Strategic Agents and Vertical Relationship in Media Markets*

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Abstract

This paper introduces two modifications to standard models of two-sided media markets. In the first modification, we consider strategic agents by allowing advertisers to invest in the quality of their ads. This leads to qualitatively different econometric specifications to estimate group externality parameters. Moreover, relative to the case of passive agents, prices on both sides are lower, benefiting the agents at the cost of platforms. In the other modification, we introduce independent distributors between platforms and readers/viewers. We show that this modification has no impact on estimating the group externality parameters. However, equilibrium prices on either side depend on group externality parameters of both sides. In the special case where each platform is split into two independent divisions, in the equilibrium all 4 divisions charge a common price which depends on the product of the group externality parameters of the two sides.

Keywords: Two-sided media markets; Strategic agents; Ad quality; Vertical relationship.

JEL Classification Codes: D43, L13

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1 Introduction

Two-sided markets have attracted increasing attention among economists. In a two-sided market, platforms serve two (or multiple) sides, and the utility of an agent joining a platform depends on how many agents on the other side join the same platform. The most studied two-sided markets are probably media markets where the group externality is negative on one side but positive on the other side. For example, in the case of TV (magazine) markets, the two sides of agents are viewers (readers) and advertisers. For advertisers, the value of placing an ad increases when the TV program or magazine reaches more readers (viewers). In contrast, it is often assumed that ads lead to nuisance cost (in turn lower utility) for viewers (readers).

Earlier studies on two-sided markets have analyzed various pricing issues (fixed fee vs per-unit price, free vs. paid etc.) and market features (single-homing vs. multi-homing).[1] There are two common assumptions in these studies. First, all agents are *passive* in the sense that their only decision is which platform(s) to join (participation decision). Second, platforms usually serve the agents directly (no middleman).[2] Both assumptions may be violated in practice. In media markets, advertisers typically are not passive, and need to make strategic decisions such as ad quality. This is because, advertisers are interested in not only how many readers will see their ads but also how effective their ads will be in influencing sales. The success of ads depend heavily on "the actual content of your commercial, the production quality".[3] And production quality comes at a cost. According to Chron, production costs of a 30-second commercial in 2008 range in price from free, to $200-$1,500 produced by local television stations, to $342,000 on average produced by an advertising agency for national commercial.[4] Second, platforms may rely on distributors to reach agents, rather than serving them directly. For example, magazines may be sold to retailers first, who then resell the magazines to final readers. Similarly, ESPN sells its content to cable providers who as middlemen then sell to viewers.

In this paper, we first consider the case of strategic agents (e.g., advertisers who make strategic decisions such as ad quality). We identify two complications if this strategic agent feature is ignored. First, wrong estimates of group externality parameters will be obtained. Advertisers' optimal ad quality is endogenous, and depends on the anticipated market share of the platform on the reader side. Once this is taken into account, the econometric model will differ qualitatively from the one under passive agents. Therefore, the passive agents model will lead to model specification errors,

[1] See, for example, Caillaud and Jullien (2003), Armstrong(2006) and Choi (2006).

[2] Recently there have been studies exploring vertical structure in two-sided markets. See, for example, Kind et. al. (2016) and Gabrielsen et. al. (2018).

[3] "How Much Does TV Commercial Production Cost?", Kelly McCaughey, August 8, 2016, http://www.greyskyfilms.com/tv-commercial-production-cost/. Accessed on January 25, 2017.

[4] "How Much Does Television Advertising Really Cost?," by Nancy Wagner, http://smallbusiness.chron.com/much-television-advertising-really-cost-58718.html. Accessed on January 25, 2017.

and in turn wrong estimates.

Second, the equilibrium prices will differ. In particular, prices on both sides of the market will be upward biased so taking into account the strategic feature of advertisers lowers equilibrium prices on both sides. On the reader side, competition is now more intense – more readers affect advertiser utility not only directly, but also indirectly through its impact on ad quality. Price must be lower with more intense competition. On the advertiser side, endogenous ad quality essentially reduces the magnitude of group externality parameter on the advertiser side. Since this group externality parameter is negative, it raises equilibrium prices. Correspondingly, a reduction in the magnitude of group externality parameter leads to a smaller increase of price, relative to the case of passive agents.

We also analyze the case of vertical relationship where there are middlemen between platforms and agents on the reader side.[5] That is, platforms sell to (dedicated) retailers at wholesale prices, and retailers then sell to readers at retail prices. Our results show that ignoring this vertical structure has no impact on estimating the group externality parameters. In particular, even if one uses wholesale prices rather than retail prices on the reader side, one can still obtain the correct group externality parameters. However, equilibrium price on either side generally depends on the group externality parameters at both sides. This is in sharp contrast to findings in standard two-sided market models where equilibrium price on either side depends on the group externality on the other side only. Interestingly, when we consider the case where each platform is split into two independent divisions, we find that the two divisions would command the same equilibrium price, which involves the product of the group externality parameters of both sides.

