first consider the airline’s net benefit from the service, that is,

\[ u = \Pi^* (\theta, s, r) - \pi^* (\theta, s) - C_{reg} (s, r), \]

which is in fact the net benefit of the airline when it is asked to pay the total cost of providing the service. By a large number of simulations, we can find that, for each parameter value combination, there exists a threshold \( \hat{r} \), which is the highest degree the regulator can provide without using public funds and determined by \( u = 0 \). We can see \( \hat{r} \), for instance, in Figure 5. Note that the curve in Figure 5 seems smooth but in fact it is not. Because it only fluctuates in extremely small intervals, we can regard it as a smooth curve. Moreover, because the fluctuation of the curve, some values smaller than \( \hat{r} \) may also be the intersections. However, this does not affect our following analysis because other possible intersections are always smaller than \( \hat{r} \) at the scale \( 10^{-12} \).

![Figure 5: \( \hat{r} \), the highest degree regulator can provide without using public funds](image)

Denote the marginal benefit of society from the service by \( MB \) and the marginal cost of providing the service by \( MC \). Then, if \( MB (\hat{r}) \leq MC \), to make the marginal benefit and marginal cost equal, the regulator will decrease the degree and the optimal degree \( r \) will the one such that \( r \leq \hat{r} \). In this case, the airline’s benefit from the service is enough to cover the total cost where the degree is \( r \) and thus the regulator will not use public funds. However, if \( MB (\hat{r}) > MC \), the regulator will optimally respond by increasing the degree but at the cost of public funds and, under incomplete
information, an information rent for the airline with a higher value of time. Then, the optimal degree $r$ will be the one such that $r > \hat{r}$. In this case, the airline’s benefit from the service is not enough to cover the total cost where the degree is $r$ and thus the regulator has to use public funds.

Moreover, we can also illustrate the problem by considering two important parameters, the passenger’s value of time $\alpha$ and the effectiveness of the service $\sigma$. Specifically, we have Figure 6.

![Figure 6: How $\alpha$ and $\sigma$ affect the decision about public funds?](image)

In Figure 6, If $\sigma$ is large, that is, the service is very effective, the airline can always obtain enough benefit from the service. Then, no matter what $\alpha$ it is, the airline can cover the total cost of providing the service and thus the regulator will not use public funds. If $\sigma$ is small, that is, the service is very ineffective, the airline can only obtain very limited benefit from the service, which is always unable to cover the total cost of providing the service, no matter what $\alpha$ it is. In this case, the regulator has to use public funds. If $\sigma$ falls into intermediate interval, when $\sigma$ decreases, the airline can still cover the total cost only if $\alpha$ becomes larger because a higher passenger’s value of time can compensate the loss of benefit resulting from the decrease of the effectiveness of the service.
4.2 The effect of pooling contract on airline

In the scenario where the regulator can provide the service without using public funds, we have shown that, under incomplete information, the airline with $\theta$ will always not mimic the airline with $\theta$ while the airline with $\theta$ may want to mimic the airline with $\theta$. In this part, we first give an example about the airline’s choices (See Table 1 and 2). Note that all the second-order conditions can be satisfied and further details are in Appendix C.2.1.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\Gamma^{FB}$</th>
<th>$\mu^{FB}$</th>
<th>$\theta$</th>
<th>net mimicking benefit of airline with $\theta$</th>
<th>outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.38073</td>
<td>0.00233513</td>
<td>1.1</td>
<td>0.001374</td>
<td>not mimic</td>
</tr>
<tr>
<td>1</td>
<td>0.38073</td>
<td>0.00233513</td>
<td>1.2</td>
<td>-0.000112916</td>
<td>not mimic</td>
</tr>
<tr>
<td>1</td>
<td>0.38073</td>
<td>0.00233513</td>
<td>1.3</td>
<td>-0.00205387</td>
<td>not mimic</td>
</tr>
<tr>
<td>1</td>
<td>0.38073</td>
<td>0.00233513</td>
<td>1.4</td>
<td>-0.0049823</td>
<td>not mimic</td>
</tr>
<tr>
<td>1</td>
<td>0.38073</td>
<td>0.00233513</td>
<td>1.5</td>
<td>-0.0070333</td>
<td>not mimic</td>
</tr>
</tbody>
</table>

Table 1: Whether airline with $\theta$ wants to mimic airline with $\theta$?

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\Gamma^{FB}$</th>
<th>$\mu^{FB}$</th>
<th>$\theta$</th>
<th>net mimicking benefit of airline with $\theta$</th>
<th>outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.488367</td>
<td>0.00474065</td>
<td>1</td>
<td>0.00506628</td>
<td>mimic</td>
</tr>
<tr>
<td>1.2</td>
<td>0.595876</td>
<td>0.00780124</td>
<td>1</td>
<td>0.00779742</td>
<td>not mimic</td>
</tr>
<tr>
<td>1.3</td>
<td>0.703273</td>
<td>0.0114694</td>
<td>1</td>
<td>0.0105286</td>
<td>not mimic</td>
</tr>
<tr>
<td>1.4</td>
<td>0.810572</td>
<td>0.0157043</td>
<td>1</td>
<td>0.0132597</td>
<td>not mimic</td>
</tr>
<tr>
<td>1.5</td>
<td>0.917785</td>
<td>0.0204699</td>
<td>1</td>
<td>0.0159909</td>
<td>not mimic</td>
</tr>
</tbody>
</table>

Table 2: Whether airline with $\theta$ wants to mimic airline with $\theta$?

