Second-best urban tolls in a monocentric city with rigid housing market regulations

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Abstract This paper investigates second-best congestion pricing in a monocentric city characterized by rigid regulatory mechanisms in the housing market: building height restrictions, zoning and property taxation. We show that, in general, welfare gains from a Pigouvian toll are non-monotonic in the extent of a quantity distortion in the housing market. This finding introduces a warning to cost-benefit analyses: the actual gains of a road tax might be 40% lower than the gains predicted by a model which disregards maximum building height restrictions and 80% higher than the gains suggested by a model which disregards zoning. Furthermore, Pigouvian toll retains its optimality in the above cases. In general, this suggests that decision making on urban road pricing can ignore quantitative restrictions in the parallel markets of land, housing and labor. However, this is not the case in the presence of a tax distortion. Introducing an ad-valorem property tax, we show that adjustments of the Pigouvian toll can lead to small, but not negligible welfare gains.

Keywords: road pricing, building height restrictions, zoning, property tax, monocentric city, cost-benefit analysis

JEL classification: R48, R52, R13, H21, H23, D61

1. Introduction

A series of recent contributions in the literature of urban economics explores the potential of land-use practices to substitute congestion pricing. These practices involve direct control of the urban sprawl or the population density over space. For instance, Brueckner (2007) and Anas and Rhee (2007) investigate the efficiency of an urban growth boundary relative to a congestion toll. More recently, Kono et al. (2012) numerically evaluated the efficiency of regulations on building size and city size relative to the gains that can be achieved by a first-best toll. Furthermore, Pines and Kono (2012) discuss second-best allocations based on space-varying property taxation and floor-to-area (FAR) regulations.1 Implicitly, this stream of research is motivated by the large implementation costs and the limited political acceptability associated with road pricing.

1 Direct density regulations such as building height restrictions (Arnott and MacKinnon, 1977; Bertaud and Brueckner, 2005), city limits determined in the sphere of politics (Brueckner, 2000) constitute land-use regulations which invoke distortions that may be associated with large welfare costs.
However, congestion pricing is already implemented in many urban areas and is likely to gain more popularity in the decades to come. This paper explores urban congestion pricing in the presence of quantity and tax-induced distortions in the housing market. Empirical observation suggests that, in a substantial part of the world, these regulations are generally stiff. The largest European cities are characterized by historical centers, where (apart from residences and workplaces) buildings function as tourist attractions. This fact is reflected to the substantial amount of money specific groups are willing to pay for their preservation. Furthermore, other cultural and social stimuli impose generic building height restrictions and zoning practices which are very likely to remain intact in the years to come. The rigidity of the above land-use practices is illustrated in the case of Stockholm where, eight years after the first congestion charge trial, floor-to-area ratios (hereafter $FAR$) inside the cordon toll have not adjusted significantly. Apart from the upper limit in the building height, which can be considered as an infinite tax for any height exceeding the limit, finite taxes on housing property and land may apply at any location, independent of building height. Compared to a road toll, these taxes might prove much more difficult to adjust, especially upwards.\(^2\)

This paper provides answers to a series of questions emerging from the above premises. That is, under rigidity in land-use regulations we examine: i) if there is any pricing scheme superior to the Pigouvian toll and ii) if the gains from the Pigouvian toll vary substantially (and to which direction) across cities with identical preferences and road technologies but different building height limits or areas subject to zoning. Answers to these questions are valuable to the transport planner: first, she needs to know to what extent road demand management can be detached from urban planning and public finance decisions (taken at different levels of government).\(^3\) Second, knowing the behavior of gains/losses (from the Pigouvian toll) across different quantity restrictions can accommodate extrapolations in cost-benefit analyses: if the gains are hardly variant and monotonic in the building height limit or the extent of the zoned areas, the expected gains from congestion charges can be extrapolated to cities with similar road technologies, independent of the differences in land-use regulations.

We build upon the earlier contribution by Bertaud and Brueckner (2005), who estimate the welfare cost from building height restrictions to be approximately $2\%$ of the household income. In the assumed framework, this is precisely the difference in commuting costs that the household in the city fringe would face if the building height restrictions could be removed. The above monocentric city framework is expanded to incorporate congestion as in Verhoef (2005) and, more recently, Tikoudis et al. (2013). A numerical version of the model is calibrated such that the benchmark equilibrium reproduces a set of stylized facts that characterize a representative monocentric city in the Western world in the absence of congestion charges.

Using the parameters of the benchmark equilibrium, we compute the welfare effect of the Pigouvian toll as a function of the maximum $FAR$ allowed. We show that this effect is not only very volatile with respect to the underlying $FAR$ regulation, but also non-monotonic: the Pigouvian toll imposed in a city without height restrictions may produce up to $40\%$ larger welfare gains relative to the

\(^2\) Other types of restrictions in the housing market involve mortgage subsidies, acquisition and property taxes, direct rent control (Arnott, 1995) and rent income taxation. Via different channels, all these regulatory mechanisms cause welfare losses through a suboptimal allocation of space across economic agents and activities (Glaeser and Luttmer, 2003).

