Dynamics of Bank Financing: the Role of Learning

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January 31, 2020

Abstract

Loan contract terms generally improve for the borrower over repeated lendings: lower lending costs and greater availability of capital. This is commonly interpreted as resulting from information accumulation and *learning* about the borrowing firms' quality over time. However, to precisely identify this learning process has been a challenge, since the lending market also features a *selection* process where risky borrowers are more likely to drop out in a relationship. Thus, to date, little is known about the importance of learning and its role in driving loan contract dynamics.

I develop a model of corporate loan market where firms' quality is unobserved to all but could be learned symmetrically over repeated lendings, and banks compete on both interest rates and loan size in each period. A firm-bank lending relationship is sticky because of switching cost, but it can be endogenously terminated by a bank.

The model shows that learning affects equilibrium loan size and interest rates not just directly through expected firm output and expected default probability, but also indirectly through expected *future values*. This is because switching cost leads to positive profits for the incumbent, and the competing bank is willing to give up current profits in exchange for expected future values, which depends on the firm's probability of entering the future. Firms with poor posterior beliefs are perceived as less likely to enter the future due to higher expected default probability and relationship termination, and have lower future values. Thus they face higher interest rates due to this interaction between imperfect competition and learning process.

I use a novel dataset on the complete panel of lending contracts between a bank and its corporate borrowers, which, crucially, also contains direct evidence of performance in each period. This allows me to identify the leaning process in presence of a selection process in a dynamic game. The equilibrium loan contract terms and termination rules take simple forms, which allows relatively simple estimation procedure. In the end I can measure the importance of learning about borrower quality, as well as its implications for firm dynamics and monetary transmission.

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1 Introduction

Bank financing is an important source of external finance for many firms. Empirical explorations of the theory of firm-bank lendings generally find that contract terms improve for the borrower over repeated lendings with banks.[1] But why is that? While theorists suggest that information accumulation during the repeated lendings could lower cost of capital and boost availability of funds to firms over time,[2] little is known about the precise process of information acquisition that could lead to the observed pattern of bank financing.

Yet understanding the dynamics of loan contract terms in bank financing is important, especially for small firms. First, small firms account for a significant share of total outputs and employment,[3] but they are also vulnerable because of their dependence on external funding, especially bank financing. Thus evolution in contract terms of bank loans can be closely related to the evolution of small firms, which has implications for the selection and evolution of industry. Second, macroeconomists have shown that the presence of incomplete information, which makes lenders imperfectly informed about their borrowers, can propagate monetary policy shocks.[4] If we could link the observed dynamics of loan contracts with the underlying process of information acquisition, then we might be able to measure the speed of such process and quantify its macroeconomic implications.

This paper develops a model of corporate loan market where borrowing firms' quality (i.e., riskiness or creditworthiness) is unobserved to all but could be learned symmetrically over repeated lendings, and banks compete on both interest rates and loan size in each period. A firm-bank lending relationship is sticky because of switching cost, but it can be endogenously terminated. I use a novel dataset on the complete panel of lending contracts between a bank and its corporate borrowers to estimate the importance of learning about borrower quality, as well as its implications for firm dynamics and monetary transmission.

In this model, the impact of learning on bank financing dynamics is twofold: First, current beliefs about the borrower quality have a *direct* impact on interest rates and loan size. The equilibrium loan size offered by a bank can be shown to equate the marginal output of the firm with the marginal cost of lending plus expected cost associated with default, so current beliefs can affect optimal loan size through expected default probability. Equilibrium interest rates also depends on expected cost of default, and thus the current belief.

More interestingly, the learning process also leads to a dynamic selection process, in which firms drop out not only due to defaults and other exogenous events, but also endogenous terminations. In other words, current beliefs of the firm's riskiness can impact the probability of exit as well. This is because as information about the firm's quality accumulate with

[1] See Petersen and Rajan (1994); Berger and Udell (1995); Berlin and Mester (1999); Bolton et al. (2016); Schwert (2018); Darmouni (2019).

[2] See Sharpe (1990); Diamond (1991); Boot, Thakor, and Udell (1991); Boot and Thakor (1994). Bhattacharya and Thakor (1993) and Boot (2000) provide excellent reviews.

[3] See Hallberg (2000); De la Torre et al. (2008); Ayyagari et al. (2011)

[4] See Hachem (2011) for a review of recent empirical evidence on the link between informational frictions and transmission of monetary shocks.

repeated loans, at some point a bank may find it unprofitable to continue offering loan contract to a firm with a poor history of loan performance realization, and would choose to terminate the relationship. The data suggests that this layer of effect is indeed present: firms are more likely to dropout after they have suspicious loan performance even if they do not end up default.

This learning and selection process can explain salient patterns observed in my data, namely, decreasing interest rates and increasing loan size over repeated lendings. Since lower-quality borrowing firms are more likely to drop out due to either defaults or endogenous exit, average quality of the remaining firms in a cohort become better over time, so do the perceived quality, or beliefs about their quality. Thus the observed contract terms, which reflect the beliefs, become more favorable over the life of a relationship.

My model also shows the cross-sectional relationship between interest rate and loan size. Unlike in mortgage market where interest rates almost for sure increases with Loan-To-Value ratio, or loan size, in corporate loan market I analyze, the opposite relationship is observed when conditional on years of relationship. This can be explained by the learning process as well. Firms with larger loan size are those believed to be better in quality, and such favorable beliefs also dictate a lower interest rate.

Furthermore, interest rates predicted by the model illuminates interactions between imperfect competition and learning. The intuition is the following. Suppose the incumbent bank (the bank that lends to the firm in the last period) also wins the competition this period. It follows that the incumbent bank's offer will match the best offer that the competing bank can make, and then plus a switching cost. Note that the competing bank's offer can actually go below its average cost of fund in this period, since the presence of switching cost confers positive profits to incumbency. So the competing bank's best offer is its cost minus future profits brought by switching costs, which depends on the firm's probability of continuing borrowing in the future. Then it implies that the incumbent's offer, which matches the competing bank's offer, reflects an interaction between switching costs and the probability of keeping borrowing in the future. Specifically, for a borrowing firm that is on the verge of exiting because of unfavorable beliefs, it is perceived to have a lower probability of making to the next period, and thus the future profits brought by the firm is lower, which makes the competing firm's offer less attractive. As a consequence, its incumbent's offer is less attractive as well. In other words, learning might amplify implications from imperfect competition.

Formally, I develop a model of the lending market for firms in which a firm's quality type is symmetrically unobserved to all, including the firm. There are two banks, which are symmetric in terms of lending technology, but might have different realizations of cost shocks in the beginning of each period. They compete via Bertrand by making a loan contract offer which consists of interest rate and loan size. Then a firm accepts one bank's offer, takes the loan, and then revenue realizes, so do loan performance and default outcome, which are observed to all and reveal information about the unknown type of the firm. Beliefs are updated following these realizations, and exit (or termination) might happen; otherwise the game carries on to the next period.

Identification of the process of learning utilizes a mandated "performance rating" that is observed in my data for each loan in every period. It provides direct evidence of the *signals* that the banks and firms use to learn about the firm's quality type. Information on performance is sufficient to identify the law of motion of beliefs about quality types. Joint information on performance and contract terms identifies the distribution of the initial prior beliefs about type when the firm first borrows from the sample bank. The initial prior together with the law of motion of beliefs identified from the signals, completely describes the learning process. Differences of interest rates observed at the initial period and in subsequent periods can identify the magnitude of switching cost. Firm's production technology can be identified from observed probability of leaving conditional on histories of performance and interest rates.

The paper is related to relationship lending literature but there is a key difference—I do not focus on asymmetric information between a firm and a bank, or between inside bank and outside bank. Darmouni (2019) attempts to explain relationship stickiness using the information gap between inside and outside banks, finding it is quantitatively insignificant in the U.S. syndicated loan market. The exercise here aims to provide an explanation of observed features in relationship banking literature *without* assuming informational frictions among banks or between firm and bank. There are good institutional reasons why such informational frictions might not be important anyways.[5] Thus, this paper can be viewed as an alternative explanation for patterns we see in relationship banking.

This paper is also relevant in the discussion of firm dynamics. This paper shares the same spirit as Jovanovic (1982) with types being unknown and time-invariant, slowed revealed through economic activity. Ericson and Pakes (1995), on the other hand, assumes firms learn through active investment. But both papers implicitly assume that the financial state of a firm is irrelevant to its production, and this is where the paper fills in. I show how imperfect competition and shocks in financing market can affect outputs and firm dynamics.

On the topic of financing frictions, Albuquerque and Hopenhayn (2004) proposes a theory of endogenous borrowing constrains arsing from limited liability and limited enforcement of debt repayment, while Clementi and Hopenhayn (2006) proposes an alternative theory of endogenous borrowing constrains arsing from limited liability and moral hazard that arises from bank's inability to observe true revenue. However, neither of these theories applies to the my setting—corporate loan market in China. The first theory predicts default never happens, and the second theory predicts debt forgiveness after bad revenue realizations, which are not true in my data. In my model, the financing friction comes mainly from the presence of collateral and the associated repossession cost, which distorts the output from the first best where firms are self-financing.

[5] In reality it is not obvious whether small and young entrepreneurs really have better assessment of their riskiness than banks who has funded numerous similar projects and accumulated expertise in financing certain industries (Manove et al., 2001).

Another strand of literature that this paper might contribute to is learning in empirical Industrial Organization. Ackerberg (2003) uses consumer panel data to estimate how consumers learn about different brand's experience values and prestige values over time. Crawford and Shum (2005) uses prescription panel data to estimate how the patient and doctor learn about different drug's curative and symptomatic match values over time. Both papers are about one agent learning about values of different objects over time, while my paper is about multiple agents learning about the value of a single object. Their identification mainly comes from observing agent's experimentation behavior, while my identification builds on direct evidence of the signals over time.

The paper that is closest to my work is Pastorino (2019), which is in the setting of labor market where firms competing for workers with unknown ability types by offering contracts that include job assignments and wages. Differences in natures of labor market and lending market translate into major differences in her model and mine. First, she needs to account for human capital accumulation process where past work experience leads to higher human capital and thus outputs, while in my setting it is not obvious that past loan size alone can lead to higher productivity in the future. Second, jobs are different in their informativeness, which lead to incentive to forgo current wages in exchange for better information acquisition opportunities. In lending market, though, this unit of fund is no different than that unit in terms of informativeness, so there is experimentation incentives. Third, in bank financing I need to account for the endogenous exit behavior since firms have other options of financing, whereas in the labor market, a worker generally does not quit unless some exogenous event (disease, injury, etc) happens.