In Table 1, the net benefit of the airline with $\theta$ is always larger than the net mimicking benefit and thus the airline with $\theta$ will always not mimic the airline with $\theta$. However, in Table 2, when $\theta = 1.1$, the net benefit of the airline with $\theta$ is smaller than the net mimicking benefit and thus the airline with $\theta$ will mimic the airline with $\theta$. 

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When $\theta$ equals to 1.2, 1.3, 1.4, and 1.5, the net benefit of the airline with $\theta$ is larger than the net mimicking benefit and thus the airline with $\theta$ will not mimic the airline with $\underline{\theta}$.

Moreover, as we have discussed, we are not sure that the airline will be better or worse because of pooling contract. Thus, we give an example where $\bar{\theta} = 1.1$ and $\underline{\theta} = 1$ (See Table 3 and 4). Note that all the second-order conditions can be satisfied and further details are in Appendix C.2.2.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\bar{\theta}$</th>
<th>$\underline{\theta}$</th>
<th>net benefit of airline with $\theta$ from pooling contract</th>
<th>$u^{FB}$</th>
<th>outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.01</td>
<td>0.3915</td>
<td>0.002265</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.2</td>
<td>1.02</td>
<td>0.402268</td>
<td>0.00218885</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.3</td>
<td>1.03</td>
<td>0.413036</td>
<td>0.00210679</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.4</td>
<td>1.04</td>
<td>0.423801</td>
<td>0.0020189</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.5</td>
<td>1.05</td>
<td>0.434566</td>
<td>0.00192326</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.6</td>
<td>1.06</td>
<td>0.445329</td>
<td>0.00182598</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.7</td>
<td>1.07</td>
<td>0.45609</td>
<td>0.00172113</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.8</td>
<td>1.08</td>
<td>0.466851</td>
<td>0.00161079</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
<tr>
<td>0.9</td>
<td>1.09</td>
<td>0.477609</td>
<td>0.00149505</td>
<td>0.00233513</td>
<td>worse</td>
</tr>
</tbody>
</table>

Table 3: The effect of pooling contract on airline with $\theta$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\bar{\theta}$</th>
<th>$\underline{\theta}$</th>
<th>net benefit of airline with $\theta$ from pooling contract</th>
<th>net mimicking benefit of airline with $\bar{\theta}$</th>
<th>outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.01</td>
<td>0.3915</td>
<td>0.00506193</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.2</td>
<td>1.02</td>
<td>0.402268</td>
<td>0.00505105</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.3</td>
<td>1.03</td>
<td>0.413036</td>
<td>0.00503374</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.4</td>
<td>1.04</td>
<td>0.423801</td>
<td>0.0050101</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.5</td>
<td>1.05</td>
<td>0.434566</td>
<td>0.00498023</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.6</td>
<td>1.06</td>
<td>0.445329</td>
<td>0.00494422</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.7</td>
<td>1.07</td>
<td>0.45609</td>
<td>0.00490217</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.8</td>
<td>1.08</td>
<td>0.466851</td>
<td>0.00485416</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
<tr>
<td>0.9</td>
<td>1.09</td>
<td>0.477609</td>
<td>0.00480029</td>
<td>0.00506628</td>
<td>worse</td>
</tr>
</tbody>
</table>

Table 4: The effect of pooling contract on airline with $\bar{\theta}$
In Table 3, the net benefit of the airline with $\theta$ from pooling contract is always smaller than the net benefit under complete information. Thus, the airline with $\theta$ becomes worse because of pooling contract. In Table 4, the net benefit of the airline with $\overline{\theta}$ from pooling contract is always smaller than the net mimicking benefit, which implies that the airline with $\overline{\theta}$ becomes worse because of pooling contract.

### 4.3 Examples for comparative-static analysis

In the last section, except $\tau$ and $z$, we have to check the thresholds for each value of a variable. Thus, in the part, we use computer simulations to see how the degree changes along the whole interval of a variable (See Figure 7-14). Note that all the second-order conditions can be satisfied and further details are in Appendix C.3.1 to C.3.8.

According to Figure 7-14, the degree monotonically changes with respect to all the variables. Specifically, the degree increases with safety standard, the exceptional event arriving rate, the delay externality parameter, the number of flights, and the passenger’s value of time and decreases with the number of available hours, the cost per seat, and the net benefit of outside option.

Note that, by theoretical analysis, except the cost per seat and the net benefit of outside option, we cannot obtain a monotonic change of the degree with respect to a variable. Moreover, we can expect that, under some parameter values, there may exist a non-monotonic change of the degree along the whole interval. However, I tried many possible parameter value combinations and I failed to find a non-monotonic change of the degree. Therefore, a further scrutiny for parameter values should be conducted in future research.

### 5 Concluding remarks

This paper studies the optimal design of delay reduction contract in EU air traffic management. In particular, this paper builds a model which is specific for EU air transport sector. Specifically, a delay function only including exceptional event delay captures slot controls and no congestion of flights at major European airports and the fixed number of flights captures grandfather right and “use it or lose it” rule.
in EU slot allocation mechanism. Moreover, this paper derives optimal contracts analytically and conducts comparative-static analysis to study the effects of some relevant exogenous variables, for instance, safety standard, the number of flights, and the passenger’s value of time, on optimal contracts, in which information rent is found to increase the possibility that contracting variables move in opposite directions. Besides, this paper also uses numerical examples to gain insights.