\(^3\) For instance, if the optimal space-varying tax turns out to be particularly exotic it might not be applicable without the introduction of another tax instrument which might need the approval of another planner. In addition, the relative efficiency of the Pigouvian toll to it might be small, even negative.
respective Pigouvian toll in a city with a mild, uniform in space, FAR restriction. Similar computations for the case of zoning suggest that the welfare gains might be 80% larger in an equilibrium with a large zoned area close to the CBD.

At the same time, employing theoretical arguments and numerical tests (based on standard direction set methods), we demonstrate that Pigouvian toll retains its optimality independent of the extent and the type of quantity restriction. Finally, we show that this optimality ceases when other relative preexisting taxes (e.g. housing property tax) are added to the model. Additional sensitivity analysis is used to monitor the efficiency of the Pigouvian toll relative to the (computed) optimal space-varying tax.

The structure of the paper is as follows. Section 2 introduces the analytical model, which is a loose synthesis of the previous models proposed in the literature: the monocentric city models by Bertaud and Brueckner (2005), with a competitive construction sector as in Muth (1969) and congestion externalities and endogenous labor supply as in Verhoef (2005) and Tikoudis et al. (2013). Section 3 presents the calibration and the stylized facts that characterize the benchmark equilibrium (absence of housing market regulations). Section 4.1 discusses the welfare effects of the Pigouvian toll at various levels of a uniform FAR restriction. Section 4.2 illustrates the optimality of the Pigouvian toll in a city with a uniform FAR restriction. Section 4.3 shows the respective results for the case of zoning. Section 4.4 quotes numerical results, i.e. the deviation of the optimal space varying-tax from the corresponding Pigouvian toll and the corresponding relative efficiency, for the case of a distortionary housing property tax. Section 5 summarizes and concludes.

2. Model

2.1. Households

Households are located anywhere within a linear monocentric city, i.e. a city with a single central business district where all jobs are located (hereafter, CBD). Let $x$ and $\bar{x}$ denote the distances of an arbitrary household and the city fringe, respectively, from CBD. Utility is derived from the consumption of a numeraire good, $C$, floor space, $s$, and leisure time, $\ell$. Assuming CES preferences, the household that located at distance $x$ from CBD maximizes:

$$U_x = \left[ (\alpha C)^\rho + (\beta s_x)^\rho + (\gamma \ell_x)^\rho \right]^{\frac{1}{\rho}}. \quad (1)$$

The total time endowment, $T$ (e.g. a year), is spent on commuting, $T_C$, working, $T_L$, and leisure, $\ell$:

$$T = T_C + T_L + \ell.$$  \quad (2)

Labor supply is inelastic throughout a working day, thus the working day is of fixed duration, $t_L$. Every trip to work requires $t_x$ units of time. For $D_{WX}$ working days the time constraint becomes:

$$T = D_{WX} t_x + D_{WX} t_L + \ell_x.$$  \quad (3)

Normalizing the duration of the working day, $t_L$, to 1, the time constraint becomes:

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4 The efficiency of such anti-sprawl tax instruments has been discussed in Bento et al. (2006, 2011)
\[ T = D_{wx} (1 + t_x) + \ell_x \Rightarrow D_{wx} = (T - \ell_x)/(1 + t_x). \]

The net wage per working day is defined as the difference between wage, \( w \), and the total pecuniary commuting cost, which consists of the total road toll charged between \( x \) and the CBD, \( \tau_{rx} \), and the operational cost (gasoline, vehicle depreciation etc.), \( mx \), which is a linear function of commuting distance. The full income of the household at distance \( x \), \( y_x \), is the maximum income which can be realized when leisure time is set to zero:

\[ y_x = B + \left( \frac{w - \tau_{rx} - mx}{1 + t_x} \right) T. \]

where \( B \) denotes a lump-sum transfer (analyzed below) from the government to the household, independent of its location. The full income can be used to buy back leisure at its shadow price, \( (w - \tau_{rx} - mx)/(1 + t_x) \), the composite good (with price normalized to one) and residential space (priced at \( p_x \) and subject to an ad-valorem tax rate, \( \tau_v \)):

\[ y_x = \left( \frac{w - \tau_{rx} - mx}{1 + t_x} \right) \ell_x + C + p_x(1 + \tau_v)s_x. \]

Maximizing (1) subject to (4), (5) and (6), and defining \( \xi = \rho/(\rho - 1) \) and \( \sigma = 1/(1 - \rho) \) yields the conditional (on \( x \)) Marshallian demand functions for the composite good, space and leisure time respectively:

\[ C_x^* = y_x \frac{(1/\alpha^p)^{-\sigma}}{(1/\alpha)^\xi} + \left[ \frac{p_x(1 + \tau_v)}{\beta} \right]^\xi + \left[ \frac{w - \tau_{rx} - mx}{\gamma(1 + t_x)} \right]^\xi, \]

\[ s_x^* = y_x \frac{(p_x(1 + \tau_v)/\beta^p)^{-\sigma}}{(1/\alpha)^\xi} + \left[ \frac{p_x(1 + \tau_v)}{\beta} \right]^\xi + \left[ \frac{w - \tau_{rx} - mx}{\gamma(1 + t_x)} \right]^\xi, \]

\[ \ell_x^* = y_x \frac{\frac{w - \tau_{rx} - mx}{\gamma^p(1 + t_x)}^{-\sigma}}{(1/\alpha)^\xi} + \left[ \frac{p_x(1 + \tau_v)}{\beta} \right]^\xi + \left[ \frac{w - \tau_{rx} - mx}{\gamma(1 + t_x)} \right]^\xi. \]

