Cournot, Conduct, and Homing in Two-Sided Markets

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Abstract

Firms in traditional markets often compete in output *à la* Cournot or in settings where competition can be explained by a conduct parameter. In this paper, we consider both Cournot competition and the conduct parameter approach for platforms in two-sided markets. We find that with Cournot competition, the markup and markdown terms are distorted toward zero for greater levels of platform competition and for increased single-homing. Allowing for any homing allocation, we also find that both the level of platform competition and agent homing decisions determine side specific conduct parameters and, similar to the traditional market, we show that these side specific conduct parameters can be derived from elasticity-adjusted Lerner indices.

Keywords: Platforms, conduct parameter, network externality, Lerner index.

1 Introduction

The study of how firms compete was first introduced in the seminal work by Cournot (1838) and Bertrand (1883). Since then, both models remain fundamental to the study of competition with several generalizations between the two. One of these generalizations is the conduct parameter approach which provides a single parameter that characterizes the level of competition between firms (see Corts (1999), d'Aspremont and Ferreira (2009) and Weyl and Fabinger (2013)). This allows for the consideration of many competition structures in a simple framework.[1]

Given the importance of comparing outcomes across a variety of competition structures, it is natural to consider such a comparison for platforms in two-sided markets. While two-sided markets have received considerable attention (Rochet and Tirole (2003), Caillaud and Jullien (2003), Armstrong (2006), Hagiu (2006) and Weyl (2010)), a direct comparison across different types of competition structures has yet to be considered. This is largely since a substantial amount of the literature considers horizontally differentiated platforms, a model pioneered by Armstrong (2006), where differentiated platforms compete in prices and market structure is effectively fixed due to tractability concerns.

Given the extent to which competition structures vary across platform industries, the consideration of alternative forms of platform conduct is important. In terms of Cournot competition amongst platforms, the studies are few. Gabszewicz and Wauthy (2014) use Cournot competition to consider vertical differentiation between platforms where agents single-home. White and Weyl (2016) show that the Cournot game can be interpreted as a special case of an insulated tariff game between platforms. However, both models consider competition between two platforms and neither investigates how changes to platform market structure impact equilibrium pricing. One paper that considers changes to platform market structure is Correia-da Silva et al. (2019). They consider a model of Cournot competition

[1] For example, the conduct parameter approach can describe symmetric Cournot or Bertrand competition of either the homogeneous or differentiated type.

between more than two platforms and they determine the cases for which a platform merger harms consumers and sellers. Unfortunately, their model assumes that all agents single-home on both sides of the market. Instead, we allow for any homing allocation to better understand how both the number of platforms and agent homing decisions impacts platform conduct on each side of the market.[2]

One model that considers platform competition across homing allocations is that of Jeitschko and Tremblay (2019). They consider price competition between homogeneous platforms in a two-sided market where every agent, on each side of the market, makes endogenous homing decisions. The main focus of their paper is endogenous homing decisions and the resulting equilibrium allocations that arise when platforms compete in prices.[3] In terms of pricing, they find that the Bertrand Paradox occurs with platforms setting prices on each side of the market equal to the corresponding marginal costs. However, they only focus on the extreme case of homogeneous price competition between platforms and they do not consider Cournot competition or a conduct parameter approach to platform competition.

To consider Cournot competition and the conduct parameter approach in two-sided markets, we extend the model of Jeitschko and Tremblay (2019). This allow us to make comparisons across platform conduct for any homing allocation. For the competitive bottleneck allocation (the commonly analyzed allocation where consumers single-home and sellers multi-home) with Cournot competition, we find that competition reduces the markup in the consumer price as well as the markdown in the seller price but the markup to sellers and the markdown to consumers are consistent with a monopoly platform. In the limit, this implies that greater platform competition results in consumers being subsidized while sellers

[2]Several studies consider platform competition between N platforms using a Salop circle approach: Reisinger et al. (2009), Alexandrov et al. (2011), Anderson et al. (2012) and Anderson et al. (2019). However, these models consider media or search platforms where indirect network externalities do not exist between the two sides of the market. In contrast, Baranes et al. (2019) consider how four platforms on the Salop circle compete, or merge, when network externalities are present on both sides; however, they assume all agents single-home.

[3]For example, they show that the competitive bottleneck allocation where all consumers single-home and all firms multi-home is always an equilibrium. In addition, they show that mixed-homing allocations where single-homing and multi-homing consumers as well as single-homing and multi-homing firms can also exist in equilibrium.

face markups. This pricing result coincides with optimal pricing in competitive bottleneck models where the single-homing side of the market faces a more competitive price than the multi-homing side (see Belleflamme and Peitz (2019) for details).

This phenomenon, pricing distortions that depend on homing decisions, generalizes to any homing allocation. In particular, we find that a greater proportion of single-homing consumers or sellers increases the extent to which competition distorts markups and markdowns toward zero. This implies that with mixed-homing on both sides of the market, greater platform competition results in both platform prices converging to their respective marginal cost (a result that is consistent with the traditional Cournot model).

