Preemption with a Second-Mover Advantage*

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Abstract

We examine innovation in a market-entry timing game with complete information and observable actions when there is a second-mover advantage. Allowing for heterogenous payoffs between players, and for both leader's and follower's payoff functions to be multi-peaked and non-monotonic, we find that there are at most two pure-strategy subgame perfect equilibria. Sometimes these resemble familiar second-mover advantage equilibria from the literature. However, we show that despite there being a follower advantage at all times, there can be a preemption equilibrium with inefficient early entry. In fact, immediate entry is possible in a continuous analogue of the centipede game. These results are related to the observed premature entry and product launches in various markets.

Key words: timing games, second-mover advantage, preemption.
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1 Introduction

Timing can be everything. The release date can make (or break) a new product. While this is a non-trivial decision for a monopolist, the choice of when to enter

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the market is complicated immensely by the presence of a rival. If the market leader makes more than the follower, there will be an incentive to preempt one's rival and enter the market early (**?**). Take two rival movie studios, for example, both contemplating when to release a new action blockbuster in the upcoming season.[1] If the first film on the market will capture the public's imagination and the lion's share of viewers, each studio will vie to be the leader, possibly resulting in both studios releasing their movie at the very beginning of the season. But being second is sometimes best. In fact, **?** show that in a variety of markets, early imitators often end up doing better than market pioneers. A follower can learn from the leader's mistakes and free-ride on their investment; they can imitate, then better, the pioneer in terms of quality or cost. Waiting might also allow a firm to outmaneuver their rival; delaying the release of a movie might allow a studio to develop more effective marketing strategy, for instance. Recent examples of second movers outperforming their pioneer rivals abound. Google was not the first search engine, nor was Chrome the first web browser. Similarly, iPhone and the Samsung Galaxy were not the first smart phones. In this paper we focus on entry games when there is a second-mover advantage allowing for a very general structure of payoffs.

The basic features of our model are as follows. Two firms can make an irreversible and one-off decision to enter a market. Time is continuous and all previous actions (entry or not) are observable; consistent with this, we focus on closed-loop equilibria. While we place few restrictions on the payoffs, at any point in time a firm earns more as a follower than as a market leader, herein capturing the second-mover advantage aspect of our model.

In **?** and others, the entrants are ex ante identical and have access to the same potential innovation. But usually firms are not all the same. Drawing inspiration from **?**, we study two heterogenous firms that can have different payoffs from entering at a given time.[2] This assumption of heterogeneity is applicable in many

[1]For an analysis of the timing of movie release dates see **?**; also see **?**.

[2]**?** analyze an innovation game with heterogenous firms when there is licensing (by the leader) and imitation (by the follower). They find that industry leaders (who are more efficient) need not be the firm that innovates, as they might prefer to free ride on the public good (innovation) provided by its rival. **?** uses a similar framework to examine the impact of regulation of technological adoption.

situations. Firms might differ in their ability to exploit market opportunities. Expected payoffs from launching a new phone handset could differ between two rivals, given their preexisting reputation, network or tie-in products. The same can be said for a process (cost-saving) innovation – its payoff depends on access to markets, how the new technology meshes with a firm's existing practices, and so forth. As noted above, often firms also have to choose which technology to implement. Returning to the smartphone example, Samsung made a choice to switch its cell phone operating system from its own in-house system to an Android platform; Sony also made an equivalent choice. Despite its closed system, in many ways Apple faces a similar tradeoff when contemplating the timing of a new iOS for its devices.

Implicit in this is that not all technologies are available immediately; rather, some technologies are only available (or worth considering) later. This potentially changes the payoff structure. Unlike in **?**, a leader's or follower's entry payoff can be multi-peaked with respect to entry time, reflecting when a new technology becomes available, and as its profitability wanes. This payoff structure, generated by the choice between multiple technologies, combined with the asymmetric payoffs between players, creates a new strategic entry environment not previously analyzed.

The key results are as follows. Adapting the technique of **?**, we show that there can be either one or two pure-strategy subgame perfect equilibria. Depending on the payoffs, it could be the case that either firm enters when its leader payoff is maximized, providing its rival with the advantage of being the follower. It can also be the case that only one of these *leader-maximized* equilibria – that is, an equilibrium in which one firm's leader payoff is maximized – exists. In this case, one of the firms, anticipating a lower follower payoff at the time that its rival would have entered as a leader, preempts and opts to enter earlier as the market pioneer. Note that this preemptive incentive exists even though follower payoffs at any given time are higher for those of a leader.

It could be, however, that neither of these *leader-maximized* equilibria exist. This is only possible when the leader and follower payoffs for at least one of the firms is non-monotonic. In this *leader-constrained* equilibrium, a firm preemptively enters the market even before its leader payoff is maximized, so as to avoid an anticipated lower profit in the future if it waited. As an example of this, consider a continuous market-entry version of the centipede game in which payoffs are increasing (overall) with later entry but that the local peaks for one firm coincide with a relatively lower payoff for its rival. This generates an iterative process in which each firm has an incentive to enter earlier, fearful of a lower follower payoff it could receive if its rival was allowed to enter as a leader later. In equilibrium, entry occurs inefficiently early – in fact, entry could occur immediately in this example. This outcome is indicative of the products being launched when they are really not ready, squandering opportunities for more mature and socially efficient innovation. This type of equilibrium is relevant, capturing some of the interplay between technology companies such as the decision as to when to launch a new smartphone by rivals like Apple and Samsung (and others).[3,4] It also suggests that even though we observe rival studios release their new movies at the start of the season, there could still be a follower advantage; moreover, both firms might be better off if they could commit to not release their films until later.[5]

This paper draws on an extensive literature on innovation timing games.[6] Our analysis of an irreversible investment decision with complete information and observable actions (closed-loop equilibria) follows **?**, **?**, **?** and **?**. This framework has been used to study a range of applications. For example, **???** adopt a variant of **?** to examine the order of market entry, clustering and delay. They show that with many potential entrants the most efficient firm need not be the first to enter the market and that delays are non-monotonic with the number of firms. In addition, they suggest a new justification for clustering of entry. Others have studied similar issues. Extending **?**, **?** shows that a second-mover advantage is increasing in the costs of R&D.[7] **?** study delays and rushes into a market in a stopping game with a continuum of players.

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[3]See 'Phone tag; Apple v Samsung' in *The Economist*, September 16 2017.

[4]This sort of scenario is reminiscent of deterrence/accommodation models, such as **?**. For empirical studies of strategic entry see **?** (US drive-in cinemas), **?** (MRI technology adoption in US hospitals) and **?** (pharmaceuticals with expiring patents).

