Information Design in Common Value Auction with Moral Hazard

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[PRELIMINARY AND INCOMPLETE]

1 Introduction

This paper studies the extent to which information design can alleviate moral hazard in a common value auction. In our setting, an auctioneer holds a first-price sealed bid auction on a common-value object. After firms submit cash bids, the auctioneer then decides how to release information about bids to the winning bidder. The winning bidder then takes a costly action, which is not contractible, but affects the auctioneer's payoff.

This theoretical setting is motivated by the empirical setting of the U.S. offshore oil auctions, which features a very low exploratory drilling rate (10%). The government's revenue comes from both the cash bids and a fraction of the oil revenue during the lease period. Under the current information structure, the government releases all information on bids to the winning bidder.

Our data, which contain information on all auctions held and subsequent drilling activities between 1983 and 2019, show that the winning bidder tends to not undertake any exploratory drilling activity when the second bid is lower, conditional on the winning bid. This suggests that firms do update their beliefs about the oil potential of a tract based on other firms' bids.

Our theoretical results show two main insights. First, any information structure that is not fully revealing will lower the bids. Second, in the absence of the effect on equilibrium bids, full information revelation is never optimal, and providing no information is always optimal. The intuition is the following. Since drilling is costly, a bidder will only submit bids if his prior about the profit from drilling is sufficiently high. By providing additional information after the auction stage, the auctioneer incurs a positive probability of the winning bidder choosing not to drill after observing more information, whereas under the no information case, the winning bidder will choose to drill.

We then build an empirical model of the offshore oil auctions in which firms have private signals about the quantity of oil in each tract and some expectation about the cost of drilling exploration wells. Firms then compete in the first-price sealed bid auction. The government releases information about all bids to the winner, and the cost of drilling is realized. The winner then decides whether to drill. We use an instrument for the cost of drilling (the number of oil rigs that were in the area in the previous 5 years) and assume that the cost of drilling and the latent oil quantity are independent. We are then able to identify the signals as a function of the bids and the value of winning the auction as a function of realized bids and conduct counterfactual exercises on alternative information structures.

2 Literature Review

Our paper is related to the literature on information spillovers. In the context of oil drilling, Hendricks and Porter (1996) studies the determinants of firms' exploratory drilling decision using data on wildcat tracts off the coasts of Texas and Louisiana between 1954 and 1979. They found that leaseholders do not respond much to the information by their rivals' bids. The difference between their finding and ours could be due to the difference in the institutional setting, since our data cover a later period in which most of the available tracts were offered for sale, whereas before 1983, a tract has to be nominated by at least one firm to be offered for auction. The nomination process helped to eliminate information asymmetry between bidders as the fact that a tract has been nominated contains valuable information about the tract (Haile, Hendricks, and Porter (2010)).

3 Institutional Setting and Data

For each tract, we define 'neighboring tracts' using a similar metric as in Hendricks, Pinske, and Porter (2003), which are tracts within 0.11 degree of lattitude and 0.12 degress of longitude. We first conduct a similar exercise as in Hendricks and Porter (1996) using data from 1996 to 2010. Although we have data until 2019, the data is truncated to take into account lease length, which could be up to 10 years.

Table 3.1: Summary Statistics

Statistic	Mean	N	St. Dev.
# of Neighboring Leases	69.51	13,637	68.51
# of Productive Neighboring Leases	12.32	13,637	26.42
# of Explored Neighboring Leases	4.98	13,637	12.43
# of Bids	1.47	13,637	1.04
Percentage of Rejected Bids	0.09	13,637	0.28
Average Winning Bids of Neighboring Leases	912,830.90	10,713	1,012,019.00
Explored Leases	0.05	13,637	0.23

If we restrict the attention to leases that have at least 2 bids and were not rejected by the government

Table 3.2: Summary Statistics

Statistic	Mean	N	St. Dev.
# of Neighboring Leases	82.85	3,375	74.43
# of Productive Neighboring Leases	15.15	3,375	30.30
# of Explored Neighboring Leases	6.40	3,375	13.87
# of Bids	2.79	3,375	1.35
Percentage of Rejected Bids	0.00	3,375	0.00
Average Winning Bids of Neighboring Leases	979,136.90	2,776	1,004,633.00
Explored Leases	0.11	3,375	0.31

Conduct similar analysis as to Hendricks and Porter (1996)