2 Data and Institutional Background

2.1 China's Banking Industry

The banking sector in China originated from a centralized system in 1949 when the People's Bank of China (PBC), as China's central bank, governed both commercial bank businesses (e.g., deposits, lending, and foreign exchange) and central bank functions. Along with economic opening policies being instituted by Deng Xiaoping in 1978, the banking system entered a period of reform. In 1983, the PBC began to focus on national macroeconomic policy, monetary stability, and economic development. At the same time, the big four commercial banks (i.e., the ICBC, ABC, BOC and, CCB) started to take over commercial bank businesses, and each was specialized in a specific area. The Bank of Communications' experience in reform and development has paved the way for the development of shareholding commercial banks in China and exemplified banking reforms in China (Gao et al., 2019a).

Between 1988 and 2005, twelve joint equity banks were established, mostly as SOEs or institutions transformed from local financial companies. Although joint equity banks are also national banks, unlike big five commercial banks, they usually focus on local business and operate on a much smaller scale. By the end of the year 2013, as reported by China Banking Regulatory Commission (CBRC)'s annual reports, the big five commercial banks dominate the market and control for approximately 43.3% of the market share. On the other hand, joint equity banks are much smaller and control for about 17.8% of the market share. The rest of the financial institutions belong to the third tier such as municipal commercial banks.

2.2 Deregulation of Credit Controls and Interest Rates

The first step in deregulation of credit control is taken was 1998. Until then, the central bank had controlled the lending of Chinese banks through binding credit quotas. This binding credit plan system was formally abolished in 1998, replaced with an indicative non-binding credit target. In other words, commercial banks in China are no longer required to provide loans in compliance with state directives or policy targets. Instead, they are encouraged to allocate funds "on the basis of proper credit assessments" and lend based on economic and commercial considerations. This change in policy has been hailed by Chinese monetary authorities as an important initial step in transforming the credit culture of Chinese banks (Xu et al., 2016).[6]

Lending rates in China have been substantially more liberalized than deposit rates throughout the path of interest rate deregulation, starting with the interbank offered rate in the capital market to be fully market-priced in 1996. From 1998 onward, the People's Bank of China (PBC) started to widen the floating band on banks' interest rates. In 2004, the deposit rate floor and the lending rate ceiling were eliminated for the major banks. The remaining lending rate floor was gradually widened and eventually completely removed.[7] In practice, the lending rate floor was largely non-binding even before it was removed (Xu et al., 2016). Deposit ceiling was binding (He and Wang, 2012), and it was not removed until October 23, 2015.

The day following October 23, 2015 marked the last change of benchmark lending rates until this day. The benchmark lending rate refers to the official reference for lending rate published by PBC. It served as a non-binding "guidance" on lending rates in the market, and had been an active policy instruments. Prior to October 2015, changes in the benchmark lending rates had been made by PBC[8] at random dates, typically seven or eight times a year. After October 24, 2015, however, it seems that the benchmark lending rate has ceased to function as an active policy instrument since no adjustment has been made so far; instead, the focus of PBC is increasing on short-term money market rates, namely the 7-day interbank pledged repo rate (DR007).[9] This move is generally seen as part of interest rate liberalization that

[6]There are still signs of quantitative controls on bank credit, as the central bank employs an array of quantitative instruments aimed at controlling credit growth, such as yearly aggregate target levels for new loans and the use of so-called window guidance which can be described as a form of moral persuasion aimed at controlling the sectoral direction of lending (Okazaki, 2007).

[7]The lending rate floor was reduced to 0.9 times the benchmark official lending rate in October 2004, 0.8 times the benchmark lending rate in June 2012, 0.7 times in July 2012, followed by a complete removal in July 2013.

[8]These changes need to be approved by the State Council (China's equivalent to a government cabinet) as well (McMahon et al., 2018).

[9]In the 2016, third-quarter Monetary Policy Executive Report, the PBC stated that "DR007 moves around the open market operation 7-day reverse repo rate. The DR007 can better reflect the liquidity

allowed PBC to improve its policy framework (McMahon et al., 2018).

2.3 Data Description

This paper utilizes three sources of data from a Chinese bank: 1) corporate loan contracting data, 2) (anonymized) corporate borrower data, and 3) annual snapshots of loan rating data. The corporate loan contracting data contains detailed information on loan contracts with corporate borrowers, including a loan identifier, a firm identifier, date of origination, contract interest rates, loan size, loan term, types of pledged collateral, appraised value of each type of pledged collateral, branch and sub-branch identifier, and most importantly, an indicator of whether the loan is classified as a nonperforming asset (NPA) as of June 2018. NPAs are listed on the balance sheet of the bank after a prolonged period of non-payment and evidence of extremely low repayment probability. They are typically viewed as loans that are in default.

The corporate borrower data used in this paper include the the borrowing firm's industry code, size, ownership type, date of incorporation, initial capital, and an internal credit rating. This credit rating system has 16 categories, where three of them is A-level, 9 of them is B-level, three of them is C-level, and the last one is D-level. A firm is assigned a credit rating at the initial screening conducted when it borrows from the bank for the first time. The classification is roughly based on the creditworthiness of the firm, although detailed criterion is not disclosed.

Another source of data is snapshots of loan ratings on all of the bank's outstanding loans the end of each year. For each loan, banks in China are required to report a loan rating to CBRC on a monthly bases until the loan is off the bank's balance sheet. The loan ratings are assigned according to a five-category loan classification system, in which there are five levels: 1 is the highest rating for the "normal" loans, 2 is for the "special mentioned", 3 is for the "substandard", 4 is for the "doubtful" and 5 is for the "loss". In this paper, loan rating is defined as a dummy variable for whether the loan rating is 1 or others (i.e., ratings from 2 to 5), as in Gao et al. (2019b). This method of classification is mainly based on borrowers' *repayment ability*, that is, their *actual* ability to repay principal and interests. Assessing this ability entails, for example, analyzing the borrower's changes in revenue and profits, cash flow, financial position, management efficiency, etc. One bank can get access to records of a borrowing firm's past abnormal loan ratings from CBRC in a credit inquiry.

The loan data is merged with firm data by the firm identifier, which is then merged with the loan rating data with loan identifier. The sample period is from January 2010 to June 2018, during which there were almost no binding ceilings or floors on the lending rate. This is also a period when the sample bank started to actively lend to small firms and form lending relationships with them, which enables me to track the evolution of lending contracts from the very beginning, avoiding the left-censoring problem often present in other studies.

condition in the banking system and has an active role to cultivate the market base rate".

Sample Restriction and Aggregation

For the purpose of this paper I restrict the sample to only include "small" and "young" firms that first borrow from the bank only after the beginning of the sample period, Jan 2, 2010, where "small" is defined according to a national criteria[10], and "young" is defined as less than 10 years old. In total, there are 9543 such firms in my sample.

There are several salient feature of firms' borrowing behavior in my sample. First, on average 78.6% of firms borrow only once in a given year, and among those who borrow more than once, the interest rates on multiple loans originated in the same year are similar, in contrast with the relatively large year-to-year variations on lending rates within a firm.[11] Second, the vast majority of the loans are short-term loans: 80.63% of loan terms are 1-year and 8.7% are six-month. Lastly, 70.09% of the firms who repaid the last year's loan come back and take out a new loan, which continues their lending relationships with the bank.

These patterns together implies the lending contracts are negotiated annually, which also confirms to anecdotal evidence from local loan officials. Thus, instead of treating each individual loan as an observation, I use firm-year level observations by taking the median interest rates and sum of loan sizes within each year for each firm. In total, there are 25731 observations on firm-year level.

Relationship Duration

A lending relationship begins when a firm borrows from the sample bank for the first time, and ends when either 1) the firm defaults on its last loan, or 2) it has not borrow from the bank for more than two consecutive years. This definition of the ending of the relationship is inspired by the following facts observed in my data: First, the sample bank never lends to a firm who just defaulted on its previous loan; and second, among the firms who has not defaulted, 90.6 % has been borrowing annually without a gap, and 7.45% has had only one gap year in their borrowing history, with only 1.95% firms have a two-year gap or longer. This suggest that an active lending relationship rarely has a gap that is longer than two years, so I define a relationship to be ended if no loan has been taken for more than two years since the last time the firm borrowed.[12]

[10] See the administrative rule at `http://www.gov.cn/zwgk/2011-07/04/content_1898747.htm`. This classification rule defines the range of number of employees and annual revenues to qualify for a small-sized company, which varies by industry. For example, in the retail industry, a company with employment 5 to 20 and annual revenue 10 to 50 million RMB (1.4 to 7.2 million USD) is classified as a small-sized company.

[11] On average the coefficient of variation of interest rates on multiple loans within the same year for the same firm is 0.07.

[12] In other words, a relationship is defined as ended if the last loan is taken prior to Jun 2016.

