The marginal social cost of service reliability in public transit

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With kind support from Réseau Ferré de France
1. Motivation and background

2. Modelling framework

3. Equilibrium conditions in the general case

4. Equilibrium solution in a simplified case

5. Conclusion
Motivation

As congestion increases, transport systems become less and less reliable:

- OECD (2010), Benezech (2013) for public transit

In PT, service reliability has 2 main impacts:

- Waiting time
- Comfort

Research questions: for heavily congested systems, should the operator improve service frequency or service reliability?
Background research

Value of reliability & scheduling models

- Car users: substantial literature
- Transit users: fewer works, based on vehicle timetables
- Benezech, Coulombel (2013) : the value of service frequency and reliability for PT with high frequency

Common assumption: **exogenous congestion**

Bottleneck models

- Car traffic case with travel timel variability : Coulombel & de Palma (2014)
- Little research into the PT case
Aim of the paper

Adapt the bottleneck model to public transport

– Bottleneck model with discrete vehicle departure times
– High frequency $\Rightarrow$ frequency-based services
– Similar to Monchambert, Lindsey, de Palma (ITEA 2014), but irregularity in headways

Research objectives

– What is the impact of service frequency and service reliability on:
  • Departure time profile
  • Equilibrium cost
Modelling framework – Transport supply

Direct transit service between A and B

Headway-based services
  – Passengers cannot adapt to departure times
  – Travel time $T_V$ is assumed constant
  – Headways $H$ and waiting time $W$ are random variables

Relation between $H$ and $W$

$$\varphi_W(x) = \frac{1 - \Phi_H(x)}{\mu_H}$$

$\varphi_W$ distribution of $W$
$\Phi_H$ cumulative distribution function of $H$
Modelling framework – Travel demand

\( P \) passengers with preferred arrival time \( t^* \)

**Scheduling preferences**

- \((\alpha, \beta, \gamma)\) preferences
- Distinction between waiting time and in-vehicle time
- Congestion penalty linear with (stochastic) vehicle load \( N \)

\[
C(t) = \alpha_W W + \alpha_V (1 + kN)T_V + \beta SDE + \gamma SDL
\]

**Schedule delay penalties**

- Schedule Delay Early \((t^* - t - W - T_V)^+\)
- Schedule Delay Late \((t + W + T_V - t^*)^+\)
Modelling framework – Travel demand

Headstart (or safety margin)

\[ m = t^* - t - T_V \]

– Relates to waiting time, not in-vehicle travel time:

• If \( W \leq m \), the passenger is early
• If \( W \geq m \), the passenger is late

Minimisation of the expected cost

\[ \Gamma(m) = E[C(m)] = \alpha_w \mu_w + \alpha_v (1 + kE[N])T_V \]

\[ + \beta \int_0^m (m - u) \phi_w(u)du + \gamma \int_m^{+\infty} (u - m) \phi_w(u)du \]
Equilibrium conditions (1)

User equilibrium with regards to the expected cost

Passenger departure rate $n(t)$ such that

i. For all $t$ such that $n(t) > 0$, $\Gamma(t)$ is constant. Let $\Gamma_n$ be this value.

ii. For all $t$ such that $n(t) = 0$, $\Gamma(t) \geq \Gamma_n$.

iii. The total number of passengers satisfies $\int n(t) \, dt = P$.

iv. If $\tilde{n}$ is another function satisfying i, ii and iii, then $\Gamma_{\tilde{n}} > \Gamma_n$. 
Equilibrium conditions (2)

Exploration of the first condition

– Equivalent to: for all \( t \) such that \( n(t) > 0 \), \( \Gamma'(t) = 0 \).

– Information about the expected passenger load:

\[
\frac{d \mathbb{E}[N]}{d m} = \frac{\gamma (1 - \Phi_w(m)) - \beta \Phi_w(m)}{k \alpha_v T_v}
\]

<table>
<thead>
<tr>
<th>( m )</th>
<th>(-\infty)</th>
<th>0</th>
<th>( m^* )</th>
<th>(+\infty)</th>
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</thead>
<tbody>
<tr>
<td>( \frac{d \mathbb{E}[N]}{d m} )</td>
<td>( \frac{\gamma}{k \alpha_v T_v} )</td>
<td>( \uparrow )</td>
<td>0</td>
<td>( \downarrow )</td>
</tr>
<tr>
<td>( \mathbb{E}[N] )</td>
<td>0</td>
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<td>0</td>
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– Since \( \mathbb{E}[N] \) is positive and scheduling costs go to \(+\infty\) as |\( m \)| goes to \(+\infty\), the set of \( t \) such that \( n(t) > 0 \) is necessarily bounded.
Equilibrium conditions (3)

