Bailouts, risk taking and systemic risk in a stochastic dynamic game

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Abstract

Bailouts increase moral hazard and exacerbate risk taking (strategic effect). However, they also decrease the probability of actual failure, thereby increase firm value, which decreases the individual incentive to take risk (charter value effect). In my model, the interplay of these countervailing effects determine market structure, while market structure dynamics in turn drives the interplay. The strategic effect dominates in concentrated markets, but firms take less risk in fragmented markets in the presence of bailouts. The overall effect of bailouts on systemic risk in steady state depends on competitive and entry conditions. Contrary to conventional wisdom, bailouts can reduce systemic risk overall.

Keywords: Systemic Risk, Bailouts, Dynamic Game, Oligopoly, Industry Dynamics, Numerical Analysis

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1 Introduction

Systemic risk is the threat that the market becomes dysfunctional. When policy makers consider a market to be fundamentally important (for macroeconomic and/or political reasons), they often

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bail out firms to prevent the market from collapsing. Systemic bailouts are not limited to the banking industry: in the past the state has provided financial support to e.g. money market mutual funds, insurance companies (AIG) or the "big three" automobile manufacturers (Chrysler, Ford, General Motors).[1] Furthermore, state support does not have to be financial assistance, it can take other forms. For instance, consider the audit market: since its indictment brought Arthur Andersen down in 2002, it is often speculated that the remaining four auditors are just "too-few-to-fail" and hence regulators have shied away from firm actions against Big Four auditors on a number of occasions (e.g. Business Week 2003, The Economist 2004 and 2005, WSJ 2004, NYT 2005, Forbes 2013, FT 2015 and 2017).[2] While the forms of systemic bailouts can be different, bailouts are a fundamental concern in all markets for the same reason: insulating firms from the risk of failure exacerbates moral hazard and thereby increases systemic risk.

Policy makers have long argued that anticipated systemic bailouts increase moral hazard.[3] Indeed, strategic complementarities can emerge, because firms anticipating bailouts understand that their ultimate survival also depends on rivals' survival strategies: "if we all fail, we don't fail... so if we all take much risk, we don't actually take much risk".[4] However, there is a countervailing effect at work: systemic bailouts decrease the probability of actual failure and thereby increase the (charter) value of firms, which in turn reduces the individual incentive to take risk.[5] The overall effect of bailouts is therefore ambiguous at both individual firm and systemic level. At firm level, I show that which effect dominates depends on market structure: in the presence of anticipated systemic bailout, firms in concentrated markets take more risk, but they take less in fragmented markets compared to what they would do if governments never bailed out firms. At the systemic level, risk taking fuels firm failures, which (along with entry) drives market structure

[1] There is a push at the Financial Stability Board to extend the existing framework of 'Systematically Important Financial Institutions', currently covering banks and insurers, to all types of financial institutions (FSB 2015).

[2] One cannot help but wonder whether the three leading Credit Rating Agencies, which apply the exact same business model as auditors, emerged unharmed from the recent financial crisis because of similar concerns.

[3] For instance, Brandao-Marques et al (2013) find empirical evidence for higher expected government support increasing banks' risk taking.

[4] There is much anecdotal evidence that firms take risk in a strategic space by taking into account rivals' behaviour. In banking for instance, the famous comment of Chuck Prince (former CEO of Citigroup) in 2007 is a much quoted example: "As long as the music is playing, you've got to get up and dance".

[5] The so-called charter value effect dates back to the seminal work of Keeley (1990), which argued that the introduction of competition reduced charter values and caused banks to take more risk. See also, e.g. Cordella and Yeyati (2003) or Gropp et al (2011), which emphasise the role of the charter value effect in mitigating the effect of moral hazard in banking.

dynamics in my model. I calculate measures of expected systemic risk in steady state and find that in some parameter regions, the model confirms conventional wisdom: bailouts make the market more systematically risky. However, the model also cautions in two important ways. First, in some important parameter regions (low demand and sluggish entry) a market with bailouts can exhibit *less* systemic risk. Second, even when the market with bailouts is more systematically risky, the difference in expected systemic risk is often not large, which highlights the important counteracting effect of firm value. Overall, the analysis suggests that bailouts may not be overly concerning from a systemic risk point of view.[6]

To fully analyse the implications of systemic bailouts, several modelling features are indispensable. First, because risk taking involves a dynamic trade off between instantaneous profits and survival in a strategic context, the framework of a game with infinite horizon is important. Second, competition, entry, market structure, and risk taking must all be determined endogenously in order to capture the rich set of mechanisms at play and to develop a deeper understanding of industry dynamics.[7] Therefore, I employ and extend the model of Tóth (2012), where firms face a trade-off typical in financial markets: lower profits today can ensure higher probability of survival tomorrow. In my model, the benefit of survival depends on future profits, which are in turn determined by how many other firms will likely compete with the firm in the future. In short, a firm "gambles" on its survival knowing that rivals do just the same: it chooses its survival probability taking into account its current and future rivals' surviving strategies, the entry process and the resulting industry dynamics. I introduce systemic bailouts by assuming that if the system (i.e. all incumbent firms) fails, then firms will be bailed out and can start afresh in the next period. I look at how the presence of bailouts changes strategies and show that bailouts introduce strategic complementarities in risk taking. That is, when rivals take more risk, the firm anticipating systemic

[6]The paper analyses the effect of bailout policies on (individual and systemic) risk and leaves aside important issues, such as the direct costs of systemic bailouts, which are typically (but not always) large.

[7]There is long standing research into how the triad of competition, market structure and risk are related in banking. Empirical as well as the theoretical literature have conflicting findings (see surveys e.g. Beck 2008 or VanHoose 2017). Schaeck et al (2009) suggests that both market structure and competition can have independent effects on risk taking. The relationships among these three factors are further complicated by the fact that in banking competition and market structure may be related in an unusual way: Claessens and Laeven (2004) found a positive relationship between competition and market structure. It is therefore important to analyse the incentives of risk taking in a setup, where competition and market structure are two distinct concepts and determined endogenously, as in my model.

bailout also has an incentive to increase its risk exposure. I call this reaction the strategic effect. It turns out that the balance between the strategic and charter value effects critically depends on market structure.

Risk creates a dynamic trade off between instantaneous profit and survival. In the analysis, I focus on this trade off in a highly stylised model and do not model the banking business in detail for several reasons.[8] First, a bank can take risk in numerous ways without altering the basic nature of the dynamic trade-off. For instance, reducing the stock of liquid assets increases profits in the short run, but will also make the bank more exposed in a future liquidity crisis and thereby reduces the probability of survival. Likewise, choosing to lend to riskier borrowers would yield higher interest revenue, but would naturally increase the probability of bankruptcy tomorrow. Similarly, increasing the maturity mismatch between liabilities and assets would increase interest margin, but also the probability of default. And so on. Singling out any of these channels would appear ad hoc; modelling them all would be too complex. Importantly, the channel through which the bank builds up its risk exposure is immaterial in what follows, what matters is that taking more risk today increases current profits at the expense of the probability of survival tomorrow. The second reason for not modelling the banking business in detail is that systemic bailouts are not limited to banking. For instance, money market mutual funds, insurance companies (AIG) or the "big three" automobile manufacturers (Chrysler, Ford, General Motors) were also provided government support in the past.[9] Looking further away, state support does not have to be financial assistance, it can take many forms. Consider the audit (or Credit Rating Agency, CRA) market, where auditors (CRA) face a very similar dynamic trade off: they can increase current fees by yielding to client pressure and issuing biased opinions (ratings), while also risking their reputation tomorrow and ultimately their market survival.[10] Since its indictment brought Arthur Andersen down in 2002, it is often speculated that the remaining four auditors are just "too-few-to-fail" and hence regulators have shied away from firm actions against Big Four auditors on a number of

[8]Although I do offer a simple model of banking in the Appendix.

[9]To some extent, all firms face the dynamic trade off discussed above: e.g. for an automobile manufacturer, saving on R&D expenditure increases profits today, but jeopardises the firm's survival in tomorrow's market.

[10]Both auditors and credit rating agencies are paid by the companies which they audit and rate. As a result of this conflict of interest, auditors and credit rating agencies have an incentive to provide dishonest audit reports and inflated ratings.

occasions (e.g. Business Week 2003, The Economist 2004 and 2005, WSJ 2004, NYT 2005, Forbes 2013, FT 2015 and 2017). Similarly to banking, this is a fundamental concern, because insulating accounting firms (CRA) from the risk of failure exacerbates moral hazard and deteriorates the quality of audit reports (ratings), which play a fundamental role in financial markets.

The current model can be considered a much simplified version of Ericson and Pakes (1995), who developed a framework for the empirical analysis of dynamic oligopoly models with heterogeneous firms. The models in this literature are designed for empirical work and therefore have to be rich enough to be taken to data. For this reason, firm heterogeneity, uncertainty at both firm and market levels, and entry and exit are crucial ingredients in these studies. As a result, the computation of the Markov Perfect Equilibria (MPE) is typically highly complex and thus many of these models allow only for a handful of firms in practice. To ease computational burden, there have been much development in this literature (see for a survey Aguirregabiria 2019). I take a fundamentally different approach. Observed and unobserved heterogeneity are of course essential parts of any empirical framework, but I do not take my model to data, so I can sacrifice one of the major hurdles in computation, firm heterogeneity, and assume firms are homogeneous and focus on symmetric equilibrium. Also, in my model the only source of uncertainty external to the firm is rivals' entry costs, the distribution of which is fixed over time. Market conditions (i.e., prices, technology, etc.) are deterministic at the outset, and hence the stochastic evolution of the market is solely governed by firms' survival strategies and the entry that ensuing failures generate. These assumptions vastly simplify computations and I can analyse the effect of market structure with arbitrary number of firms. Importantly, these assumptions are also crucial to deliver meaningful analytical results.[11]

Previous articles have analysed the effect of systemic risk and bailouts in banking: see e.g. Suarez (1994), Cordella and Yeyati (2003), Acharya and Yorulmazer (2007, 2008), Diamond and Rajan (2012) and Farhi and Tirole (2012), Allen et al (2018), Dell'Aricciaa and Ratnovski (2019). However, none of these articles analyse an oligopolistic setting (imperfect competition), where the effect of market structure could be studied. In this article, I show that market structure is key

[11] The dynamic stochastic games which are designed for empirical analysis of industry dynamics are usually too complex for analytical purposes: they deliver "very little in the way of analytical results of applied interest; i.e. just about anything can happen." (Doraszelski and Pakes 2007)

to understand the effect of bailouts on systemic risk.[12] The current study is not the first that analyses the banking sector in a stochastic dynamic game. Corbae and D'Erasmo (2013) study, inter alia, the effect of bailout on risk taking. Their study is different from the present paper in two important respects. First, they endogenously impose some features on market structure by defining national and regional banks along with a (non-strategic) competitive fringe, a framework developed in Ifrach and Weintraub (2017) in order to alleviate computational burden. Second, in their study, they analyse the effect of "too-big-to-fail" bailout policy: national banks are bailed out if they produce negative profits. They find that while national banks become riskier, smaller banks decrease their risk exposure leading to a net effect of overall reduction in risk. The concept of systemic risk that I am investigating is very different. In Corbae and D'Erasmo (2013) the strategic aspect of systemic risk build-up due to bailouts is missing (the bailout of a national bank does not depend on the failure of other banks), whereas this strategic effect is the focus in my study.

