Trading Up: Dynamic Pricing With Multiple Varieties*

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Abstract

We study dynamic monopoly pricing with two varieties. We show that (i) the seller cannot do better than set static monopoly prices, (ii) a commitment problem arises if the seller can 'trade up' consumers to more valuable options, (iii) without commitment, the seller engages in price discrimination within and across periods by lowering prices to trade up consumers. Our analysis allows for varieties to be durable or perishable and to be sold or rented.

Keywords: TBD
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1 Introduction

A large literature has developed that studies the sale of durable goods over time without capacity constraints. Following influential work by Stokey (1979), this literature has demonstrated the importance of a sellers commitment ability in dynamic pricing problems. Contrary to what intuition might suggest, it is profit-maximizing for a seller that can restock inventory to charge the same price at all times. Yet, without commitment ability, the seller cannot stop herself from lowering the price to induce previous non-buyers to buy the good and engage in dynamic pricing. In its most extreme form when buyers are infinitely patient, this commitment problem can lead a seller to obtain zero profits (Coase, 1972).

In recent work, Nava and Schiraldi (2019) have demonstrated that the zero profit insight of the 'Coase conjecture' does not extend to the case of a monopoly seller of a durable good with multiple varieties. Instead, the non-commitment equilibrium is characterized by a market-clearing condition. That is, optimal prices must ensure that all buyers above marginal cost choose to purchase one of the two varieties. This condition necessarily implies that no positive profit can be obtained if only one variety is on offer.

In this paper, we build on the framework developed in Nava and Schiraldi (2019) and analyze dynamic pricing of a monopoly with two varieties when the varieties may be durable or perishable and can be sold or rented. We demonstrate that the driving force behind the commitment problems of a monopoly is what we term 'trading up'. The possibility to offer an option valued more by a given buyer than the previously chosen one causes the seller to engage in dynamic pricing to induce the buyer to change to the higher-valued option. In doing so, the seller can extract a higher rent from the transaction, since the value of the option to the buyer is greater. However, consumers are only willing to switch to the higher-valued option, if they obtain more rent in turn from doing so. Hence, trading up buyers induces some consumers to make strategic consumption decisions, which causes profits with varying prices to be below profits with a constant price over time.

We derive the following three results. First, we show that the highest profit attainable by the monopolist requires the dynamic cutoffs to be equal to the static profit-maximizing cutoffs. This can be implemented using constant static monopoly profit-maximizing prices. Hence, the seller cannot do better than set constant static monopoly prices. Second, we show that a commitment problem arises when trading up opportunities exist at any point in the dynamic setting, since the seller cannot resist the temptation to trade up consumers without commitment and doing so implies that the commitment solution can no longer be reached. Third, we show that prices are discriminated across periods and across consumer segments within a given period, when the seller engages in trading up.

Our work in this paper provides a simple way to check for a commitment problem in a particular application of dynamic pricing. It is sufficient to consider whether (i) the static optimal solution contains at least one segment of consumers, in which at least some individuals do not select the consumption option for which they have the highest valuation, and (ii) if the setting allows for consumption paths (i.e. sequences of consumption decisions) in which consumers switch from the option associated with the static segment to the higher valued one. If these two conditions are satisfied, a monopoly seller has a commitment problem.

For example, in the classic sale of a single durable good, there exist two consumption options: the good and the outside option. By construction, consumers that choose the outside option have a higher valuation for the good and can switch from non-consumption in previous rounds to consumption at a later stage. Hence, the monopolist has a commitment problem. Similarly, with two varieties of a durable good on sale as in Nava and Schiraldi (2019), individuals can be traded up from the outside option and the monopolist has a commitment problem. However, if the static solution no longer contains individuals in the sole option from which they could switch to a higher-valued one, the commitment problem disappears. This is the case in Board and Pycia (2014). Lastly, in a setting with a single durable good being rented or a perishable good being sold and exit being absorbing as in Tirole (2016), there is no commitment problem since buyers may not switch from the lower-valued option of exit to purchase by assumption.

We contribute to a large literature on the sale of a durable good over time (e.g Coase, 1972; Gul et al., 1986; Sobel, 1991) and of multiple varieties of a durable good (e.g. Nava and Schiraldi, 2019; Board and Pycia, 2014). Our work differs by allowing for the renting of a durable or sale of a perishable good. In doing so, we also contribute to the literature on behavior-based pricing (e.g. Acquisti and Varian, 2005; Armstrong, 2006; Fudenberg and Villas-Boas, 2007; Taylor, 2004). In contrast to recent work in Rochet and Thanassoulis (2019), we do not allow varieties to be sold as a bundle. More precisely, in our setting, the two goods can never be bought both *at the same time*, irrespective of a bundling price. Hence, our setup may be understood as offering two varieties of the same good over time.

2 General Setup

Consider a measure of non-atomic consumers with unit-demand who make a discrete choice between two varieties of a good, a and b, and an outside option, o. Consumers are characterized by a fixed value profile $v = (v_a, v_b, v_o)$, where v_i denotes the value for the consumption option $i \in \{a, b, o\}$. The value of the outside option v_o is normalized to zero across all types and common knowledge. Value profiles for a and b are private information. A measure $\mathcal{F}$ defined on the unit square $[\underline{v}, \bar{v}]^2 = [0,1] \times [0,1]$ describes the distribution of value profiles for varieties a and b among buyers. The measure $\mathcal{F}$ is common knowledge. F denotes the associated cumulative distribution and V the support. We place the following assumptions on the distribution functions and measure.

Assumption 1 (distribution) *(i) The support V is convex.*

(ii) $\mathcal{F}$ is absolutely continuous on $\underline{v}, \bar{v}$.

(iii) The density distribution f is bounded and strictly positive, such that

$0 < \underline{f} \leq f(v_a, v_b) \leq f$ for any $v \in V$.

