Vickrey Meets Alonso: Commute Scheduling and Congestion in a Monocentric City

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Dynamic congestion and the bottleneck model

- Bottleneck model (Vickrey (1969), Arnott, de Palma, Lindsey (1990, 1993)) is the standard economic model that analyzes congestion dynamics.

- Commuters pass a bottleneck with a fixed level of capacity (maximum travelers who can pass the bottleneck in a time unit) to arrive at the destination.

- Commuters choose their optimal departure time, but congestion may arise in equilibrium because the rate of travelers arriving at the bottleneck may exceed the bottleneck capacity.

- Time-varying tolls can be imposed such that the queue is totally removed.
Congestion dynamics and space

- Standard bottleneck model misses an important aspect of urban traffic congestion: Space.

- Since scheduling preference is *linear* in travel time and cost or being late of early, distance does not matter for commute scheduling.

- Fosgerau and de Palma (2012) use a scheduling preference that is (not strict) a generalization of the customary $\alpha - \beta - \gamma$ preference.

- Consider a city, where there is a bottleneck at the entrance to the central business district (CBD).
Adopting the monocentric city model framework to extend the bottleneck model

• We follow Fosgerau and de Palma (2012)’s framework, which however is incomplete in the sense that the spatial distribution of population is *exogenous*.

• We endogenize the spatial distribution of population by adopting the Alonso (1964) - Mills (1967) - Muth (1969)s’ monocentric city model framework.

• Gubins and Verhoef (2014) consider bottleneck congestion in the same spatial framework as ours. But, they rely on an ad hoc assumption to link the time preference to the consumption preference.

• Our model is a straight unification of the bottleneck model and the monocentric model!
There is no congestion from home to bottleneck. So, \( a = d + x \).

\( t(a) = a \) holds if there is no congestion at the bottleneck. But, \( t(a) > a \) when there is queue.

Waiting time at the bottleneck is \( t(a) - a \).
Scheduling preference

- Consumers have identical scheduling preferences, with the cost function given by $c(d, t)$, with $c_1 < 0$ and $c_2 > 0$, meaning that commuter prefers to depart later and arrive earlier, ceteris paribus.

- We impose only weak assumptions on $c(d, t)$. See paper for specific assumptions on the second-order partial derivatives of $c(d, t)$.

- Write $c(d, t)$ as a function of $a$ (arrival time at the bottleneck) using $d = a - x$ and $t = t(a)$ (with $t'(a) \geq 0$) to have 

  $$c(a - x, t(a)).$$
Incorporating commute scheduling in the monocentric model

- City is linear with unit width and the CBD at its left end. City is *open* with no (or any) spatial boundary.

- Budget constraint is $y = e + pq$, where $y$ is income earned at CBD and $p$ is price per unit of land (housing).

- Scheduling cost, $c(d, t)$ takes a money metric form. So, $c$ is directly subtracted from $e$, so that $e - c$ becomes a component in utility. (Equivalently, $c$ subtracted from $y$ in budget constraint.)

- Consumer utility function is well-behaved and written as $U(e - c, q)$. Consumer maximizes utility by choices of $e$, $q$, and $a$. 
Features of urban spatial equilibrium

- Consumers attain a common utility level, a key condition for spatial equilibrium.

- $c'(x) > 0$ holds, since the consumer residing farther away must depart earlier, ceteris paribus.

- Equal-utility giving $p$, called bid-rent function, declines with $x$, so that $p'(x) < 0$.

- Individual land consumption rises with $x$, so that $q'(x) > 0$.

- Let $f(x)$ denote population density at $x$, which equals $1/q(x)$ (assuming a linear city); $q'(x) > 0$ implies $f'(x) < 0$, a falling population density with $x$. 
Commute scheduling in laissez-faire equilibrium

- Queuing begins immediately after the first commuter arrives at the bottleneck at time $a_0$.
- Commuter at $x$ arrives at the bottleneck at time $a(x)$, with differential equation given by

  $$a'(x) = -\frac{c_2 \left( a(x) - x, t(x) \right) f(x)}{c_1 \left( a(x) - x, t(x) \right) \psi} > 0.$$  

  So, commuter located farther away arrives later.
- Arrival time at the destination for resident at $x$ is

  $$t(x) = a_0 + \frac{F(x)}{\psi},$$

  where $\psi$ is bottleneck capacity and $F(x)$ is number of residents within $x$.
- It shows how commute schedule at a particular location interacts with urban spatial equilibrium.
• $t(x) - a(x)$ gives waiting time for resident at $x$.
• Residents located beyond $x_1$ do not queue.
• $a_*(x)$ gives the optimal arrival time in the absence of queue as a (sole) function of $x$.