[5]It could also be the case that there are two equilibria, with one the iterative *leader-constrained* equilibrium, as described above, and the other *leader-maximized.*

[6]See **?** or **?**, Chapter 5 for a survey of the literature. Further, **?** consider innovation when the firms make one irreversible decision (to enter) in a simple timing-game framework (see Sections 4.5 and 4.12).

[7]For related papers also see **?**, **?**, **?**, and **??**.

While we assume that previous actions of a rival are observable, an alternative approach to study innovation is to assume players' actions are unobservable as in **??**, where unobservable actions are equivalent to each firm being able to pre-commit. Reinganum shows that in the open-loop equilibria there will be diffusion in the sense that firms adopt the technology at different dates, even though all firms are ex ante identical. Similarly, **?** develop an innovation game with unobservable actions that permits any firm (in terms of the order of entry) to receive the highest payoff. This allows for a war-of-attrition, with higher payoffs for late movers, a pre-emption game with higher payoffs for early movers, or a combination of both. An important point of comparison is that in our model firms use feedback rules to determine their strategy at any particular point in time; this means that they are unable to commit to their strategy at the beginning of the game.

Information also plays a key role in the players' entry strategies. **?** show that when two potential rivals are uncertain about their entry costs, competition leads to inefficient entry that is too early. Other authors consider inefficiencies in innovation when there is asymmetric information. For example, **?**, **?** and **?** assume that a firm's capability to innovate is private information. In these models, delay allows a firm to get better information about the potential innovation (its costs, value, and so on), but waiting runs the risk that a rival will innovate first, capturing most of the returns.

2 The model

Assume two firms ($i = 1, 2$) are in a continuous-time stopping game starting at $t = 0$ until some terminating time $T \in (0, \infty]$. Firm i's one-off decision to stop (that is, 'enter' the market) at $t_i \geq 0$ is irreversible and observable immediately by the other firm.[8] The game ends when one of two firms has stopped/entered the market. The payoff to each firm depends on the stopping time. If the game ends with player i stopping at time t_i, the payoffs of the leader and the follower are $L_i(t_i)$ and $F_j(t_i)$, respectively, where $i, j = 1, 2$ and $i \neq j$.

We make the following assumptions.

[8]When there is no ambiguity, we refer to payoffs as a function of t rather than t_1.

Assumption 1. *Time is continuous in that it is 'discrete but with a grid that is infinitely fine'.*

Assumption 2. *Firms always choose to stop earlier rather than later in payoff-equivalent situations.*

Assumption 3. *If more than one firm chooses to stop (enter) at exactly the same time, one of these firms is selected to stop (each with an ex ante probability of $\frac{1}{2}$).*

Entry models in the literature adopt equivalent assumptions. Assumption 1 invokes **?** who show that under certain conditions a continuous-time strategy profile is the limit of a discrete-time game with increasingly fine time grids. It also replicates A1 of **?**.[9] Assumption 2, which is similar to A3 in **?**, allows us to focus on just one (payoff-equivalent) equilibrium in the case of indifference between early and late entry.[10] This simplifies our analysis so as to focus on the timing of entry rather than on issues of equilibrium selection.

Assumption 3 – part of A3 in **?** and Assumption 5 in **?** – avoids potential coordination failures involving simultaneous entry. The intuition underlying this assumption warrants further discussion. In some situations, as a practical matter, if two firms try to enter the market at the same time there might be some capacity constraint or institutional requirement that prevents joint entry – consequently, one firm becomes the leader and the other firm is relegated to the role of second entrant. For instance, in a particular market there could be a bureaucratic rule that requires the leadership role be allocated to the firm that has the first email registered in a designated inbox. Even if both firms simultaneously send their messages, only one email can arrive first. As a consequence, with simultaneous moves, each firm has some probability of being the leader.[11] Here, Assumption 3 gives either firm an equal chance being first when there is simultaneous entry. Note, we invoke this assumption for completeness, as simultaneous entry is not

[9] See **?**, footnote 4 for a further discussion.

[10] **?** assume that if the follower is indifferent between two alternative entry times, it chooses the earliest time of entry.

[11] Equivalent intuition applies to any (bureaucratic) tie-breaking rule that determines the winner in what seems to be a dead heat. **?** present a similar rationale for this assumption, suggesting there could be small random delays between when a decision is made and when a new technology is adopted, meaning that there is a positive probability that either firm will be first in the event of joint adoption.

an issue on the equilibrium path given our assumption of a follower advantage, as explicitly defined below.

The following two assumptions ensure that the leader stops in finite time. The first element of this is that leaders' payoff functions reach their respective global maxima at a finite point in time; this means that both firms will not delay entry indefinitely. **?** (Assumption 3), **?** (Assumption 2(ii)) and **?** (Assumption 4) all make equivalent assumptions. Secondly, we assume that entering provides a higher payoff than each firm's respective outside option of zero, thus ensuring that our analysis is not unnecessarily complicated by having to consider whether one or both firms never enter the market. Again, this mirrors assumptions made previously in the literature; Assumption 4 in **?**, Assumption 2(ii) in **?** and Assumption 5 in **?**.

Assumption 4. *There exists a finite* $m_i < T$*, which is the earliest time at which* $L_i(t)$ *attains its global maximum. Specifically,* $L_i(m_i) > L_i(\tau)\ \forall\ \tau < m_i$*, and* $L_i(m_i) \geq L_i(\tau)\ \forall\ \tau \geq m_i$ *where* $i = 1, 2$*.*

In other words,

$$m_i = \min \arg \max_t L_i(t) \text{ and } m_i < T. \tag{1}$$

Assumption 5. *Each firm's outside (non-entry) payoff is normalized to 0, and* $L_i(t) \geq 0$ *and* $F_i(t) \geq 0\ \ i = 1, 2$*.*

So far, the model outlined is quite general, extending the symmetric game of **?**. It could incorporate all sorts of market-entry outcomes, such as first-mover advantages or preemption games with an equalization of rents. However, our focus here is on interesting – and empirically relevant – case when there is a second-mover advantage. This scenario is summarized in the following assumption.

Assumption 6. $L_i(t) < F_i(t)$ *for all* t *and* $i = 1, 2$*.*

Second-mover advantage here refers to the fact that at any given time each firm earns a greater return as a follower than it does as a leader.