2 The model

First, I describe the environment for the baseline model, the market without bailout. The model with bailout will be a straightforward variation of this framework.

I analyse dynamic strategic interaction among firms. In particular, the firms' current decisions affect their own as well as their rivals' future payoffs and they take into account the implications of their decisions on their own and their rivals' future behaviour, which naturally affect future payoffs. I follow the standard structure of the Ericson-Pakes (1995) framework and assume that competition among firms has two different dimensions. In every period, firms engage in an (unmodelled) static "market game" from which they get a symmetric equilibrium payoff $\pi(n)$, where n is the number of

[12]Following the recent financial crisis, policy makers considered and implemented structural reforms, which have mostly concentrated on the business structure of banking, as opposed to the structure of the market. For instance, the Volcker rule in the USA prohibits commercial banks from proprietary trading and limits their activities in hedge funds and private equity. In the UK, the Independent Commission on Banking recommended ring fencing of core services (ICB 2011). Liikanen (2012) also set out its proposals to the EU of separating trading from commercial banking. These structural measures directly target the business models of banking and they may only have an indirect impact on market structure. The current paper, however, does not engage with these proposals, but focuses instead on the effects of market structure and competition on risk taking and systemic risk.

firms in the market (static dimension). Crucially, I assume the decisions taken in this market game have no dynamic implications and as a result $\pi(n)$ can be calculated independently and imported into the computation of dynamic policies. Then firms make forward looking investment decisions when they choose their risk exposures (dynamic dimension).

2.1 Baseline model: market with no bailout

Time is discrete and infinite, firms discount the future with a common factor $\beta \in (0,1)$. The number of firms present in the market is denoted by $n \in \{0, 1, 2, ...\}$. Each period consists of two phases, a production phase and an entry phase.

Production Phase

At the beginning of each production phase, firms engage in an unmodelled price or quantity competition and realise instantaneous profit $\pi(n)$ in a symmetric equilibrium of this market game. Similarly to previous literature on industry dynamics (for a survey, see Doraszelski and Pakes 2007 or Aguirregabiria 2019), I assume that the distribution of future states, conditional on the current state and investments, is independent of the prices (or quantities) that firms set in the market game. This allows me to model the market game in a static framework and simply feed the reduced form profit function $\pi(n)$ into the dynamic optimization problem as a primitive of the stochastic dynamic game. The function $\pi(n)$ essentially captures exogenous factors such as demand conditions, product substitutability, production costs, regulation, etc.

Assumption 1. *$\pi(n) \leq \pi(n-1)$ for $n > 2$, $\pi(2) < \pi(1)$, $0 \leq \pi(\cdot) < \infty$, and $\lim_{n\to\infty} \pi(n) = 0$.*

Assumption 1 conforms with standard models of homogeneous as well as differentiated product (Bertrand or Cournot) competition and hence this specification essentially captures all cases typically considered in the literature. As discussed in the introduction, I will use this reduced form profit function in the analysis and leave the market game unmodelled in order to allow the setup to encompass many potential market settings. The focus of my analysis will be on the dynamic trade off between profits and survival.

In each period, each firm $i = 1, ..., n$ chooses its probability of survival $x_i \in [0,1]$ at a cost of $g(x_i)$ and hence its per-period net profit is $\pi(n) - g(x_i)$.

Assumption 2. *$g : [0,1) \to [0,\infty]$, $g(h), g'(h), g''(h) > 0$ for $h > 0$ and $g(0) = g'(0) = 0$, $\lim_{h\to 1} g(h) = \infty$.*

This simple formulation captures the basic trade off that many firms face in the financial industry: at the cost of sacrificing current profits, the firm can increase its probability of survival. In earlier work, I offered a microfoundation for the reduced form profit function $\pi(n) - g(x_i)$, using a standard framework of unobserved quality (see Appendix A in Tóth 2012). This would plausibly fit most applications (in the audit or CRA markets quality, i.e. the credibility of audits and ratings provided, is fundamentally unobserved), including banking where risk is not observed.[13] Alternatively, in banking, it is possible to think of $g(x_i)$ as monitoring cost, where more monitoring ensures a higher probability of survival, similarly to e.g. Dell'Ariccia et al (2014) or Martinez-Miera and Repullo (2017).[14] The fixed cost nature of $g(x_i)$, i.e. that it enters additively into the profit function, is a common modelling feature both in the literature spearheaded by Ericson-Pakes (1995) and also in studies on unobserved quality.[15] However, one may not find the fixed cost nature realistic in general and in models of banking in particular. Therefore, in the Appendix I offer an alternative microfoundation of the market game based on liquidity risk, which, while still simple, is more closely aligned with traditional models of banking and also provides a useful robustness check.

If the production phase started with n firms, then the probability that at the end of the production phase firm i faces a market structure consisting of $n - k$ firms in total is $x_i \Pr(k|x_{-i})$, where k is the number of rivals who have just failed and $\Pr(k|x_{-i})$ is the probability mass function of

[13] E.g. at the time of purchase, Arthur Andersen clients did not observe the quality of other audits (e.g. Enron's) and thus could not observe the credibility that Andersen's audit report lent to their financial statements. In other words, a company does not know whether today's audit report will turn out to be worthless tomorrow if the auditor loses its reputation.

[14] For fixed cost of monitoring, see e.g. Dell'Ariccia and Ratnovski (2019). In other studies, the cost of monitoring is usually related to the number of projects that a bank finances. In reality, monitoring costs have both fixed and variable components. Implicitly, I assume that the marginal cost of monitoring is constant and captured in $\pi(n)$.

[15] This assumption has important technical advantages and also serves an important general purpose. In any static game, firms would never choose to produce negative profits. While this is perfectly reasonable in a static setting, firms in a dynamic environment may find it optimal to operate at a loss today in the hope of profits tomorrow. In order to allow for this possibility, the dynamic leg of the optimisation problem enters additively.

the convolution of $n-1$ Bernoulli distributions with success probabilities x_{-i}, where $x_{-i} = [x_j]_{j \neq i}$.

Entry Phase

In each period, production is followed by an entry phase, where $N \in Z^+$ potential entrants may enter sequentially.[16] Let the fixed costs of entry be F, which are identically and independently distributed across potential entrants with CDF $\rho(\cdot)$ and support $[0, \infty]$. Each entrant knows her own fixed cost of entry *before* entering. An entrant's fixed cost is private information, but all entrants know the distribution of fixed costs. Thus, entrants are heterogeneous. This property somewhat complicates the space of strategic interactions because, in addition to the game between incumbents and entrants, it allows for a non-trivial game among the entrants themselves. In particular, entrants are not symmetric, so, despite the sequential structure, they are unable to foresee the entry process with certainty within a period. Thus, n firms in the market (incumbents surviving in the production phase and entrants who have already entered in this period) will expect the lth entrant to enter with (endogenous) probability $\rho_{l,n}$, where $l = 0, .., N$. Note that heterogeneity before entry induces a non-degenerate distribution of the number of entrants in each state.[17] Let the equilibrium value function at the end of the lth entry round be $W_l(n)$. (That is, n includes the lth entrant if it has entered the market.) Then,

$$W_l(n) = W_{l+1}(n+1)\rho_{l+1,n} + W_{l+1}(n)(1 - \rho_{l+1,n}) \tag{1}$$

The probabilities $\rho_{l,n}$ are determined endogenously: given that the lth entrant enters if $W_l(n+1) - F > 0$, the probability of entry is $\rho_{l,n} = \rho(W_l(n+1))$. The sequential nature of entry facilitates a simple recursive formulation of the entry process. In particular, for the first entrant the value of being in the market is just the expected equilibrium firm value over the distribution of the second entrant's entry decision, which in turn depends on the third entrant's decision and so on until the last entry round. As there is no further entry after the last entry round in a

[16] Time "stops" within each period; there is discounting only across periods.

[17] An alternative way to obtain a non-degenerate distribution of entrants in each state would be simultaneous entry, where entrants use mixed strategies. In general, there is a considerable variety of entry models in applied work, which is mostly the result of the fact that there is little empirical guidence on the subject (Doraszelski and Pakes 2007).

period, $W_N(n) = V(n)$, where $V(n)$ is the (symmetric) equilibrium value function of the firm in the production phase without bailout.

The strategies are assumed to be Markov and I focus on symmetric Markov Perfect Equilibrium (MPE) in pure strategies. That is, strategies depend only on payoff relevant information. The payoff relevant information can be conveniently condensed into a state variable, which is the number of firms n. The dynamic program of firm i is thus

$$v(n; x_{-i}) = \max_{0 \leq x_i \leq 1} \left\{ \pi(n) - g(x_i) + \beta x_i \sum_{k=0}^{n-1} W_0(n-k) \Pr(k|x_{-i}) \right\} \tag{2}$$

and the resulting value function in a symmetric equilibrium is

$$V(n) = \pi(n) - g(x(n)) + \beta x(n) \sum_{k=0}^{n-1} W_0(n-k) \binom{n-1}{k} (1-x(n))^k (x(n))^{n-1-k} \tag{3}$$

where $W_0(n-k)$ is the value of a (surviving) firm at the end of the production phase, just before the first entrant makes its entry decision, and $x(n)$ are the (symmetric) equilibrium survival probabilities.

Assumption 1 and 2 are maintained throughout the paper. The summary of timing within a period is as follows. Production phase: 1. Firms choose prices/quantities and $\pi(n)$ is determined in a symmetric equilibrium; 2. firms choose x;[18] 3. failures and exits occur. Entry phase: 4. N firms enter sequentially, the lth entrant enters with probability $\rho_{l,n}$ when there are n firms already in the market (surviving incumbents and previous entrants).

The following results hold in this model:

Proposition 1. *In the market without bailout, there exists a unique symmetric equilibrium in pure strategies in which,*

1. $V(n), W_l(n), x(n), \rho_{l,n}$ are strictly decreasing in n. Furthermore, $W_l(n)$ and $\rho_{l,n}$ are strictly increasing in l. If entry occurs, it will unambiguously lower the probability of further entry: $\rho_{l,n} > \rho_{l+1,n+1}$.

