Auctioning Annuities*

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Abstract

We develop a model of demand and supply for annuities to estimate welfare in a market for annuities, and to identify better mechanisms. In our model, retirees are *rationally-inattentive* with preferences for both pecuniary and non-pecuniary attributes, and insurance companies who sell annuities have private information about their costs and compete in multi-attribute auctions. We use a rich data from Chile, where annuities are bought and sold in a market through two-period competitive tenders with selective entry. We find (i) significant heterogeneity in preferences and costs; (ii) information processing cost decreases with savings; (iii) but is highest for retirees who have sales-agents as intermediaries; (iv) these retirees also exhibit stronger dislike for bad risk-ratings; but (v) the annuitization costs do not vary with savings. Through counterfactual simulations, we find that simplifying the mechanism to a one-period English auction increases welfare by T%, and decreases average markup from T% to T%. If we use a sequential auction, where companies with better risk-ratings offer first, the total welfare increases by T% and consumer surplus by T%.

Keywords: Annuity, Asymmetric Information, Rational Inattention

JEL: D14, D44, D91, C57, J26, L13.

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1 Introduction

According to the U.S. Social Security Administration's 2015 actuarial life table, a man who is 65 years old in 2015 is expected to live until he is 84 yrs., and for a women it is 85.5 yrs. In Chile, these numbers are 83 yrs. and 86.5 yrs., respectively. So a retiree has to ensure that his/her savings last for almost two decades *after* retirement. And as we live longer still, this "cake-eating" problem will become more difficult to manage, and, all else equal, it increases the risk of running out of savings. A hedge against this risk is to buy longevity derivatives.

Annuity is one such instrument that provides insurance against outliving ones savings. In a standard annuity, for example, a 65-year-old man turns over, i.e., annuitizes, his saving of $120,000 to an insurance company, in exchange for $630 every month for life. If he dies at 68, he would have collected $22,680 from the company, while the remainder is kept by the company and his heirs get nothing.[1] An advantage of annuity over defined contribution plans, e.g., 401(k) in the U.S., is that with annuity insurance companies bear all the risk; see Yaari (1965); Davidoff, Brown, and Diamond (2005) and Reichling and Smetters (2015). And so, if insurance companies have lower annuitizing costs than the public sector, then a private market for annuities is socially desirable. But it is important to know answers to questions like "How should such a market be structured, should we use posted prices or auctions?" "How should we promote competition?" and "Should we give preferences to companies with better risk-ratings?" among others. Our objective is to answer these questions.

Answering these questions require comparing costs and conducts under different market-structure, which is difficult because only companies know their own costs. In example above, this cost refers to the expected amount of dollars required to finance a stream of payments of one dollar until death of the retiree, which is known as the *Unitary Necessary Capital* (henceforth, UNC). So if the cost is $0.95 per one dollar, then the total monthly cost for the insurance company of providing a pension of $630 is $598.5. Thus the UNC reflects the opportunity cost for a company, and it depends on its capital market portfolios and investment strategies and its own expectation about the mortality rate of the retiree, all of which are company's private information.

So as a first step we would rely on a model of demand and supply for annuities, and estimate these UNCs, in the vein of classic Bresnahan (1989). Any model, however, must reflect the information companies have about retirees and also allow for *adverse selection* because those with higher savings tend to live longer (Zaninotto et al., 2020) and have a higher UNC (Einav, Finkelstein, and Schrimpf, 2010; Fajnzylber, Willington, and Pizarro, 2019). But because most papers either focus on demand (Lockwood, 2018; Shu, Zeitham-

[1] To break even, he would have to live until he is 81 years old.

mer, and Payne, 2016; Brown et al., 2017; Illanes and Padi, 2018) or on a monopoly seller (Einav, Finkelstein, and Schrimpf, 2010), we know little the effect of imperfect competition on pensions, allocations, and welfare in annuity markets, and how they interact with companies' private information. Our objective is to fill this gap, by estimating a rich but tractable model of demand and oligopoly supply for annuities using a dataset from Chile.

In 1981, Chile replaced its old public pay-as-you-go pension system with a new system of privately managed individual accounts (Júaregui, 2010). Since 2004, retirees have to use a centralized electronic system, known as the SCOMP, to buy annuities from insurance companies, and the system is structured like an auction market (Alcalde and Vial, 2016, 2017; Morales and Larraín, 2017). There are few advantages of using SCOMP data. We observe demographic characteristics, savings for all retirees, including all the offers they received including their final choices. In fact we observe everything companies observe about a retiree before they make their first-round offers, so we can model the supply side accurately.

Our findings will directly inform policy debate in Chile to improve the market. While the Chilean pension system has reached a large share of the elderly (91% receive some pension income), the level of retirement income is insufficient to maintain the pre-retirement standard of living. The median replacement rate in Chile (ratio of initial pension to last wage) is 44%. This is substantially lower than 70% replacement rate that the ILO recommends. There are many causes for the rate to be this low: the compulsory contribution rate of 10% of covered wages is low, minimum retirement age has remained at low levels (60 years old for women and 65 for men), 53.4% of individuals contribute only for a short fraction of their careers all the while the life expectancy at retirement is among the highest in the world. We take these factors as given, and focus on the supply side so that we can determine if the current market system is efficient, and if not, recommend an alternative mechanism that improves welfare.

We also contribute to our understanding of how to design a new market for annuities, which is an important topic of debate in many other countries. For example, in the U.S. the H.R. 1994, *Setting Every Community Up for Retirement Enhancement Act of 2019* incentivizes small businesses and communities to band together to create retirement plans. Our results can speak to directly to this policy by showing what kind of market-structure works.

We begin by presenting few key salient features of our sample that our model captures. Even though a retiree can choose to initiate a second-round offer, after receiving the first-round offers, close to 17% of retirees do not. All else equal, we find that a retiree who initiates second-round gets higher pensions than who does not. Moreover, retirees do not always choose an annuity that gives the highest pension, suggesting that they care about other non-pecuniary attribute, such as risk-rating of a company. But even if we compare offered pensions across companies with same or better risk-ratings, close to 28% of employees leave *money on the table*, where the average money left on the table is worth approximately 9-10 months of pensions. This suggests that some retirees, possibly the ones with lower education and hence lower savings, do not even know how to evaluate risk-ratings. Retirees can use an intermediary to "help" them choose an annuity, but we find that a retiree that uses sales-agent is more, not less, likely to make mistakes. But, we find that those with higher savings are less likely to use a sales-agent.

To capture these "demand side" features we posit that at the time of retirement retirees do not know how much they value the risk-rating of a company. They, however, have a prior about their preference which can depend on their age and gender, and if they want to know more about how much they "should" care about risk-rating they have to process some information, which is costly. To capture the effect of sales-agent and savings on the likelihood of not choosing the highest pension, we allow the information processing cost to vary with intermediaries. Based on this interpretation of the decision process, we model retirees as *rationally inattentive* decision makers, à la Sims (1998). In the first-round, retirees face a discrete choice problem: to choose one of the offers or to go to the second-round for negotiations. We follow Matějka and McKay (2015) to model the resulting choice probabilities, and conditional on the second-round, they bargain with companies for a better offer.

On the supply side, there are 19 companies in our sample. But not all of them make offers to all retirees. The average participation rate is approximately 73%. This suggest that companies make endogenous participation decision, which results in selection, i.e., only those with low enough annuitization cost, i.e., UNC, make an offer. Upon entry, firms participate in a multi-attribute first-price auction, with a possibility of going to the second-round which we model as a multi-attribute English auction. The multi-attribute aspect arises from the fact that highest "bidder" is not always the winner. This means, a company who has better risk-rating than others has a competitive advantage, because it can offer lower pension and still win. And more the retirees care about risk-ratings larger will be this advantage.

To identify the information processing costs, we use the empirical elasticity of choice probability with respect to the offered pensions. Intuitively, the elasticity is inversely proportional to the information processing cost, such that those who are less responsive to pensions are ascribed to have higher cost of processing information. If we estimate the elasticities separately by intermediary channels and savings, we can identify channel and savings specific costs. Our estimates suggest that those who have higher savings have lower information processing costs, and those who use sales-agent have larger costs, which are consistent with the fact that those with larger savings tend to be more educated and that those with sales-agent are more likely to leave money-on-the-table.

To identify the preference parameters and the distribution of companies returns, we only use the data on the second round. It is helpful to consider the standard english auction as a benchmark. Without the company characteristics and without bequests, the observed chosen pension is simply the second-order statistic of the UNCs, which in turn identifies the "parent" distribution of UNC. In fact, if we group retirees by their savings, we can nonparametrically identify the conditional distributions of costs, given savings. An advantage of our approach is that if we focus on the second-period only, we can express chosen pension as a linear combination of differences in risk-rating between the two most competitive firms and the annuitization cost, that is easy to estimate.

We find substantial variation in the distribution of UNCs both across companies and retirees and across income. For instance we find that the cost distributions do not change with savings. Thus we do not find evidence of adverse selection in the intensive margin, i.e., those who expect to live longer might have higher savings, which can cost more for the insurance companies because they are making longer commitments. But this cost can vary across companies, based on their financial positions, which is consistent with our estimates.[2]

In terms of the preferences for risk-ratings, our estimates suggest a very interesting pattern supporting evidence of steering. We find that those who use sales-agent or directly contact insurance companies, have larger coefficient for risk-rating than those who use the other two channels. Given that bankruptcy is very rare in Chile–there has been only one bankruptcy in the past four decades–it stands to reason that retirees should not put too much weight on risk-ratings. But this "weight" decreases with savings, suggesting that those with higher savings tend to be more educated and so are less likely to use sales-agents, but even among those who do, they put less weight on risk-ratings.

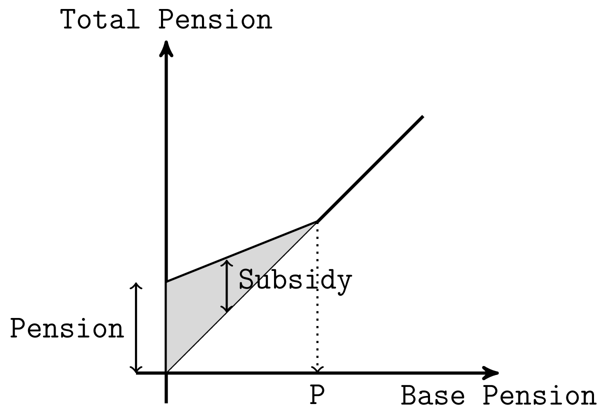
These estimates suggest some interesting counterfactual exercises that can help us improve the market. In this paper we address the following question. First, what happens to welfare if we changed the rule to have only one period auction instead of two? Close to 17% choose annuities in the first-round, even though the companies who know that there might be second-round offer lower pension in the first-round. Second, what happens if the risk-ratings were not used? Third, given that companies have to incur entry cost, can we improve the welfare by implementing simultaneous auction instead of sequential auction as shown by Bulow and Klemperer (2009) and Roberts and Sweeting (2013).

COUNTERFACTUAL RESULTS: To Be Written

In the remainder of the paper we proceed as follows. In Section 2, we introduce the institutional detail, and in Section 3 we introduce our data. Section 4 presents our model

[2]There is also selection in the extensive margin that have been document previously in the literature, that those who have health concerns, say, typically choose to self-insure late-life risks. Therefore those who prefer annuity over PW will live longer, and therefore should cost more to annuitize.

Figure 1: **Subsidies and Pensions**



Notes. The figure shows the subsidies that the government gives to those whose *base-pension*, the sum of all contributory pensions, is less than a threshold P. In September 2018, this threshold was $463 per month.

and Section 5 discusses identification. Section 6 and Section 7 present the estimation method, the estimation results and the counterfactual analysis, respectively. Section 8 concludes. The Appendix includes additional details and proofs not included in the main text.

2 Institutional Background

The Chilean pension system went through a major reform in the early 1980s, when it transited from a *pay-as-you-go* system to a fully funded capitalization in individual accounts run by private pension funds (AFPs). Under the system, workers must contribute 10% of their monthly earnings, up to a pre-determined maximum which in 2018 was U.S. $2,319, into accounts that are privately managed by the AFPs, and grow over time.[3]

The Chilean government regulates and supervises the AFPs. And at the time of retirement guarantees minimum pensions for workers who fail to save enough during their work-lives (savings less than P in Figure 1). Those with savings less than the threshold receive subsidies that are inversely proportional to their savings (Fajnzylber, 2018). We focus only on those with savings above P, who can and must participate in the electronic annuity market. Those with excess savings may withdraw and not annuitize some of their savings.