Denote the consumption decision that a given consumer makes by d, where $d \in \{(1,0,0), (0,1,0), (0,0,1)\}$. That is, when a consumer chooses to purchase variety a, he/she chooses $d = (1,0,0)$. Call this consumption decision d_a. Similarly $d_b = (0,1,0)$ and $d_o = (0,0,1)$. The game is played repeatedly over T periods. In each period $t = 0, ..., T$, consumers make a consumption decision. Over the course of the game, the sequence of decisions a given consumer makes describes a *consumption path*. Denote the consumption paths available to a given consumer at time $t = 0, ..., T$ and that can therefore be followed from t onward by $\mathbf{d}_k^t \in \{\mathbf{d}_0^t, ..., \mathbf{d}_K^t\}$, where each path specifies a consumption decision d_k^τ for each period $\tau = t, ..., T$ along path k such that $\mathbf{d}_k^t = (d_k^t, d_k^{t+1}, ..., d_k^T)$ and $d_k^t \in \{d_a, d_b, d_o\}$. The set of consumption paths available for a given buyer at time t is given by $\mathbf{D}_K^t$. Similarly, for a given consumer that followed path $\mathbf{d}_k^0$ from the first period up until period t, the sequence (or history) of consumption decisions from the first period until time t is denoted by $\mathbf{d}_k^{-t}$. For convenience, we will henceforth suppress the time index on consumption paths and the set of paths when only considering those that start at the beginning of the game at $t = 0$.

Our setup allows for any one of the three consumption options a, b, o to be *absorbing*. That is, once a consumer chooses an absorbing option, in every future period he/she must continue to choose this option. For an option $i \in \{a, b, o\}$ denote the consumption path that always contains only decision d_i in every period from t onward by $\mathbf{d}_i^t = (d_i^t, d_i^{t+1}, ..., d_i^T)$.

Definition 1 (absorbing state) *An option $i \in (a, b, o)$ is absorbing, if for any buyer v that makes consumption decision d_i at any time $t-1$, $\mathbf{D}_K^t = \mathbf{d}_i^t$.*

When one of the two varieties a and b is absorbing, said variety is a *durable* good being sold. Similarly, if a given variety is non-absorbing, it is a durable good being rented for a period or *perishable* good. Lastly, if the outside option is absorbing, we are in a setting with *absorbing exit* as considered in Tirole (2016). Note that by construction, requiring one (or more) of the options to be absorbing restricts the set of consumption paths available to buyers over the course of the game. In our analysis, we consider the general case of none of the three states being absorbing, implying that consumers may transition out of any state and into any of the other two at any time in the game. Staying in the same state is always possible. This general framework is depicted in Figure 1.

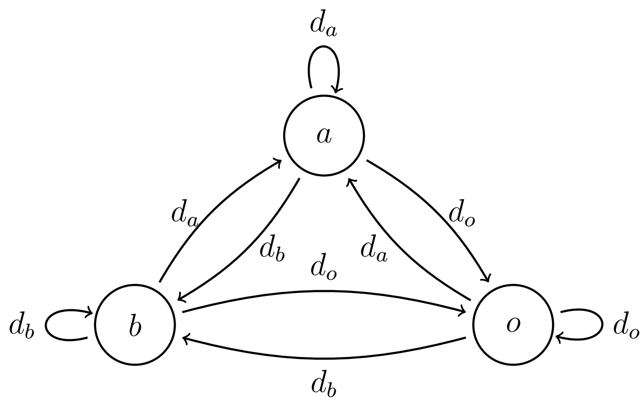

Figure 1: No absorbing states

The payoff to a buyer choosing option i in period t at the associated price p_i^t is $\delta^t(v_i - p_i^t)$, where δ denotes the discount factor. Hence, an individual's total payoff is the discounted stream of per-period payoffs obtained over the course of the game. The price for the outside option is always equal to zero for all types across all periods, $p_o^t = p_o = 0$. Denote the set of prices on path k from t onward by $\mathbf{p}_k^t = (p_k^t, p_k^{t+1}, ..., p_k^T)$ where $\mathbf{p}_k^t \in (\mathbf{p}_0^t, ..., \mathbf{p}_K^t)$ and $p_k^t = (p_{a,k}^t, p_{b,k}^t, p_o)$ and the set of all prices a buyer faces from t onward for all possible paths K and all periods $\tau = t, ..., T$ by $\mathbf{P}_K^t$. Then we may write the discounted per-period payoff to a buyer of type v at time t on path k as $\delta^t(v - p_k^t)d_k^t$.

It follows that from the perspective of a consumer with type v, the utility of a consumption path $\mathbf{d}_k^t$ from time t onward is given by

$$U_t(v, \mathbf{d}_k^t) = \sum_{\tau=t}^{T} \delta^\tau (v - p_k^\tau) d_k^\tau \equiv \mathbf{v}_{k,t} - \mathbf{p}_{k,t},$$

where with a slight abuse of notation $\mathbf{v}_{k,t}$ and $\mathbf{p}_{k,t}$ denote the discounted stream of valuations and prices respectively along path k from time t on. As before, for convenience we will henceforth omit the time index on $\mathbf{v}_{k,t}$ and $\mathbf{p}_{k,t}$ when the consumption path under consideration begins at $t = 0$.

Then, the demand for variety a in period t for a set of buyers denoted by S and facing the same set of consumption paths $\mathbf{D}_K^t$ and associated prices $\mathbf{P}_K^t$ is given by

$$\begin{aligned}\mathcal{D}_a^t(\mathbf{P}_K^t|S) = \mathcal{F}\big((v-p_a^t)d_a + \max\{U_{t+1}(v,\mathbf{D}_{k,a}^{t+1})\}\\ \geq \max\{(v-p_b^t)d_b + \max\{U_{t+1}(v,\mathbf{D}_{k,b}^{t+1})\},\\ (v-p_o^t)d_o + \max\{U_{t+1}(v,\mathbf{D}_{k,o}^{t+1})\}\}|S\big)\\ \equiv \mathcal{F}(\mathbf{P}_K^t, A|S),\end{aligned}$$

where depending on the choice in period t three different sets of consumption paths are available in period $t+1$, $\mathbf{D}_K^{t+1} = \mathbf{D}_{K,a}^{t+1} \cup \mathbf{D}_{K,b}^{t+1} \cup \mathbf{D}_{K,o}^{t+1}$. Similarly, the demand for variety b is

$$\begin{aligned}\mathcal{D}_b^t(\mathbf{P}_K^t|S) = \mathcal{F}\big((v-p_b^t)d_b + \max\{U_{t+1}(v,\mathbf{D}_{k,b}^{t+1})\}\\ \geq \max\{(v-p_a^t)d_a + \max\{U_{t+1}(v,\mathbf{D}_{k,a}^{t+1})\},\\ (v-p_o^t)d_o + \max\{U_{t+1}(v,\mathbf{D}_{k,o}^{t+1})\}\}|S\big)\\ \equiv \mathcal{F}(\mathbf{P}_K^t, B|S),\end{aligned}$$

and the residual demand of non-buyers is

$$\begin{aligned}\mathcal{D}_o^t(\mathbf{P}_K^t|S) = \mathcal{F}\big((v-p_o^t)d_o + \max\{U_{t+1}(v,\mathbf{D}_{k,o}^{t+1})\}\\ \geq \max\{(v-p_b^t)d_b + \max\{U_{t+1}(v,\mathbf{D}_{k,b}^{t+1})\},\\ (v-p_a^t)d_a + \max\{U_{t+1}(v,\mathbf{D}_{k,a}^{t+1})\}\}|S\big)\\ \equiv \mathcal{F}(\mathbf{P}_K^t, O|S).\end{aligned}$$

