Local Power Markets*

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Abstract

In current power markets, the bulk of electricity is sold wholesale and transported to consumers via long-distance transmission lines. Recently, decentralized power markets have evolved, especially in developing countries where small networks based on solar generation have increased access to electricity in rural areas. We analyze the private and social implications of such local power markets. We show that local power markets with peer-to-peer trading can be competitive and provide efficient investment despite a small number of parties. We find that network externalities arise from stochastic power demand, and derive conditions under which standardization of network types and controlled market entry enhance welfare.

Keywords: Market Design, Networks, Peer-to-Peer Markets, Electricity.

JEL-Classification: D16, D26, D47, L94.

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1 Introduction

Electricity access for all has become a major topic for international energy, climate, and regulatory policy. Especially in developing countries, small networks based on solar generation increasingly provide rural areas with electricity. However, micro-grids for local electricity, eventually separated from large-scale power grids, and local peer-to-peer trading have also been field-tested in established electricity systems. The emergence of local power markets follows advances in information technology, that accompanies physical trades by internet-based exchange of data, allowing for frequent market clearing among peers and neighbors.

In this paper, we provide a first model for peer-to-peer trading in local power markets. The results that we derive are substantially different from typical equilibrium models for the electricity sector. For example, local power markets can yield competitive market prices despite a low number of market participants (i.e., households) and despite the fact that all participating households exercise strategic behavior. Furthermore, investment incentives are efficient. Yet, we find that network externalities exist and warrant regulatory scrutiny.

A report by the National Association of Regulatory Utility Commissioners and the US AID Agency (US AID, 2017) lists decentralized vs. centralized planning, ownership, and technical standards as main topics for design policy and regulation of local power markets. As such, our model attempts to answer the following questions. Do decentralized investment incentives among households yield welfare-optimal investment in local power markets? Are trade and equilibrium power prices competitive? Do technical standards of local power markets require regulatory oversight of market entry and compatibility of standards? Indeed, networks in remote rural areas often have started at low levels of power and quality, typically using simple and easily accessible direct current (DC) technology. Then, with increasing demands and capital costs, alternating current (AC) network standards become feasible, allowing for higher voltage levels, and higher peak and average electricity consumption.

In our model, households can decide to participate in a local market for electricity. Households participate by demanding electricity from the local network, or by investing in generating plants, here solar plants, that add supply to the local network. Households that have invested in generation capacity can then be net-selling or net-buying from the market, depending on whether their generation covers more or less than their consumption, respectively. Given aggregate household demand and aggregate solar supply, the market constitutes an equilibrium price for electricity. We model market clearing using a demand function approach. We also model storage devices, that are used in local networks to balance demand and supply and to soften price variance.

We also analyze implications that arise from the upstream market structure in the market for solar plants and network operation. We consider firms that offer to households the bundle of solar plants and network connection and operation in the local market. Specifically, we investigate if an upstream market that consists of different quality and standards (e.g., AC or DC) leads to inefficient market outcomes due to network externalities.

We find that investment incentives for each participating household are sufficient to obtain optimal market investment. In turn, also the equilibrium market price for electricity in the local market is perfectly competitive. While both net-buying and net-selling households exercise strategic pricing, their strategies cancel out and hence the market price is competitive even with only a few participating households. In essence, households engage in demand reduction in a similar fashion as in Ausubel et al. (2014), but inflate their demand and prices in case they have invested in production and are net-sellers. Next to this positive efficiency result, we find that positive externalities exist that make larger markets relatively more attractive. As a consequence, standardization of network technology can enhance welfare, as can controlled market entry.

Our findings contribute to the large literature on electricity market architecture. Since the deregulation of this sector (Wilson, 2002), this literature has paid significant attention to wholesale market design (e.g., Newbery, 1998; Wolfram, 1999; Borenstein et al., 2000; Fabra et al., 2006; Bushnell et al., 2008; Reguant, 2014; Holmberg and Wolak, 2018), retail market efficiency (e.g., Giulietti et al., 2005; Joskow and Tirole, 2006; Allcott, 2011), as well as network regulation (e.g., Sappington et al., 2001; Joskow, 2008; Tangerås, 2012). We add to this literature by providing a first model on the efficiency of local power markets, where strategic households simultaneously act as producer, consumer, and trader, and where we abstract from the canonical producer to consumer, wholesale to retail market structure.

We also relate to the recent literature on electrification of rural areas. Dinkelman (2011) finds positive effects on female employment of a massive grid connection plan in South Africa in the 1990's. In a recent paper, Lee et al. (2018) provide experimental evidence on the effect of electrification in rural Kenya. They find considerable scale economies of connecting households to the power grid, but that demand falls sharply in price and that overall consumption remained low in their setting.[1] We add with a theoretical study to this growing empirical literature on electrification. To our knowledge, no model has been developed yet to understand efficiency and regulatory requirements for electrification through

[1] Even if at low levels of consumption, Aklin et al. (2016) report that a few hours of additional electricity supply to households nonetheless increased household satisfaction in rural India.

local power markets.

Last, our model also adds to the literature on market design. Beginning with Wilson (1979) and Klemperer and Meyer (1989), this strand has started analyzing market interaction via demand functions and supply functions, respectively. In many cases, the supply function framework has been used to study wholesale power markets (e.g., Green and Newbery, 1992; Baldick et al., 2004; Hortacsu and Puller, 2008; Hortaçsu et al., 2019). Recently, demand function equilibria have also been used to model strategies of traders in sequential double auctions (Du and Zhu, 2017). For our model, we draw from the demand function equilibrium framework, where a household's investment then determines the initial position with which households enter the power trading game. We also introduce the analysis of network externalities into the demand function framework, and illustrate the benefits of compatibility and standardization (e.g., Katz and Shapiro, 1985; Economides, 1996; Baake and Boom, 2001).

