Deceptive Advertising, Regulation and Gullible Consumers*

Aastha Gupta†

March 2020

Abstract

This paper is an attempt to formulate a more effective regulation against deceptive practices by incorporating consumer behavior. I model "gullible" consumers who easily believe any advertising claim (false or otherwise) made by the monopolist – who can either be a high-type or low-type depending on the product quality – and impose a fixed penalty as a form of regulation against false advertising. With the presence of gullible consumers along with rational/sophisticated buyers in the market, I show that there exists a unique fully-revealing equilibrium where the low-type firm is able to deceive gullible consumers and at the same time, high-type firm is able to signal its quality to sophisticated consumers. I further show that the socially optimal level of penalty is higher than the penalty required to just avoid deception and is increasing with the proportion of sophisticated consumers. This outcome contradicts the expectation that gullibility must strengthen the case for regulation. However, it is the increased buyer sophistication that strengthens the case for regulation from a welfare perspective.

JEL Classification: D11, D42, D82, L12, L15

Keywords: Regulation; Asymmetric Information; Deception; False Advertising; Signaling; Product Quality

*I am grateful to my advisor, Prof. Santanu Roy for his constant support and guidance. I would also like to thank the participants at Texas Economic Theory Camp and SMU Brown Bag Seminar for insightful comments.

†Ph.D Candidate, Department of Economics, Southern Methodist University. Email: aagupta@smu.edu

1 Introduction

False/deceptive advertising refers to the seller's act of communicating misleading information about a product or service to the buyers, whether intentional or not. For example, New Balance and Reebok falsely claimed that their toning shoes can help burn calories; Volkswagen cheated on performance tests to falsely claim that their cars are environmental friendly; Dannon misled consumers about the health benefits of its Activa yogurt. So the question arises why do firms engage in such deceptive practices? Recent literature (Rhodes and Wilson 2018, Piccolo et al 2015, 2017) has attempted to answer this question which show that low-quality firm sends out the same advertising claim as its high-quality counterpart resulting in successful deception of rational buyers. In response, governments in many countries have formed regulatory bodies – Federal Trade Commission in USA, Competition Bureau in Canada, European Advertising Standards Alliance in Europe – to monitor and prevent deceptive advertising. They have formulated laws and regulations to make it difficult for the firms to lie about their product's characteristics. They do so by imposing penalties or some expected cost on firms engaging in deceptive advertising. These regulatory bodies have successfully cracked down on many deceptive advertising cases. Retailers/manufacturers who were found guilty (like New Balance, Volkswagen to name a few) had to pay multi-million dollar penalties to settle charges by the regulatory bodies. But still several cases go unnoticed and undetected.

One of the stated objectives of these regulations is to protect buyers. Buyers can be deceived for various reasons. One of the reasons is that they are not sophisticated enough to interpret the market signals. If a firm is charging a very low price for its product and at the same time advertising it as a high-quality product, some consumers will be deceived. Those are gullible consumers, who easily believe any claim made by the seller, whether false or not. I want to examine false advertising in a setup where not all consumers are sophisticated, some are gullible. Then the question arises that in markets with gullible consumers present alongside sophisticated consumers, what is the optimal regulation? What is the role of such regulation? Note that regulation is implemented to avoid deception but will also have effect on other market features (Glaeser and Ujhelyi 2010, Janssen and Roy 2019). Therefore, regulation should take into account overall market welfare. For this purpose, I also examine the impact on social welfare outcome.

A simple static framework with quality as private information can be used to develop on this economic argument, where multiple signals are used to signal quality to potential consumers.[1] I study a monopoly case where only the monopolist has private information about the quality (high or low) of its product. The marginal cost of production is dependent on the product quality. Each firm-type can signal its quality with a combination of price and an advertising

[1] Examples are cited as a vindication of the statement that firms engage in false advertising or make false claims. Some examples may not fit the model but it makes sense to focus on these models where price signaling works and even then there is a possibility for deception.

message. I assume no cost of sending out the advertising message. Of course, if cost of sending the message is large, no one sends it. But I abstract from issues related to cost of advertising. However, a false advertising message incurs a fixed penalty. There are two types of consumers – sophisticated and gullible. Both types of consumers have same prior probability of product being high quality but they update their beliefs differently. While the sophisticated consumers update using both price and advertising signals of the firm, the gullible consumers only update if the firm advertises as high-type. This generates separate demand curves for sophisticated and gullible consumers. I consider a multistage game where the nature determines the product quality. Only the firm observes its true quality and decides upon a price signal and an advertising message, followed by consumers making purchase decisions. I solve for perfect Bayesian equilibrium which satisfies Intuitive Criterion (Cho and Kreps, 1987).

I characterize a unique separating perfect Bayesian equilibrium wherein for a low level of penalty and a certain range for proportion of sophisticated consumers, the low-type falsely advertises its product as high-quality and only serves the gullible consumers. On the other hand, the high-type signals its true quality and serves both types of consumers. For an intermediate level of penalty, the high-type signals its quality by charging a high price. The low-type also charges a high price when gullible consumers are dominant in the market but reduces the price as the proportion of sophisticated consumers begins to increase. However, it does not falsely advertise irrespective of which consumer type is dominant in the market. For a high level of penalty, the equilibrium converges to a full information outcome.

Not surprisingly, the result shows that the low-type indulges in deceptive advertising for a certain range of penalty level as well as proportion of sophisticated consumers. Therefore, what is its implication for the optimal regulation? Some may argue that optimal regulation should set the penalty at an arbitrarily high level to scare away the firms completely. However, expected penalty denotes the probability of enforcement or detection in case the firm is guilty. There is an upper bound on nominal penalty which cannot exceed the total consumer damages as a consequence of false advertising. Note that court is the one awarding damages, not the regulators and thus, out of proportion penalty cannot be justified in the court of law. Moreover, increasing penalty or probability of detection is costly as monitoring cost is involved (such as cost to gather evidence). Hence, I focus on the minimum level of penalty which serves the purpose.

The optimal regulation depends on the type of policy goal: whether to just avoid deception or to maximize overall social welfare. The minimum level of penalty needed to avoid deceptive advertising is low and decreasing in the proportion of sophisticated consumers. In fact, no regulation is needed if the proportion of sophisticated consumers is very high in the market because the low-type loses out on all the incentive to falsely advertise its quality and hence, no fear of deception. However, if the policy goal is to maximize social welfare, then a much higher level of penalty is required. Such a level of penalty leads to full

information outcome and also removes signaling distortion which is present in the case of high proportion of sophisticated consumers (Bagwell and Riordan, 1991). An interesting observation is that this optimal penalty level is increasing in the proportion of sophisticated consumers, that is, the presence of sophisticated consumers increases the importance for regulation. This contradicts the expectation that the presence of gullible consumers should strengthen the case for regulation. Instead, buyer sophistication requires stronger regulatory effort from a welfare perspective.

