Optimal timetable under unreliable travel conditions

N. Coulombel, A. de Palma

Corresponding Author: Nicolas Coulombel
Eastern Paris University, LVMT
✉️: nicolas.coulombel@enpc.fr
PRESENTATION OUTLINE

1. **Motivation**
2. **The model**
3. **The single train problem**
4. **The multiple trains problem**
5. **Conclusion**
Reliability is a key issue in transport

- as congestion increases, systems become less and less reliable:
  - OECD (2010), Fosgerau (2010), Benezech (2013) for public transit

Large body of literature on car users and the value of travel time variability

- Exogenous congestion: Fosgerau and Karlström (2010), Fosgerau and Engelson (2011), ...
- Endogenous congestion: de Palma and Coulombel (2013, 2014)
Motivation (2)

Less literature specific to public transit

- Value of travel time variability: Benezech and Coulombel (2013)
- Optimum level of reliability: Monchambert and de Palma (2014)

Some questions remains unaddressed → when service reliability declines:

- 1) Are timetables robust?
- 2) How important is it to provide information to users regarding the level of reliability?
- Relevant literature: De Palma and Lindsey (2001), ...
The model
One transit line, no intermediate stops

Timetable

- \( L \) trains, official timetable \((h_i)_{i \in \{1, \ldots, L\}}\)

![Diagram showing Advertised travel time \( T_V \) between stations A and B with time points \( h_{i-1}, h_i, h_{i+1} \)]
(1) In-vehicle travel time (IVTT)
   - the train leaves on time but is delayed en route: \( \widetilde{T}_V = T_V + \varepsilon_i \)

(2) Train departure time
   - the actual train departure time is not the scheduled one: \( \widetilde{h}_i = h_i + \omega_i \)
   - 2 differences with the previous case
     - waiting time instead of additional IVTT: \( \widetilde{T}_W = \min_{\widetilde{h}_i > t_d} \widetilde{h}_i - t_d \)
     - possibility to miss the train

This presentation focuses on case 1
\section*{Demand}

\textit{N} travelers

- Heterogeneous preferred departure time $d^* = t^* - T_V$
  - PDF $f$, CDF $F$
  - mean $\mu_d$, standard deviation $\sigma_d$

**Standard generalized cost function**

$$\tilde{c}(t_d) = \alpha_V \tilde{T}_V + \beta (t^* - \tilde{t}_a)^+ + \gamma (\tilde{t}_a - t^*)^+$$

with $\tilde{t}_a = t_d + \tilde{T}_V$

- $\alpha_V$ : unit cost of in-vehicle travel time
For each agent type, 2 possibilities:

- Optimal Strategy (OS)
- Naïve Strategy (NS)

<table>
<thead>
<tr>
<th>Transit Operator</th>
<th>Transit Users</th>
</tr>
</thead>
<tbody>
<tr>
<td>Op-NS / Usr-NS</td>
<td>Op-OS / Usr-NS</td>
</tr>
<tr>
<td>Op-NS / Usr-NS</td>
<td>Op-OS / Usr-OS</td>
</tr>
</tbody>
</table>

Question: relative efficiency of the 4 strategies?
The Single Train Problem
Train departure time: $h$

**Individual cost**  (when preferred departure time = $d^*$)

$$c(h) = \alpha_V T_V + \beta (d^* - h)^+ + \gamma (h - d^*)^+$$

**Total cost**  (integration over $d^*$)

$$C_D(h) = N \alpha_V T_V + N \left( \beta \int_{d^*}^{+\infty} (u - h) f(u) du + \gamma \int_{-\infty}^{h} (h - u) f(u) du \right)$$
The optimal departure time $h^*$ is given by the F.O.C. 

*(de Palma and Lindsey, 2001)*

\[ \gamma \int_{-\infty}^{h^*} f(u)du = \beta \int_{h^*}^{+\infty} f(u)du \]  \hspace{1cm} (1)

\[ \iff F(h^*) = \frac{\beta}{\beta + \gamma} \]  \hspace{1cm} (2)

Minimum cost

\[ C_D(h^*) = N \left( \alpha T_V + (\beta + \gamma) \int_{h^*}^{+\infty} (u - \mu_d) f(u)du \right) \]  \hspace{1cm} (3)

Problem formally equivalent to Fosgerau and Karlström (2010)
CASE 1 - STOCHASTIC TRAVEL TIME

Random delay $\varepsilon_i$ (for reminder $\tilde{T}_V = T_V + \varepsilon_i$)

- PDF $\varphi$, CDF $\Phi$
- Mean $\mu_\varepsilon$, scale $\sigma_\varepsilon$

For users, only one train $\Rightarrow$ naïve strategy = optimal strategy

Expected cost for one individual with PDT = $d^*$

$$E[\tilde{c}(h)] = \alpha_V(T_V + \mu_\varepsilon) + \beta \int_{-\infty}^{d^*-h} (d^* - h - x)\varphi(x)dx + \gamma \int_{d^*-h}^{+\infty} (h + x - d^*)\varphi(x)dx$$

Expected total cost (integration over $d^*$)

$$C_s(h) = N\alpha_V(T_V + \mu_\varepsilon) + N \int_{-\infty}^{+\infty} \left\{ \beta \int_{-\infty}^{u-h} (u - h - x)\varphi(x)dx + \gamma \int_{u-h}^{+\infty} (h + x - u)\varphi(x)dx \right\} f(u)du$$
For convenience, we use the «random advance»: \( \psi(t) = \varphi(-t) \)

- intuition: the operator internalizes the random delay in the PDT
- when \( \varphi \) is the \( \delta \) Dirac distribution, \( \psi = \varphi = \delta \) and \( \psi \ast f = f \) ...
Normalizing the random delay yields the following results:

- **Optimum departure time**
  - linear in $\mu_\varepsilon$: $\tau^* = \tau^*_{\mu_\varepsilon=0} - \mu_\varepsilon$
  - influence of $\sigma_\varepsilon$: no specific pattern, **everything can happen**!