3 Applications

Coasian dynamics: In this setting there is only one variety, say a, and the outside option is normalized to 0. Furthermore a is an absorbing state (a durable good being sold), as in e.g. Stokey (1979); Coase (1972); Gul et al. (1986); Sobel (1991).

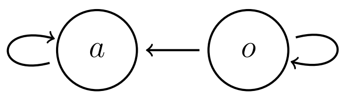

Figure 2: One durable good

Two absorbing varieties: In this setting, there are two varieties which are both absorbing states (durable goods being sold) and the outside option is normalized to zero, as in Nava and Schiraldi (2019); Board and Pycia (2014).

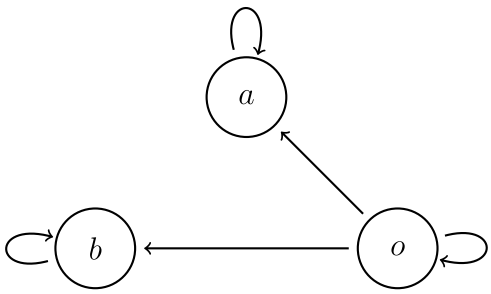

Figure 3: Two durable goods

Positive Selection: In this setting, there is one durable good being rented or perishable good being sold, say a, the outside option is normalized to zero and exit (i.e. the outside option) is absorbing. This is the setting of Tirole (2016).

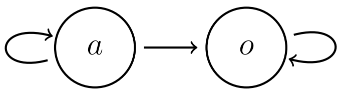

Figure 4: One rental good with absorbing exit

Behavior-based pricing: In this setting, there is one variety that is non-absorbing and the outside option is normalized to zero, as in (e.g. Acquisti and Varian, 2005; Armstrong, 2006; Conitzer et al., 2012; Taylor, 2004; Fudenberg and Villas-Boas, 2007).

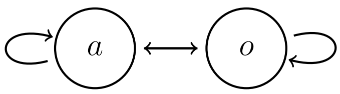

Figure 5: One rental good

4 Static Illustration

We now pre-empt the logic of the dynamic analysis in the static version of our setting. Suppose there is only one period and denote the price vector by $\mathbf{p} = \{p_o, p_a, p_b\}$.Then the demand for variety i simplifies to

$$\mathcal{D}_i(\mathbf{p}) = \mathcal{F}(v_i - p_i > \max\{v_j - p_j, v_o - p_o\}), \;\; i, j = a, b, i \neq j.$$

The total revenue arising from all consumers across the three consumption options is given by

$$\begin{aligned} R(\mathbf{p}) &= p_a \mathcal{D}_a(\mathbf{p}) + p_b \mathcal{D}_b(\mathbf{p}) + p_o \mathcal{D}(\mathbf{p}) \\ &= p_a \mathcal{F}(v_a - p_a > \max\{v_b - p_b, v_o - p_o\}) \\ &\quad + p_b \mathcal{F}(v_b - p_b > \max\{v_a - p_a, v_o - p_o\}). \end{aligned}$$

Figure 6 illustrates the resulting market segmentation when prices are such that all three segments are non-empty.

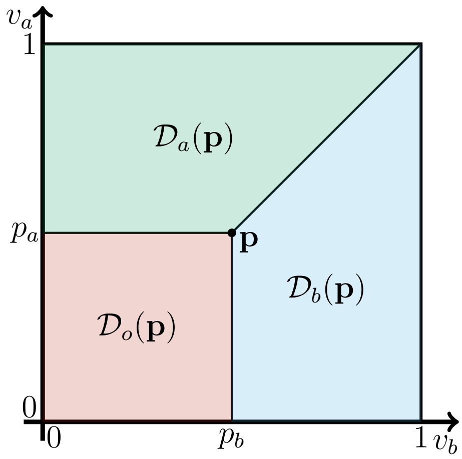

Figure 6: Static Illustration

To anticipate the logic of the dynamic analysis below, we proceed to rewrite the revenue $R(\mathbf{p})$ using the fact that the consumption options can be ordered by their price. Since this ordering is invariant across consumer types, we can express the static total revenue as the sum of the 'baseline' payment for the less expensive product made by all active buyers, plus the 'price premium' paid by buyers of the more expensive product. Without loss of generality, suppose that prices are ordered such that $p_o \leq p_a \leq p_b$. Then, total revenue may be written as consisting of the 'baseline' price v_a^* for product a paid by all active buyers, plus the 'price premium' $(v_b^{**} - v_a^{**})$ paid by buyers of product b. This alternative way of expressing revenue is formalized in Lemma 1.

Lemma 1 (static revenue) *Total revenue* $R(\mathbf{p})$ *arising in the static setting may* *be written as*

$$\begin{aligned}R(\mathbf{p}) &= v_o \times [\mathcal{F}(\mathbf{p},0)+\mathcal{F}(\mathbf{p},A)+\mathcal{F}(\mathbf{p},B)]\\&\quad + (v_a^* - v_o)\times[\mathcal{F}(\mathbf{p},A)+\mathcal{F}(\mathbf{p},B)]\\&\quad + (v_b^{**}-v_a^{**})\times\mathcal{F}(\mathbf{p},B)\\&= v_a^*\times[\mathcal{F}(\mathbf{p},A)+\mathcal{F}(\mathbf{p},B)]\\&\quad + (v_b^{**}-v_a^{**})\times\mathcal{F}(\mathbf{p},B),\end{aligned}$$

where v_a^*, v_a^{**} and v_b^{**} denote the valuations of the indifferent types.