Retirement Process

Step 0. The process of buying an annuity starts when a worker communicates her decision

[3]Officially, the maximum is expressed in *Unidad de Fomento* (UF), which is a unit of account used in Chile. UF follows the evolution of the Consumer Price Index and is widely used in long-term contracts. UF is adjusted every year, since 2008, depending on the real growth rate of wages. In 2018 the UF was 78.3 UF.

of considering retirement to her designated AFP. She must also disclose all legal beneficiaries and their ages.[4] The AFP then generates a Balance Certificate that contains information about the total saving account balance (henceforth, just savings), and the demographics.

Step 1. The retiree is associated with one of the 4 intermediaries or "channels" to enter the SCOMP system to request quotes. The two channels AFP or to approach a participating insurance company directly, are free, but sales-agent and independent advisor charge fees.

Step 2. The retiree requests offers for some, or all pension products. Upon request, insurance companies in the system have 8 business-days to make offer(s).

Step 3. These bids or offers are collated by the SCOMP and presented to the retiree as "certificate of quotes." The certificate is in the form of a table, one for each type of annuity (e.g., deferred or immediate annuity) sorted from the highest to the lowest pensions along with the name of the company, its risk-rating, and, in the case of a product with a guaranteed period, a discount rate.[5]

Step 4. The retiree can either (i) decide not to retire; or (ii) fill a new Request for Quotes (presumably for different annuities); or (iii) choose PW; or (iv) accept one of the first-round offers; or (v) negotiate with companies by requesting second-round offers.

Step 5. If the retiree requests second-round offers then the participating companies observe all first-round offers, and can decide to increase the offer, and match the best standing offer on the table. This process closely resembles an ascending auction, and throughout the paper we assume that the top-two firms in the first-round remain top-two in the second round.

Types of Annuities

There are three main pension "products," Programmed Withdrawal (PW), immediate annuity and deferred annuity. Under PW, the savings are kept in an AFP administered account, which pays actuarially fair pensions every month, and in the event of death, remaining funds are used to pay survivorship pensions or, if there are no eligible beneficiaries, form part of the inheritance of the retiree. A Programmed Withdrawal is a way to self-insure the late-life risks, and barring extraordinarily high returns, the pension steadily decreases year after year.

Under both immediate annuity and deferred annuity, the funds are transferred to an insurance company (chosen by the retiree) that will provide a constant (in real terms) monthly

[4] Eligible beneficiaries include spouse, children under age 24 or for life if they are disabled. In some cases worker's parent(s) can also qualify as beneficiaries.

[5] The discount rate is relevant only if the retiree dies before the guaranteed period. Then the beneficiaries may choose to receive the present value of those payments in one payment rather than the monthly payments. In that case the offered discount rate is used to calculate the net present value. The document also includes a separate table with the first-year pension under the programmed withdrawal (PW) option, and a graph that shows the expected evolution of the pension under the PW option.

pension.[6] Immediate annuity with no guaranteed period pays some fixed pension every month immediately until the retiree's death, and after that the contract ends. In a deferred annuity with no guaranteed period, pension payments begin in a future date (specified in the contract) and until then the retiree can withdraw (some fixed amount) from her savings.

Annuities may also have two special coverage conditions: a *guaranteed-period* and an *increased-percentage*. Suppose an annuity has a guaranteed period of 10 years, then the retiree is guaranteed to be paid for at least 10 years, even in the event of her death. And if the annuity has increased-percentage condition then in case of death the spouse will receive a survivorship pension equivalent to a 100% of the retiree's pension, instead of the 50% or 60% established by law. Both these clauses leave bequests to the retiree's beneficiaries. Almost every retiree, 99.9% of our sample, choose either 0, 10, 15 or 20 years of guaranteed periods.

3 Data

In this section we introduce our data on the Chilean retirement system between January 2007 and December 2017. For this period, we observe everyone who used SCOMP to buy an annuity or choose PW, and their gender, age and savings. More importantly, we observe everything companies observe about these retirees prior to making the first-round offers, suggesting that the possibility of unobserved heterogeneity is less of a concern for us. For everyone, we know all the offers they received, whether they decided to request the second-round offer, and their final choice. Although a retiree can request offers for many types of annuities, we focus on the offers for the type that was finally chosen, unless they choose PW.

3.1 Retirees

We focus on individuals without eligible children, who are considering retirement within 10 years of normal retirement age (NRA), which is 60 years for a woman and 65 years for a man. Looking to maintain cases with similar competitive behavior from the part of insurance companies, we kept only the most common annuity options: we kept products with either 0, 10, 15 or 20 years of guaranteed periods and with at most 3 years of deferment. The result is a data set with 238,891 retirees, with an almost even split between PW, immediate annuities and deferred annuities. Table 1 shows the shares of each type. Less than 1% of retirees choose annuity *with* PW, and so we exclude them, leaving a total of 238,548 retirees.

[6] The main trade-off when choosing an annuity or a PW is between getting an insurance against longevity and financial turmoil and bequeathing the funds in case of an early death. Another relevant difference is that while annuitization is an irreversible decision, a person can go from PW to annuity.

Table 1: **Market Share, by type of pension product**

Product	Obs.	%
PW	78,161	32.7
Immediate annuity	87,115	36.4
Deferred annuity	73,272	30.6
Annuity with PW	343	0.9
Total	238,891	100

Notes. The table shows the number of retirees choosing different annuity products.

Table 2: **Age Distribution, by Gender and Marital Status**

Retiring Age	S-F	M-F	S-M	M-M	Total
Before NRA	1,871	1,771	4,714	22,142	30,498
At NRA	20,789	22,475	17,114	72,572	132,950
Within 3 years after NRA	14,470	16,797	4,447	19,086	54,800
At least 4 years after NRA	6,900	6,715	1,251	5,434	20,300
Total	44,030	47,758	27,526	119,234	238,548

Notes. The table shows the breakdown by marital status and retirement age. The first letter denotes marital status (Single or Married) and the second letter denotes gender (Male or Female), e.g., 'S-F' refers to a single female. NRA is the normal retirement age, which is 60 years for a female and 65 years for a male.

In Table 2 we present statistics about age, marital status, and gender at retirement. As we can see, approximately 56% retire at their NRA, and close to 79% retiree within three years of their NRA, and married men (MM) make up half of all retirees. Retirees also vary in terms of their savings; see Table 3. The mean savings in our sample is \$112,471, while the median savings is \$74,515 and the first and third quartiles are \$46,449 and \$132,356, respectively. Savings are higher for men, and those who retire before NRA tend to have larger savings. These savings are transferred to the insurance company if retirees buy annuities.

First-Round Offers

A retiree receives approximately 10.6 offers for every type of annuity, and the number increases with savings. For instance, those with savings at the 75^{th} percentile of our sample get an average of 12.4 offers and those at the 25^{th} percentile get an average of 7.8 offers. There is substantial variation in the offered pensions across companies and retirees; see Table 4. On average, for immediate annuity, retirees get an offer of \$570 and for deferred annuities the average offer is \$446. Women, on average get an offer of \$479 for immediate annuities and \$412 for deferred annuities, while for men they are \$631 and \$473, respectively, that are

Table 3: **Savings, by Retirement Age and Gender**

	Mean	Median	P25	P75	N
Retiring Age					
Before NRA	185,660	129,637	73,104	245,857	30,498
At NRA	89,907	60,023	41,521	103,680	132,950
Within 3 years after NRA	115,666	87,126	54,353	135,562	54,800
At least 4 years after NRA	141,673	101,594	58,815	168,202	20,300
Overall	112,471	74,515	46,449	132,356	238,548
Gender					
Female	97,308	81,180	51,817	121,633	91,788
Male	121,955	69,372	43,818	147,184	146,760
Overall	112,471	74,515	46,449	132,356	238,548

Notes. Summary statistics of savings, in U.S. dollars, by retiree's age at retirement, and by retiree's gender.

consistent with men having higher savings and shorter life expectancy than women.

Table 4: **Monthly Pension Offers, by Annuity type and Gender**

Annuity Type	Gender	Mean	P25	Median
Immediate	Female	479	288	414
	Male	631	272	435
	Overall	570	278	423
Deferred	Female	412	269	374
	Male	473	252	356
	Overall	446	259	365

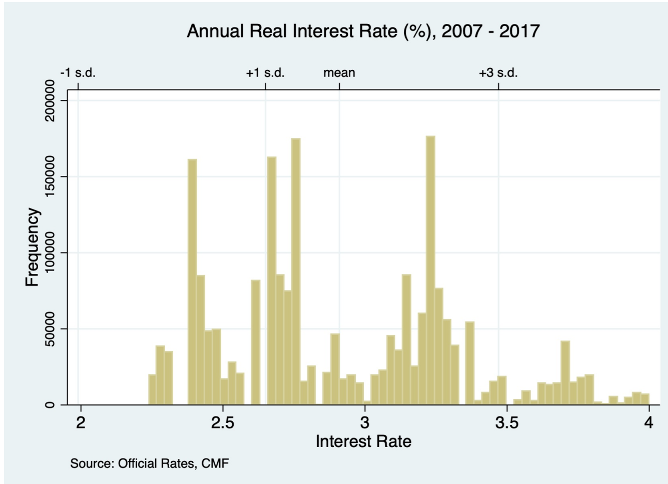
Notes. Summary of all monthly pensions (in U.S. dollars) offers received in the first round.

We observe a lot of variation in offers, both within companies (across retirees) and across companies, in the offers, which in turn suggests a variation in the cost of annuitization, i.e., the UNCs. And one of the factors that can influence UNCs of a company is the prevailing interest rates in the market. In our sample we find a lot of variation in the real interest rates (see Figure 2), which affects companies differently based on their portfolios, and provides a nice exogenous variation in the data.

Chosen Annuities

Throughout this paper we restrict ourselves to immediate or deferred annuities, with either 0, 10, 15 or 20 years of guaranteed period. In Table 5 we present the "transition" rate from the first-round to the second-round. Among those who choose PW, 98.1% of them choose

Figure 2: **Histogram of Interest Rate**



Note. Histograms of the interest rates in our sample.

PW in the first-round. Among those who choose an annuity, 86.9% requested and accepted second-round offers, 11.2% accepted a 1^{st}-round offer without requesting second-round offers.

Table 5: **Number of Retirees who choose in First- or Second-Round**

Round/Choice	PW	1^{st} round	2^{nd} round	Total
1^{st} round	76,690	18,001	0	94,691
2^{nd} round	1,471	2,979	139,407	143,857
Total	78,161	20,980	139,407	238,548

Notes. Round refers to whether retirees chose in the first- or in the second-round.

In Table 6 we present several information about the chosen annuities: (i) the total number of accepted offers by the type of annuity; (ii) the average number of first-round and second-round offers received for the annuity that was eventually chosen; (iii) the number of accepted second-round offers; (iv) the average percentage increase in pension offers from first-round to second-round (only for the accepted choice); (v) the percentage of retirees who requested at least one second-round offer; (vi) the percentage of retirees who chose the highest paying alternative; and (vii) the percentage of retirees who chose a dominated option, in terms of either pensions (with same risk-rating) or risk-ratings (with same pension) or both.

Intermediary Channel

When the incentives of an intermediary do not align with those of a retiree, then it is possible that the former will "steer" the latter away from the optimal decision. The misalignment

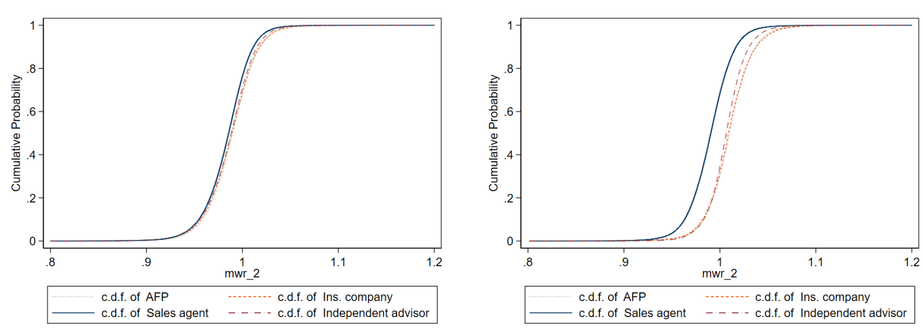
Table 6: **Summary of Accepted Annuities**

GP Years	N Accepted	Average # of 1^{st} Round Offers	# Accepted in 2^{nd} Round	Increase	Average % Requested 2^{nd} Round	Best	Dominated
			Inmediate				
0	21,292	11.3	16,357	1.5	80	59	22
120	26,907	11.1	23,463	1.3	89	51	28
180	24,452	11.6	22,070	1.4	92	49	29
240	14,464	11.8	13,020	1.5	92	51	29
Total	87,115	11.4	74,910	1.4	88	53	27
			Deferred				
0	11,703	10.9	8,919	1.5	79	53	23
120	26,119	11.0	23,390	1.4	91	46	31
180	26,775	11.4	24,324	1.4	92	42	34
240	8,675	11.0	7,864	1.3	92	42	34
Total	73,272	11.1	64,497	1.4	90	45	31

Notes. The table shows the number of chosen annuities by type of product, the average number of first-round offers received for the chosen annuity, the number of accepted offers that resulted from second-round offers, the average percentage increase between the first-round and second-offers (for the accepted choice), the percentage of individuals who requested at least one second-round offer, the percentage of retirees who chose the highest paying alternative option and the percentage of individuals who chose an offer that was dominated by another alternative with same (or better) credit rating.

of incentives may be particularly relevant for sales-agents, who receive their intermediation fee only if the retiree chooses the sales-agent's firm. In other words, it is possible and very likely that those with sales-agent would appear to value non-pecuniary benefits of a company more than the pecuniary benefits. In Figure 3 we present evidence that shows that using sales-agents leads to accepting (stochastically) lower offers.