To our knowledge, this is the first paper exploring the issue of strategic agents. Some existing studies have analyzed the role of quality in platform markets. For example, Greiner and Sahm (2018) considers content quality, while Hagiu (2011) allows user utility to depend on the average quality of users in the network. In their models quality is not a choice variable by the agents, who remain passive. Related to our second modification, there are more recent studies exploring vertical relationship in two-sided markets. For example, Kind et. al. (2016) introduces vertical relationship in one side of the market, with a single distributor serving both platforms. This is different from our setting with two (dedicated) distributors each serving one platform exclusively. Gabrielsen et. al. (2018) analyzes competing platforms' incentive to use resale price maintenance (RPM) in two types of vertical relationship: exclusive agents as in our model and a common agent as in Kind et. al. (2016). However, they model two-sided markets quite differently from Kind et. al. and our paper. In addition, their focus is the comparison of with vs. without RPM, with vertical relationship in both cases. In contrast, our focus in the second modification is to compare

[5]Many two-sided markets have middlemen. In our setting, readers may buy magazines from a retailer rather than the publisher (manufacturer) directly. In terms of credit cards, a consumer typically receives credit card from an issuer (e.g., banks, airlines and hotels etc.) rather than a credit card agency (e.g., Visa).

with vs. without vertical relationship.

Our paper is most closely related to the literature estimating two-sided market models. Most look at media markets where one of the two sides is advertisers. See, for example, Kaiser and Wright (2006) and Kaiser and Song (2009) for magazine market, Argentesi and Filistrucchi (2007) for newspaper market and Wilbur (2008) for TV advertising.[6] Kaiser and Wright use a model similar to Armstrong (2006) with two platforms and specific consumer distribution (Hotelling model). They solve the model, and also construct several instrument variables to deal with endogeneity issues. Our benchmark model largely follows that in Kaiser and Wright with some simplification (see Section 2 for more details). Our focus is on the two modifications of strategic agents and vertical relationship, and how they affect estimation and comparative statics. Neither of the aforementioned studies considers these modifications.

There are also various theory studies analyzing media markets. See, for example, Anderson and Coate (2005), Kind, Nilssen and Sorgard (2007) and Reisinger (2012). Similar to the empirical literature mentioned above, in these studies, agents only make platform participation decisions (passive agents) and platforms usually serve agents directly (no middleman). Advertisers in our model viewing the competing platforms as differentiated, but they all value consumers the same. In contrast, in Athey, Calvano and Gans (2013), advertisers have heterogeneous valuations for reaching consumers.[7]

The rest of the paper is organized as follows. We present and analyze the benchmark model in Section 2. Section 3 considers the case of strategic agents where agents on one side (advertisers) make investment decisions which directly affect the utility level of readers joining the same platform. In Section 4 we analyze a vertical structure where platforms sell to middlemen (e.g. retailers) instead of serving the agents directly. We conclude in Section 5. Proofs of lemmas and propositions can be found in the appendix.

2 The benchmark model

Our benchmark model follows Kaiser and Wright (2006) closely. There are two platforms (e.g., magazines) located at the two end points of a Hotelling line with platform 1 located at 0. Platforms serve two groups of agents: readers and advertisers. There is a continuum of each group of agents

[6]There are also earlier studies estimating networks effects, for example, Rysman (2004). There, advertisers choose the size of their ad. But this is more like the platform chooses multiple qualities (versions) or their product and let the advertisers self-select. This is qualitatively different from our strategic agent case where advertisers determine how much to invest in making their ad, choosing ad quality.

[7]This is also assumed in various other studies. See Anderson and Jullien (2016) for a survey of this literature. Note that while advertisers may have different valuations for reaching viewers, each viewer values all advertisers the same. In contrasts, in our model, a viewer values different ads differently, depending on the quality of these ads.

(with mass 1), uniformly distributed on the Hotelling line. Transport cost is linear in distance traveled and unit transport cost t is the same for the two groups of agents. Following Kaiser and Wright, the unit transport cost is assumed to be $t = \frac{1}{2}$. We also make several simplifying assumptions, leading to cleaner utility functions for the readers and advertisers, which we describe next.[8]

Consider a reader located at x. If she joins platform 1, she enjoys a utility of

$$u_1^r = \theta^r + \gamma N_1^a - p_1 - \frac{1}{2}x.$$

The superscripts 'r' and 'a' denote readers and advertisers respectively.

In the above expression, θ^r is the reservation value from consuming the content provided by the platform. p_1 is the price a reader has to pay to joining platform 1 (e.g., buying a copy of its magazine). N_1^a is the number of advertisers who join platform 1. γ measures the externality of ads on readers. We follow the common assumption $\gamma < 0$ in the literature on media markets. That is, going through ads is a nuisance cost which viewers/readers need to bear to access the content provided by the platforms.

If this reader joins platform 2 instead, her utility will be

$$u_2^r = \theta^r + \gamma N_2^a - p_2 - \frac{1}{2}(1 - x).$$

Similarly, an advertiser located at x will have the following utilities

$$u_1^a = \theta^a + \rho N_1^r - a_1 - \frac{1}{2}x,$$

$$u_2^a = \theta^a + \rho N_2^r - a_2 - \frac{1}{2}(1 - x),$$

where θ^a is the reservation utility and the group externality parameter is $\rho > 0$.

By looking at the u_i^j expressions, $i = 1, 2,\ j = a, r$, we can see that for agents on either side, only group externality parameter on the same side enters into the utility function, and enters in a linear fashion. For example, only ρ enters into the u_i^a expressions and does so linearly.