The remainder is organized as follows. Section two presents a model for local power markets. In section three, we study network competition. Section four presents extensions, and section five concludes.

2 A model of local power markets

Structure and technology. We study a local power market with n households. Each household has demand for electricity and can invest in generation assets, this is a solar plant, to generate and consume electricity. In case of excess electricity, households can sell to all $n-1$ neighbors via a common micro-grid. Vice versa, in case own generation capabilities are exhausted, each household can also buy electricity from its neighbors. Depending on the aggregate amount of installed plants and relative demands, local power trade establishes an equilibrium price for electricity.

In expectation of the equilibrium price and resulting consumption utility and costs (profit) from buying (selling) electricity, households decide on their optimal amount of plants to be installed and connected to the local grid. We model the outside option from not buying plants as the costs of purchasing electricity from the local market. Hence, the outside option is endogenous to the choices of neighbors.

The utility of each household i is denoted as $U_i(x_i, \varepsilon_i)$ and is positively related to power consumption x_i. In addition, utility depends on the realization of an idiosyncratic error term, ε_i, known only to household i. As such, local trade and prices are shaped by uncertainty about the demand of neighboring households. Furthermore, because the type, i.e., realized demand, of neighboring households is unknown, pricing strategies must be Bayesian-Nash optimal.

Local consumption and trade. We investigate a market architecture where households announce their demand for electricity, and where supply is determined by all pooled generating resources connected to the micro-grid. This is, all production units are connected to the local grid and thereafter non-strategic assets. Only demand of households can be strategic. Aggregate demand and supply then determine the price for electricity.[2]

Households simultaneously decide on their consumption schedule $X_i(p)$ that specifies their demand from the local grid at each price p. In equilibrium, each household consumes $x_i = X_i(p^*)$, where p^* is the equilibrium price that equates supply and demand. Formally, the equilibrium price is

$$p^* : \sum X_i(p, \varepsilon_i) = \sum q_i, \tag{1}$$

where q_i is the installed generation capacity of each household i. A household's profit from trading power becomes $p(q_i - X_i(p))$. Therefore, quasi-linear utility from consumption and trade becomes

$$U_i\left(X_i(p), \varepsilon_i\right) + p(q_i - X_i(p)). \tag{2}$$

Suppose the idiosyncratic consumption shock ε_i is drawn from distribution F ex-ante of submitting demand schedules. The support of ε_i is equal for all households, finite with $\varepsilon_i \in [-\varepsilon_o, \varepsilon_o]$, and symmetric around $E[\varepsilon_i] = 0$. Because households do not know their neighbors' demand realization prior to market clearing, the power price becomes a random variable. Consequently, households must maximize expected utility and, before deciding on $X_i(p)$, form an expectation on the power price as a function of all neighbors' demand shocks.

To capture the randomness in price, conditional on household i's demand function, we draw from the auction literature (Wilson, 1979; Hortacsu and Puller, 2008) and use the market clearing condition to map randomness from demand to price. Specifically, the distribution function of the equilibrium electricity price p^*, given household i's demand X_i at

[2] Detailed parameters for announcing demand may vary in practice. Households could submit dynamic demand functions over time or condition demand on other variables than price. We abstract from multi-dimensional parameters or demand heuristics and focus on the standard case where households submit demand functions that specify demand during a certain period of time.

some price p, becomes

$$\begin{aligned} H_i(p, X_i(p)) &= Pr\left(p^* \leq p \mid X_i\right) \\ &= Pr\left(\sum_{j\neq i} X_j(p,\varepsilon_j) + X_i \leq \sum q_i \;\middle|\; X_i\right), \end{aligned} \tag{3}$$

where j indexes all households except household i. Thus, H_i denotes the probability that $p^* \leq p$, and the support of H_i on $[\underline{p}, \bar{p}]$ depends on the support of all idiosyncratic demand shocks. Intuitively, the probability that the equilibrium price p^* is below some price p relates to the probability that supply is larger than demand at price p, keeping X_i fixed.

Using this probability measure, the expected utility of household i can be written as

$$EU_i = \int_{\underline{p}}^{\bar{p}} \left[U_i\left(X_i(p), \varepsilon_i\right) + p(q_i - X_i(p))\right] dH_i(p, X_i(p)). \tag{4}$$

Households seek to find a demand function that specifies optimal demand over the range of possible prices. Optimality is therefore given by the Euler-Lagrange first order condition, which after rearranging yields:

$$\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p = \left(q_i - X_i(p)\right) \frac{H_{X_i}(p, X_i(p))}{H_p(p, X_i(p))}, \tag{5}$$

where H_{X_i} and H_p are the derivatives of H_i with respect to X_i and p. To interpret the optimality condition in equation (5), note that H_p is the probability density function of price and must be positive. In contrast, H_{X_i} must be negative, because additional demand decreases the likelihood that price is below any given value. Consequently, $\frac{H_{X_i}}{H_p} < 0$, and in equilibrium households that are net-sellers to the local market must have marginal utility from consumption below the market price. We summarize this finding in the following Proposition.

Proposition 1. *Households that are net-sellers from the local market in equilibrium mark-up sales above marginal utility of consumption. Households that are net-buyers from the local market in equilibrium mark-down demand below marginal utility of consumption, i.e.,*

$$\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} < p \iff q_i > X_i(p)$$

$$\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} > p \iff q_i < X_i(p).$$

Proof. The result follows from equation (5). The formal proof is in Appendix A. □

In principle, the equilibrium condition in equation (5) suffices for computing the equilibrium strategies, given model primitives for utility. Yet, H_i depends on the functional form of the equilibrium demand functions, and the probability distribution of price can only in few cases be evaluated analytically (Hortaçsu, 2011).