Figure 3: **CDFs of Offered and Accepted MWR, by Channel**



Notes: The figures display the distributions of offered (left) and chosen (right) `mwr`, by channel: (1) AFP, (2) Insurance company, (3) Sales agent or (4) Independent advisor.

To account for observed differences among retirees we present the distributions of the money's worth ratio (henceforth, `mwr`), which factors in the observed differences among retirees. In particular, `mwr` is the ratio of the net present value of the lifetime of payments and the savings transferred to the insurance company, where the individual demographic characteristics affect the numerator. Heuristically, `mwr` is defined as the expected return to the employee per premium dollar invested. The figures display the distributions of the `mwr` offered in the first-round and `mwr` accepted by the retirees, respectively. The mean and the median `mwr` of the offers, by channels, are $(0.989, 0.988, 0.984, 0.987)$ and $(0.990, 0.989, 0.986, 0.988)$, respectively. These means and medians change for accepted offers to $(1.010, 1.010, 0.990, 1.007)$ and $(1.010, 1.009, 0.991, 1.007)$, respectively; the final accepted offers are better than the first-round offers. And those with sales-agents have lower `mwr`.

Table 7: **Intermediary Channel - Estimates from Multinomial Logit**

	Insurance Company	**Sales-Agent**	**Advisor**
Balance ($million)	0.629***	-0.857***	-0.130***
	(0.128)	(0.0436)	(0.0447)
Age	0.0131	-0.0408***	-0.0816***
	(0.00857)	(0.00189)	(0.00218)
Female	0.437***	-0.0588***	-0.124***
	(0.0546)	(0.0120)	(0.0140)
Married	0.0245	0.0620***	0.0874***
	(0.0491)	(0.0107)	(0.0127)
Constant	-5.029***	2.333***	4.326***
	(0.560)	(0.123)	(0.142)
N	238,548	238,548	238,548

Notes. Estimates of Multinomial Logit regression of channel choices on individual covariates. Standard errors are reported in the parentheses. $^{***}p < 0.01, ^{**}p < 0.05, ^{*}p < 0.1$.

To explore if there is there is any evidence of selection on retirees' observables we estimate the log-odds ratio of choosing one of three intermediary channel, relative to choosing AFP using a multionomial Logit model. The results (in Table 7) suggest that while some retiree characteristics are correlated with the choice of channel, for instance sales agent are more likely to be chosen by those with lower savings, or those who are young, or male or unmarried, the model fit is low (pseudo R^2 of 0.4%). So we treat the channel choice as exogenous.

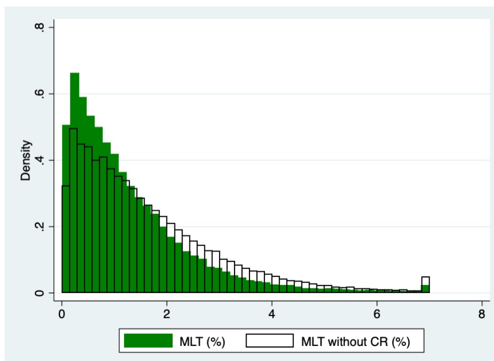
But the channel influences the outcomes, as can bee seen in Table 8. Out of 109,786 retirees who choose AFP, only 25.1% went to the second-round, whereas 85.2%, 92.0% and 87.8%, of retirees who had Insurance Company, Sales Agents or Advisors, respectively, used the second-round. Most of those who choose PW had AFP, and those who have a sales-agent are the least likely to choose a PW.

Table 8: **Retiree choices, by Intermediary Channel**

	N	Requests 2^{nd} Round	Chooses PW	Chooses in 2^{nd} Round
AFP	109,786	0.251	0.661	0.235
Company	2,169	0.852	0.066	0.817
Sales-agent	79,120	0.920	0.030	0.907
Advisor	47,473	0.878	0.066	0.846
Total	238,548	0.603	0.328	0.584

Notes. Proportion of retirees by their choices, separated by their intermediary channel.

Figure 4: **Money Left on the Table**



Note. Histograms of the money left on the table (MLT), defined as the difference between the chosen pension and the maximum pension offered by a company with same or better credit-rating as the chosen company expressed as the percentage of the chosen pension, and if we ignore the risk-rating, we get "MLT without CR." The values and top-coded at 7%.

Dominated Choices

As we saw in Table 6 not all retirees choose the annuity that pays the highest pension and in many cases they choose a strictly dominated alternative (in terms of pension and rating combination). In other words, there is "money-left-on-the-table" (MLT) which we define to be the difference between the chosen pension and the highest pension offered by a company with an equal or better credit-rating, expressed as a percentage of the chosen pension. For those who choose PW, their MLT is set to zero. In Figure 4 we present the histogram of the MLT (for those who did not choose PW and have positive MLT) calculated using the first-round offers. We find that 27.3% of annuitants left money on the table, and among those the mean MLT is 1.2% and the 99^{th}-percentile is 5.3%. For comparison, we have also calculated the MLT without the credit-rating constraints and is presented in Figure 4. As expected the MLT increases, but the increase is not too large. The mean MLT without the credit-rating constraint is 1.84% and its 99^{th}-percentile at 6.7%. We calculated the number of months of retirement postponement it would take the median worker who left money on the table (according to the definition above) to recover the 1.265% of the balance that was "lost." The result was approximately 9.4 months for men and 8.4 months for women. Why do they leave money on the table? Are retirees with sales-agent more likely to have larger MLT? In Table 9 we display the share of dominated options, by channel and product type. Interestingly, we find that those who have sales-agent do tend to choose dominated alternatives. Between 46% and 49% of those with sales-agent choose a pension that is dominated in terms of the amount, the credit rating or both. Even among those who choose an independent advisor 9 to 11% choose a dominated option.

Table 9: **Dominated Choices, by Channel**

Channel	**Type**	**Obs.**	**% Dominated**
AFP	Inmediate	23,213	16
	Deferred	14,041	13
Ins. company	Inmediate	1,118	15
	Deferred	907	14
Sales agent	Inmediate	37,203	46
	Deferred	39,567	49
Advisor	Inmediate	25,581	11
	Deferred	18,757	9
Total		160,387	29

Notes. The table displays the proportion of retirees who choose a dominated pension, separated by the intermediary channel and product type.

Next, we explore if any observed retiree characteristics are associated with positive MLT. The results are presented in Table 10. Column (1) displays the estimated marginal effects from a logistic regression of a dummy $\mathbb{1}\{MLT > 0\}$ on retiree characteristics, including (chosen) insurance company specific fixed effects (not reported). Column (2) displays the estimates of linear regression of level MLT on the same set of controls as in column (1). The estimates in column (1) suggest that: (i) retirees who either have lower savings, or those who are older, male or who are unmarried are more likely to have positive MLT; (ii) choosing sales-agent increases the probability of positive MLT by 28 percentage points compared to those who choose AFP; and (iii) the corresponding increase in probabilities for insurance company is not significant and for independent advisor the probability is lower by 2.7 percentage points. The results from column (2) are qualitatively the same.

As we mentioned above (in Tables 8 and 9) most retirees with AFP do not negotiate, but the share of retirees who choose dominated options varies with the channel. Next, we investigate if MLT would be smaller if instead of choosing in the first-round, retirees chose in the second-round after negotiations. To that end, we re-estimate the marginal effects on the

Table 10: **Determinants of MLT**

Regressorss	(1) Pr(**MLT** > 0)	(2) **MLT**	(3) Pr(**MLT** > 0)	(4) **MLT**
Balance($million)	-0.169***	-1.461***	-0.142***	-1.385***
	(0.0122)	(0.0686)	(0.0122)	(0.0685)
Age	0.00424***	0.00709***	0.00421***	0.00713***
	(0.000392)	(0.00215)	(0.000390)	(0.00214)
Female	-0.00737***	-0.0762***	-0.00572**	-0.0741***
	(0.00248)	(0.0129)	(0.00247)	(0.0129)
Married	-0.0113***	-0.0813***	-0.0105***	-0.0810***
	(0.00225)	(0.0117)	(0.00224)	(0.0117)
Second round			-0.0974***	-0.234***
			(0.00345)	(0.0180)
Ins.company	-0.00121	0.109*	0.00857	0.139**
	(0.00741)	(0.0633)	(0.00734)	(0.0634)
Sales agent	0.278***	0.279***	0.293***	0.358***
	(0.00257)	(0.0157)	(0.00256)	(0.0169)
Advisor	-0.0275***	-0.206***	-0.0122***	-0.114***
	(0.00249)	(0.0188)	(0.00250)	(0.0204)
Company F.E.	✓	✓	✓	✓
Observations	160,387	43,717	160,387	43,717
R-squared		0.0850		0.0887

Notes. Estimates from a Logit regression of the marginal effects on the probability of leaving money on the table (columns 1 and 3) and the OLS coefficients for the percentage of MLT, conditional on MLT greater than zero (columns 2 and 4), using subsample of accepted annuities. Standard errors are reported in the parentheses. $^{***}p < 0.01,^{**} p < 0.05,^{*} p < 0.1$.

probability of positive MLT and the level of MLT amount but with an additional dummy variable that controls for whether there was a second-round offer. The results are presented in columns (3) and (4) in Table 10. We find that receiving a second-round offer reduces $\Pr(MLT > 0)$ by 9.74% and reduces MLT by 0.23 percentage points, and the association between sales-agent and MLT increases.

We also compute the average probability of having positive MLT and expected MLT using current second round status and assuming that everyone requests second-round offers. The results are presented in Table 11. We can see that negotiating in the second-round reduces the probability of leaving money on the table by 8.06% (subtracting column (2) from (1), for row $D_2 = 0$), which is consistent with the Logit regression in Table 10, column (3). In the second half of Table 11 we present these estimates, by channel. The regressions compares those who went to the second-round with those who did not. The coefficient reflects that when there is a second round a retiree is less likely to have positive MLT, or, even if MLT is positive, the level is smaller.

Table 11: **Average Predicted** $\Pr(MLT > 0|\texttt{second-round})$

	$\Pr(\textbf{MLT} > 0|\textbf{2nd Round})$	$\Pr(\textbf{MLT} > 0|\textbf{2nd Round} = 1)$	$\mathbb{E}(\textbf{MLT}|\textbf{2nd Round})$	$\mathbb{E}(\textbf{MLT}|\textbf{2nd Round} = 1)$
$D_2 = 0$	25.80	17.75	1.32	1.09
$D_2 = 1$	27.53	27.53	1.23	1.23
		Channel		
AFP	12.51	10.75	1.00	0.91
Ins. company	13.58	12.72	1.19	1.12
Sales agent	46.01	45.21	1.33	1.32
Independent advisor	8.10	7.73	0.68	0.67
Total	27.34	26.44	1.24	1.22

Notes. $\Pr(MLT > 0|\texttt{2nd Round})$ is the predicted probability of $\{MLT > 0\}$ using the second-round dummy $D_2 \in \{0, 1\}$. $\Pr(MLT > 0|\texttt{2nd Round} = 1)$ is the predicted probability of $\{MLT > 0\}$ given $\texttt{2nd Round} = 1$.

3.2 Insurance Companies

As we mentioned earlier, on average, there are 11 companies participating in a given auction, which is remarkable. The quarterly Herfindahl-Hirschman Index (HHI) at the product level and at the channel level are always below 1900 for most of the time, suggesting stiff competition in the market. From the perspective of the retirees, companies vary in terms of their risk-ratings, which signals the likelihood of a company filing for bankruptcy. These ratings are reported by the SCOMP along with the first-round offers.

We collected quarterly data on risk-ratings for most companies for our entire sample period; see Table 12. At the end we have 629 company-quarter observations, and what stands out is that although there is some variation, these ratings do not change that often and most companies have high ratings.