2. The process can be described as a birth and death process with a unique stationary steady

[18] Note that firms can also choose prices (or quantities) and x simultaneously.

state distribution P. The probability mass function is $P_n = \frac{\lambda_{n-1}}{\mu_n} P_{n-1}$ *for* $n \geq 1$ *with* $P_0 = \left(1 + \sum_{n=1}^{\infty} \prod_{i=0}^{n-1} \frac{\lambda_i}{\mu_{i+1}}\right)^{-1}$*, and with the property* $E(n) < \infty$*, where the parameters* $\mu_n = n(1-x(n))$ *(average number of failures, "deaths") and* $\lambda_n = \sum_l \rho_{l,n}$ *(average number of entries, "births") are the intensity parameters of the Markov process.*

Proof. See Tóth (2012). □

Firms choose to invest more in survival in more concentrated markets, despite the fact that market concentration frustrates survivors in three fundamental ways: it can bring less profit ($\pi(n) - g(x(n))$ can be decreasing), it generates rivals that are harder to outlive, and also attracts more entry. Yet, in equilibrium these effects are dominated by the fact that there are fewer rivals to outlast, hence the investment profile increasing with market concentration. The model describes very intuitive industry dynamics. In fragmented markets, the equilibrium value of the firm is low and thus the intensity of entry is low too, while firms invest little. The market experiences a shake-out. However, as the industry becomes concentrated, firms invest more, fail with smaller probability and entry also intensifies, which prevents the market from becoming too concentrated for a prolonged time. The model exhibits continuous turbulence in the form of failures and entry even in the limit.

2.2 The market with bailout

In the previous model, there was no government intervention, the market evolved undisturbed. As a result, the state when there is no firm in the market (i.e. state zero) happens with positive probability in the steady state. Now, I introduce the concept of bailout into the model. I define systemic failure as the event when all incumbent firms fail (i.e. there is no firm at the end of the production phase) and assume that the government bails out all failing firms. In other words, if there are n firms in the market who all invest $y(n)$ in survival in a symmetric equilibrium, then in the event of a systemic failure, which happens with probability $(1 - y(n))^n$, the production phase ends as if no firm has failed, i.e. with n firms again.[19]

[19]In principle, one could define systemic failure as the state when there is no firm in the market at the end of the entry, rather than the production, phase. In other words, a systemic crisis would happen when each incumbent

Thus, the modified dynamic program is as follows:

$$\widetilde{v}(n; y_{-i}) = \max_{0 \leq y_i \leq 1} \left\{ \pi(n) - g(y_i) + \beta y_i \sum_{k=0}^{n-1} \widetilde{W}_0(n-k) \Pr(k|y_{-i}) \right.$$
$$\left. + \beta \widetilde{W}_0(n) (1 - y_i) \Pr(n-1|y_{-i}) \right\} \tag{4}$$

where the entrants' values and entry probabilities are defined analogously to the case without bailout.[20] The resulting value function in a symmetric equilibrium is then

$$\widetilde{V}(n) = \pi(n) - g(y(n)) + \beta y(n) \sum_{k=0}^{n-1} \widetilde{W}_0(n-k) \binom{n-1}{k} (1 - y(n))^k (y(n))^{n-1-k}$$
$$+ \beta \widetilde{W}_0(n) (1 - y(n))^n \tag{5}$$

A crucial difference between the models with and without bailout is the strategic effect that emerges in the presence of bailout. This effect can be easily illustrated in the simplest case, a duopoly without entry:

$$v(2; x_j) = \max_{0 \leq x_i \leq 1} \{\pi(2) - g(x_i) + \beta x_i (x_j V(2) + (1 - x_j) V(1))\}$$
$$\widetilde{v}(2; y_j) = \max_{0 \leq y_i \leq 1} \left\{\pi(2) - g(y_i) + \beta y_i \left(y_j \widetilde{V}(2) + (1 - y_j) \widetilde{V}(1)\right) + \beta (1 - y_i)(1 - y_j) \widetilde{V}(2)\right\}$$

Differentiating the first order condition and using the implicit function theorem yields

$$\frac{\partial x_i}{\partial x_j} = \frac{V(2) - V(1)}{g''(x_i)} < 0$$

This is always negative because the value function is decreasing (Proposition 1) and the cost

firm has failed and no one has entered the market. This definition, however, would arguably be in conflict with the concept of a bailout: de novo entry is typically slow in practice and bailouts happen because the market cannot be without firm even for a short period of time.

[20] That is, $\widetilde{W}_l(n) = \widetilde{W}_{l+1}(n+1)\widetilde{\rho}_{l+1,n} + \widetilde{W}_{l+1}(n)(1 - \widetilde{\rho}_{l+1,n})$, where $\widetilde{\rho}_{l,n} = \rho(\widetilde{W}_l(n+1))$ for $l = 0, .., N$ and $n \in \{0, 1, 2, ...\}$.

function is convex (Assumption 2). That is, investments in survival are strategic substitutes in the model without bailout, i.e. a firm has an incentive to take less risk if the rival takes more. In the model with bailout, however, we have that

$$\frac{\partial y_i}{\partial y_j} = \frac{2\widetilde{V}(2) - \widetilde{V}(1)}{g''(y_i)}$$

This expression need not be negative, and indeed in most cases it is not (see below), even when the value function is decreasing. When $\partial y_i / \partial y_j$ is positive, investments are strategic complements: if a firm takes more risk, then its rival also has an incentive to decrease its probability of survival. This is because the firm counting on bailout recognises that its ultimate survival depends on the rival's survival strategy too: "if we both fail, we don't fail... so if we both take much risk, we don't actually take much risk". This has important implications for risk taking behaviour as well as systemic risk.

I make a few comments on existence. In both models, the second order conditions are simply $-g''(\cdot) < 0$ (Assumption 2), so the best replies are unique. Moreover, in the model without bailout, the reaction functions are continuous, downward sloping and linear ($\partial x_i / \partial^2 x_j = 0$) and thus a symmetric equilibrium in pure strategies exists (which is also unique when $\partial x_i / \partial x_j \neq -1$).[21] However, existence in pure strategies in the model with bailout is not ensured, despite that the reaction functions are continuous and linear here too ($\partial y_i / \partial^2 y_j = 0$). This is the result of strategic complementarities, because in general nothing guarantees that the reaction functions intersect. In the computations I did not come across instances when the equilibrium in pure strategies did not exist. If non-existence arises, changing the specification of the function $g(\cdot)$ would presumably resolve the problem.

3 Computation

The computed Markov Perfect Equilibrium in pure strategies is such that given optimal policies the value functions satisfy the Bellman equations (3) and (5), and given the value functions investment

[21] See Tóth (2012) and the proof of Proposition 1 therein for the general case.

policies satisfy the first order conditions, up to some sufficiently small error.[22] This implies that firms choose optimal policies based on their beliefs on future industry structure, and these beliefs are consistent with rivals' behaviours. For the numerical analysis I use the following cost function:

$$g(h)=\frac{h^2}{1-h},\quad g'(h)=\frac{1}{(1-h)^2}-1,\quad 0\leq h<1$$

This function conforms to Assumption 2 and leads to simple analytical first order conditions to the programmes (2) and (4), which (after rearranging and imposing symmetry) yield, respectively:

$$x(n)=1-\frac{1}{\sqrt{1+\beta\sum_{k=0}^{n-1}W_0(n-k)\binom{n-1}{k}(1-x(n))^k(x(n))^{n-1-k}}}\tag{6}$$

$$y(n)=1-\frac{1}{\sqrt{1+\beta\sum_{k=0}^{n-1}\widetilde{W}_0(n-k)\binom{n-1}{k}(1-y(n))^k(y(n))^{n-1-k}-\beta\widetilde{W}_0(n)(1-y(n))^{n-1}}}\tag{7}$$

The equilibrium profit function from the market game and the entry probability function that I use for the calculations are as follows, respectively,

$$\pi(n)=\alpha/n,\qquad 0<\alpha<\infty$$
$$\rho(a)=1-e^{-\gamma a},\qquad 0<\gamma<\infty$$

As mentioned before, $\pi(n)$ captures exogenous factors in general and hence α can be thought of in numerous ways. In what follows, I refer to α as a demand parameter (high α means strong demand), but equally one can consider it as a parameter for the level of product differentiation in the market game (high α means more differentiation, and thus more market power and higher profits). The parameter γ shifts the distribution of the fixed cost of entry. I set the discount factor $\beta=0.97$. As customary in the literature (see e.g. Aguirregabiria 2019), I set the maximum number of firms that can be in the market at any time equal to 25 and allow for 10 potential entrants in each period. The qualitative results are not sensitive to the pre-set maximum number

[22] This error is typically smaller than 10^{-10} after 30 iterations and after about 100 iterations the error reaches machine precision (10^{-16}).

of firms, the values of the discount factor or the functional forms chosen for $g(h), \pi(n), \rho(a)$. The only reason for displaying results for (maximum) 25 firms is visual presentation.[23]

Computations are done in Matlab R2018b. The algorithm that solves the model iterates over the value and policy functions until convergence.[24] In the computations, I set all initial values for the value and policy functions equal to zero, results are completely invariant to starting values, so the equilibrium can be regarded as "numerically unique". An iteration consists of the following steps for e.g. the case without bailout (the other case is analogous):

1. Using the values $V(n)$ for all n from the previous iteration, calculate $W_l(n)$ backwards for all l, n, starting at $W_N(n) = V(n)$, and then $W_l(n) = W_{l+1}(n+1)\rho_{l+1,n} + W_{l+1}(n)(1-\rho_{l+1,n})$ for all $l < N$, where $\rho_{l,n} = \rho(W_l(n+1))$. At the end, one has $W_0(n)$ for all n.

2. Using the values of $W_0(n)$ from step 1 above and the values of $x(n)$ from the previous iteration, update the policy function using the first order condition (6) for all n.

3. Using $W_0(n)$ and $x(n)$ from step 1 and 2, update the value function $V(n)$ for all n, using equation (3).

The steady state probabilities are computed according to the formula in Proposition 1 for the case without bailout.[25] For the case with bailout one needs to account for the fact that state zero (i.e. no firm in the market) is never reached by design (i.e. there is no entry into and exit from state zero), so the modified steady state probabilities are $\widetilde{P}_n = \left(\widetilde{\lambda}_{n-1}/\widetilde{\mu}_n\right)\widetilde{P}_{n-1}$ for $n \geq 2$ with $\widetilde{P}_1 = \left(1 + \sum_{n=1}^{\infty}\prod_{i=1}^{n-1}\left(\widetilde{\lambda}_i/\widetilde{\mu}_{i+1}\right)\right)^{-1}$, where the parameters of average "births" (entry) $\widetilde{\lambda}_i$ and average "deaths" (failure) $\widetilde{\mu}_i$ are defined analogously to λ_i and μ_i in Proposition 1. The simple structure of the stochastic dynamic game ensures that the algorithm calculates the equilibrium in a matter of seconds, even with hundreds of firms.