We therefore assume utility that exhibits saturation, resulting in linear demand. Assuming standard linear demand then allows for a full derivation of the market equilibrium. Indeed, we view linear demand as a good approximation for consumers that stick to simple rather than more complex, nonlinear demand heuristics.[3] Formally, we assume utility of

$$U_i\left(x_i, \varepsilon_i\right) = (\theta_i + \varepsilon_i)x_i - \frac{1}{2}x_i^2, \tag{6}$$

with $\frac{\partial U_i(x_i,\varepsilon_i)}{\partial x_i} = \theta_i + \varepsilon_i - x_i$ and where θ_i represents a household's individual utility with $\theta_1 > \theta_2 > ... > \theta_n$. Given the above, we allow each household to submit a linear demand specification

$$X_i(p, \varepsilon_i) = \alpha_i + \beta_i\varepsilon_i - \gamma_i p, \tag{7}$$

where α_i, β_i, and γ_i are choice variables for each household.[4] That is, when announcing demand, households can shade their reservation value as well as their sensitivity to the error term and price.

Using equation (7), the probability of affecting the market price can be written as $H_{X_i} = -1$. In addition, the density function of price yields $H_p = \sum \gamma_{-i}$ with $\sum \gamma_{-i} = \sum_{j \neq i} \gamma_j$. We provide the full derivation in the Appendix A. Substituting $\frac{H_{X_i}}{H_p}$ into the first order condition in (5) yields

$$\theta_i + \varepsilon_i - (\alpha_i + \beta_i\varepsilon_i - \gamma_i p) - p = (q_i - (\alpha_i + \beta_i\varepsilon_i - \gamma_i p))\,\frac{-1}{\sum \gamma_{-i}}. \tag{8}$$

Coefficient matching then yields equilibrium demand $X_i^*(p)$ with

$$\alpha_i = \frac{q_i + \theta_i \sum \gamma_{-i}}{1 + \sum \gamma_{-i}} \quad \text{and} \quad \beta_i = \gamma_i = \frac{\sum \gamma_{-i}}{1 + \sum \gamma_{-i}}. \tag{9}$$

[3] Linear bid functions are in line with solutions proposed by Baldick et al. (2004) on supply function equilibria as well as Du and Zhu (2017) who model linear demand equilibria for traders who operate in sequential double auctions.

[4] Note that a different factor for the squared term in $U_i(x_i, \varepsilon_i)$ and resulting different slopes for marginal utility only adds a scaling parameter to the following.

The above results show that households who strategically choose demand parameters α_i, β_i, and γ_i in equilibrium respond to the steepness of their neighboring households' demand $\sum \gamma_{-i}$. Put differently, in equilibrium each household optimizes its demand schedule vis-à-vis the slope of household-specific residual supply. This equilibrium feature also reveals the complementarity in demand strategies: The more household i's neighbors shade demand to reduce price, the more can household i likewise shade demand without giving up on its equilibrium share of the network's total power consumption. As stated in the following Proposition, this complementarity results in symmetric equilibrium demand functions.

Proposition 2. *In equilibrium, households submit symmetric demand functions, conditional on size θ_i and demand shock ε_i. With $\gamma_i = \frac{\sum \gamma_{-i}}{1+\sum \gamma_{-i}} = \frac{(n-1)\gamma_i}{1+(n-1)\gamma_i}$ $\forall$ i, the equilibrium demand function is symmetric with*

(1) $\gamma_i^* = \frac{n-2}{n-1}$

(2) $\beta_i^* = \frac{n-2}{n-1}$

(3) $\alpha_i^* = \frac{q_i+\theta_i(n-2)}{n-1}$.

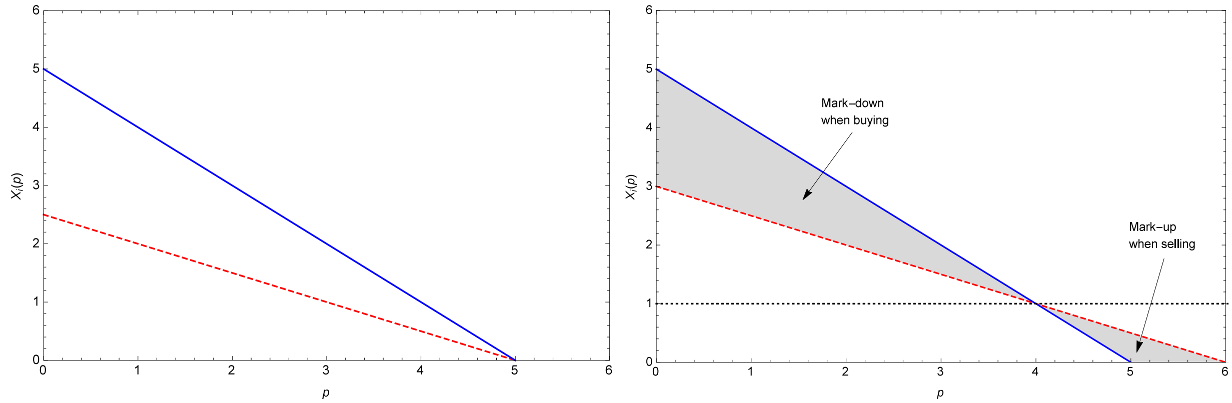
Proof. In Appendix B, we show that asymmetric strategies cannot exist if $\gamma_i > 0$. □

What is the magnitude of demand reduction? Note that the truth-telling demand schedule implies $\alpha_i = \theta_i$ and $\beta_i = \gamma_i = 1$. From Proposition 2 it follows that $\lim_{n\to\infty} \alpha_i^* = \theta_i$ and $\lim_{n\to\infty} \beta_i^* = \lim_{n\to\infty} \gamma_i^* = 1$. Hence strategic demand shading ceases for a large number of neighbors.[5] Both β_i and γ_i approach 1 from below. In addition, for any $q_i \in [0, \theta_i]$ (as reasonable), α_i approaches θ_i from below. Consequently, households increasingly shade demand for higher consumption levels.