In summary, the first five assumptions are standard in the market-entry timing game literature with complete information and observable actions; see for example

?. Our last assumption allows us to consider second-mover advantage games only.[12] Unlike most of the literature we incorporate the possibility of heterogenous players. Moreover, our set-up is more general than **?** as payoffs can be multi-peaked. This allows for new interesting equilibria not possible previously.[13]

2.1 Equilibrium concept

Following **?**, we use subgame perfection. A history h_t is defined as the knowledge of whether or not firm $i = 1, 2$ previously stopped at any time $\tau < t$, and if so when. A strategy of firm i, denoted by $\sigma_i(h_t)$, indicates at each history h_t whether firm i stops at t ($\sigma_i(h_t) = 1$) or does not stop at t ($\sigma_i(h_t) = 0$). A strategy pair (σ_1, σ_2) maps every history to an outcome, which is the minimum of stopping times t_1 and t_2. As usual, a strategy profile (σ_1^*, σ_2^*) constitutes a subgame perfect equilibrium (SPE) if the strategies are sequentially rational after every history. Note here that with this representation we only need to specify the strategies when there has been no entry in the history of the game, because we assume that once one firm has entered, the game ends (**?**). This allows us, for ease of exposition, to refer to each firm's entry strategy as a function of time only, $\sigma_i(t)$.

3 Characterization of equilibria

In this section we first describe equilibria in the case of symmetric firms (Section 3.1), before exploring market entry when the firms potentially have different payoffs (Section 3.2).

[12] Our working paper **?** deals with more general scenarios, in particular when Assumption 6 does not hold.

[13] It is worth noting that while all of the examples provided here use continuous functions, our results also apply when there is a finite number of discontinuities. This is because a discontinuity only has an impact on the equilibrium outcome when there is a change in the relative position of the leader and follower curves for the same firm. An implication of Assumption 6 is that this is not possible. For a detailed analysis of entry games with discontinuous (but symmetric) payoffs see **?**.

Figure 1: Second-mover advantage equilibria with symmetric firms

3.1 Symmetric firms

To outline a benchmark for the analysis that follows, first assume that both firms are the same in terms of their potential payoffs, $L(t)$ and $F(t)$. Let us define the following analog to Assumption 4:

$$m = \min \arg \max_{t} L(t) \text{ and } m < T. \tag{2}$$

The proposition below describes the method for determining the entry time of the leader in the symmetric case.

Proposition 1. *[**?**] There are two second-mover advantage pure-strategy SPE of the symmetric model. In either equilibrium, the leader always enters the market at $t = m$.*

The intuition for this Proposition is illustrated in Figure 1. In this case there are two second-mover advantage equilibria in which one of the firms enters at $t = m$, whilst the other enjoys a higher payoff as a follower. The first firm to enter the market will choose the earliest time at which the leader's payoff is maximized. While both firms prefer to be the follower, one will have to play the role of pioneer. First, take the equilibrium in which firm 2 never enters first. Given this, firm 1's best option is to enter at m. The other equilibrium involves firm 1 always waiting, and entry by firm 2 at m. The underlying incentives to enter and wait are not affected if the two payoffs functions take on more complicated shapes – while one firm waits, the leader will come in at the earliest time at which the return to the leader is maximized.

This is an example of a *leader-maximized* equilibrium. While it earns less than the follower, the leader enters at a time that maximizes its return, conditional on it being first into the market.

3.2 Asymmetric firms

As noted previously, firms are more often than not different from one another. In this section we develop a method of determining the leader's entry time in all pure-strategy SPE, allowing for asymmetric payoff functions. Firstly, to find the pure-strategy SPE we note that any equilibrium with player i entering at time t_i must satisfy two necessary conditions:

Condition 1. *No preemption by the leader i (NPL):* $L_i(t_i) > L_i(\tau), \ \forall \ \tau \in (0, t_i)$.

Condition 2. *No preemption by the follower j (NPF):* $F_j(t_i) > L_j(\tau), \ \forall \ \tau \in (0, t_i)$.

If the *NPL* does not hold, the leader (player i) will deviate by entering earlier. Similarly, the *NPF* must hold in any SPE, otherwise the follower (player j) has an incentive to preempt and enter slightly earlier than the leader, as in **?**.[14] Even if these conditions hold, they do not in of themselves guarantee that a specific entry time is part of an SPE, because both only compare payoffs at a particular time relative to their historic values. These conditions, by definition, do not make any comparisons with future potential payoffs. Of course, such a consideration is necessary when determining any SPE.

To solve for the leader's entry time, let us eliminate all points that do not satisfy either of these conditions (the *NPL* and the *NPF*) by constructing sets $A_1(t', t'')$ and $A_2(t', t'')$. For each firm $i \in \{1, 2\}$, $j \neq i$ and $T \geq t'' > t' \geq 0$, define the following set:

$$A_i(t', t'') = \{ \ t \in (t', t''] \ | \ L_i(t) > L_i(\tau) \ \& \ F_j(t) > L_j(\tau) \ \forall \ \tau \in (t', t)\}. \quad (3)$$

By definition, a point belongs to set $A_i(t', t'')$ if it satisfies both *NPL* and *NPF*. By way of comparison, to solve the symmetric-player entry game **?** construct one set that is applicable to both firms. Here, asymmetry requires the construction of a set $A_i(.)$ for each firm and for any truncated game played on interval $[t', t'']$.

[14]**?** adopt similar conditions, which they refer to as the *Leader Preemption Constraint* and the *Follower Preemption Constraint*.

For each firm $i \in \{1,2\}$ define the following time:

$$t_i^* = \begin{cases} \arg\max_t A_i(0,T) & \text{when} \quad A_i(0,T) \neq \emptyset, \\ 0 & \text{when} \quad A_i(0,T) = \emptyset. \end{cases} \tag{4}$$

In addition, assume without loss of generality that $t_1^* \geq t_2^*$. Moreover, define recursively for $s \geq 2$ the following times $t_{1,s}^*$ and $t_{2,s}^*$:

$$t_{1,s}^* = \begin{cases} \arg\max_t A_i(0,t_{2,s-1}^*) & \text{when} \quad A_i(0,t_{2,s-1}^*) \neq \emptyset, \\ 0 & \text{when} \quad A_i(0,t_{2,s-1}^*) = \emptyset; \end{cases} \tag{5}$$

and

$$t_{2,s}^* = \begin{cases} \arg\max_t A_i(0,t_{1,s}^*) & \text{when} \quad A_i(0,t_{1,s}^*) \neq \emptyset, \\ 0 & \text{when} \quad A_i(0,t_{1,s}^*) = \emptyset. \end{cases} \tag{6}$$

with initial values $t_{1,1}^* = t_1^*$ and $t_{2,1}^* = t_2^*$.