[23] The maximum number of firms is immaterial, because in the parameter range investigated states close to 25 are not reached in the steady state (see Figures 1 and 2). If one wishes to investigate other parameter ranges (and/or functional forms) the maximum number of firms can be adjusted so that it is not "binding" (i.e. it is not reached in the steady state).

[24] It is possible in principle to solve for the equilibrium by solving the system of non-linear equations, which define the equilibrium. However, this method struggles when the discount factor is high and the number of firms is large ($n > 10$).

[25] The steady state probabilities are calculated by using the equilibrium properties of the discrete time game above to derive transition probabilities when each period of the game is infinitely small. In essence, the discrete time Markov transition kernel is transformed into a continuous time analogue and the game produces a birth and death process. For further discussion and proofs, see Tóth 2012.

4 Numerical Results

I focus on two parameters, α and γ, to assess the effect of competition on the effect of individual risk taking and systemic risk. I will interpret α as a demand parameter, higher values indicating stronger demand and leading to higher potential profits $\pi(n)$ at all market structures. However, one can equally interpret it, for instance, as a general market power parameter (e.g. product differentiation). The other parameter γ measures the intensity of entry: higher γ increases the probability of entry at all market structures and the competitive threat that incumbent firms face ("outside competition").

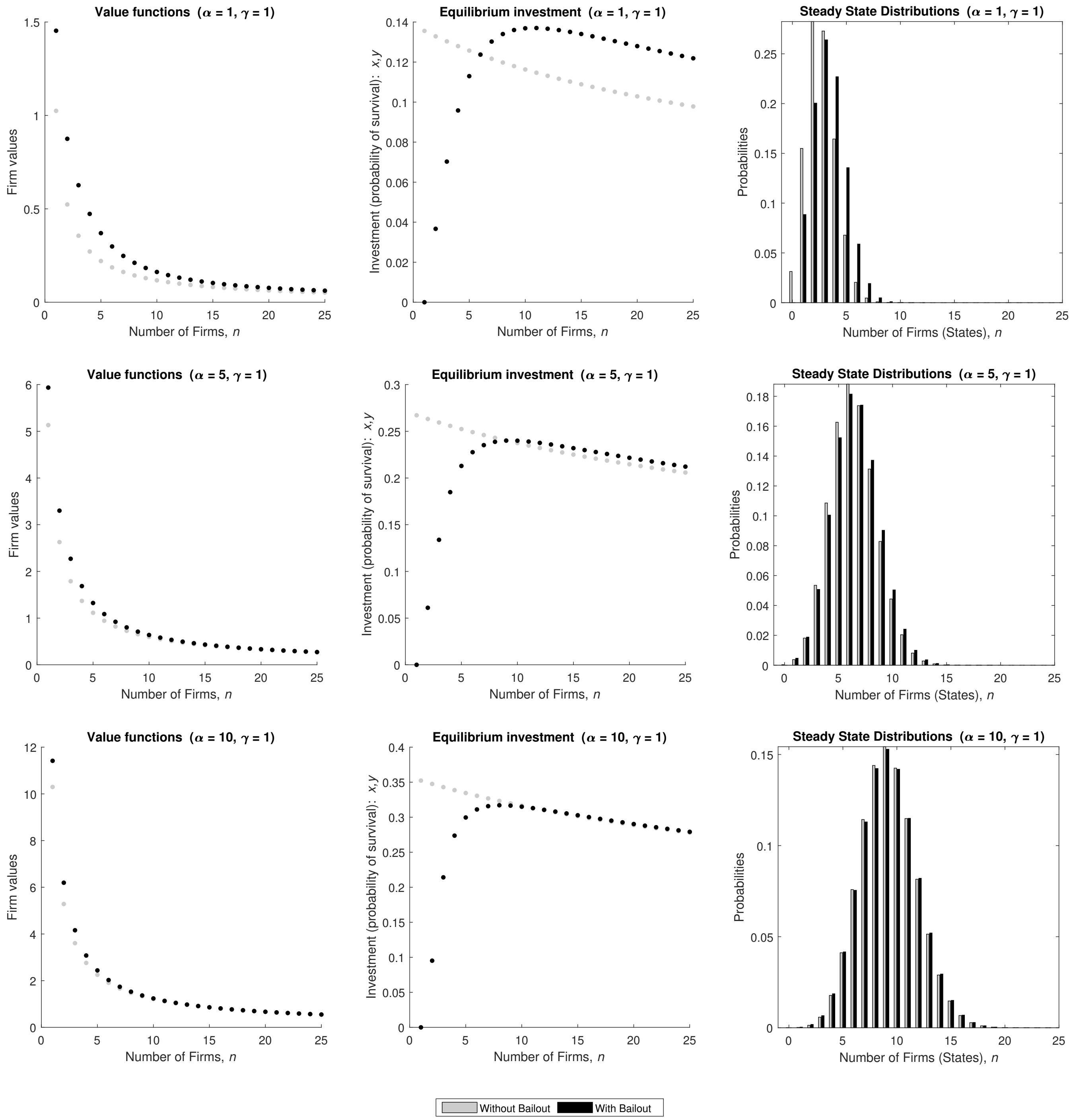
From the graphs in Figures 1 and 2, several general observations can be made, which are robust across all parameters and parameter ranges. First, there is a clear charter value effect: firms in markets with bailout always have higher values than the firms which operate in a market without bailout. This is very intuitive: by reducing the (expected) cost of failure, the government increases the continuation value of the firm, which naturally increases firm value. Also, observe that firm values decrease with the number of firms, as expected (see the first panels of Figures 1 and 2). Second, the differences between the individual risks (measured by $x(n), y(n)$, the equilibrium investments in survival) that firms take in the two market settings crucially depend on market structure. In particular, in more concentrated markets firms with bailout take more risk; but as the number of firms grow, they tend to take less risk than the firms in the market without bailout. This counterintuitive result is the product of the interplay of the charter value and the strategic effects. *Strategic effect* is the effect due to a firm having an incentive to choose lower probability of survival when rivals do the same. *Charter value effect* stems from the fact that a firm has an incentive to choose higher probability of survival when the (continuation) value of the firm is higher. In Figures 1 and 2, when there are few firms in the market the strategic effect dominates, while in more fragmented markets the reverse is true. A monopoly counting on bailout always invests zero in survival, because its failure is always a systemic event and thus it will survive with certainty, regardless of whether it has failed (and got bailed out) or not.[26] But the probability of survival

[26] It is possible to take the monopoly out of the game (by saying e.g. if there is only one firm left, then the government nationalises the firm), set $\widetilde{V}(1) = V(1)$ and feed this exogenous value into the optimisation problems. The results are qualitatively the same, as the problem of strategic complementarities does not disappear in the

is less than certain for a duopoly, because now the firm can de facto fail if there is no systemic failure (i.e. its rival survives). Therefore, a duopolist has some incentive to invest in survival, but typically invests less and takes more risk than a duopolist would in the market without bailout. This is the strategic effect. However, the event that everyone fails and gets bailed out becomes ever less likely in more fragmented markets. As a result, the strategic effect diminishes with the number of firms. Because the strategic effect diminishes faster than the charter value effect (firm values decrease with n), the charter value effect becomes dominant for sufficiently fragmented markets and consequently firms take less risk when they can count on government bailout compared to the case when they cannot. This result is surprising, primarily because conventional wisdom would not suggest that firm value can play such a pivotal role when it's so small. Lastly, notice that the market with bailout is always less concentrated in steady state (see the third panels of Figures 1 and 2). This is the result of entry (higher firm values attract more entry) and of course the fact that systemic failures and the resulting bailouts work against market concentration by setting the market back to its original state.

Figure 1 analyses the effect of demand conditions. As α and thus the potential profit from the market game $\pi(n)$ increases, firm values naturally increase (first column of graphs). This has the unsurprising effect of firms taking less risk in general (i.e. they increase investments in survival, second columns of graphs). Consequently, the strategic effect plays a much more muted role, because the event of a systemic failure and the resulting government bailout becomes more remote for any given $n > 1$. This in turn means that the difference in firm values across the two market settings narrows as α increases (first column of graphs). As a result, in the region when the charter value starts to dominate the strategic effect (i.e. in fragmented markets), the difference in charter values are so tiny that the difference between the investment profiles across the two market settings vanishes (although it is still true that $x(n) < y(n)$ for n large enough). In sum, the two types of markets still exhibit important differences in concentrated markets, but for more fragmented market structures, the two market types look very much alike for large α. Especially so, when we consider the steady state distributions in more detail (third column of graphs). For low values of α, strategic effects play an important role, hence the apparent differences in firm values, investment

market with bailout.