Proof. The set of prices $\mathbf{p}$ induces the following indifference conditions among the consumption options

$$\begin{aligned}v_a^* - p_a &= v_o - p_o,\\v_b^{**} - p_b &= v_a^{**} - p_a,\\v_b^{***} - p_b &= v_o - p_o.\end{aligned}$$

Solving the indifference conditions for prices, we find $p_a = p_o + (v_a^* - v_o)$ and $p_b = p_a + (v_b^{**} - v_a^{**}) = p_o + (v_a^* - v_o) + (v_b^{**} - v_a^{**}) = p_o + (v_b^{***} - v_o)$. Since the value and the price of the outside option satisfy $v_o = p_o = 0$, we immediately have that $p_a = v_a^*$ and $p_b = v_a^* + (v_b^{**} - v_a^{**})$. ■

Lemma 1 shows that the total revenue $R(\mathbf{p})$ arising in the game can be rewritten in a way that will turn out to be convenient for the analysis of the dynamic setting. Note that Lemma 1 does not depend on the ordering of prices p_a and p_b. For $p_b > p_a$, the revenue as written in Lemma 1 can be interpreted as the 'baseline' payment of the cheaper product a for all buyers, and the 'price premium' for buyers of the more expensive product b. For $p_a > p_b$, Lemma 1 continues to apply, however one may now think of the 'baseline' payment as the payment for the more expensive product a, from which the 'price reduction' for the cheaper product b is subtracted.[1] For later reference, let $\mathbf{p}^m = (p_a^m, p_b^m)$ denote the profile of optimal revenue-maximizing prices that satisfy $\mathbf{p}^m \in \arg\max_{\mathbf{p}} R(\mathbf{p})$, and let $\mathbf{v}^m = (v_1^m, v_2^m, v_3^m)$ be the profile of optimal static cutoffs associated with revenue-maximizing prices.

[1] In this case, we may instead choose to write revenue using $p_b = v_b^{***}$ and $p_a = p_b + (v_b^{**} - v_a^{**})$ to retain the intuitive form of revenue where part of all buyers pay a 'price premium.'

5 Consumer Demand

Lemma 2 (skimming) *Consider two different consumption paths at time τ of $\mathbf{d}_k^\tau$ and $\mathbf{d}_l^\tau$, with $\mathbf{d}_k^\tau \neq \mathbf{d}_l^\tau$. If a buyer with type v prefers $\mathbf{d}_k^t$ to $\mathbf{d}_l^t$, $U_\tau(v, \mathbf{d}_k^\tau) \geq U_\tau(v, \mathbf{d}_l^\tau)$, then so does any buyer with type $\tilde{v} \neq v$, such that*

$$(\tilde{v} - v) \sum_{t=\tau}^{T} (d_k^\tau - d_l^\tau)\delta^\tau = \tilde{\mathbf{v}}_{k,\tau} - \mathbf{v}_{l,\tau} - (\tilde{\mathbf{v}}_{l,\tau} - \mathbf{v}_{l,\tau}) \geq 0. \quad (1)$$

Proof. If a buyer with type v prefers consumption path $\mathbf{d}_k^\tau$ to consumption path $\mathbf{d}_l^\tau$, $k \neq l$, from time τ onward, we must have

$$\sum_{\tau=t}^{T} \delta^\tau (v - p_k^\tau) d_k^\tau \geq \sum_{\tau=t}^{T} \delta^\tau (v - p_l^\tau) d_l^\tau.$$

Now consider some type $\tilde{v} \neq v$. Then we have

$$\begin{aligned} \sum_{\tau=t}^{T} \delta^\tau (\tilde{v} - p_k^\tau) d_k^\tau &= \sum_{\tau=t}^{T} \delta^\tau (v - p_k^\tau) d_k^\tau + (\tilde{v} - v) \sum_{\tau=t}^{T} d_k^\tau \\ &\geq \sum_{\tau=t}^{T} \delta^\tau (v - p_l^\tau) d_l^\tau + (\tilde{v} - v) \sum_{\tau=t}^{T} d_k^\tau \end{aligned}$$

since we have assumed that type v prefers path d_k to d_l. For type $\tilde{v}$ to have the same preferences then requires that

$$\sum_{\tau=t}^{T} \delta^\tau (v - p_l^\tau) d_l^\tau + (\tilde{v} - v) \sum_{\tau=t}^{T} d_k^\tau \geq \sum_{\tau=t}^{T} \delta^\tau (\tilde{v} - p_l^\tau) d_l^\tau,$$

which can be rearranged to yield (1). ■

Condition (1) in Lemma 2 shows that classic forms of skimming are not guaranteed to hold in the general setting we consider. It is not sufficient for type $\tilde{v}$ to have strictly higher valuations than type v in order to have the same preferences. Instead, we find that whether skimming holds for some higher type depends on the consumption paths available to buyers, as it is the extent of the differences in consumption decisions on two different paths that determines if skimming applies. Intuitively, the more the set of admissible consumption paths is restricted, the easier it is to satisfy the skimming condition.

It should not come as a surprise then, that if one (or more) of the three available options is assumed to be an absorbing state, the skimming condition becomes easier to satisfy. Indeed, in the most restrictive setting with a single absorbing variety, condition (1) in Lemma 2 will always be satisfied. To see this, first note that with a single absorbing variety, say a, where v is the valuation of type v for variety a, and denoting by $\mathbf{d}_k^\tau$ the path that includes consumption at $\tau = t$, we must have that $d_k^\tau = d_a$ for any $\tau = t, ..., T$ since a is absorbing. Second, if we normalize the value of the outside option to zero, it immediately follows that if $d_l^\tau \neq d_a$ for at least one of $\tau = t, ..., T$, then at any such time that the two paths contain diverging consumption choices we have $(\tilde{v} - v)(d_a - d_o) = \tilde{v} \geq 0$. Then it follows that $(\tilde{v} - v)(\mathbf{d}_k^\tau - \mathbf{d}_l^\tau) = (\tilde{v} - v)(\mathbf{d}_a - \mathbf{d}_l^\tau) > 0$ for any $\tilde{v} > v$ if $v_o = 0$ and hence the skimming condition is always satisfied for a higher type. This is the familiar skimming result established in the literature for a single durable good (for details see e.g. Fudenberg, Levine, Tirole 1984).