Finally, substituting (7)-(9) into the objective function yields the conditional indirect utility:

\[ V_x^* = \left[ B + \frac{(w - \tau_{rx} - mx)}{1 + t_x} T \right] \left\{ \left( \frac{1}{\alpha} \right)^\xi + \left[ \frac{p_x(1 + \tau_v)}{\beta} \right]^\xi + \left[ \frac{w - \tau_{rx} - mx}{\gamma(1 + t_x)} \right]^\xi \right\}^{-1/\xi}. \]

The spatial equilibrium is characterized by locational indifference, i.e. constant utility, \( u \), over space. Equation (10) can then be reformulated in order to express the floor space as a function of the equilibrium
utility:

\[ p_x(u) = \frac{\beta}{(1 + \tau_y)} \left\{ u^{-\xi} \left[ B + \frac{(w - \tau_Rx - mx)}{1 + t_x} T \right]^{\xi} - \left( \frac{1}{\alpha} \right)^\xi - \frac{[w - \tau_Rx - mx]}{\gamma(1 + t_x)} \right\}^{\frac{1}{\xi}} \]. \tag{11}

Now, equation (11) can be plugged into (7), (8) and (9) to yield the compensated demands for the composite good, space and leisure at any arbitrary distance \( x \), which we denote by \( C_x(u) \), \( s_x(u) \), and \( \ell_x(u) \) respectively.

2.2. Developers

Developers use constant returns to scale technology to convert land and capital into residential space. The capital intensive form of the Cobb-Douglas production function is:

\[ \hat{s}_x = g\hat{k}_x^\delta, \tag{12} \]

with corresponding profit:

\[ \pi = p_x g\hat{k}_x^\delta - p_K\hat{k}_x - p_{Lx}, \tag{13} \]

where \( g \) is a technology constant, \( \hat{k} \) the units of capital over one unit of land, \( p_K \) the exogenous price of capital and \( p_{Lx} \) the per-unit price of land. Note that \( \hat{s}_x \) can be interpreted as the floor (space)-to-area ratio (hereafter, FAR) at location \( x \). Dividing the supplied space in (12) with the compensated space demand at distance \( x \), \( s_x \), yields the household density at point \( x \):

\[ n_x = \frac{g\hat{k}_x^\delta}{s_x}. \tag{14} \]

From the first-order condition for profit maximization, we derive the optimal capital intensity:

\[ \hat{k}^* = \left( \frac{p_K}{\delta p_x g} \right)^{1/(\delta-1)}. \tag{15} \]

Inserting (15) into (13) and solving for zero profits, yields the land rent:

\[ p_{Lx} = (p_K)^{\delta/(\delta-1)}(p_x g)^{1/(1-\delta)}(\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)}). \tag{16} \]

2.3. Firms

A representative firm, which is located at the CBD, operates under constant returns to scale, producing the composite good with capital and labor:

\[ Y = K^\zeta L^{1-\zeta}. \tag{17} \]

From cost minimization we derive the conditional factor demands for labor:
\[ L = Y \left( \frac{(1 - \zeta)p_K}{\zeta w} \right)^{\zeta}, \quad (18) \]

and capital:

\[ K = Y \left( \frac{\zeta w}{(1 - \zeta)p_K} \right)^{1-\zeta}. \quad (19) \]

Using (18) and (19), it can be shown that the marginal cost is:

\[ p_k^\zeta w^{1-\zeta} (\zeta^{\zeta/(1-\zeta)} - \zeta^{1/(1-\zeta)}). \quad (20) \]

### 2.4. Commuting

Households commute from any given location \( x \) to the CBD through a single radial road. Letting \( n_x \) denote the residential density at location \( x \), we can write the travel time per unit of distance as:

\[ t_0 + t_1 \int_{z}^{\hat{x}} n_x \, dx, \quad (21) \]

i.e. the sum of the free-flow travel time per unit of distance, \( t_0 \), and a term that represents the congestion delay caused by the traffic volume at \( z \). This flow contains all households that are located between \( z \) and the city limit, \( \bar{x} \). Multiplying this with the sensitivity parameter, \( t_1 \), yields the time delay per unit of distance at point \( z \). Integrating (21) over the interval \((0, z)\) yields the commuting time for the household at \( z \):

\[ t_z = z t_0 + t_1 \int_{0}^{\hat{x}} min\{z, x\} \ n_x \, dx. \quad (22) \]