Another interesting observation is that the model has no pooling perfect Bayesian equilibrium. This is in contrast with the past literature which states that false advertising occurs only as a pooling equilibrium outcome. The difference arises because my framework requires pooling on both the price and advertising message which neither of the firm types find profitable. At least one type always has an incentive to deviate (formal proof given in the text). It is the Intuitive Criterion that rules out pooling outcome, even when there is no regulation. Thus, regulation does not destroy pooling. In fact, it makes it even harder to sustain pooling PBE.

This paper contributes to multiple strands of literature. There is an extensive literature on product quality signaling (Ellingsen, 1997; Daughety and Reinganum, 2008a,b; Adriani and Deidda, 2009; Janssen and Roy, 2010), rationally inattentive consumers (Grubb, 2009, 2015; Armstrong and Huck, 2010; Eliaz and Spiegler, 2006) and false advertising (Hattori and Higashida, 2012; Piccolo et al, 2015; Rhodes and Wilson, 2018; Janssen and Roy, 2019). This paper is closely related to Rhodes and Wilson (2018), which studies a monopoly model to characterize optimal policy against false advertising but they rule out price signaling by definition. In similar lines, Piccolo et al (2015, 2017) study a duopoly model where one firm is of high-quality and other of low-quality but the consumers do not know which one is high- and which one is low-quality. In both the papers, it is assumed that costs are symmetric across firm types and focuses only on rational buyers. My paper, on the other hand, incorporates both rational and gullible buyers in the framework with cost dependent on product quality.

Corts (2013) analyzes the presence of both false claim ban and need for specific performance/quality tests instead of only one of the two regulations to achieve better outcomes on a social welfare basis. Corts (2014) defines socially optimal level of penalty when a single false claim penalty exists and exerts that finite penalty is a desirable outcome. Both the papers are different from this paper for various reasons: the monopolist does not know its product quality but can learn it at a cost, the production cost is not dependent on quality, price signaling is assumed to not exist; and they consider only rational consumers. A few papers focus on the welfare effects of regulation. Janssen and Roy (2019) examines the impact of policy penalizing false claim on market outcome when no false advertising occurs in equilibrium. Their model also allows firm to signal quality through prices as well as direct communication but latter involves a cost.

Glaeser and Ujhelyi (2010) analyze and compares the impact of three different government policies on consumer welfare: false claim ban, counter-advertising and taxing the product.

Rest of the paper is organized as follows: Section 2 presents the model. Section 3 solves the firm's problem and fully characterizes the equilibrium concept. Section 4 analyzes optimal regulation under two cases - (i) to avoid deception and (ii) to maximize overall social welfare. Section 5 concludes.

2 Model

Consider a one-period product market with one firm and many potential buyers of the product. The product can either be of high quality or low quality, $q = \{H, L\}$. It is produced at a constant marginal cost, $c_q \geq 0$, which depends on the true quality of the product. The marginal cost of a high-quality product is $c_H > 0$ and of a low-quality product is $c_L = 0$ (without loss of generality). Nature determines the product quality for the firm. The product quality is only observed by the firm and is not revealed to consumers. Each type of firm sets a price p and an advertising message, $a \in \{0, 1\}$. Message $a = 0$ implies that the firm did not advertise its product whereas $a = 1$ implies that the firm advertised its product as high quality. If a firm lies about its product quality, then it incurs a fixed penalty cost of d.

The potential consumers of the product are given by a unit mass, each with a unit demand. The consumers have homogeneous valuation, $v > 0$, for a low-quality product and heterogeneous valuation for a high-quality product which is uniformly distributed between v and $(1 + v)$. Further, these consumers are divided into two categories:

1. Sophisticated consumers believe that the product is of high-quality with probability r. They observe the price and advertising message set by each firm and update their beliefs to r'.

2. Gullible consumers have the same prior probability as the sophisticated consumers. However, upon observing an advertising message, $a = 1$, these consumers update their beliefs to product being high quality, irrespective of the price. Their beliefs are unaffected by any other factor (price and/or $a = 0$) because of limited reasoning abilities.

Let s denote the fraction of sophisticated consumers and $(1 - s)$ the fraction of gullible consumers in the market.

The timeline of the game is as follows: First, nature determines the quality of the firm which is drawn from an independent distribution that assigns probability r to high type. Firm observes its quality and simultaneously sets a price and choses an advertising message. Finally, all the consumers decide whether to purchase the product or not by maximizing their utility/expected utility. The

firm's payoff is equal to its profit. The payoff of each consumer is his/her ex-post utility.

The solution concept used is that of perfect Bayesian equilibrium where the out-of-equilibrium beliefs satisfy the intuitive criterion (Cho and Kreps, 1987) in every subgame.

3 Equilibrium

In equilibrium, consumers maximize their expected utility in order to make a purchase decision. Given a consumer's beliefs, expected utility is equal to the expected reservation price *minus* the price set by the monopolist. This results in separate demand curves for each type of consumer, as shown in *Figures 1 and 2*. At price P and $a = 1$, $(1 + \frac{v-P}{r'})$ fraction of sophisticated consumers buy for any price $P\epsilon\,[v, r' + v]$ and $(1+v-P)$ fraction of gullible consumers buy for any price $P\epsilon\,[v, 1 + v]$. When $a = 0$, all the sophisticated consumers buy as long as $P \leq v$ and $(1 + \frac{v-P}{r})$ fraction of gullible consumers buy for any $P\epsilon\,[v, r + v]$.

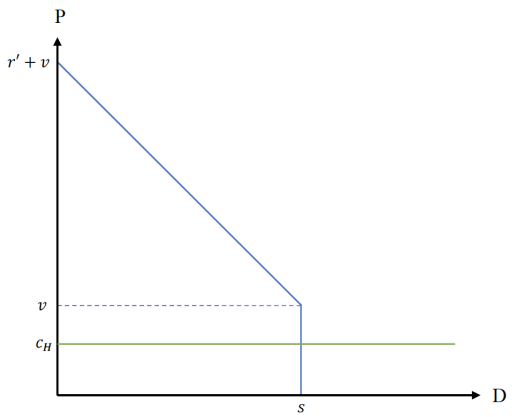

Figure 1: Demand curve of Sophisticated Consumers

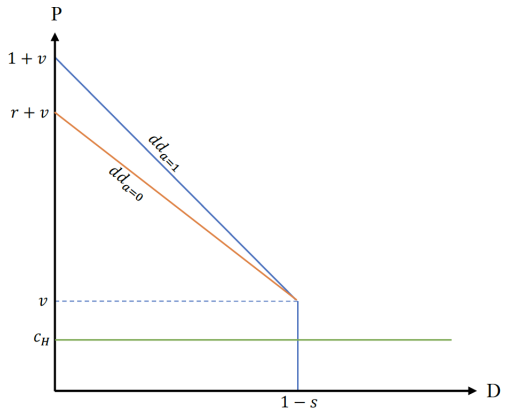

Figure 2: Demand curve of Gullible Consumers

A separating equilibrium defines price(s) for which firms of similar type charge the same price and signal their true quality, at least to the sophisticated consumers. A low-type firm can serve all the consumers in the market by simply charging v. But when the gullible consumers dominate the market, it has an opportunity to charge a higher-than-valuation price and lie about its quality given the penalty is low. Ideal price for the low-type to charge in such a scenario would be the one that maximizes its profit:

$$max\ \pi_L(P^L) = P^L(1 + v - P^L)(1 - s) - d$$

$$\Rightarrow\ P^L = \frac{1+v}{2}, a = 1$$

However, if the penalty is high, the low-type can still charge a high price even without lying about its quality because gullible buyers do not understand price signals and therefore, maximize their expected utility. Ideal price in such a case is:

$$max\ \pi_L(P^L) = P^L\left(1 + \frac{v - P^L}{r}\right)(1 - s)$$

$$\Rightarrow\ P^L = \frac{r+v}{2}, a = 0$$

Assuming $r > v$, the above price is greater than v, generating a higher profit to low-type even without lying about its quality. Therefore, when the gullible consumers are dominant in the market, the low-type chooses to lie about its quality if and only if:

$$\pi_L(P^L = \frac{1+v}{2}, a = 1) \geq \pi_L(P^L = \frac{r+v}{2}, a = 0)$$

$$\Rightarrow \ d \leq \frac{(1-s)(1-r)(r-v^2)}{4r} = \bar{d_1}$$

As long as $d \leq \bar{d_1}$, low-type charges $P^L = \frac{1+v}{2}$ and indulges in deception. For $d \geq \bar{d_1}$, low-type still charges a higher price, $P^L = \frac{r+v}{2}$, but does not lie about its quality due to an increased penalty. In both the scenarios, it only serves the gullible consumers. However, when the proportion of sophisticated consumers increases in the market, the low-type deviates to charge $v$because loss of sophisticated consumers is more than the gain from gullible consumers. If $d \leq \bar{d_1}$, such deviation occurs when:

$$\pi_L(P^L = \frac{1+v}{2}, a = 1) \leq \pi_L(P^L = v, a = 0)$$

$$\Rightarrow \ s \geq \frac{(1-v)^2 - 4d}{(1+v)^2}$$

If $d \geq \bar{d_1}$:

$$\pi_L(P^L = \frac{r+v}{2}, a = 1) \leq \pi_L(P^L = v, a = 0)$$

$$\Rightarrow \ s \geq \frac{(r-v)^2}{(r+v)^2}$$

The strategy of the low-type is summarized in the following lemma:

Lemma 1. *The pricing and advertising strategy of the low-type monopolist is as follows:*

1. When $d \leq \bar{d_1}$, $P^L = \frac{1+v}{2}, a = 1$ *if* $s \leq \frac{(1-v)^2-4d}{(1+v)^2}$ *and* $P^L = v, a = 0$ *if* $s \geq \frac{(1-v)^2-4d}{(1+v)^2}$.

2. When $d \geq \bar{d_1}$, $P^L = \frac{r+v}{2}, a = 0$ *if* $s \leq \frac{(r-v)^2}{(r+v)^2}$ *and* $P^L = v, a = 0$ *if* $s \geq \frac{(r-v)^2}{(r+v)^2}$.

The high-type monopolist's ideal price is $P^H = \frac{1+v+c_H}{2}$, the full information price. This price along with the message $a = 1$ is the best response of the

high-type when gullible consumers are dominant in the market (s is very low) because advertising message is enough for the gullible consumers to believe that the product is of high-quality. However, when the proportion of sophisticated consumers increases in the market, the fear of mimicry from the low-type increases as well. Therefore, with increasing s, the high-type should deviate to a higher signaling price to separate itself from the low-type firm. It can do so by choosing a price, P that is not profitable for the low-type to mimic. When $d \leq \bar{d_1}$ and $0 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2}$,

$$\pi_L(P^L = \frac{1+v}{2}, a = 1) \ \geq \ P(1+v-P) - d$$

$$\Rightarrow \ P \geq \bar{P}_{a=1} = (1+\sqrt{s})\left(\frac{1+v}{2}\right) \ \ or \ \ P \leq \underline{P}_{a=1} = (1-\sqrt{s})\left(\frac{1+v}{2}\right)$$

As $\pi_H(\bar{P}_{a=1}) > \pi_H(\underline{P}_{a=1})$, all the prices $P \leq \underline{P}_{a=1}$ are ruled out. Intuitive criterion suggests that the high-type chooses the least-cost signaling price which is just enough to prevent low-type to mimic high-type's prices. This is satisfied by $P^H = \bar{P}_{a=1}$ because $\pi_H(\bar{P}_{a=1})$ is the highest for the price range $P \geq \bar{P}_{a=1}$ and sufficient to separate from the low type. Therefore, the high type's best response is to charge $P^H = max\left\{\frac{1+v+c_H}{2}, \bar{P}_{a=1}\right\}$. $P^H = \frac{1+v+c_H}{2}$ when:

$$\frac{1+v+c_H}{2} \geq \bar{P}_{a=1} \ \Rightarrow \ s \leq \left(\frac{c_H}{1+v}\right)^2$$

Hence, $P^H = \bar{P}_{a=1}$ when $\left(\frac{c_H}{1+v}\right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2}$. That is, the high-type charges full information price when s is very low and switches to a higher signaling price, $\bar{P}_{a=1}$ for intermediate values of s.

Similarly, when $d \geq \bar{d_1}$ and $0 \leq s \leq \frac{(r-v)^2}{(r+v)^2}$,

$$\pi_L(P^L = \frac{r+v}{2}, a = 0) \geq P(1+v-P) - d$$

$$\Rightarrow \ P \geq \bar{P}_{a=0} = \frac{1+v}{2} + \frac{\sqrt{(1+v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d}}{2}$$

or

$$P \leq \underline{P}_{a=0} = \frac{1+v}{2} - \frac{\sqrt{(1+v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d}}{2}$$

Following the similar argument as above, the intuitive criterion suggests that the high type charges $P^H = max\left\{\frac{1+v+c_H}{2}, \bar{P}_{a=0}\right\}$. This generates a second threshold for d:

$$\frac{1+v+c_H}{2} \geq \frac{1+v}{2} + \frac{\sqrt{(1+v)^2 - \frac{(1-s)(r+v)^2}{r} - 4d}}{2}$$

$$\Rightarrow \;\; d \geq \left(\frac{1-s(1-r)}{4r}\right)\left[\left(\frac{r}{1-s(1-r)} - v\right)^2 - c_H^2\right] = \bar{d}_2$$

Therefore, if $d \geq \bar{d}_2$, the high-type always charges the full information price, irrespective of s. However, if $d \leq \bar{d}_2$, the high type charges $P^H = \frac{1+v+c_H}{2}$ when $s \leq \frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2}$ (solving the above inequality for s). But when $\frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \frac{(r-v)^2}{(r+v)^2}$, high-type switches to signaling price : $P^H = \bar{P}_{a=0}$.