However, this result does not carry over to a setting with two absorbing varieties. While the set of consumption paths available to consumers remains restricted, the skimming condition may not always hold for a higher type, since there are now two different divergences in consumption choices possible. Consider path $\mathbf{d}_k^\tau$ to contain consumption of variety i at time τ where $i \in \{a, b\}$. Then, $\mathbf{d}_k^\tau = \mathbf{d}_i^\tau$. As before, when $v_o = 0$, we can see that if path $\mathbf{d}_l^\tau$ diverges from path $\mathbf{d}_k^\tau$ by containing some d_o, the skimming condition (1) is satisfied for any type $\tilde{v}_i > v_i$. However, if instead the paths diverge because path $\mathbf{d}_l^\tau$ contains some d_j, where $j \neq i \in \{a, b\}$, we have at the period of divergence $(\tilde{v} - v)(d_i - d_j) = \tilde{v}_i - v_i - (\tilde{v}_j - v_j)$ which is weakly positive only if $\tilde{v}_i - v_i \geq \tilde{v}_j - v_j$. In sum, we find that in a setting with two absorbing varieties, skimming applies to any type that satisfies $\tilde{v}_i - v_i \geq \max\{0, \tilde{v}_j - v_j\}$. This is the skimming condition established in Nava & Schiraldi (2019) for two durable goods.

In fact, we can see that the skimming condition will always be satisfied for a higher type in any setting that features only two states of which one is absorbing and one is not. The case of durable goods above has established this result if the non-absorbing state is the outside option. Now consider the reverse. Suppose there is one variety that is non-absorbing and an absorbing outside option. Consider path $\mathbf{d}_l^\tau$ to contain choosing the outside option at time τ. Then, $\mathbf{d}_l^\tau = \mathbf{d}_o^\tau$. Path $\mathbf{d}_k^\tau$ must diverge at least once by choosing some d_a (supposing that a is the variety on offer). Then we immediately have at that period of divergence $(\tilde{v} - v)(d_a - d_o) = \tilde{v} \geq 0$ and hence the skimming condition strictly applies to any type that satisfies $\tilde{v} > v$.

We now show that demand segments in the dynamic game and the total revenue that arises from a given set of consumers can be reorganized and expressed using the same logic as in Lemma 1 in the static version of our setting.

Lemma 3 (segmentation) *For any set of consumers $S \subseteq \mathcal{F}$ at any time $t \in$*

$0, ..., T$ *that are facing the same set of consumption paths* $\mathbf{D}^t_{K,S}$ *and associated prices* $\mathbf{P}^t_{K,S}$*, the segments consumers sort themselves into and the total revenue* $R_S(\mathbf{P}^t_{K,S})$ *arising from consumers in set* S *can be written as*

$$\begin{aligned} R_S(\mathbf{P}^t_{K,S}) &= \sum_{t=\tau}^{T} \delta^{\tau-t} p_0^\tau \times \mathcal{F}\left(\sum_{k=0}^{K_S} \mathbf{d}_k^\tau\right) \\ &\quad + \sum_{k=1}^{K_S} v^k \sum_{t=\tau}^{T} \delta^{\tau-t}(d_k^\tau - d_{k-1}^\tau) \times \mathcal{F}\left(\sum_{k}^{K_S} \mathbf{d}_k^\tau\right) \\ &= \mathbf{p}_{0,t} \times \mathcal{F}\left(\sum_{k=0}^{K_S} \mathbf{d}_k^t\right) + \sum_{k=1}^{K_S} (\mathbf{v}^k_{k,t} - \mathbf{v}^k_{k-1,t}) \times \mathcal{F}\left(\sum_{k}^{K_S} \mathbf{d}_k^t\right), \end{aligned} \tag{2}$$

where $\mathbf{D}^t_{K,S}$ *has been sorted by the discounted stream of prices, such that* $\mathbf{p}_{0,t}$ *is the lowest discounted stream of prices from* t *onward and* $\delta^{\tau-t}p_0^\tau d_0^\tau$ *its element at time* τ*, and* $\mathcal{F}(\mathbf{d}_k^t)$ *denotes the measure of all consumers on path* $\mathbf{d}_k^t$ *in set* S*, and* v_k^k *the set of types indifferent between consumption path* $\mathbf{d}_k^t$ *and* $\mathbf{d}_{k-1}^t$*.*

Proof. Consider some set of consumers $S \subseteq \mathcal{F}$ at time $\tau \in t, ...T$ facing the same set of consumption paths $\mathbf{D}^\tau_{K,S}$ and associated prices $\mathbf{P}^\tau_{K,S}$ available to them. Sort the set of available consumption paths according to their discounted stream of prices from t onward $\mathbf{p}_{k,t}$, such that $\mathbf{p}_{0,t} \leq \mathbf{p}_{1,t} \leq ... \leq \mathbf{p}_{K_S,t}$, where $\mathbf{p}_{k,t} = \sum_{t=\tau}^{T} \delta^{\tau-t} p_k^\tau d_k^\tau$. Let v^* denote the type who is indifferent between the lowest consumption path $\mathbf{d}_0^t$ and the next-higher consumption path $\mathbf{d}_1^t$. By construction, this indifferent type must satisfy

$$\mathbf{v}^*_{1,t} - \mathbf{p}_{1,t} = \mathbf{v}^*_{0,t} - \mathbf{p}_{0,t}$$

or

$$\mathbf{p}_{1,t} = \mathbf{p}_{0,t} + (\mathbf{v}^*_{1,t} - \mathbf{v}^*_{0,t}),$$

where the asterisk indicates the indifferent type's discounted stream of values along consumption path 0 and 1, respectively. Similarly, let v^{**} denote the type who is indifferent between $\mathbf{d}_1^t$ and $\mathbf{d}_2^t$. For this type we have that $\mathbf{p}_{2,t} = \mathbf{p}_{1,t} + (\mathbf{v}^{**}_{2,t} - \mathbf{v}^{**}_{1,t})$. It follows that

$$\mathbf{p}_{2,t} = \mathbf{p}_{0,t} + (\mathbf{v}^*_{1,t} - \mathbf{v}^*_{0,t}) + (\mathbf{v}^{**}_{2,t} - \mathbf{v}^{**}_{1,t}).$$

Iterating this procedure for all consumption paths $k = 0, ..., K_S$ yields

$$\mathbf{p}_{K_S,t} = \mathbf{p}_{0,t} + \sum_{k=1}^{K_S} (\mathbf{v}^k_{k,t} - \mathbf{v}^k_{k-1,t}),$$

where the subscript denotes the consumption path and the superscript indexes the indifferent type. The discounted stream of payments along consumption path $\mathbf{d}^t_{K_S}$ can thus be expressed as the stream of baseline payments $\mathbf{p}^t_0$ along consumption path $\mathbf{d}^t_0$ plus the premia paid by indifferent types for each higher consumption path $1, 2, ..., K_S$.