Figure 1 illustrates equilibrium demand functions for three households (i.e., for $\beta = \gamma = \frac{1}{2}$), one of which owns solar generation assets. Panel (a) depicts true demand (solid line) and strategic demand (dashed) for a representative household with $\theta_i = 4, \varepsilon_i = 1$, and $q_i = 0$. Panel (b) shows true demand (solid line), strategic demand (dashed line), and supply (dotted line) for one household with $\theta_i = 4, \varepsilon_i = 1$, and $q_i = 1$. As can be seen, households that do not produce electricity shade their demand function by announcing lower demand at any price. Also the household with $q_i = 1$ strategically reduces its demand. However, it demands additional quantities at each price, thereby increasing the market price to its favor. For all

[5] There exists a second, trivial equilibrium with $\alpha_i = q_i$ and $\beta_i = \gamma_i = 0$. For $n \leq 2$, this is the only equilibrium candidate that survives. This type of equilibrium also appears in the analysis of supply functions. In this equilibrium, all trade breaks down.

$X_i(p) > q_i = 1$ this household demands electricity at prices lower than marginal utility, while for $X_i(p) < q_i = 1$ it is willing to sell electricity at a mark-up to marginal utility.



(a) $\theta_i = 4, \varepsilon_i = 1, q_i = 0$ (b) $\theta_i = 4, \varepsilon_i = 1, q_i = 1$

Figure 1: Demand functions for a local market with three households.

Panel (a) in Figure 1 also shows the similarity to standard demand reduction equilibria (e.g., Ausubel et al., 2014), where bids for the first unit demanded equal true marginal utility, and demand reduction occurs for all additional units.

Using Proposition 2 and equation (7), the equilibrium demand schedule yields

$$X_i^*(p) = \frac{n-2}{n-1}(\theta_i + \varepsilon_i - p) + \frac{1}{n-1}q_i, \tag{10}$$

and the market clearing condition in (1) becomes

$$\sum_i^n \left(\frac{n-2}{n-1}(\theta_i + \varepsilon_i - p) + \frac{1}{n-1}q_i \right) = \sum_i^n q_i. \tag{11}$$

The equilibrium price therefore is

$$p^* = \frac{1}{n}\sum_{i=1}^n (\theta_i + \varepsilon_i - q_i). \tag{12}$$

and depends only on market fundamentals. We summarize this finding in the next corollary.

Corollary 1. *The equilibrium market price is independent of demand reduction strategies and only depends on market fundamentals, i.e., θ_i, ε_i, and q_i. As a result, demand functions only determine market shares in consumption at the competitive market price.*

The equilibrium price is independent of bidding shading, because strategies for net-buyers and net-sellers level out. To see this, consider again the case of three households, two of which have zero supply. In market equilibrium, we have net-supply from household i of $q_i - X_i(p)$ equaling demand of the two buying households, $\sum X_{-i}(p)$. The demand reduction of the buying households will be exactly off-set by the selling household. This requires that the selling household reduces its net-supply by consuming more electricity, i.e., by announcing higher demand at any price. The household consumes more electricity, because the cost of consumption reduces when by the same action the price of its supply increases.

As becomes apparent, for trade to take place, the market price must be above the price at which at least one household i has $X_i(p) < q_i$, else there is no trade. In turn, at this price, household i must meet demand by some households j with marginal utility higher than this price. Without making these conditions explicit, we assume that the distributions of θ_i, ε_i, and q_i satisfy trading requirements, else the market breaks down.[6]

Last, the expected price that determines investment follows when recalling that $E[\varepsilon_i] = 0$. Note that while the expected price is always competitive, the distribution of generation assets will determine the amount of consumption. As such, investment incentives only stem from the game of competing for market shares.

Investment and network effects. Next, we derive investment incentives and investigate possible externalities in this market. Prior to market clearing, households may invest in generation assets. Hence, at the investment stage, households do not know the realization of their consumption shock ε_i. Therefore each household expected utility in (4), subtracting investment costs, by weighting over all possible demand shocks, including their own.

To separate household i's demand shock from the equilibrium price, define $\sum_{j \neq i} \varepsilon_j := \Psi_i$ with $g_i(\Psi_i)$ being the density function of the cumulative demand shock. Then, the equilibrium price in (12) can be written as $p^* = \frac{1}{n} [\Psi_i + \varepsilon_i + \sum_i (\theta_i - q_i)]$. Furthermore, recalling that $\varepsilon_i \in [-\varepsilon_o, \varepsilon_o]$, g is distributed between $[-(n-1)\varepsilon_o, (n-1)\varepsilon_o]$. A household therefore finds its optimal investment by maximizing

$$\mathbb{E}[EU_i] = \int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[U_i(X_i(p^*), \varepsilon_i) + p^*(q_i - X_i(p^*))\right] g(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i - p_s q_i \qquad (13)$$

where p_s is the market price for solar generation assets. The first order condition yields an

[6] This is, we rule out intersections of net-supply and net-demand at prices below zero.

optimal choice of q_i as a function of all other household's investment decisions:

$$q_i = \theta_i - (n-1)p_s - \frac{n-2}{n}\sum_i (q_i - \theta_i)\,. \tag{14}$$

Equation (14) shows that, for household i, its investment in solar plants is a strategic substitute to the aggregate market investment. Summing this optimality condition over all n households, i.e., $\sum q_i = \sum \theta_i - n\left((n-1)p_s - \frac{n-2}{n}\sum(q_i - \theta_i)\right)$ and solving for $\sum q_i$ yields aggregate equilibrium investment of

$$\sum_i q_i = \sum_i \theta_i - np_s. \tag{15}$$

Breaking down the above aggregate investment then shows that an individual household in equilibrium invests $q_i = \theta_i - p_s$. Hence, any household only invests in solar plants if its expected maximum valuation of electricity, θ_i, is above the price of solar assets. We summarize this finding in the next Proposition.