When $d \leq \bar{d}_1$ and $\frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1$ or $d \geq \bar{d}_1$ and $\frac{(r-v)^2}{(r+v)^2} \leq s \leq 1$, the sophisticated consumers are dominant in the market and the low-type charges v. The high type signals its quality such that:

$$\pi_L(P^L = v, a = 0) \geq P(1+v-P) - d$$

$$\Rightarrow \;\; P \geq \overline{\overline{P}} = \frac{1+v}{2} + \frac{\sqrt{(1-v)^2 - 4d}}{2} \;\; or \;\; P \leq \underline{\underline{P}} = \frac{1+v}{2} - \frac{\sqrt{(1-v)^2 - 4d}}{2}$$

Again, intuitive criterion suggests that the high type's best response is $P^H = max\left\{\frac{1+v+c_H}{2}, \overline{\overline{P}}\right\}$. This generates a threshold which is a modified version of $\bar{d}_2$:

$$\frac{1+v+c_H}{2} \geq \frac{1+v}{2} + \frac{\sqrt{(1-v)^2 - 4d}}{2}$$

$$\Rightarrow \;\; d \geq \frac{[(1-v)^2 - c_H^2]}{4} = \bar{\bar{d}}_2$$

Note that $\bar{d}_2 = \bar{\bar{d}}_2$ at $s = 1$. If $d \geq \bar{\bar{d}}_2$, the high-type continues to charge the full information price, irrespective of s, because a high penalty is enough to prevent low-type to mimic high-type's price. If $d \leq \bar{\bar{d}}_2$, high-type switches to signaling price, $\bar{\bar{P}}$, irrespective of s. The strategy of the high-type is summarized in the following lemma:

Lemma 2. *The pricing and advertising strategy of the high-type monopolist is as follows:*

1. *When* $d \leq \bar{d}_1$,
   (a) $P^H = \frac{1+v+c_H}{2}, a = 1$ *and if* $0 \leq s \leq \left(\frac{c_H}{1+v}\right)^2$
   (b) $P^H = \bar{P}_{a=1}, a = 1$ *if* $\left(\frac{c_H}{1+v}\right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2}$
   (c) $P^H = \bar{\bar{P}}, a = 1$ *if* $\frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1$.
2. *When* $\bar{d}_1 \leq d \leq \bar{d}_2$,
   (a) $P^H = \frac{1+v+c_H}{2}, a = 1$ *if* $0 \leq s \leq \frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2}$
   (b) $P^H = \bar{P}_{a=0}, a = 1$ *if* $\frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \frac{(r-v)^2}{(r+v)^2}$
   (c) $P^H = \bar{\bar{P}}, a = 1$ *if* $\frac{(r-v)^2}{(r+v)^2} \leq s \leq 1$.
3. *When* $\bar{d}_2 \leq d \leq \bar{\bar{d}}_2$,
   (a) $P^H = \frac{1+v+c_H}{2}, a = 1$ *if* $0 \leq s \leq \frac{(r-v)^2}{(r+v)^2}$
   (b) $P^H = \bar{\bar{P}}, a = 1$ *if* $\frac{(r-v)^2}{(r+v)^2} \leq s \leq 1$.
4. *When* $d \geq \bar{\bar{d}}_2$, $P^H = \frac{1+v+c_H}{2}, a = 1$ *for all* s.

Before stating the complete separating equilibrium, I first define a benchmark case with no regulation ($d = 0$), as shown in *Figure 3*. When gullible consumers are dominant in the market, the high-type charges full-information monopoly price as advertising message is enough to signal its true quality. On the other hand, the low-type indulges in false advertising and sets a profit-maximizing price. While all consumers purchase the good in the high-quality state, only the gullible consumers purchase in the low-quality state. As the proportion of sophisticated consumers starts to increase, the high-type slowly starts to increase its price to signal its true quality and the low-type continues to false advertise but its demand is falling with increasing s. When sophisticated consumers become dominant in the market, the high-type continues to signal its quality through high prices but the low-type shifts to reservation price. The low-type

can no longer afford to ignore the sophisticated consumers and hence, switches to $a = 0$ to serve all the consumers in the market.

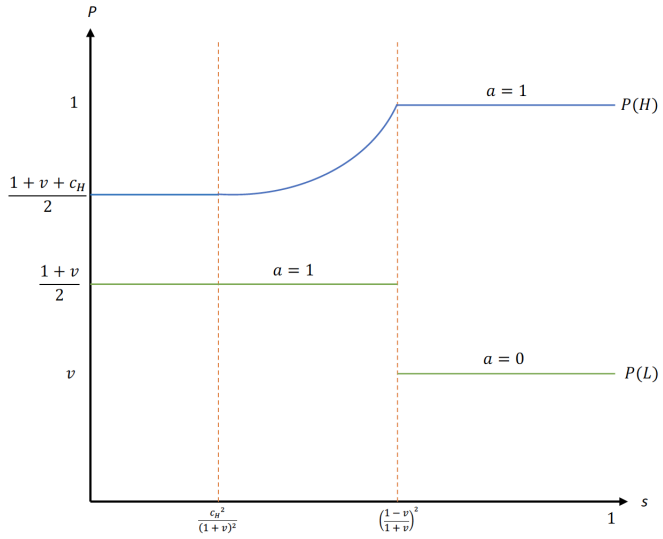

Figure 3: Separating Perfect Bayesian Equilibrium under No Regulation

With regulation ($d \geq 0$), the complete separating equilibrium satisfying the intuitive criterion is shown in *Figure 4* and summarized in the following proposition:

Proposition 3. *The unique separating equilibrium satisfying the intuitive criterion is defined as:*

1. *When* $d \leq \bar{d}_1$,
   (a) $P^H = \frac{1+v+c_H}{2}, a = 1$ *and* $P^L = \frac{1+v}{2}, a = 1$ *if* $0 \leq s \leq \left(\frac{c_H}{1+v}\right)^2$
   (b) $P^H = \bar{P}_{a=1}, a = 1$ *and* $P^L = \frac{1+v}{2}, a = 1$ *if* $\left(\frac{c_H}{1+v}\right)^2 \leq s \leq \frac{(1-v)^2-4d}{(1+v)^2}$
   (c) $P^H = \overline{\overline{P}}, a = 1$ *and* $P^L = v, a = 0$ *if* $\frac{(1-v)^2-4d}{(1+v)^2} \leq s \leq 1$.
2. *When* $\bar{d}_1 \leq d \leq \bar{d}_2$,
   (a) $P^H = \frac{1+v+c_H}{2}, a = 1$ *and* $P^L = \frac{r+v}{2}, a = 0$ *if* $0 \leq s \leq \frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2}$
   (b) $P^H = \bar{P}_{a=0}, a = 1$ *and* $P^L = \frac{r+v}{2}, a = 0$ *if* $\frac{r(c_H^2+4d)-(1-r)(r-v^2)}{(r+v)^2} \leq s \leq \frac{(r-v)^2}{(r+v)^2}$
   (c) $P^H = \overline{\overline{P}}, a = 1$ *and* $P^L = v, a = 0$ *if* $\frac{(r-v)^2}{(r+v)^2} \leq s \leq 1$.