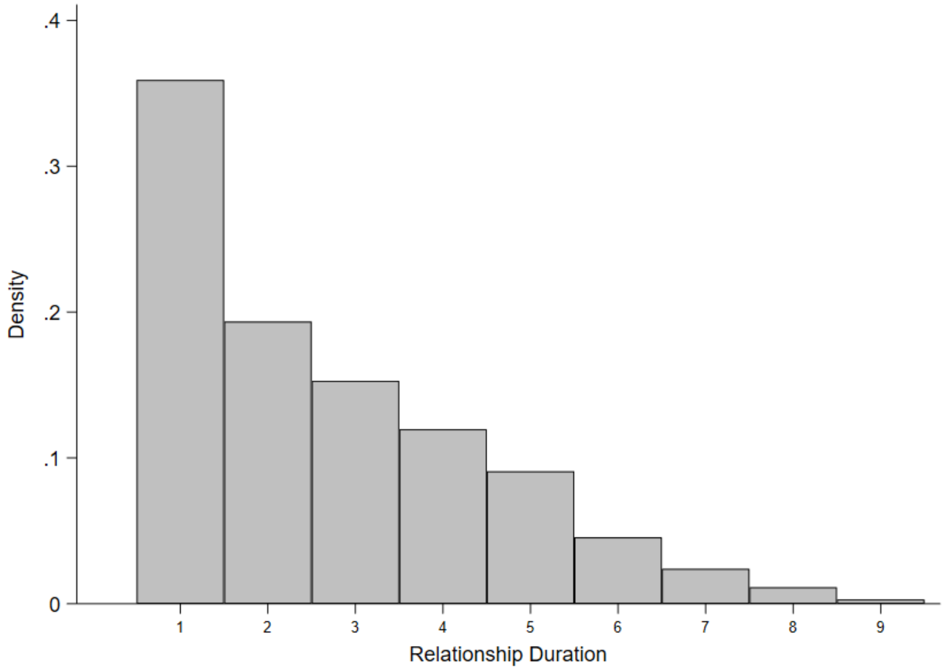


Figure 1: Distribution of Relationship Duration

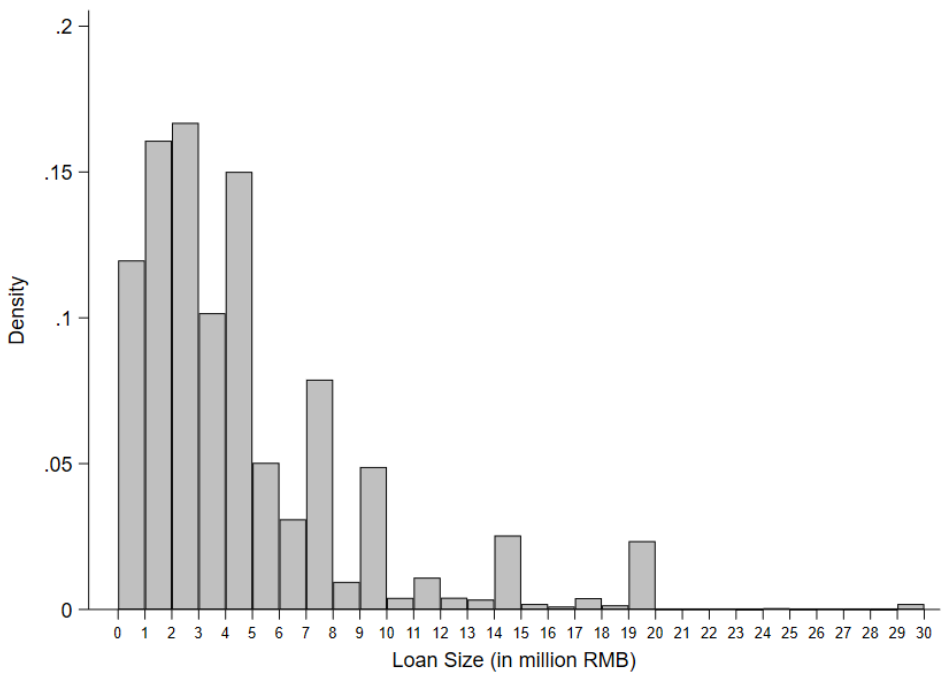


Figure 2: Histogram of Loan Size (in millions RMB)

Figure 1 shows the distribution of relationship duration. The distribution are similar if conditional on non-defaulting firms. Repeated lendings are common: more than 64% of borrowing firms borrowed more than once. And there is a considerable fraction of firms who has borrowed for more than 3 times.

Loan Contract Terms

Figure 2 shows the distribution of loan size. Most of the loan sizes is whole millions (in RMB). I categorize loan size into eight class: < 2, $[2, 4)$, $[4, 6)$,$[6, 8)$,$[8, 10)$,$[10, 20)$,$[20, 50)$, > 50.

There are three types of collateral: securities, fixed-assets, and third-party guarantee, which are ordered in their perceived liquidability. In the sample, 35.5% of loans only have third-party guarantee, 28.4% of loans only pledge fixed-assets, 23.97% of loans pledge both fixed-assets and third-party guarantee, and the remaining 12% uses securities as collateral.[13] I use the value of the most liquidable type pf collateral that is pledged, divided by the loan size, as a measure of collateral coverage ratio. More than 98% of the loans are secured (meaning coverage ratio > 1).

Average interest rate on the 1-year loans in my sample is 6.8%, and 78.4% of them are repaid monthly, with 21.5% repaid quarterly. In the following analysis, to better reflect the part of interest rate variation that is not merely due to changes in cost of fund, I define the *lending spread* to be the difference between the observed raw interest rates and 3-month Shanghai Interbank Offered Rate (SHIBOR) at the date of origination. SHIBOR is the reference rate based on the interest rates at which banks offer to lend unsecured funds to other banks in the Shanghai wholesale money market, and is often used as a measure of cost of fund for banks in China (McMahon et al., 2018). The average spread is 2.6%, with standard deviation 1.02%.

[13] The way collateral works in this bank is that, if default happens, the bank first capitalizes the type of pledged collateral that is most liquidable, for example, if a firm pledged both fixed-assets and third-party guarantee, then the bank would liquidate fixed-assets first, and then claim the third-party guarantee if the proceedings from fixed-assets liquidation is not enough to recover the loss. Third-party guarantee is notoriously hard to claim once the default happens, according to local loan officers, and fixed-assets often face substantial depreciation in the liquidation process. That is why the value of pledged collateral is sometimes higher than loan amount.

Table 1 shows the summary statistics of lending spreads over year by different cohorts.[14]

Table 1: Summary of Lending Spreads over Year by Cohorts

Year \ Cohort	2010	2011	2012	2013	2014	2015	2016	2017
2010	3.744	2.131	2.892	3.207	2.361	2.700	2.960	1.706
	(0.398)	(1.190)	(1.187)	(0.737)	(0.958)	(0.888)	(0.617)	(0.545)
2011		1.978	2.816	3.199	2.364	2.774	3.023	1.689
		(1.228)	(1.154)	(0.820)	(0.954)	(0.902)	(0.575)	(0.575)
2012			2.832	3.265	2.465	2.807	3.064	1.681
			(1.203)	(0.813)	(0.895)	(0.892)	(0.597)	(0.580)
2013				3.247	2.491	2.871	3.095	1.789
				(0.773)	(0.884)	(0.845)	(0.548)	(0.557)
2014					2.317	2.648	3.039	1.803
					(0.952)	(0.963)	(0.578)	(0.592)
2015						2.409	2.983	1.747
						(0.963)	(0.606)	(0.660)
2016							2.776	1.555
							(0.530)	(0.535)
2017								1.505
								(0.557)

[1] Different columns correspond to different borrowing cohorts, and rows correspond to calendar years.
[2] The standard deviations are in parentheses.

Loan Rating, Default, and Attrition

The unconditional default rate in my sample is 6.31%. Loan ratings, which is recorded during the repayment period, are closely related to the subsequent default, as shown in Figure 3.

Attrition here is defined as ending of the relationship without default, and it is more common in the data than default. The data does not provide any information about the reason why a firm stops borrowing. What we can learn from data about attrition is that, the rate of attrition decreases over the year of relationships, as shown in Figure 4.

[14] Cohort of year Y are defined as the group of firms that first borrow from the sample bank in year Y.