Table 3.3: Determinants of Exploratory Drilling Decision

	Not Yet Explored	
# Nearby Leases	0.000	0.00
	(0.000)	(0.00
# Nearby Explored Leases	-0.002***	-0.002
	(0.000)	(0.00
# Nearby Productive Leases	-0.000	-0.00
	(0.000)	(0.00
No Nearby Explored Leases		-0.07
		(0.14
Log Area Bid	0.000	0.00
	(0.002)	(0.00
Log Ratio of First and Second Bids	0.026***	0.02
	(0.009)	(0.01
Log Acre	0.058**	0.05
	(0.023)	(0.02
Log of First Bid	-0.045***	-0.047
	(0.006)	(0.00
Number of Bids	-0.017***	-0.021
	(0.005)	(0.00
Log Ratio of First and Second Bids x No Nearby Explored Leases		-0.00
		(0.01
Log of First Bid x No Nearby Explored Leases		0.00
		(0.01
Number of Bids x No Nearby Explored Leases		0.00
		(0.01
Constant	0.690**	0.82
	(0.344)	(0.34
Sample Average	0.106	0.10
Observations	3,375	3,37
R^2	0.093	0.09
Adjusted R^2	0.087	0.09
Residual Std. Error	0.294 (df = 3352)	0.294 (df =
F Statistic	15.673*** (df = 22; 3352)	13.763*** (df =

Table 3.3 suggests that winning bidders are less likely to drill exploratory wells when the second bid is significantly lower than the first bid and when the number of bids is lower, controlling for the first bid as well as the characteristics of nearby leases prior to the auction date.

4 Model

We start by analyzing the model with full information revelation. Since our empirical setting features full information revelation by the auctioneer, the analysis here will be useful for the identification results. We then generalize the model to the case of a general information structure.

4.1 Benchmark: Full Information Revelation

We consider a first-price sealed bid auction for tract t associated with observed characteristics X_t and oil deposit Q_t, which is the common value of the tract to all bidders. For each tract t, there are N_t potential bidders. N_t is known to all bidders prior to the auction.[1] Bidders in tract t have a common belief about the cost of exploration drilling for tract t, $F_c(c_t)$. In the bidding stage, each firm will choose a bidding strategy $B_{it}(S_{it}, F_{S,Q}, F_c, N_t)$. Prior to the auction, each bidder receives a private signal S_{it} about Q_t, but all bidders know about the ex-ante joint distribution $F_{S,Q}$ of (S_t, Q_t).

In the exploration stage, the cost c_t is realized, and the winning bidder decides whether to drill an exploration well after observing c_t, the submitted bids, and identity of the bidders. Let $d_{it} \in \{0, 1\}$ denote whether the winning bidder i in tract t drill an exploration well. Firm i's expected payoff is given by

$$\mathbb{E}_{Q_t}\left(V_{it}(d_i t, c_t, Q_t, B_{it})|B_t\right) = \mathbb{E}_{Q_t}\left(d_{it}((1-r)Q_t - c_t)|B_t\right)$$

4.1.1 Exploration Stage

We assume (for now) that the winner is able to deduce the signals of other bidders after winning bids. We will show later that this is WLOG given the current information structure (full information revelation). The winner will choose to drill $d_{it} = 1$ if

$$\mathbb{E}\left(Q_t|B_t\right) = \mathbb{E}\left(Q_t|S_t\right) \geq \frac{c_t}{1-r} \tag{4.1}$$

$F_{S,Q}$ can be reformulated into two components (1) the prior (which is common across all bidders, this is identical to the marginal distribution of Q in $F_{S,Q}$), and (2) the signal structure, which is more informative for the informed bidder (which is the conditional distribution $F_{S|Q}$). where

$$\begin{aligned}
\mathbb{E}\left(Q_t|S_t\right) &= \int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ \\
\implies \underbrace{\mathbb{E}_{Q_t}\left(V_{it}(d_{it}, c_t, Q_t, B_{it})|B_t\right)}_{V_t^*} &= \max\left\{0, (1-r)\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ - c_t\right\}
\end{aligned}$$

4.1.2 Bidding Stage

In the bidding stage, each firm will choose their bidding strategy $B_{it}(S_{it})$ based on the belief about the joint distribution of S_t and Q_t. There is no reservation price.