Figure: Commute schedule outcomes as a function of location
Socially optimal toll

- **Conjecture**: Socially optimal toll eliminates queuing and induces full capacity utilization. Let $\Theta$ denote the set of tolls satisfying this condition.

- In $\Theta$, the planner chooses the toll that maximizes:

$$\int p(x)q(x)f(x)dx + \int \tau(a(x))f(x)dx,$$

which can be potentially redistributed to the city residents.

- **Toll is additive to scheduling cost.** With $a_\tau = t_\tau$, utility is written

$$U[y - c (a_\tau - x, a_\tau) - \tau(a_\tau) - p_\tau q_\tau, q_\tau],$$

where $\tau(a_\tau)$ is time-varying optimal toll.
Comparison between social optimum and laissez-faire

- First traveler's arrival takes place earlier in social optimum than under laissez-faire, i.e., $a_{\tau 0} \leq a_0$.
- Scheduling cost is higher in social optimum than under laissez-faire for those located at $x_0$, i.e.,

$$c(a_{\tau 0} - x_0, a_{\tau 0}) \geq c(a_0 - x_0, a_0),$$

since $a_{\tau 0} \leq a_0 \leq a_*(x_0)$.
- Location of the last tolled person is nearer to the CBD than the location of the last queuing person under laissez-faire, i.e., $x_{\tau 1} \leq x_1$.
- So, under tolling, residents located between $x_{\tau 1}$ and $x_1$ arrive without either meeting the queue or paying the toll. So, they benefit from tolling:

$$c(a_{\tau}(x) - x, a_{\tau}(x)) \leq c(a(x) - x, t(x)),$$

for $x \in [x_{\tau 1}, x_1]$. 
Arrival schedules under socially optimal toll

Figure: Commute schedule outcomes as a function of location

\[ a(x) \]
\[ a^*(x) \]
\[ t(x) \]
\[ a_{\tau}(x) \]
Scheduling cost with and without tolling

Figure: Scheduling-cost profiles with and without tolling
Underlying mechanism

- This asymmetry outcome arises due to the fact that more distant residents suffer disproportionately from congestion.

- Central residents can depart quite late and still become the first in the queue.

- This implies that people tend to arrive too late under laissez-faire.

- Tolling fixes this distortion and induces earlier arrivals than under laissez-faire.

- By adjusting arrival schedules, tolling gives a benefit (in terms of scheduling cost) to suburban residents at the expense of central residents’ scheduling costs.
Density with and without tolling

- Housing price is a compensating differential for scheduling costs. So, when toll revenues are not redistributed, the followings hold.

- Housing price is lower in social optimum than under laissez-faire in the city center. There is the opposite pattern in the suburb.

- Due to the price effect, population density is lower in social optimum than under laissez-faire in center. The opposite pattern in the suburb.

- Optimal toll shifts the central population to the suburb, inducing a city that is less dense in the center and more dense in suburb.
Effect of socially optimal toll on densities

Figure: Density profiles with and without tolling

- $f$
- $f_{\tau}$

Distance from the bottleneck ($x$)
Comparison to previous studies

- This result is contrasted to that in the congested city models adopting static congestion assumption (e.g., Wheaton (1998), Brueckner (2007)).
  - In these models, external congestion cost is proportional to distance traveled.

- Gubins and Verhoef (2014) derive a similar result to ours. But, mechanism is fundamentally different.
  - Tolling lowers densities by allowing residents to spend more time at home by eliminating the queue, which gives a stronger incentive for residents to have a larger house.
Conclusion

• Paper has presented a unified framework of the bottleneck congestion model and the monocentric city model; Congestion dynamics and the urban spatial equilibrium interact.

• Tolling induces a city that is less dense in center but more dense in suburb. The result itself and/or the mechanism are different from previous studies.

• Theory generates a number of empirical implications: Density effect of tolling, relationship between commute scheduling and residential location, queuing time varying with location, and comparative static results.