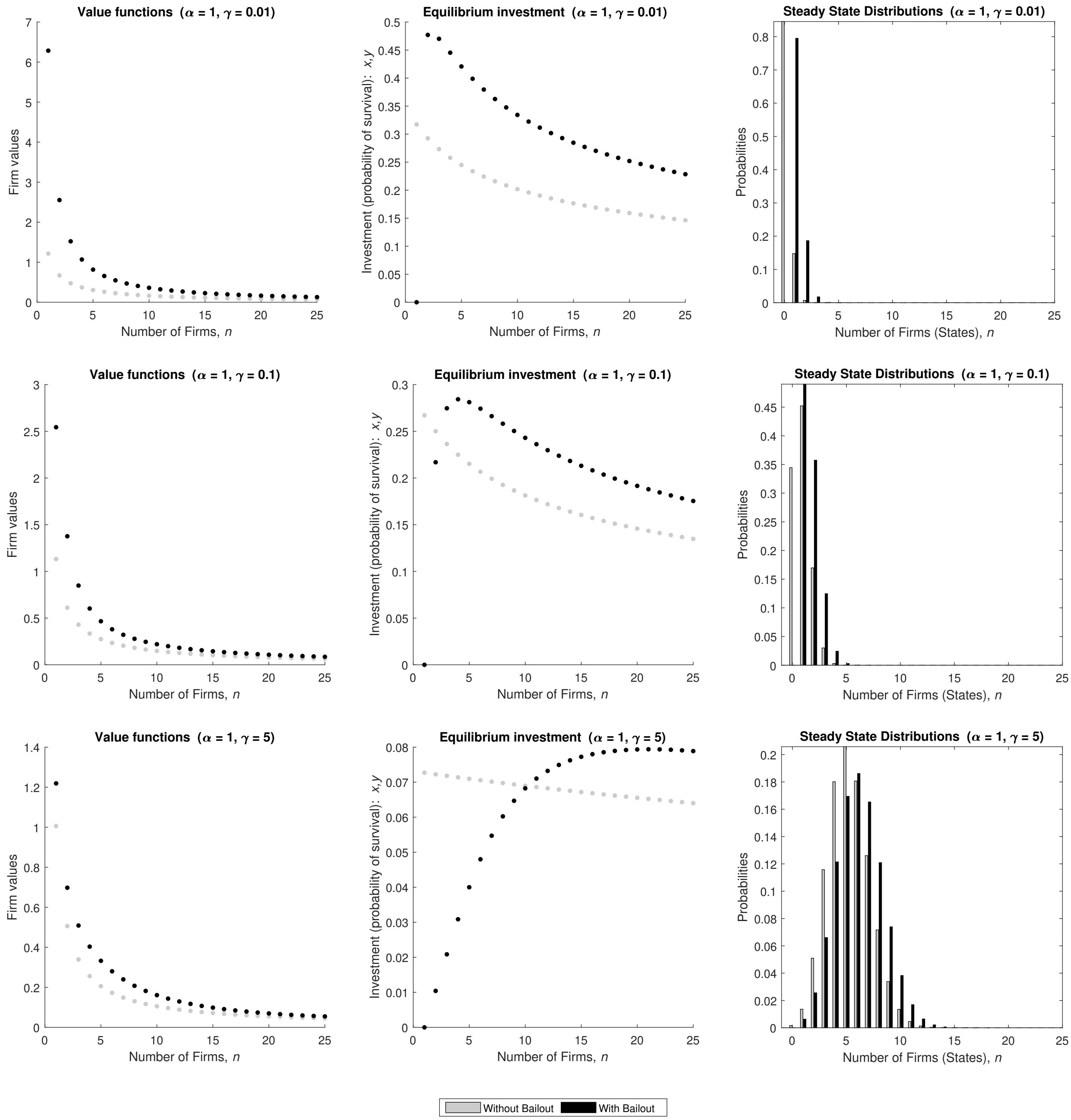
Notes: static profit function: $\pi(n) = \alpha/n$, $\alpha = 1, 5, 10$ (for the first, second and third row of graphs, respectively); probability function of entry: $\rho(a) = 1 - \exp(-a)$; $\beta = 0.97$; maximum number of firms: 25 (potential entrants: 10).

Figure 1: The effect of demand ($\alpha = 1, 5, 10$)

profiles, and the resulting steady state distributions. However, as α increases, firm values and investments increase, which in turn result in both higher entry and lower failure rates, leading to relatively fragmented markets in both settings. As discussed, when the markets are fragmented, the differences across the two market types are very small, hence the almost identical steady state distributions. In other words, for high α the two types of markets look strikingly alike and bailout policy does not seem to make (much) difference.

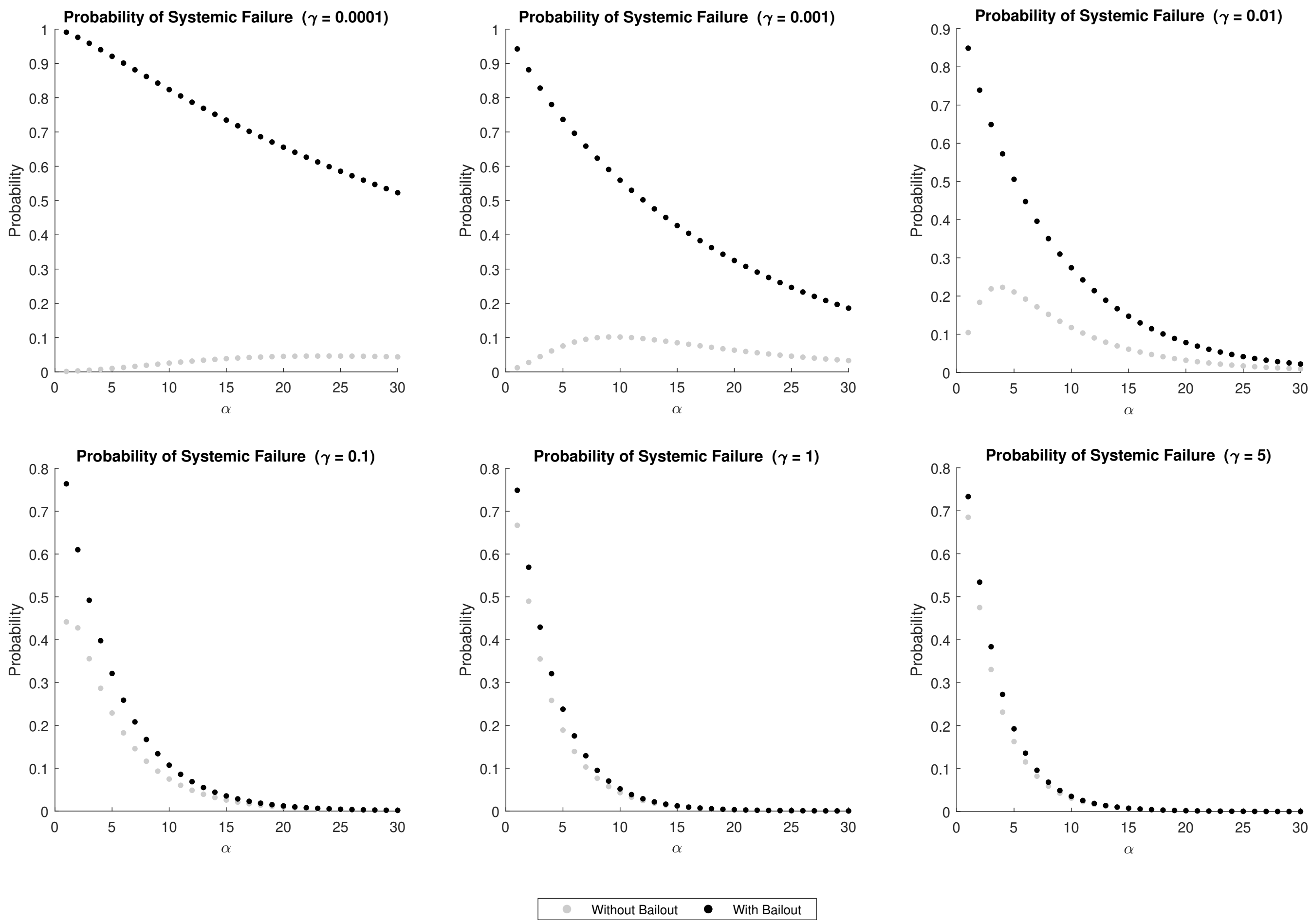
Figure 2 examines the effect of outside competition. For low γ, entry is costly in general and thus happens infrequently. This increases firm values in general, because firms can hold on to their positions longer in expectation (e.g. it's more probable that a surviving monopolist can start the next period as a monopolist again). Because firm values are high, investments are relatively high, which means that systemic failure is less likely, and hence the strategic effect is muted and the charter value effect dominates even when markets are more concentrated. However, as entry intensifies (i.e. γ increases), the charter value effect starts to dominate only in more fragmented markets while the difference in the investment profiles between the two market types narrows too due to the smaller difference in firm values at these states.

I analyse systemic risk in Figures 3 and 4. In particular, for different parameter values I calculate the weighted average of systemic risk, where the weights are the steady state probabilities. From the analyses above, the effect on the average systemic risk (i.e. the probability that all firms fail) is unclear. On the one hand, firms in the market with bailout invest less and fail with higher probability when the market is concentrated. On the other hand, they invest more and fail less often in more fragmented markets, although the difference is typically fairly small. Moreover, the market with bailout is less concentrated in steady state, suggesting the investment profiles in fragmented markets would weigh more in the average of systemic risks across states. In Figure 3, I define systemic risk for the case without bailout as the probability of *systemic failure*, i.e. the probability that the market *enters* (in the case with bailout: *would enter*) into state zero at the end of a production phase and it is calculated as $P(1)(1-x(1))+P(2)(1-x(2))^2+...$, where $P(j)$ is the steady state probability of state j and $x(j)$ is the equilibrium investment in state $j \geq 1$. The difference between the two types of markets in Figure 3 is large when entry is very sluggish



Notes: static profit function: $\pi(n) = 1/n$; probability function of entry: $\rho(a) = 1 - \exp(-\gamma a)$ for $\gamma = 0.01, 0.1, 5$ (for the first, second and third set of three graph, respectively); $\beta = 0.97$; maximum number of firms: 25 (potential entrants: 10).

Figure 2: The effect of entry ($\gamma = 0.01, 0.1, 5$)



Notes: Systemic failure is defined as the expected probability of *entering* into the state with zero firms at the end of the production phase.

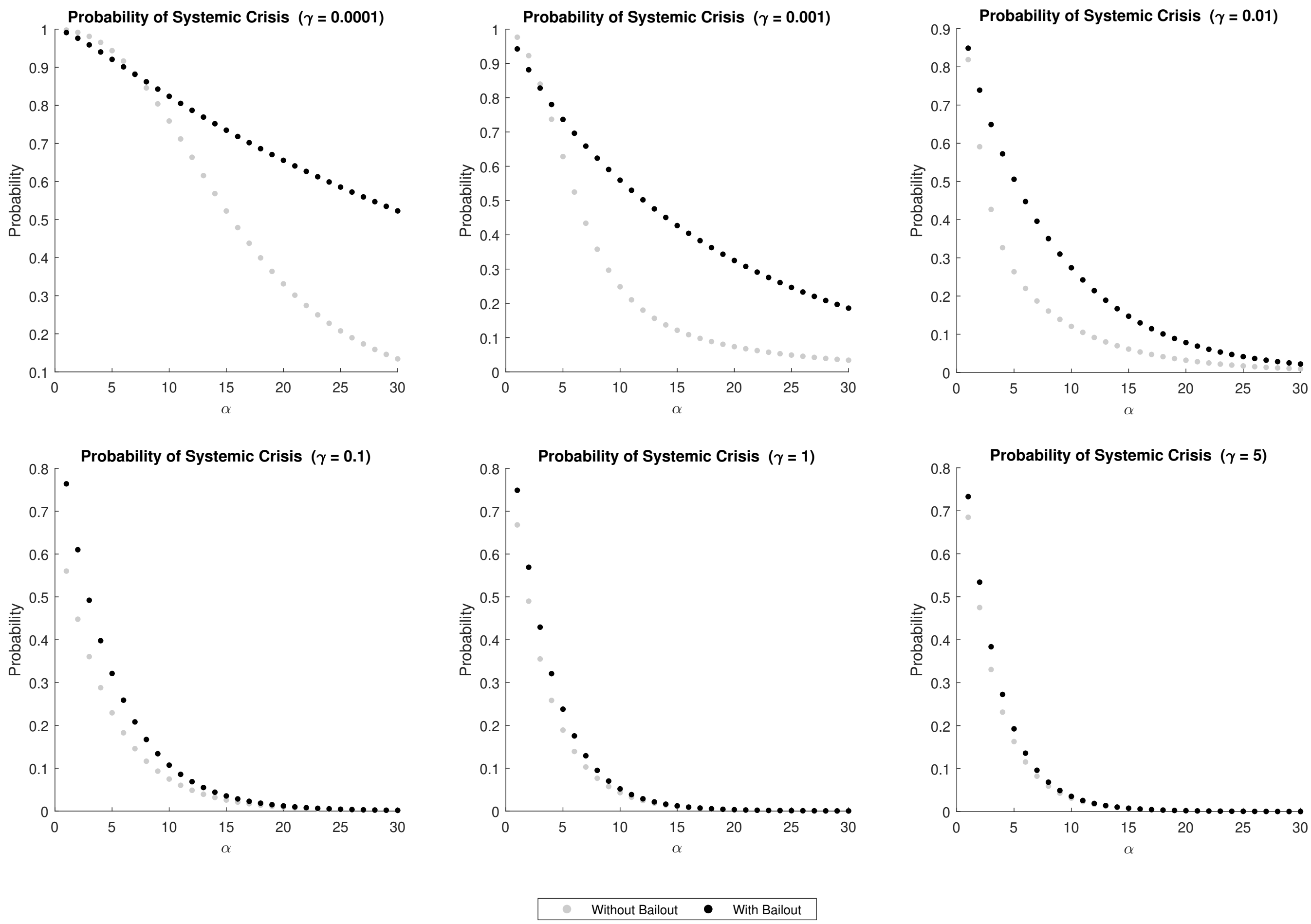
Figure 3: Expected probability of systemic failure