3. *When* $\bar{d}_2 \leq d \leq \bar{\bar{d}}_2$,

   (a) $P^H = \frac{1+v+c_H}{2}, a = 1$ *and* $P^L = \frac{r+v}{2}, a = 0$ *if* $0 \leq s \leq \frac{(r-v)^2}{(r+v)^2}$

   (b) $P^H = \bar{\bar{P}}, a = 1$ *and* $P^L = v, a = 0$ *if* $\frac{(r-v)^2}{(r+v)^2} \leq s \leq 1$.

4. *When* $d \geq \bar{\bar{d}}_2$, $P^H = \frac{1+v+c_H}{2}, a = 1$ *and* $P^L = v, a = 0$ *for all* s.

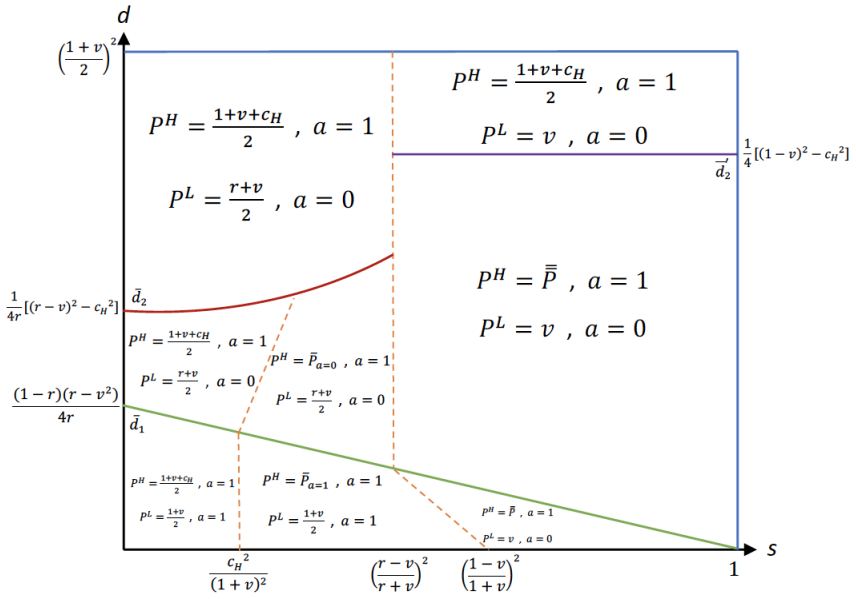

Figure 4: Separating Perfect Bayesian Equilibrium with Regulation

Pooling equilibrium exists when both types of monopolist decide to charge the same price and send the same advertising message.

Lemma 4. *No pooling equilibrium where both types of monopolist send* $a = 0$ *message satisfies the intuitive criterion.*

Proof: If $a = 0$, the monopolist faces an average demand: $Q(P) = 1 + (\frac{v-P}{r})$. Given this demand and marginal cost c_H, $P^* = \frac{r+v+c_H}{2}$ is the profit-maximizing price for the high-type monopolist which generates $\pi_H^* = (P^* - c_H)(1 + \frac{v-P^*}{r}) =$

$\frac{(r+v-c_H)^2}{4r}$. Consider any pooling equilibrium where $P^H = P^L = P^{'}$, $a = 0$ and buyers believe the monopolist to be high-type with probability r. The high- and low-type earn profits $\pi_H = (P^{'} - c_H)(1 + \frac{v-P^{'}}{r}) \leq \frac{(r+v-c_H)^2}{4r}$ and $\pi_L = (P^{'})(1 + \frac{v-P^{'}}{r})$. Suppose the sophisticated consumers believe that any firm deviating from such a strategy is a high-type monopolist. If high-type deviates to a higher price P along with $a = 1$, its profit changes to $\pi_H(P) = (P - c_H)(1 + v - P)$ as both the sophisticated and gullible consumers update their beliefs to $r = 1$. Construct a price $\hat{P}$ such that $\pi_H(P) = (P - c_H)(1 + v - P) = \frac{(r+v-c_H)^2}{4r}$ which yields $\hat{P} = \frac{1+v+c_H}{2} + \sqrt{(\frac{1+v-c_H}{2})^2 - \frac{(r+v-c_H)^2}{4r}}$. At $\hat{P}$, the low-type earns $\pi_L(\hat{P}) = (\hat{P})(1 + v - \hat{P}) < (P^{'})(1 + \frac{v-P^{'}}{r})$, by the definition of pooling equilibrium. Therefore, any price $\hat{P} + \varepsilon$ generates $\pi_H(\hat{P} + \varepsilon) > \frac{(r+v-c_H)^2}{4r}$ and $\pi_L(\hat{P} + \varepsilon) < (P^{'})(1 + \frac{v-P^{'}}{r})$. Hence, the high-type can profitably deviate to a higher price which low-type would never mimic and the intuitive criterion fails.

Lemma 5. *No pooling equilibrium where both types of monopolist send $a = 1$ message satisfies the intuitive criterion.*

Proof: If $a = 1$, the monopolist faces an average demand only from the sophisticated consumers: $Q(P) = 1 + (\frac{v-P}{r})(s + r(1 - s))$. Given this demand and marginal cost c_H, $P^* = \frac{v+c_H}{2} + \frac{r}{2(s+r(1-s))}$ is the profit-maximizing price for the high-type monopolist which generates $\pi_H^* = (P^* - c_H)(1 + \frac{v-P^*}{r}(s + r(1 - s))) = \frac{(v-c_H)^2(s+r(1-s))}{4r} + \frac{r}{4(s+r(1-s))}$. Consider any pooling equilibrium where $P^H = P^L = P^{'}$, $a = 1$ and sophisticated buyers believe the monopolist to be high-type with probability r. The high- and low-type earn profits $\pi_H = (P^{'} - c_H)(1 + \frac{v-P^{'}}{r}(s + r(1 - s))) \leq \pi_H^*$ and $\pi_L = (P^{'})(1 + \frac{v-P^{'}}{r}(s + r(1 - s)))$. Suppose the sophisticated consumers believe that any firm deviating from such a strategy is a high-type monopolist. If high-type deviates to a higher price P, its profit changes to $\pi_H(P) = (P - c_H)(1 + v - P)$ as the sophisticated consumers update their beliefs to $r = 1$. Construct a price $\hat{P}$ such that $\pi_H(P) =$