- **Minimum cost**
  - linear in $\mu_\varepsilon$: $\bar{C}_s(\mu_\varepsilon, \sigma_\varepsilon) = \bar{C}_s(0, \sigma_\varepsilon) + N\alpha_V \mu_\varepsilon$
  - increases with $\sigma_\varepsilon$ in a **convex manner**
  - case of naïve strategy: same behavior in $\sigma_\varepsilon$, but with a steeper increase than for the optimal strategy
APPLIED CASE - NORMAL D* / NORMAL DELAY DISTRIBUTION FUNCTION $\psi * f$

**Parameters**

$\mu_d = 0$

$\sigma_d = 1/\sqrt{3}$

$\mu_\varepsilon = 0$
Parameters

\[ \mu_d = 0 \]
\[ \sigma_d = 1/\sqrt{3} \]
\[ \mu_\varepsilon = 0 \]
\[ \alpha_V = 10 \text{€/h} \]
\[ \beta = 10 \text{€/h} \]
\[ \gamma = 30 \text{€/h} \]
\[ T_V = 0.5 \text{ h} \]
Parameters

\( \mu_d = 0 \)
\( \sigma_d = 1 \)
\( \mu_\epsilon = 0 \)
\( \alpha_V = 1 \)
\( \beta = 1 \)
\( \gamma = 3 \)
\( T_V = 1 \)
UNIFORM D*- EXPONENTIAL DELAY : DISTRIBUTION FUNCTION

Parameters

\[ \mu_d = 0 \]
\[ \sigma_d = 1/\sqrt{3} \]
\[ \mu_\varepsilon = 0 \]
OPTIMAL TRAIN DEPARTURE TIME

Parameters

\( \mu_d = 0 \)
\( \sigma_d = 1/\sqrt{3} \)
\( \mu_\varepsilon = 0 \)
\( \alpha_V = 10\,\text{€}/\text{h} \)
\( \beta = 10\,\text{€}/\text{h} \)
\( \gamma = 30\,\text{€}/\text{h} \)
\( T_V = 0.5 \,\text{h} \)
Nice result: the « equivalence theorem »

With poorer service reliability, the optimal strategy (for the transit operator) entails:

- non-monotonic adjustments in the optimal departure time...
  - strong dependence to the shapes of the PDFs
- ...for a very limited abatement of the cost compared to the naïve strategy.

Ignorance is bliss?
The Multiple Trains Problem
Users can choose between several trains

- **Optimal Strategy ≠ Naïve Strategy**

In case 2 (random train departure time), missing the train is no longer the end → possibility to catch the next train

- Treated in forthcoming paper

The « equivalence theorem » falls!

- F.O.C are more complex

- ⇒ must resort to numerical analysis
**UNIFORM D* – UNIFORM DELAY**

**IMPACT OF $\sigma_{\epsilon}$ ON OPTIMAL TIMETABLE**

**Parameters**

- $\mu_d = 0$
- $\sigma_d = 1/\sqrt{3}$
- $\mu_{\epsilon} = 0$
- $\alpha_V = 10\,€/h$
- $\beta = 10\,€/h$
- $\gamma = 30\,€/h$
- $T_V = 0.5 \, h$

Influence of $\sigma_{\epsilon}$ on the optimal timetable

$L = 3$
IMPACT OF $\sigma_\varepsilon$ ON EXPECTED TOTAL COST

Influence of $\sigma_\varepsilon$ on the expected total cost
$L = 3$

Cost (€)

$\sigma_\varepsilon$

Graph showing the influence of $\sigma_\varepsilon$ on the expected total cost with $L = 3$ for different costs of operations and users.

- Red: Cost Op-NS/Usr-NS
- Orange: Cost Op-OS/Usr-NS
- Yellow: Cost Op-NS/Usr-OS
- Green: Cost Op-OS/Usr-OS
NUMBER OF TRAINS AND MINIMUM COST - HIGH SERVICE RELIABILITY

\[ \sigma_e = 0.15 \]
NUMBER OF TRAINS AND MINIMUM COST - LOW SERVICE RELIABILITY

\[ \sigma_\varepsilon = \frac{1}{\sqrt{3}} \]
No « equivalence theorem »...

Ranking of the strategies (by increasing cost)

1. **User:** Optimizer & **Operator:** Optimizer
2. **User:** Optimizer & **Operator:** Naïve
3. **User:** Naïve & **Operator:** Optimizer
4. **User:** Naïve & **Operator:** Naïve

When users are naïve: increasing the number of trains increases the expected total cost → Braess’s paradox
Conclusion
For the transit operator, our results tend to show that:

- providing information to users is more efficient than adjusting the timetable
- especially true when service frequency is important \(\Rightarrow\) adding a train raises the mean user cost when users are naïve
- adjusting the timetable mitigates the cost of reliability, but not to a great extent

Single train case

- Nice result: equivalence theorem

Points missing and future extensions

- Case 2: random train departure time
- Other utility functions, including risk aversion