Note that this result allows for the possibility that, along consumption path $\mathbf{d}^t_k$, some consumers may be indifferent to some consumption path $\mathbf{d}^t_{k-l}$, with $l > 1$, rather than $\mathbf{d}^t_{k-1}$. For such consumers, we have

$$\mathbf{p}_{k,t} = \mathbf{p}_{k-l,t} + (\mathbf{v}'_k - \mathbf{v}'_{k-l}).$$

by construction, where v' denotes the indifferent type. For the remaining measure of consumers along $\mathbf{d}^t_k$, we instead have

$$\mathbf{p}_{k,t} = \mathbf{p}_{k-l,t} + \sum_{i=k-l}^{k} (\mathbf{v}^i_{i+1} - \mathbf{v}^i_i),$$

where v^i denotes the type that is indifferent between $\mathbf{d}^t_{i+1}$ and $\mathbf{d}^t_i$. Hence,

$$(\mathbf{v}'_k - \mathbf{v}'_{k-l}) = \sum_{i=k-l}^{k} (\mathbf{v}^i_{i+1} - \mathbf{v}^i_i),$$

such that we can write the payments along $\mathbf{d}^t_k$ as claimed before. Then it follows that we may write the total revenue arising in set S from t onward $R_S(\mathbf{P}^t_{K,S})$ as in equation 2. ■

Lemma 3 shows that the rewriting of the revenue arising from a given set of consumers carries over from the static setting to the dynamic game. The intuition behind the result is simple. Instead of writing the total revenue as a stream of prices discounted and multiplied with the appropriate segment, we can express it as a stream of price differences. That is, every consumer in the set S pays the lowest discounted stream of prices of all available consumption paths plus each price difference to the next more expensive path, until we reach the path he/she actually follows. The price differences can then be replaced by the valuation differences of the corresponding indifferent types, as done in Lemma 3. The benefit of doing so, is that it allows us to express the segments buyers sort themselves into corresponding to each price/valuation difference (or the lowest stream of prices) as all consumers that lie above the indifferent type in the entire set S. That is, the measure of consumers for each term in equation (2) is solely defined by being the entire subset of S that begins at the cutoff of the indifferent type that corresponds to the price/valuation difference paid by this segment. This turns out to be very convenient for the remaining analysis.

This alternative way of expressing the total revenue that can arise highlights the driving force of pricing in our general setting. It demonstrates that the rent that can be obtained in the market depends on the discounted stream of valuations of the respective indifferent types. Therefore, a consumption path that includes choosing a consumption option for which a given set of consumers has a higher valuation more often than some other path will increase the overall rent in the market. This may not seem very surprising, but it turns out to be the sole logic driving dynamic pricing in our setting. We formalize our notion of increasing the overall rent by 'trading up' consumers in the following definition.

Definition 2 (trading up) *Consider a type-v buyer who followed consumption path $\mathbf{d}_k$ until period $t-1$. There is a 'trading up opportunity' for this buyer if there exists another consumption path $\mathbf{d}_l^t$, $l \neq k$ with $\mathbf{v}_{l,t} > \mathbf{v}_{k,t}$. The seller 'trades up' this buyer in period t if prices in period t are chosen such that the buyer switches from path $\mathbf{d}_k^t$ to path $\mathbf{d}_l^t$.*

The notion of trading up captures the idea that a seller may benefit from switching a consumer to a higher-valued consumption path than the currently chosen one. This is most easily understood in the static setting. Consider a type-v consumer who faces two varieties a and b, with $v_a < v_b$. Then, if prices are such that the consumer prefers to purchase variety a rather than b, there is a trading up opportunity: since the consumer has a greater willingness-to-pay for variety b, a firm that sells both varieties can gain from switching this buyer to variety b and adjusting prices accordingly. The same logic applies in the dynamic setting where forward-looking consumers compare the values of consumption paths rather than consumption options. Note that for consumption path $\mathbf{d}_l^t$ to provide a greater discounted stream of valuations than path $\mathbf{d}_k^t$ at period t, $\mathbf{v}_{l,t} > \mathbf{v}_{k,t}$, the purchase decision along path $\mathbf{d}_l^t$ must provide greater value than that along path $\mathbf{d}_k^t$ at least once.

6 Pricing

All of our results so far have made no assumption about the prices chosen and hence are independent of the market structure on the supply side. We have merely assumed that prices exist that can be associated with each respective consumption path, irrespective of what they look like. Intuitively, consumers care about where prices come from only insofar as it determines the level of prices. We now analyse optimal pricing, given Lemmas 1-3. Note that we will explicitly allow the seller to condition prices on the consumption path, so that prices may differ across different paths. That is, a buyer that has followed path $\mathbf{d}_k$ and at time t has a history of $\mathbf{d}_k^{-t}$ faces prices $p_k^t = (p_a^t(\mathbf{d}_k^{t-1}), p_b^t(\mathbf{d}_k^{t-1}), p_o)$. We first analyze pricing under commitment by the seller.