Firm i with signal S_{it} will choose B_{it} to maximize its payoff.

$$\begin{aligned}
B_{it} &= \arg\max_b \mathbb{E}\left(V_t^* - b|b > \max_{j\neq i} B_{jt}\right)\Pr\left(b > \max_{j\neq i} B_{jt}\right) \\
\implies B_{it} &= \arg\max_b \int_{-\infty}^b (\mathbb{E}_{S_{jt}, j\neq i}\left(V_t^*\left(S_t, c_t|S_{it} > S_{jt}\forall j \neq i\right)\right) - b)g_{M_{it}|B_{it}}(m_{it})dm_{it} \\
\implies 0 &= (\mathbb{E}_{S_{jt}, j\neq i}\left(V_t^*\left(S_t, c_t|S_{it} > S_{jt}\forall j \neq i\right)\right) - b)g_{M_{it}|B_{it}}(b) - G_{M_{it}|B_{it}}(b)
\end{aligned}$$

where $M_{jt} = \max_{j\neq i} B_{jt}$. Therefore

$$\mathbb{E}_{S_{jt}, j\neq i}\left(V_t^*\left(S_t, c_t|S_{it} > S_{jt}\forall j \neq i\right)\right) = B_{it} + \frac{G_{M_{it}|B_{it}}(B_{it})}{g_{M_{it}|B_{it}}(B_{it})} \tag{4.2}$$

4.2 Equilibrium under a more general information structure

In a standard auction framework without ex-post moral hazard, the signal is often normalized to be the expectation of the posterior belief $\mathbb{E}_{S_{jt},j\neq i}\left(V_t^*\left(S_t,c_t|S_{it}>S_{jt}\forall j\neq i\right)\right)$, conditional on winning. In our context, however, we are interested in how information design affects firms' incentives, and the posterior belief becomes an endogenous object that cannot be arbitrarily normalized. In this section, we show how the equilibrium bidding strategies are affected by strategic information disclosure by the auctioneer. In this more general case, in the exploration stage, the cost c_t is realized, and the winning bidder decides whether to drill an exploration based on *a signal about other firms' bids* and its own signal.

We first show that it is without loss of generality to consider an information structure based on the firms' signals (instead of firms' bids).

Let Σ_t denote the signal about firms' private signals. The information structure $\pi(\Sigma_t|S_t)$, which is the probability distribution of Σ_t being realized conditional on S_t, is commonly known to all bidders. We denote the posterior $\Pr(S_t|\Sigma_t)$ as $\rho(S_t|\Sigma_t)$ where $\rho(S_t|\Sigma_t)=\frac{\pi(\Sigma|S)f_S(S)}{f_\Sigma(\Sigma)}$.

In the exploration stage, the posterior belief now becomes

$$\mathbb{E}(Q_t|\Sigma_t)=\int_{supp(S)}\int_0^\infty Q_t\frac{f_Q(Q_t)f_{S|Q}(S_t|Q_t)}{f_S(S_t)}dQ_t\rho(S_t|\Sigma_t)dS_t$$

The firm will choose to drill if $\mathbb{E}(Q_t|\Sigma_t)\geq\frac{c_t}{1-r}$. Therefore

$$V_t^{\Sigma^*}=\max\left\{0,(1-r)\int_{supp(S)}\int_0^\infty Q_t\frac{f_Q(Q_t)f_{S|Q}(S_t|Q_t)}{f_S(S_t)}dQ_t\rho(S_t|\Sigma_t)dS_t-c_t\right\}$$

In the bidding stage, the firm takes into account the information structure π and chooses bids B_t to maximize its expected payoff

$$B_{it}=\arg\max_b\mathbb{E}\left(V_t^{\Sigma^*}-b|b>\max_{j\neq i}B_{jt}\right)\Pr\left(b>\max_{j\neq i}B_{jt}\right)$$

Our focus is on a symmetric Bayes Nash equilibrium in which all players play according to a strategy $B(S_{it})$. Let $Y_{it}=\max_{j\neq i}S_{it}$, and we denote F_Y as the conditional probability of $Y_{it}|S_{it}$.