Note that this paper focuses on a monopoly case. Thus, an extension to oligopoly case and a study for strategic interactions among airlines will be a direction of future research.

Figure 7: The effect of $s$ on $r_{SB}^S$ and $r_{SB}^B$

Figure 8: The effect of $\beta$ on $r_{SB}^S$ and $r_{SB}^B$

Figure 9: The effect of $\gamma$ on $r_{SB}^S$ and $r_{SB}^B$

Figure 10: The effect of $T$ on $r_{SB}^S$ and $r_{SB}^B
Appendix A: Discussion of solutions

A.1 Complete information in Scenario 3

We discuss solutions according to the value of $\alpha$ and the signs of $LR$’s and $LR$’s intercept on horizontal axis.

For convenience, let

$$I_1 = 4 \left( \bar{f}D(s) \sigma - m(s) \right) + 3\eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma,$$
\[ I_2 = 4(\theta f D (s) \sigma - m (s)) + 3\eta (\xi - \tau + a (s) - z - \alpha D (s)) \alpha D (s) \sigma. \]

**Case 1:** \( 0 < \alpha \leq \frac{2}{D(s)\sigma} \sqrt{\frac{m(s)}{3\eta}} \) and \( I_2 \geq 0 \)

In this case, \( 0 < \alpha \leq \frac{2}{D(s)\sigma} \sqrt{\frac{m(s)}{3\eta}} \) implies \( 3\eta \alpha^2 D (s) \sigma^2 \leq 4m (s) \) and thus that the slope of \( LR \) (LR) is larger than that of \( LOGR \) (LOGR) for any \( r \). Moreover, because of \( \ddot{\bar{r}} > \ddot{\theta} \), \( I_2 \geq 0 \) implies that both the signs of \( \dddot{LR} \)'s and \( \dddot{LR} \)'s intercept on horizontal axis are positive. Then, we can show \( \dddot{F}_B \) and \( \dddot{L}_B \) in Figure 15.

Consider the second-order conditions. According to \( I_2 \geq 0 \) and \( \ddot{\bar{r}} > \ddot{\theta} \), we can obtain

\[
\frac{\partial^2 W}{\partial r^2} \bigg|_{r=r_B} = \frac{1}{4 \left(1 + \bar{r}_B^2\right)^2} \left\{ 3\eta \alpha^2 D (s) \sigma^2 \left[1 - \ln \left(1 + \bar{r}_B\right)\right] - 3\eta (\xi - \tau + a (s) - z - \alpha D (s)) \alpha D (s) \sigma - 4\theta f D (s) \sigma \right\} < 0.
\]

Then, by \( 0 < 3\eta \alpha^2 D (s) \sigma^2 \leq 4m (s) \), we have \( \frac{\partial^2 W}{\partial r^2} \bigg|_{r=r_B} < 0 \).

Following the same way, we can also find \( \frac{\partial^2 W}{\partial r^2} \bigg|_{r=r_B} < 0 \).

**Case 2:** \( \frac{2}{D(s)\sigma} \sqrt{\frac{m(s)}{3\eta}} < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)} \) and \( I_2 \geq 0 \)

In this case, \( \frac{2}{D(s)\sigma} \sqrt{\frac{m(s)}{3\eta}} < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)} \) implies \( 4m (s) < 3\eta \alpha^2 D (s) \sigma^2 \) and thus that the slope of \( LR \) (LR) is first smaller and then larger than that of \( LOGR \) (LOGR). Moreover, both the signs of \( \dddot{LR} \)'s and \( \dddot{LR} \)'s intercept on horizontal axis are positive. Then, we can show \( \dddot{F}_B \) and \( \dddot{L}_B \) in Figure 16.

Following the same way as Case 1, we can obtain

\[
\frac{\partial^2 W}{\partial r^2} \bigg|_{r=r_B} < \frac{1}{4 \left(1 + \bar{r}_B^2\right)^2} \left\{ 3\eta \alpha^2 D (s) \sigma^2 \left[1 - \ln \left(1 + \bar{r}_B\right)\right] - 4m (s) \right\}.
\]

According to Figure 16, we can obtain

\[
3\eta \alpha^2 D (s) \sigma^2 \frac{1}{(1 + r_{tan})} = 4m (s).
\]
Because of $\tau^{FB} > r_{tan}$, we can obtain
\[
\ln (1 + \tau^{FB}) > \ln \left[ \frac{3\eta \alpha^2 D (s)^2 \sigma^2}{4m (s)} \right].
\]

Thus, we have
\[
\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{FB}} < \left. \frac{1}{4(1 + \tau^{FB})^2} \right\} \left\{ \frac{3\eta \alpha^2 D (s)^2 \sigma^2}{4m (s)} \left\{ 1 - \ln \left[ \frac{3\eta \alpha^2 D (s)^2 \sigma^2}{4m (s)} \right] \right\} - 4m (s) \right\}
= \left. \frac{m (s)}{(1 + \tau^{FB})^2} \right\} \left\{ \frac{3\eta \alpha^2 D (s)^2 \sigma^2}{4m (s)} \left\{ 1 - \ln \left[ \frac{3\eta \alpha^2 D (s)^2 \sigma^2}{4m (s)} \right] \right\} - 1 \right\}.
\]

Then, we consider a function $x (1 - \ln x) - 1$, where $x > 1$. The derivative of this function with respect to $x$ is given by
\[
\frac{d \left[ x (1 - \ln x) - 1 \right]}{dx} = -\ln x < 0.
\]

Thus, we have $[x (1 - \ln x) - 1]_{\sup} = 0$.

