Inference in strategic-interaction models with censored action spaces

Abstract

While econometric methods have been developed to perform inference in models that is robust to phenomena such as the existence of multiple solutions, there exists a glaring gap in the literature: the possibility that players' underlying action space is censored (truncated), so that agents are unable to select their *interior* optimal action. This could happen, for example, when firms have capacity constraints, or when economic agents have financial constraints, or when there exist frictions that prevent them to adjust their choices to their optimal levels in the short-run. While censoring and truncation phenomena have received vast attention in single-agent microeconometric models, it has received no attention in strategic-interaction econometric models. This paper aims to fill in this gap by introducing the possibility of censoring in action spaces in a very general way. I focus on static games first. Testable implications in these models involve inequalities of conditional moments, that can be analyzed with existing methods.

1 Introduction

The paper concentrates on econometric inference in static games. This literature is vast and a partial list includes Bjorn and Vuong (1984), Bresnahan and Reiss (1990, 1991a, 1991b), Berry (1992), Tamer (2003), Bajari, Hong, and Ryan (2010), Ciliberto and Tamer (2009), Aradillas-López (2011), Kline (2015), Kline (2016), citeNBajari/Hong/Krainer/Nekipelov:09, Aradillas-López (2010), de Paula and Tang (2012), Xu and Wan (2014), Xu (2014), Lewbel and Tang (2015), Liu, Xu, and Vuong (2017) and Xiao (2018). While these papers have made tremendous strides to handle issues such as multiple equilibria and different forms of behavior, so far the literature has ignored the possibility of censoring in strategy spaces, reflecting phenomena such as capacity constraints or frictions that prevent players to adjust their actions in the short-run. This paper is motivated by the widespread prevalence of such constraints in the real world and by the resulting need to have inference methods that can be robust to these phenomena. Simply put, ignoring the possibility of censoring results in incorrect estimation and counterfactual results. And while this has long been recognized and incorporated into single-agent econometric models, this has not been the case in the econometrics of games. This paper aims to fill in this gap and develop inferential methods that are robust to the possible presence of censoring.

2 The implications of censoring in agents' action spaces: a two-player example

I illustrate the issues that arise with censoring in these models through a simple two-player example. This highlights how drastically the testable implications of the model can change, as well as the risks of ignoring this issue. From there, I describe testable implications of the model in this new richer environment as well as an inferential strategy based on these implications.

2.1 Description of the model

Consider a simultaneous game played between two players, which could represent for example two competing firms who have to choose an action, labeled as y_1 and y_2 respectively (subscript p refers to player $p \in \{1, 2\}$). Depending on the specific application, y_p could refer to actions such as: price, advertising expenditure, production level, investment, etc. For illustration purposes, suppose actions are continuous (e.g, perfectly divisible production or some financial decision such as price). My approach to censoring can be readily extended to discrete-choice games. Suppose payoff functions for both players are given by

$$u_1(y_1, y_2) = \left(X_1'\beta_1 - \eta_1 \cdot y_1 - \Delta_1 \cdot y_2 - \varepsilon_1\right) \cdot y_1 \quad \text{for player 1,}$$
$$u_2(y_1, y_2) = \left(X_2'\beta_2 - \eta_2 \cdot y_2 - \Delta_2 \cdot y_1 - \varepsilon_2\right) \cdot y_2 \quad \text{for player 2.}$$

Suppose (X_1, X_2) are observed by the econometrician, but $(\varepsilon_1, \varepsilon_2)$ are payoff shocks that are unobserved. Suppose, however that both players observe (X_1, X_2) and $(\varepsilon_1, \varepsilon_2)$. Let $\theta_1 \equiv (\beta_1, \eta_1, \Delta_1)$, $\theta_2 \equiv (\beta_2, \eta_2, \Delta_2)$ and $\theta \equiv (\theta_1, \theta_2)$. Here, θ denotes a vector of parameters (treated here as constant, unknown quantities). The object of interest for inference is θ. Estimating it can allow us to learn about the nature of the strategic interaction (indicated by the properties of (Δ_1, Δ_2)) and conduct counterfactual analysis[1].