(small γ) and it disappears as I allow for a more intense entry process (increasing γ). However, this is deceptive. Consider the first panel in Figure 3, where the probability of systemic failure in the market without bailout is so low (near zero). Here the market (almost) never enters into state zero, simply because the market is (almost) always in state zero (see e.g. the third panel in the top row in Figure 2).

For this reason, I also calculated the probability of *systemic crisis* in Figure 4, i.e. for the case without bailout the probability that the market *is* (in the case with bailout: *would be*) in state zero at the end of a production phase and it is calculated as $P(0) \cdot 1 + P(1)(1 - x(1)) + P(2)(1 - x(2))^2 + ...$[27] This shows a dramatically different picture (first graph in Figure 4): surprisingly, systemic crisis is more probable without bailout in adverse demand conditions (low α) and when the market faces little threat of entry (low γ). This is primarily because, under these market conditions, firms with bailout invest substantially more in survival even in concentrated markets as discussed above. However, in good demand conditions (high α) conventional wisdom is restored and the market without bailout is substantially less (systematically) risky. This is because high α ensures that firms in both markets invest a lot more in survival and the difference between survival probabilities are fairly small - except in state 1, where the monopoly in the market with bailout still invests zero. Because entry is suppressed, state 1 occurs with high probability in steady state and thus has a large impact on the average probability of systemic crisis. As we allow for more entry, however, the difference between the two types of markets disappear at all market structures (last graph in Figure 4).

What is clear from both Figure 3 and 4 is that stronger demand or more intense entry result in lower systemic risk in general and also that the differences between the two markets vanish with α and γ. This is the result of the fact that higher α and γ, together or alone, lead to an (expected) market structure that is fragmented in steady state, and this in turn keeps the strategic effect at bay. The key, therefore, appears to be (endogenous) market structure. As long as there is government bailout, more concentrated markets will typically exhibit more systemic risk, which

[27] For the case with bailout, both for systemic failure and systemic crisis the probability is $\widetilde{P}(1)(1 - y(1)) + \widetilde{P}(2)(1 - y(2))^2 + ...$ (where $\widetilde{P}(j)$ is the steady state probability of state j and $y(j)$ is the equilibrium investment in state j), because state zero is of course never reached.



Notes: Systemic crisis is defined as the expected probability of *being* in the state with zero firms at the end of the production phase.

Figure 4: Expected probability of systemic crisis

is primarily due to the strategic effect at work. However, if circumstances in the market, such as benign demand conditions, lax competition (e.g. high product differentiation, high switching costs, etc), or intense entry nudge the market towards a fragmented structure, then the fact that firms can count on bailouts is not overly concerning from a systemic risk perspective.[28]

5 Conclusions

The stochastic dynamic game presented highlights a complex relationship between market structure, competition and risk taking, which has important implications for systemic risk. I analyse two channels through which bailout policies affect risk taking behaviour: the strategic effect increases the incentive to take risk, while the charter value effect reduces it. The interplay of these two effects determine market structure, while market structure in turn drives the interplay between them. Regarding systemic risk, conventional wisdom is challenged in some parameter regions where bailout policy *reduces* systemic risk; in others it is confirmed that the presence of bailout increases systemic risk, as policy makers and academics often argue in public debates. However, the effect of bailout policies on systemic risk seem to be small overall, which points to the important countervailing role of charter value.

The model and the accompanying Matlab code present a useful framework for future research and policy analyses. This framework, for instance, can be used to assess and compare different bailout policies. In particular, this paper analyses the most conservative policy option, where bailout occurs only when all incumbent firms fail. It would be interesting to consider the effect of alternative policies, when the government bails out firms under less dramatic scenarios, e.g. when 50% of incumbents fail. The effect of a more generous bailout policy is uncertain, because it amplifies both the charter value and the strategic effects in an unpredictable way, while it also pushes the market towards a more fragmented structure. Furthermore, after introducing firm heterogeneity and uncertainty at both firm and market levels, one can conduct a structural estimation of the model and estimate the actual change in systemic risk due to bailouts. The

[28] It is perhaps important to emphasise that high α does not necessarily mean that firms make (much) profit in a period, as profit (i.e. $\pi(n) - g(x)$) also depends on investment and (endogenous) market structure.

implications of the game can also be tested across markets in a reduced form fashion.

References

Allen, F. , Carletti, E. , Goldstein, I. , Leonello, A. , 2018. Government guarantees and financial stability. J. Econ. Theory 177, 518–557 .

Aguirregabiria (2019): Empirical Industrial Organization, Models, Methods and Applications, mimeo

Brandao-Marques L; Correa R; Sapriza H (2013): "International evidence on government support and risk taking in the banking sector," International Finance Discussion Papers 1086, Board of Governors of the Federal Reserve System (U.S.).

Acharya VV, Yorulmazer T. (2007): Too many to fail: an analysis of time-inconsistency in bank closure policies. J. Financ. Intermed. 16:1–31

Acharya VV, Yorulmazer T. (2008): "Cash-in-the-Market Pricing and Optimal Resolution of Bank Failures," Review of Financial Studies, 21, 2008, 2705-2742

Acharya VV, Hanh T. Le, Shin HS (2017): Bank Capital and Dividend Externalities, Review of Financial Studies, Volume 30, Issue 3, 1 March 2017, Pages 988–1018

Beck, T. (2008): “Bank Competition and Financial Stability: Friends or Foes?” World Bank Policy Research, Working Paper no. 4656, 2008.

Business Week (2003): The big four: too few to fail?, BusinessWeek. 9/1/2003, Issue 3847, p34-34. 1p.

Claessens S.; Laeven L. (2004): “What Drives Bank Competition? Some International Evidence.” Journal of Money, Credit and Banking, Vol. 36 (2004), pp. 563–583.

Cordella, T., Levy Yeyati, E., 2003. Bank bailouts: moral hazard vs value effect. J. Financ. Intermed. 12 (4), 300–330.

Corbae D, D’Erasmo P (2013): A Quantitative Model of Banking Industry Dynamics, mimeo

Dell’Ariccia, G. , Laeven, L. , Marquez, R. , 2014. Real interest rates, leverage, and bank risk-taking. J. Econ. Theory 149, 65–99 .

Dell'Ariccia G and L. Ratnovski (2019): Bailouts and systemic insurance, Journal of Banking and Finance 105 (2019) 166–177

Diamond, DW. and Rajan R (2012): Illiquid Banks, Financial Stability, and Interest Rate Policy, Journal of Political Economy, Vol. 120, No. 3 (June 2012), pp. 552-591

Doraszelski, U. and Pakes, A. "A framework for applied dynamic analysis in IO." Chapter 30 in Armstrong, M. and Porter, R., eds., *Handbook of Industrial Organization*, Vol. 3, Amsterdam: Elsevier, 2007

Economist (2004): Called to account - The future of auditing; London Vol. 373, Iss. 8402, (Nov 20, 2004): 71-73.

Economist (2005): Crime and punishment; London Vol. 375, Iss. 8432, (Jun 25, 2005): 96.

Erickson, R. and Pakes, A. "Markov-Perfect Industry Dynamics: a Framework for Empirical Work." *Review of Economic Studies*, Vol. 62 (1995), pp. 53-82.

Farhi, E and Tirole J (2012): "Collective Moral Hazard, Maturity Mismatch and Systemic Bailouts." American Economic Review 102 (1): 60-93.

Financial Stability Board (2015): Assessment Methodologies for Identifying Non-Bank Non-Insurer Global Systemically Important Financial Institutions, Financial Stability Board, https://www.fsb.org/methodologies-for-identifying-non-bank-non-insurer-global-systemically-important-financial-institutions/

Financial Times (2017): "Fab Four' accountancy groups are too few to fail, FT.com; London (Sep 22, 2017).

Financial Times (2015): Professional services: Accounting for change, FT.com; London (Aug 27, 2015).

Forbes (2013): Are the 'Big Four' Audit Firms Too Big to Fail? By: Lagace, Martha, Forbes.com, 2/4/2013

Freixas, X and Rochet, JC (2008): The Microeconomics of Banking, MIT Press

Gropp, R., Hakenes, H., Schnabel, I. (2011) Competition risk-shifting and public bail-out policies. Review of Financial Studies 24 (6), 2084–2120.

Ifrach B, Weintraub GY (2017): A Framework for Dynamic Oligopoly in Concentrated Industries, Review of Economic Studies, Volume 84, Issue 3, 1 July 2017, Pages 1106–1150

Independent Commission on Banking (2011). Final Report: Recommendations, London.

Keeley, MC (1990): "Deposit Insurance, Risk, and Market Power in Banking," American Economic Review, vol. 80(5), pp 1183-1200

Liikanen, E. (2012). Final Report of the High-level Expert Group on Reforming the Structure of the EU Banking Sector, Brussels.

Martinez-Miera D; Repullo R (2017): Search for Yield, Econometrica, 85 (2017), pp. 351-378

New York Times (2005): 'Auditors: Too Few to Fail', http://www.nytimes.com/2005/06/25/business/audi too-few-to-fail.html

Schaeck K., Cihak M., Wolfe, S. (2009): "Are Competitive Banking Systems More Stable?" Journal of Money, Credit and Banking, Vol. 41 (2009), pp. 711–734.