Table 12: **Risk-Ratings**

Rating	**Frequency**	**%**	**Cumulative %**
AA+	155	24.64	24.64
AA	245	38.95	63.59
AA-	171	27.19	90.78
A+	2	0.32	91.1
A	15	2.38	93.48
BBB+	1	0.16	93.64
BBB	6	0.95	94.59
BBB-	15	2.38	96.98
BB+	19	3.02	100
Total	629	100	

Notes: The table shows the distribution of quarterly credit-ratings from 2007-2018.

As we mentioned earlier, even though there are 19 unique companies, not every company participates in every auction. In Table 13, we present the summary statistic of the number of participating insurance companies in the first round, grouped together by retirees savings (deciles). As we can see the range is wide, with a maximum of 15 companies and a minimum

Table 13: **Number of Participating Companies, by Savings**

Savings-Deciles	Min	P5	P25	Median	P75	P95	Max
1	1	1	5	7	8	10	14
2	1	6	8	9	10	12	15
3	1	8	10	11	12	13	15
4	1	9	10	11	12	13	15
5	1	9	11	12	12	13	15
6	1	9	11	12	13	14	15
7	1	10	12	13	14	15	15
8	1	10	11	13	14	15	15
9	1	9	11	12	13	15	15
10	1	9	11	12	13	14	15
Overall	1	5	9	11	12	14	15

Notes. The table presents summary statistics of the number of participating companies, by savings.

of 1 company across all savings deciles, and the number of participating companies increases, albeit only slightly, with higher income. We define the set of potential entrants to be the set of all companies that have participated in at least one auction in the same month.

The participation rate, which is the ratio of the number of actual bidders to the number of potential bidders, varies a across our sample from as low as 0.08 to as high as 1, with mean and median rates of 0.73 and 0.78, respectively, and a standard deviation of 0.18. We posit that the decision to participate depends on various financial positions of an insurance company, which can change the opportunity cost of participating in an auction. For our empirical application, we use an auction model with endogenous entry by companies. Although we do not present the estimation results here, using Poisson regression we find that savings is an important predictor of the number of participants, which is consistent with selective entry. One standard deviation increase in savings, which is approximately $87,000, is associated with roughly 1 additional participating company. And women have 0.61 additional participating companies than men, while sales agents and advisors are associated with approximately 0.19 fewer participants than the other 2 channels.

4 Model

In this section we introduce the model of demand for, and supply of, annuities. For the demand, we consider a decision problem facing a retiree who uses SCOMP to first solicit pension offers from companies, and then chooses one of these offers or requests external offers in a second-round. As we saw, retirees do not always choose the offer with the highest pension, and we posit that they also care about the risk-ratings of a company, but they do not know their preference for risk-ratings, only some prior. They can learn their preference and reduce uncertainty, but that requires some due-diligence and information processing, which is costly. To capture this tradeoff between pension, risk-ratings and due-diligence, we model the retiree as a rationally inattentive decision maker (Sims, 1998). If, and when the retiree chooses to go to the second-round, they know their preferences before choosing.

On the supply side, we can view a retiree as an auctioneer, and we begin with a set of life insurance companies who observe all relevant characteristics of the retiree, and their cost of annuitizing the said retiree. Based on this set of information they decide whether to participate in this "auction," as in Samuelson (1985). We model the bidding process as an extensive form game, where the first round is a modified first-price auction, and conditional on going to the second-round it is a modified English-auction, where the "modification" comes from the fact that unlike in the standard auction model, here the rule of selecting a winner is stochastic, where the probability is derived by Matějka and McKay (2015).

4.1 Demand

An annuity has two components: monthly pension P, and bequest B that depends on the type of annuity. Let UNC_i denote retiree $i \in I$'s *unitary necessary capital* of self-annuitizing, which is the dollar amount necessary to finance a pension of one dollar until death, which is defined by $UNC_i \equiv \sum_{t=0}^{T} \frac{q_{it}}{(1+\delta_t)^t}$, where q_{it} is the probability of being alive in month t, T is the maximum number of months i is expected to be alive and δ is the common (objective) discount factor. To determine q_{it} we use the official mortality table, where the probability depends on i's demographic characteristics $\tilde{X}_i$, and we use the market risk-free rate of return as the discount factor δ.

At the time of making the decision, i evaluates the net present expected value of an annuity offered by company $j \in J$ that offers P_j every month, that is $\mathbf{A}_{ij} \equiv P_j \times UNC_i$, where J is the set of companies who participate in i's auction. Once we know $\mathbf{A}_{ij}$, we can determine the net-present expected value of bequest B_{ij}, which is always a fixed α_i fraction of $\mathbf{A}_{ij}$, i.e., $\mathbf{B}_{ij} = \alpha_i \times \mathbf{A}_{ij}$. It is important to note that α_i depends on the type of annuity and $\tilde{X}_i$, and is known to us. Let i's indirect utility from buying annuity $(\mathbf{A}_{ij}, \mathbf{B}_{ij})$ from j, in leu of her savings, S_i, be

$$\tilde{U}_{ij} = \beta_i^{\top} \tilde{Z}_{ij} + \mathbf{A}_{ij} + \theta_i \mathbf{B}_{ij} - S_i, \tag{1}$$

where $\tilde{Z}_{ij} \in \mathbb{R}$ denotes j's risk-ratings, which can vary across retirees, $\beta_i \in \mathbb{R}$ is i's preference for $\tilde{Z}_{ij}$, and θ_i is i's bequest motive. To analyze the decision problem facing i, it is easier to work with a normalized utility (1) per dollar of savings. Dividing $\tilde{U}_{ij}$ by i's savings S_i gives

$$U_{ij} \quad \equiv \quad \frac{\tilde{U}_{ij}}{S_i} = \beta_i^\top \frac{\tilde{Z}_{ij}}{S_i} + \frac{P_j \times UNC_i}{S_i} + \theta_i \times \frac{\mathbf{B}_{ij}}{S_i} - 1 := \beta_i^\top Z_{ij} + \rho_{ij} + \theta_i \times b_{ij} - 1, \quad (2)$$

which is the indirect utility per dollar, and where ρ_{ij} is the `mwr`. Thus (2) shows that there is a tradeoff between higher pensions and lower risk-ratings, but we assume that i does not know her (β_i, θ_i), only their distribution. In particular we follow Matějka and McKay (2015) and assume that before the retirement process begins, i has a correct belief that $\boldsymbol{\beta}_i \equiv (\beta_i, \theta_i) \overset{i.i.d}{\sim} F_{\boldsymbol{\beta}}(\cdot)$ with support $[\underline{\boldsymbol{\beta}}, \overline{\boldsymbol{\beta}}]$, and if i wants to learn her preference has to incur some information processing cost, which costs $\lambda > 0$ per unit of information.

So, i has to first decide how much to spend learning about $\boldsymbol{\beta}_i$, and after that make the decision. Matějka and McKay (2015) consider similar discrete choice decision problem facing a rationally inattentive decision maker and determine the optimal decision rule, and we use their solution. Let $\sigma : [\underline{\boldsymbol{\beta}}, \overline{\boldsymbol{\beta}}] \times \mathcal{A} \times \mathcal{B} \to \Gamma := \Delta([0,1]^{J+1})$ denote the strategy of a retiree with preference parameter $\boldsymbol{\beta}$, with offered pensions $\boldsymbol{\rho} := (\rho_1, \ldots, \rho_J) \in \mathcal{A}$, and bequests $\mathbf{b} := (b_1, \ldots, b_J) \in \mathcal{B}$. The strategy is a vector $\sigma(\boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}) \equiv (\sigma_1(\boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}), \ldots, \sigma_J(\boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}), \sigma_{J+1}(\boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}))$ of probabilities, where $\sigma_j(\boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}) = \Pr(i \texttt{ chooses } j | \boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}) \in [0,1]$. For notational simplicity, we suppress the dependence of choice probabilities on the offers $(\boldsymbol{\rho}, \mathbf{b})$.

Let i's expected utility from j be given by $\int U_{ij} \sigma_j(\boldsymbol{\beta}) dF_{\boldsymbol{\beta}}(\boldsymbol{\beta})$, and we further assume that information processing cost has to be paid only in the first-round. By the time i decides to go to the second-round i knows her $\boldsymbol{\beta}_i$. Let $\mathbb{E}U_i$ be the ex-ante expected utility from second-round. Then i's maximization problem can be stated as:

$$\max_{\{\sigma(\boldsymbol{\beta}) \in \Gamma\}} \Big\{ \sum_{j=1}^{J} \int U_{ij}(\boldsymbol{\beta}) \sigma_j(\boldsymbol{\beta}) dF_{\boldsymbol{\beta}}(\boldsymbol{\beta}) - (\texttt{information cost}) + \sigma_{J+1}(\boldsymbol{\beta}) \times \mathbb{E}U_i \Big\}, \quad (3)$$

where the information cost is equal to the reduction in uncertainty times λ, where we use relative entropy to measure information and uncertainty.[7] In other words, the total information cost of updating prior from $F_{\boldsymbol{\beta}}(\cdot)$ to $F'_{\boldsymbol{\beta}}(\cdot)$ is $\lambda \times \{\texttt{entropy of } F_{\boldsymbol{\beta}} - \texttt{entropy of } F'_{\boldsymbol{\beta}}\}$.

Let $\sigma_j^0 := \int_{\underline{\boldsymbol{\beta}}}^{\overline{\boldsymbol{\beta}}} \sigma_j(\boldsymbol{\beta}) dF_{\boldsymbol{\beta}}(\boldsymbol{\beta})$ be the unconditional probability of choosing option j. Then the expected reduction of entropy of i conditional on $\boldsymbol{\beta}$ is

$$I(\sigma, F_{\boldsymbol{\beta}}) = -\sum_{j=1}^{J} \sigma_j^0 \log \sigma_j^0 + \int_{\underline{\boldsymbol{\beta}}}^{\overline{\boldsymbol{\beta}}} \left(\sum_{j=1}^{J} \sigma_j(\boldsymbol{\beta}) \log \sigma_j(\boldsymbol{\beta}) \right) dF_{\boldsymbol{\beta}}(\boldsymbol{\beta}),$$

and the information cost is $\lambda \times I(\sigma, F_\beta)$; see Matějka and McKay (2015). Substituting this

[7] Entropy of a continuous random vector $\boldsymbol{\beta}$ with density $f_{\boldsymbol{\beta}}(\cdot)$ is $\mathbb{E}[-\ln(f_{\boldsymbol{\beta}}(\boldsymbol{\beta})]$.

cost in (3), we can re-write i's optimization problem as

$$\max_{\{\sigma_j(\boldsymbol{\beta})\}_{j=1}^{J+1}} \left\{ \sum_{j=1}^{J} \int_{\underline{\boldsymbol{\beta}}}^{\overline{\boldsymbol{\beta}}} U_{ij}\sigma_j(\boldsymbol{\beta}) dF_{\beta}(\boldsymbol{\beta}) - \lambda \times I(\sigma, F_{\boldsymbol{\beta}}) + \sigma_{J+1}(\boldsymbol{\beta})\mathbb{E}U_i \right\}. \tag{4}$$

Then by adapting Matějka and McKay (2015)'s choice formula to two-periods, we can show that the probability that i chooses j is given by

$$\sigma_{ij}(\boldsymbol{\beta}, \boldsymbol{\rho}, \mathbf{b}) = \begin{cases} \frac{\exp\left(\log \sigma_j^0 + \frac{U_{ij}}{\lambda}\right)}{\sum_{k=1}^{J} \exp\left(\log \sigma_k^0 + \frac{U_{ik}}{\lambda}\right) + \exp\left(\frac{\mathbb{E}U_i}{\lambda}\right)}, & j = 1, \ldots, J \\ \\ \frac{\exp\left(\frac{\mathbb{E}U_i}{\lambda}\right)}{\sum_{k=1}^{J} \exp\left(\log \sigma_k^0 + \frac{U_{ik}}{\lambda}\right) + \exp\left(\frac{\mathbb{E}U_i}{\lambda}\right)}, & j = J + 1. \end{cases} \tag{5}$$

4.2 Supply

Next, we present the supply side, where J insurance companies participate in an auction run by "auctioneer" i with characteristics $X_i \equiv (S_i, \tilde{X}_i)$. For notational simplicity we suppress the dependence on X_i, and treat J as exogenous, but in our empirical application we make the dependence explicit and allow J to be endogenous. We further assume that the companies know $\boldsymbol{\beta}_i$, and therefore the probability of being a winner is given by (5).