An additional trip made by the household at distance \( z \) increases the travel time of a commuter located at any \( x \geq z \) by \( t_1 z \), and the travel time of each commuter located at \( x \leq z \) by \( t_1 x \). This delay can be multiplied by the shadow value of time at \( x \):

\[ \frac{(w - \tau_{Rx} - mx)}{1 + t_x}, \quad (23) \]

to provide a measure for the marginal external cost of an additional trip generated by the commuter at \( z \), imposed to the commuter at \( x \). Integrating over the interval \((0, \bar{x})\) yields the marginal external cost of congestion, generated by the household at \( z \):

\[ g(z) = t_1 \int_{0}^{\bar{x}} \min\{z, x\} \ n_x \ \frac{(w - \tau_{Rx} - mx)}{1 + t_x} \, dx. \quad (24) \]
2.5. Government and public budget

The government can tax (or subsidize) road use, and recycle the tax revenue and land rents in a manner that ensures a balanced budget. The total land revenue collected is the sum:

$$RL = Ra + R = \bar{x}r_A + \int_0^{\bar{x}} (plx - r_A) \, dx,$$

(25)

of which the first term, $Ra$, is transferred to an absentee land owner and the second term, $R$, i.e. the excess land rent, is returned to the consumers in a lump-sum manner.\(^5\) This assumption ensures that the city cannot expand without cost and that the excess rents remain within the urban economy.

When imposed, a road toll generates the revenue:

$$G = \int_0^{\bar{x}} n_x Dwx r_Rx \, dx,$$

(26)

which is endogenous, since it depends on both density, $n_x$, labor supply, $Dwx$, and city size, $\bar{x}$. Furthermore, the revenue from the ad-valorem tax on property is:

$$Q = \int_0^{\bar{x}} px(1 + \tau_v)gk^\delta \, dx,$$

(27)

The toll and tax revenue is also returned in the form of a lump-sum transfer. Therefore:

$$B = \frac{G + R + Q}{N}.$$

(28)

2.6. Equilibrium without constraints

In equilibrium, the (closed) city must accommodate the exogenous population, $N$, thus:

$$\int_0^{\bar{x}} n_x \, dx = N.$$

(29)

Also, at the city fringe the -endogenously determined- land rent must be equal to the opportunity cost of land, i.e. the agricultural rent:

$$r_A = (p_R)^{\delta/(\delta - 1)}(pxg)^{1/(1-\delta)}(\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)}).$$

(30)

The rest of the equations comprise the clearing of the labor market:

\(^5\) The model is a general equilibrium analogue of the closed-city under public ownership model (CCP) proposed in Fujita (1989).
\[
\int_0^\bar{x} \left( \frac{T - \ell_x}{1 + t_x} \right) n_x \, dx = Y \left( \frac{(1 - \zeta)p_K}{\zeta w} \right) \zeta, \tag{31}
\]

the zero profit condition for the representative firm:
\[
p_K^\zeta w^{1-\zeta} (\zeta \zeta/(1-\zeta) - \zeta^{1/(1-\zeta)}) = 1, \tag{32}
\]

and the closing identity:
\[
Y - \int_0^\bar{x} C_x n_x \, dx = \bar{x}r_A + p_K \left[ Y \left( \frac{\zeta w}{(1 - \zeta)p_K} \right)^{1-\zeta} + \int_0^\bar{x} \left( \frac{p_K}{\delta p_x g} \right)^{1/(\delta-1)} \, dx \right], \tag{33}
\]

which requires the value of the city export (left hand side) to be equal to the opportunity cost of land, \(\bar{x}r_A\), and the value of imported capital; the latter is the product of its exogenous price, \(p_K\), and the sum of: i) the demanded quantity by the representative firm, given in equation (19), and ii) the demanded quantity by the construction sector developers, i.e. the integral of (15) across space.

Equations (29)-(32) define a non-linear system in four unknowns: \(Y, \bar{x}, w, u\). The rest of the endogenous variables (\(y, C, s, \ell, p, p_L, \hat{s}, n, G, K, L, R, B, t, g, \hat{K}\)) can be completely determined for any value of the above four unknowns.

2.7. Equilibrium with a uniform building height restriction

In this paper, we consider the case of a uniform floor-to-area ratio restriction. This introduces a new endogenous variable, \(\bar{x}\), which is the distance from the CBD at which the maximum height constraint ceases binding (hereafter the binding limit). Equation (12) becomes:
\[
\hat{s}_x = \begin{cases} 
\bar{h} & \text{if } x < \bar{x} \\
gk_x \delta & \text{if } x > \bar{x} 
\end{cases} \tag{34}
\]

where \(\bar{h}\) is the maximum number of floors (floor-to-area ratio) permitted at any point in the city. The equilibrium population density in (14) becomes:
\[
n_x = \begin{cases} 
\frac{\bar{h}}{s_x} & \text{if } x < \bar{x} \\
gk_x \delta s_x & \text{if } x > \bar{x} 
\end{cases} \tag{35}
\]

And the equilibrium condition in (29) is now written as:
\[
\left[ \int_0^\bar{x} \frac{\bar{h}}{s_x} \, dx + \int_{\bar{x}}^\bar{x} \frac{gk_x \delta}{s_x} \, dx \right] = N. \tag{36}
\]