Note that t_i^*, $t_{1,s}^*$ and $t_{2,s}^*$ are always unique. t_i^* describes the potential optimal entry time if player i has to be the leader for $t \leq T$. $t_{1,s}^*$ describes the potential optimal entry time if player 1 has to preempt player 2, who is expected to enter after $t_{2,s-1}^*$. Similarly, $t_{2,s}^*$ describes the potential optimal entry time if player 2 has to preempt player 1, who is expected to enter after $t_{1,s}^*$. Moreover, by construction of $t_{i,s}^*$ and earlier assumption $t_1^* \geq t_2^*$, it follows that

$$t_{1,1}^* \geq t_{2,1}^* \geq t_{1,2}^* \geq t_{2,2}^* \geq \cdots \geq t_{1,s}^* \geq t_{2,s}^* \geq \ldots. \tag{7}$$

Next, define recursively for $s \geq 2$ the following times $m_{1,s}$ and $m_{2,s}$:

$$m_{1,s} = \min \arg\max_t L_1(t) \;\text{ for }\; t \leq t_{2,s-1}^*; \tag{8}$$

and

$$m_{2,s} = \min \arg\max_t L_2(t) \;\text{ for }\; t \leq t_{1,s}^*. \tag{9}$$

with initial values $m_{1,1} = m_1$ and $m_{2,1} = m_2$.

As a consequence of the way they are defined, $m_{1,s}$ and $m_{2,s}$ are always unique. $m_{1,s}$ describes the earliest time at which $L_1(t)$ attains its maximum for entry times $t \leq t^*_{2,s-1}$. In the same way, $m_{2,s}$ describes the earliest at which $L_2(t)$ attains its maximum for $t \leq t^*_{1,s}$.

The following lemma allows to make the following comparison of different times.

Lemma 1. *The following inequality always holds:* $t^*_{i,s} \leq m_{i,s}$.

Proof: See Appendix C.

Take t^*_1 and m_1, for example, and assume that firm 1 will be the leader and firm 2 the follower in any equilibrium. m_1 is the time at which firm 1's leader payoff is maximized. This means that, for firm 1 itself, the *NPL* condition is satisfied at m_1, but not beyond this time. Furthermore, t^*_1 is constructed to be the maximum time that satisfies both the *NPL* and *NPF* conditions. Consequently, t^*_1 cannot exceed m_1 as it belongs to a more restrictive set. Depending on whether the *NPF* is satisfied at m_1 will determine whether t^*_1 is equal to or less than m_1. This logic applies to the comparison between any pair of $t^*_{i,s}$ and $m_{i,s}$.

Finally, we define the following set[15]

$$S = \{i = 1, 2 \;\; \& \;\; s = 1, 2, \ldots, s_m\}. \tag{10}$$

Second-mover advantage equilibria

We first explore situations in which the familiar second-mover advantage equilibria arise in the case with asymmetric firms. To do so, while we place no restrictions on the leader payoff functions (other than continuity), in a similar way to **?** and **???**, first consider the situation when both $F_1(t)$ and $F_2(t)$ are non-increasing. This could be the case, for instance, when later entry by the leader affords it to enter the market with a better (less costly) production technology or product, or possibly both, which in turn exerts greater competitive pressure on the second entrant.

The following proposition summarises all potential equilibria in this case.

[15]To avoid unnecessary complications, we assume that there is a finite s_m, such that $m_{1,s_m} = m_{2,s_m} = 0$. This assumption would hold in virtually all economic applications.

Figure 2: when both $F_1(t)$ and $F_2(t)$ are decreasing

Proposition 2. *Consider the SPE of the two-player asymmetric timing game when both $F_1(t)$ and $F_2(t)$ are non-increasing.*

1. *If $t_1^* = m_1$, there are two SPE, one with firm 1 entering at $t = t_1^* = m_1$ and the other with firm 2 entering at $t = t_2^* = m_2$;*

2. *If $t_1^* < m_1$, there is a unique SPE with firm 2 entering at $t = t_2^* = m_2$.*

Proof: See Appendix C.

To garner some intuition for these results, with the help of Figure 2, first consider Proposition 2(1). Note that $t_1^* = m_1$ guarantees that $L_2(t_2^*) < F_2(t_1^*)$. As illustrated in the top panel of the Figure, in this case there are two SPE in which both firms prefer to be the follower. Specifically, in each of these equilibria one of the firms enters when they attain their highest leader payoffs m_i (and the other firm always waits, unless entering strictly dominates waiting). Note that this example is a small perturbation of the symmetric-players case illustrated in Figure 1.

The second scenario is illustrated in the example shown in Figure 2(b). If the game reaches $t_1^* < m_1$ without entry, firm 1 would not enter at this time; rather it has an incentive to wait and enter at m_1. Understanding firm 1's incentive, as $F_2(m_1) < L_2(m_2)$, firm 2 has an incentive to preempt its rival by entering at $t_2^* = m_2$.[16] Consequently, there is a unique equilibrium with firm 2 entering at t_2^*.

Note that $t_1^* = m_1$ is equivalent to $A_1(t_1^*, T) = \emptyset$, while $t_1^* < m_1$ is equivalent to $A_1(t_1^*, T) \neq \emptyset$. If $A_1(t_1^*, T) = \emptyset$, truncating the game at t_1^* does not affect the set, nor firm 1's decision to enter at t_1^*. In other words, the binding condition that determines $A_1(0, T)$ is the *NPL*. On the other hand, if $A_1(t_1^*, T) \neq \emptyset$, truncating the set at t_1^* affects firm 1's entry decision. This is because the *NPF* is the binding condition for t_1^*. This means that if the game were to reach t_1^* without entry, firm 1 would opt to wait until m_1. Anticipating the lower payoff it would receive as a follower at m_1, firm 2 chooses to enter the market earlier as a leader at m_2.

[16] In Figure 2(b) $A_1(0, T) = (0, t_1^*]$, where $t_1^* < m_1$ as t_1^* is determined by historical maximum of L_2. On the other hand, $A_2(0, T) = (0, t_2^*]$, where $t_2^* = m_2$.

Figure 3: Non-monotonic payoffs

The equilibria in Proposition 2, illustrated in Figure 2, are all examples of *leader-maximized* equilibria. When there are two pure-strategy equilibria, as in Proposition 2(1), either firm enters at the time at which their payoff is maximized as a leader (depending on which one of them is the market pioneer). In Proposition 2(2), firm 2 enters at the time at which its leader payoff is maximized. This outcome can be thought of as a type of preemption equilibria – firm 2, anticipating a lower payoff as a follower at t_1^*, instead enters early to preempt firm 1.