Companies differ in terms of their cost of annuitizating, i.e., their UNCs. Thus, if j can annuitize i cheaper than j', then, j has an advantage over j' because all else equal j can offer higher pension. Let UNC_j be j's unitary necessary capital, defined as $UNC_j \equiv \sum_{t=0}^{T} \frac{q_{jt}}{(1+\delta_{jt})^t}$ where q_{jt} is j's estimate of the probability that the retiree will be alive in month t, and δ_{jt} is j's discount rate, all of which are j's private information. Firm j's UNC_j differs from retiree i's UNC_i because i's estimate of the mortality and discount can be different from the official mortality and market risk-free rate. In fact, UNC_j is j's private information, with the interpretation that $(P_j \times UNC_j)$ is the net present value of the cost of offering a pension of P_j every month. Thus, the ratio of UNC_j to UNC_i captures j's margin from selling annuity to i, and henceforth we call this ratio i's relative cost of annuitizing a dollar.

If the annuity under consideration includes bequests, then the cost should also include the cost of in providing bequests. In particular, if the annuity provides a net present value of pension $\mathbf{A}_{ij}$ and net present value of bequest $\mathbf{B}_{ij}$, then from the perspective of the company j, $\mathbf{B}_{ij} = \alpha_{ij} \times \mathbf{A}_{ij}$, where α_{ij} depends on retiree i and company j. This is because now when it comes to bequests, company must also have an expectation about dependent(s)' mortality.

The supply side departs in many dimensions when compared to the standard auction model. So, to understand the equilibrium offers, we first consider the first-period bidding, then we consider the second-period bidding and finally introduce the differences across risk-rating. Let $\mathbf{A}_j$ and $\mathbf{B}_j$ denote the net present expected value of pensions and bequest associated with j's offer of monthly pension P_j, with some bequest. If we ignore the second-round, j's net present expected profit from offering $\{\mathbf{A}_j, \mathbf{B}_j\}$, to a retiree i with S_i is

$$\begin{aligned}
\mathbb{E}\Pi_{ij} &= (S_i - (\mathbf{A}_j + \mathbf{B}_j)) \times \Pr(\texttt{j is chosen by offering } P_j|\mathbf{P}_{-j}) \\
&= (S_i - (\mathbf{A}_j + \alpha_{ij}\mathbf{A}_j)) \times \Pr(\texttt{j is chosen by offering } P_j|\mathbf{P}_{-j}) \\
&= (S_i - (P_j \times UNC_j \times (1+\alpha_{ij}))) \times \Pr(\texttt{j is chosen by offering } P_j|\mathbf{P}_{-j}) \\
&= S_i \times \left(1 - \frac{P_j \times UNC_j}{S_i} \times (1+\alpha_{ij})\right) \times \Pr(\texttt{j is chosen by offering } P_j|\mathbf{P}_{-j}) \\
&= S_i \left(1 - \frac{P_j \times UNC_i}{S_i} \times \frac{UNC_j}{UNC_i} \times (1+\alpha_{ij})\right) \times \Pr(\texttt{j is chosen by offering } P_j|\mathbf{P}_{-j}) \\
&\equiv S_i \left(1 - \rho_{ij} \times r_{ij}\right) \times \sigma_j(\rho_j, \boldsymbol{\rho}_{-j}), \qquad (6)
\end{aligned}$$

where ρ_{ij} is the `mwr` and $r_{ij} := \frac{UNC_j}{UNC_i} \times (1+\alpha_{ij})$ is j's cost of annuitizing every dollar for i, relative to what costs i to self-insure. Thus $(1 - \rho_{ij} \times r_{ij})$ is j's markup, per-dollar of savings.

We assume the cost r_{ij} is private and is distributed independently and identically across companies as $W_r(\cdot|S)$, with density $w_r(\cdot|S)$ that is strictly positive everywhere in its support $[\underline{r}, \overline{r}]$. Allowing the cost distribution to depend on savings, S, captures the essence of adverse selection in the intensive margin, i.e., those who have higher wages tend to be healthy, and therefore expect to live longer and therefore cost more for the company to annuitize.

Let $\Psi : [\underline{r}, \overline{r}] \to \mathbb{R}_+$ denote a symmetric behavioral (bidding) strategy such that $\Psi(r_j)$ is the pension submitted by j. And let $\tilde{\Psi}(\cdot)$ denote the bidding strategy used in the (modified) English auction, and let $\tilde{\rho}_j$ denote the corresponding bid. Under the assumption that all firms other than j use the pair $\{\Psi(\cdot), \tilde{\Psi}(\cdot)\}$ to choose their offers, j solves

$$\max_{\rho_j \geq 0, \tilde{\rho}_j \geq \rho_j} S_i \times \left\{(1 - \rho_j \times r_j) \times \sigma_j(\boldsymbol{\rho}) + S_i \times \sigma_{J+1}(\boldsymbol{\rho}) \times \mathbb{E}\Pi_j^{II}(\tilde{\rho}_j | r_j, \boldsymbol{\rho})\right\},$$

where $\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|v_j, \rho)$ is j's expected profit from choosing $\tilde{\rho}_j$ in the English auction, given that the first-round offers were $\boldsymbol{\rho}$ and $j's$ cost is r_j. To solution depends on $\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j, \boldsymbol{\rho})$, which is the expected profit from going to the second-round.

To get the intuition, let us consider a simpler (hypothetical) case without Z, so that conditional on reaching the English auction, the highest offer wins. This will allow us to determine the effect of having second-round on the first-round offers, and once we understand that, extending it to allow retirees to use both pecuniary and non-pecuniary aspects before choosing, becomes easier.

Solving backwards, consider the English auction where the offers from the first stage, $\boldsymbol{\rho}$, are commonly known. Although, companies can update their beliefs about their competitors' value, it is still a weakly dominant strategy to choose the dropout pension to be one's own value as long as it is greater than the first-stage offer. Thus, the weakly dominant strategy is $\tilde{\Psi}(r_j|\boldsymbol{\rho}) = \tilde{\rho}_j = \max\{1/r_j, \rho_j\}$. Using this information, we can now determine the expected payoff $\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j, \boldsymbol{\rho})$. Define $L(\xi|\rho) = \{r : \max\{1/r, \rho\} \leq \xi\}$ and $\tilde{W}_r(\xi|\rho) = \int_{L(\xi|\rho)} dW_r(r)$, and let $G(\cdot) = (\tilde{W}_r(\cdot|\rho))^{J-1}$ with density $g(\cdot)$. The expected payment is

$$
\begin{aligned}
R(r_j|\boldsymbol{\rho}) &= \Pr(\texttt{win}|\boldsymbol{\rho}) \times \mathbb{E}(\texttt{second highest bid}|\texttt{second highest bid} \leq \tilde{\rho}) \\
&= \Pr(\texttt{win}|\boldsymbol{\rho}) \times \int_{1/\bar{r}}^{\tilde{\rho}} y \frac{g(y)}{\Pr(\max\{1/r_j, \rho_j\} \geq \max_{k\neq j} \max\{1/r_k, \rho_k\})} dy \\
&= \Pr(\texttt{win}|\boldsymbol{\rho}) \times \int_{\underline{r}}^{\tilde{\rho}} y \frac{g(y)}{\Pr(\texttt{win}|\boldsymbol{\rho})} dy = \int_{1/\bar{r}}^{\max\{r_j,\rho_j\}} yg(y)dy,
\end{aligned}
$$

and so the ex-ante expected payoff is

$$
\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j, \boldsymbol{\rho}) = \int_{1/\bar{r}}^{\max\{1/r_j,\rho_j\}} ydG(y) - G(\tilde{\rho}_j)r_j. \tag{7}
$$

Lemma 1. *For every $j \in J$, the first-stage offer satisfies $\rho_j < 1/r_j$, and the second-stage offer satisfies $\tilde{\rho}_j = 1/r_j$. And as a consequence, the pensions offered in the first-round when there are two-rounds is smaller than the pension offered when there was only first-round.*

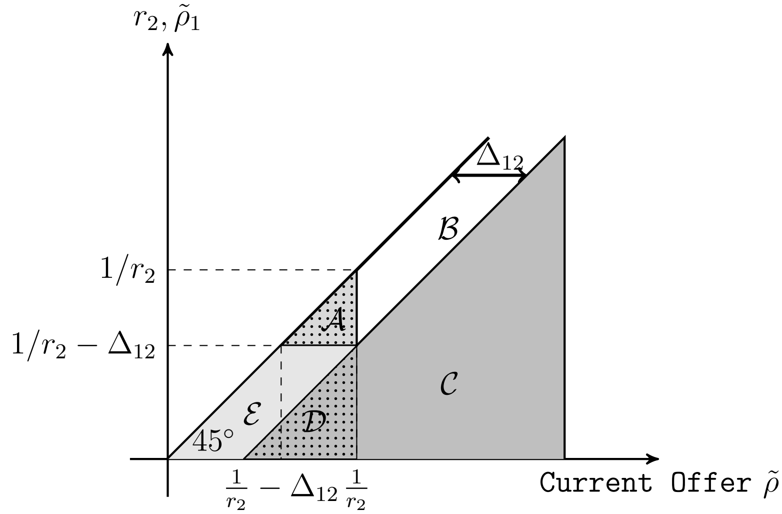
In our empirical application, we can focus only on the second-round, which is easier to characterize and use than the first-round. But before we do that we have to incorporate companies' risk-ratings in the choice. Retirees care about the risk-ratings, besides the pensions, so we have to modify the English auction to allow for the fact that the "winner" company may not offer the highest pension. Thus, fixing i, if $\Delta_{jk} := (\beta_i^\top Z_j + \theta_i b_{ij}) - (\beta_i^\top Z_k + \theta_i b_{ik})$ is positive then it gives j an "advantage" over k in choosing the pension $\tilde{\rho}_j$, and vice versa.

To understand the intuition, it suffices to consider only two companies. Let there be $J = 2$ firms with costs r_1 and r_2 and that, all else equal, the retiree prefers 1. This is captured by a parameter $\Delta_{12} > 0$, and suppose this parameter is commonly known. The retiree will choose to buy an annuity from 1 if and only if $\tilde{\rho}_1 + \Delta_{12} > \tilde{\rho}_2$. To understand the intuition of how the equilibrium of the English auction changes with Δ_{12} let us start by assuming that $\{r_1, r_2, \Delta_{12}\}$ are commonly known. Then the second-stage game becomes:

1. The game starts with each company pushing a button while a clock automatically and continuously raises the pension.

2. To choose its bid, a company simply stops pushing the button when the clock reaches the desired pension.

Figure 5: **Best Response Correspondence**



Notes. The figure is a schematic representation of best response correspondence with $J = 2$. The x-axis is the current (highest) pension offer, and the y-axis is the return for company 2 and the pension offered by company 1.

3. The clock rises continuously until all firms release their buttons.

4. The retiree evaluates the utilities and chooses the one she prefers.

The Subgame Perfect Nash Equilibrium (SPNE) outcome of this game is as follow: If $1/r_2 \leq 1/r_1 + \Delta_{12}$, then 1 wins, and 2 pushes the bid up to $1/r_2$, and 1 pushes the bid up to $1/r_2 - \Delta_{12}$. And if $1/r_2 > 1/r_1 + \Delta_{12}$, then 2 wins, and 1 pushes up to $1/r_1$, and player 2 pushes up to $1/r_1 + \Delta_{12}$.

Now, suppose the costs r_1 and r_2 are companies' private information, but Δ_{12} is commonly known. Even in that case, the Weak Perfect Bayesian Equilibrium of the game is not complicated; see Lemma 2. The intuition is very simple. Suppose $\Delta_{12} + 1/r_1 > 1/r_2$. Irrespective of $r_1 > r_2$ or $r_1 \leq r_2$ the maximum 2 can offer is $\tilde{\rho}_2 = 1/r_2$, which 1 can match and win with an offer of $\tilde{\rho}_1 = 1/r_2 - \Delta_{12}$. If, however, $\Delta_{12} + 1/r_1 \leq 1/r_2$, then the best 1 can offer without making a loss is $\tilde{\rho}_1 = 1/r_1$, which 2 can match and win with bid $\tilde{\rho}_2 = \Delta_{12} + 1/r_1$. Figure 5 helps illustrate the best-response correspondence of Firm 2, as a function of the current clock (or existing price) $\tilde{a}$, Firm 1's offer $\tilde{\rho}_1$, Firm 2's cost r_2 and a fixed $\Delta_{12} > 0$. In region $\mathcal{E}$, 2 wants to win and there the optimal response is to choose $\tilde{\rho}_2 = \tilde{\rho}_1 + \Delta_{12}$. In $\mathcal{D}$, Firm 2 wants to win but the optimal response is to leave the auction immediately because firm 2 is already winning in that region and hence chooses $\tilde{\rho}_2$ to be the current highest pension offer. And thus the region $\mathcal{D}$ is never reached in equilibrium.