By this result, we have $\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{FB}} < 0$. We can also show $\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{FB}} < 0$ by exactly the same way.

**Case 3:** $\frac{2}{D(s)\sigma} \sqrt{\frac{m(s)}{3\eta}} < \alpha \leq \frac{\xi - \tau + \alpha (s) - z}{D(s)}$, $I_1 \geq 0$, and $I_2 < 0$

In this case, the slope of $LR$ ($LR$) is first smaller and then larger than that of $LOGR$ ($LOGR$). Moreover, the sign of $LR$’s intercept on horizontal axis is positive while the sign of $LR$’s is negative. Then, we can show $\tau^{FB}$ and $\tau^{FB}$ in Figure 17.

Following the same way as Case 2, we have $\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{FB}} < 0$.

According to Figure 17, we have
\[
4m (s) = \frac{3\eta \alpha^2 D (s)^2 \sigma^2 \ln (1 + r_{tan})}{J + r_{tan}}.
\]

Because $LR$’s intercept on horizontal axis is negative, there exists a solution only when $J > |\tau_{int}|$. Thus, we have
\[
-3\eta (\xi - \tau + \alpha (s) - z - \alpha D (s)) \alpha D (s) \sigma - 4\theta f D (s) \sigma < 3\eta \alpha^2 D (s)^2 \sigma^2 \left\{ \ln \left[ \frac{3\eta \alpha^2 D (s)^2 \sigma^2}{4m (s)} \right] - 1 \right\}.
\]
Then, we can obtain
\[
\frac{\partial^2 W}{\partial r^2} \bigg|_{r = r^{FB}} = \frac{1}{4 (1 + r^{FB})^2} \left\{ 3\eta \alpha^2 D(s)^2 \sigma^2 \left[ 1 - \ln (1 + r^{FB}) \right] - 3\eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma - 4\theta f D(s) \sigma \right\} \\
< \frac{3\eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + r^{FB})^2} \left\{ \ln \left[ \frac{3\eta \alpha^2 D(s)^2 \sigma^2}{4m(s)} \right] - \ln (1 + r^{FB}) \right\} \\
= \frac{3\eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + r^{FB})^2} \ln \left[ \frac{3\eta \alpha^2 D(s)^2 \sigma^2}{4m(s)} \right].
\]

At the optimum, \(3\eta \alpha^2 D(s)^2 \sigma^2 \frac{1}{(1 + r^{FB})} < 4m(s)\) always holds. Thus, we have
\[
\frac{\partial^2 W}{\partial r^2} \bigg|_{r = r^{FB}} < 0.
\]

**Case 4:** \(\frac{2}{D(s)\sigma} \sqrt{\frac{m(s)}{3\eta}} < \frac{\xi - \tau + a(s) - z}{D(s)}\) and \(I_1 < 0\)

In this case, the slope of \(LR\) (\(LR\)) is first smaller and then larger than that of \(LOGR\) (\(LOGR\)). Moreover, because of \(\overline{\theta} > \theta\), \(I_1 < 0\) implies that both the signs of \(LR\)'s and \(LR\)'s intercept on horizontal axis are negative. Then, according to the information above, we can show \(r^{FB}\) and \(r^{FB}\) in Figure 18.

Following the same way as Case 3, we can find \(\frac{\partial^2 W}{\partial r^2} \bigg|_{r = r^{FB}} < 0\) and \(\frac{\partial^2 W}{\partial r^2} \bigg|_{r = r^{FB}} < 0\).

### A.2 Incomplete information in Scenario 3

Following the same way as complete information, we can show \(\tilde{r}\) in Figure 19-23 and confirm all the second-order conditions.

### A.3 Incomplete information in Scenario 4

We discuss solutions according to the value of \(\alpha\) and the signs of \(\overline{LR}\)'s and \(LR\)'s intercept on horizontal axis.

For convenience, let
\[
I_3 = 4 (1 + \lambda) \left( \overline{\theta} f D(s) \sigma - m(s) \right) + (3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma,
I_4 = 4 (1 + \lambda) (\theta f D(s) \sigma - m(s)) + (3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma
\]
\[-4 \frac{\mu}{1 - \mu} \lambda \Delta \theta f D(s) \sigma.
\]
Case 1: $0 < \alpha < \frac{2}{D(s)\sigma} \sqrt{\frac{(1+\lambda)m(s)}{(3+2\lambda)\sigma^2}} + I_4 \geq 0$

In this case, $0 < \alpha < \frac{2}{D(s)\sigma} \sqrt{\frac{(1+\lambda)m(s)}{(3+2\lambda)\sigma^2}}$ implies $(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \leq 4 (1 + \lambda) m(s)$ and thus that the slope of $LR$ ($LR$) is larger than that of $LOGR$ ($LOGR$) for any $r$. Moreover, because of $\theta > \theta$, $I_4 \geq 0$ implies that both the signs of $LR$’s and $LR$’s intercept on horizontal axis are positive. Then, we can show $\tau^{SB}$ and $\tau^{FB}$ in Figure 24. Note that $\tau^{FB}$ and $\tau^{FB}$ are also included in the figures henceforth.