Finally, note in a different context, **?** generate qualitatively equivalent equilibria. But the outcome could be different with more complex payoff structures. We turn our attention to this situation now.

Preemption equilibria

So far we have assumed that the follower payoff functions are non-increasing with the time of entry by the leader. Even in that case, a preemption equilibrium with one iteration is possible. Here we show that any number of these strategic leapfrogging iterations are possible in the general case.

To facilitate the following analysis, define the following critical time:

$$t^{**} = \max_{i,s \in S} \{t^*_{i,s} = m_{i,s}\}, \tag{11}$$

where S is defined in (10). In addition, let i^* be firm i for which $t^*_{i,s} = m_{i,s}$ is maximized. We derive the following proposition.

Proposition 3. *Consider the SPE of the two-player asymmetric timing game.*

1. *If $t_1^* = m_1$, there are two SPE, one with firm 1 entering at $t = t_1^*$ and the other with firm i^* entering at $t = t^{**}$;*

2. *If $t_1^* < m_1$, there is a unique SPE with firm i^* entering at $t = t^{**}$.*

Proof: See Appendix C.

To work through the intuition, begin with part (2) of the Proposition when $t_1^* < m_1$. An example of this situation is shown in Figure 3(a) and (b). If the game reaches t_1^* without entry, firm 1 will have an incentive to continue to wait until m_1. However, firm 2 prefers to come in as a leader early (at m_2), because its *NPF* condition does not hold beyond t_1^*. In other words, anticipating firm 1 would enter at m_1, firm 2 would preempt this by coming in early at m_2. This is the first preemptive iteration in this example. But this is not the equilibrium entry time in this case, due to the complex structure of payoffs. Rather, firm 1, anticipating its follower payoff at m_2 will assess its highest payoff from entry as a leader at any time between 0 and t_2^*. By definition, this time is $m_{1,2}$. Here, given that firm 1's leader payoff $L_1(m_{1,2}) > F(m_2)$, firm 1 would preempt firm 2's (m_2) entry by entering itself at $m_{1,2}$. This can be thought of as a second preemptive iteration. Finally, firm 2 uses the same reasoning, considering whether it is better off waiting to be the follower at $m_{1,2}$ or a leader earlier. In this case preemptive leadership is better, so we see firm 2 entering at $t_{2,2}^* = m_{2,2}$. This gives rise to three preemptive iterations, and firm 2 plays the role of the leader.

Note here that each firm is always better off being a follower at any point in time than being a leader. However, if a firm anticipates a lower return as a follower at its rival's entry time as leader in the future, a firm may well have an incentive to preempt. Following this logic, this preemptive entry may well itself be preempted by earlier entry from its rival. Moreover, the incentive to preempt one's rival in this case leads to the situation in which the leader's payoff is not maximized; we denote this situation as a *leader-constrained* equilibria. This is not possible with relatively simple payoff functions, for example with non-increasing follower payoffs as in Section 3.2 above, or with concave leader payoffs, as in the **?**. Moreover, this incentive to preempt can lead to lower total surplus; both the leader and the follower could be better off with later entry if they could commit to do so.

Now turn our attention to Proposition 3(1), illustrated in Figure 3(b) and (c). This has an additional layer of complexity as compared to the case discussed previously because there are two possible pure-strategy SPE. The first involves firm 1 acting as the leader entering at t_1^* with firm 2 playing the role of follower. The other equilibrium is similar to that in part (2), discussed above. Let us work through the intuition of the preemptive iterations again. If we reach t_2^* without entry there is an equilibrium with firm 2 entering at m_2. However, as in the example shown, firm 1 prefers to be a leader earlier at $m_{1,2}$ than a follower at m_2. Of course, firm 2 will consider its options. If, as shown, its follower payoff at $m_{1,2}$ is less than the payoff it can get from preempting, firm 2 will enter at $t^*_{2,2} = m_{2,2}$ as the leader. Note that the equilibrium with entry at t^*_1 is always *leader-maximized*, while the other equilibrium can be either *leader-maximized* or *leader-constrained*.

Strategies

In an attempt to minimize confusing notation, for the truncated entry game starting at some time t', there are associated points $t^*_{i,s}(t')$ and $m_{i,s}(t')$, as explicitly defined in Appendix B.

In part 1 of Proposition 3 there are two equilibria. First consider when firm 1 is the leader at t^*_1. The firms' strategies in the SPE are:

$$\sigma_1(t) = \begin{cases} 1 \quad \text{if} \quad [t = \max_{i,s \in S}\{t^*_{i,s}(t) = m_{i,s}(t)\}] \ \& \ [i^* = 1], \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise;} \end{cases} \tag{12}$$

$$\sigma_2(t) = \begin{cases} 1 \quad \text{if} \quad [t = \max_{i,s \in S}\{t^*_{i,s}(t) = m_{i,s}(t)\}] \ \& \ [i^* = 2], \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{13}$$

These strategies require that firm i^* enters when it has no further incentive to wait in the hope of a higher return later, taking into account the iterative process described earlier – that is $t = \max_{i,s \in S}\{t^*_{i,s}(t) = m_{i,s}(t)\}$ – and the other firm prefers to be a follower. Note that, for example, when both *leader-maximized* equilibria are present, this strategy profile selects the equilibrium in which firm 1 is the leader.

Now consider the firm strategies in the other type of equilibrium in Proposition 3(1), which can be either *leader-maximized* or *leader-constrained*:

$$\sigma_1(t) = \begin{cases} 1 \quad \text{if} \quad [t = \max_{i,s \in S \setminus \{i=1,s=1\}}\{t^*_{i,s}(t) = m_{i,s}(t)\}] \ \& \ [i^* = 1], \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise;} \end{cases} \tag{14}$$

$$\sigma_2(t) = \begin{cases} 1 \quad \text{if} \quad [t = \max_{i,s \in S \setminus \{i=1,s=1\}} \{t^*_{i,s}(t) = m_{i,s}(t)\}] \;\; \& \;\; [i^* = 2], \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{otherwise.} \end{cases} \quad (15)$$

The main difference from the previous strategies given by equations 12 and 13 is that the firms play the earlier *leader-maximized* equilibrium, rather than the one with later entry. This is captured in the strategies above by $i, s \in S \setminus \{i = 1, s = 1\}$, which means that firms are iterating backwards, adjusting their preemption strategies accordingly, from the possible entry time with $t = t^*_2$.