Proposition 3. *Equilibrium investment in generation assets of household i is $q_i = \theta_i - p_s$.*

Proof. The result follows from equation (15) and is derived in full in Appendix C. □

That is, small consumers with valuation for electricity lower than the price of solar generation assets will not invest and contribute supply to the local market. The maximum willing to pay for generation assets equals θ_i for the first unit of q_i and decreases thereafter. Households who do not invest instead buy from the (competitive) price from the local market.

Corollary 2. *Individually optimal investment maximizes welfare.*

Corollary 2 follows from equation (15) and by noting that welfare is independent of who owns the generation assets. We formally proof this result in Appendix C.

Using Proposition 3 and equation (12), the realized equilibrium power price —given optimal investment— eventually becomes $p^* = \frac{1}{n}\sum_i(\varepsilon_i + p^s)$. With $E[\varepsilon_i] = 0$, the expected power price of the local market then simply is

$$\mathbb{E}[p^*] = p_s, \tag{16}$$

so that optimal investment and the price for electricity are determined by the costs of generation assets.

Lastly, substituting the equilibrium investment into equation (13), the expected utility of household i obtains as

$$\mathbb{E}[EU_i^*] = \frac{1}{2}\left(\theta_i^2 - p_s^2\right) + \frac{n-2}{2(n-1)}\sigma, \tag{17}$$

where σ denotes the variance of demand shock ε_i. The first expression represents consumer surplus, while the second expression signals positive network effects.[7] Network effects exist, because the expected utility depends positively on the number of market participants. Where do these network effects come from? The stochastic power demand results in a stochastic power price, imposing shocks to utility. The more market participants there are, the smaller are these shocks and thus the smaller is the resulting dis-utility.

Corollary 3. *In local peer-to-peer electricity markets, positive network externalities exist. A household's utility increases for a higher number of participating households.*

As a result of positive network externalities, market entry by possibly differentiated suppliers of solar assets with limited compatibility can be harmful. We analyze competition by different suppliers in the next section.

3 Competition among networks

In this section, we illustrate how network competition among different suppliers results in potentially separated networks with different types of households. We seek to answer whether network competition decreases welfare by weakening positive network externalities. To fix ideas, assume that a quasi-monopolistic firm exists that offers higher quality devices than some competitive fringe suppliers. This is, there exists a monopoly that offers solar plants incl. connection to and operation of the grid, and a competitive fringe, whose solar plants are not compatible with the monopolist's quality. We introduce the difference in quality as

$$\theta_i = \begin{cases} \theta_i & \text{if buying from monopolist} \\ \omega\theta_i & \text{if buying from competitive fringe} \end{cases} \tag{18}$$

with $0 < \omega < 1$. As the monopoly supplier offers higher quality, its marginal costs are likewise higher. Assume that its marginal costs are c. The monopolist charges prices of

[7] For instance, with ε_i being uniformly distributed (and Ψ_i following an Irwin-Hall distribution), expected utility equals $\frac{1}{2}\left(\theta_i^2 - p_s^2\right) + \frac{n-2}{6(n-1)}\varepsilon_0^2$.

$p_s = p_s^M > c$, has n^m buyers (network participants) and sales of $q_i^M = \theta_i - p_s^M$. Using our previous results, expected utility of its customer i becomes

$$\mathbb{E}[EU_i^M] = \frac{1}{2}\left(\left(\theta_i^2 - p_s^2\right) + \frac{n^m - 2}{n^m - 1}\sigma\right). \tag{19}$$

We assume that the fringe suppliers charge competitive prices normalized to zero, $p_s^F = c = 0$. The competitive suppliers have in total n^f buyers (network participants), and equilibrium sales per customer of $q_i^F = \omega\theta_i$. Suppose there exist n potential customers that choose between high and low cost, respectively quality, solar plants, with $n = n^m + n^f$. Therefore, we can write expected utility of buying from competitive suppliers as

$$\mathbb{E}[EU_i^F] = \frac{1}{2}\left(\omega\theta_i^2 + \frac{n - n^m - 2}{n - n^m - 1}\sigma\right). \tag{20}$$

Monopoly problem. Next, we show how that monopolist chooses its price p_s^M and how this price determines the division of households among high quality and low quality, incompatible networks. Note that if household $i = m$ with θ_m is indifferent between the two networks, then all smaller households with $\theta_i < \theta_m$ must prefer the competitive network, while all larger households $\theta_i > \theta_m$ prefer the monopolist's network. The monopolist therefore maximizes profit by finding the marginal household m that by a margin finds it worthwhile to pay the monopoly price and join the high quality network.