To close the model, a new equation is required. This states that the building height at point \(\bar{x}\) is equal to the restricted height limit, \(\bar{h}\). Inserting (15) into (12), this condition can be written as:
\[ \hat{s}_x = g \left( \frac{p_k}{s g p_x g} \right)^{\delta/(\delta - 1)} = \bar{h}. \]  

(37)

2.8. Equilibrium under zoning

Under zoning, and in absence of other restrictions, the structural density is free to adjust anywhere except for an interval \((x_L, x_U)\) where it is restricted to be zero. That is:

\[
\hat{s}_x = \begin{cases} 
   g \bar{k}_x^\delta & \text{if } x < x_L \text{ or } x > x_U \\
   0 & \text{if } x_L \leq x \leq x_U
\end{cases}
\]  

(38)

Furthermore, travel times in (22) and marginal external costs in (24) are adjusted for the fact that the flow at any point in the interval \((x_L, x_U)\) remains constant. As in the unintervened case, the rent at the city fringe is given by (30); no specific restrictions apply for points \(x_L\) and \(x_U\). Despite land between these two points is forced to be vacant, it is still acquired from the absentee landlord at the land opportunity cost.

3. Calibration

3.1. Unregulated equilibrium

The unregulated equilibrium\(^6\) (hereafter, the free equilibrium) characterizes a quite sprawled (40 kilometers), congested metropolitan area without any type of government intervention. The distance between the CBD and the city fringe can be covered in 47 minutes. The average speed ranges between 15 and 50 kilometers per hour, depending on the location from which the commuter departs. The average speed of the median household (approximately 37 km per hour) is roughly consistent with the average commuting speed reported for large US cities in the national household travel survey. The speed in CBD is one-fifth of the speed in the city fringe.

The floor to area ratio ranges between 25 (CBD) and 0.2 floors (fringe). Housing prices in the CBD are roughly 3.5 times larger than those in the fringe. This price variation is accompanied by a respective variation in the household size (2.5 times larger in the fringe) and a steeper variation in the land prices and the structural density (capital-land ratio).

Households work between 288 (CBD)\(^7\) and 233 (fringe) days per year. The (endogenous) income of the representative household is 43000 €, almost all of which (92%) is earned by labor. This income is spent on consumption (57%), housing (39%) and transport (4%). The maximum marginal external congestion cost, i.e. this generated by the most remote household is 4000€, slightly above 9% of the mean income. The solid lines in Figure 1 summarize the key variables of interest.

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\(^6\) The base equilibrium parameter vector values are: \(\alpha = 7.0, \beta = 0.11, \gamma = 0.9, \rho = 0.2, \tau_0 = 0.02, \tau_1 = 0.12,\n\)
\( g = 0.01, N = 1, m = 417.5, r_A = 10.0, p_K = 1.0, \delta = 0.8, \zeta = 0.2.\)

\(^7\) 0.79 of a total time endowment normalized to one.
3.2. Pigouvian toll

The dashed lines in Figure 1 indicate how the key variables of interest are affected (in the long-run) when a road charge equal to the marginal external cost of congestion, i.e. the scheme in (24), is imposed at each location in the metropolitan area.

Clearly, floor-to-area ratio adjusts upwards in locations fairly close to the CBD and downwards in locations further away. This adjustment, accompanied by a similar adjustment in prices, reflects the changes in relative accessibility of locations that occurred after the introduction of the policy. The small elasticity of labor supply with respect to pecuniary commuting costs underlies negligible adjustments in labor supply. In contrast, with adjustments through the behavioral margin of relocation, significant changes take place in commuting time and speed. The reduction in the total external costs in the model is associated with a welfare gain; the compensating variation from the Pigouvian toll is approximately 66 € per household, annually. Next section investigates the performance of the Pigouvian toll and other policies in a city where the floor-to-area ratio is regulated.

![Figure 1](image)

Figure 1. Floor-to-area ratio (upper left panel), commuting time (upper right), residential space prices (lower left), and marginal external congestion cost (lower right) in the two equilibria without building height regulations: unregulated (solid) and Pigouvian (dashed).

4. Policy analysis

4.1. Pigouvian toll across cities with different maximum FAR

A natural question arising in the context of this paper is to what extent welfare gains of the Pigouvian toll vary across different levels of a preexisting, stiff maximum building height regulation. In general, a more stringent FAR restriction is associated with a more sprawled city. The left panel of Figure 2 illustrates the population density in a series of cities emerging from the same parameters used in the benchmark equilibrium of section 3.1, but under various maximum floor-to-area ratios imposed by the planning
When labor supply responds mildly to commuting distance, someone can expect the total number of vehicle kilometers to raise with urban sprawl. The same holds for the marginal external cost of congestion and commuting time, which are summarized in the middle and right panel of Figure 2 respectively. On the other hand, the scheme will increase demand for floor space in locations where the maximum FAR restriction is already binding. Furthermore, as the last two columns of Table 1 suggest, for mild to medium FAR restrictions, Pigouvian toll expands the inner sprawl of the city (the extent of the area in which FAR constraint is binding), augmenting the total distortion generated in the housing market.