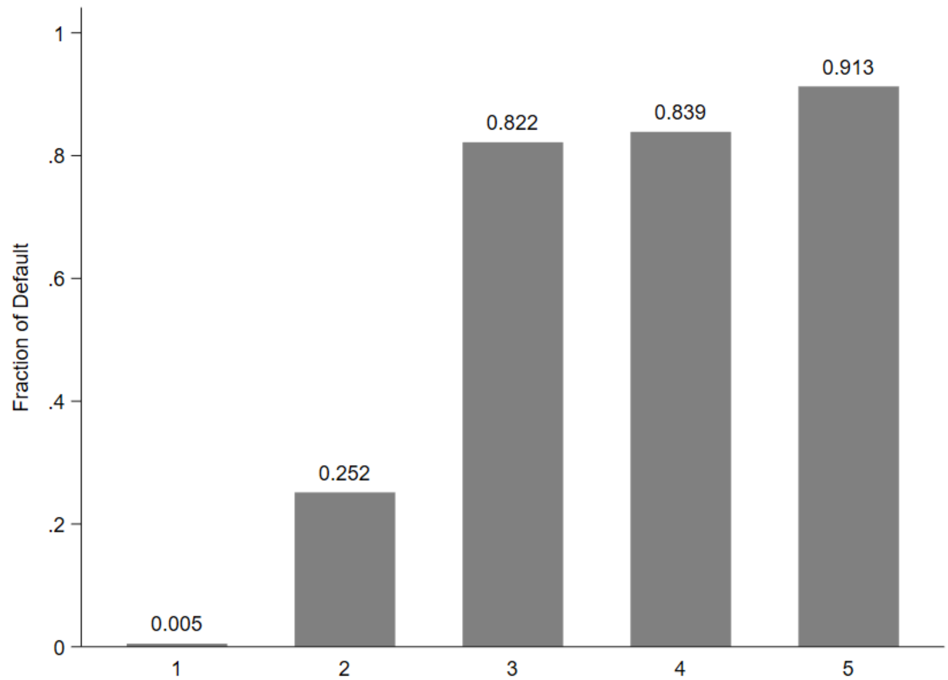


Figure 3: Fraction of Subsequent Default Conditional on Loan Rating

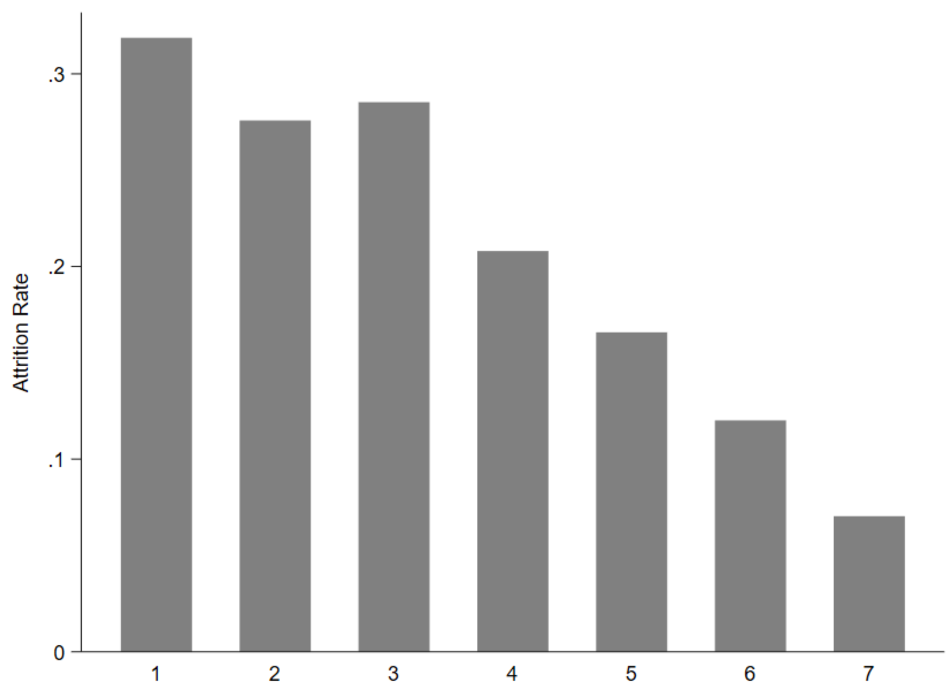


Figure 4: Attrition Rates over Years of Relationship

3 Descriptive Analysis

Early papers on creditor-firm relationships often examine how a firm's cost of borrowing change over the course of its relationship with a bank. A commonly accepted argument is, the longer a borrower has been servicing its loans, the more likely the business is viable and its owner trustworthy (Diamond, 1991). And conditional on its past experience with the borrower, the lender now expects loans to be less risky, which should reduce its expected cost of lending and increase its willingness to provide funds (Petersen and Rajan, 1994).[15] In other words, a combination of selection and learning could lead to decreasing interest rates over the course of the relationships. I find that this generally holds true in my data. Moreover, the cross-sectional dispersion of lending rates also decreases, which suggestive of a strong selection effect. These finds are summarized in Observation 1.

Observation 1. *The mean and standard deviation of lending rates decrease over the course of the relationship.*

Table 2 shows the correlation between lending spreads and year in relationship controlling for firm fixed effects, monthly time fixed effects and collateral information. From column (1), on average a firm sees a decrease of 30 basis point in lending spreads as they keep a functional lending relationship.

Table 2: Pricing Regression with Firm Fixed Effects

	(1) All	(2) Duration=2	(3) Duration=3	(4) Duration=4	(5) Duration=5
Years of Relationship	-0.315***	-0.293***	-0.305***	-0.307***	-0.319***
	(-26.73)	(-5.88)	(-9.46)	(-7.45)	(-9.53)
Other Controls	Monthly time fixed effect, types of collateral				
Observations	22006	3210	3774	4032	3818

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

To better isolate the effects of attrition, I focus on fixed groups of firms who share the same relationship duration. Column (2)-(5) show results from the pricing regression for the subset of borrowing firms with a certain relationship duration. From left to right, it is clear that the coefficient on years of relationship becomes bigger as the duration becomes longer. This could be explained by differences in average quality of groups of firms with the same duration. Firms who stick longer are arguably the *ex post* better firms, and they are more likely to see larger decreases in interest rates as banks learn about their quality over time.

To see the change of dispersion over years of relationship, ideally we want to look at time

[15] Although whether the cost savings will pass through and how much will pass-through might depend on the competitiveness and information structure of the capital market for small firms.

series of standard deviation for lending spreads among observationally identical[16] firms. I implement this idea by first residualizing lending spreads conditional on firm observables, year of relationship, monthly time effects and branch fixed effects, and then checking how does the standard deviation of residuals change over years of relationship. This approach can tease out variations of lending spreads coming from differences in observed firm characteristics as well as macroeconomic environment, allowing us to examine the part of dispersion resulting from the evolution of bank's belief.

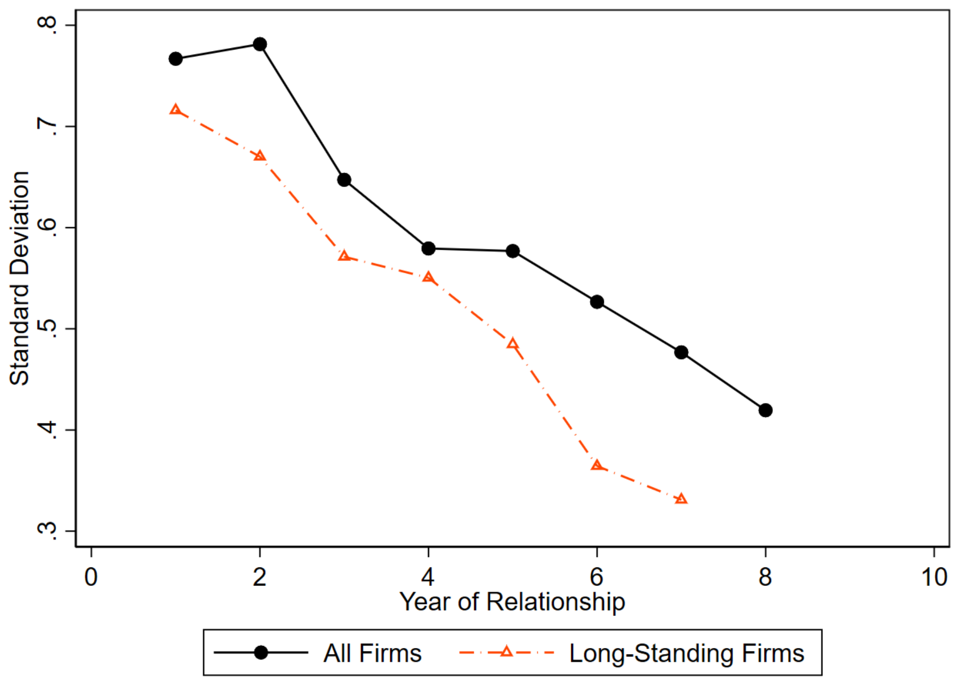


Figure 5: Standard Deviation of Residualized Lending Spreads

The solid connected line in Figure 5 shows the time series of standard deviations for the residuals from regressing lending spreads on observables in the whole sample. The dispersion slightly increases from the first to the second year of relationship, and declines afterwards, though the rate of decline is not monotonic. Note that composition of private types of firms are changing from year to year due to firms leaving or default. To better control for this source of variation, I then focus on a fixed group of firms: those whose relationship last for 7 years without gaps. The dash-dot line in Figure 5 shows the time series of standard deviation of residuals within the this group. Overall the standard deviation declines over years of relationship.

Observation 1 is consistent with a selection process where bad borrowers are more likely to leave the sample so the composition of the remaining borrower pool become less diverse and better in quality, so offered rates for them become lower on average and also less dispersed.

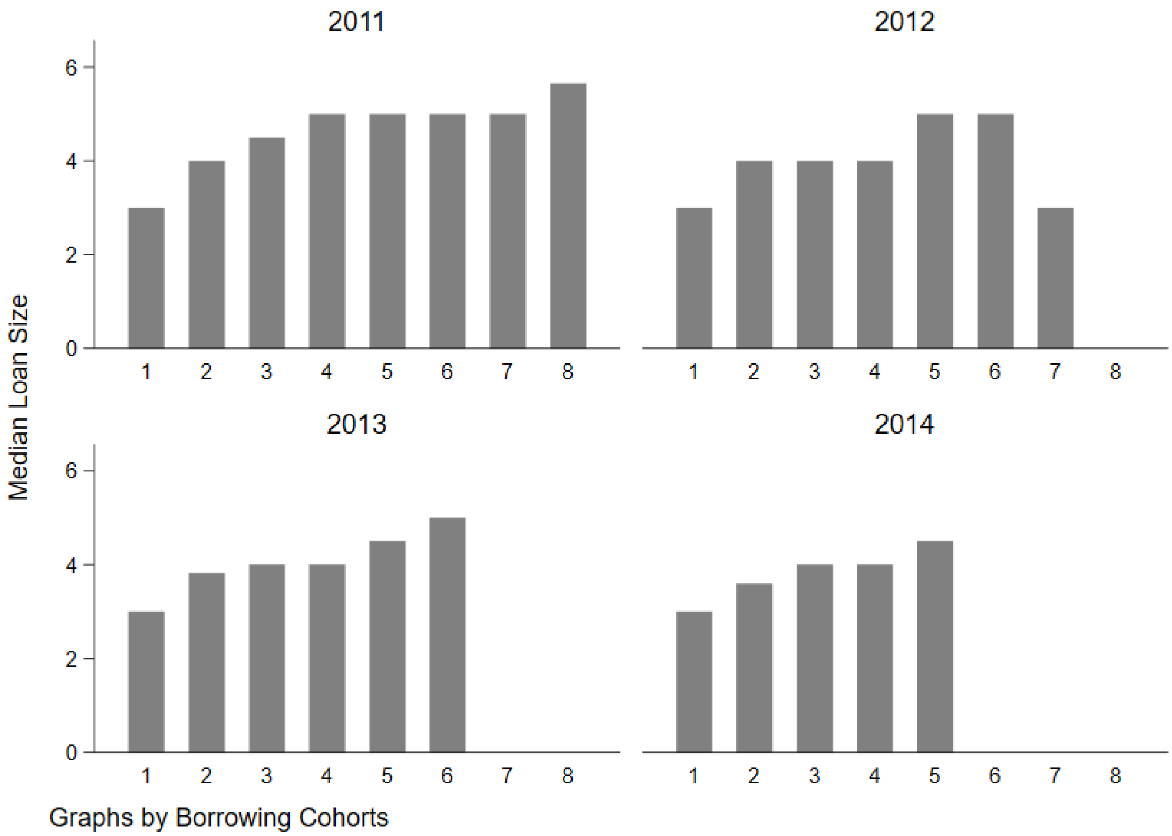
[16]from the bank's point of view

So what about other aspects of loan contracts, say loan size?

Observation 2. *Median loan size increases over the course of the relationship.*

This can be seen in Figure 6, which plots the median loan size over year of relationship by different borrowing cohorts. However, this graph cannot control for the effects of time, so I also regress loan size on year of relationship, firm fixed effects, monthly fixed effects, and types of collateral. The coefficient on years of relationship is 0.161 with t-statistics 2.49.