The results from the general homing model show that both platform competition and agent homing decisions impact the extent of platform monopolization and cross-subsidization in two-sided markets. To gain a better understanding of this intricacy, we generalize our model of Cournot platform competition to a model that utilizes the conduct parameter approach for platforms. To the best of our knowledge, we are the first to attempt such an exercise. We show that both platform conduct and agent homing allocations generate side specific conduct parameters, and we find that greater single-homing on a particular side decreases platform monopolization over that side while greater platform market power increases monopolization over that side. We also show that, similar to the traditional market, the side specific conduct parameters can be derived from elasticity-adjusted Lerner indices and this highlights the tractability of the conduct parameter approach to platform competition.

The rest of this paper is organized as follows. Section 2 briefly summarizes the Cournot and conduct for the traditional market. Our approach to the two-sided market is developed in Section 3. The Cournot equilibrium pricing for the competitive bottleneck and mixed-homing allocations are determined in Section 4. Section 5 develops our conduct parameter approach for two-sided markets and there we derive our side specific conduct parameters that ensure that the traditional relationship between Cournot and conduct also holds for platforms. Finally, Section 7 concludes.

2 A Traditional Market

2.1 Cournot Competition

First consider Cournot competition in a traditional market. Suppose that there are N competing firms, denoted by $X = 1, 2, ..., N$, that sell homogeneous goods. Each firm chooses their output q^X for $X = 1, 2, ..., N$. Demand for the product is given by: $p = p(Q)$, where p is the price, $Q = \sum q^X$ is the total quantity, $p(Q)$ is the downward sloping inverse demand function, and $\epsilon = \frac{\partial Q}{\partial p} \cdot \frac{p}{Q} < 0$ is the elasticity of demand. Each firm has profit given by: $\pi^X = [p(Q) - c] \cdot q^X$, where c denotes the marginal cost. Maximizing profits with respect to a firm X's output yields the first-order condition given by $\frac{\partial \pi^X}{\partial q^X} = p'(Q) \cdot q^X + [p(Q) - c] = 0$. Symmetry implies that $q^X = q = \frac{Q}{N}$ for all $X = 1, 2, ..., N$, and so we have

$$p = c \ + \underbrace{\frac{1}{N} \cdot \frac{p}{-\epsilon}}_{\text{Markup}} . \tag{1}$$

2.2 The Conduct Parameter Approach

The conduct parameter approach offers a general model of symmetric imperfect competition where, instead of explicitly defining interaction between firms, we require that firms set the elasticity-adjusted Lerner index, $L \cdot \epsilon = \frac{p-c}{p} \cdot (-\epsilon)$, equal to a conduct parameter, $\theta \in [0, 1]$, that implicitly defines competition. Thus, to solve the firm's problem when market conduct is given by θ, we have that the equilibrium price is given by:

$$p = c \ + \underbrace{\theta \cdot \frac{p}{-\epsilon}}_{\text{Markup}} . \tag{2}$$

Notice that conduct equals zero under perfect competition so that $p = c$, equals one under monopoly or full collusion, and equals $\frac{1}{N}$ when N firms compete in output.

3 A Two-Sided Market

In a two-sided market, two groups of agents (consumers and sellers)[4] benefit from the indirect network externalities that exist across the two groups. For example, gamers benefit from greater video game availability and game developers benefit from greater console ownership. Let the consumer side be denoted as Side 1 and the seller side as Side 2, and suppose that there are N competing platforms.

3.1 Consumers and Sellers

First consider the consumer side of the market. Consumers benefit from interaction with the seller side of the market, and some consumers benefit more from sellers than others. For example, some consumers benefit more from apps (teens) than others (parents).[5] Let consumer types be denoted by $\tau_1 \in [0, \overline{\tau}_1]$ and let consumers be distributed uniformly. The utility from joining platform X, $X = 1, 2, ..., N$, for a consumer of type τ_1 is given by:

$$u_1^X(\tau_1) = \alpha_1(\tau_1) \cdot q_2^X - p_1^X, \tag{3}$$

where q_2^X denotes the number of sellers on platform X, p_1^X denotes the consumer price of platform X, and $\alpha_1(\cdot)$ denotes the network externality function for consumers. Without loss of generality, suppose that consumers with lower τ_1 types have greater network benefits than consumers with higher τ_1 types. This implies that $\alpha_1(\cdot)$ is decreasing.[6]

Note that platforms are homogeneous from the consumers perspective. That is, the network externality function, $\alpha_1(\cdot)$, does not differ across platforms. This implies that platforms can only differ in the consumer price or in the amount of sellers provided. Thus, platforms are ex ante homogeneous.

[4]Depending on the platform industry, a seller might be a content provider, game developer, book publisher, etc.