$(P - c_H)(1 + v - P) = \pi_H^*$ which yields $\hat{P} = \frac{1+v+c_H}{2} + \sqrt{(\frac{1+v-c_H}{2})^2 - \pi_H^*}$. At $\hat{P}$, the low-type earns $\pi_L(\hat{P}) = (\hat{P})(1 + v - \hat{P}) < (P^{'})(1 + \frac{v-P^{'}}{r}(s + r(1-s)))$, by the definition of pooling equilibrium. Therefore, any price $\hat{P} + \varepsilon$ generates $\pi_H(\hat{P} + \varepsilon) > \frac{(r+v-c_H)^2}{4r}$ and $\pi_L(\hat{P} + \varepsilon) < (P^{'})(1 + \frac{v-P^{'}}{r})$. Hence, the high-type can profitably deviate to a higher price which low-type would never mimic and the intuitive criterion fails.

Hence, no pooling equilibrium exists that satisfies the intuitive criterion for any s and d. This result is in contrast with the findings of the previous literature (Rhodes and Wilson, 2018; Piccolo et al, 2015, 2017). The polarity in results occurs from the fact that previous literature ruled out any possibility of signaling. Whereas in my paper, some consumers (sophisticated) always update their beliefs. Also, the pooling is required on both the price and the message, which can never be the same for both types of firms even when all consumers are gullible.

4 Social Welfare Analysis and Policy Implications

Given the above scenario, an optimal policy can have two types of goals: either to prevent false advertising/deception or to maximize overall social welfare. Under the separating equilibrium, each goal results in a different minimal required penalty. To prevent false advertising, a minimum penalty of $\bar{d_1} = \frac{(1-s)(1-r)(r-v^2)}{4r}$ is sufficient for any level of s because the low-type has an incentive to lie about its quality only below $\bar{d_1}$ threshold. Therefore, we do not need to set a high value for d to avoid deception. Moreover, as s increases, $\bar{d_1}$ falls implying that the presence of high proportion of sophisticated consumers reduces the need to have such kind of regulation. As $s$becomes very large, there is no fear of deception which implies zero penalty should suffice.

For the second type of goal, I first calculate the social welfare in the low-quality and high-quality states under the separating equilibrium, shown in *Figures 5 and 6*. The social welfare is the sum of consumer surplus and the monopolist's profit. It can be calculated by the difference in the valuation and the cost of producing of a unit of the good *times* the total demand at a price P.

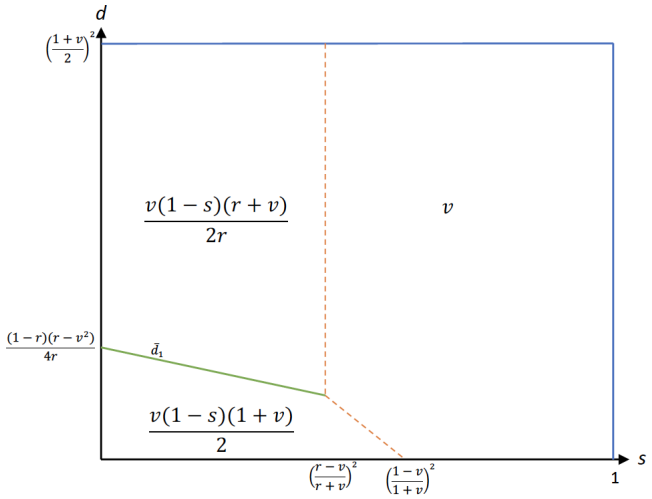

Figure 5: Social Welfare Function in Low-quality State

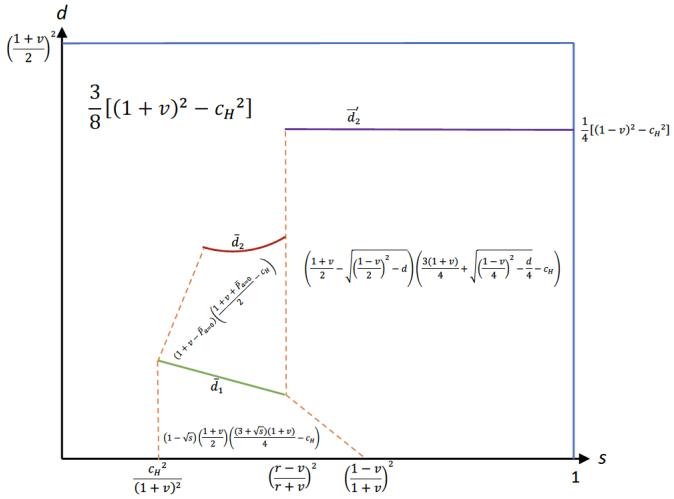

Figure 6: Social Welfare Function in High-quality State

The social welfare function in the low-quality state shows monotone behavior for small and intermediate values of s. It is decreasing as s increases because the low-type monopolist only serves the gullible consumers for the initial values of s. For high values of s, the social welfare jumps to become constant throughout because the low-type simply charges the reservation price and hence serves all the consumers in the market.

The social welfare function in the high-quality state is a decreasing function of s because higher proportion of sophisticated consumers implies high signaling price which negatively effects demand and consumer surplus. However, the social welfare function is always increasing in d. Higher penalty for false advertising means lower incentive for the high-type to signal its true quality as the high penalty is enough to restrict low-type to mimic high-type prices.

To maximize social welfare, the optimal penalty is affected by the fraction of sophisticated consumers in the market. For a low s, $\bar{d_1} = \frac{(1-s)(1-r)(r-v^2)}{4r}$ is optimal as the low-type does not lie about its quality and the underlying distortion is minimum. For an intermediate value of s, the social welfare in the low-quality state is decreasing initially as the low-type lies about its quality. Also in the high-quality state, the presence of signaling distortion negatively affects social welfare. When d increases to $\bar{d_1}$, it first corrects the low-type's price and advertising signal. As d increases further up to $\bar{d_2} = \left(\frac{1-s(1-r)}{4r}\right)\left[\left(\frac{r}{1-s(1-r)} - v\right)^2 - c_H^2\right]$, it then corrects the high-type's prices, removing entire signaling distortion thereby increasing overall welfare. This indicates the U-shaped effect of d on the social welfare function. Therefore, for an intermediate s, $\bar{d_2}$ is optimal penalty level. For a high value of s, there is no deception possible but signaling distortion is still present for all $d < \vec{d_2}$. Therefore, $\vec{d_2} = \frac{[(1-v)^2 - c_H^2]}{4}$ is optimal as the entire signaling distortion goes away, thereby increasing overall welfare. Hence, maximizing social welfare involves a lower penalty initially but this threshold increases with increasing fraction of sophisticated consumers or equivalently, decreasing fraction of gullible consumers. Generally, it would be expected for the gullible consumers to strengthen the case for higher penalty/stronger regulation. But the above result implies the opposite: that the presence of gullible consumers weakens the case for strong regulation. This occurs because the penalty is not only disincentivizing false advertising for the low-type but also removes signaling distortion created by the high-type prices. Under the case of high proportion of sophisticated consumers, there is no deception but the signaling distortion is at its peak. Note that the monopoly distortion will always be present.