The stage game is as follows. In stage 1, platforms choose prices p_i and a_i simultaneously. Observing these prices, readers and advertisers choose which platform to join simultaneously in stage 2.

The marginal advertiser x^a, who is indifferent between joining either platform, can be derived as

$$u_1^a = u_2^a \Rightarrow x^a = \frac{1}{2} + \rho(N_1^r - N_2^r) - (a_1 - a_2). \tag{1}$$

[8] Our objective is to illustrate that, even in the simplest model, having strategic agents or vertical structure change the results qualitatively.

The number of advertisers joining either platforms is given by

$$N_1^a = x^a, \quad N_2^a = 1 - x^a.$$

Similarly we can obtain the marginal reader

$$x^r = \frac{1}{2} + \gamma(N_1^a - N_2^a) - (p_1 - p_2), \tag{2}$$

and

$$N_1^r = x^r, \quad N_2^r = 1 - x^r.$$

Estimating group externality parameters:

Let $n_1^a = \frac{N_1^a}{N_1^a + N_2^a}$ and $n_1^r = \frac{N_1^r}{N_1^r + N_2^r}$. Since $N_i^j + N_{-i}^j = 1$ (unit mass of agents on either side), we have

$$n_1^a = \frac{1}{2} + \rho(N_1^r - N_2^r) - (a_1 - a_2), \tag{3}$$

$$n_1^r = \frac{1}{2} + \gamma(N_1^a - N_2^a) - (p_1 - p_2). \tag{4}$$

With data on sales N_i^j, market share n_i^j and prices (p_i and a_i), these two equations can be used to estimate group externality parameters ρ and γ respectively.[9]

Equilibrium prices and comparative statics:

Platforms' profit maximization problems are,

$$\max_{p_i, a_i} \quad \pi_i = a_i N_i^a + p_i N_i^r, \quad i = 1, 2.$$

Solving the FOCs, we can obtain equilibrium prices and profit as the following:

$$p_i = -\rho + \frac{1}{2}, \quad a_i = -\gamma + \frac{1}{2}, \quad \pi_i = -\frac{1}{2}\gamma + \frac{1}{2} - \frac{1}{2}\rho. \tag{5}$$

Taking derivatives, we have

$$\frac{\partial p_i}{\partial \gamma} = 0, \quad \frac{\partial p_i}{\partial \rho} = -1; \quad \frac{\partial a_i}{\partial \gamma} = -1, \quad \frac{\partial a_i}{\partial \rho} = 0. \tag{6}$$

That is, own derivatives ($\frac{\partial p_i}{\partial \gamma}$ and $\frac{\partial a_i}{\partial \rho}$) are always zero and cross derivatives ($\frac{\partial p_i}{\partial \rho}$ and $\frac{\partial a_i}{\partial \gamma}$) are always -1 (similar to the results in Armstrong (2006)).

[9] Prices and sales on the right hand side are likely to be endogenous. Kaiser and Wright propose a series of instruments to solve this problem.

All these results are straight from Kaiser and Wright (2006). Next, we introduce two new modifications to the model, one at a time. First, we allow agents on one side to make a strategic choice, which affects their own utility as well as the utility of agents on the other side. Second, we allow a vertical structure on one side with an intermediary between the platform and the agents. For example the magazines may be sold to retailers first which then sell the magazines to readers.

3 Strategic agents

The benchmark model assumes that all agents are passive, i.e., they only determine which platform to join. What if some agents make strategic decision (beyond the simple participation decisions), which in turn affect the well being of agents on the other side and the platforms?

Consider advertisers for example. Suppose that they also decide what types of ads to show to readers on a platform, measured by ad quality $\kappa \geq 0$. It is intuitive that better quality ads, likely more entertaining and/or less annoying, would benefit the readers. We assume that better quality ads benefit the advertisers as well, because readers are likely to pay more attention to these ads, making them more effective. Investment on ad quality is costly, entailing a fixed investment cost of $c(\kappa)$, where $c'(\kappa) > 0$ and $c''(\kappa) > 0$. For tractability, we assume that ad cost takes the form $c(\kappa) = \frac{1}{2}\kappa^2$. With strategic agents, the stage game changes slightly. Stage 1 is the same as in the benchmark model where the platforms choose prices p_i and a_i. In stage 2, in addition to platform participation decisions by the advertisers and readers, advertisers also make ad quality decisions simultaneously and independently.

Consider an arbitrary advertiser $x \in [0, 1]$. Its utility from joining platform 1 is

$$u_1^a = \theta^a + f(\rho, \kappa, N_1^r) - a_1 - \frac{1}{2}x - \frac{\kappa_1^2}{2},$$

where $f(\rho, \kappa, N_1^r)$ increases with ρ, κ and N_1^r. For simplicity, we assume that

$$f(\rho, \kappa, N_1^r) = \rho(1 + \kappa)N_1^r.$$

Note that if the advertiser does not invest in ad quality ($\kappa = 0$), then we recover the utility in the benchmark model. The advertiser's utility of joining either platform is,

$$\begin{aligned} u_1^a &= \theta^a + \rho(1 + \kappa)N_1^r - a_1 - \frac{1}{2}x - \frac{\kappa_1^2}{2}, \\ u_2^a &= \theta^a + \rho(1 + \kappa)N_2^r - a_2 - \frac{1}{2}(1 - x) - \frac{\kappa_2^2}{2}. \end{aligned} \quad (7)$$

The advertiser chooses κ optimally to maximize u_i^a. Solving $\frac{\partial u_i^a}{\partial \kappa} = 0$, we obtain the results in the following lemma.