Consider the second-order conditions. According to $I_4 \geq 0$ and $\bar{\theta} > \theta$, we can obtain

$$\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{SB}} = \frac{\mu}{4(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left[1 - \ln \left(1 + \tau^{SB}\right)\right] \right.$$  

$$- (3 + 2\lambda) \eta \left(\xi - \tau + a(s) - z - \alpha D(s)\right) \alpha D(s) \sigma - 4 (1 + \lambda) \bar{\theta} f D(s) \sigma \right\}$$  

$$< \frac{\mu}{4(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left[1 - \ln \left(1 + \tau^{SB}\right)\right] - 4 (1 + \lambda) m(s) \right\}.$$

Then, by $0 < (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \leq 4 (1 + \lambda) m(s)$, we have $\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{SB}} < 0.$

Following the same way, we can also obtain

$$\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{SB}} = \frac{1 - \mu}{4(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left[1 - \ln \left(1 + \tau^{SB}\right)\right] \right.$$  

$$- (3 + 2\lambda) \eta \left(\xi - \tau + a(s) - z - \alpha D(s)\right) \alpha D(s) \sigma - 4 (1 + \lambda) \bar{\theta} f D(s) \sigma$$  

$$+ \frac{\mu}{1 - \mu} \lambda \Delta \theta f D(s) \sigma \right\}$$  

$$< \frac{1 - \mu}{4(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left[1 - \ln \left(1 + \tau^{SB}\right)\right] - 4 (1 + \lambda) m(s) \right\} < 0.$$

Case 2: $\frac{2}{D(s)\sigma} \sqrt{\frac{(1+\lambda)m(s)}{(3+2\lambda)\sigma^2}} < \alpha < \frac{\xi - \tau + a(s) - z}{D(s)}$ and $I_4 \geq 0$

In this case, $\frac{2}{D(s)\sigma} \sqrt{\frac{(1+\lambda)m(s)}{(3+2\lambda)\sigma^2}} < \alpha < \frac{\xi - \tau + a(s) - z}{D(s)}$ implies $4 (1 + \lambda) m(s) < (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2$ and thus that the slope of $LR$ ($LR$) is first smaller and then larger than that of $LOGR$ ($LOGR$). Moreover, both the signs of $LR$’s and $LR$’s intercept on horizontal axis are positive. Then, we can show $\tau^{SB}$ and $\tau^{SB}$ in Figure 25.

Following the same way as Case 1, we can obtain

$$\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{FB}} < \frac{\mu}{4(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left[1 - \ln \left(1 + \tau^{SB}\right)\right] - 4 (1 + \lambda) m(s) \right\}. $$
According to Figure 25, we can obtain

\[(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \frac{1}{(1 + r_{tan})} = 4(1 + \lambda) m(s).\]

Because of \(\tau^{SB} > r_{tan}\), we can obtain

\[\ln(1 + \tau^{SB}) > \ln \left[ \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4(1 + \lambda) m(s)} \right].\]

Thus, we have

\[\partial^2 W \bigg|_{\tau = \tau^{SB}} < \frac{\mu}{4(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left\{ 1 - \ln \left[ \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4(1 + \lambda) m(s)} \right] \right\} \right\] - 4(1 + \lambda) m(s) \right\}

\[= \frac{\mu(1 + \lambda) m(s)}{(1 + \tau^{SB})^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left\{ 1 - \ln \left[ \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4(1 + \lambda) m(s)} \right] \right\} \right\} - 1 \}.

Then, we consider a function \(x(1 - \ln x) - 1\), where \(x > 1\). The derivative of this function with respect to \(x\) is given by

\[\frac{d[x(1 - \ln x) - 1]}{dx} = -\ln x < 0.\]

Thus, we have \([x(1 - \ln x) - 1]_{\sup} = 0\).

By this result, we have \(\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{SB}} < 0\). We can also show \(\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{SB}} < 0\) by exactly the same way.

**Case 3:** \(\frac{2D(s)\sigma}{(3+2\lambda)\eta} < \alpha \leq \frac{\xi - (\tau + a(s) - z)}{D(s)}, I_3 \geq 0, \text{ and } I_4 < 0\)

In this case, the slope of \(LR\) (\(LR\)) is first smaller and then larger than that of \(LOGR\) (\(LOGR\)). Moreover, the sign of \(LR\)'s intercept on horizontal axis is positive while the sign of \(LR\)'s is negative. Then, considering the position of \(LR\) under complete information, we can show \(\tau^{SB}\) and \(\tau^{SB}\) in Figure 26 and 27.

Following the same way as Case 2, we have \(\frac{\partial^2 W}{\partial \tau^2} \bigg|_{\tau = \tau^{SB}} < 0\).