Figure 6: Median Loan Sizes over Year of Relationship by Cohorts



Observation 3. *Larger loan size is associated with lower interest rates.*

Following the first two observations, which state that interest rates decrease and loan size increase over the course of relationship, it is then natural to consider the correlation between interest rates and loan size. I regress interest rates on loan size conditional on monthly time fixed effects, cohorts fixed effects, region fixed effects and firm observables (industry code, size, ownership type, date of incorporation, initial capital, internal credit rating) given different year in relationship, and the results are shown in Table 3.[17]

[17] I get very similar results when using lending spread instead of interest rates

Column (1) shows the correlation between interest rates and loan size for the initial period. On average, every one million RMB in loan size is associated with 1 basis point decrease in interest rate. Similar results hold for subsequent periods.

Table 3: Regression of interest rates on loan size

	(1)	(2)	(3)
Year in Relationship:	1st Year	2nd Year	3rd Year
Loan Size (in million RMB)	-0.0109***	-0.00989***	-0.0114***
	(-10.18)	(-6.53)	(-8.71)
Observations	8166	5150	3625

Other controls: Monthly FE, Cohort FE, Region FE, Firm observables
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Observation 1-3 supports the selection process interpretation. As the relationship progresses, firms with better quality remains, and they incrementally get better loan terms, like lower cost of borrowing and access to more credit. However, the next observation shows that the progression of loan terms does not happen totally automatically as time pass by; it depends on loan performance, which is summarized in the observed loan rating.

Observation 4. *Loan terms deteriorates and attrition probability increases or following a bad loan rating in the previous year.*

If there is a bad rating (meaning loan rating > 1) in $t-1$ but the firm does not end up defaulting on it, then how will the contract terms on the next period's loan change? I define dummy variable $SizeDown = \mathbf{1}\{Loansize_t < Loansize_{t-1}\}$ and $SpreadUp = \mathbf{1}\{Spread_t > Spread_{t-1}\}$ where spread refers to the lending spread defined previously. Then I use linear regression to check how does the occurrence of a bad loan rating change the probabilities of these two events, respectively.

Table 4: Regression of direction of change in contract terms on bad loan rating

	(1)	(2)
	$SizeDown_t$	$SpreadUp_t$
$BadRating_{t-1}$	0.164***	0.0520*
	(7.23)	(2.33)
Observations	9926	9926

Other controls: Monthly FE, Cohort FE, Region FE, Firm observables, Year in relationship
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4 shows the results. Column (1) shows that the probability that a firm faces a cut on loan size increases by 16.4 percent when the loan rating in previous period is bad, and the probability that a firm faces a higher lending spread increases by 5 percent. Results are similar when I use raw interest rates.

Alternatively, instead of using binary outcomes, I could use the continuous outcome measured by difference between loan size in $t+1$ and t, as well as difference between lending spread in $t+1$ and t. Table 5 shows the results. Column (1) shows that the loan size on average will shrink 455 thousands RMB and lending spread will go up 13.9 basis points after a bad rating happens.

Table 5: Regression of changes in contract terms on bad loan rating

	(1) D.Loansize	(2) D.Spread
L.Bad Loan Rating	-0.455*	0.139**
	(-2.43)	(2.76)
Observations	12242	10895

Other controls: Monthly FE, Cohort FE, Region FE, Firm observables, Year in relationship
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Another aspect regarding occurrence of bad ratings is that, the attrition probability, i.e., the probability that the relationship between a firm and a borrower ended without default, increases immediately after the loan rating turns bad. This can be seen in a regression of probability of attrition (whether the firm no long show up after period t although no default happened) on whether the loan in period t has a bad rating. Table 6 column (1) shows results from this regression. On average, the probability of never showing up after t increases drastically (by 52.3 percent) when the loan rating in period t is bad. I also use the sum of occurrence of bad ratings in the past as explanatory variable in the regression as an alternative specification and find that occurrence of bad ratings in the past in general makes a firm more likely to disappear.

Table 6: Regression of changes in contract terms on bad loan rating

	(1) Attrition	(2) Attrition
Bad Loan Rating	0.523***	
	(49.80)	
Sum of Bad Loan Rating		0.247***
		(36.06)
Observations	24569	24569

Other controls: Monthly FE, Cohort FE, Region FE, Firm observables, Year in relationship
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

These findings reveal loan performance as an important learning channel through which banks updates its perceived riskiness of borrowers and adjust loan contract terms accordingly. Also, it seems at least part of the reason we see such high attrition rates is associated with past loan performance.

Observation 5. *Relatively high past spreads lead to higher probability of attrition.*

Following speculation from the previous observation that probability of attrition might be related to the history of loan ratings and thus perceived riskiness, here I check the association of past lending spreads and probability of attrition, since lending spreads can reflect perceived riskiness. Ideally, I need to measure the relative level of a lending spread compared to what an observationally similar firm would get within the same time and location. Here I use the Empirical Cumulative Distribution Function (ECDF) within lending spread observed in the same calendar year as the measurement, same credit rating class, and same branch (I exclude observations where there are less than 10 loans in a bin).

I conduct logit regressions of the leaving probability on ECDFs of past lending spreads using three specifications: ECDF of the last lending spreads, ECDFs of last two lending spreads, and ECDFs of the first and last lending spreads. Other controls include monthly time effects, years into relationship and firm characteristics. Table 7 presents results from these specifications[18].

Table 7: Logit Regression of Leaving on ECDFs of Past Lending Rates

	(1)	(2)	(3)	(4)
Year of Relationship	-0.0404	-0.0405	-0.0445	-0.0403
	(-0.92)	(-0.92)	(-1.02)	(-0.92)
L.Empirical cdf of lending spreads	0.488***	0.475**		0.493**
	(3.32)	(3.19)		(3.17)
L2.Empirical cdf of lending spreads		0.0697		
		(0.48)		
Initial Empirical cdf of lending spreads			0.143	-0.0150
			(1.02)	(-0.10)
Observations	3602	3602	3602	3602

Other controls: Monthly FE, Cohort FE, Region FE, Firm observables
t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7 shows a relatively high lending spreads on last period's loan is positively correlated with the firm's leaving probabilities. In other words, borrowers with higher perceived risk, which is reflected in lending spreads, are more likely to drop out, even before default happens.

[18]The regression simple is restricted to firms with duration of relationship at least 3 years. I conduct same regressions on lending rates, which yield highly similar results.

4 Model

4.1 Model Description

I describe here the lending market, lending contract, firm's output technologies, firms and bank's information, the timing of decisions, and equilibrium. Briefly, the lending market consists of firms that generate revenues using capital provided by a lender (bank) as the only input, and a short-term lending contract specifies loan size and interest rate. Firms differ in their efficiency type, which scholastically determines their revenue realizations and loan performance in each period. A firm's efficiency type is unobserved to all. But whether this period's revenue realization is successful or not, or *performance*, is observed to all and provides information about firm's ability. Based on updated beliefs after performance realization, the lending relationship can be terminated at the end of each period.

Banks and Firms

I consider a lending market for corporate loans in which two banks, $j = 1, 2$ compete for a firm by offering lending contracts which will be specified below. Time is discrete and indexed by $t \geq 1$. Banks and firms discount the future by the factor $\beta \in [0, 1]$.

Bank j's cost of lending k unit of fund, is $c(x_t, k) + \epsilon_{jt}$ where vector x_t contains exogenous firm characteristics, exogenous loan characteristics (including value of collateral), time and region. The first term $c(x_t, k)$ is the deterministic part of cost of fund which is common to both banks, and ϵ_{jt} is a mean-zero idiosyncratic cost shock specific to the firm-bank-period combination.[19] Distributions of cost shock are identical to both banks. In other words, banks have same lending technology.

A firm is characterized by one of two efficiency types, $\Theta = \{\theta^h, \theta^l\}$ with $\theta^h, \theta^l \in [0, 1]$ and $\theta^h > \theta^l$. Types are unobserved to all (including the firm). Each efficiency level corresponds to a success probability, where the "success" will be described below. I refer to θ^h as the high-efficiency type and θ^l as the low efficiency type.

Lending Contracts

A lending contract in period t consists of (r_t, k_t), where $r_t \in [\underline{r}, \bar{r}]$ denotes 1 plus interest rate, and $k_t \in \mathbb{R}^+$ denotes loan size. Here lending contracts refers to endogenous contract terms, so it does not include value of collateral pledged. The value of pledged collateral per dollar, z_t, is treated as one element of x_t; in other words, I treat value of collateral as given and abstract myself away from modeling collateral choice, since I do not observe the total pledgeable assets of firms. Collateral will be transferred to the bank and then liquidated in the case of default. But only a fraction $\chi < 1$ of the collateralized value could be recovered in the end, which means default is costly to banks. The value of pledged collateral is observed in the data, but not the recovered/recoverable value.

[19] k term and ϵ are additive, which is for estimation convenience. As we will see in Section 5, the value of total matched surplus will also in additive in ϵ.

Revenue and Performance

Output of a firm depends on the amount of capital borrowed from the bank, referred to as *revenue*. I assume that a firm's revenue per capital has two parts: a stochastic part whose support is fixed[20] but probabilities of different realizations are determined by the intrinsic type of the firm; and a deterministic part as a function of observables (say firm characteristics and length of relationship). Specifically, revenue in period t, denoted as y_t, can be expressed as[21]

$$y_t = [a_t + g(x_t)]k_t \tag{1}$$

where a_t is stochastic and its distribution depends on the firm's type, and vector x_t is a deterministic, containing firm characteristics, time and region.

I further assume that realizations of the stochastic part, a_t, is binary: a_t equals $\bar{a}$ with probability θ and $\underline{a}$ otherwise. The event $v_t = \mathbf{1}\{a_t = \bar{a}\}$ is referred to as a "success" performance, and high type has a higher probability of success performance than low type since $\theta^h > \theta^l$. The outcome of v_t is observed to all, which can be used to infer the revenue. In other words, performance serves as a "signal" about the true type of the firm, based on which the posterior beliefs are updated via Bayes rule.

Information and Belief Updating

At the beginning of the first period, banks and firms share a common initial prior belief p_1 that a given firm is of high type. Although here I consider the case of a single prior, in estimation I allow for different priors for different firms. Since performance is observed by all, learning about a firm's type based on a firm's performance is symmetric: banks and firms share the same information, and thus the same belief, p_t at any t.