Intuitively, for higher prices, the monopolist increases its margin, however, at the same time decreases the utility of its customers by reducing network effects. In turn, for lower prices, and capitalizing network effects, the utility increases and hence willingness to pay. To find the optimal monopoly price, first write the utility of the marginal household in the monopoly network as

$$\mathbb{E}[EU^M(\theta_m, m, p_s^M)] = \frac{1}{2}\left(\theta_m^2 - (p_s^M)^2 + \frac{m-2}{m-1}\sigma\right), \tag{21}$$

with $m > 2$. In contrast, the utility of the 'largest' household $m+1$ with θ_{m+1} in the competitive network with $p_s^W = 0$ yields

$$\mathbb{E}[EU^F(\theta_{m+1}, n^f, p_s^M)] = \begin{cases} \frac{1}{2}\left(\omega^2\theta_{m+1}^2 + \frac{n-m-2}{n-m-1}\sigma\right) & \text{if } n - m > 2 \\ \frac{1}{2}\omega^2\theta_{m+1}^2 & \text{if } n - m \leq 2. \end{cases} \tag{22}$$

Note that for $n^f = n - m \leq 2$, no trade occurs in the competitive market, see footnote

5. Hence, for very low monopoly prices that only leave $n^f = n - m \leq 2$ households in the competitive network, all trade in this network ceases.

The stability condition for household m to remain in the monopolist's networks requires that the difference between the expected utilities, $\Delta E^M = \mathbb{E}[EU^M(\theta_m, n^f)] - \mathbb{E}[EU^F(\theta_m, n^f + 1)]$, is at least positive:

$$\Delta E^M(\theta_m, m, p_s^M) = \begin{cases} \frac{1}{2}((1-\omega^2)\theta_m^2 - (p_s^M)^2 - \frac{n+1-2m}{(m-1)(n-m)}\sigma) \geq 0 & \text{if } \; n - m \geq 2 \\ \frac{1}{2}((1-\omega^2)\theta_m^2 - (p_s^M)^2 + \frac{m-2}{m-1}\sigma) \geq 0 & \text{if } \; n - m < 2. \end{cases} \tag{23}$$

If the indifference condition in (23) holds, household m prefers to stay in the monopoly network. In addition, we also require a stability condition for the competitive network, where household $m+1$ must prefer to remain in the competitive network. This stability condition becomes

$$\Delta E^F(\theta_{m+1}, m, p_s^M) = \frac{1}{2}\left((\omega^2 - 1)\theta_{m+1}^2 + (p_s^M)^2 - \frac{2m+1-n}{m(n-m-1)}\sigma\right) \geq 0. \tag{24}$$

Solving $\Delta E^M(\theta_m, m, p_s^M) = 0$ for $p_s^M(m)$ yields

$$p_s^M(m, \sigma) = \sqrt{(1-\omega^2)\theta(m)^2 + \frac{n+1-2m}{(n-m)(m-1)}\sigma}, \tag{25}$$

which is the price the monopolist can charge to attract household m into the monopoly network. For this price to implement an interior solution, the monopoly price has to decrease in m, this is, for smaller households. To guarantee an interior solution, where both the monopoly and the competitive network co-exist, we in abuse of notation make θ_i continuous over i and define $\theta_i = \theta(i)$ with $\theta'(i) < 0$. For the interior solution to exist we must therefore have

$$\frac{\partial \Delta E^M(\theta(m), \cdot)}{\partial m} < 0 \quad \Leftrightarrow \quad (1-\omega^2)\theta(m)\theta'(m) < -\frac{1}{2}\left(\frac{1}{(n-m)^2} + \frac{1}{(m-1)^2}\right)\sigma. \tag{26}$$

The left hand side is negative and presents the decrease in relative consumer surplus between the two networks as θ_m declines. The right hand side presents the change in relative network externalities between the two networks as θ_m declines. Equation (26) needs to hold for the competitive network having any customers that trade electricity. As can be seen, for critically high demand shocks, the interior solution ceases and all customers join the monopolist's network.

The monopoly problem for $m \leq n-2$ then can be written as

$$max_m \Pi^M = (p_s^M(m,\sigma) - c^M)\left[\sum_{i=1}^{m} \theta(i) - m p_s^M(m,\sigma)\right]. \tag{27}$$

What is the role of stochasticity for the equilibrium monopoly price? We have that

$$p_s^M(m,0) = \sqrt{(1-\omega^2)}\theta(m);\ sign\left[\frac{\partial p_s^M(m,0)}{\partial \sigma}\right] = sign[2m-n-1] \tag{28}$$

Therefore, the monopoly price increases in the volatility and network effects if more than half of all households are in the monopolist's network, else the volatility decreases the monopoly price.

4 Extensions

As a first extension, we include storage into the model. Storage capacities allow for storing unnecessary production in times of low demand realizations and for selling stored generation in times of high demand. Especially where local markets rely on weather-dependent solar generation, storage typically accompanies the architecture of local power markets.

In essence, we show that storage does not change equilibrium prices and investment. However, storage cushions demand shocks to the market and thereby increases welfare. To incorporate storage activity, we model storage as an additional player on the market. This can be thought of as either a centralized storage device or, likewise, as the aggregate reactions of all individual households' storage devices.