According to Figure 26 and 27, we have

\[4(1 + \lambda) m(s) = \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \ln(1 + r_{tan})}{J + r_{tan}}.\]
Because \( LR \)'s intercept on horizontal axis is negative, there exists a solution only when \( J > |\text{int}| \). Thus, we have

\[
-(3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma - 4 (1 + \lambda) \theta f D(s) \sigma + 4 \frac{\mu}{1 - \mu} \lambda \Delta \theta f D(s) \sigma
\]

\[
< (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left\{ \ln \left[ \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + \lambda) m(s)} \right] - 1 \right\}.
\]

Then, we can obtain

\[
\frac{\partial^2 W}{\partial r^2} \bigg|_{r=rSB} = \frac{1 - \mu}{4 (1 + rSB)^2} \left\{ (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \left[ 1 - \ln (1 + rSB) \right] 
-(3 + 2\lambda) \eta (\xi - \tau + a(s) - z - \alpha D(s)) \alpha D(s) \sigma - 4 (1 + \lambda) \theta f D(s) \sigma
+4 \frac{\mu}{1 - \mu} \lambda \Delta \theta f D(s) \sigma \right\}
\]

\[
\leq \frac{(1 - \mu) (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + rSB)^2} \left\{ \ln \left[ \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + \lambda) m(s)} \right] - \ln (1 + rSB) \right\}
\]

\[
= \frac{(1 - \mu) (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + rSB)^2} \ln \left[ \frac{(3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2}{4 (1 + \lambda) m(s) (1 + rSB)} \right].
\]

At the optimum, \( (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma^2 \frac{1}{(1 + rSB)} < 4 (1 + \lambda) m(s) \) always holds. Thus, we have \( \frac{\partial^2 W}{\partial r^2} \bigg|_{r=rSB} < 0 \).

**Case 4:** \( \frac{2}{D(s)\sigma} \sqrt{\frac{(1 + \lambda) m(s)}{(3 + 2\lambda) \eta}} < \alpha \leq \frac{\xi - \tau + a(s) - z}{D(s)} \) and \( I_3 < 0 \)

In this case, the slope of \( LR \) (\( LR \)) is first smaller and then larger than that of \( LOGR \) (\( LOGR \)). Moreover, because of \( \overline{\theta} > \hat{\theta} \), \( I_3 < 0 \) implies that both the signs of \( LR \)'s and \( LR \)'s intercept on horizontal axis are negative. Then, according to the information above, we can show \( rSB \) and \( rSB \) in Figure 28.

Following the same way as Case 3, we can find \( \frac{\partial^2 W}{\partial r^2} \bigg|_{r=rSB} < 0 \) and \( \frac{\partial^2 W}{\partial \sigma^2} \bigg|_{r=rSB} < 0 \).
Figure 15: $\tau^{FB}$ and $\underline{\tau}^{FB}$

Figure 16: $\tau^{FB}$ and $\underline{\tau}^{FB}$
Figure 17: $\tau^{FB}$ and $\xi^{FB}$

Figure 18: $\tau^{FB}$ and $\xi^{FB}$
Figure 19: $\pi^{FB}$, $\tau^{FB}$, and $\tilde{r}$

Figure 20: $\pi^{FB}$, $\tau^{FB}$, and $\tilde{r}$
Figure 21: $\tau^{FB}$, $\underline{\tau}^{FB}$, and $\bar{\tau}$

Figure 22: $\tau^{FB}$, $\underline{\tau}^{FB}$, and $\bar{\tau}$
Figure 23: $\tau^{FB}$, $\tau^{FB'}$, and $\tilde{\tau}$

Figure 24: $\tau^{FB}$, $\tau^{FB'}$, $\tau^{SB}$, and $\tau^{SB'}$
Figure 25: $\tau^{FB}$, $\tau^{FB}$, $\tau^{SB}$, and $\tau^{SB}$

Figure 26: $\tau^{FB}$, $\tau^{FB}$, $\tau^{SB}$, and $\tau^{SB}$
Figure 27: $r^{FB}$, $r^{FB}$, $r^{SB}$, and $r^{SB}$

Figure 28: $r^{FB}$, $r^{FB}$, $r^{SB}$, and $r^{SB}$
Appendix B: Proofs in the section of comparative-static analysis

B.1 Proof of Lemma

Proof. According to $\Omega (r, s, \beta, \gamma, T, f, \alpha, \tau, z) = 0$, we have $d\Omega = 0$, that is,

$$\frac{\partial \Omega}{\partial r} dr + \frac{\partial \Omega}{\partial s} ds + \frac{\partial \Omega}{\partial \beta} d\beta + \cdots + \frac{\partial \Omega}{\partial z} dz = 0.$$

We know $r = h (s, \beta, \gamma, T, f, \alpha, \tau, z)$ and

$$dr = \frac{\partial r}{\partial s} ds + \frac{\partial r}{\partial \beta} d\beta + \cdots + \frac{\partial r}{\partial z} dz.$$

Plugging $dr$ into the first equation, we can obtain

$$\left( \frac{\partial \Omega}{\partial r} \frac{\partial r}{\partial s} + \frac{\partial \Omega}{\partial s} \right) ds + \left( \frac{\partial \Omega}{\partial r} \frac{\partial r}{\partial \beta} + \frac{\partial \Omega}{\partial \beta} \right) d\beta + \cdots + \left( \frac{\partial \Omega}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial \Omega}{\partial z} \right) dz = 0.$$

Because each variable $x$ can vary independently, we must have

$$\frac{\partial \Omega}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial \Omega}{\partial x} = 0.$$

Therefore, we have

$$\frac{\partial r}{\partial x} = -\frac{\partial \Omega / \partial x}{\partial \Omega / \partial r}.$$