[5]Similar arguments can be made for consumer heterogeneity in game content for the gaming platform industry and in video content for the video streaming platform industry.

[6]Furthermore, suppose that $\alpha_1(\cdot)$ is twice continuously differentiable.

Consumers might also have the option to multi-home (join all platforms). The utility for a consumer of type τ_1 that multi-homes is given by:

$$u_1^M(\tau_1) = \alpha_1(\tau_1) \cdot [Q_2 - q_2^M] - P_1, \tag{4}$$

where $Q_2 = \sum_{X=1}^{N} q_2^X$, $P_1 = \sum_{X=1}^{N} p_1^X$, and q_2^M denotes the number of sellers that multi-home. This implies that consumers only benefit once from a seller that multi-homes. For example, if a video game is available on the Xbox, then a multi-homing gamer does not receive an additional benefit if that game is also available on the Playstation. All together, a consumer that multi-homes benefits from the unique sellers available on each platform, and this additional access comes at a price of paying for all platforms. Lastly, suppose that a consumer's outside option from not joining either platform is valued at zero.

Now consider the seller side of the market. Sellers benefit from greater consumer participation on a platform. Like consumers, sellers are heterogeneous in their network gains. That is, some sellers are more successful than others. Let seller types be denoted by $\tau_2 \in [0, \overline{\tau}_1]$ and let sellers be distributed uniformly. The utility from joining platform X, $X = A, B$, for a seller of type τ_2 is given by:

$$u_2^X(\tau_2) = \alpha_2(\tau_2) \cdot q_1^X - p_2^X, \tag{5}$$

where q_1^X denotes the number of consumers on platform X, p_2^X denotes the seller price of platform X, and $\alpha_2(\cdot)$ denotes the network externality function for sellers. Without loss of generality, suppose that sellers with lower τ_2 types have greater network benefits than sellers with higher τ_1 types. This implies that $\alpha_2(\cdot)$ is decreasing.[7]

Sellers may multi-home. The utility for a seller of type τ_1 that multi-homes is given by:

$$u_2^M(\tau_2) = \alpha_2(\tau_2) \cdot [Q_1 - q_1^M] - P_2, \tag{6}$$

[7]Furthermore, suppose that $\alpha_2(\cdot)$ is twice continuously differentiable.

where $Q_1 = \sum_{X=1}^{N} q_1^X$, $P_2 = \sum_{X=1}^{N} p_2^X$, and q_1^M denotes the number of consumers that multi-home so that sellers only benefit from the opportunity to offer their product to new consumers that would otherwise not have access to their product. In addition, suppose that a seller that does not join a platform earns a value of zero.

3.2 The Platforms

Platforms generate revenues from each side of the market so that profits for platform X, $X = 1, 2, ..., N$, are given by:

$$\Pi^X = [p_1 - c_1] \cdot q_1^X + [p_2 - c_2] \cdot q_2^X, \tag{7}$$

where c_i denotes the marginal cost to the platform for an additional Side i agent. If platforms compete in output on Side i, then platform X chooses q_i^X to maximize profits taking the output of the opposing platform, q_i^Y with $Y = 1, 2, ..., N$, and $Y \neq X$, as given.

4 The Cournot Equilibrium

With more than one platform available to consumers and sellers, assumptions regarding the homing decisions must be specified. We consider two allocations: the competitive bottleneck, where consumers single-home and sellers multi-home, which is commonly studied in the literature and the general case of any homing allocation. The equilibrium concept is the Fulfilled Expectations Cournot Equilibrium, developed by Katz and Shapiro (1985) for one-sided network markets. We extend the concept to two-sided markets so that consumers and sellers have expectations for outputs by each platform, platforms have expectations for agent homing decisions, and in equilibrium, expectations are fulfilled.[8]

[8]In this Section we focus on interior solutions. However, in Section 6 we consider specific functional forms that allow for a richer analysis of comparative statics and there we discuss how such a focus impacts our results.

4.1 The Competitive Bottleneck Allocation

First consider the competitive bottleneck allocation where consumers single-home (they join only one platform), and firms multi-home (they join all platforms). This allocation implies that $q_1^M = 0$ so that the market output is given by $Q_1 = \sum_{X=1}^{N} q_1^X$. Similarly, on the seller side we have that the market output is given by $Q_2 = q_2^M = q_2^X$ for $X = 1, 2, ..., N$. Solving for the platform equilibrium in light of this allocation yields the following result:

Proposition 1 (The Cournot Competitive Bottleneck Prices)**.** *If consumers single-home, sellers multi-home, and the N platforms compete in output, then the equilibrium prices for consumers and sellers are*

$$p_1^{CB} = c_1 + \frac{1}{N} \cdot \frac{p_1^{CB}}{-\epsilon_1} - \alpha_2(Q_2^{CB})Q_2^{CB}, \tag{8}$$

$$p_2^{CB} = c_2 + \frac{p_2^{CB}}{-\epsilon_2} - \frac{1}{N} \cdot \alpha_1(Q_1^{CB})Q_1^{CB}. \tag{9}$$

where $\epsilon_i = \frac{\partial Q_i}{\partial p_i} \cdot \frac{p_i}{Q_i} < 0$ is the demand elasticity of Side i for the platform and Q_i^{CB} denotes the implicitly defined equilibrium total participation by users on Side i when the competitive bottleneck allocation occurs.