$$\begin{aligned}
&B_{it} = \arg\max_b \mathbb{E}\left(V_t^{\Sigma^*} - b | b > \max_{j\neq i} B_{jt}\right) \Pr\left(b > \max_{j \neq i} B_{jt}\right) \\
\implies &B_{it} = \arg\max_b \int^{B^{-1}(b)} \int_{supp(\Sigma)} \int_{supp(c)} \left(V_t^{\Sigma^*} - b\right) dF_{\Sigma|Y}(\Sigma|y, S_{it}) dF_c(c) dF_Y(y|S_{it}) \\
\implies &0 = \frac{d}{db} B^{-1}(b) \int_{supp(\Sigma)} \int_{supp(c)} \left(V_t^{\Sigma^*} - b\right) dF_{\Sigma|y}(\Sigma|y = S_{it}, S_{it}) dF_c(c) f_Y(S_{it}|S_{it}) \\
&\quad - \int^{B^{-1}(b)} \int_{supp(\Sigma)} \int_{supp(c)} dF_{\Sigma|Y}(\Sigma|y, S_{it}) dF_c(c) dF_Y(y|S_{it}) \\
\implies &0 = \frac{1}{B'(S_{it})} \int_{supp(\Sigma)} \int_{supp(c)} \left(V_t^{\Sigma^*} - B(S_{it})\right) dF_{\Sigma|y}(\Sigma|y = S_{it}, S_{it}) dF_c(c) f_Y(S_{it}|S_{it}) \\
&\quad - \int^{B^{-1}(b)} \int_{supp(\Sigma)} \int_{supp(c)} dF_{\Sigma|Y}(\Sigma|y, S_{it}) dF_c(c) dF_Y(y|S_{it}) \\
\implies &0 = \int_{supp(\Sigma)} \int_{supp(c)} V_t^{\Sigma^*} dF_{\Sigma|y}(\Sigma|y = S_{it}, S_{it}) dF_c(c) f_Y(S_{it}|S_{it}) - B(S_{it}) f_Y(S_{it}|S_{it}) - B'(S_{it}) F_Y(S_{it}|S_i \\
\implies &\frac{f_Y(S_{it}|S_{it})}{F_Y(S_{it}|S_{it})} \int_{supp(\Sigma)} \int_{supp(c)} V_t^{\Sigma^*} dF_{\Sigma|y}(\Sigma|y = S_{it}, S_{it}) dF_c(c) = B(S_{it}) \frac{f_Y(S_{it}|S_{it})}{F_Y(S_{it}|S_{it})} + B'(S_{it})
\end{aligned}$$

Let $L(x|s) = \exp\left(-\int_x^s \frac{f_Y(\alpha|\alpha)}{F_Y(\alpha|\alpha)} d\alpha\right)$

$$B^{\Sigma}(s) = \int_{\underline{S}}^{s} \int_{supp(c)} \int_{supp(\Sigma)} V_t^{\Sigma^*} dF_{\Sigma|y}(\Sigma|Y_{it} = x, S_{it} = x) dF_c(c) dL(x|s) \tag{4.3}$$

Under the special case of full information revelation

$$B^{F}(s) = \int_{\underline{S}}^{s} \int_{supp(c)} \int_{supp(S)} V_t^{F^*} dF_{S_{-i}|y}(S_{-i}|Y_{it} = x, S_{it} = x) dF_c(c) dL(x|s) \tag{4.4}$$

where

$$V_t^{F^*} = \max\left\{0, (1-r) \int_0^{\infty} Q \frac{f_Q(Q) f_{S|Q}(S|Q)}{f_S(S)} dQ - c_t\right\}$$

and

$$V_t^{\Sigma^*} = \max\left\{0, (1-r) \int_{supp(S)} \int_0^{\infty} Q_t \frac{f_Q(Q_t) f_{S|Q}(S_t|Q_t)}{f_S(S_t)} dQ_t \rho(S_t|\Sigma_t) dS_t - c_t\right\}$$

Proposition 4.1 $B^{\Sigma}(s) \leq B^{F}(s) \; \forall \; s$.

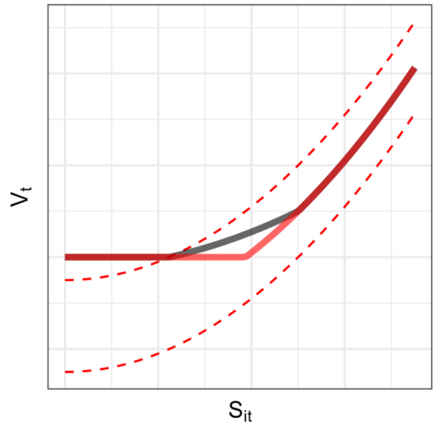

Figure 4.1: Example of $V_t^{\Sigma *}$ and V_t^{F*}. The dashed red lines are $V_t^F(S_{-i})$ and $V_t^F(S_{-i'})$, respectively. Their expectation is the solid red line. The black line is $V_t^{\Sigma *}$ when the information structure splits the posterior belief into S_{-i} and S'_{-i}.