\qed

B.2 Proof of Corollary 1

Proof. For the terms in (10) and (12), we have

$$4 (1 + \lambda) m (s) \left(1 + r^{SB}\right) \geq 4 (1 + \lambda) m (s),$$

$$4 (1 + \lambda) m (s) \left(1 + r^{SB}\right) \geq 4 (1 + \lambda) m (s),$$

$$(3 + 2\lambda) \eta \alpha^2 D (s)^2 \sigma \left[1 - \sigma \ln \left(1 + r^{SB}\right)\right] \leq (3 + 2\lambda) \eta \alpha^2 D (s)^2 \sigma,$$

$$(3 + 2\lambda) \eta \alpha^2 D (s)^2 \sigma \left[1 - \sigma \ln \left(1 + r^{SB}\right)\right] \leq (3 + 2\lambda) \eta \alpha^2 D (s)^2 \sigma.$$
If \( \alpha \leq \frac{2}{D(s)} \sqrt{\frac{(1+\lambda)m(s)}{(3+2\lambda)\eta \sigma}} \), then

\[
4 (1 + \lambda) m(s) \geq (3 + 2\lambda) \eta \alpha^2 D(s)^2 \sigma
\]

and thus \( \frac{\partial r}{\partial D(s)} \geq 0 \) and \( \frac{\partial r}{\partial D(s)} \geq 0 \).

B.3 Proof of Corollary 3

Proof. From (20), we have

\[
2\alpha D(s) \sigma \ln (1 + r^{SB}) - \alpha D(s) + (\xi - \tau + a(s) - z - \alpha D(s)) \geq \xi - \tau + a(s) - z - 2\alpha D(s).
\]

If \( \alpha \leq \frac{\xi - \tau + a(s) - z}{2D(s)} \), then

\[
\xi - \tau + a(s) - z - 2\alpha D(s) \geq 0
\]

and thus \( \frac{\partial r^{SB}}{\partial \alpha} \geq 0 \) and \( \frac{\partial r^{SB}}{\partial \alpha} \geq 0 \).

Appendix C: Numerical examples

C.1 When regulator uses public funds?

We use the following parameter values: \( \beta = 0.01, \gamma = 120, \theta = 1, N = 2, \xi = 3, \zeta = 1, s = 2, \tau = 0.8, T = 1.5, f = 1, z = 2 \). Moreover, we also specify \( a(s), g(s), \) and \( m(s) \) as

\[
a(s) \equiv \frac{\ln s}{2\ln 2},
\]

\[
g(s) \equiv \frac{2^s}{100},
\]

\[
m(s) \equiv \frac{3}{50} + \frac{s}{100},
\]

where \( a(s) \) is a concave function of \( s \), \( g(s) \) is a convex function of \( s \), and \( m(s) \) is a linear function of \( s \).

For the example in Figure 5, we use \( \alpha = 0.35 \) and \( \sigma = \frac{1}{\ln 2} \). For the examples in
Figure 6, we use the values of $\alpha$ ranging from 0 to 2.5 and the following values of $\sigma_1$, $\frac{9}{10\ln 2}$, $\frac{1}{2\ln 2}$, $\frac{11}{20\ln 2}$, $\frac{3}{5\ln 2}$, $\frac{13}{10\ln 2}$, $\frac{7}{4\ln 2}$, $\frac{3}{5\ln 2}$, $\frac{17}{20\ln 2}$, $\frac{9}{10\ln 2}$, $\frac{19}{20\ln 2}$, and $\frac{1}{\ln 2}$. Table 5 and 6 contain the simulations in Figure 6. In these two tables, the number is the difference between $MB(\hat{r})$ and $MC$. If it is positive, the regulator has to use public funds. Otherwise, the regulator will not use public funds.

C.2 The effect of pooling contract on airline

C.2.1 Mimicking

We use the following parameter values: $\alpha = 0.6$, $\beta = 0.01$, $\gamma = 1.2$, $\sigma = \frac{9}{10\ln 2}$, $\lambda = 0.1$, $N = 2$, $\xi = 3$, $\zeta = 1$, $s = 2$, $\tau = 0.8$, $T = 1.5$, $f = 1$, $z = 2$. Moreover, we fix $\theta_1 = 1$ and consider the values of $\theta$ ranging from 1.1 to 1.5.

C.2.2 Airline’s welfare change because of pooling contract

We use the following parameter values: $\alpha = 0.6$, $\beta = 0.01$, $\gamma = 1.2$, $\sigma = \frac{9}{10\ln 2}$, $\lambda = 0.1$, $N = 2$, $\xi = 3$, $\zeta = 1$, $s = 2$, $\tau = 0.8$, $T = 1.5$, $f = 1$, $z = 2$. Moreover, we fix $\theta_1 = 1$ and $\theta = 1.1$ and consider the values of $\mu$ ranging from 0.1 to 0.9.

C.3 Comparative-static analysis

C.3.1 The effect of safety standard on degree

We use the following parameter values: $\alpha = 0.5$, $\beta = 0.01$, $\gamma = 120$, $\theta_1 = 1$, $\bar{\theta} = 1.02$, $\sigma = \frac{7}{10\ln 2}$, $\lambda = 0.1$, $\mu = 0.4$, $N = 2$, $\xi = 3$, $\zeta = 1$, $\tau = 0.8$, $T = 1.5$, $f = 1$, $z = 2$. Moreover, we consider the values of $s$ ranging from 2 to 2.15. All of the simulations are in Table 7. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.