First notice that each price equals the sum of three terms: (i) marginal cost, (ii) plus a markup term, (iii) plus a markdown term that incorporates the network benefit to the other side. And, if there is a monopoly platform ($N = 1$), then platform pricing is consistent with standard monopoly models: $p_i^{CB} = c_i + \frac{-p_i^{CB}}{\epsilon_i} - \alpha_j(Q_j^{CB})Q_j^{CB}$ for $i, j = 1, 2$ and $i \neq j$. However, with competition across N platforms we see that homing decisions distort prices across the two sides. On the consumer side where single-homing occurs, greater competition reduces the markup term. In contrast, on the seller side where multi-homing occurs, greater competition reduces the markdown term. Thus, as competition increases, prices diverge so that the consumer price includes no markup and the seller price includes no markdown. Such pricing distortions across homing decisions is consistent with the previous literature where platforms compete for single-homers and have market power over the multi-homers (e.g., Rochet and Tirole (2003), Armstrong (2006) and Hagiu (2006)).

Considering the competitive bottleneck results altogether, we see that Cournot competing platforms are able to both (i) invest in network effects and (ii) charge markup. This implies that Cournot competing platforms avoid the Bertrand Paradox and earn profits. In addition, as platform competition becomes perfect, $N \to \infty$, platform prices do *not* approach marginal costs. Instead, platforms apply no markup to consumers while fully subsidizing them while charging sellers a full markup and no subsidy. In this case, Cournot competition results in a straddle pricing equilibria which Jeitschko and Tremblay (2019) show to be the only pricing strategy where platforms competing in prices can earn profits. Thus, even in the limit, N Cournot competing platforms might earn profits.

4.2 General Homing Allocations

The competitive bottleneck equilibrium shows how differences in agent homing decisions on each side of the market can affect equilibrium pricing and the nature of platform competition. This suggests that it is important to consider other allocations between agents. Furthermore, many platform industries exhibit allocations where some agents on each side of the market single-home while others multi-home. For example, some consumers own a single video game console while other consumers own multiple consoles; at the same time, some video games are available across all consoles while other games are only produced for a single console. Similarly, some consumers subscribe to a single video streaming service (Amazon Video, HBO Go, Hulu, or Netflix) while other consumers join multiple; simultaneously, some video content is exclusive to a streaming platform while other content is available across several platforms.

To consider a general homing environment, suppose that the proportion of consumers (sellers) that single-home is given by $\phi_1(\phi_2) \in [0, 1]$. For simplicity, suppose that the homing distribution of consumers (ϕ_1) and the homing distribution of sellers (ϕ_2) are known to the platforms prior to choosing output. While this assumption may appear strong, there are many industries where platforms might know the distribution of agent homing. For example, when gaming platforms like Microsoft, Nintendo, and Sony produce a new video game console, they likely have an idea of the homing distribution based on the homing distribution for the previous console generation.[9]

Suppose that, without loss of generality, we have that the first platform ($X = 1$) produces the lowest output: $q_i^1 = \min_{X=1,\ldots,N}\{q_i^X\}$. Given agent homing decisions, this implies that Q_i is given by:

$$Q_i = \underbrace{(1-\phi_i)q_i^1}_{\text{Multi-homers}} + \underbrace{\phi_i \cdot q_i^1}_{\text{Low } q \text{ Single-homers}} + \underbrace{\sum_{X=2}^{N} q_i^X - (1-\phi_i)q_i^1}_{\text{High } q \text{ Single-homers}}. \tag{10}$$

Note that if platforms choose the same q_i, then $Q_i = (1-\phi_i)q_i + \phi_i \cdot q_i + (N-1)\phi_i \cdot q_i = 1 + (N-1)\phi_i q_i$. That is, to identify the unique number of Side i agents, the $(1-\phi_i)q_i$ multi-homers are counted once but the $\phi_i \cdot q_i$ single-homers are counted for each of the N platforms so that $Q_i = (1-\phi_i)q_i + N\phi_i q_i = 1 + (N-1)\phi_i q_i$. Also notice that if $\phi_1 = 1$ and $\phi_2 = 0$, then the competitive bottleneck equilibrium occurs. Given the amount of single-homers on each side of the market, we have the following equilibrium prices:

Proposition 2 (The Cournot General Homing Prices)**.** *If a ϕ_1 portion of consumers single-home, a ϕ_2 portion of sellers single-home, and platforms compete in output, then the equilibrium prices are given by:*

$$p_1^G = c_1 + \frac{1}{1+(N-1)\phi_1} \cdot \frac{p_1^G}{-\epsilon_1} - \frac{1}{1+(N-1)\phi_2} \cdot \alpha_2(Q_2^G)Q_2^G, \tag{11}$$

$$p_2^G = c_2 + \frac{1}{1+(N-1)\phi_2} \cdot \frac{p_2^G}{-\epsilon_2} - \frac{1}{1+(N-1)\phi_1} \cdot \alpha_1(Q_1^G)Q_1^G, \tag{12}$$

where $Q_i^G = 1 + (N-1)\phi_i q_i^G$ denotes the implicitly defined equilibrium total participation by

[9] Similarly, smartphone platforms and video streaming platforms likely know the amount of consumer and content multi-homing.

users on Side i when the general allocation occurs.

Proposition 2 generalizes the results from Proposition 1 and highlights how the extent of single-homing vs. multi-homing impacts pricing for Cournot competing platforms. In particular, if there is greater consumer single-homing (ϕ_1 increases), then the markup to consumers is reduced for each platform. Furthermore, greater consumer single-homing implies that each platform's investment in consumers — the $-\frac{1}{1+(N-1)\phi_1} \cdot \alpha_1(Q_1^G)Q_1^G$ in Equation (12) — also reduces. Thus, greater single-homing on either side of the market will increase competition by reducing markups and network investment. This implies that if all agents single-home on both sides, then $N \to \infty$ results in $p_i^G \to c_i$ so that the Cournot equilibrium leads to the Bertrand Paradox of setting prices equal to marginal costs.

5 The Conduct Parameter Approach to Platforms

In the traditional market there is an isomorphic relationship between the conduct parameter approach and the Cournot game. More explicitly, we see that the traditional market equilibrium prices given by Equations (1) and (2) coincide when $\theta = \frac{1}{N}$. Given that the conduct parameter captures the extent of monopolization, our results from Section 4 suggest that homing decisions will impact conduct across the two sides of the market. To investigate the relationship between conduct and homing, we first consider platform competition with the competitive bottleneck allocation prior to generalizing our results to any homing allocation.

5.1 The Competitive Bottleneck Allocation

To maintain consistency between the Cournot and conduct models, we require that $\theta = \frac{1}{N}$ as in the traditional market. Thus, our results from Proposition 1 imply that

Corollary 1 (The Conduct Competitive Bottleneck Prices)**.** *If consumers single-home, sellers multi-home, and platform conduct is given by* $\theta \in [0, 1]$*, then the equilibrium prices for* *consumers and sellers are*

$$p_1^{CB} = c_1 + \theta \cdot \frac{p_1^{CB}}{-\epsilon_1} - \alpha_2(Q_2^{CB})Q_2^{CB}, \tag{13}$$

$$p_2^{CB} = c_2 + \frac{p_2^{CB}}{-\epsilon_2} - \theta \cdot \alpha_1(Q_1^{CB})Q_1^{CB}. \tag{14}$$

In this case, platform conduct impacts the two sides differently because consumers single-home and sellers multi-home. In particular, we see that platform competition only impacts how the consumer markup $\left(\frac{p_1^{CB}}{-\epsilon_1}\right)$ and markdown $\left(-\alpha_1(Q_1^{CB})Q_1^{CB}\right)$ carry through to final prices. In contrast, the seller markup $\left(\frac{p_2^{CB}}{-\epsilon_2}\right)$ and markdown $\left(-\alpha_2(Q_2^{CB})Q_2^{CB}\right)$ are not impacted by platform competition as platforms always maintain monopoly power over the group of agents that multi-home.

5.2 General Homing Allocations

Similarly, if we maintain consistency between Cournot and conduct so that $\theta = \frac{1}{N}$, then our results from Proposition 2 imply that

Corollary 2 (The Conduct General Homing Prices). *If a ϕ_1 portion of consumers single-home, a ϕ_2 portion of sellers single-home, platforms choose outputs, and platform conduct is given by $\theta \in [0,1]$, then the equilibrium prices for consumers and sellers are*

$$p_1^{G} = c_1 + \theta_1 \cdot \frac{p_1^{G}}{-\epsilon_1} - \theta_2 \cdot \alpha_2(Q_2^{G})Q_2^{G}, \tag{15}$$

$$p_2^{G} = c_2 + \theta_2 \cdot \frac{p_2^{G}}{-\epsilon_2} - \theta_1 \cdot \alpha_1(Q_1^{G})Q_1^{G}, \tag{16}$$

where $\theta_i = \frac{1}{1+\left(\frac{1}{\theta}-1\right)\phi_i}$ represents the conduct parameter for side i for $i = 1, 2$.