[1] I do not assume whether actions are strategic complements or substituts.

2.1.1 Optimal choices an equilibrium when the action space is unrestricted

Let us maintain throughout that $\eta_p > 0$ for $p = 1, 2$, so that payoff functions are *strictly concave*. Also, let us assume that the action spaces $\mathcal{Y}_1$ and $\mathcal{Y}_2$ are unrestricted (i.e, unbounded). In this case, players' optimal choices can be obtained from the first-order conditions

$$\frac{\partial u_1(y_1, y_2)}{\partial y_1} = 0 \quad \text{and} \quad \frac{\partial u_1(y_1, y_2)}{\partial y_1} = 0$$

Solving the first-order condition for player p in terms of y_{-p} yields the *best-response function* for player p. These best-response functions are

$$\begin{aligned} BR_1(y_2, X_1, \varepsilon_1 | \theta_1) &= \frac{X_1'\beta_1 - \Delta_1 \cdot y_2 - \varepsilon_1}{2\eta_1}, \\ BR_2(y_1, X_2, \varepsilon_2 | \theta_2) &= \frac{X_2'\beta_2 - \Delta_2 \cdot y_1 - \varepsilon_2}{2\eta_2}. \end{aligned} \tag{1}$$

Best-responses are unique and well-defined in this example due to the concavity of payoffs. I refer to the functions described in (1) as players' *unrestricted best-response functions*. The solution concept of Nash equilibrium (NE) assumes that both players best-respond to each other simultaneously, so the observed choices (y_1^*, y_2^*) would be the solution to the system of equations

$$y_1^* = BR_1(y_2^*, X_1, \varepsilon_1 | \theta_1), \quad \text{and} \quad y_2^* = BR_2(y_1^*, X_2, \varepsilon_2 | \theta_2).$$

Assuming that $4\eta_1\eta_2 - \Delta_1\Delta_2 \neq 0$, equilibrium choices are given by

$$\begin{aligned} y_1^* &= \frac{2\eta_2 \cdot X_1'\beta_1 - \Delta_1 \cdot X_2'\beta_2 + \Delta_1 \cdot \varepsilon_2 - 2\eta_2 \cdot \varepsilon_1}{4\eta_1\eta_2 - \Delta_1\Delta_2} \equiv X_1'\theta_{11} + X_2'\theta_{12} + \nu_1, \\ y_2^* &= \frac{2\eta_1 \cdot X_2'\beta_2 - \Delta_2 \cdot X_1'\beta_1 + \Delta_2 \cdot \varepsilon_1 - 2\eta_1 \cdot \varepsilon_2}{4\eta_1\eta_1 - \Delta_1\Delta_2} \equiv X_1'\theta_{21} + X_2'\theta_{22} + \nu_2, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \theta_{11} &\equiv \frac{2\eta_2\beta_1}{4\eta_1\eta_2 - \Delta_1\Delta_2}, \quad \theta_{12} \equiv \frac{\Delta_1\beta_2}{4\eta_1\eta_2 - \Delta_1\Delta_2}, \quad \nu_1 \equiv \frac{\Delta_1 \cdot \varepsilon_2 - 2\eta_2 \cdot \varepsilon_1}{4\eta_1\eta_2 - \Delta_1\Delta_2} \\ \theta_{21} &\equiv \frac{2\eta_1\beta_2}{4\eta_1\eta_2 - \Delta_1\Delta_2}, \quad \theta_{22} \equiv \frac{\Delta_2\beta_1}{4\eta_1\eta_2 - \Delta_1\Delta_2}, \quad \nu_2 \equiv \frac{\Delta_2 \cdot \varepsilon_1 - 2\eta_1 \cdot \varepsilon_2}{4\eta_1\eta_2 - \Delta_1\Delta_2} \end{aligned}$$