I interpret the symmetric learning setting as a plausible approximation to the lending market in which the sample bank, with its long history and well-recognized expertise in funding numerous projects for business in various industries, lends to firms. As argued in Manove, Padilla, and Pagano (2001), banks and other financial institutions that fund large numbers of investment projects in a specific sector of the economy are well placed to appraise the potential performance of those projects. In my setting, the initial screening that the bank meticulously conducts at the beginning is an extensive review and assessment of the viability and creditworthiness of the firm, often requiring interview and examination of comprehensive documents. As the relationship commences, the bank has an entire department specializing in actively monitoring and updating information about the conditions of borrowing firms; they are even allowed to send bankers inside the firm to verify important information. Thus information asymmetry between a bank and a firm might be limited. As for potential inter-bank information asymmetry, the credit report provided by the regulator is one important source of information that banks rely on, and such reports are directly aimed at eliminating asymmetry of credit information possessed by different banks.

[20]If support depends on types then types will be perfectly revealed just after one period.

[21]There is no productivity shock, since shocks to revenue would be not distinguishable from the cost shock since we do not directly observe either of them.

At the end of period $t-1$ after revenue (or performance) realizes, banks and firms update beliefs about a firm's type according to Bayes rule, which, for given p_{t-1}, leads to two possible values of p_t depending on performance at $t-1$, v_{t-1}:

$$p_t = \frac{p_{t-1}\theta^h}{p_{t-1}\theta^h + (1-p_{t-1})\theta^l} \qquad \text{if } v_{t-1} = 1; \tag{2}$$

$$p_t = \frac{p_{t-1}(1-\theta^h)}{p_{t-1}(1-\theta^h) + (1-p_{t-1})(1-\theta^l)} \qquad \text{if } v_{t-1} = 0. \tag{3}$$

Note that the difference between θ^h and θ^l implies the *speed of learning* about types, ranging from no learning if $\theta^h = \theta^l$ and complete learning if $\theta^h = 1$ and $\theta^l = 0$.

Default

Let $d_t = 1$ denote default at the end of period t. I assume the conditional probability of default given a bad revenue realization ($v_t = 0$), is an *exogenous* function of firm observables, $p^d(x_t)$; and the conditional probability of default following a successful realization ($v_t = 1$) is zero. Thus the expected default probability at the beginning of period t can be expressed as $(1-p_{t-1})p^d(x_t)$, where $(1-p_{t-1})$ is the expected probability of a bad revenue realization ($v_t = 0$) based on current beliefs p_{t-1}.

At first it may seem strange that interest rates do not effect default probabilities, but in some cases it can be seen as a reasonable approximation of the reality: suppose the return on loan in the case of a bad revenue realizations (i.e., $\underline{a}$) is even lower than 1, then firms have to resort to other financing channels if they want to repay this loan; the probability that they are able to re-finance and repay may depend mostly on luck, connections, other assets, cash reservoir, etc, and the interest rate itself may not lie on the margin of whether the firm can repay or not.

Endogenous Termination and Exogenous Separation

It is possible that, for a firm perceived as somewhat suspicious at the beginning of period t, another bad performance realization in period t would turn posterior beliefs about its type so adverse that its current bank no longer wants to lend to it in the future. Here I allow the bank to deny future credit access based on this period's performance, which I call the endogenous termination.

Let $e_t \in \{0, 1\}$ denote endogenous termination, which indicates whether the bank will deny all future loans to the firm when this period's performance turns out to be bad ($v_t = 0$). In that case, the firm must turn to the other bank j' for loans, which makes j' the monopoly lender. Then j' would make contract such that the firm is indifferent between borrowing and not borrowing, where the utility of not borrowing is normalized to 0. So the firm's value of losing one bank is 0, same as not borrowing. Thus termination of lending relationship with one bank is *as if* the firm lost all access to future loans. This endogenous termination is helpful in explaining the observed declining dropout rates among no-defaulting firms over time, because the longer the relationship is, the less sensitive the posterior belief is to new information, and thus the endogenous termination is less likely to happen.

This model also allows for exogenous separation that is due to reasons other than default or endogenous termination, which is denoted by an indicator variable ζ_t. For example a firm might switch to other channels of financing for some exogenous reasons. In data it is not uncommon to see some firms leave the sample without any bad loan performance. To account for separations that cannot be fully explained by endogenous termination as predicted by the model, I assume an idiosyncratic separation probability ρ in each period. Separation is permanent and the value of separation is normalized to 0 to both the firm and banks.

Switching cost

The observed "stickiness" of relationships is hard to rationalize if a firm could switch costlessly between the two banks, since the probability that one bank's cost shock is lower than the other for several consecutive periods is low. Also, switching cost in banking industry is a well-documented feature, and shown to be a determinant for maintaining the long-term relationship between customers and banks (Kiser, 2002; Calem et al., 2006; Ho, 2015). So in this model I assume that if the firm chooses a bank different from the one it chose last period, it has to pay a switching cost ω upfront.

Timing

The timing assumption is the following. In the beginning of each period, cost shocks are realized, and the two banks simultaneously[22] submit offers to borrowing firms which consists of an interest rate and loan size. Then, each firm decides which offer to accept. Next, the loan is taken, revenue is realized, beliefs are updated, loan outcome (default or not) and exogenous separation is realized. (In the case of default, the pledged collateral will be liquidated and bank financing is terminated.) Lastly, the bank decides whether it will terminate the relationship. If none of the termination, default, or separation happens, then the game carries on to the next period.

Without loss, I focus on the competition between the two banks for one firm. In this component game, the events in t are $(\epsilon_t, r_t, k_t, l_t, v_t, d_t, \zeta_t, e_t,)$: $\epsilon_t = \{\epsilon_{jt}\}$ is the vector of both bank's cost shocks; $(r_t, k_t) = \{r_{jt}, k_{jt}\}$ is the vector of each bank's lending contract offer; $l_t = \{l_{jt}\}$ is the vector of the bank's decisions to accept bank j ($l_{jt} = 1$), or reject ($l_{jt} = 0$); performance v_t is an indicator of whether realized revenue is high ($v_t = 1$) or low ($v_t = 0$); d_t is an indicator for whether default happens; ; ζ_t is an indicator of whether exogenous separation happens; $e_t = \{e_{jt}\}$ is the vector of bank's decisions to terminate the relationship at the end of t ($e_{jt} = 1$) or not ($e_{jt} = 0$). The component games ends if any one of d_t, ζ_t, and e_t equals 1.

[22] This means that I abstract away from search and do not allow first-mover advantage to the incumbent. This assumption might be rationalized by thinking of all firms as savvy shoppers of credit: they actively reach out to multiple banks in each period, presenting a comprehensive document of their credit history and financial status to them and letting them compete.

4.2 Equilibrium

The state that banks face each period when they make lending contract offers is (s_t, ϵ_t), where $s_t = (p_{t-1}, x_t, l_{t-1})$, p_{t-1} the latest posterior belief that the firm is of high type, x_t is observed characteristics, $l_{t-1} = \{l_{jt-1}\}$ is the firm's bank choice in last period, and $\epsilon_t = \{\epsilon_{jt}\}$ collects all realized cost shocks. The state the firm faces when choosing among lending contracts consists of (s_t, ϵ_t) and banks' strategy, $(r_t, k_t, e_t) = \{r_{jt}, k_{jt}, e_{jt}\}$.

The equilibrium concept used here is *cautious* Markov perfect equilibria. An equilibrium consists of offer strategies $r_{jt} = r_j(s_t, \epsilon_t)$, $k_{jt} = k_j(s_t, \epsilon_t)$ and termination policy $e_{jt} = e_j(s_t, \epsilon_t)$ for each bank j, and an acceptance strategy $l_t = l(s_t, \epsilon_t, r_t, k_t, e_t)$ for the firm with element $l_{jt} = l_j(s_t, \epsilon_t, r_t, k_t, e_t)$ for each j, and belief updating rules $p_{t+1}(p_t, v_t)$ such that in each period:

(i) the firm maximizes the (expected present discounted) value of profits;

(ii) both banks maximize the (expected present discounted) value of profits;

(iii) the non-lending bank is indifferent between lending and not lending to the firm at the loan size that maximizes its (expected present discounted) value of profits;

(iv) beliefs are update as in (2)-(3).

A last piece of notation that needs to be introduced before analyzing the firm and bank's equilibrium strategies is the expected output $y(s_t, k_{jt}) \equiv E\left[y_t | p_{t-1}, x_t, k_{jt}\right]$, where the expectation is taken over this period's revenue performance given posterior belief p_{t-1}, characteristics x_t and bank j's offer of loan size k_{jt}.

Firm's Strategy

Given the banks' strategies, the firm's strategy satisfies

$$
\begin{aligned}
W(s_t, r_t, k_t, e_t) = \max_{j=1,2} \sum_f l_f \Bigg\{ & y(s_t, k_{jt}) - \omega(1 - l_{jt-1}) \\
& + p_t \Big[- r_{jt} k_{jt} + \beta\rho EW(s_{t+1}, r_{t+1}, k_{t+1}, e_{t+1} | s_t, v_t = 1, l_{jt} = 1) \Big] \\
& + (1 - p_{t-1}) \left[1 - p^d(x_t)\right] \Big[- r_{jt} k_{jt} \\
& \qquad + \beta\rho(1 - e_{jt}) EW\big(s_{t+1}, r_{t+1}, k_{t+1} | s_t, v_t = 0, l_{jt} = 1\big) \Big] \\
& - (1 - p_{t-1}) p^d(x_t) z_t k_{jt} \Bigg\}
\end{aligned}
$$

where the conditional expectation $EW(\cdot | s_t, v_t, l_{jt})$ is over next period's cost shocks conditional on this period's performance and bank choice. The beginning-of-period state next period is $s_{t+1} = \left(p_{t+1}\left(p_t, v_t = 1\right), x_{t+1}, l_t\right)$ or $s_{t+1} = \left(p_{t+1}\left(p_t, v_t = 0\right), x_{t+1}, l_t\right)$ if good or bad performance is realized.

The expression above shows net revenues for the firm under different scenarios. The first line is the expected revenue minus switching cost if the firm switches, the second line corresponds to the case of successful realization where the firm would repay the principal and interests and enjoy the continuation value of the lending relationship conditional on successful performance, the third and forth line is the case of unsuccessful realization without default where the firm still repay the loan but only continues if the relationship is not terminated, and the fifth line is the default case where the firm would lose the value of pledged collateral and no future bank financing would be available.