For Proposition 3(2) the strategy pairs given in either set of equations support the same entry time as $t^*_1 < m_1$. As a result, there is no *leader-maximized* equilibrium with entry at t^*_1.

Continuous version of the centipede game

Now we describe a specific example of Proposition 3(2) with several (many) preemptive iterations, leading to inefficient early entry. To do so, consider the following continuous (market-entry) version of the centipede game illustrated in Figure 4. Note that while there is a positive trend in the payoffs to each firm, they experience several peaks and troughs along the way. These mountainous looking payoffs could represent the seasonally changing payoffs associated with new versions of two smartphones as they become available. Important also in this example is that the local peak for one firm roughly corresponds to a time of a local trough for its rival. In the Figure, firm 1's potential payoffs are shown in panel (a) with the top line being its follower payoff F_1 and the bottom line its leader payoff L_1, whilst panel (b) shows the follower and leader payoffs for firm 2. Consider the equilibrium outcome of this centipede entry model. As with the previous equilibria, if the game reaches t^*_1 without entry, firm 1 would continue waiting until m_1, as its leader payoff is increasing and there is no credible threat of entry by firm 2. This is a situation that firm 2 wishes to avoid, so it would be willing to enter at m_2 to preempt firm 1's opportunistic actions after this time. But, given the non-monotonic nature of the payoffs, this entry time provides firm 1 with a lower follower payoff than it could have got as a leader previously. Moreover, entry by firm 2 at t^*_2 is not credible, because firm 2's leader payoff is increasing at this time. To gazump this, firm 1 will come in at $m_{1,2}$, if there had been no entry prior. This preemptive

Figure 4: Continuous version of the centipede game

process of iterating backwards moves through all of the peaks (and corresponding rival troughs) – before entry in equilibrium occurs immediately at $t^*_{1,6} = m_{1,6} = 0$. The intuition for this equilibrium is the same as above – to avoid a lower future payoff a firm will enter earlier, a move which itself induces even earlier entry by its rival, and so on. In this game, the payoffs to the two firms need not be equalized, as in the classic preemption equilibrium. Moreover, entry here is inefficient – both firms could be made better off if they could wait and enter later.

4 Concluding comments

The decision when to launch a new product or production process is a critical question for many firms; it can help determine profit, firm survival and the shape of markets. More generally, it drives economic development. Given its importance, innovation has received a great deal of attention from economists, such as in the seminal work of **?**. Getting the timing of innovation right is difficult enough for any firm, let alone when it has a rival breathing down its neck. In this paper we examine innovation as a market entry game with duopoly rivals in a very general framework; our main model only assumes that payoffs are continuous and that there is a second-mover advantage. Most importantly, inspired by **?**, we allow for asymmetries between firms. This incorporates the situation when firms have different capabilities or technologies. Moreover, allowing for non-monotonic leader payoffs captures situations where multiple technologies or products become available for adoption at different times.

The second-mover advantage equilibria we observe can be familiar, replicating those in the literature (**???**). We dub these equilibria *leader-maximized* equilibria, as the pioneer chooses a time of entry that maximizes its payoff, given that it has to be the first into the market. The more general payoffs in our game also allows for the possibility for different types of equilibria. For example, with non-monotonic payoffs there can be equilibria in which there are several iterations, with each successive preemption designed to avoid a lower payoff in the future. This incentive to preempt, even with the ever-present follower advantage, can lead to a *leader-constrained* equilibrium in which the leader enters at a time before its leader payoff function is maximized. This type of preemption equilibrium does not arise in the previous literature. This analysis suggests that the incentive to preempt does not rely on a first-mover advantage. Moreover, as our continuous version of the centipede (entry) game illustrates, the timing of entry can be inefficiently early.

Finally, our focus has been on entry (or innovation). Our analysis applies equally well to a stopping game considering the time to exit a market. It could be that each firm is better of remaining in the market at any point in time rather than exiting, but anticipating future outcomes a firm may opt to exit earlier, possibly inefficiently so.

Appendix A - Example of a game with a Second-Mover Advantage

To provide some further intuition for the main results in the paper, and to allow for a closer comparison with the previous literature, we construct the following modification of **?**. Essentially, we augment their example to allow 1) for more than one potential innovation that firms can put into practice; and 2) for the second-mover advantage. As noted in the introduction, firms are often faced with the choice between two or more alternative technologies. Examples of competing technologies for tablets, phone handsets and computer hardware come to mind, but a similar choice often has to be made when considering adopting cost-reducing technologies. Each technology will typically come with its own advantages and, moreover, the relative advantages of a given technology can change over time. Our framework is general enough to capture all of these scenarios.

Here, we completely characterize all SPE of this two-player innovation game using the algorithm outlined in this paper. By doing so, we show how an augmented example of **?** can provide micro-foundations for the *leader-constrained* equilibrium with multiple iterations, as illustrated in Figure 3 and discussed above.

Consider the case when two firms are contemplating when to upgrade to a new technology, which they can implement at some time $t_i \in [0, \infty)$ for $i = 1, 2$. Each firm can choose to implement one of the two options $k = 1, 2$ available. For each firm, the old (null) technology generates a flow of profit normalized to zero; that is, $\pi_{i0} = 0$, $i = 1, 2$. After adoption, the new technology k affords firm i a flow of profit $\pi_{ik} > 0$.

The payoffs are discounted by a common discount factor e^{-rt}, so that the net-present value of profits for the leader (firm i) entering at t_i with technology k is:

$$L_i(t_i, k) = \int_{t_i}^{\infty} e^{-rt}\pi_{ik}dt - K(t_i, k) = \frac{e^{-rt_i}}{r}\pi_{ik} - K(t_i, k). \tag{16}$$

Here, we use the exponentially declining development cost function, $K(t_i, k) = K_0 e^{-\lambda_k t_i} + K_{ik}$, with $\lambda_k > r$.

The payoff to firm i if firm j enters with technology k at t is:

$$F_i(t_i, k) = L_i(t_i, k) + \Delta, \tag{17}$$

where $\Delta > 0$. The assumption here is that the leader needs to incur some sunk costs to develop the market. Once these set-up costs have been incurred, the follower can enter, free riding on the leader's investment. Consequently, the difference between being a leader or a follower for each firm is equal to these set-up costs Δ.