Suppose the econometrician observes a random sample $(Y_{1i}, Y_{2i}, X_{1i}, X_{2i})_{i=1}^{n}$. Assuming that the actions observed in the data are the realization of a NE implies that

$$Y_{1i} = BR_1(Y_{2i}, X_{1i}, \varepsilon_{1i}|\theta_1), \quad \text{and} \quad Y_{2i} = BR_1(Y_{1i}, X_{2i}, \varepsilon_{2i}|\theta_2). \tag{3}$$

And therefore the stochastic relationship between (Y_{1i}, Y_{2i}) and (X_{1i}, X_{2i}) is described by the system (2). From here, the parameters of the model can be estimated based on qualitative assumptions about the statistical relationship between the unobserved shocks $(\varepsilon_1, \varepsilon_2)$ and the observed payoff shifters (X_1, X_2). But notice that *the key to the validity of* (2) *is the underlying assumption that the action spaces are unrestricted, so that both players can always play their best-response choices described in* (1).

While this is a simple example, it accurately describes the state-of-the-art treatment of action spaces in the econometrics of games literature, which generically assumes that it is unrestricted and that optimal choices are given by interior solutions. This paper produces econometric inference methods where action spaces are randomly censored, and where the econometrician cannot observe whether the choices in the data come from unrestricted best-responses or if they were censored. In the next section I describe how even this simple model changes dramatically once I introduce the possibility of censoring.

3 Testable implications with censored action spaces

3.1 Introducing censoring in action spaces

Suppose the action space of player p is bounded above by $\overline{\xi}_p$, which I treat as random and unobserved (to the econometrician). By concavity of payoffs, this immediately means that players' restricted best-responses are given by

$$\begin{aligned}\overline{BR}_1(y_2, X_1, \varepsilon_1, \overline{\xi}_1|\theta_1) &= \min\left\{BR_1(y_2, X_1, \varepsilon_1|\theta_1),\ \overline{\xi}_1\right\},\\ \overline{BR}_2(y_1, X_2, \varepsilon_2, \overline{\xi}_2|\theta_2) &= \min\left\{BR_2(y_1, X_2, \varepsilon_2|\theta_2),\ \overline{\xi}_2\right\},\end{aligned} \tag{4}$$

$\overline{\xi}_p$ represents, for example, production (capacity) constraints or it may reflect in general the presence of frictions that prevent players from choosing any action from within their action spaces.

3.1.1 The need for random endogenous censoring

The only realistic way to model censoring is to treat these censoring points as unobserved random variables (to the econometrician) and to allow them to be correlated with the remaining variables (payoff shifters) in the model. My goal is also to perform inference without assuming a parametric distribution for unobserved shocks. Thus, I allow for $(\overline{\xi}_1, \overline{\xi}_2)$ to be correlated with $(X_1, X_2, \varepsilon_1, \varepsilon_2)$, although I make some qualitative restrictions about this stochastic relationship later on.

3.2 Equilibrium properties with censoring

The presence of censoring replaces the unconstrained best-response functions in (1) with their constrained (censored) counterparts in (4). Assuming that the data observed is an equilibrium from the constrained game implies now

$$Y_{1i} = \overline{BR}_1(Y_{2i}, X_{1i}, \varepsilon_{1i}, \overline{\xi}_{1i}|\theta_1) \quad \text{and} \quad Y_{2i} = \overline{BR}_2(Y_{1i}, X_{2i}, \varepsilon_{2i}, \overline{\xi}_{2i}|\theta_2).$$

Thus, in particular,

$$Y_{1i} \leq BR_1(Y_{2i}, X_{1i}, \varepsilon_{1i}|\theta_1), \quad \text{and} \quad Y_{2i} \leq BR_2(Y_{1i}, X_{2i}, \varepsilon_{2i}|\theta_2). \tag{5}$$

These inequalities must hold *with probability one* (w.p.1), and they generalize the unconstrained equilibrium condition in (3). Crucially, notice that the inequalities in (5) are still expressed in terms of the unconstrained best-response functions and they hold for any realization of the censoring points $(\overline{\xi}_1, \overline{\xi}_2)$, and they hold whether or not a particular (Y_{1i}, Y_{2i}) is censored (an event that I assume to be unobservable in the data). This allows me to design inferential procedures that do not require an explicit model of the censoring data-generating process.