Proposition 1 (monopoly commitment) *The monopolist cannot do better than implement the static optimal market segmentation* (v_1^m, v_2^m, v_3^m) *by setting static multi-product monopoly prices and refraining from dynamic price discrimination. Maximum profit is given by* $\Pi(\mathbf{p}^m) = \sum_{t=0}^{T} \delta^t \pi(\mathbf{p}^m)$*, where* $\pi(\mathbf{p}^m)$ *denotes the static monopoly maximizing profit.*

Proof. By Lemma 3 we can write the profit function of the multi-product monopolist with commitment from $t = 0$ onward as

$$\Pi(\mathbf{P}_K) = \mathbf{v}_0 \times \mathcal{F}\left(\sum_{k=0}^{K} \mathbf{d}_k\right) + \sum_{k=1}^{K} (\mathbf{v}_k^k - \mathbf{v}_{k-1}^k) \times \mathcal{F}\left(\sum_{k=1}^{K} \mathbf{d}_k\right)$$

with the set of all consumption paths $\mathbf{D}_K$ and associated prices $\mathbf{P}_K$, where we have omitted the time index on the discounted streams of prices and valuations, as well as paths, and where $\mathbf{p}_0 = \mathbf{v}_0$ follows from the observation that the monopolist will not leave any rent to the lowest type selecting consumption path $\mathbf{d}_0$. Observe that for each respective mass of consumers $\mathcal{F}(\cdot)$ we have

$$(\mathbf{v}_k^k - \mathbf{v}_{k-1}^k) \times \mathcal{F}\left(\sum_{k=1}^{K} \mathbf{d}_k\right) = (\mathbf{v}_k^k - \mathbf{v}_{k-1}^k) \times \mathcal{F}\left(v : \mathbf{v} \in [\mathbf{v}_k^k - \mathbf{v}_{k-1}^k, \mathbf{1}]\right)$$

where the measure of consumers is time-invariant and the lower bound of the segment is the cutoff, which can be separated into the per-period discounted differences in valuations, while the upper bound is $\mathbf{1} = \sum_{t=0}^{T} \delta^t v^{max}$ where v^{max} is the type $(1, 1, v_o)$. By construction, each valuation difference appropriately discounted enters the lower bound of the interval for the segment. Then each term is maximized at the same value of the valuation difference. It follows that each term in the profit function must be maximized at the static revenue-maximizing market segmentation (v_1^m, v_2^m, v_3^m). These cutoffs can be implemented by choosing constant static revenue-maximizing or multi-product monopoly maximizing prices $\mathbf{p}^m$ in each period $t = 0, ..., T$. It follows that maximum profit is given by $\Pi(\mathbf{p}^m) = \sum_{t=0}^{T} \delta^t \pi(\mathbf{p}^m)$, where $\pi(\mathbf{p}^m)$ denotes the static monopoly maximizing profit. ■

Proposition 1 shows that a monopolist seller cannot do better than induce the optimal static market segmentation by setting static revenue-maximizing and hence multi-product monopoly prices and refraining from price discrimination. The intuition is as follows: by varying prices over time, a seller provides incentives to consumers to exploit dynamic price discrimination by making strategic purchase decisions. The foregone profit from strategic consumer behavior is large enough to outweigh the benefits from dynamic price discrimination.

This finding is in line with the classic result that it is optimal for a monopolist not to discriminate prices with commitment when consumer types are fixed (e.g. Stokey, 1979; Hart and Tirole, 1988; Acquisti and Varian, 2005; Fudenberg and Villas-Boas, 2007). Proposition 1 demonstrates that this insight extends to multiple varieties and holds across any combination of absorbing states.

Now assume that the monopolist lacks commitment ability. In this case, the seller chooses prices anew in every period. Then we can show that the seller faces a commitment problem only if trading up opportunities exist.

Proposition 2 (monopoly commitment problem) *The multi-product monopolist only faces a commitment problem if trading up opportunities exist.*

Proof. Suppose the seller induced the commitment solution of optimal static revenue-maximizing cutoffs $\mathbf{v}^m = \{v_1^m, v_2^m, v_3^m\}$ in every previous period $0, ..., t-1$, such that there exists at least one segment of buyers who all chose option $i \in \{a, b, o\}$ in every previous period. Then the path they have followed until t is $\mathbf{d}_i$ with the associated sequence of past purchase decisions at time t of $\mathbf{d}_i^{-t}$. Then the measure of consumers with history $\mathbf{d}_i^{-t}$ and being priced anew is given by

$$\mathcal{F}(\mathbf{d}_i^{-t}) = \mathcal{F}\left(\sum_{k=0}^{K_i} \mathbf{d}_k^t\right),$$

where the consumption paths available to these consumers are index by $0, ..., K_i$. By Lemma 3, we can write the monopolists profit from segment i at time t as

$$\Pi_i^t\left(\mathbf{P}_{K,i}^t\right) = \sum_{k=0}^{K_i}(\mathbf{v}_{k,t}^k - \mathbf{v}_{k-1,t}^k) \times \mathcal{F}\left(\sum_{k=0}^{K_i} \mathbf{d}_k^t\right),$$

where $\mathbf{v}_{-1,t}^0 = 0$.

Suppose that the segment of buyers under consideration contains some types v that satisfy $v_j > v_i, j \neq i, i, j \in \{a, b, o\}$. At least one such segment must always exists if $\{v_1^m, v_2^m, v_3^m\}$ was implemented in each previous period. Then there exists a trading up opportunity for these types and overall rent and monopoly profits in this segment can be strictly increased by setting prices such that $K_i \geq 1$. Denote by v^* the subset of types that satisfy both $v_j > v_i$ and

$$\mathbf{v}_{i,\tau}^* - \mathbf{p}_{i,\tau} = \mathbf{v}_{l,\tau}^* - \mathbf{p}_{l,\tau},$$

in any previous period $\tau \in 0, ..., t-1$, where $\mathbf{d}_l$ denotes the consumption path they were indifferent to. These types always exist if the static optimum includes at least two options being chosen and since we have assumed that $\{v_1^m, v_2^m, v_3^m\}$ was implemented in every period τ, we know that they must have been indifferent to the same $\mathbf{d}_l$ in every previous period. If instead the static optimum is to induce the same consumption decision for all types in $[\underline{v}, \bar{v}]$, we must have $\mathcal{F}(\mathbf{d}_i^{-t}) = \mathcal{F}$. Hence, optimally trading up in t and having implemented optimal static cutoffs in previous periods is inconsistent in this case. In any other case, by construction, when $K_i \geq 1$ in period t then for types v^* we must instead have had

$$\mathbf{v}_{k,\tau}^* - \mathbf{p}_{k,\tau} = \mathbf{v}_{l,\tau}^* - \mathbf{p}_{l,\tau},$$

since we have sorted the consumption paths $\mathbf{D}_{K,i}^{\tau}$ appropriately to define $\Pi_i^t(\mathbf{P}_{K,i}^t)$ and where $\mathbf{d}_k \in \mathbf{D}_{K,i}^{\tau} \setminus \mathbf{d}_0$. But if trading up opportunities exist, we must have that $\mathbf{v}_{k,\tau} > \mathbf{v}_{i,\tau}$. In addition, if $\mathbf{v}_{k,\tau}^* - \mathbf{p}_{k,\tau} = \mathbf{v}_{i,\tau}^* - \mathbf{p}_{i,\tau}$, it follows that the mass of types that choose path $\mathbf{d}_k$ is empty. Hence, setting $\mathbf{p}_{k,\tau} > \mathbf{p}_{i,\tau}$ to implement $\mathbf{v}_{k,\tau}^* - \mathbf{p}_{k,\tau} > \mathbf{v}_{i,\tau}^* - \mathbf{p}_{i,\tau}$ and induce a positive mass of types to choose path $\mathbf{d}_k$ must be profit-increasing.