As firm i maximizes its payoff, the net-present value of profits for the leader (firm i) entering at t_i with the best technology available is:

$$L_i(t_i) = \max_{k=1,2}\left[\frac{e^{-rt_i}}{r}\pi_{ik} - K(t_i, k)\right]; \tag{18}$$

which results in

$$F_i(t_i) = L_i(t_i) + \Delta. \tag{19}$$

The market demand in each period is 1 unit at a constant price of 1. We assume that firms share the market equally; the profits before and after entry are

$$\pi_{i0} = (1 - c_i^0)/2, \;\; \pi_{ik} = (1 - c_{ik}^i)/2,$$

where $c_{i0} = 1$ and $c_{ik} < 1$ are the costs corresponding to the old and new technology cases.

Several points are worth noting here in relation to this example and the analysis of the model in the paper. Firstly, the follower's payoff function $F_i(t)$ is always higher than the leader's payoff function $L_i(t)$ and the parameters are such that the payoffs are always positive. This means that Assumptions 5 and 6 are satisfied, and that we can apply the framework developed. Secondly, the curves – $L_1(t)$, $L_2(t)$, $F_1(t)$ and $F_2(t)$ – in Figure 3 are all derived using equations (18) - (19). In this way, we are able to construct an entry game with non-monotonic payoff functions and a second-mover advantage with only a slight augmentation to an established example in the literature. Moreover, we are able to use this example to illustrate our novel *leader-constrained* equilibrium (with multiple iterations) when there is a second-mover advantage.

Appendix B

Let us redefine points $t^*_{i,s}(t')$ and $m_{i,s}(t')$ for a truncated game starting from t'. For each firm $i \in \{1, 2\}$ define the following time:

$$t^*_i(t') = \begin{cases} \arg\max_t A_i(t', T) & \text{when} \quad A_i(t', T) \neq \emptyset, \\ t' & \text{when} \quad A_i(t', T) = \emptyset. \end{cases} \tag{20}$$

In addition, assume without loss of generality that $t^*_1(t') \geq t^*_2(t')$. Moreover, define recursively for $s \geq 2$ the following times $t^*_{1,s}(t')$ and $t^*_{2,s}(t')$:

$$t^*_{1,s}(t') = \begin{cases} \arg\max_t A_i(t', t^*_{2,s-1}) & \text{when} \quad A_i(t', t^*_{2,s-1}) \neq \emptyset, \\ t' & \text{when} \quad A_i(t', t^*_{2,s-1}) = \emptyset; \end{cases} \tag{21}$$

and

$$t^*_{2,s}(t') = \begin{cases} \arg\max_t A_i(t', t^*_{1,s}) & \text{when} \quad A_i(t', t^*_{1,s}) \neq \emptyset, \\ t' & \text{when} \quad A_i(t', t^*_{1,s}) = \emptyset. \end{cases} \tag{22}$$

with initial values $t^*_{1,1}(t') = t^*_1(t')$ and $t^*_{2,1}(t') = t^*_2(t')$.

Next, define recursively for $s \geq 2$ the following times $m_{1,s}(t')$ and $m_{2,s}(t')$:

$$m_{1,s}(t') = \min \arg \max_t L_1(t) \text{ for } t' \leq t \leq t^*_{2,s-1}(t'); \tag{23}$$

and

$$m_{2,s}(t') = \min \arg \max_t L_2(t) \text{ for } t' \leq t \leq t^*_{1,s}(t'). \tag{24}$$

with initial values $m_{1,1}(t') = m_1(t')$ and $m_{2,1}(t') = m_2(t')$.

Appendix C

Proof of Lemma 1

Compare $t^*_{i,s}$ and $m_{i,s}$. First, they both are defined on the same time range. For $i = 1$ the range is $[0, t^*_{2,s-1}]$, while for $i = 2$ the range is $[0, t^*_{1,s}]$. Second, $m_{i,s}$ is the time at which firm i's leader payoff is maximized. This means that, for firm i itself, the *NPL* condition is satisfied at $m_{i,s}$, but not beyond this time. Furthermore, $t^*_{i,s}$ is constructed to be the maximum time that satisfies both the *NPL* and *NPF* conditions. Consequently, $t^*_{i,s}$ cannot exceed $m_{i,s}$ as it belongs to a more restrictive set. Depending on whether the *NPF* is satisfied at $m_{i,s}$ will determine whether $t^*_{i,s}$ is equal to or less than $m_{i,s}$. $\square$

Proof of Proposition 2

This proof consists of four parts: (A), (B), (C) and (D). In part (A) we show that all SPE with positive entry times must belong to either $A_1(0, T)$ if firm 1 enters first or $A_2(0, T)$ if firm 2 enters first. In part (B) we prove that t^*_1 and t^*_2 given by (4) are well defined. Part (C) proves that if $A_1(t^*_1, T) = \emptyset$ then there are two SPE with firm 1 entering at $t = t^*_1$ and firm 2 entering at $t = t^*_2$. Finally, part (D) considers the scenario when $A_1(t^*_1, T) \neq \emptyset$. In this case there is a unique SPE with firm 2 entering at $t = t^*_2$.

(A) As a preliminary step, let us prove all SPE with positive entry times must belong to either $A_1(0, T)$ if firm 1 enters first or $A_2(0, T)$ if firm 2 enters first. Assume, on the contrary, that there is an SPE with a positive entry time

$t_i^* \notin A_i(0,T)$. It must be the case that both NPL and NPF conditions are satisfied. If condition NPL is not satisfied, the leader (player i) will have an incentive to enter earlier at τ. On the other hand, if condition NPF is not satisfied, the follower (player j) will have an incentive to preempt the leader (player i) and enter slightly earlier, as in **?**. Neither of these situations are possible in equilibrium. Consequently, there is a contradiction, proving the statement that all SPE with positive entry times must belong to either $A_1(0,T)$ if firm 1 enters first or $A_2(0,T)$ if firm 2 enters first.

(B) Next, let us prove that t_i^* for $i = 1, 2$ given by (4) is well defined. Specifically, there exists a unique t_i^* at which either $L_i(t)$ is maximized over $A_i(0,T)$ or $t_i^* = 0$ when $A_i(0,T) = \emptyset$. When $A_i(0,T) = \emptyset$, entering at $t_i^* > 0$ can not be an SPE; so the only potential entry time for player i is $t_i^* = 0$.