Lemma 1 *Optimal ad quality is given by* $\kappa_i^* = \rho N_i^r$ *where* N_i^r *is platform* $i = 1, 2$*'s expected market share on the reader side.*

We can see that optimal ad quality depends on which platform the advertiser joins (i.e., N_i^r), but is independent of the exact advertiser location x. As a result, all advertisers joining platform i will choose the same ad quality κ_i, even though they make ad quality decisions independently. Consequently, the average ad quality is also κ_i for platform i.

3.1 Estimating group externality parameters

Having derived advertiser's optimal investment decisions, next we investigate how such investment decisions affect the estimation of group externality parameters.

Substituting $\kappa_i^* = \rho N_i^r$ into advertiser's utility functions, we can obtain

$$\begin{aligned} u_1^a &= \theta^a + \rho N_1^r + (\rho N_1^r)^2 - a_1 - \frac{1}{2}x - \frac{1}{2}(\rho N_1^r)^2 \\ &= \theta^a + \rho N_1^r + \frac{\rho^2}{2}(N_1^r)^2 - a_1 - \frac{1}{2}x; \\ u_2^a &= \theta^a + \rho N_2^r + \frac{\rho^2}{2}(N_2^r)^2 - a_2 - \frac{1}{2}(1-x). \end{aligned} \tag{8}$$

On the advertiser side, it remains that only the own group externality parameter ρ enters into the utility function, except that it now enters in a nonlinear fashion.

Similar to the benchmark model, we derive the marginal advertiser x^a and the number of advertisers joining either platform N_i^a. We can then obtain[10]

$$n_1^a = \frac{1}{2} + \rho(N_1^r - N_2^r) + \underbrace{\frac{\rho^2}{2}\left[(N_1^r)^2 - (N_2^r)^2\right]}_{\text{additional term}} - (a_1 - a_2), \tag{9}$$

where $n_1^a = \frac{N_1^a}{N_1^a + N_2^a}$.

Different from equation (4) in the benchmark model, n_1^a is now linear-quadratic (rather than linear) in N_i^r. This can also be seen with general $f(\rho, \kappa_i, N_i^r)$ and $c(\kappa)$. κ^* in general will be a function of N_i^r. Once the κ_i^* value is substituted, n_i^a will not be linear in N_i^r anymore. In addition, ρ appears twice in equation (9), including in the form of $\frac{\rho^2}{2}$.

[10] For the econometric model in equation (9) to be defined, there needs to be variation across the platforms, in particular, $N_1^r \neq N_2^r$. In contrast, in the theory model, with platform symmetry (no disturbance), the equilibrium always has $N_1^r = N_2^r = \frac{1}{2}$ and $N_1^a = N_2^a = \frac{1}{2}$.

Next, we move on to the reader side which remains passive. Consider an arbitrary reader $x \in [0,1]$. His/her utility from joining platform $i = 1,2$, taking into account ad quality, is given by

$$u_i^r = \theta^r + g(\gamma, \bar{\kappa}_i, N_i^a) - p_i - \frac{1}{2}x,$$

where $\bar{\kappa}_i$ is the average ad quality on platform i, and $g(\gamma, \bar{\kappa}_i, N_i^a)$ increases with γ and $\bar{\kappa}_i$ but decreases with N_i^a.

For simplicity, assume that

$$g(\gamma, \bar{\kappa}_i, N_i^a) = \gamma(1 - \bar{\kappa}_i)N_i^a, \quad i = 1,2.$$

Because all advertisers on the same platform make the same investment, we have $\bar{\kappa}_i = \kappa_i^*$. Reader utility from joining the platforms are given by,

$$\begin{aligned} u_1^r &= \theta^r + \gamma(1 - \kappa_1^*)N_1^a - p_1 - \frac{1}{2}x, \\ &= \theta^r + \gamma(1 - \rho N_1^r)N_1^a - p_1 - \frac{1}{2}x; \\ u_2^r &= \theta^r + \gamma(1 - \rho N_2^r)N_2^a - p_2 - \frac{1}{2}(1 - x). \end{aligned} \tag{10}$$

From equation (10), we can see that on the reader side, now group externality parameters of both sides (ρ and γ) enter into their utility functions. In particular, it contains the same linear term γN_i^a, but also an interaction term $\rho\gamma N_i^r N_i^a$.

Using the new u_i^r expressions, we can derive the marginal reader x^r and N_i^r, which then lead to

$$\begin{aligned} n_1^r &= \frac{1}{2} + \gamma[(1 - \kappa_1^*)N_1^a - (1 - \kappa_2^*)N_2^a)] - (p_1 - p_2) \\ &= \frac{1}{2} + \gamma(N_1^a - N_2^a) - \underbrace{\gamma \cdot \rho \cdot (N_1^r \cdot N_1^a - N_2^r \cdot N_2^a)}_{\text{additional term}} - (p_1 - p_2). \end{aligned} \tag{11}$$

We can easily see that it differs from (3) in the benchmark model. Combining results for both the advertiser and reader sides, we have the following proposition.