5 Identification

The object for identification includes the distribution of exploration costs F_{c_t}, and the signal structure $F_{S|Q}$. We observe data on bids, drilling decisions, and oil quantity if development takes place.

We first show that F_{c_t} is identified when an instrument for the cost distribution exists and when c_t is uncorrelated with the oil value of the tract. Once F_{c_t} is identified, the mean of the posterior belief about the net value of winning the auction is also identified.

Our first identification result relies on the assumption that bidders do not update their belief about the oil value of the tract when c_t is realized. This does not rule out ex-ante correlation between c_t and Q_t based on the observed characteristics of the tracts. This assumption is needed because we do not observe data on exploration cost.

Theorem 5.1 *Under the following assumptions*

1. *The realization of the cost of exploration is independent of the oil value of the tract.*

$$c_t \perp Q_t | X_t$$

2. *There exists an instrument Z_t for the distribution of cost:*

$$c_t = c_{0t} + g(Z_t)$$

where $c_{0t} \perp Z_t$, $g(Z_t) \in \mathbb{R}^+$, and $Z_t \perp Q_t$ with unknown g, where g is strictly increasing and twice-differentiable.

F_{c_0}, g are identified, and the mean of the posterior belief $\int_0^\infty \int_{[0,S_{it}]^{N_t-1}} Q \frac{f_Q(Q) f_{S|Q}(S|Q)}{f_S(S)} f_{S_{-i}|S_i}(S_{-i}) dS_{-i} dQ$ is point-wise identified.

Proof. In what follows, we omit X_t for notational convenience.

Equation (4.2) under the assumptions of Theorem 5.1 becomes:

$$\int_c \max\left\{0, (1-r)\int_0^\infty \int_{[0,S_{it}]^{N_t-1}} Q \frac{f_Q(Q) f_{S|Q}(S|Q)}{f_S(S)} f_{S_{-i}|S_i}(S_{-i}) dS_{-i} dQ - c\right\} dF_c(c) = B_{it} + \frac{G_{M_{it}|B_{it}}(\cdot}{g_{M_{it}|B_{it}}(\cdot}$$

For notational convenience, let $\Delta_{it} = (1-r)\int_0^\infty \int_{[0,S_{it}]^{N_t-1}} Q \frac{f_Q(Q) f_{S|Q}(S|Q)}{f_S(S)} f_{S_{-i}|S_i}(S_{-i}) dS_{-i} dQ$, we then have

$$
\begin{aligned}
\int_0^{\Delta_{it}} (\Delta_{it} - c_t) dF_c(c_t|Z_t) &= B_{it} + \frac{G_{M_{it}|B_{it}}(B_{it})}{g_{M_{it}|B_{it}}(B_{it})} \\
\implies \int_0^{\Delta_{it} - g(Z_t)} (\Delta_{it} - c_{0t} - g(Z_t)) dF_{c_0}(c_{0t}) &= B_{it} + \frac{G_{M_{it}|B_{it}}(B_{it})}{g_{M_{it}|B_{it}}(B_{it})} \\
\implies (\Delta_{it} - g(Z_t)) F_{c_0}(\Delta_{it} - g(Z_t)) - \int_0^{\Delta_{it} - g(Z_t)} c_{0t} dF_{c_0}(c_{0t}) &= B_{it} + \frac{G_{M_{it}|B_{it}}(B_{it})}{g_{M_{it}|B_{it}}(B_{it})}
\end{aligned} \tag{5.2}
$$

In addition, a firm will choose not to bid if the following constraint holds (assuming 0 reserve price—to be discussed later):

$$
\int_0^{\Delta_{it} - g(Z_t)} (\Delta_{it} - c_{0t} - g(Z_t)) dF_{c_0}(c_{0t}) \leq 0 \tag{5.3}
$$

Note that the left hand-side of (5.2) is strictly increasing in $(\Delta_{it} - g(Z_t))$. To see this:

$$
\frac{d}{dx}\left(x F_{c_0}(x) - \int_0^x c_{0t} dF_{c_0}(c_{0t}) \right) = x f_{c_0}(x) + F_{c_0}(x) - x f_{c_0}(x) > 0
$$