Combining the two effects yields the relative efficiencies suggested in the second column of Table 1. At mild levels of a maximum FAR regulation, the housing market effect dominates: although more vehicle kilometers (and thus externalities) are produced compared to the city without a maximum FAR, inner sprawl increases drastically after the implementation of the Pigouvian toll (almost 80% for the case of twenty floors). As a result, Pigouvian welfare gains can fall below 60% of the respective gains in a city without FAR regulations (59% in the case of fifteen floors).

Table 1. Pigouvian toll in a series of cities with different FAR regulations. Relative efficiency, pre and post-tax inner sprawl and city size.

<table>
<thead>
<tr>
<th>Road toll scheme</th>
<th>Relative efficiency</th>
<th>$\hat{x}_0$</th>
<th>$\hat{x}_1$</th>
<th>$\hat{x}_0$</th>
<th>$\hat{x}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum FAR allowed</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>20.0</td>
<td>0.741</td>
<td>40.1</td>
<td>38.3</td>
<td>0.95</td>
<td>1.7</td>
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<td>15.0</td>
<td>0.587</td>
<td>40.8</td>
<td>39.3</td>
<td>2.8</td>
<td>3.3</td>
</tr>
<tr>
<td>10.0</td>
<td>0.640</td>
<td>42.9</td>
<td>41.7</td>
<td>6.6</td>
<td>6.8</td>
</tr>
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<td>0.884</td>
<td>47.6</td>
<td>46.4</td>
<td>14.0</td>
<td>13.5</td>
</tr>
<tr>
<td>4.00</td>
<td>1.060</td>
<td>53.0</td>
<td>51.8</td>
<td>22.3</td>
<td>21.0</td>
</tr>
</tbody>
</table>

Notes: subscripts 0 and 1 denote the equilibria before and after the introduction of the Pigouvian toll respectively. All distances expressed in km; all relative efficiency measures expressed as compensating variations relative to the Pigouvian toll in the benchmark equilibrium.

However, at more severe levels of a maximum FAR, the inner sprawl prior to the Pigouvian toll occupies a large fraction of the total built area (42% in the case of four stories), with the vast majority of the population residing already in it. In fact, the introduction of a Pigouvian toll can reduce the inner sprawl through a general reduction of housing consumption in the city. Our findings suggest that, in such cases, the welfare gains not only recover, but they might exceed the benchmark gains in a city without FAR regulations. That is, the road externality effect dominates.

The above finding has significant policy implications. The second column of Table 1 indicates that the same policy in a series of seemingly identical cities (in terms of road technology, preferences, and population) can generate very diverse welfare effects. Therefore, cost-benefit analyses based on extrapolations of welfare gains from road pricing can be sloppy if they disregard the regulations in the housing market. Given the above, the question of whether the Pigouvian toll is indeed the optimal toll in a FAR regulated city emerges.
4.2. Is Pigouvian toll suboptimal?

To test the optimality of the Pigouvian toll, we perform a series of numerical tests based on the idea that the toll function in (24) can be approximated with the use of a piecewise linear interpolant (see (39) in Appendix B1). Then, the optimal tax at the selected points used in the interpolation (distances from CBD) can be computed with standard numerical optimization techniques. We use starting values that account for 70% of the Pigouvian toll in the interpolation points. Table 2 displays the efficiency of the Pigouvian toll relative to the optimized tax interpolant.

Table 2. Relative efficiency of the Pigouvian toll (benchmark: optimal tax interpolant).

<table>
<thead>
<tr>
<th>Elasticity of substitution (σ)</th>
<th>0.40</th>
<th>1.25</th>
<th>1.54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum FAR allowed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.0</td>
<td>1.014</td>
<td>1.035</td>
<td>1.025</td>
</tr>
<tr>
<td>10.0</td>
<td>1.009</td>
<td>1.011</td>
<td>1.010</td>
</tr>
<tr>
<td>6.00</td>
<td>1.006</td>
<td>1.008</td>
<td>1.002</td>
</tr>
</tbody>
</table>

Notes: all results in the table produced with linear piecewise interpolation and Newton-Raphson with line search.

The results on the third column of Table 2 suggest that Pigouvian toll retains its optimality. This finding is underpinned by the important distinction between a *tax-induced distortion* and *quantity rationing*. While the extent of the former can be affected in the presence of a tax interaction (see section 4.3), the extent of the latter cannot. That is, for any arbitrary tax scheme imposed on the road, the minimum difference between the equilibrium and the first-best FAR in any arbitrary location x where the constraint is binding is the one observed in the Pigouvian toll equilibrium. To establish that the result does not depend on the assumed elasticity of substitution, the computations are repeated for two different cities in which σ is lower (higher). In each case, the model parameters are recalibrated in order for the base equilibrium to loosely resemble the unregulated equilibrium of section 3.1. The results are displayed in the second (fourth) column of Table 2.

---

9 Various different levels of initial perturbations have been tried and resulted in slightly different relative efficiencies; however, the general result remains intact: in none of the cases has the relative efficiency been observed to fall below 1.0.
The above finding has a clear policy implication: decisions on road pricing can in general ignore building height restrictions at the background, allowing a detachment of the decisions regarding road pricing from urban planning decisions with respect to the maximum FAR. This implies lower costs in the design and implementation of an urban road toll system. However, as shown in the next section, this detachment ceases in the presence of tax interactions.