Suarez, J. (1994). Closure rules, market power, and risk-taking in a dynamic model of bank behavior. London School of Economics/Financial Markets Group Paper No. 196.

Tóth, Á. (2012), The great industry gamble: market structure dynamics as a survival contest. RAND Journal of Economics, 43: 348-367.

VanHoose D. (2017) Bank Competition, Stability, and Regulation. In: The Industrial Organization of Banking. Springer, Berlin, Heidelberg

Wall Street Journal (2004): Ernst & Young Faces Tax-Shelter Inquiry, Eastern edition; New York, N.Y. [New York, N.Y]24 May 2004: C.1.

Zigraiova, D. and Havranek, T. (2016): Bank Competition and Financial Stability: Much Ado About Nothing?. Journal of Economic Surveys, 30: 944–981.

Appendix

Banking model: liquidity risk

In the analysis above, the cost function of survival $g(\cdot)$ enters additively into the profit function. The fixed cost nature of $g(\cdot)$ is a common modelling feature both in the literature spearheaded by Ericson-Pakes (1995) and also in studies on unobserved quality. This assumption has technical advantages and also serves an important general purpose. In any static game, firms would never choose to produce negative profits. While this is perfectly reasonable in a static setting, firms in a dynamic environment may find it optimal to operate at a loss today in the hope of profits tomorrow. In order to allow for this possibility, the dynamic leg of the optimisation problem enters additively in the main body of the paper. However, one may not find the fixed cost nature realistic in general and in models of banking in particular. Therefore, in what follows I discuss an alternative specification.

The primary goal of this Appendix is to provide a microfoundation of the market game that, while still simple, is more closely aligned with traditional models of banking. The secondary objective is robustness check: we will see that while the model presented here produces fundamentally different firm behaviour and market structure dynamics, the main conclusions are unchanged in terms of systemic risk.

The environment

As is often the case in the banking literature, I assume banks compete for deposits a la Cournot and the loan market is completely competitive, i.e. all banks can lend as much as they want at rate $R > 0$.[29] I abstract from equity capital and other forms of liabilities, the banks are funded solely by deposits. The deposit rate is a simple function of total deposits collected, $r_D = \sum_{i=1}^{n} d_i$. Banks take risk by deciding what proportion of deposits to invest in loans and what proportion to leave in liquid assets (cash).[30] Investing more in loans increases current profits, but exposes the bank more in a future liquidity crisis and thus reduces the probability of survival tomorrow. In particular, bank i in the market without bailout (the market with bailout is analogous) can invest $\sqrt{l\left(1-x_i\right)}$ portion of its deposits in loans and leave the rest in liquid cash, which ensures it will survive with probability $f(x_i)$, where $l\left(\cdot\right), f\left(\cdot\right) : [0,1] \rightarrow [0,1]$, $l'\left(\cdot\right), f'\left(\cdot\right) > 0$. As before, I assume that the distribution of future states, conditional on the current state and x_i, is independent of deposits (and r_D) that banks set in the market game.

[29] The model could just as easily be set in the framework of differentiated products, such as in Freixas and Rochet (2008), Chapter 3.3.

[30] Note that it should be clear from the profit function (8) that the formulation is also consistent with a story of credit risk, where banks take risk by e.g. choosing higher interest rate and financing more risky projects at the expense of the probability of survival.

Then, in a period bank i in the market without bailout makes profit on $\sqrt{l(1-x_i)}$ portion of its deposits d_i and makes a loss on the rest of its deposits, because cash earns zero interest. The per period profit of the bank in the market game is thus

$$
\begin{aligned}
\Pi(d_i, x_i; n, R, d_{-i}) &= (R - r_D)\, d_i \sqrt{l(1-x_i)} - r_D d_i \left(1 - \sqrt{l(1-x_i)}\right) \\
&= \left(R\sqrt{l(1-x_i)} - r_D\right) d_i \qquad (8)
\end{aligned}
$$

Differentiating with respect to d_i, imposing symmetry, solving for d and substituting results in,

$$
\Pi(x_i; n, R) = \left(\frac{R}{n+1}\right)^2 l(1-x_i)
$$

The dynamic programs thus become

$$
\begin{aligned}
v(n; x_{-i}) &= \max_{0 \le x_i \le 1} \left\{ \pi(n)\, l(1-x_i) + \beta f(x_i) \sum_{k=0}^{n-1} W_0(n-k) \Pr(k | f(x_{-i})) \right\} \\
\widetilde{v}(n; y_{-i}) &= \max_{0 \le y_i \le 1} \left\{ \pi(n)\, l(1-y_i) + \beta f(y_i) \sum_{k=0}^{n-1} \widetilde{W}_0(n-k) \Pr(k | f(y_{-i})) \right. \\
&\quad \left. + \beta \widetilde{W}_0(n)(1 - f(y_i)) \Pr(n-1 | f(y_{-i})) \right\}
\end{aligned}
$$

where $\pi(n) = (R/(n+1))^2$ and $W_0(n-k), \widetilde{W}_0(n-k)$ are determined according to the entry process described in (1). For the second order conditions to hold, it is sufficient that $l''(\cdot), f''(\cdot) \le 0$. The choice of functional forms in the numerical calculations are based on convenience as customary (see, e.g. Doraszelski and Pakes 2007). The following specification leads to simple analytical first order conditions, which always ensure interior solutions: $l(\cdot) = f(\cdot) = \sqrt{(\cdot)}$.[31] In particular, in a symmetric equilibrium the first order conditions yield:

[31] Any of the functions $l(\cdot), f(\cdot)$ can be linear. I prefer the current specification, because it always yields interior solutions. However, the results for $l(1-x_i) = \sqrt{(1-x_i)}$ and $f(x_i) = x_i$ are very similar and are available upon request.

$$x\left(n\right)=\frac{\left(\beta\sum\limits_{k=0}^{n-1}W_0(n-k)\binom{n-1}{k}(1-\sqrt{x(n)})^k(\sqrt{x(n)})^{n-1-k}\right)^2}{\left(\pi\left(n\right)\right)^2+\left(\beta\sum\limits_{k=0}^{n-1}W_0(n-k)\binom{n-1}{k}(1-\sqrt{x(n)})^k(\sqrt{x(n)})^{n-1-k}\right)^2}$$

$$y\left(n\right)=\frac{\left(\beta\sum\limits_{k=0}^{n-1}\widetilde{W}_0(n-k)\binom{n-1}{k}(1-\sqrt{y(n)})^k(\sqrt{y(n)})^{n-1-k}-\beta\widetilde{W}_0(n)(1-\sqrt{y(n)})^{n-1}\right)^2}{\left(\pi\left(n\right)\right)^2+\left(\beta\sum\limits_{k=0}^{n-1}\widetilde{W}_0(n-k)\binom{n-1}{k}(1-\sqrt{y(n)})^k(\sqrt{y(n)})^{n-1-k}-\beta\widetilde{W}_0(n)(1-\sqrt{y(n)})^{n-1}\right)^2}$$

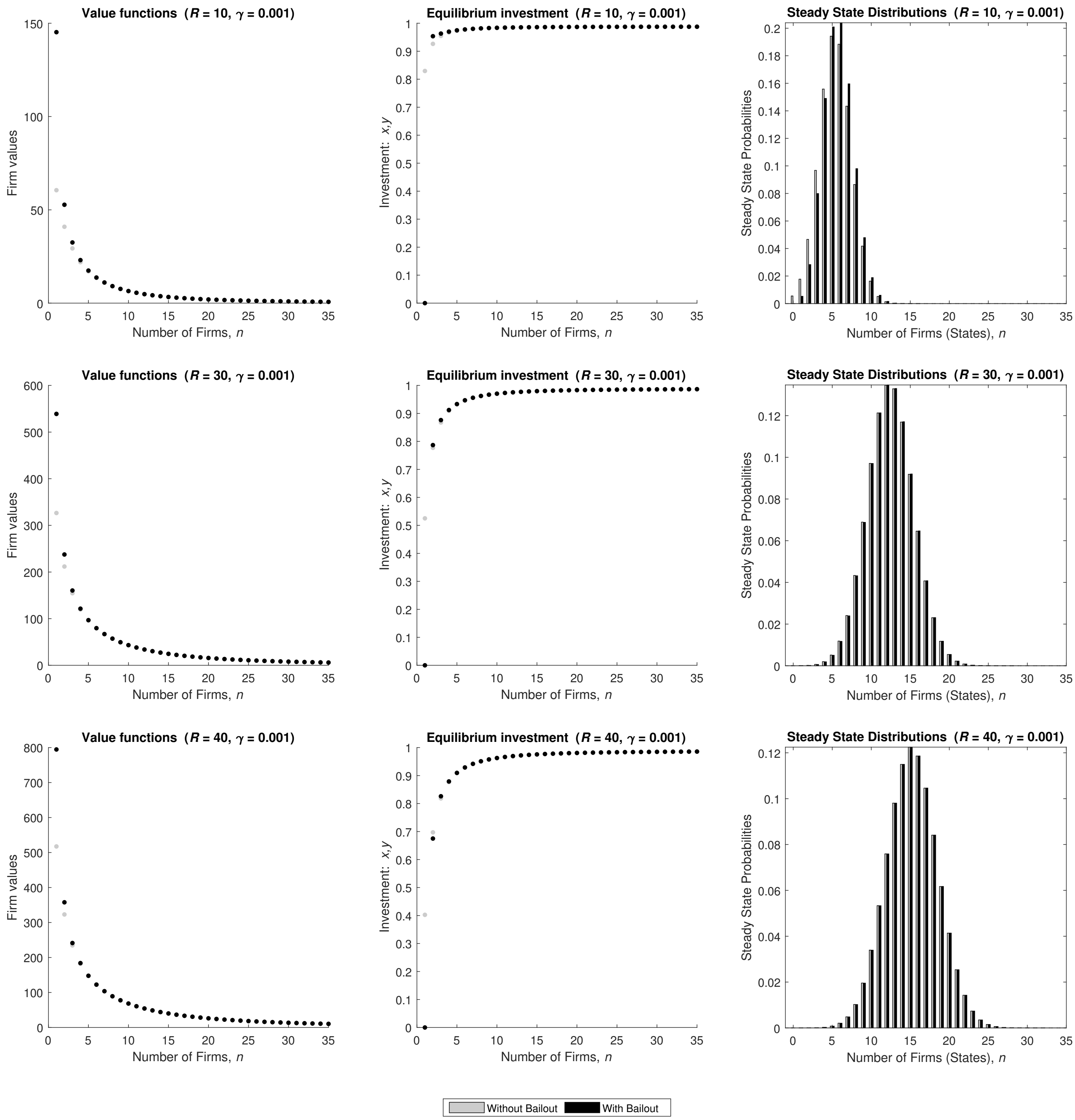
I solve the game using the same algorithm as discussed in the Computation section.