For any $a \in [0,1]$, let $\phi(a, Z_t)$ be the corresponding quantile at a of $B_{it} + \frac{G_{M_{it}|B_{it}}(B_{it})}{g_{M_{it}|B_{it}}(B_{it})}$. $\phi(a, Z_t)$ is identified from the data. We assume that ϕ_2 exists. Let $\Delta^{(a)} = F_\Delta^{-1}(a)$. Equation (5.2) implies

$$
\phi_2(a, Z_t) + F_{c_0}\left(\Delta^{(a)} - g(Z_t)\right) g'(Z_t) = 0 \tag{5.4}
$$

Suppose there exists $\tilde{g}, \tilde{\Delta}, \tilde{F}_{c_0}$ and g, Δ, F_{c_0} that both satisfy (5.4). Since g and $\tilde{g}$ are both strictly increasing and differentiable, there exists $h \in C^1$ such that $\tilde{g} = h \circ g$.

If $\tilde{g} = g + k$, where k is a constant, $\tilde{\Delta} = \Delta + k$, and $\tilde{F}_{c_0} = F_{c_0}$.

We now consider the case in which $h'(x) \neq 0$.

$$
\begin{aligned}
F_{c_0}\left(\Delta^{(a)} - g(Z_t)\right) g'(Z_t) &= \tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - \tilde{g}(Z_t)\right) g'(Z_t) h'(g(Z_t)) \\
\implies F_{c_0}\left(\Delta^{(a)} - g(Z_t)\right) &= \tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - h(g(Z_t))\right) h'(g(Z_t)) \\
\implies F_{c_0}\left(\Delta^{(a)} - x\right) &= \tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h'(x) \\
\implies f_{c_0}\left(\Delta^{(a)} - x\right) \frac{d}{da}\Delta^{(a)} &= \tilde{f}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h'(x) \frac{d}{da}\tilde{\Delta}^{(a)} \\
\text{and } - f_{c_0}\left(\Delta^{(a)} - x\right) &= -\tilde{f}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h'^2(x) + \tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h''(x) \\
\implies \frac{1}{\frac{d}{da}\Delta^{(a)}} &= \frac{h'(x)}{\frac{d}{da}\tilde{\Delta}^{(a)}} + \frac{\tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h''(x)}{\tilde{f}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h'(x) \frac{d}{da}\tilde{\Delta}^{(a)}}
\end{aligned}
$$

Therefore,

$$
\frac{\partial}{\partial x}\left(h'(x) + \frac{\tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h''(x)}{\tilde{f}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right) h'(x)} \right) = 0 \forall x, a
$$

Therefore, $h''(x) = 0$, and $h'(x) = k \neq 0$, where k is a constant. Without loss of generality, assume that $k < 1$. In this case, however, since $F_{c_0}\left(\Delta^{(a)} - x\right) = \tilde{F}_{c_0}\left(\tilde{\Delta}^{(a)} - h(x)\right)k$, $F_{c_0} > 1$ with positive probability, which is a contradiction.

Therefore, g and Δ are identified up to a constant, and F_{c_0} is identified.

To show that Δ is identified, note that

$$\mathbb{E}_{S_{it}}\Delta_{it} = (1-r)\int_0^\infty Q f_Q(Q)dQ$$

Therefore, $\int_0^\infty \int_{[0,S_{it}]^{N_t-1}} Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)} f_{S_{-i}|S_i}(S_{-i})dS_{-i}dQ$ is identified, and g is also identified. ▢

In the drilling stage, we observe $\mathbb{I}\left(0 \leq (1-r)\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ - c_t\right)$.

$$\Pr(d_t = 1|S) = \Pr\left(c_t \leq (1-r)\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ\right) = F_c\left((1-r)\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ\right)$$

Therefore, $\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ$ is also identified.