Proposition 6. *The optimal penalty levels are summarized as follows:*

1. *Socially optimal level of penalty is higher than the penalty required to avoid deception.*
2. *To avoid deception, $\bar{d_1}$ is the minimum penalty level for all s.*

(a) $\bar{d}_1$ *is decreasing in* s*: sophisticated consumers reduce the possibility of deception.*

3. *To maximize social welfare, the minimum optimal penalty is increasing with* s*:*

   (a) $\bar{d}_1$ *is optimal if* $0 \leq s \leq \left(\frac{c_H}{1+v}\right)^2$

   (b) $\bar{d}_2$ *is optimal if* $\left(\frac{c_H}{1+v}\right)^2 \leq s \leq \frac{(r-v)^2}{(r+v)^2}$

   (c) $\bar{d}_2'$ *is optimal if* $\frac{(r-v)^2}{(r+v)^2} \leq s \leq 1$.

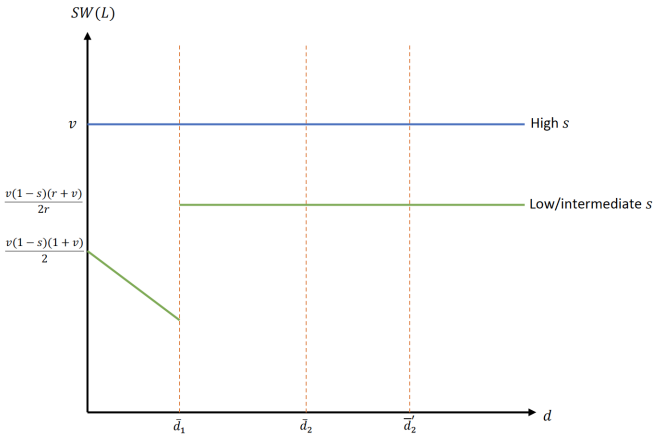

Figure 7: Social Welfare in Low-quality State as a function of d

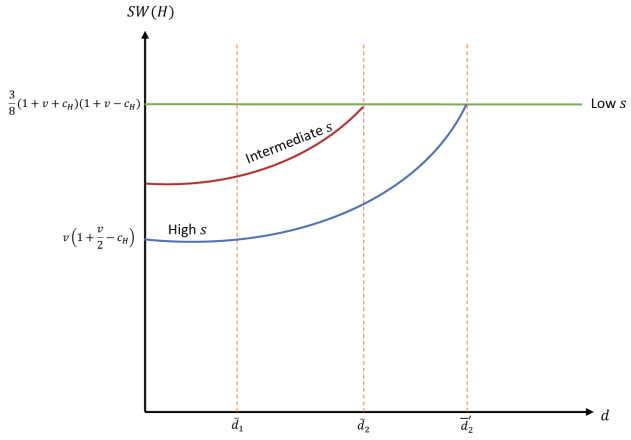

Figure 8: Social Welfare in High-quality State as a function of d

Next, I calculate the profit functions of each type of monopolist under the separating equilibrium, as shown in *Figures 9 and 10*. The profit function of the low-type monopolist is given by $\pi_L = P(1+v-P)(1-s)-d$ if it lies about its quality and $\pi_L = P(1+\frac{v-P}{r})(1-s)$ if it does not lie about its quality. Observe that for low and intermediate values of s, the low-type monopolist always lobbies against the regulation because it finds false advertising extremely profitable despite serving only the gullible consumers. For high values of s, $\pi_L = v$ and the low-type is indifferent about regulation as false advertising is not profitable with few gullible consumers present in the market. Hence, lower the s, stronger the intensity with which low-type lobbies against regulation.

On the other hand, the profit function of the high-type monopolist is given by $\pi_H = (P - c_H)(1 + v - P)$ which is increasing with higher regulation. For low values of s, the high-type is indifferent about regulation as gullible consumers only understand the advertising signal to update their beliefs. For intermediate values of s, the high-type monopolist strongly lobbies for regulation and for high values of s, high-type lobbies for even higher regulation as it gives incentive to charge the full-information monopoly price. With increasing s, the high-type has to charge a high signaling price which adversely affects its profits. A regulation as high as $\bar{d}_2$ for intermediate values of s and $\vec{d}_2$ for high values of s can prevent such a fall in high-type's profits. Hence, higher the s, stronger the intensity with which high-type lobbies for regulation.

Lemma 7. *As expected, with increasing s,*

1. *the incentive of the high-type to lobby for regulation increases.*
2. *the incentive of the low-type to lobby against the regulation falls.*