4.3. Pigouvian toll in a city with zoning

Like a maximum FAR, zoning constitutes a quantity distortion. In fact, it can be seen as a special case of a building height restriction, in which FAR is forced to be zero in a specified area. Unlike a uniform maximum FAR, however, the extent of the distortion in the housing market remains intact after the introduction of a Pigouvian toll. As a result, road externalities determine completely the extent of welfare gains in any arbitrary choice of zoning area \((x_L, x_U)\).

When the lower bound, \(x_L\), is placed close to CBD, expanding the zoned area forces households to relocate further away. With mild adjustments in labor supply, total external costs of congestion rise. Then, Pigouvian toll produces gradually larger welfare gains as the extent of the vacant area increases (first row of Table 3). The opposite is true when the bound is placed close to city fringe. In this case, extension of zoning causes relocation closer to CBD, decreasing the welfare gains from a Pigouvian toll (third row). Apart from being diametrically opposite, the above effects also differ in magnitude due to the initial distribution of population over space, which is far from uniform. Therefore, in a cost-benefit analysis that disregards zoning effects, welfare gains of a Pigouvian toll are much more likely to be underestimated rather than the opposite. Between the two polar cases, gradual expansion of the upper zoning bound, \(x_U\), produces non-monotonic results; that is, the direction of household relocation becomes ambiguous.

Similar tests to those employed in section 4.2. show that Pigouvian toll retains its optimality at any arbitrary choice of zoning area, generalizing the result to any quantity distortion in the model’s parallel markets, i.e. land, housing and labor.

Table 3. Pigouvian toll welfare gains in cities with zoning.

<table>
<thead>
<tr>
<th>Extent of zoned area (km)</th>
<th>1.00</th>
<th>2.00</th>
<th>3.00</th>
<th>4.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zoning lower bound (km from CBD)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>1.134</td>
<td>1.338</td>
<td>1.597</td>
<td>1.870</td>
</tr>
<tr>
<td>4.00</td>
<td>1.021</td>
<td>1.023</td>
<td>0.998</td>
<td>0.954</td>
</tr>
<tr>
<td>30.00</td>
<td>0.995</td>
<td>0.992</td>
<td>0.989</td>
<td>0.986</td>
</tr>
</tbody>
</table>

Notes: All welfare gains are expressed in terms of compensating variations relative to the case without zoning.

4.4. Optimal toll in the presence of ad valorem property taxation

We now investigate the role a pre-existing, ad-valorem tax per unit of housing property, paid by the consumer. We consider a wide range (0-8%) in order to capture the various levels of property taxation across Europe and North America. Because congestion is present in the benchmark city, this tax (at low

\[^{10}\] Note that the welfare effects of zoning per se are negative. That is, equilibria with a larger zoning area, \(x_U - x_L\), are associated with lower utility, despite Pigouvian toll might produce larger welfare gains.
levels) may be welfare improving: it generally reduces dwelling size, resulting in a more compact city where fewer vehicle kilometers are produced. However, the equilibrium tax per square meter is higher closer to CBD; this generates a *push-out effect* which increases urban sprawl. Therefore, the total impact of the ad-valorem tax on the size of the city is ambiguous.

The presence of this policy implies a clear tax interaction: a location-specific tax for the use of road coexists with a location-specific tax on property. However, this interaction is relatively weak compared to others investigated in relevant literature: for instance, a labor tax and a road toll might be perfect substitutes in settings where labor supply is inelastic in the intensive margin and there is no commuting substitute to car (Parry and Bento, 2001).

As shown in the second column of Table 4, the welfare gains from a Pigouvian toll are generally stable across different levels of the ad-valorem tax. The third (fourth) column displays the gains (percentage difference in welfare gains) from an alternative road pricing scheme based on the optimal tax interpolant discussed in section 4.2. Despite the difficulty to differentiate between the Pigouvian and the optimal tax at low levels of property taxation, column 4 shows that the former ceases to be optimal even at moderate levels of an ad-valorem tax.

As figure 3 suggests, the optimal road tax turns to be lower everywhere in the city. Note that, despite piecewise linear interpolation yields a strictly monotonous, concave price scheme, the total toll is negative in a considerable space interval, in all three cases. In fact, forcing the tax to be non-negative in the entire interval \((0, \bar{x})\) exhausts almost all of the additional gains. This implies that these gains stem to a large extent from household relocation closer to CBD, as the last column of Table 4 suggests.

<table>
<thead>
<tr>
<th>Road toll scheme</th>
<th>Pigouvian toll</th>
<th>Optimal interpolant</th>
<th>% change</th>
<th>% change in city size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing property</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0%</td>
<td>1.0</td>
<td>1.0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>2%</td>
<td>1.011</td>
<td>1.011</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>4%</td>
<td>1.021</td>
<td>1.028</td>
<td>0.7%</td>
<td>-0.87%</td>
</tr>
<tr>
<td>6%</td>
<td>1.030</td>
<td>1.059</td>
<td>2.8%</td>
<td>-1.05%</td>
</tr>
<tr>
<td>8%</td>
<td>1.038</td>
<td>1.095</td>
<td>5.2%</td>
<td>-1.30%</td>
</tr>
</tbody>
</table>

Notes: all values are expressed relative to the compensating variation of a Pigouvian toll in the base, unintervened equilibrium.