We have the following:

$$\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)}dQ = H(\boldsymbol{B}_t)$$
$$\int_{[0,S_{it}]^{N_t-1}}\int_0^\infty Q\frac{f_Q(Q)f_{S|Q}(S|Q)}{f_S(S)} f_{S_{-i}|S_i}(S_{-i})dQdS_{-i} = G_i(\boldsymbol{B}_t)$$

Theorem 5.2 (Identification of B^Σ) *B^Σ is identified.*

Proof (Proof of Theorem 5.2).

$$\begin{aligned}
L(x|s) &= \exp\left(-\int_x^s \frac{f_Y(\alpha|\alpha)}{F_Y(\alpha|\alpha)}d\alpha\right)\\
&= \exp\left(-\int_x^s \frac{f_{B^F(Y)}(B^F(\alpha)|B^F(\alpha))}{F_{B^F(Y)}(B^F(\alpha)|B^F(\alpha))}(B^{F'}(\alpha))d\alpha\right)\\
&= \exp\left(-\int_{B^F(x)}^{B^F(s)} \frac{f_{B^F(Y)}(B^F(\alpha)|B^F(\alpha))}{F_{B^F(Y)}(B^F(\alpha)|B^F(\alpha))}d(B^F(\alpha))\right)\\
&= L(B^F(x)|B^F(s))
\end{aligned}$$

Therefore

$$\begin{aligned}
&B^\Sigma(s) = \int_{\underline{S}}^s \int_{supp(c)} \mathbb{E}\left(V_t^{\Sigma^*}(\Sigma|Y = x, S_{it} = x)\right)dF_c(c)dL(x|s)\\
\implies &B^\Sigma(s) = \int_{\underline{S}}^s \int_{supp(c)} \mathbb{E}\left(V_t^{\Sigma^*}\left(\Sigma|B^F(Y) = B^F(x), B^F(S_{it}) = B^F(x)\right)\right)dF_c(c)dL(x|s)\\
\implies &B^\Sigma(s) = \int_{\underline{S}}^s \int_{supp(c)} \mathbb{E}\left(V_t^{\Sigma^*}\left(\Sigma|B^F(Y) = B^F(x), B^F(S_{it}) = B^F(x)\right)\right)dF_c(c)dL(x|s)\\
&= \int_{B^F(\underline{S})}^{B^F(s)} \int_{supp(c)} \mathbb{E}\left(V_t^{\Sigma^*}\left(\Sigma|B^F(Y) = B^F(x), B^F(S_{it}) = B^F(x)\right)\right)dF_c(c)B^{F^{-1'}}(B^F(x))dL\left(B^F(x
\end{aligned}$$

We can now compute $B^{\Sigma}\left(B^{F^{-1}}(B^F(x))\right)$ if $B^{F^{-1'}}(B^F(x))$ can be computed. The following lemma shows that $B^F(s)$ is indeed identified.

Lemma 5.1 $B^F(s)$ *is identified.* ▢

Proof (Proof of Lemma 5.1). Under full information revelation:

$$B^F(s) = \int_{B^F(\underline{S})}^{B^F(s)} \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}\left(B^F(S)|B^F(Y) = B^F(x), B^F(S_{it}) = B^F(x)\right)\right) dF_c(c) B^{F^{-1'}}(B^F(x)) dL\,($$
$$= \int_{B^F(\underline{S})}^{B^F(s)} \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}\left(B^F(S)|B^F(Y) = B^F(x), B^F(S_{it}) = B^F(x)\right)\right) dF_c(c) B^{F^{-1'}}(B^F(x)) dL\,($$

where

$$\mathbb{E}\left(V_t^{F^*}\left(B^F(X)|B^F(Y) = B^F(x), B^F(S_{it}) = B^F(x)\right)\right)$$
$$= \int_0^{B_F(x)} \max\left\{0, H(B^F(X)|B^F(Y) = B^F(x))) - c\right\} dF_{B^F}(B^F(X))$$

Equation (5.5) is a first-kind Volterra integral equation. We will first transform it into a second-kind Volterra integral equation. For notational convenience, let us denote $B^F(s) = a$, $B^F(x) = \alpha$, and $V_t^F\left(B^F(S)|B^F(Y) = \alpha, B^F(s) = \alpha\right) = V_t^F(\alpha, \alpha)$

$$\frac{d}{da}a = \frac{d}{da}\int_0^a \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(\alpha,\alpha)\right) dF_c(c) B^{F^{-1'}}(\alpha) dL\,(\alpha|a)$$
$$= \left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c)\right] B^{F^{-1'}}(a) L'\,(a|a) +$$
$$\int_0^a \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(\alpha,\alpha)\right) dF_c(c) B^{F^{-1'}}(\alpha) \frac{\partial^2}{\partial(\alpha)\partial(a)} L\,(\alpha|a)$$
$$\implies B^{F^{-1'}}(a) = \left[1 - \int_0^a \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(\alpha,\alpha)\right) dF_c(c) B^{F^{-1'}}(\alpha) \frac{\partial^2}{\partial(\alpha)\partial(a)} L\,(\alpha|a)\, d(\alpha)\right]$$
$$\left[\left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c)\right] \frac{d}{d\alpha} L\,(a|a)\right]^{-1} \tag{5.6}$$