Now note that to keep the indifference condition for v^* at the static monopoly optimum in any period τ therefore requires that $\mathbf{p}_{l,\tau} < \mathbf{v}_{l,\tau}^*$. However, leaving rent for the lowest type v^* in the adjacent segment must be non-profit-maximizing in that segment. ■

Proposition 2 clarifies that the driving force behind dynamic pricing of a monopolist in our general setting is trading up. The possibility of the seller to switch consumers to a consumption path for which they have a higher overall willingness-to-pay allows the seller to charge prices that extract part of this additional rent. However, consumers will only be willing to switch if they obtain a strictly greater rent from doing so. This induces some consumers to strategically self-select into consumption paths in order to be traded up in future periods. As shown in Proposition 1, this reduces the monopolists profit. But without the ability to commit to prices ex-ante, the monopolist cannot resist the temptation to discriminate prices and trade up consumers.

The logic of the proof of Proposition 2 implies that it is straightforward to check whether in a particular application a monopolist seller has a commitment problem. It is sufficient to consider (i) if the static optimal solution contains at least one segment of consumers, in which at least some individuals do not select the consumption option for which they have the highest valuation, and (ii) if the setting allows for consumption paths in which consumers switch from the option associated with the static segment to the higher valued one.

These conditions are satisfied for example in classic papers on coasian dynamics. In such a setting, there exists one absorbing variety (the durable good) and an outside option. Clearly, consumers that choose the outside option have a higher valuation for the good by construction and are allowed to switch from non-consumption in previous rounds to consumption at some later stage. Hence, the monopolist has a commitment problem (e.g. Coase, 1972). Similarly, with two absorbing varieties, individuals can still be traded up from the outside option and the seller has a commitment problem (Nava and Schiraldi, 2019), but if the static solution does not contain any individuals in the sole non-absorbing option, the commitment problem disappears (Board and Pycia, 2014). Indeed, this is the reason why Nava and Schiraldi (2019) find that with two durable goods, the equilibrium without seller commitment must clear the market: in this case, there are no consumers left in the sole non-absorbing state of the outside option and hence there no longer is a commitment problem. Lastly, if we are in a setting with one non-absorbing variety and an absorbing outside option, there is again no commitment problem since no consumption paths are allowed in which consumers can be traded up from the lower-valued option of exiting the market to the purchase option (Tirole, 2016).

We now characterize optimal dynamic prices of a multi-product monopolist that lacks commitment ability.

Proposition 3 (monopoly non-commitment) *A seller that lacks commitment ability engages in dynamic price discrimination when trading up opportunities exist. Prices are discriminated within periods (that is, across segments) and across periods. Optimal dynamic prices are constant for consumers that have been traded up to their highest-value option and decreasing for all other consumers.*

Proof. Consider a consumer segment $j \in (1, ..., S_t)$ with history $h_j^t \in \mathcal{H}^t$ and mass

$$\mathcal{F}(\mathbf{p}_t^j, v : \mathbf{v} \in [\underline{\mathbf{v}}_{j,t}, \bar{\mathbf{v}}_{j,t}] | h_j^t),$$

for which the seller's profit can be written as

$$\Pi_j^t(\mathbf{p}_t^j) = \sum_{k_j=0}^{K_j} (\mathbf{v}_{k_j,t}^{k_j} - \mathbf{v}_{k_j-1,t}^{k_j}) \times \mathcal{F}(\mathbf{p}_t^j, v : \mathbf{v} \in [\mathbf{v}_{k_j,t}^{k_j} - \mathbf{v}_{k_j-1,t}^{k_j}, \bar{\mathbf{v}}_{j,t}] | h_j^t).$$

First, profit-maximization requires that prices charged to consumers in any segment j are interior in the sense that $p_{i,j}^{\tau} \in [\underline{v}_{i,j}, \bar{v}_{i,j}]$, $i \in \{a, b\}$, for $\tau \geq t$, since the firm will never want to leave any rent to the lowest type, and hence no consumer will ever be willing to buy at price $p_{i,j}^{\tau} > \bar{v}_{i,j}$.

Second, when trading up opportunities exist, $K_j \geq 1$, we must have that $\mathbf{v}_{K_j,t} > \mathbf{v}_{j,t}$. As before it follows that setting $\mathbf{p}_{K_j,t} > \mathbf{p}_{j,t}$ to implement $\mathbf{v}_{K_j,t}^{*} -$

$\mathbf{p}_{K,t} > \mathbf{v}^*_{j,t} - \mathbf{p}_{j,t}$ and inducing a positive mass of types to choose path C_K must be profit-increasing. Hence, the seller can increase its profit by charging prices for path C_K that are strictly interior.

Third, since optimal prices for each segment with trading up opportunities are strictly interior, optimal prices for consumers that do not always purchase their highest-value option must fall. For consumers that always purchase their highest-value option, there are no trading up opportunities, and optimal prices are constant. ■

Proposition 3 shows that the monopolist engages in trading up consumers by price discriminating. The seller offers price discounts to buyers that have not selected their highest valued consumption option in order to trade them up. In doing so, the seller exploits the fact that he/she can condition on the entire purchase history and hence prices are discriminated across consumer segments (i.e. consumption paths) and across periods.

The result shows that the classic notion of coasian dynamics as falling prices for non-buyers generally characterizes dynamic pricing. However, once a given set of consumers has been traded up to their respective highest-value option, prices remain constant. This reflects Tiroles (2016) key insight that under positive selection optimal pricing is constant. The reason is that there are no more trading up opportunities left for such a set of buyers.

7 Conclusion

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