Proposition 1 *(Estimating group externality parameters) Ignoring advertisers' strategic investment decisions will lead to wrong econometric models for estimating the group externality parameters on both the advertiser and reader sides.*

If the underlying model features strategic agents yet it is not modeled, then the econometric models will be misspecified, leading to incorrect estimates.

3.2 Equilibrium prices and comparative statics

We solve platforms' FOCs and obtain equilibrium prices and profits, as given in the next Proposition.

Proposition 2 *(Strategic agents) When advertisers make strategic investment in ad qualities, the unique SPNE is characterized by*

$$p_i = \frac{1}{2} - \rho + \frac{1}{2}\rho(\gamma - \rho), \quad a_i = \frac{1}{2} - \gamma + \frac{1}{2}\rho\gamma,$$

$$\pi_i = \frac{1}{2}\gamma\rho - \frac{1}{2}\gamma + \frac{1}{2} - \frac{1}{2}\rho - \frac{1}{4}\rho^2, \quad i = 1, 2.$$

Proof. See the Appendix. ■

Using the equilibrium price expressions, one can easily verify that

$$\begin{aligned} \text{Own derivatives} \quad &: \quad \frac{\partial p_i}{\partial \gamma} = \frac{\rho}{2} > 0, \quad \frac{\partial a_i}{\partial \rho} = \frac{\gamma}{2} < 0; \\ \text{Cross derivatives} \quad &: \quad \frac{\partial p_i}{\partial \rho} = -\rho + \frac{\gamma}{2} - 1 < -1, \quad \frac{\partial a_i}{\partial \gamma} = \frac{\rho}{2} - 1 \in (-1, 0), \end{aligned}$$

which lead to the following Corollary.

Corollary 1 *(Strategic agents) Different from the benchmark case, with strategic agents, own derivatives ($\frac{\partial p_i}{\partial \gamma}$ and $\frac{\partial a_i}{\partial \rho}$) are not zero and the cross derivatives ($\frac{\partial p_i}{\partial \rho}$ and $\frac{\partial a_i}{\partial \gamma}$) are not -1.*

In standard two-sided markets, own partial derivatives are always zero and cross partial derivatives are always -1. This is not true anymore under strategic agents. Why? The answer lies in the different forms of utility functions. First, note that ρ enters into u_i^a expressions nonlinearly. Second, both γ and ρ enters into the u_i^r expressions.[11]

We can also explore welfare impacts of strategic agents. The results are summarized in the next Proposition.

Proposition 3 *Relative to the benchmark case of passive agents, with strategic agents,*

(i) Equilibrium prices are lower on both sides.

(ii) Platforms are worse off while advertisers and readers are better off.

[11] To see this, we consider a hypothetical situation where advertiser's ad quality affects their own utilities but not readers' utilities. It can be shown that equilibrium prices are then $p_1 = -1/2\rho^2 - \rho + 1/2$ and $a_1 = 1/2 - \gamma$. Note that own derivatives are still zero, since only group externality parameter of the same side enters into an agent's utility function. However, ρ now enters into p_1 nonlinearly, because ρ enters into u_i^a expressions nonlinearly.

Proof. See the Appendix. ■

(i) may seem a bit counterintuitive, since investment in ad quality helps the advertisers and readers on the platform, which in turn should help that platform. Of course, the problem is that advertisers on both platforms are strategic agents (i.e., prisoners' dilemma). Let us see why. When platforms choose prices on the reader side, there is more to lose when advertisers are strategic – ρ affects u_i^a not only directly, but also indirectly by affecting advertiser investment. This gives each platform more incentive to attract readers which intensifies competition at the reader side and leads to lower prices. Prices on the advertisers side are determined by group externality at the reader side. We can see it from reader's utility function,

$$\begin{aligned} u_1^r &= \theta^r + \gamma(1-\kappa_1)N_1^a - p_1 - \frac{1}{2}x \\ &= \theta^r + \tilde{\gamma}N_1^a - p_1 - \frac{1}{2}x, \end{aligned}$$

where $\tilde{\gamma} \equiv (1-\kappa_1^*)\gamma$. It look just like the u_1^r except γ is replaced by $\tilde{\gamma}$. Correspondingly, on the advertiser side, platforms will raise prices by $|\tilde{\gamma}|$. Prices on the advertiser side are also lower than those in the benchmark model since

$$|\tilde{\gamma}| = |(1-\kappa_1^*)\gamma| = \left|\left(1-\rho\cdot\frac{1}{2}\right)\gamma\right| < |\gamma|.$$

With lower prices on both sides, platforms must be worse off. However, advertisers and readers are better off because they both enjoy lower prices. Additionally, advertisers can mimic the benchmark case by choosing zero investment. The fact that they choose positive investment must make them strictly better off. For readers, investment by advertisers directly raises their utilities, everything else the same. Combined with lower prices, readers must be better off as well.

To summarize, if the strategic agent aspect is ignored, we find that: (i) Estimates of the group externality parameters will be biased; (ii) Equilibrium prices are upward biased, and their derivatives with respect to group externality parameters differ; (iii) Platform profits are over-reported while advertiser and reader surplus are under reported.