3.2.1 Observable implications of equilibrium with censoring

The inequalities in (5) can be readily translated into testable implications which can ultimately used to perform inference on θ. I present below a simple version of the type of restrictions that my I exploit in censored games. There are two relevant cases, depending on whether best-response functions are increasing ($\Delta_p \leq 0$) or decreasing ($\Delta_p \geq 0$) functions of the opponent's action.

Implications in the strategic-complement case

Let us focus on Player 1 (the exact same analysis follows for Player 2). Suppose that, for a given (X_1, ε_1) the best-response function $BR_1(y_2, X_1, \varepsilon_1|\theta_1)$ is increasing in y_2. Next, take any $y_2 \in \mathcal{Y}_2$ and suppose $Y_2 \leq y_2$. Then,

$$\underbrace{Y_1 \leq BR_1(Y_2, X_1, \varepsilon_1|\theta_1)}_{\text{by censoring and (5)}} \underbrace{\leq BR_1(y_2, X_1, \varepsilon_1|\theta_1)}_{\text{by strategic-complementarity}} \implies Y_1 \leq BR_1(y_2, X_1, \varepsilon_1|\theta_1)$$

Therefore, using the expressions for best-response functions in (1), in the strategic complement case,

$$\forall\ y_2 \in \mathcal{Y}_2,\ Y_2 \leq y_2 \implies \varepsilon_1 \leq X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot Y_1 \tag{6-C}$$

Implications in the strategic-substitutes case

Now suppose Player 2's action is a *strategic substitute* for Player 1, which corresponds to the case where, for a given (X_1, ε_1), the best-response function $BR_1(y_2, X_1, \varepsilon_1|\theta_1)$ is decreasing in y_2. Take any $y_2 \in \mathcal{Y}_2$ and suppose $Y_2 \geq y_2$. Then,

$$\underbrace{Y_1 \leq BR_1(Y_2, X_1, \varepsilon_1|\theta_1)}_{\text{by censoring and (5)}} \underbrace{\leq BR_1(y_2, X_1, \varepsilon_1|\theta_1)}_{\text{by strategic-substitutability}} \implies Y_1 \leq BR_1(y_2, X_1, \varepsilon_1|\theta_1)$$

And so, in the strategic-substitutes case,

$$\forall\ y_2 \in \mathcal{Y}_2,\ Y_2 \geq y_2 \implies \varepsilon_1 \leq X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot Y_1 \tag{6-S}$$

4 An inferential method

4.1 A median-independence assumption

My goal is to describe methods such that I can do inference on θ without having to assume a parametric distribution for the unobservable payoff shocks $(\varepsilon_1, \varepsilon_2)$. I illustrate how this can be done based on the inequalities described in (6-C) and (6-S). Robust inference can be conducted invoking the type of assumptions made in single-agent, quantile regression models.

> **Assumption M:** Let $F_p(\epsilon_p|X_1, X_2)$ denote the joint distribution function of ε_p conditional on (X_1, X_2). I maintain that $F_p(0|X_1, X_2) = \frac{1}{2}$ w.p.1 in (X_1, X_2).

The previous assumption basically assumes that ε_p is symmetrically distributed around zero w.p.1, for any (X_1, X_2). As is the case in quantile regression models, this allows for correlation between ε_p and (X_1, X_2) while only requiring that the median of ε_p to be constant[2]. From now on, let $\mathbf{1}[A]$ denote the indicator function for the event A (i.e, $\mathbf{1}[A] = 1$ of A is true and $\mathbf{1}[A] = 0$ otherwise).