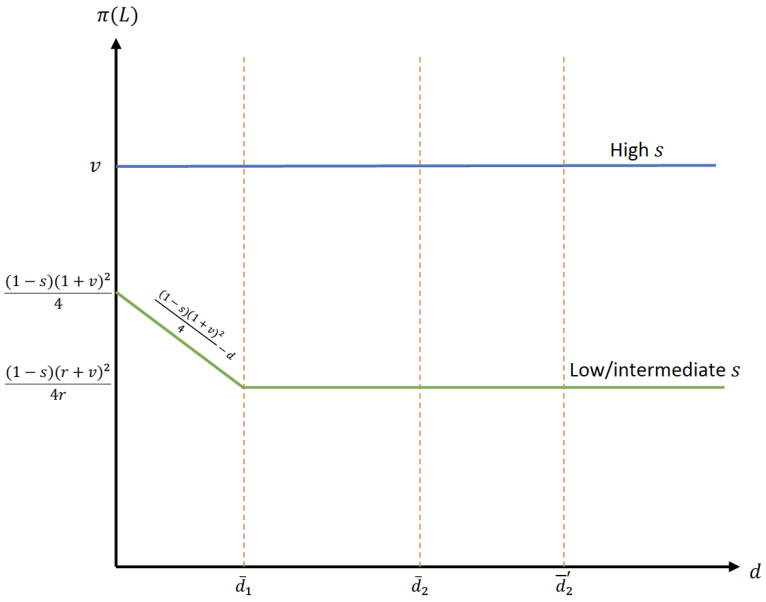

Figure 9: Profit Function of Low-type Monopolist, as a function of d

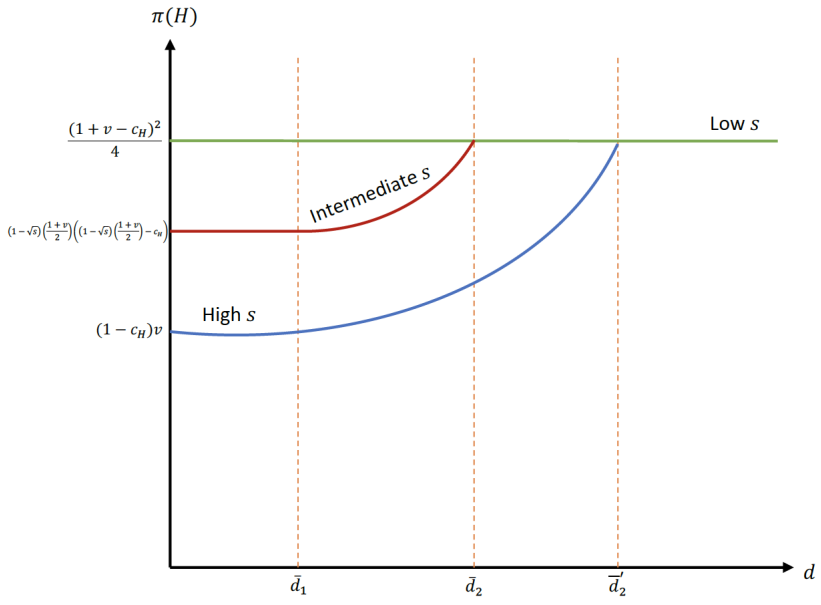

Figure 10: Profit Function of High-type Monopolist, as a function of d

5 Conclusion

False advertising needs immediate attention now, especially with the emergence of online retail which has made deceiving consumers extremely easy. Moreover, not all consumers are sophisticated enough to understand the price and advertising signals and in fact, believe everything at face value. I consider a setting where gullible consumers are, by definition, deceived by a low-type monopolist in equilibrium. The results imply that a low level of penalty is adequate to avoid false advertising practices. However, a higher level of penalty ensures full information outcome which maximizes social welfare. The main aspect of the paper that separates it from the past literature is that false advertising does not necessarily occur in a pooling equilibrium outcome. A low-type can successfully separate itself from high-type and still deceive some consumers. A future work in this area can incorporate competition in the above model to formulate optimal regulation and analyze its welfare implications.

References

[1] Adriani, F. and L. G. Deidda (2011), Competition and Signaling Role of Prices, *International Journal of Industrial Organization* 29, pp. 412-425.

[2] Aköz, K., C. Arbatli and L. Celik (2020), Manipulation Through Biased Product Reviews, *The Journal of Industrial Economics, Forthcoming.*

[3] Anderson, S. and R. Renault (2006), Advertising Content, *American Economic Review* 96, pp. 93-113.

[4] Armstrong, M. and S. Huck (2010), Behavioral Economics as Applied to Firms: A Primer, CESifo Working Paper Series No. 2937.

[5] Bagwell, K. and M. Riordan (1991), High and Declining Prices Signal Product Quality, *American Economic Review* 81, pp. 224-239.

[6] Caves, R.E. and D.P. Greene (1996), BrandsQuality Levels, Prices, and Advertising Outlays: Empirical Evidence on Signals and Information Costs, *International Journal of Industrial Organization* 14, pp. 29-52.

[7] Cho, I. K., and D. M. Kreps (1987), Signaling Games and Stable Equilibria, *Quarterly Journal of Economics* 102, pp. 179-221.

[8] Corts, K. S. (2014), Finite Optimal Penalties For False Advertising, *The Journal of Industrial Economics, 62,* pp. 661-681.

[9] Daughety, A. F., and J. F. Reinganum (1997), Everybody Out of the Pool: Product Liability, Punitive Damages and Competition, *Journal of Law, Economics and Organization* 13, pp. 410-432.

[10] Daughety, A. F., and J.F. Reinganum (2008a), Products Liability, Signaling & Dis- closure, *Journal of Institutional and Theoretical Economics*, 164, pp.106-126.

[11] Daughety, A. F., and J. F. Reinganum (2008b), Communicating Quality: A Unified Model of Signaling & Disclosure, *RAND Journal of Economics* 39, pp. 973-89.

[12] Eliaz, K. and R. Spiegler (2006), Contracting with Diversely Naive Agents, *The Review of Economic Studies* 73, pp. 689-714.

[13] Ellingsen, T. (1997), Price Signals Quality: The Case of Perfectly Inelastic Demand, *International Journal of Industrial Organization* 16, pp. 43-61.

[14] Erdem, T., M.P. Keane, and B. Sun (2008), A Dynamic Model of Brand Choice When Price and Advertising Signal Product Quality, *Marketing Science* 27, pp. 1111-1125.

[15] Grubb, M. (2009), Selling to Overconfident Consumers, *American Economic Review* 99, pp. 1770-1807.

[16] Grubb, M. (2015), Behavioral Consumers in Industrial Organization: An Overview, *Review of Industrial Organization* 47, pp. 247-258.

[17] Hattori, K. and K. Higashida (2012), Misleading Advertising in Duopoly, *Canadian Journal of Economics* 45, pp. 1154-1187.

[18] Janssen, M. and S. Roy (2010), Signaling Quality through Prices in an Oligopoly, *Games and Economic Behavior* 68, pp.192-207.

[19] Janssen, M. and S. Roy (2015), Competition, Disclosure and Signaling, *Economic Journal* 125, pp. 86-114.

[20] Janssen, M. and S. Roy (2019), Regulating Product Communication, http://faculty.smu.edu/sroy/JRpaper2019.pdf

[21] Kartik, N. (2009), Strategic Communication with Lying Costs, *Review of Economic Studies* 76, pp. 1359-95.

[22] Milgrom, P. and J. Roberts (1986), Price and Advertising Signals of Product Quality, *Journal of Political Economy* 94, pp. 796-821.

[23] Piccolo, S., G. Ursino, and P. Tedeschi (2015), How Limiting Deceptive Practices Harms Consumers, *RAND Journal of Economics* 46, pp. 611-624.

[24] Piccolo, S., G. Ursino, and P. Tedeschi (2017), Deceptive Advertising with Rational Buyers, *Management Science* 64, pp. 1291-1310.

[25] Rao, A.R. and K.B. Monroe (1989), The Effect of Price, Brand Name, and Store Name on Buyers Perceptions of Product Quality: An Integrative Review, *Journal of Marketing Research* 26, pp. 351-357.

[26] Rao, A and E. Wang (2017), Demand for "Healthy" Products: False Claims and FTC Regulation, *Journal of Marketing Research* 54, pp. 968-989.

[27] Rhodes, A., and C. Wilson (2018), False Advertising, *Rand Journal of Economics* 49, pp. 348-369.