Now consider the situation when $A_i(0,T)$ is not empty. Let us prove the existence of the solution to this problem of maximizing $L_i(t)$ over $A_i(0,T)$ when $A_i(0,T) \neq \emptyset$. Note that set $A_i(0,T)$ is bounded because m_i is finite, where m_i is the time at which $L_i(t)$ reaches its global maximum (Assumption 4). We need to show that set $A_i(0,T)$ always contains its supremum. Assume that it does not. This means that there is a sequence $\{t_k\}$ contained in $A_i(0,T)$ that converges to some limit t_i^* that is not contained in set $A_i(0,T)$. This requires that either NPL or NPF is not satisfied for t_i^*. As sequence $\{t_k\}$ belongs to $A_i(0,T)$, it means that both NPL and NPF hold for sequence $\{t_k\}$. As both $L_i(t)$ and $F_j(t)$ are continuous functions, it means t_i^* also belongs to $A_i(0,T)$. This leads to a contradiction, proving existence.

The uniqueness follows immediately from the way set $A_i(0,T)$ is constructed. If two entry times were to maximize $L_i(t)$ over $A_i(0,T)$, then the later time would not belong to $A_i(0,T)$.

(C) Next, let us consider case (1) when $A_1(t_1^*, T) = \emptyset$. In this scenario there are two SPE with firm 1 entering at $t = t_1^*$ and firm 2 entering at $t = t_2^*$. Before proceeding, let us prove that $A_2(t_2^*, T) = \emptyset$. Note that two assumptions 1) non-increasing F_1 and 2) a smaller payoff to the leader (second-mover advantage) imply that the follower payoff $F_1(t_2^*)$ dominates any leader payoff $L_1(t)$ for $t \leq t_2^*$. This means the NPF is satisfied at and beyond t_2^* and this critical time is determined the by the NPL condition. Consequently, $A_2(t_2^*, T) = \emptyset$.

Now consider an SPE where firm i is the leader and firm j is the follower ($i, j = 1, 2$ and $i \neq j$). First, given $t_i^* \in A_i(0, T)$, if the follower deviates by entering at some time $\tau < t_i^*$, it will get a payoff of $L_j(\tau) < F_j(t_i^*)$. If it deviates by entering at t_i^*, it will get a payoff of $(L_j(t_i^*)+F_j(t_i^*))/2$, which is not greater than $F_j(t_i^*)$. If the follower enters at $\tau > t_i^*$, there will be no change to the equilibrium outcome. Consequently, there is no profitable deviation for the follower.

Second, in part (A) we proved that there is no equilibrium with the leader entering at $\tau > t_i^*$. Given $t_i^* \in A_i(0, T)$ and $A_i(t_i^*, T) = \emptyset$, if the leader deviates by entering earlier at some time $\tau < t_i^*$, it will get a payoff of $L_i(\tau) < L_i(t^*)$. There is no profitable deviation for the leader.

There is no other equilibria as entering at t_i^* dominates entering at any other time. Consequently, we have proved that there are two equilibria in case (1).

(D) Next, let us consider case (2) when $A_1(t_1^*, T) \neq \emptyset$. In this scenario relative to part (C), firm 1 entering at $t = t_1^*$ is not an equilibrium as at t_1^* firm 1 has an incentive to wait. As a result firm 2 will have incentive to preempt firm 1's entry. The other equilibrium is not affected as the same logics as in part (C) applies. Consequently, there exists only one SPE with firm 2 entering at $t = t_2^*$. This observation concludes the proof of this Proposition. $\square$

Proof of Proposition 3

Similar to the proof of Proposition 2, this proof consists of four parts: (A), (B), (C) and (D). We borrow part (A) from Proposition 2; that is, below we will rely on the result that all SPE with positive entry times must belong to either $A_1(0, T)$ if firm 1 enters first or $A_2(0, T)$ if firm 2 enters first. In part (B) we prove that $t_{1,s}^*$ and $t_{2,s}^*$ given by (5) and (6) are well defined. Part (C) proves that if $A_1(t_1^*, T) = \emptyset$ then there are two SPE with firm 1 entering at $t = t_1^*$ and firm i^* entering at $t = t^{**}$. Finally, part (D) considers the scenario when $A_1(t_1^*, T) \neq \emptyset$. In this case there is a unique SPE with firm i^* entering at $t = t^{**}$.

(B) Here we want to extend the part (B) proof from Proposition 2; that is, we need to prove that $t_{i,s}^*$ for $i = 1, 2$ and $s > 1$ given by (5) and (6) are well defined. Note that the only difference from earlier proof is that we construct sets $A_i(.)$, that are over shorter horizons. $A_i(0, T)$ is replaced with either $A_i(0, t_{2,s-1}^*)$

or $A_i(0, t^*_{2,s-1})$. The same logics directly applies.

(C) Next, let us consider case (1) when $A_1(t^*_1, T) = \emptyset$. In this scenario there are two SPE: one with firm 1 entering at $t = t^*_1$ and the other with firm i^* entering at $t = t^{**}$. Note that when $A_2(t^*_2, T) = \emptyset$, the part (C) proof from Proposition 2 directly applies. So we will only deal with the case when $A_2(t^*_2, T) \neq \emptyset$. Note also that even when $A_2(t^*_2, T) \neq \emptyset$, the equilibrium with firm 1 entering at $t = t^*_1$ is not affected by the differences between two propositions. Consequently, we only concentrate on the second equilibrium.

Consider an SPE where firm i^* is the leader and the other firm j is the follower. First, given $t^{**} \in A_{i^*}(0, T)$, if the follower deviates by entering at some time $\tau < t^{**}$, it will get a payoff of $L_j(\tau) < F_j(t^{**})$. If it deviates by entering at t^{**}, it will get a payoff of $(L_j(t^{**}) + F_j(t^{**}))/2$, which is not greater than $F_j(t^{**})$. If the follower enters at $\tau > t^{**}$, there will be no change to the equilibrium outcome. Consequently, there is no profitable deviation for the follower.

Second, given $t^{**} \in A_{i^*}(0, T)$ and $A_{i^*}(t^{**}, T) = \emptyset$, if the leader deviates by entering earlier at some time $\tau < t^{**}$, it will get a payoff of $L_{i^*}(\tau) < L_i(t^{**})$, which is not profitable. There is also no equilibrium with the leader entering at $\tau > t^{**}$. If the leader decides to wait at t^{**}, the follower will enter at its next convenient time. As a result there is no profitable deviation for the leader. Consequently, we have proved that there are two equilibria in case (1).

(D) Next, let us consider case (2) when $A_1(t^*_1, T) \neq \emptyset$. In this scenario relative to part (C), firm 1 entering at $t = t^*_1$ is not an equilibrium as at t^*_1 firm 1 has an incentive to wait. As a result firm 2 will have incentive to preempt firm 1's entry. The other equilibrium is not affected as the same logics as in part (C) applies. Consequently, there exists only one SPE with firm i^* entering at $t = t^{**}$. This observation concludes the proof of this Proposition. □

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