In the general setting for any homing allocation, we see that side specific conduct parameters emerge. More specifically, we see that conduct on a particular side depends on that side's homing allocation (ϕ_i) and platform conduct (θ): $\theta_i = \frac{1}{1+\left(\frac{1}{\theta}-1\right)\phi_i}$. Furthermore, greater single-homing on a particular side decreases monopolization on that side $\left(\frac{\partial\theta_i}{\partial\phi_i} < 0\right)$, and greater platform market power increases monopolization on that side $\left(\frac{\partial\theta_i}{\partial\theta} > 0\right)$.

Recall that the conduct parameter was given by elasticity-adjusted Lerner index in the traditional market: $\left(L \cdot (-\epsilon_i) = \frac{p-c}{p} \cdot (-\epsilon) = \theta\right)$. Naturally, we would like to derive a similar relationship for the two-sided market case. To do so, note that Armstrong (2006) and Weyl (2010) derive a modified version of the Lerner index that applies to platforms in two-sided markets. In the context of our model, they show that a monopoly platform sets prices so that $L_i \cdot (-\epsilon_i) = \frac{p_i - c_i + \alpha_j(Q_j)Q_j}{p_i} \cdot (-\epsilon_i) = 1$ for $i, j = 1, 2$ and $i \neq j$. Adapting our pricing results from Corollary 2 we see that conduct on each side can be expressed by the following:

$$\theta_i = \frac{p_i - c_i + \theta_j \cdot \alpha_j(Q_j)Q_j}{p_i} \cdot (-\epsilon_i). \tag{17}$$

Notice that this equation closely resembles the elasticity-adjusted Lerner index for platforms that was derived by Armstrong (2006) and Weyl (2010). The key difference is that the markdown term must account for conduct on the other side of the market. Hence, conduct on the other side of the market (θ_j) multiplies with the markdown term ($\alpha_j(Q_j)Q_j$) in Equation 17. Comparing the conduct formula in the traditional market with the conduct formulas in the two-sided market we see a striking resemblance where the difference between the relationships is captured by the indirect network effect that distorts platform pricing. Altogether, we see that the conduct parameter approach from traditional markets successfully expands to two-sided markets which gives researchers an additional tool for analyzing platform competition.

6 Merger and Welfare Implications

The approach that we have selected enables us to consider comparative statics relating to platform mergers. In particular, we are interested in how changes in the number of platforms, N, impacts equilibrium pricing and welfare. It is well known within the platform literature that greater platform competition need not increase welfare as more platforms can decrease lower prices (improving welfare) while reducing individual network size (reducing welfare). How network sizes change with the number of platforms will naturally depend on agent homing allocations and so we first consider the competitive bottleneck allocation prior to general homing allocations.

To definitively sign comparative statics, we make simplify assumptions on the $\alpha_i(\cdot)$ functions and the marginal costs: the $\alpha_i''(\cdot) = 0$ and $c_1 = c_2 = 0$. Allowing for a richer environment is likely to change our results quantitatively. At the end of this section, we offer some discussions on this.

6.1 The Competitive Bottleneck Allocation

For the competitive bottleneck allocation, we find that consumer participation increases while seller participation decreases with more platforms. This result is consistent with our pricing results from Proposition 1 where platforms have incentives to price more competitively to consumers while extracting rents from sellers. We see this explicitly in the comparative static results for prices: the price to consumers decreases while the price to sellers increases when the number of platforms increases. These results are summarized in the following lemma:

Lemma 1. *In the competitive bottleneck equilibrium, equilibrium consumer participation increases and seller participation decreases as the number of platforms increases:* $\frac{\partial Q_1^{CB}}{\partial N} > 0$ *and* $\frac{\partial Q_2^{CB}}{\partial N} < 0$*. In addition, the price to consumers decreases while the price to sellers is ambiguous with an increase in platforms:* $\frac{\partial p_1^{CB}}{\partial N} < 0$ *and* $\frac{\partial p_2^{CB}}{\partial N} > 0$.

In terms of welfare, with all consumers interacting with all sellers in the competitive bottleneck allocation, total welfare is given by:

$$W^{CB} = \int_0^{Q_1^{CB}} \alpha_1(\tau_1) Q_2^{CB} \, d\tau_1 + \int_0^{Q_2^{CB}} \alpha_2(\tau_2) Q_1^{CB} \, d\tau_2. \tag{18}$$

At first glance, the pricing and participation comparative statics from Lemma 1 do not provide a clear implication for how the number of competing platforms will impact welfare. That is, with more platforms resulting in greater consumer participation but fewer sellers, the impact to welfare is ambiguous at first glance. Thus, we consider how greater platform competition impacts welfare directly and we find that welfare in the competitive bottleneck allocation decreases in the number of platforms so that a monopoly platform generates more welfare than platform competition. Formally, we have the following:

Lemma 2. *In the competitive bottleneck allocation, welfare is decreasing in the number of platforms:* $\frac{\partial W^{CB}}{\partial N} < 0$ *for all* $N \geq 2$.