4.2 The strategic-complement case

Equation (6-C) derived an implication of equilibrium behavior with censoring in the strategic-complement case. Recall that strict concavity of payoffs requires $\eta_1 > 0$. Therefore, for any $y_1 \in \mathcal{Y}_1$, $Y_1 \geq y_1 \implies X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot Y_1 \leq X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot y_1$. Combining this with (6-C),

$$\forall\ (y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2,\ \mathbf{1}\big[Y_1 \geq y_1,\ Y_2 \leq y_2\big] \leq \mathbf{1}\Big[\varepsilon_1 \leq X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot y_1\Big].$$

A testable implication can be derived from here combined with Assumption M by choosing (y_1, y_2) such that $X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot y_1 = 0$. Fix $y_2 \in \mathcal{Y}_2$ and let $y_1 = \frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1}$. It follows that, in the strategic-complements case,

$$\forall\ y_2 \in \mathcal{Y}_2\colon\ \frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1} \in \mathcal{Y}_1, \quad \mathbf{1}\left[Y_1 \geq \frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1},\ Y_2 \leq y_2\right] \leq \mathbf{1}\big[\varepsilon_1 \leq 0\big].$$

Take any positive "weight function" $\phi(X_1, X_2) > 0$ chosen by the econometrician and define

$$M_\phi^c(y_2|X_1, X_2, \theta_1) =$$
$$E\left[\left(\mathbf{1}\left[\frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1} \in \mathcal{Y}_1\right] \cdot \mathbf{1}\left[Y_1 \geq \frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1},\ Y_2 \leq y_2\right] - \frac{1}{2}\right) \cdot \phi(X_1, X_2) \middle| X_1, X_2\right]$$

Then, equilibrium behavior with censoring and strategic-complements predicts that

$$M_\phi^c(y_2|X_1, X_2, \theta_1) \leq 0 \quad \forall\ y_2 \in \mathcal{Y}_2, \ \text{ w.p.1. in } (X_1, X_2). \tag{7}$$

A confidence set for θ_1 can be constructed based on the conditional moment inequality described in (7). I sketch below how this can be done.

[2]As I show below, the assumption can be relaxed to the condition $F_p(0|X_1, X_2) \leq \frac{1}{2}$ w.p.1 in (X_1, X_2) (replacing the equality with an inequality).

4.3 The strategic-substitute case

Using the same reasoning as in the complement-case and combining in this case with (6-S),

$$\forall\ (y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2,\ \mathbf{1}\big[Y_1 \geq y_1,\ Y_2 \geq y_2\big] \leq \mathbf{1}\Big[\varepsilon_1 \leq X_1'\beta_1 - \Delta_1 \cdot y_2 - 2\eta_1 \cdot y_1\Big].$$

The moment function is now

$$M^s_\phi(y_2|X_1, X_2, \theta_1) =$$
$$E\left[\left(\mathbf{1}\left[\frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1} \in \mathcal{Y}_1\right] \cdot \mathbf{1}\left[Y_1 \geq \frac{X_1'\beta_1 - \Delta_1 \cdot y_2}{2\eta_1},\ Y_2 \geq y_2\right] - \frac{1}{2}\right) \cdot \phi(X_1, X_2) \middle| X_1, X_2\right]$$

Equilibrium behavior with censoring and strategic-substitutes predicts that

$$M^s_\phi(y_2|X_1, X_2, \theta_1) \leq 0 \quad \forall\ y_2 \in \mathcal{Y}_2, \ \text{ w.p.1. in } (X_1, X_2). \tag{8}$$

4.4 Combining both cases

Whether the game is of strategic substitutes or complements depends entirely on the sign of Δ_1. Both cases can be combined straightforwardly by using

$$M_\phi(y_2|X_1, X_2, \theta_1) = M^c_\phi(y_2|X_1, X_2, \theta_1) \cdot \mathbf{1}\left[\Delta_1 < 0\right] + M^s_\phi(y_2|X_1, X_2, \theta_1) \cdot \mathbf{1}\left[\Delta_1 \geq 0\right].$$

Equilibrium behavior with censoring predicts that

$$M_\phi(y_2|X_1, X_2, \theta_1) \leq 0 \quad \forall\ y_2 \in \mathcal{Y}_2, \ \text{ w.p.1. in } (X_1, X_2). \tag{9}$$