To simplify the equations above, define probability of continuing to the next period as

$$\eta(s_t, e) \equiv p_t + (1 - p_{t-1}) \left[1 - p^d(x_t)\right] (1 - e) \tag{4}$$

then we can re-write the firm's value function as:

$$\begin{aligned} W(s_t, r_t, k_t, e_t) = \max_{j=1,2} \sum_f l_f \Big\{ & y(s_t, k_{jt}) - \omega(1 - l_{jt-1}) + (1 - p_{t-1}) p^d(x_t) \left[r_{jt} - z_t\right] k_{jt} \\ & - r_{jt} k_{jt} + \beta \rho \eta(s_t, e_{jt}) EW\big(s_{t+1}, r_{t+1}, k_{t+1} | s_t, d_t = 0, l_{jt} = 1\big) \Big\} \end{aligned}$$

where the conditional expectation $EW(\cdot|s_t, d_t = 0, l_{jt})$ is over both this period's performance and next period's cost shocks conditional on this period does not default.

Banks' Strategy

Note that in this model, the choice of interest rate and loan size is static, i.e., they only affect the bank's current profit (conditional on other players' strategies). Loan offers also do not change the (expected) probability of bad performance or default. In other words, the decision of loan offer and the decision of termination are independent of each other. Thus, an equivalent representation of a bank's problem is one where the bank chooses r, k and e *simultaneously* at the beginning of each period.

Given the firm's and the competitor's strategies, bank j's strategy satisfies

$$\begin{aligned} \Pi^j(s_t, \epsilon_{jt}) = \max_{e,r,k} l_{jt} \Big\{ & - c(x_t, k) - \epsilon_{jt} + k_{jt} \Big[r_{jt} - (1 - p_{t-1}) p^d(x_t) \big(r_{jt} - \chi z_t\big)\Big] \\ & + \beta \rho \eta(s_t, e) E\Pi^j\big(s_{t+1}, \epsilon_{jt+1} | s_t, d_t = 0, l_{jt} = 1\big) \Big\} \\ + l_{j't} \Big\{ & \beta \rho \eta(s_t, e_{j't}) E\Pi^j\big(s_{t+1}, \epsilon_{jt+1} | s_t, d_t = 0, l_{j't} = 1\big) \Big\} \end{aligned} \tag{5}$$

where $l_{jt} = l_j(s_t, \epsilon_t, r_t, k_t, e_t)$ is the firm's acceptance decision in t, $\eta(\cdot)$ is the probability of continuing to the next period defined in (4), and $E\Pi^j\big(\cdot\,|s_t, d_t = 0, l_t\big)$ is the expectation over performance and next period's cost shocks conditional on the firm's bank choice (either choose bank j or not) and default does not occur this period.

The last line in (5) corresponds to the situation where the firm chooses the other bank j'. More specifically, the bank takes into account the option value of not lending to the firm in the current period and attracting him back in some future period, which arises from the information revealed by performance at the competitor.

The tradeoff around choosing loan size is that, larger loan size means larger total cost, but it also leads to larger total revenue for the firm which implies the bank can charge higher interest rates. The choice of interest rates follows the familiar intuition from Bertrand competition. And the termination might be desirable when the belief about a firm's type after getting one more bad realization will be so bad that expected profits on future loans will be negative. In that case the bank would rather terminate the lending relationship avoid expected future loss.

Cautious MPE

The equilibrium loan contract offered by the lending bank (or winning bank), say j, must makes its value of lending no lower than not lending, because otherwise j would not want to lend. But the losing contract can not be uniquely pinned down without further refinements. This is because in equilibrium, given the firm's strategy of not choosing j', bank j' can choose a contract with interest rate low enough that its value of lending is even lower than not lending, but not too low to cause j not willing to match. This is the same problem of multiplicity of equilibria as in the static Bertrand game with different costs.[23]

The refinement I adopt here is called a *cautious* equilibrium, as proposed by Bergemann and Välimäki (1996). The idea is simple, the losing bank should be cautious enough to offer a contract for which he does not regret should he be chosen against all expectations. Formally, it means that the losing bank should be indifferent between lending and not lending. That is,

$$
\begin{aligned}
&\beta\eta(s_t, e_{j't})E\Pi^j\big(s_{t+1}, \epsilon_{jt+1}|s_t, d_t = 0, l_{j't} = 1\big) \\
&= \max_{e,r,k}\Big\{ - c(x_t, k) - \epsilon_{jt} + k_{jt}\Big[r_{jt} - (1 - p_{t-1})p^d(x_t)\big(r_{jt} - \chi z_t\big)\Big] \\
&\qquad + \beta\rho\eta(s_t, e)E\Pi^j\big(s_{t+1}, \epsilon_{jt+1}|s_t, d_t = 0, l_{jt} = 1\big)\Big\}
\end{aligned} \tag{6}
$$

where the left hand side of (6) is the bank j's value if the firm chooses the other bank, j', and the right hand side is bank j's value if the firm chooses bank j.

This requirement rules out uninteresting rejected offers by the losing bank in which the left hand side of (6) is strictly higher than the right side; that is, the losing bank strictly prefers losing to winning the firm. (Offers such that the right side is strictly higher than the left side are already ruled out by the profit maximization principle of the losing bank.) By pinning down the losing bank's offer (and imposing regularity assumptions on production function), this requirement implies that equilibrium is unique.

[23]In a static Bertrand game where firms have different costs $c_1 < c_2$, the "standard" equilibrium is $p_1 = p_2 = c_2$ and the low cost firm is making the sale. However any price combination $p_1 = p_2 \in [c_1, c_2)$ can also be sustained as an equilibrium if the consumer chooses the low cost producers with probability one.

4.3 Characterization of Equilibrium

The cautious MPE of the dynamic duopoly model outlined above takes simple forms. This may not seem surprising given this model has two important features: i) banks are symmetric; and ii) loan size and interest rates do not affect loan outcomes or future values. Based on these assumptions and equilibrium conditions, it can be shown that in equilibrium a bank chooses loan size and termination *as if* it was maximizing the total matched value between the bank and the firm conditional on the bank wins. And given equilibrium loan size and termination rule, interest rates are determined via a similar logic as static Bertrand competition with a twist from incumbency advantage induced by interaction between switching cost and learning process. The choice of bank boils down to compare difference of cost shock with switching cost.

Equilibrium Loan Size and Termination Rule

The duopoly competition between the bank together with refinement (6), induces each bank to correctly internalize the firm's revenue and so to maximize the combined surplus regardless of whether the firm actually chooses the bank or not. The combined surplus, or total matched value, between the firm and bank j conditional on bank j is chosen can be expressed as:

$$\bar{V}^j(s_t, \epsilon_{jt}) = \max_{e,k} \Big\{ y(s_t, k_{jt}) - \omega(1 - l_{jt-1}) - c(x_t, k) - \epsilon_{jt} - (1 - p_{t-1})p^d(x_t)\,(1 - \chi)\, z_t k$$
$$+ \beta\rho\eta(s_t, e) E\big[\bar{V}^j(s_{t+1}, \epsilon_{jt+1}) \big| s_t, d_t = 0, l_{jt} = 1\big] \Big\} \tag{7}$$

In (7), the first line shows the matched value in the current period, which is revenue minus switching cost (if there is any), cost of lending, and expected cost associated with default $(1 - p_{t-1})p^d(x_t)\,(1 - \chi)\, z_t k$. And the second line is the continuation value.

It is clear from (7) that the optimal loan size is chosen statically, which gives equilibrium k_{jt} a closed-form solution. Specifically, the equilibrium loan size equate the marginal revenue, $\partial y(s_t, k)/\partial k$, with the marginal cost of lending, $mc(x_t, k) \equiv \partial c(x_t, k)/\partial k$, as well as the expected cost associated with collateral repossession, $(1 - p_{t-1})p^d(x_t)(1 - \chi)z_t$.

The termination rule that maximize $\bar{V}^j$ is determined by whether the continuation value conditional on bad realization in this period ($v_t = 0$) is negative or not. Intuitively, when posterior beliefs are unfavorable enough, it is better to stop financing than to carry on the relationship, because the latter would imply negative future surplus for them.

Formally, the results are summarized in the following proposition:

Proposition 1 (Equilibrium Loan Size and Termination Rule)**.** *Given x_t, bank j's the equilibrium loan size k_{jt} and termination rule e_{jt} solve the problem in (7). Specifically, the equilibrium loan size offered by bank j is given by*

$$k_{jt} = mc^{-1}\Big(p_t\bar{a} + (1 - p_t)\underline{a} + g(x_t) - (1 - p_{t-1})p^d(x_t)(1 - \chi)z_t; x_t\Big) \tag{8}$$

where $mc^{-1}(\cdot;x_t)$ is the inverse function of $mc(x_t,k)\equiv \partial c(x_t,k)/\partial k$ with respect to k.

And the equilibrium termination policy offered by bank j is given by

$$e_{jt} = \mathbf{1}\Big\{E\big[\bar{V}^j(s_{t+1},\epsilon_{jt+1})\big|s_t, v_t=0, l_{jt}=1\big]<0\Big\} \tag{9}$$

It is worth noting that although the model allows competition on both interest rates and loan sizes, *in equilibrium* banks offer the same loan size and differ only in interest rates. Realizations of shocks and incumbency advantage are only reflected in differences of interest rates, not on loan sizes.

Equilibrium Interest Rate

The planning problem in Proposition 1 eliminates interest rate since it is an internal transfer that does not affect the total matched value between a bank and a firm conditional on the bank is chosen. Determination of equilibrium interests comes from the price competition together with the requirement of cautious equilibrium. Specifically, the price competition between banks in each period makes the stage game similar to a static Bertrand pricing game in which banks have different costs. The equilibrium price for the lending bank is thus sufficiently high that the competitor cannot match it and obtain positive profits in this period. The competitor is also cautious so that the losing offer match its value of not lending.