Specifically, storage bids into the market at the same time as households do: the storage device supplies whenever the clearing price is above the expected market price, and charges from the grid whenever the price is below the expected price. Formally, we have a supply function of the storage units of

$$X_s(p) = s(p - E[p]). \tag{29}$$

To analyze the effect of storage, first note that storage as additional buyer (seller) in the market only impacts the expected utility of a household via the equilibrium market price p^* and its distribution function H. In detail, the distribution of the clearing price now obtains from

$$
\begin{aligned}
H(p, X_i(p)) &= Pr\left(p^* \leq p \mid X_i(p)\right) \\
&= Pr\left(\sum_{j \neq i} X_j(p, \varepsilon_j) + X_i(p) - X_s(p) \leq \sum q_i \;\middle|\; X_i(p)\right).
\end{aligned} \tag{30}
$$

Using this price distribution, the equilibrium strategy of household i for buying or selling on the local market changes to

$$
\alpha_i = \frac{q_i + \theta_i \sum(s + \gamma_{-i})}{1 + s + \sum \gamma_{-i}} \quad \text{and} \quad \beta_i = \gamma_i = \frac{s + \sum \gamma_{-i}}{1 + s + \sum \gamma_{-i}}. \tag{31}
$$

While demand strategies change with storage, the expected equilibrium price does not. This is because storage only operates if prices realize above or below the expected market price. Therefore, in expectation storage does not operate, and only buys (sells) if realized prices are below (above) the expected price. As a result, with an unchanged equilibrium power price it follows that storage also does not change the equilibrium investment.

However, storage alters the distribution of realized equilibrium prices. In Appendix D, we show that the realized equilibrium power price with storage becomes

$$
p^* = \sum_{i=1}^{n} \left(\frac{1}{n}(\theta_i - q_i) + \tau \varepsilon_i \right) \tag{32}
$$

with

$$
\tau = \frac{2 + n + s - \sqrt{(n-2)^2 + 2ns + s^2}}{2(2n + s)} < \frac{1}{n}. \tag{33}
$$

Because $\tau < \frac{1}{n}$, price shocks in either direction caused by net positive or net negative aggregate demand shocks are cushioned as compared to the no storage equilibrium price of $\sum_{i=1}^{n} \frac{1}{n} \left(\theta_i - q_i + \varepsilon_i\right)$. We further find that storage can increase the utility of the participating household, if this household's consumption shock is higher than the average consumption shock.

Future extensions could focus on the workings of flat tariffs for local consumption instead of prices that vary with demand, and the effect of algorithms that bid on behalf of households, i.e., that potentially act as an aggregated bidding and operating unit for several households.

5 Conclusion

In this article, we provide a first model for studying efficiency, welfare, and network externalities in local power markets. We have shown that local power markets, that are emerging in rural areas in developing economies, but are also field-tested in established electricity systems, can provide electricity at competitive prices. This result holds even for a low number of connected households. Furthermore, the market arrangement, where each household's supply is connected to the common local grid and households thereafter compete in demand functions, provides efficient investment incentives. Individually optimal investment guarantees that welfare is optimal. However, despite these positive results for the efficiency of these market types, we find that network externalities exist. Therefore, market entry and standardization of network technologies enhances welfare and should be placed under regulatory oversight.

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A Equilibrium demand functions

Integrating (4) by parts yields

$$
\begin{aligned}
EU_i &= \int_{\underline{p}}^{\overline{p}} \left[U_i\left(X_i(p), \varepsilon_i\right) + p(q_i - X_i(p))\right] dH(p, X_i(p))dp \\
&= \left[U_i\left(X_i(p), \varepsilon_i\right) + p(q_i - X_i(p))\right] H(p, X_i(p))|_{\underline{p}}^{\overline{p}} \\
&- \int_{\underline{p}}^{\overline{p}} \frac{d}{dp}\left[U_i\left(X_i(p), \varepsilon_i\right) + p(q_i - X_i(p))\right] H(p, X_i(p))dp
\end{aligned}
$$

and because $H(\underline{p}) = 0$ and $H(\bar{p}) = 1$ we obtain

$$
\begin{aligned}
EU_i = U_i\left(X_i(\overline{p}), \varepsilon_i\right) + \overline{p}(q_i - X_i(\overline{p})) \\
- \int_{\underline{p}}^{\overline{p}} \left[\left(\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p\right) X_i'(p) + q_i - X_i(p)\right] H(p, X_j(p))dp.
\end{aligned}
$$

Therefore households maximize

$$
\max_{X_i(p)} \left[U_i\left(X_i(\overline{p}), \varepsilon_i\right) + \overline{p}(q_i - X_i(\overline{p})) - \int_{\underline{p}}^{\overline{p}} \mathcal{L}(p, X_i(p), X_i'(p))dp\right]
$$

with

$$
\mathcal{L}(p, X(p), X_i'(p)) := \left[\left(\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p\right) X_i'(p) + q_i - X_i(p)\right] H(p, X_i(p)).
$$

With free salvage value (Kamien and Schwartz, 2012), the first order condition for an unspecified $X_i(\overline{p})$ becomes

$$
\frac{d}{dp}\mathcal{L}_{X'} = \mathcal{L}_X
$$

with

$$
\left(\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p\right) - \mathcal{L}_{X'} = 0 \text{ for } p = \overline{p}.
$$

Taking derivatives yields

$$\mathcal{L}_{X'} = \left(\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p\right) H(p, X_i(p))$$

$$\mathcal{L}_X = H_{X_i}\left[\left(\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p\right) X_i'(p) + q_i - X_i(p)\right] + \left[\frac{\partial^2 U_i(X_i(p), \varepsilon_i)}{\partial X_i^2} X_i'(p) - 1\right] H(p, X_j(p))$$

and

$$\frac{d}{dp}\mathcal{L}_{X'} = (H_p(p, X_j(p)) + H_{X_i}(p, X_j(p)) X_i'(p)) \left(\frac{\partial U_i(X_i(p), \varepsilon_i)}{\partial X_i} - p\right) + \left(\frac{\partial^2 U_i(X_i(p), \varepsilon_i)}{\partial X_i^2} X_i'(p) - 1\right) H(p, X_i(p)).$$