Expected passenger load

– Found by integrating over all possible values of headways and time elapsed since last departure (*age* in renewal theory A)

\[
\mathbb{E}[N] = \int_0^{+\infty} \left[ \int_0^{x_w} n(t + u)du \right] \varphi_w(x_w)dx_w + \int_0^{+\infty} \left[ \int_0^{x_a} n(t - u)du \right] \varphi_a(x_a)dx_a
\]

– Direct derivation gives the derivative in an operational way, as the convolution of \( n \) with

\[
\psi(x) = \begin{cases} 
\varphi_a(x) & \text{pour } x > 0 \\
-\varphi_w(-x) & \text{pour } x < 0 
\end{cases}
\]
Equilibrium conditions (4)

**Equilibrium conditions**

i. For all $t$ such that $n(t) > 0$

$$ (n \ast \psi)(t) = \frac{\gamma(1 - \Phi_w(t^* - T_v - t)) - \beta \Phi_w(t^* - T_v - t)}{k\alpha_v T_v} $$

ii. For all $t$ such that $n(t) = 0$, $\Gamma(t) \geq \Gamma_n$.

iii. The total number of passengers satisfies $\int n(t) \, dt = P$.

**Comments**

– The conditions on the boundary is an inequality, which makes things complex (what is $E[N]$ when $n(t) = 0$?)

– We hope/think that there is only one viable solution so condition 4 (minimality) does not need to be verified.
Simplification

Assumption: during a headway, variations of \( n(t) \) are negligible

\[
E[N] = n(t)E[H_u] = n(t)\mu_H \left(1 + \frac{\sigma_H^2}{\mu_H^2}\right)
\]

This gives the derivative of \( n \)

\[
n'(t) = \frac{\beta \Phi_w(t^*-T_v-t) - \gamma (1 - \Phi_w(t^*-T_v-t))}{k\alpha_v T_v \left(1 + \frac{\sigma_H^2}{\mu_H^2}\right)}
\]

<table>
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<th>( n'(t) )</th>
<th>(-\infty)</th>
<th>(+)</th>
<th>0</th>
<th>(-)</th>
<th>(+\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi_w )</td>
<td>( \frac{\beta}{k\alpha_v T_v} )</td>
<td>+</td>
<td>0</td>
<td>-</td>
<td>( - \frac{\gamma}{k\alpha_v T_v} )</td>
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- Similar to the standard bottleneck model
- Easy to integrate if \( \Phi_w \) is known.
Applied case - Exponentially distributed headways

Assumption: shifted exponential distribution of headway

\[
\begin{align*}
\phi_h(x) &= e^{-(x+1)} \\
\Phi_h(x) &= 1 - e^{-(x+1)}
\end{align*}
\]  \quad \text{for } x \geq -1

Very good fit of empirical data:
Applied case – Equilibrium solution

**Parameters**

- $P = 1000$
- $\mu_H = 2 \text{ min}$
- $\alpha_V = 10\text{€/h}$
- $\alpha_W = 20\text{€/h}$
- $\beta = 10\text{€/h}$
- $\gamma = 30\text{€/h}$
- $T_V = 0.5 \text{ h}$
Applied case – Equilibrium cost

**Parameters**

- $P = 1000$
- $\alpha_V = 10\,€/h$
- $\alpha_W = 20\,€/h$
- $\beta = 10\,€/h$
- $\gamma = 30\,€/h$
- $T_V = 0.5\,h$

![Graph showing User Cost (€) vs. σ (min) with lines for μH=2 and μH=10.](image-url)
Applied case –
Marginal social cost of service reliability

**Parameters**

- $P = 1000$
- $\alpha_V = 10€/h$
- $\alpha_W = 20€/h$
- $\beta = 10€/h$
- $\gamma = 30€/h$
- $T_V = 0.5\ h$
Conclusion

Intermediate results

– When service reliability decreases, departures are more spread over the rush hour $\Rightarrow$ vehicles are less crowded
– This equilibrium mechanism mitigates the cost of TTV $\Rightarrow$ the exogenous congestion assumption leads to overestimating the cost of TTV
– The marginal social cost of service reliability strongly depends on service frequency (and vice-versa)

Still ongoing work

– Derivation of the marginal social cost of service frequency and service reliability in the general case?
– Push the economic analysis further: optimal levels of service frequency and reliability, pricing,...