While many have speculated welfare comparisons across competition structures for the competitive bottleneck allocation, to the best of our knowledge, this is the first *formal* result for how welfare changes with the number of platforms in the competitive bottleneck setting that has become the benchmark model in the platform literature. And, in terms of potential mergers, our results suggest that a merge to monopoly will improve welfare in platform industries where the competitive bottleneck allocation occurs.

While a merge to monopoly has yet to occur in platform industries, we have observed several industries where the number of platforms has changed due to entry and exit. For example, Microsoft's entry into the video game market effectively increased the number of platforms from two (Sony and Nintendo) to three. Similarly, Google entered the smartphone market that was originally developed by Apple and Microsoft followed suit; however, Microsoft appears to be exiting the market in 2020. Our findings from Lemma 2 suggest that these entries would actually harm welfare if these industries adhere to the competitive bottleneck allocation. The smartphone market is the most likely to fit the competitive bottleneck bill as most consumers own a single smartphone while app providers make their apps available across platforms. This suggests that Microsoft's upcoming exit from the smartphone market will improve welfare in the smartphone industry.

6.2 General Homing Allocations

While the case of the competitive bottleneck generates unambiguous comparative statics, it is important to consider general homing allocations to determine how robust those results, and their corresponding policy recommendations, actually are. By considering general homing allocations, we also determine the extent for which single- and multi-homing distributions impact our findings; something that is often overlooked in the literature.

Recall that in the general homing setup, ϕ_i captures the extent of single-homing on Side i so that ϕ_i closer to one (zero) corresponds to the case where the majority of Side i agents single-homing (multi-home). This implies that the competitive bottleneck allocation corresponds to the special case where $\phi_1 = 1$ and $\phi_2 = 0$. In terms of prices and participation, we expect that the comparative statics might hinge on agent homing decisions. We find that this is indeed the case:

Lemma 3. *For general homing allocations characterized by $\phi_1, \phi_2 \in [0, 1]$, equilibrium consumer (seller) participation increases in the number of platforms if and only if $2\phi_1 > \phi_2$ ($2\phi_2 > \phi_1$): $\frac{\partial Q_1^G}{\partial N} > 0$ if and only if $2\phi_1 > \phi_2$ and $\frac{\partial Q_2^{CB}}{\partial N} > 0$ if and only if $2\phi_2 > \phi_1$.*

Thus, if the extent of single-homing on Side i is atleast half the extent of single-homing on Side j ($\phi_i > 0.5\phi_j$), then greater platform competition results in greater equilibrium participation on Side i. However, if there is too little single-homing (relative to the other side), then participation decreases with greater platform competition. These results echo our results from the competitive bottleneck allocation where greater platform competition increases participation on the single-homing side but decreases participation on the multi-homing side.

One important conclusion from our findings in Lemma 3 is that there is a large mass of homing allocations where participation on *both* sides increases. A result that is not possible when focusing on the competitive bottleneck allocation. This occurs when the $\phi_1, \phi_2 \in [0, 1]$ are such that $2\phi_2 > \phi_1 > 0.5\phi_2$. Thus, agent homing decisions drastically impact the effect that greater platform competition has on the resulting equilibrium, and this is especially important when considering the welfare effects from increased platform competition.

7 Conclusion

The study of platform competition often presents researchers with difficulties that do not arise in the study of competition amongst traditional firms. As a result, there is limited variety for models that consider platform competition. In particular, and, to the best of our knowledge, we are the first to integrate the conduct parameter approach with platform competition.

We show that if the competitive bottleneck occurs, then platform competition reduces the markup to consumers while also reducing the markdown to sellers; at the same time, the markup to sellers and the markdown to consumers are consistent with monopoly platform pricing. By generalizing to any homing allocation, we find that greater single-homing increases the extent to which competition distorts markups and markdowns toward zero. This implies that with mixed-homing on both sides of the market, greater platform competition results in both platform prices converging to their respective marginal cost (a result that is consistent with a traditional market).

In terms of results specific to conduct, we show that both platform conduct and agent homing allocations generate side specific conduct parameters, and we find that greater single-homing on a particular side decreases platform monopolization over that side while greater platform market power increases monopolization over that side. We also show that, similar to the traditional market, the side specific conduct parameters can be derived from elasticity-adjusted Lerner indices and this highlights the tractability of the conduct parameter approach to platform competition. In future research, we intend to consider platform merges and welfare using the conduct parameter approach.