The optimal tax interpolant provides a general downward adjustment and local corrections which account for the inefficiency of the property tax. These adjustments, which may not be feasible without the use of an additional policy instrument, can provide considerable improvements in terms of welfare gains.
6. Concluding remarks

This paper has investigated road pricing policies under complete rigidity in the housing market regulations. The topic is highly relevant because empirical evidence shows that policy adjustments in the housing market are relatively slow.

We focused on the cases of a maximum floor-to-area ($FAR$) ratio and zoning. We numerically demonstrated that the welfare gain from a Pigouvian toll is non-monotonic function of the building height regulation stringency. Conceptually, this function can be composed by two major effects of a Pigouvian toll: the welfare gains on the road, which are increasing as the maximum building height decreases, and the losses in the housing market, which are more severe at milder levels of the $FAR$ restriction. In the case of zoning, the second result is absent: all land parcels (in which structural density is allowed to be positive) operate efficiently both prior and after the introduction of road pricing. Then, total welfare effects are entirely determined on the road. In both cases, numerical tests show that the optimal road toll coincides with the Pigouvian.

These findings have significant policy implications. Our computations suggest that the Pigouvian toll imposed in a city without height restrictions may produce up to 40% larger welfare gains relative to the respective Pigouvian toll in a city with a mild, uniform in space, $FAR$ restriction. Similar computations for the case of zoning suggest that the welfare gains might be 80% larger in an equilibrium with a large zoned area close to the CBD. Therefore, cost-benefit analyses based on extrapolations of welfare gains from road pricing can be sloppy if they disregard the regulations in the housing market. Furthermore, the optimality of the Pigouvian toll in the above cases suggests that decision making on urban road pricing can ignore quantitative restrictions in the parallel markets of land, housing and labor.

Finally, the paper investigates the case in which a distortionary property tax replaces the above quantity distortions. In the presence of a concrete tax interaction (Pigouvian road tax and property tax), it is shown that Pigouvian toll is relatively inefficient: a general downward adjustment and local corrections which account for the inefficiency of the property tax are required. These adjustments can provide small but considerable improvements in terms of welfare.
6. Appendices

A. Notation

Table A. Basic model variables and parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>distance from CBD</td>
<td>p</td>
</tr>
<tr>
<td>(\bar{x})</td>
<td>city size (sprawl)</td>
<td>(p_L)</td>
</tr>
<tr>
<td>(\hat{x})</td>
<td>binding boundary (inner sprawl)</td>
<td>(p_K)</td>
</tr>
<tr>
<td>(\hat{k})</td>
<td>structural density</td>
<td>(r_A)</td>
</tr>
<tr>
<td>(\delta)</td>
<td>floor-to-area (FAR) ratio</td>
<td>(R)</td>
</tr>
<tr>
<td>(y)</td>
<td>disposable income</td>
<td>(R_L)</td>
</tr>
<tr>
<td>(m)</td>
<td>pecuniary commuting cost (per km)</td>
<td>(G)</td>
</tr>
<tr>
<td>(\tau_x)</td>
<td>total road tax from (x) to CBD</td>
<td>(n)</td>
</tr>
<tr>
<td>B</td>
<td>lump-sum income</td>
<td>(K)</td>
</tr>
<tr>
<td>(\ell)</td>
<td>leisure time</td>
<td>(N)</td>
</tr>
<tr>
<td>t</td>
<td>commuting time</td>
<td>(Q)</td>
</tr>
<tr>
<td>s</td>
<td>apartment size</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>composite good (numeraire)</td>
<td></td>
</tr>
</tbody>
</table>

B. Computational details


We have divided the road intervals \((0,\bar{x})\) and \((\bar{x},\bar{x})\) in two and (respectively) three segments of equal length. The points that define these segments are given by the vector \(\delta'=[0, 0.33 \bar{x}, 0.66 \bar{x}, \bar{x} + 0.33 (\bar{x} - \bar{x}), \bar{x} + 0.66 (\bar{x} - \bar{x}), \bar{x}]\). Define any pricing scheme in the paper as a vector \(c'\) of equal size as \(\delta'\), and let \(c_i\) denote the total price charged to the household located at distance \(\delta_i\). At any \(\delta_i < x < \delta_{i+1}\), the charge is given by the piecewise linear interpolant:

\[
c_x = c_i + \frac{x-\delta_i}{\delta_{i+1}-\delta_i}(c'_{i+1}-c'_i). \tag{39}
\]

Throughout the paper, the expression in (39) is called the tax interpolant. Any optimization technique employed in the paper attempts to approximate the vector \(c'\) which belong to the policy space and maximizes (10) subject to the equilibrium conditions.

References