Using the above and rearranging $\frac{d}{dp}\mathcal{L}_{X'} = \mathcal{L}_X$ yields equation (5). For $p = \overline{p}$ we have

$$\left(\frac{\partial U_i(X_i(\overline{p}), \varepsilon_i)}{\partial X_i} - \overline{p}\right) - \mathcal{L}_{X'} = \left(\frac{\partial U_i(X_i(\overline{p}), \varepsilon_i)}{\partial X_i} - \overline{p}\right) - \left(\frac{\partial U_i(X_i(\overline{p}), \varepsilon_i)}{\partial X_i} - \overline{p}\right) = 0.$$

B Symmetry

We show that only symmetric strategies exist for all positive γ_i. We proceed by induction arguments. Consider two households i and j, plus the remaining $(n-2)$ households with (ex-ante possibly distinct) γ_k. From the derivation of bidding strategies we know that $\gamma_i = \frac{\sum \gamma_{-i}}{1+\sum \gamma_{-i}}$. Hence, we can write the optimality conditions for each household in equation (9) as

$$C_i = \gamma_i - \frac{\gamma_j + \sum_{-i,-j}^{n-2} \gamma_k}{1 + \gamma_j + \sum_{-i,-j}^{n-2} \gamma_k}$$

$$C_j = \gamma_j - \frac{\gamma_i + \sum_{-i,-j}^{n-2} \gamma_k}{1 + \gamma_i + \sum_{-i,-j}^{n-2} \gamma_k}$$

and solve for γ_i and γ_j that guarantee $C_i = C_j = 0$. The only solution with positive γ_i is for $\gamma_i = \gamma_j$. Hence $s = 2$ households (i and j) exist with symmetric strategies $\gamma_i = \gamma_j$. Now taking one additional household l out of $\sum_{-i,-j}^{n-2} \gamma_k$ we can write

$$C_i = \gamma_i - \frac{(s-1)\gamma_i + \gamma_l + \sum_{-i,-l}^{n-3} \gamma_k}{1 + (s-1)\gamma_i + \gamma_l + \sum_{-i,-l}^{n-3} \gamma_k}$$

$$C_l = \gamma_l - \frac{s\gamma_i + \sum_{-i,-l}^{n-3} \gamma_k}{1 + s\gamma_i + \sum_{-i,-l}^{n-3} \gamma_k}$$

with $s = 2$. We again solve for γ_i and γ_l that guarantee $C_i = C_l = 0$ and again find only one solution with positive parameters, which is the symmetric solution. Continuing with $\gamma_i = \gamma_j = \gamma_l$ and $s = 3$ yields one additional symmetric household. Continue until $s = n - 1$ for which all households' γ_i are symmetric.

C Equilibrium investment

Starting from expected utility

$$\mathbb{E}[EU_i] = \int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[U_i(X_i(p^*), \varepsilon_i) + p^*(q_i - X_i(p^*))\right] g(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i - p_s q_i$$

we take the first order condition and arrive at

$$\int_{-\varepsilon_o}^{\varepsilon_o} \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \left[\frac{2(n-1)(\theta_i + \varepsilon_i - q_i) + (n-2)\left(\sum \theta_{-i} + \Psi_i - \sum q_{-i}\right)}{n(n-1)}\right] g(\Psi_i) d\Psi_i f(\varepsilon_i) d\varepsilon_i = p_s$$

We then solve the inner integral by parts using that $G(-(n-1)\varepsilon_o) = 0$ and $G((n-1)\varepsilon_o) = 1$. Further, we use that for a symmetric PDF g with mean zero, the anti-derivative of its CDF, $\overline{G}(\Psi_i) = \int G(\Psi_i) d\Psi_i$, evaluated at the bound of the support yields $\overline{G}((n-1)\varepsilon_o) = \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} G(\Psi_i) d\Psi_i = \Psi_i G(\Psi_i)|_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} - \int_{-(n-1)\varepsilon_o}^{(n-1)\varepsilon_o} \Psi_i g(\Psi_i) d\Psi_i = (n-1)\varepsilon_o$ and $\overline{G}(-(n-1)\varepsilon_o) = 0$. Using a corresponding procedure for the outer integral yields equation (14) in the main text.

To find investment that maximizes welfare we evaluate $\frac{\sum_i \mathbb{E}[EU_i]}{\partial q_i} = 0$ and obtain

$$q_i = \theta_i - (n-1)^2 p_s - (n-2) \sum_i (q_i - \theta_i).$$

Note that this differs from the optimal investment that maximizes only household i's expected utility. Summing over all households we obtain the aggregate investment

$$\sum q_i = n \left(\theta_i - (n-1)^2 p_s - (n-2) \sum_i (q_i - \theta_i) \right).$$

When solving for $\sum q_i$ we again obtain

$$\sum_i q_i = \sum_i \theta_i - np_s,$$

as for the case when a households only maximizes its own expected utility, rather than welfare as here.

D Demand functions with storage

Starting from the equilibrium strategies in equation (31), one easily obtains the equilibrium aggregate market demand $\sum_i X_i(p, s, \varepsilon_i)$. Expected market demand is $\sum_i X_i(p, s, 0)$. Solving the clearing condition $\sum_i X_i(p, s, 0) - X_s(p) = \sum_i q_i$ for the equilibrium price we obtain the expected market price with storage, which is $\sum_i \frac{1}{n}(\theta_i - q_i)$ and hence the same as for the no storage case. Using the expected price in the storage supply function yields $X_s(p) = s(p - E[p]) = s(p - \sum_i \frac{1}{n}(\theta_i - q_i))$. In turn, this allows to solve for the realized price $\sum_i X_i(p, s, \varepsilon_i) - X_s(p) = \sum_i q_i$ which yields equation (32) in the main text.