Numerical Results

As is clear from the Figures 5-8, the main difference is that banks in these models behave differently, investment in survival increases with market structure. This is simply because the cost of survival $l\left(1-x\right)$ enters now multiplicatively, rather than additively. As a result, lower $\pi(n)$ naturally reduces the cost of survival, which in turn induces banks to invest more in fragmented markets.

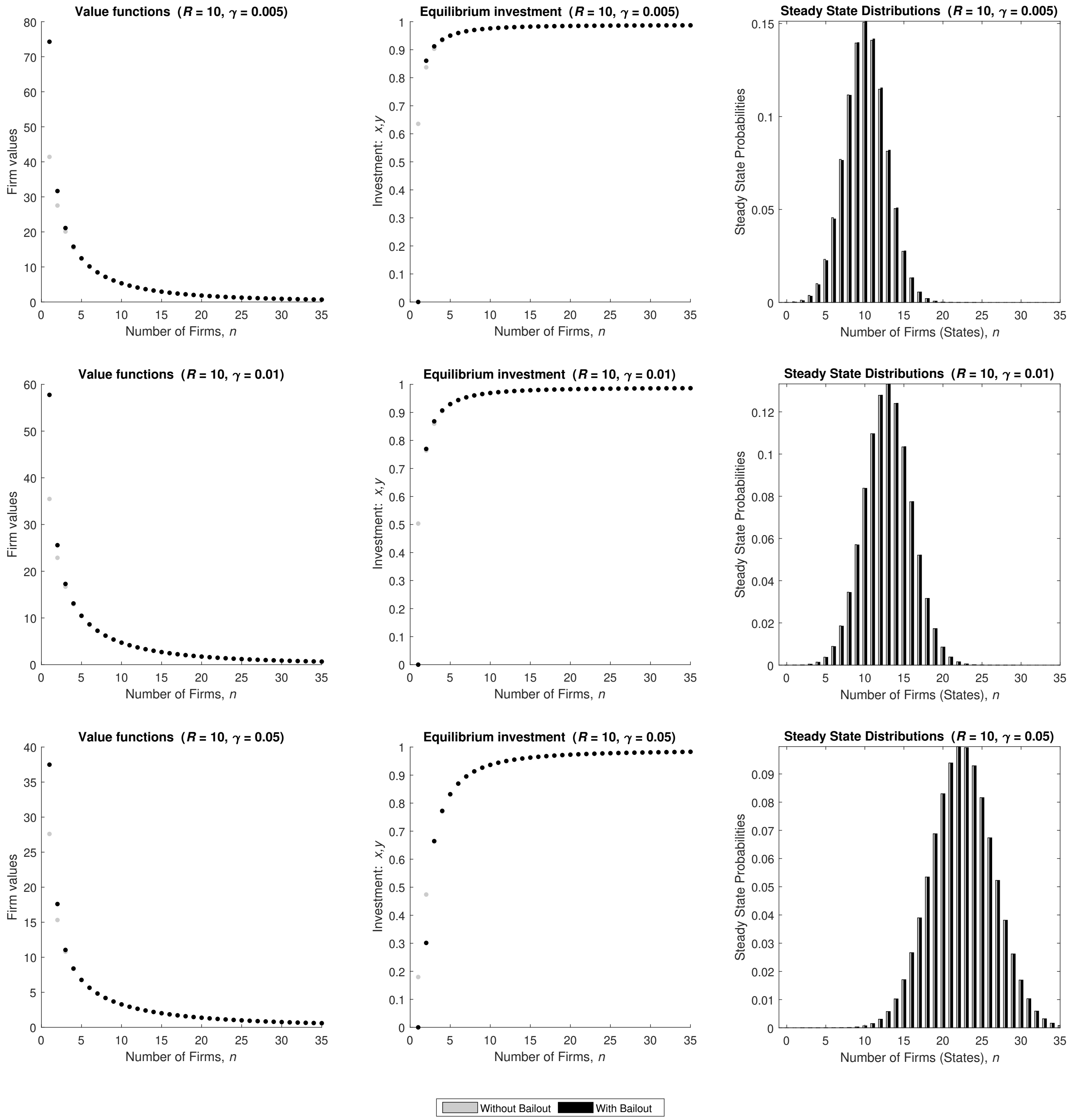
I present the graphs without detailed analyses, I only highlight the most important features. Similarly to the model discussed in the body of the paper, there is always a charter value effect ($\widetilde{V}(n)>V(n)$), banks counting on bailout invest less in concentrated and invest more in fragmented markets (albeit the difference is vanishingly small), and expected systemic crisis is lower for low demand (low R) with bailout than without it.[32] Therefore, in terms of systemic risk, this model produces very similar conclusions.

[32]Note that the parameter values in these graphs are different to those in the main body, primarily because I had to negotiate visualisation. Typically, the same parameter values would have required more firms to ensure that the maximum number of firms is not reached in steady state, in which case it would have been difficult to see anything in the graphs. However, the graphs across the two models are not directly comparable anyway, because the models as well as the functional forms are different, only tendencies could be meaningfully compared and assessed.



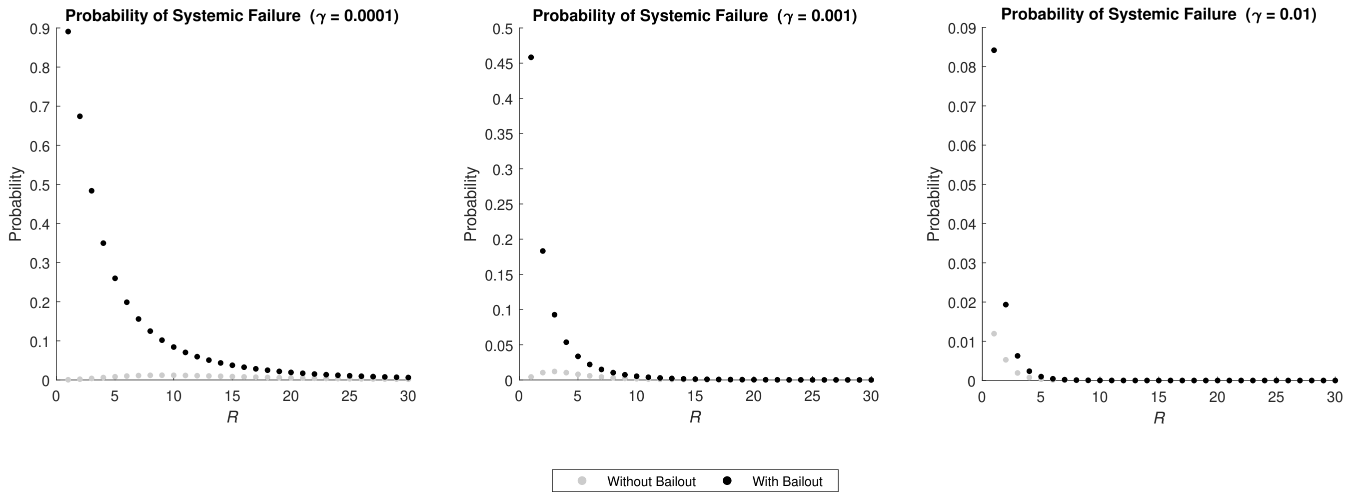
Notes: static profit function: $\pi(n) = (R/n+1)^2$, $R = 10; 30; 40$ (for the first, second and third row of graphs, respectively); probability function of entry: $\rho(a) = 1 - \exp(-0.001a)$; $\beta = 0.97$; maximum number of firms: 35 (potential entrants: 5).

Figure 5: Banking model: The effect of demand ($R = 10; 30; 40$)



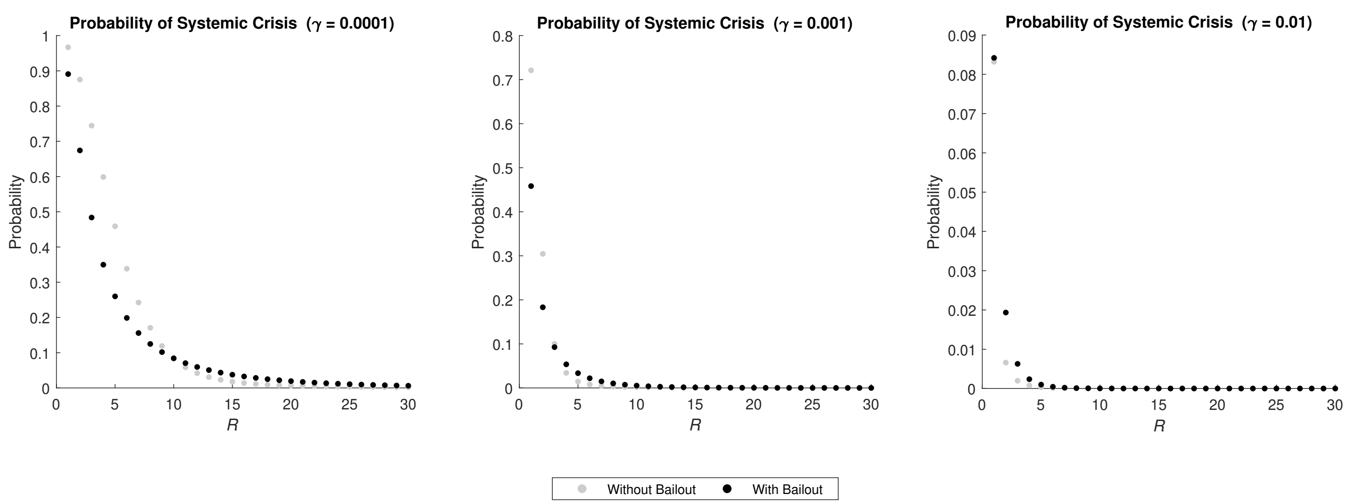
Notes: static profit function: $\pi(n) = (10/n+1)^2$; probability function of entry: $\rho(a) = 1 - \exp(-\gamma a)$ for $\gamma = 0.005; 0.01; 0.05$ (for the first, second and third set of three graph, respectively); $\beta = 0.97$; maximum number of firms: 35 (potential entrants: 5).

Figure 6: Banking model: The effect of entry ($\gamma = 0.005; 0.01; 0.05$)



Notes: Systemic failure is defined as the expected probability of *entering* into the state with zero firms at the end of the production phase.

Figure 7: Banking model: Expected probability of systemic failure



Notes: Systemic crisis is defined as the expected probability of *being* in the state with zero firms at the end of the production phase.

Figure 8: Banking model: Expected probability of systemic crisis