Now consider region $\mathcal{C}$, where Firm 2 is winning but is overpaying. The best response is to leave immediately, i.e., $\tilde{\rho}_2 = \tilde{a}$. In region $\mathcal{B}$, Firm 2 does not want to win because the existing price $\tilde{a} > 1/r_2$, so $\tilde{\rho}_2$ can be anything between $\tilde{a}$ and $\tilde{\rho}_1$, and similarly in region $\mathcal{A}$, Firm 2 does not want to win because the existing price $\tilde{\rho}_1 + \Delta_{12} > 1/r_2$, so that $\tilde{\rho}_2$ can be anything between $\tilde{a}$ and $\tilde{\rho}_1$. Thus, in regions $\mathcal{A}, \mathcal{B}, \mathcal{C}$ and $\mathcal{D}$, Firm 2 can optimally choose $\tilde{\rho}_2 = 1/r_2$ and in $\mathcal{E}$ it can choose $\tilde{\rho}_1 + \Delta_{12}$. With $J > 2$, if j_i^* is the chosen firm by i, then

$$\beta_i^\top Z_{ij_i^*} + \theta_i b_{ij_i^*} + \tilde{\rho}_{j_i^*} = \max_{k \neq j} \left\{ \beta_i^\top Z_{ik} + \theta_i b_{ik} + 1/r_k \right\}. \tag{8}$$

5 Identification

In this section, we study the identification of our parameters of interest, which are the distribution of preferences, $F_{\boldsymbol{\beta}}(\cdot)$, the distribution of costs $W_r(\cdot|S)$, and the information processing cost λ that depends on the channel and savings. We observe outcomes of the annuity process described above for N retirees where N is large. We observe everything about retirees that companies observe before making the first-round offers, so unobserved heterogeneity in companies information is less of a concern with our sample.

If they do not choose PW, then we observe the set of companies $J_i \geq 2$, their characteristics $Z_j \in \mathbb{R}$ and their monthly pension offers $\boldsymbol{\rho}_i := (\rho_1, \ldots, \rho_{J_i})$. From the offers $\mathbf{A}_i$ and the savings S_i we can determine the corresponding bequests $\mathbf{B}_i \in \mathbb{R}_+^{J_i}$. Let $D_i^1 \in \{1, \ldots, J+1\}$ denote i's choice in the first-stage, such that $D_i^1 = j$ means i chose from j, and $D_i^1 = (J+1)$ means i chose to go to the second-round. Conditional on $D_i^1 = (J+1)$, we also observe j's final choice and the identity of the chosen company.

5.1 Information Processing Cost

For simplicity, let us assume that every company participates in all auctions, such that for all $i \in I, J_i = J$. This assumption is without loss of generality, but allows us to focus on the main source of identification. From the data, we can identify the conditional choice probability (known as the share) for $j \in J$ being chosen, given $X = x$, $Z = z$ and $(\boldsymbol{\rho}, \mathbf{b})$ as

$$\begin{aligned} \hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}|J) &= \sum_i \frac{\mathbb{1}[D_i^1 = j, X_i = x, Z = z, \boldsymbol{\rho}, \mathbf{b}]}{\sum_i \mathbb{1}[X_i = x, Z = z, \boldsymbol{\rho}, \mathbf{b}]}, \\ \hat{\sigma}_{J+1}(\tilde{x}, z, \boldsymbol{\rho}, \mathbf{b}|J) &= 1 - \sum_{j=1}^{J} \hat{\sigma}_j(x, z, \boldsymbol{\rho}, \mathbf{b}). \end{aligned} \tag{9}$$

But, we observe that J_i varies with i, and let $\mathcal{J}$ denote the unique values of J. Then we can "bin" our data into $|\mathcal{J}|$ subsets of different size. Using the relevant subsample, we can first identify $\{\hat{\sigma}_j(x, z, \boldsymbol{\rho}|J)\}_{j\in J}$ and $p(J) = \Pr(\texttt{J firms participate})$, for each $J \in \mathcal{J}$, which gives us $\hat{\sigma}_j(x, z, \boldsymbol{\rho}) = \sum_{J\in\mathcal{J}} \hat{\sigma}_j(x, z, \boldsymbol{\rho}|J) \times p(J)$, where $\hat{\sigma}_j(x, z, \boldsymbol{\rho}|J)$ is defined in (9).

Integrating (5) with respect to $F_{\boldsymbol{\beta}}$ and using the definition of $\hat{\sigma}_j(x, z, \boldsymbol{\rho})$ gives

$$\hat{\sigma}_j(x, z, \boldsymbol{\rho}) = \int \frac{\exp\left(\alpha_j + \frac{U_{ij}}{\lambda}\right)}{\sum_{k=1}^{J} \exp\left(\alpha_k + \frac{U_{ij}}{\lambda}\right) + \exp\left(\frac{\mathbb{E}U_i}{\lambda}\right)} dF_{\boldsymbol{\beta}}(\boldsymbol{\beta}). \tag{10}$$

For any $X = x$ and $Z = z$, the derivative of (10) with respect to ρ_j identifies

$$\lambda = \frac{\hat{\sigma}_j(x, z, \boldsymbol{\rho})(1 - \hat{\sigma}_j(x, z, \boldsymbol{\rho}))}{\frac{\partial \hat{\sigma}_j(x,z,\boldsymbol{\rho})}{\partial \rho_j}}.$$

Thus, the information processing cost depends on how sensitive (elasticity) is the choice probability with respect to `mwr`. Consider an extreme case when choice for j is insensitive to changes in premium, i.e., $\frac{\partial \hat{\sigma}_j(x,z,\boldsymbol{\rho})}{\partial \rho_j} \approx 0$ then $\lambda \approx +\infty$. So the only way to rationalize the fact that retirees do not respond to changes in pension is that their information processing cost is very high. If the demand for insurance company j is very sensitive to pension then that will mean that the cost of processing information is low, and vice versa. To identify the cost as a function of the channel and savings, we can simply use the appropriate subsample, and follow the same logic as above.

5.2 $F_{\boldsymbol{\beta}}$ and W_r

To identify both the preference distribution $F_{\boldsymbol{\beta}}$ and the cost distribution W_r it is sufficient to consider only those who buy annuities from the second-round. The advantage of using the second-round data is that the second-round model is simpler than the first-round model. Heuristically, we use the equilibrium bidding condition in (8) to express the chosen (and observed) `mwr` as a linear additively separable function of the difference in risk-ratings of the top-two firms, differences in bequests and the cost of the runner-up firm. Conditional on us determining the runner-up company (for which we exploit first-stage data where we observe offers of all the firms; see Section 6), we can treat this equation as a random-coefficient model, which we know is identified (Hoderlein, Klemelä, and Mammen, 2010).

We, however, cannot identify the bequest motive because we have focused only on one type of annuity for each retiree, and as a result there is no variation in bequests. Second, from the random-coefficient model what we identify is the distribution of the order statistic of costs. To identify the parent distribution, however, we use the fact that there is a one-to-one relationship between the distribution of order statistic and the (truncated) cost distribution

(Arnold, Balakrishnan, and Nagaraja, 1992).[8] The distribution is truncated because of the endogenous entry decision of the companies. To identify the full distribution, we can use the variation in the number of participants, which identifies the likelihood that the cost will be above the entry threshold, which in turn identifies the cost distribution.

Before we continue, we first have to introduce new notation, that allows us to track the chosen and the runner-up company for retirees. Let j_i^* be the company chosen by i in the second-round, and let $\tilde{\rho}_{j_i^*}$ be the chosen offer. Then from (8) $\tilde{\rho}_{j^*}$ satisifes

$$\tilde{\rho}_{j_i^*} \quad = \quad \max_{k \neq j_i^*, k \in J_i} \{\beta_i^\top Z_{ik} + \theta_i b_k + 1/r_k\} - \beta_i^\top Z_{ij_i^*} - \theta_i b_{j_i^*}. \tag{11}$$

Let k_i^* denote the runner-up company in i's auction, i.e., k_i^* is the strongest opponent of the "winner" company j_i^*. Next we show that conditional on determining k_i^* for every $i \in N$, we can identify $F_{\boldsymbol{\beta}}$ and W_r. Using these notations we can simplify (11) to get

$$\tilde{\rho}_{j_i^*} \quad = \quad \beta_i^\top (Z_{ik_i^*} - Z_{ij_i^*}) + \theta_i (b_{k_i^*} - b_{j_i^*}) + 1/r_{k_i^*}, \tag{12}$$

where the preference parameters $\boldsymbol{\beta}_i$ and the "error" $1/r_{k_i^*}$, which is the reciprocal of the cost of runner-up firm, are unknown random variables that need to be identified.

But, the bequest is $b_{ij} = \alpha_i \times \rho_{ik}$ for any k, which means that our regressors do not satisfy the necessary rank conditions for the identification. So, we write (12) as

$$\tilde{\rho}_{j_i^*} \quad = \quad \frac{\beta_i^\top}{1 + \alpha_i \theta_i} (Z_{ik_i^*} - Z_{ij_i^*}) + 1/r_{k_i^*} := \tilde{\beta}_i^\top (Z_{ik_i^*} - Z_{ij_i^*}) + 1/r_{k_i^*}, \tag{13}$$

and identify the distributions of $\tilde{\beta}_i$ and $1/r_{k_i^*}$. And we know from the random coefficient literature, e.g., Hoderlein, Klemelä, and Mammen (2010), that these distribution are nonparametrically identified as long as $(Z_{ik_i^*} - Z_{ij_i^*})$ and $1/r_{k_i^*}$ are uncorrelated and there is sufficient variation in $(Z_{ik_i^*} - Z_{ij_i^*})$, both of which hold in our case.

Selective Entry

To model selective entry we follow Samuelson (1985). Let $\tilde{J}$ be the set of companies that are interested in selling annuities to i with characteristics X_i. When i requests offers for a product, company $j \in \tilde{J}$ observes its cost r_j and all firms simultaneously decide whether or not to participate, and it costs (the same) $\kappa \geq 0$ for each company to participate. This cost captures the opportunity cost to participate, and it can vary across retirees. Let $J \subset \tilde{J}$

[8] For example, the distribution G_l for the l-th order statistic out of L random variables, each of which is distributed as G_0 can be expressed as $G_l(y) = \frac{L!}{(l-1)!(L-l)!} \int_0^{G_0(y)} s^{l-1}(1-s)^{L-l} ds := \phi(G_0(y))$. This mapping $G_l(y) = \phi(G_0(y))$ is strictly increasing in $G_0(y)$ hence this is invertible.

denote the set of participating companies. After the participation decision, every $j \in J$ knows J and makes an offer.

We restrict attention to symmetric Perfect Bayesian-Nash equilibrium where the entry decision is characterized by a unique threshold $r^* \in [\underline{r}, \overline{r}]$ such that a company decides to participate only if $r_j \leq r^*$. Suppose we observe all the bids, then the cost distribution among participating companies is $W_r^*(r; \tilde{J}) := W_r(r|r \leq r^*; \tilde{J}) = W_r(r)/W_r(r^*; \tilde{J})$. Upon entry, the equilibrium bidding strategies are the same as under exogenous entry. Let $r^*_{\tilde{J}}$ be the threshold with $\tilde{J}$ potential bidders, and suppose $\tilde{J} \in \mathcal{J} := \{\underline{J}, \ldots, \overline{J}\}$, where $\overline{J}$ is the maximum number of potential bidders and $\underline{J}$ is the smallest number of potential bidders. All else equal, $r^*_{\tilde{J}}$ decreases with $\tilde{J}$, so $W_r(r)$ is identified on the support $[\underline{r}, r^*_{\underline{J}}]$.

But we only observe the chosen pension, which means instead of the W_r, we identify the distribution of order statistics.[9] Then, we can identify the parent distribution W_r using the results in (Arnold, Balakrishnan, and Nagaraja, 1992) on the support $[\underline{r}, r^*_{\underline{J}}]$.

6 Estimation

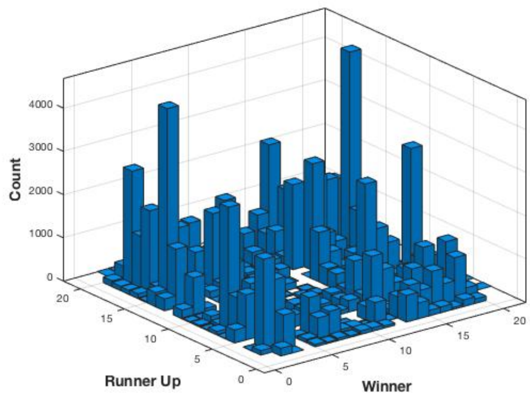
Runner Up

Our identification argument relies on the assumption that if j_1 and j_2 are two firms that provides the highest stage one utility for retiree i, then j_1 and j_2 are also the top-two companies in the second-round. If we see that i chose j_1 in the second-period then we know that j_1 is the company that provides the highest utility in the second-stage. But we still have to determine j_2 in order to use (13) for estimation. In this section, we propose to use data from the first-round and identify j_2 to be the firm that has the second-largest probability of being chosen, which depends on all the offers made in the first-round. To construct a measure of the probability of being selected in the first-round, we estimate series of "alternative-specific conditional Logit model," in the line of McFadden (1974)'s choice models.