4 Vertical relationship

In the previous section, we have explored the modification of strategic agents. In this section, we go back to passive agents, and introduce an extra layer on the reader side of the market. We assume that platforms do not directly sell to readers, but rather through independent retailers. In particular, there are two (dedicated) retailers, with retailer $i = 1, 2$ serving platform $i = 1, 2$ only.[12]

[12]There are alternative ways to model the retail sector. For example, there may be a single retailer serving both platforms. Or there are may be two undedicated retailers, each serving both platforms. The former case introduce

The stage game is as follows. In stage 1, platforms choose wholesale prices (w_i) simultaneously. In stage 2, retail prices on both sides (p_i and a_i) are chosen simultaneously. In stage 3, readers and advertisers decide what platforms to join.

The advertiser and reader's utilities from joining either platform are the same as in the benchmark model,

$$u_i^a = \theta^a + \rho N_i^r - a_i - t_i(x), \quad u_i^r = \theta^r + \gamma N_i^a - p_i - t_i(x), \quad i = 1, 2,$$

where $t_1(x) = \frac{1}{2}x$ and $t_2(x) = \frac{1}{2}(1-x)$. Note that retail prices are now chosen by retailers, not platforms.

Let w_i denote platform i's wholesale price charged to retailer i. Retailer i's profit maximization problem is

$$\max_{p_i} \quad \pi_i^R = (p_i - w_i)N_i^r, \quad i = 1, 2.$$

Platform i's profit is

$$\pi_i^M = w_i N_i^r + a_i N_i^a, \quad i = 1, 2.$$

4.1 Estimating group externality parameters

First note that the agents' utility functions are the same as in the benchmark model case, we still have

$$n_1^a = \frac{1}{2} + \rho(N_1^r - N_2^r) - (a_1 - a_2),$$
$$n_1^r = \frac{1}{2} + \gamma(N_1^a - N_2^a) - (p_1 - p_2).$$

If one uses final retail prices (p_i and a_i), having a vertical structure would have no impact on the correct estimation of the group externality parameters γ and ρ.

We solve the game backwards, starting with stage 2 where platforms and retailers maximize their respective profits given w_i:

$$\max_{p_i} \ \pi_i^R, \quad \max_{a_i} \ \pi_i^M.$$

Solving the retailers' FOC, we can obtain $p_i(w_1, w_2)$. They lead to[13]

$$p_1 - p_2 = \frac{(8\gamma^2 + 3)(w_1 - w_2)}{9 - 16\gamma\rho}. \tag{12}$$

double marginalization, but also the problem of sharing a single downstream distribution. In the latter case, one will need to introduce retailer differentiation on top of platform differentiation. Our specification of two dedicated retailers introduces double marginalization only.

[13] More details are provided in Proof of Proposition 4.

What if one uses wholesale prices (w_i) instead of the retail prices (p_i) on the reader side to estimate γ? To see this, we substitute the p_i expressions as functions of w_1 and w_2, into the n_1^r expression above. We can obtain

$$\begin{aligned} n_1^r &= \frac{1}{2} + \gamma(N_1^a - N_2^a) - (p_1 - p_2). \\ &= \frac{1}{2} + \gamma(N_1^a - N_2^a) - \frac{(8\gamma^2 + 3)(w_1 - w_2)}{9 - 16\gamma\rho}. \end{aligned}$$

If one ignores the endogeneity of wholesale prices, then group externality parameters can still be correctly estimated even with wholesale prices.

4.2 Equilibrium prices and comparative statics

In the previous section, we have solved for the optimal $p_i(w_1, w_2)$ from retailers' FOCs. From platforms' FOC, we can obtain $a_i(w_1, w_2)$. Substituting the optimal $p_i(w_1, w_2)$ and $a_i(w_1, w_2)$ expressions into π_i^M, we can solve for the optimal wholesale prices w_i^* and in turn p_i^* and a_i^*. The results are presented in the next Proposition.

Proposition 4 *(Vertical structure) With retailers on the readers side, equilibrium prices are given by:*

$$w_i = \frac{9 - 4\rho - 6\gamma - 16\rho\gamma}{16\gamma^2 + 6}, \tag{13}$$

$$p_i = w_i + \left(\frac{1}{2} - 2\rho\gamma\right), \tag{14}$$

$$a_i = -2\gamma w_1 + \left(\frac{1}{2} - 2\rho\gamma\right). \tag{15}$$

Proof. See the Appendix. ■

In the equilibrium, platforms will choose $w_i > 0$. However, it is interesting to investigate the case where $w_i = 0$ is imposed. Substituting $w_i = 0$ into equations (14)-(15), we can obtain $p_i = a_i = \frac{1}{2} - 2 \cdot \rho \cdot \gamma$. We can see that there is a common price charged by the two platforms on the two sides of the market. With $w_i = 0$, the platforms make profits from the advertiser side only, while retailers profit from the reader side only. Platforms and retailers maximize their own profits, and do not take into account theirs impacts on the other side. This is the same as the case where a platform splits itself into two independent divisions (presented in the next Corollary), one for each side, and each division maximizes its own profit.