5 Constructing a confidence set (CS) for the parameters of the model

The *identified set* of parameters would be all those elements $\theta_1 \in \Theta_1$ (where Θ_1 denotes the parameter space) such that the inequality in (9) is satisfied. That is,

$$\Theta_1^I = \{\theta_1 \in \Theta\colon\ M_\phi(y_2|X_1, X_2, \theta_1) \leq 0 \quad \forall\ y_2 \in \mathcal{Y}_2, \ \text{ w.p.1. in } (X_1, X_2).\} \tag{10}$$

Constructing a CS based on (10) using existing econometric methods based on inequalities of moments (expectations). These include, among others, Chernozhukov, Hong, and Tamer (2007), Romano and Shaikh (2010), Andrews and Soares (2010), Bugni (2010), Andrews and Jia-Barwick (2010), Chernozhukov, Lee, and Rosen (2013), Andrews and Shi (2013), Romano, Shaikh, and Wolf (2014), Chetverikov (2012), Andrews and Shi (2013), Armstrong (2015), Armstrong (2014), Pakes, Porter, Ho, and Ishii (2015). In Kosenkova (2018), I exploit testable implications that, like (10), take the form of a functional inequality (an inequality of conditional functionals). Following my approach in that paper, I first proceed to re-express Θ_1^I in terms of an unconditional, mean-zero restriction. Let

$$\mathcal{M}(\theta_1) = \int_{y_2 \in \mathcal{Y}_2} E_{X_1,X_2}\left[\max\{M_\phi(y_2|X_1,X_2,\theta_1),\, 0\}\right] dW(y_2),$$

where $W(\cdot)$ is a weight function chosen by the researcher. The identified set in (10) can be re-expressed as

$$\Theta_1^I = \{\theta_1 \in \Theta\colon\ \mathcal{M}(\theta_1) = 0\} \tag{11}$$

In Kosenkova (2018), I estimate a functional similar to $\mathcal{M}(\theta_1)$

$$\widehat{\mathcal{M}}(\theta_1) = \int_{y_2 \in \mathcal{Y}_2} \left\{ \frac{1}{n} \sum_{i=1}^{n} \widehat{M}_\phi(y_2|X_1,X_2,\theta_1) \cdot \mathbf{1}\left[\widehat{M}_\phi(y_2|X_1,X_2,\theta_1) \geq -b_n\right] \right\} dW(y_2),$$

where b_n is a bandwidth sequence converging to zero at an appropriate rate. In Kosenkova (2018), I characterize the asymptotic properties of estimators such as $\widehat{\mathcal{M}}(\theta_1)$. Once this is done, a CS for θ_1 can be estimated as

$$CS(\theta_1)_{1-\alpha} = \left\{\theta_1 \in \Theta_1\colon\ \varphi(n) \cdot \widehat{\mathcal{M}}(\theta_1) \leq \widehat{c}_n(\theta_1)\right\},$$

where $\varphi(n)$ is a function of the sample size n, and $\widehat{c}_n(\theta_1)$ is a critical value which is derived from the asymptotic properties of $\widehat{\mathcal{M}}(\theta_1)$ and is selected such that

$$\lim_{n\to\infty} \inf_{\theta \in \Theta_1^I} \Pr\left[\theta_1 \in CS(\theta_1)_{1-\alpha}\right] \geq 1-\alpha,$$

where $1-\alpha$ is the desired target coverage probability. Under assumptions on the rate of convergence of the tuning parameters, the limiting distribution is standard normal. But the method also allows bootstrap approach.

6 Monte Carlo simulations

——TBD——-

This section shows Monte Carlo simulations that illustrates that ignoring censoring in action space can lead to severely biased estimation. Bias is larger in cases when restriction of the action space is more binding. The size of the CS also depends on the biding properting of the restrictions.

7 Application to real data

—–-TBD——

8 Conclusion

——TBD———

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