To allow for the most general estimation, we divided the sample in 175 different groups, defined as a function of the year of retirement, three age groups (below, at, and above NRA), two gender groups and three channel groups (we combine insurance companies and sales-agents into one). For each group, we estimate the model where the choice of an individual depends on firms characteristics such as the ratio of reserves to assets, the fraction of sellers employed by each firm, the ratio between the fraction of complaints and premium of each firm, and the risk rating and also the `mwr`. The random utility associated with j's offer to i

[9] We identify the order statistic distribution (13)'s RHS first and then using $F_{\tilde{\beta}}$ the cost distribution.

Figure 6: **Histogram of Winner and Runner Up**



Notes: This is a histogram of the winning company (x-axis) and the runner-up (y-axis). The winner is observed and the runner-up is determined using logistic regression (14).

is given by the following expression

$$\eta_{ij} = \gamma_j^0 + \gamma_j^1 \times Z_j + \gamma^2 \times \texttt{mwr}_{ij} + \varepsilon_{ij}, \tag{14}$$

where γ_j^0 is a company-specific constant, and γ_j^1 is also a company-specific coefficient for firm specific variables. For each of the 175 group, we estimate company specific parameters $\{\gamma^0, \gamma^1\}$ for every company that made at least one offer.[10] Then the probability of observing a particular choice is then given by $\Pr(D_i^1 = j) = \frac{\exp(\hat{\eta}_{ij})}{\sum_{j=1}^{J} \exp(\hat{\eta}_{ij})}$. Using these estimated probabilities for a retiree i, we say that a company j is the runner-up if j provides the second highest utility to individual i among the set of companies ultimately chosen by i.

Estimating $F_{\tilde{\beta}}(\cdot|X)$ and $W_r(\cdot|S)$

In this section, we consider the problem of estimating the distribution of $\tilde{\beta}$ before considering the problem of estimating conditional distribution of r. In the previous section we have shown that the joint distribution F_δ is nonparametrically identified, from which we can identify $F_{\tilde{\beta}}(\cdot|X)$ and $W_r(\cdot|S)$, where the conditioning variable X is age, gender, savings and intermediary channel. But, to estimate $F_{\tilde{\beta}}(\cdot|X = x)$ while being agnostic about the relationship between X and $\tilde{\beta}$, we can only use the subsample with $X = x$, and this subsample might not have enough variation to estimate the distribution; see Gaillac and Gautier (2019).

In light of this, we make two parametric assumptions: (i) $\tilde{\beta}_i$ in (13) depends on i's gender (male or female), the retirement age group (earlier than NRA, at NRA, after NRA)

[10] For normalization, one of the firm-specific set of coefficients is set to zero.

and savings quintiles; and (ii) $\tilde{\beta}_i$ is Normally distributed, i.e., $\tilde{\beta}_i \sim \mathcal{N}(\beta_{g(i)}, \sigma_{g(i)})$ where $g(i) \in G$ denotes the group that i belongs to, and G is the set of these 15 groups. Thus, while we assume that within each group the preferences are normally distributed with group specific mean and variance, we allow them to differ arbitrarily across groups.

Following the same reasoning as above, in view of our sample size, we assume that $W_r(\cdot|S_q)$, where S_q denotes the $q^{th}-$ quintile of savings in our sample. We assume that conditional on i's savings S_i belonging to the q-quintile, the cost $r \sim W_r(\cdot|S_q)$, where $q = 1, \ldots, 5$. In our sample, potential bidders can be either 13, 14 or 15. Let $N_{q,J}$ denote the subset of retirees whose savings is in the $q^{th}-$quintile and have $J \in \{13, 14, 15\}$ potential bidders. Then, we can re-write our estimation Equation (13) with group-specific coefficients for each $(q, J) \in \{1, \ldots, 5\} \times \{13, 14, 15\}$ pair as

$$\tilde{\rho}_{j_i^*} \;\; = \;\; \tilde{\beta}_{g(i)}^{\top}(Z_{k^*} - Z_{j^*}) + 1/r_{k_i^*}; \quad i = 1, 2, \ldots, N_{q,J}, \tag{15}$$

and $\tilde{\beta}_g = \tilde{\beta} + \upsilon_g$ where $\mathbb{E}(\upsilon_g) = 0$ and $\mathbb{E}(\upsilon_g^2) < \infty$. Therefore, for each (q, J) pair, using GLS we estimate six $\hat{\tilde{\beta}}_g$ parameters and $1/\hat{r}_{k_i^*}$ for all i in that group. Then, for each q we determine the weighted average over J, and estimate the CDF of $(J-1 : J)$ order statistics of $W_r^*(\cdot|S_q)$.

6.1 Empirical Findings

Here, we present our estimation results. There are three key parameters: preferences for risk-ratings ($\tilde{\beta}$), information processing costs (λ) and the distributions and densities of (relative) cost of annuatization $(W_r(\cdot|S_q), w_r(\cdot|S_q))$ where q denotes the quintile.

In Table 14 we present the estimates of $\tilde{\beta}$, for each group. We can see, as expected, that the estimates are all negative, which means retirees dislike higher risk-rating. Moreover, there is a lot of variation across groups, and more importantly, those who use sales agents dislike the risk-ratings more. This is consistent with the role of the sales-agent that we envision in this market -that they would (over) emphasize the importance of risk-rating. In other words, our estimates suggest that those who use sales agent forego larger pensions from companies with lower risk-rating, which our model will interpret as caring more about risk-rating. As we mentioned in the introduction, given how unlikely a bankruptcy is, retirees should put less weight on risk-ratings. And as we consider the preferences across savings-quintiles, we see that those in largest quintiles have significantly stronger dislike towards lower risk-ratings. These estimates are consistent with the hypothesis that should a company go bankrupt, it is those with larger savings who stand to lose the most, because for the rest there is guaranteed payment by the government in the unlikely event of a bankruptcy.

Table 14: **Estimates of Preferences for Risk-Ratings**

	Age	Gender	Q1	Q2	Q3	Q4	Q5
AFP	Pre	M	-243	-195	-20	-25	-1541
		F	n.a.	-204	-169	-137	-649
	At	M	-185	-83	-2	-482	-1852
		F	-202	-90	-99	-611	-788
	Post	M	-160	-66	-207	-581	-2102
		F	-202	-156	-319	-597	-1131
Sales-Agent	Pre	M	-375	-194	-178	-553	-1260
		F	-574	-254	-327	-726	-1183
	At	M	-278	-150	-322	-833	-1938
		F	-412	-336	-591	-982	-1223
	Post	M	-265	-212	-363	-1090	-2523
		F	-379	-365	-669	-1064	-1568
Advisor	Pre	M	-296	-251	-207	-317	-765
		F	-465	-321	-373	-584	-431
	At	M	-335	-213	-202	-249	-1055
		F	-555	-354	-455	-651	-878
	Post	M	-331	-219	-302	-592	-1405
		F	-493	-385	-472	-564	-925

Notes. Estimates of preferences for risk-rating, by gender, age of retirement and savings quintiles.

On top of that, when we consider the information processing cost, we find that those who use sales agent have higher information processing cost. The estimates of the information processing costs are displayed in Table 15. These estimates are also consistent with sales agents emphasizing not only the importance of risk-rating but also trying to steer retirees towards their own companies. If that process leads to positive MLT, our model interprets that as though the retirees are less sensitive to changes in pensions. And finally, we find that the information processing cost decreases with savings, except among those who use private advisors. This is consistent with the observation that those with higher savings tend to be those with higher education, who in turn have lower information processing costs. The fact that the costs do not vary with savings among those who with PFAs suggest that PFAs are doing their "job" and helping retirees choose their pensions.

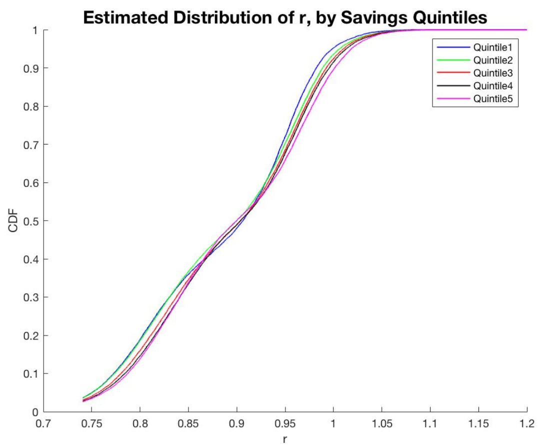
Finally, in Figure 7 we present the estimates of conditional distributions and densities of costs given saving-quintile. Our results indicate that the costs do not vary by savings, suggesting that there is no evidence of selection on savings, i.e., the intensive margin. Nonetheless, we find that in almost all cases the cost is less than 1, which means the companies have lower UNC than the public. In other words, the private sector can cheaply provide annuities

Table 15: **Information Processing Cost, by Savings and Channel**

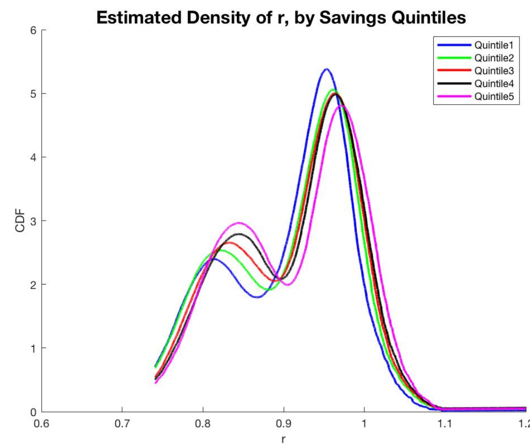
Savings/Channel	PFA	Sales Agent	Advisor	Overall
Q1	0.009	0.027	0.006	0.021
Q2	0.006	0.019	0.004	0.016
Q3	0.005	0.013	0.003	0.013
Q4	0.005	0.012	0.003	0.005
Q5	0.005	0.012	0.003	0.006
Overall	0.005	0.013	0.003	0.009

Notes. Estimates of information processing cost, by savings quintiles and intermediary channel.

Figure 7: **Conditional Distributions and Densities of Costs**



(a) Conditional CDFs, $W_r(\cdot|S_q)$ on $[\underline{r}, r^*_{12}]$



(b) Conditional PDFs, $w_r(\cdot|S_q)$ on $[\underline{r}, r^*_{12}]$.

than the government.

7 Counterfactual Analysis

To Be Completed

8 Conclusion

To Be Completed

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Appendix

A.1 Proof of Lemma 1

Proof. Suppose not, and suppose $\rho_j \geq 1/r_j$. If $\rho_j > 1/r_j$ then in the second period too, the offer will be greater than the return. This means the expected payoff from the auction is negative. In such case it is better to submit $\rho_j = 0$ and $\tilde{\rho}_j = 0$ which guarantees zero payoff. If $\rho_j = 1/r_j$ then $\tilde{\rho}_j = \rho_j = 1/r_j$ when the expected payment will be zero, where as the insurance company can deviate to $\rho_j = 1/r_j - \varepsilon$ with $\varepsilon > 0$ and $\tilde{\rho}_j = 1/r_j$ that generates strictly positive payoff for j, given that $\sigma_j > 0$ and $\sigma_{J+1} > 0$.