Corollary 2 *(Independent division) If each platform is split into two independent divisions, one for each side, then in the unique equilibrium, prices are given by*

$$p_i = a_i = \frac{1}{2} - 2\rho\gamma.$$

In standard two-sided market models, prices on either side depend on the group externality parameter of the other side, not its own side. As a result, prices in the two sides differ from each other ($p_i \neq a_i$). In contrast, under independent division, prices on the two sides are the same and depend on group externalities of both sides. Let us see why. Let π_r^1 and π_a^1 denote the profit of platform 1's reader and advertiser division respectively,

$$\pi_1^r = p_1 n^r, \quad \pi_1^a = a_1 n^a.$$

Their respectively FOCs are

$$\frac{\partial \pi_1^r}{\partial p_1} = n^r + p_1 \frac{\partial n^r}{\partial p_1} = 0, \quad \frac{\partial \pi_1^a}{\partial a_1} = n^a + a_1 \frac{\partial n^a}{\partial a_1} = 0.$$

It can be easily verified that

$$\frac{\partial n^r}{\partial p_1} = \frac{\partial n^a}{\partial a_1} = -\frac{1}{1 - 4\gamma\rho}.$$

In the equilibrium, $n^r = n^a = \frac{1}{2}$. It must be that

$$p_1 = a_1 = -\frac{1}{2 \cdot \frac{\partial n^r}{\partial p_1}} = \frac{1}{2} - 2\rho\gamma.$$

In standard two-sided markets, platform 1 maximizes its joint profit from the two sides,

$$\pi_1 = p_1 n^r + a_1 n^a.$$

Profit maximization requires

$$\frac{\partial \pi_1}{\partial p_1} = n^r + p_1 \frac{\partial n^r}{\partial p_1} + \underbrace{a_1 \frac{\partial n^a}{\partial p_1}}_{\text{cross impact}} = 0,$$

$$\frac{\partial \pi_1}{\partial a_1} = n^a + a_1 \frac{\partial n^a}{\partial a_1} + \underbrace{p_1 \frac{\partial n^r}{\partial a_1}}_{\text{cross impact}} = 0.$$

The platform internalizes the impact of price change in one side on the profit from the other side (cross impacts), so the cross derivatives enter into FOCs. It can be easily verified that the two cross derivatives are unequal,

$$\frac{\partial n^r}{\partial a_1} = -\frac{2\gamma}{1 - 4\gamma\rho} \neq \frac{\partial n^a}{\partial p_1} = -\frac{2\rho}{1 - 4\gamma\rho}.$$

This leads to different prices at the two sides ($p_i \neq a_i$).[14]

Using the equilibrium prices in equations (14) and (15), one can easily verify the results in the next Corollary.

Corollary 3 *(Vertical relationship) Different from the benchmark case, own derivatives ($\frac{\partial p_i}{\partial \gamma}$ and $\frac{\partial a_i}{\partial \rho}$) are not zero, and the cross derivative $\frac{\partial p_i}{\partial \rho}$ in general are not -1.*

Even with $w_i = 0$, final prices on either side depend on the product of the two group externality parameters. On top of that, optimal w_i is a function both group externality parameters and affects the prices on the reader side further. Together they are responsible for the results in Corollary 3.

5 Conclusion

This paper considers, one at a time, two modifications to the standard two-sided market models. In the first modification, agents one side of the market (advertisers) make strategic choices on the ad quality, which affects their own utility as well as utilities of the readers joining the same platform. We find that having strategic agents leads to qualitative different demand systems for the estimation of group externality parameters. As a result, if the strategic agent feature is not properly accounted for, one would obtain wrong estimates of the group externality parameters. We also solve for equilibrium prices and find that under strategic agents, equilibrium prices on both sides are lower than when agents are passive, and they depend on group externality parameters of both sides in general.

In the second modification, we introduce independent retailers between platforms and readers. We find that this modification has no impact on estimating group externality parameters. However, the equilibrium prices in general depend on group externality parameters at both sides of the market. One particularly interesting finding is for the case where each platform is split into two independent divisions. We recover a common equilibrium price, charged by all divisions. This common price depends on the product of the group externality parameters at the two sides. This is in sharp contrast to the standard two-sided models (e.g., Armstrong (2006)) where prices differ across the two sides, since price on each side depends only on the group externality parameter of the other side.

While this study focuses on the estimation of group externality parameters and equilibrium pricing, future studies can investigate the impact of these modification on other issues such as single-homing vs. multi-homing, price discrimination, merger impacts. Various studies have shown

[14] The cross partial derivatives must offset the own derivatives in a way so that the eventual equilibrium price on either side does not depend on the group externality of that side.

that policies that work well in one-sided markets may not have the desirable effects when applied to two-sided markets. The two modifications considered in this paper will add extra complexity.