These conditions imply that in equilibrium, the firm is indifferent between choosing j and j', and that the losing bank is indifferent between lending and not lending. It can be shown that equilibrium interest rates can be pinned down by these indifference conditions. And the results are summarized in the following proposition:

Proposition 2 (Equilibrium Interest Rate)**.** *Suppose bank j is chosen, i.e., $l_{jt}=1$.*

If the winning bank j is the incumbent, i.e., $l_{jt-1}=1$, then interest rate offered by j is

$$r_{jt} = \big[1-(1-p_t)p^d(x_t)\big]^{-1}\left\{\frac{1}{k_{jt}}\big[c(x_t,k_{jt})+\epsilon_{j't}+\big(1-\beta\rho\eta(x_t,e_{jt})\big)\omega\big]-(1-p_t)p^d(x_t)\chi z_t\right\} \tag{10}$$

and the losing interest rate is:

$$r_{j't} = \big[1-(1-p_t)p^d(x_t)\big]^{-1}\left\{\frac{1}{k_{jt}}\big[c(x_t,k_{jt})+\epsilon_{j't}-\beta\rho\eta(x_t,e_{jt})\omega\big]-(1-p_t)p^d(x_t)\chi z_t\right\} \tag{11}$$

If the winning bank j is not the incumbent, i.e., $l_{jt-1}=0$, then interest rate offered by j is

$$r_{jt} = \big[1-(1-p_t)p^d(x_t)\big]^{-1}\left\{\frac{1}{k_{j't}}\big[c(x_t,k_{j't})+\epsilon_{j't}-\big(1+\beta\rho\eta(x_t,e_{j't})\big)\omega\big]-(1-p_t)p^d(x_t)\chi z_t\right\} \tag{12}$$

where k_{jt} and e_{jt} are given by (8) and (9) respectively. And the losing interest rate is the same as in (11).

The first observation is that, in both cases the incumbent's offer is always higher than the non-incumbent's offer by ω, then discounted by expected default probability, i.e., the denominator $\left[1-(1-p_t)p^d(x_t)\right]^{-1}$. This is a direct result from the firm's indifference condition. Since the firm always need to pay ω when it switches, the incumbent can then exploit the switching cost.

Also, the losing offers in both cases are same, since they are both determined by the indifference condition (6), where the value of not lending does not change with whether the losing bank is the incumbent. In other words, the interest rate in (11) makes the bank indifferent between lending and not lending. Note that this interest rate is lower than the "break-even" interest rate that makes the *current* profits zero:

$$\left[1-(1-p_t)p^d(x_t)\right]^{-1}\left\{\frac{1}{k_{jt}}\left[c(x_t,k_{jt})+\epsilon_{j't}\right]-(1-p_t)p^d(x_t)\chi z_t\right\}.$$

This is because in this dynamic game banks also consider expected future value, and the incumbent has higher expected future value than the non-incumbent by exactly ω, the switching cost. Thus, a bank is willing to sacrifice some of the current profits in order to gain the incumbency status and higher future value. However, how much it is willing to sacrifice also depends on discount factor β and the expected probability that the firm even has a future, i.e., the expected probability of continuing, $\rho\eta(x_t,e_{jt})$. Thus, it is willing to lower the offer by $\beta\rho\eta(x_t,e_{jt})\omega$ (then discounted by expected default probability, i.e., the denominator) to trade off the future profits.

The interaction between imperfect competition and learning process is embodied in this term, $\beta\rho\eta(x_t,e_{jt})\omega$. Firms with adverse beliefs have lower p_{t-1}, i.e., the expected probability of a success performance, and higher probability of being terminated ($e_{jt}=1$), which makes the probability of continuing $\eta(x_t,e_{jt})$ lower. Thus, the markdown that banks are willing to offer, $\beta\rho\eta(x_t,e_{jt})\omega$, is also smaller. This means that these firms face larger consequence from imperfect competition.

Equilibrium Bank Choice

It can also be shown that, the equilibrium choice of bank is the one that generates the largest sum of values to all. Since Proposition 1 establishes that both banks have the same choice of loan size and termination rule, and that cost shocks are independent, then the optimal choice of bank is reduced to comparing the differences of temporary cost shocks with the switching cost. In other words, the firm switches bank if and only if the outside bank's temporary cost shock is low enough to justify the switching cost.

Proposition 3 (Equilibrium Bank Choice)**.** *If the firm chooses bank j in $t-1$, i.e., $l_{jt-1}=1$, then it stays with the same bank in t if $\epsilon_{jt}-\epsilon_{j't}\leq\omega$.*

5 Empirical Strategy

Here I discuss assumptions maintained in estimation, (preliminary) identification argument, and estimation strategy. Since the data is on only one bank, all the variables below refers to the ones I observe at the sample bank, so I omit subscript j here.

Loan Size

The model predicts k as a continuous function of expected output and cost, although in the data the observed loan size are mostly whole numbers with bumps around 10,15,20,etc. Also note that there is no error term in the equation of equilibrium loan size (8), since there is no randomness in marginal output and marginal cost. This means that it would be hard for the predicted k to fit the observed loan size without any further econometric assumptions.

As in common in the literature, I assume the loan size is measured with error. Specifically, denote the observed loan size at t as k_t^o, and assume the observed loans size k_t^o falls into L discrete categories $k_1^o, k_2^o, .., k_L^o$ depending on the sum of true (but unobserved) k_t and a measurement error u_t:

$$k_t^o = \begin{cases} k_1^o & \text{if } k_t + u_t \leq \mu_1 \\ k_2^o & \text{if } \mu_1 < k_t + u_t \leq \mu_2 \\ & \vdots \\ k_L^o & \text{if } k_t + u_t > \mu_{L-1} \end{cases} \tag{13}$$

The measurement error term is assumed to have standard logistic distribution, and the end points can be estimated using ordered logit.

Initial Prior Belief

As mentioned in the descriptive analysis, there is substantial variation in initial loan contract terms, which would be hard to explain if I assume the initial priors are common across all borrowers. Also, in reality the sample bank does acknowledge differences of first-time borrowing firms since the bank always performs a broad initial screen to assess the current condition of the firm. Thus, I allow firms to differ along dimensions that are known to firms and banks, possibly through the initial screening, but are unobserved to the econometrician. Examples of these sort of attributes are political connections, personal relationship between the firm's manager and the local loan officers, etc. As in Crawford and Shum (2005) and Pastorino (2019), I assume that each firm is of initial bracket $b = 1, 2, ..., B$, which determines the initial prior about the firm's efficiency type. In other words, firms with the same initial bracket b share the same prior belief p_{b1}. Let q_b be the probability of bracket b, then both $\{p_{b1}\}$ and $\{q_b\}$ are parameters to be estimated.

Performance

As in Pastorino (2019), this paper also has direct evidence of performance, which is the loan rating data. The ratings are classified according to the borrower's *actual* repayment ability at that point in time. Specifically, banks have to asses how well the borrowing firm's operation is by analyzing its cash flows, changes in revenue and profits, and other aspects of financial status. The loan ratings become abnormal when the firm show signs of financial distress, even *before* delinquency happens (Gao et al., 2019b). In other words, it directly links the observed performance ratings to how successful the revenue realizations are. Thus, I assume the performance variable v_t is observed, with $v_t = 1$ meaning the loan rating for the firm's loan in this period is good (i.e., rated level 1), and $v_t = 0$ meaning the loan rating in this period is abnormal (i.e., rated level 2 to 5).

Cost Shocks

I assume that cost shocks ϵ_{jt} are Gumbel distributed with location $-\sigma\gamma$ and scale σ.[24]

Identification

The parameters to identify are:

(i) the parameters of the learning process, which is completely described by the distribution of performance, $\{\theta^h, \theta^l\}$, and of initial priors, $\{p_{b1}\}$and $\{q_b\}$, by (2) and (3);

(ii) the parameters of expected output $\underline{a}$,$\bar{a}$, $g(\cdot)$, switching cost ω, conditional probability of default $p^d(\cdot)$ and exogenous separation probability ρ;

(iii) the parameters of cost of fund $c(\cdot)$, repossession cost χ, and distribution of cost shocks σ;

(iv) the parameters of observed loan size distribution $\mu_1, ..., \mu_{L-1}$.

A challenge is to identify a model of lending market from data on only one bank. I do so by imposing symmetry of lending technology and information acquisition between banks, which implies that the law of motion of beliefs about the firm's type can be recovered just from information on loan performance data at one bank. Conditional on each type's history of loan performance, repeated information on interest rate and loan size can be used to identify parameters related to expected output and cost of fund.

As for competition, in the Bertrand setting I focus on, the interest rate that a firm pays to the sample bank in equilibrium can reflect the cost shocks of it competitor, which allows the distribution of cost shocks to be identified. And the difference between initial interest rate and interest rate later in the relationship can identify the switching cost.

Estimation Strategy

For a convenience in estimation, I make the following functional form assumptions:

(i) cost of fund $c(x_t, k) = C_0 + C_1(x_t)k + C_2(x_t)k^2$;

[24] Then the mean of cost shocks is 0. γ is the Euler constant.

(ii) $g(x_t) = \Gamma' x_t$ is a linear function of characteristics;

(iii) conditional probability of default $p^d(\cdot)$ is a logistic function of characteristics;

An immediate implication from (i) is that $mc(x_t, k) = C_1(x_t) + 2C_2(x_t)k$ so now $mc^{-1}(\cdot; x_t)$ in (8) becomes linear, i.e., the equilibrium k_t becomes a convenient linear function of expected outputs and expect cost from default.

We can plug the equilibrium k into the Bellman equation for the total matched value between the firm and the bank, $\bar{V}^j(\cdot)$, as in (7), in which the only choice variable is termination policy and the problem becomes an optimal stopping problem. Note that it is not necessary to fully solve the value function in order to estimate related primitives, since we can utilize insights from Hotz and Miller (1993) and identify parameters using CCP approach. This can be done using the subsample of firms who has a bad loan rating but does not end up defaulting, because only for them we can tell whether $e = 1$ or $e = 0$. This sub-sample will not introduce bias, because termination rule e is chosen at the beginning of t, with only unobserved factor affecting the choice of e is this period's cost shock, and the occurrence of bad loan rating (performance) and default are all independent of this period's cost shock *conditional on* observed state, including histories of contract terms and performances.

However, $\bar{V}^j(\cdot)$ does not really contain the switching cost because I rarely see firms who have bad loan ratings in the first period they borrow. But as mentioned earlier, we can utilize equations for equilibrium interest rate (10) and (11) to form likelihood of observed interest rates in the initial period and subsequent periods. By maximizing this likelihood function, we can estimate the switching cost parameter.

Lastly, any attrition I see that cannot be explained by the switching behavior induced by cost shocks as in Proposition 2 can be attribute to exogenous separation. And this exogenous separation probability can be easily estimated using MLE after estimation of the distribution of cost shocks and switching costs.

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