Let the right hand-side of (5.6) be denoted by a mapping $T\left(B^{F^{-1'}}\right)$. Suppose that we have $B^{F^{-1'}}$ and $\tilde{B}^{F^{-1'}}$ that both satisfy (5.6).

$$\left\|T\left(B^{F^{-1'}}\right)-T\left(\tilde{B}^{F^{-1'}}\right)\right\|_\infty = \left\|\left[-\int_0^a \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(\alpha,\alpha)\right) dF_c(c) \left(B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right) \frac{\partial^2}{\partial(\alpha)\partial(a)} L(\alpha\right.$$

$$\left[\left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c)\right] \frac{d}{d\alpha} L(a|a)\right]^{-1}$$

$$\leq \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty \left[\int_0^a \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(\alpha,\alpha)\right) dF_c(c) \left|\frac{\partial^2}{\partial(\alpha)\partial(a)} L(\alpha|a)\right| d\right.$$

$$\left[\left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c)\right] \frac{d}{d\alpha} L(a|a)\right]^{-1}$$

$$= \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty \left[\int_0^a \int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(\alpha,\alpha)\right) dF_c(c) L(\alpha|a) \frac{f_Y(a|a)}{F_Y(a|a)} \frac{f_Y(}{F_Y(}\right.$$

$$\left[\left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c)\right] \frac{f_Y(\alpha|\alpha)}{F_Y(\alpha|\alpha)}\right]^{-1}$$

$$\leq \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty \left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c) \int_0^a L(\alpha|a) \frac{f_Y(\alpha|\alpha)}{F_Y(\alpha|\alpha)} d(\alpha)\right.$$

$$\left[\int_{supp(c)} \mathbb{E}\left(V_t^{F^*}(a,a)\right) dF_c(c)\right]^{-1}$$

$$\leq \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty \int_0^a L(\alpha|a) \frac{f_Y(\alpha|\alpha)}{F_Y(\alpha|\alpha)} d(\alpha)$$

$$= \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty L(\alpha|a)\Big|_0^a$$

$$= \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty \left(1-\exp\left(-\int_0^a \frac{f_Y(\epsilon|\epsilon)}{F_Y(\epsilon|\epsilon)} d\epsilon\right)\right)$$

$$< \left\|B^{F^{-1'}}(\alpha)-\tilde{B}^{F^{-1'}}(\alpha)\right\|_\infty$$

Where the second inequality holds because

$$\begin{aligned}
&\frac{\partial^2}{\partial\alpha\partial a} L(\alpha|a) \\
=&\frac{\partial^2}{\partial\alpha\partial a} \exp\left(-\int_\alpha^a \frac{f_Y(\epsilon|\epsilon)}{F_Y(\epsilon|\epsilon)} d\epsilon\right) \\
=&\frac{\partial}{\partial\alpha} \exp\left(-\int_\alpha^a \frac{f_Y(\epsilon|\epsilon)}{F_Y(\epsilon|\epsilon)} d\epsilon\right) \frac{-f_Y(a|a)}{F_Y(a|a)} \\
=&\exp\left(-\int_\alpha^a \frac{f_Y(\epsilon|\epsilon)}{F_Y(\epsilon|\epsilon)} d\epsilon\right) \frac{-f_Y(a|a)}{F_Y(a|a)} \frac{f_Y(\alpha|\alpha)}{F_Y(\alpha|\alpha)} < 0 \text{ for all } (\alpha, a)
\end{aligned}$$

$T()$ is therefore a contraction mapping. Hence, $B^{F^{-1'}}(\cdot)$ is the unique solution of (5.5). $B^F(s)$ is thus identified. ▢

References

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1. Similar to Bhattacharya, Ordin, and Roberts (2018), we will later construct N_t as the number of unique bidders that participated in leases within the past 2.5 years and within 2 km radius.↩