Using Lemma 1 in (7), the expected profit simplifies to

$$\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j,\boldsymbol{\rho}) = \int_{1/\bar{r}}^{1/r_j} ydG(y) - G(r_j)r_j. \tag{A.1}$$

Substituting this expression for the expected profit, $\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j,\boldsymbol{\rho})$ from (A.1), in j's objective function, and taking the derivative with respect to ρ_j we get

$$\begin{aligned} -\sigma_j(\boldsymbol{\rho}) \times r_j + (1 - r_j\rho_j)\frac{\partial \sigma_j(\boldsymbol{\rho})}{\partial \rho_j} + \frac{\partial \sigma_{J+1}(\boldsymbol{\rho})}{\partial \rho_j}\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j,\boldsymbol{\rho}) \\ +\sigma_{J+1}(\boldsymbol{\rho})\frac{\partial \mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j,\boldsymbol{\rho})}{\partial \rho_j} = 0, \end{aligned}$$

$$\textit{or,} \qquad -\sigma_j(\boldsymbol{\rho}) \times r_j + (1 - r_j\rho_j)\frac{\partial \sigma_j(\boldsymbol{\rho})}{\partial \rho_j} + \frac{\partial \sigma_{J+1}(\boldsymbol{\rho})}{\partial \rho_j}\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j,\boldsymbol{\rho}) = 0, \tag{A.2}$$

where the second equality follows from the fact that the derivative of (A.1) with respect to ρ_j is zero. As an aside, note that a consequence of having the second stage is that the insurance companies offer smaller pensions in the first round. To see this, suppose ρ_j^* solves (A.2)

$$-\sigma_j(\rho_j^*, \boldsymbol{\rho}_{-j})r_j + (1 - r_j\rho_j^*)\frac{\partial \sigma_j(\rho_j^*, \boldsymbol{\rho}_{-j})}{\partial \rho_j^*} + \underbrace{\frac{\partial \sigma_{J+1}(\rho_j^*, \rho_{-j})}{\partial \rho_j^*}}_{<0}\mathbb{E}\Pi_j^{II}(\tilde{\rho}_j|r_j,\boldsymbol{\rho}^*) = 0$$

from which it follows that $-\sigma_j(\rho_j^*, \rho_{-j})r_j + (1 - r_j\rho_j^*)\frac{\partial \sigma_j(\rho_j^*, \rho_{-j})}{\partial \rho_j^*} > 0$. The expected profit for insurance j is (quasi) concave in ρ_j when there is no second stage and because $\sigma_j(\boldsymbol{\rho}) + (1 - r_j\rho_j)\frac{\partial \sigma_j(\boldsymbol{\rho})}{\partial \rho_j} = 0$, we can conclude that $\rho_j^* < \rho_j$ for all $j = 1, \ldots, J$. □

A.2 Modified English Auction

When companies are differentiated, the English auction is *modified* in the sense that the winner need not be the one who offers the largest pension. Without loss of generality suppose there are $J = 2$ life insurance companies with costs r_1 and r_2 and they choose pensions (bids) $\tilde{\rho}_1$ and $\tilde{\rho}_2$, respectively. Suppose, all else equal, the retiree prefers firm 1, and this is captured by a parameter $\Delta_{12} > 0$, and will choose the annuity from 1 if and only if $\tilde{\rho}_1 + \Delta_{12} > \tilde{\rho}_2$. We can model this game by adapting Milgrom and Weber (1982)'s formulation of a Japanese auction, where a clock rises continuously and each player pushes a button until he stops pushing to inform his bid. The game ends when there is only one bidder left, and he pays the bid of the last player that released his button. Our modified English auction is similar to the Japanese auction and is described as follows:

1. Firm j privately observes $r_j \sim W_r(\cdot), j = 1, 2$.
2. Each firm pushes a button while a clock automatically and continuously raises the price. Let $\tilde{a} \geq 0$ denote the current "value" of the clock.
3. Firm j's bid is the value of the clock at which j releases the button.
4. The clock rises until both firms release their buttons.
5. The retiree observes $\{\tilde{\rho}_1, \tilde{\rho}_2\}$, and chooses one, and payoffs are realized.

Each firms's strategy can be summarized by two functions: One that indicates when to stop if the other firm hasn't stopped, and the other indicates when to stop if the other firm has stopped at some $\tilde{\rho}$. Let $\Psi_j^0(\tilde{a}, r_j) : \mathbb{R}^2 \to [\tilde{a}, \infty]$ denote the price at which to release the button, as a function of j's own cost r_j and the clock's current position $\tilde{a}$, and let $\Psi_j^1(\tilde{a}, \tilde{\rho}_{j'}, r_j) : \mathbb{R}^3 \to [\tilde{a}, \infty]$ denote the price to release the button given the value of the clock is $\tilde{a}$, its competitor $j' \neq j$ dropped out at $\tilde{\rho}_{j'}$ and j's cost is r_j.

The equilibrium concept we use is the Weak Perfect Bayesian Equilibrium (WPNE). A strategy profile Ψ and a system of beliefs μ form a WPBE of an extensive form game if Ψ is sequentially rational given μ, and μ is derived from Ψ, whenever possible. The following lemma characterizes WPBE of the modified English auction.

Lemma 2. *The WPBE outcome of this game is as follows: If* $1/r_2 - \Delta_{12} < 1/r_1$*, then Firm 1 wins and Firm 2 pushes up to* $1/r_2 - \Delta_{12}$*, and Firm 1 also pushes the price to* $1/r_2 - \Delta_{12}$*. If* $1/r_2 - \Delta_{12} > 1/r_1$*, then Firm 2 wins, Firm 1 pushes price up to* $1/r_1$*, and Firm 2 pushes price up to* $1/r_1 + \Delta_{12}$*. The equilibrium strategies and the system of beliefs are as follow:*

- $\Psi_1^0(\tilde{a}, r_1) = \begin{cases} 1/r_1 & \textit{if } \tilde{a} \leq 1/r_1 \\ \tilde{a} & \textit{otherwise.} \end{cases}$

- $\Psi_1^1(\tilde{a}, \tilde{\rho}_2, r_1) = \tilde{a}.$

- $\Psi_2^0(\tilde{a}, r_2) = \begin{cases} 1/r_2 - \Delta_{12} & \textit{if } \tilde{a} \leq 1/r_2 - \Delta_{12} \\ \tilde{a} & \textit{otherwise.} \end{cases}$

- $\Psi_2^1(\tilde{a}, \tilde{\rho}_1, r_2) = \begin{cases} \tilde{\rho}_1 + \Delta_{12} & \textit{if } \tilde{a} \leq \tilde{\rho}_1 + \Delta_{12} \quad \textit{and} \quad \tilde{\rho}_1 + \Delta_{12} \leq 1/r_2 \\ \tilde{a} & \textit{otherwise.} \end{cases}.$

- $\mu_1^0(r_2|\tilde{a}) = \begin{cases} \frac{W_r(r_2)}{1-W_r(1/(\tilde{a}+\Delta_{12}))} & \textit{if } r_2 \leq 1/(\tilde{a} + \Delta_{12}) \\ 0 & \textit{otherwise.} \end{cases}$

- $\mu_2^0(r_1|\tilde{a}) = \begin{cases} \frac{W_r(r_1)}{1-W_r(1/\tilde{a})} & \textit{if } r_1 \leq 1/\tilde{a} \\ 0 & \textit{otherwise.} \end{cases}$

- $\mu_1^1(r_2|\tilde{a}, \tilde{\rho}_2) = \begin{cases} 0 & \textit{if } r_2 < 1/(\tilde{\rho}_2 + \Delta_{12}) \\ 1 & \textit{otherwise.} \end{cases}$

- $\mu_2^1(r_1|\tilde{a}, \tilde{\rho}_1) = \begin{cases} 0 & \textit{if } r_1 < 1/\tilde{\rho}_1 \\ 1 & \textit{otherwise.} \end{cases}$

Proof. We first verify that the beliefs are consistent with Bayes rule, given the strategies. Let $\mu_k^0(r_j|\tilde{a})$ and $\mu_k^1(r_j|\tilde{a}, \tilde{\rho}_j)$ be the beliefs –specified as cdfs– that firm k holds about firm j's type when j has not yet dropped out, and when he has dropped out at a value of $\tilde{\rho}_j$ respectively, given the value of $\tilde{a}$.

Given $\Psi_1^0(\tilde{a}, r_1)$, if firm 2 observes firm 1 exiting at $\tilde{a}$ –i.e., $\tilde{\rho}_1 = \tilde{a}$–, firm 2 assigns probability 1 to $r_1 = 1/\tilde{\rho}_1$, as specified by $\mu_2^1(r_1|\tilde{a}, \tilde{\rho}_1)$. Similarly, given $\Psi_2^0(\tilde{a}, r_2)$, if firm 1 observes firm 2 exiting at $\tilde{a}$ –i.e., $\tilde{\rho}_2 = \tilde{a}$–, firm 1 assigns probability 1 to $r_2 = 1/(\tilde{\rho}_2 + \Delta_{12})$, as specified by $\mu_1^1(r_2|\tilde{a}, \tilde{\rho}_2)$. If $\tilde{\rho}_2 = 0$, firm 1 updates his beliefs knowing that $r_2 \leq \Delta_{12}$. Before anyone has exited, beliefs are simply a bayesian update of $W_r(\cdot)$ in the supports $r_1 \leq 1/\tilde{a}$ for Firm 2' beliefs and $r_2 \leq 1/(\tilde{a} + \Delta_{12})$ for Firm 1's beliefs.

Next, we verify that the strategies are sequentially rational given the beliefs. Note first that, given the assumption $\Delta_{12} > 0$, if Firm 2 exits first it is optimal for Firm 1 to leave immediately since he already won the auction; therefore $\Psi_1^1(\tilde{a}, \tilde{\rho}_2, r_1) = \tilde{a}$. Consider now $\Psi_2^1(\tilde{a}, \tilde{\rho}_1, r_2)$. If firm 1 exited and $\tilde{\rho}_1 + \Delta_{12} \leq 1/r_2$ (and $\tilde{a} \leq \tilde{\rho}_1 + \Delta_{12}$), it is optimal for firm 2 to push the "price" up to $\tilde{a} = \tilde{\rho}_1 + \Delta_{12}$ and win. If $\tilde{\rho}_1 + \Delta_{12} > 1/r_2$ then it is optimal for firm 2 to exit immediately as he can not obtain positive profits winning the auction.[11]

[11]Off the equilibrium path, if $\tilde{a} > \tilde{\rho}_1 + \Delta_{12}$, firm 2 wins so it should exit immediately.

Now let us analyze the optimality of $\Psi_1^0(\tilde{a}, r_1)$ given $\Psi_2^1(\tilde{a}, \tilde{\rho}_1, r_2)$, $\Psi_2^0(\tilde{a}, r_2)$, $\Psi_1^1(\tilde{a}, \tilde{\rho}_2, r_1)$, and Firm 1's beliefs about r_2. Note that if firm 1 exits first, it will lose the auction and get zero with probability one because Firm 2 will push up to $\tilde{\rho}_1 + \Delta_{12}$ as long as $r_2 \leq 1/(\tilde{a} + \Delta_{12})$, which has probability one as specified in $\mu_1^0(r_2|\tilde{a})$.

Therefore, if Firm 1 exits at $\rho < 1/r_1$ –rather than at $1/r_1$– the realized payoffs will differ if and only if $1/r_2 \in [\rho + \Delta_{12}, 1/r_1 + \Delta_{12}]$. Similarly, if $\rho > 1/r_1$ –rather than $1/r_1$– realized payoffs will differ if and only if $1/r_2 \in [1/r_1 + \Delta_{12}, \rho + \Delta_{12}]$.

Therefore, if Firm 2 has not exited and for any value of $\tilde{a} < \rho < 1/r_1$, the expected payment difference between exiting at $1/r_1$ or at ρ for Firm 1 is

$$[\mu_1^0(\frac{1}{r_1} + \Delta_{12}|\tilde{a}) - \mu_1^0(\rho + \Delta_{12}|\tilde{a})] \times (\frac{1}{r_1} - \mathbb{E}_{\mu_1^0(.|\tilde{a})}(\frac{1}{r_2} - \Delta_{1,2}|\frac{1}{r_2} \in [\rho + \Delta_{12}, \frac{1}{r_1} + \Delta_{12}])) > 0.$$

And the difference between exiting at $\tilde{\rho}_1 = 1/r_1$ or at $\rho > r_1$ is

$$[\mu_1^0(\rho + \Delta_{12}|\tilde{a}) - \mu_1^0(\frac{1}{r_1} + \Delta_{12}|\tilde{a})] \times (\mathbb{E}_{\mu_1^0(.|\tilde{a})}(\frac{1}{r_2} - \Delta_{1,2}|\frac{1}{r_2} \in [\frac{1}{r_1} + \Delta_{12}, \rho + \Delta_{12}] - \frac{1}{r_1}) > 0.$$

So it is optimal for Firm 1 release exactly at $1/r_1$ if Firm 2 has not exited.

Now, let us consider $\Psi_2^0(\tilde{a}, r_2)$. Because $\Delta_{12} > 0$, if firm 2 exits first he looses and gets zero. Given Firm 1's strategy, the expected payment difference between exiting at $\tilde{\rho}_2 = 1/r_2 - \Delta_{12}$ and at $\rho < 1/r_2 - \Delta_{12}$ is

$$\Pr\{1/r_1 \in [\rho - \Delta_{12}, 1/r_2 - \Delta_{12}]\}(1/r_2 - \mathbb{E}(1/r_1 + \Delta_{1,2}|1/r_1 \in [\rho - \Delta_{12}, 1/r_2 - \Delta_{12}])) > 0,$$

and the difference between exiting at $1/r_2$ or at $\rho > r_2$ is

$$\Pr\{1/r_1 \in [1/r_2 - \Delta_{12}, \rho - \Delta_{12}]\}(\mathbb{E}(1/r_1 + \Delta_{1,2}|1/r_1 \in [1/r_2 - \Delta_{12}, \rho - \Delta_{12}]) - 1/r_2) > 0.$$

□