C.3.2 The effect of exceptional event arriving rate on degree

We use the following parameter values: $\alpha = 0.5$, $\gamma = 120$, $\theta_1 = 1$, $\bar{\theta} = 1.02$, $\sigma = \frac{7}{10\ln 2}$, $\lambda = 0.1$, $\mu = 0.4$, $N = 2$, $\xi = 3$, $\zeta = 1$, $s = 2$, $\tau = 0.8$, $T = 1.5$, $f = 1$, $z = 2$. Moreover, we consider the values of $\beta$ ranging from 0.0098 to 0.01095. All of the simulations are in Table 8. Besides, note that for the parameter values used in this
part the regulator has to use public funds and all the second-order conditions can be satisfied.

C.3.3 The effect of delay externality parameter on degree

We use the following parameter values: $\alpha = 0.5, \beta = 0.01, \theta = 1, \overline{\theta} = 1.02, \sigma = \frac{7}{10\ln 2}, \lambda = 0.1, \mu = 0.4, N = 2, \xi = 3, \zeta = 1, s = 2, \tau = 0.8, T = 1.5, f = 1, z = 2$. Moreover, we consider the values of $\gamma$ ranging from 117.5 to 132. All of the simulations are in Table 9. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.

C.3.4 The effect of number of available hours on degree

We use the following parameter values: $\alpha = 0.5, \beta = 0.01, \gamma = 120, \theta = 1, \overline{\theta} = 1.02, \sigma = \frac{7}{10\ln 2}, \lambda = 0.1, \mu = 0.4, N = 2, \xi = 3, \zeta = 1, s = 2, \tau = 0.8, f = 1, z = 2$. Moreover, we consider the values of $T$ ranging from 1.37 to 1.52. All of the simulations are in Table 10. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.

C.3.5 The effect of number of flights on degree

We use the following parameter values: $\alpha = 0.5, \beta = 0.01, \gamma = 120, \theta = 1, \overline{\theta} = 1.02, \sigma = \frac{7}{10\ln 2}, \lambda = 0.1, \mu = 0.4, N = 2, \xi = 3, \zeta = 1, s = 2, \tau = 0.8, T = 1.5, f = 1, z = 2$. Moreover, we consider the values of $f$ ranging from 0.99 to 1.05. All of the simulations are in Table 11. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.

C.3.6 The effect of passenger’s value of time on degree

We use the following parameter values: $\beta = 0.01, \gamma = 120, \theta = 1, \overline{\theta} = 1.02, \sigma = \frac{7}{10\ln 2}, \lambda = 0.1, \mu = 0.4, N = 2, \xi = 3, \zeta = 1, s = 2, \tau = 0.8, T = 1.5, f = 1, z = 2$. Moreover, we consider the values of $\alpha$ ranging from 0.5 to 1.15. All of the simulations are in Table 12. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.
C.3.7 The effect of cost per seat on degree

We use the following parameter values: $\alpha = 0.5$, $\beta = 0.01$, $\gamma = 120$, $\theta = 1$, $\bar{\theta} = 1.02$, $\sigma = \frac{7}{10 \ln 2}$, $\lambda = 0.1$, $\mu = 0.4$, $N = 2$, $\xi = 3$, $\zeta = 1$, $s = 2$, $T = 1.5$, $f = 1$, $z = 2$. Moreover, we consider the values of $\tau$ ranging from 0.05 to 0.85. All of the simulations are in Table 13. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.

C.3.8 The effect of net benefit of outside option on degree

We use the following parameter values: $\alpha = 0.5$, $\beta = 0.01$, $\gamma = 120$, $\theta = 1$, $\bar{\theta} = 1.02$, $\sigma = \frac{7}{10 \ln 2}$, $\lambda = 0.1$, $\mu = 0.4$, $N = 2$, $\xi = 3$, $\zeta = 1$, $s = 2$, $\tau = 0.8$, $T = 1.5$, $f = 1$. Moreover, we consider the values of $z$ ranging from 1.225 to 2.05. All of the simulations are in Table 14. Besides, note that for the parameter values used in this part the regulator has to use public funds and all the second-order conditions can be satisfied.
Table 5: When to use public funds? Part 1

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Table 8: The effect of \( \beta \) on \( \bar{r}^{SB} \) and \( \underline{r}^{SB} \)

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Table 8: The effect of \( \beta \) on \( \bar{r}^{SB} \) and \( \underline{r}^{SB} \)
Table 9: The effect of \( \gamma \) on \( \bar{r}^{SB} \) and \( \underline{r}^{SB} \)

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Table 10: The effect of \( T \) on \( \bar{r}^{SB} \) and \( \underline{r}^{SB} \)
Table 11: The effect of $f$ on $\bar{r}^{SB}$ and $\underline{r}^{SB}$

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Table 12: The effect of $\alpha$ on $\bar{r}^{SB}$ and $\underline{r}^{SB}$

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Table 14: The effect of $z$ on $r^{SB}$ and $\tilde{r}^{SB}$

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References

Central Office for Delay Analysis (CODA) of EUROCONTROL, 2013. CODA Digest – Delays to Air Transport in Europe Annual 2012.