Appendix

Proof Proposition 2

Since we have a continuum of agents of mass 1 on either side, we have $N_i^j = n_i^j$ and $n_1^j = x^j$, $i = 1, 2$, $j = a, r$. Then equations (9) and (11) also provide the expressions for marginal advertiser and marginal reader. We have

$$x^a = \frac{1}{2} + \rho(N_1^r - N_2^r) + \frac{\rho^2}{2}\left[(N_1^r)^2 - (N_2^r)^2\right] - (a_1 - a_2),$$

$$x^r = \frac{1}{2} + \gamma(N_1^a - N_2^a) - \gamma \cdot \rho \cdot (N_1^r \cdot N_1^a - N_2^r \cdot N_2^a) - (p_1 - p_2).$$

Substituting the x^j expressions into the following

$$N_1^j = x^j, \quad N_2^j = 1 - x^j, \quad j = a, r,$$

and solve for N_i^j, we can obtain

$$N_1^a = \frac{1}{2} - \frac{\gamma(\rho+2)(a_1 - a_2) + \rho(\rho+2)(p_1 - p_2)}{\gamma\rho^3 - 3\gamma\rho + 1},$$

$$N_1^r = \frac{1}{2} + \frac{\gamma(\rho-2)(a_1 - a_2) - (p_1 - p_2)}{\gamma\rho^3 - 3\gamma\rho + 1}.$$

Platform i's problem is :

$$\max_{p_i, a_i} \quad \pi_i = p_i \cdot N_i^r + a_i \cdot N_i^a.$$

Solving firms' FOCs, the equilibrium prices are

$$p_i = \frac{1}{2} - \rho + \frac{1}{2}\rho(\gamma - \rho), \quad a_i = \frac{1}{2} - \gamma + \frac{1}{2}\rho\gamma,$$

and each platform earns a profit of

$$\pi_i = \frac{1}{2}\gamma\rho - \frac{1}{2}\gamma + \frac{1}{2} - \frac{1}{2}\rho - \frac{1}{4}\rho^2 \quad i = 1, 2.$$

■

Proof of Proposition 3

(i) Equilibrium prices at the benchmark model (with superscript 'b') and strategic agents model (with superscript 's') are,

$$p_i^b = \frac{1}{2} - \rho, \quad a_i^b = \frac{1}{2} - \gamma,$$

$$p_i^s = \frac{1}{2} - \rho + \frac{1}{2}\rho(\gamma - \rho), \quad a_i^s = \frac{1}{2} - \gamma + \frac{1}{2}\rho\gamma.$$

The difference $p_i^b - p_i^s = -\frac{1}{2}\rho(\gamma - \rho) > 0$ since γ is negative and ρ is positive. Similarly, $a_i^b - a_i^s = -\frac{1}{2}\rho\gamma > 0$. Combined, equilibrium prices on both sides are lower under strategic agents relative to the benchmark case.

(ii) With lower prices on both sides, advertisers and readers must be better off at the cost of platforms. ■

Proof Proposition 4

There is no change on the final agents relative to the benchmark case. Therefore, the agents' demand functions are the same,

$$n_1^r = \frac{1}{2} + \frac{2\gamma(a_1 - a_2) + (p_1 - p_2)}{4\gamma\rho - 1}$$

$$n_1^a = \frac{1}{2} + \frac{2\rho(p_1 - p_2) + (a_1 - a_2)}{4\gamma\rho - 1}$$

Let π_i^{ret} denote the profit of retailer $i = 1, 2$, and let π_i^{man} denote platform (manufacturer) i's profit. The profit maximization problems are

$$\max_{p_1} \quad \pi_1^{ret} = (p_1 - w_1) \cdot n_1^r, \quad \max_{p_2} \quad \pi_2^{ret} = (p_2 - w_2) \cdot (1 - n_1^r),$$

$$\max_{a_1} \quad \pi_1^{man} = w_1 \cdot n_1^r + a_1 \cdot n_1^a, \quad \max_{a_2} \quad \pi_2^{man} = w_2 \cdot (1 - n_1^r) + a_2 \cdot (1 - n_1^a).$$

Solving the FOCs, we can obtain the optimal retail prices as functions of w_1 and w_2.[15] We also verify that

$$p_1 - p_2 = -\frac{(8\gamma^2 + 3)(w_1 - w_2)}{16\gamma\rho - 9}.$$

Next, we substitute these retail prices into platforms' profit maximization problems,

$$\max_{w_1} \quad \pi_1^{man} = w_1 \cdot n^r + a_1 \cdot n^a,$$

$$\max_{w_2} \quad \pi_2^{man} = w_2 \cdot (1 - n^r) + a_2 \cdot (1 - n^a).$$

[15] They are lengthy and skipped. A maple file containing the results is available upon request.

Taking derivatives and then imposing symmetry ($w_1 = w_2$), we can obtain

$$w_1 = w_2 = \frac{9 - 4\rho - 6\gamma - 16\rho\gamma}{16\gamma^2 + 6}.$$

Substituting the wholesale prices into retail prices, we have

$$p_i = \frac{9 - 4\rho - 6\gamma - 16\rho\gamma}{16\gamma^2 + 6} + \left(\frac{1}{2} - 2 \cdot \rho \cdot \gamma\right) = w_i + \left(\frac{1}{2} - 2 \cdot \rho \cdot \gamma\right),$$

$$a_i = -2 \cdot \gamma \cdot \frac{9 - 4\rho - 6\gamma - 16\rho\gamma}{16\gamma^2 + 6} + \left(\frac{1}{2} - 2 \cdot \rho \cdot \gamma\right) = -2 \cdot \gamma \cdot w_i + \left(\frac{1}{2} - 2 \cdot \rho \cdot \gamma\right).$$

■

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