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Appendix of Proofs

Proof of Proposition 1: Given that consumer and seller outside options are valued at zero, Equation (3) implies that the marginal consumer that joins a platform, $\tau_1 = \tau_1^Y$, is given by $u_1^Y(\tau_1^Y) = \alpha_1(\tau_1^Y) \cdot q_2^Y - p_1^Y = 0$. Similarly, Equation (5) implies that the marginal seller that joins a platform, $\tau_2 = \tau_2^Z$, is given by $u_2^Z(\tau_2^Z) = \alpha_2(\tau_2^Z) \cdot q_1^Z - p_2^Z = 0$. Solving platform X's consumer and seller demand equations yields: $p_i^X = \alpha_i(\tau_i^X) \cdot q_j^X$ for $i, j = 1, 2$ and $i \neq j$. Furthermore, given that consumers and sellers are distributed uniformly, the last agent to join a platform provides the total market output on that side of the market which implies that $\tau_i^X = Q_i$. Substituting into the platform X's profit function yields:

$$\Pi^X = [\alpha_1(Q_1) \cdot q_2^X - c_1] \cdot q_1^X + [\alpha_2(Q_2) \cdot q_1^X - c_2] \cdot q_2^X.$$

Taking first-order conditions imply that

$$\frac{\partial \Pi^X}{\partial q_i^X} = \alpha_i'(Q_i^{CB}) \cdot q_j^X \cdot q_i^X + [\alpha_i(Q_i^{CB}) \cdot q_j^X - c_i] + \alpha_j(Q_j^{CB}) \cdot q_j^X = 0. \tag{19}$$

Since $p_i^X = \alpha_i(Q_i) \cdot q_j^X$, we have that $\frac{\partial p_i^X}{\partial q_i^X} = \frac{\partial p_i^X}{\partial Q_i} = \alpha_i'(Q_i) \cdot q_j^X$. Substituting each into the above first-order condition implies

$$p_i^X = c_i - \frac{\partial p_i^X}{\partial Q_i^{CB}} \cdot q_i^X - \alpha_j(Q_j^{CB}) \cdot q_j^X.$$

Given that consumers single-home and sellers multi-home, $Q_1^{CB} = \sum_{X=1}^N q_1^X$ and $Q_2 = q_2^M = q_2^X$ for $X = 1, 2, ..., N$, then by imposing symmetry so that $q_1^X = q_1$ for $X = 1, 2, ..., N$ and $\theta = \frac{1}{N}$, we have that

$$p_1^X = p_1^Y = p_1^{CB} = c_1 + \theta \cdot \frac{p_1}{-\epsilon_1} - \alpha_2(Q_2^{CB})Q_2^{CB},$$

$$p_2^X = p_2^Y = p_2^{CB} = c_2 + \frac{p_2}{-\epsilon_2} - \theta \cdot \alpha_1(Q_1^{CB})Q_1^{CB},$$

where $\epsilon_i = \frac{\partial Q_i}{\partial p_i} \cdot \frac{p_i}{Q_i} < 0$ is the elasticity of Side i demand for the platform. $\square$

Proof of Proposition 2: Given that consumer and seller outside options are valued at zero, Equation (3) implies that the marginal consumer that joins a platform, $\tau_1 = \tau_1^Y$, is given by $u_1^Y(\tau_1^Y) = \alpha_1(\tau_1^Y) \cdot q_2^Y - p_1^Y = 0$. Similarly, Equation (5) implies that the marginal seller that joins a platform, $\tau_2 = \tau_2^Z$, is given by $u_2^Z(\tau_2^Z) = \alpha_2(\tau_2^Z) \cdot q_1^Z - p_2^Z = 0$. Solving platform X's consumer and seller demand equations yields: $p_i^X = \alpha_i(\tau_i^X) \cdot q_j^X$ for $i, j = 1, 2$ and $i \neq j$. Furthermore, given that consumers and sellers are distributed uniformly, the last agent to join a platform provides the total market output on that side of the market which implies that $\tau_i^X = Q_i$. Substituting into the platform X's profit function yields:

$$\Pi^X = [\alpha_1(Q_1) \cdot q_2^X - c_1] \cdot q_1^X + [\alpha_2(Q_2) \cdot q_1^X - c_2] \cdot q_2^X.$$

Using the output functions given by Equation (10), we see that there are two possible first-order conditions for each output variable (for a low output platform and for a high output platform). These first-order conditions are

$$\frac{\partial \Pi^X}{\partial q_i^X} = \alpha_i'(Q_i^G) \cdot q_j^X \cdot q_i^X + [\alpha_i(Q_i^G) \cdot q_j^X - c_i] + \alpha_j(Q_j^G) \cdot q_j^X = 0,$$

for $q_i^X \geq q_i^Y$, and

$$\frac{\partial \Pi^X}{\partial q_i^X} = \phi_i \cdot \alpha_i'(Q_i^G) \cdot q_j^X \cdot q_i^X + [\alpha_i(Q_i^G) \cdot q_j^X - c_i] + \alpha_j(Q_j^G) \cdot q_j^X = 0,$$

for $q_i^X < q_i^Y$. Imposing symmetry implies that $q_i^X = q_i$ for all $X = 1, 2, ..., N$, and so the first-order condition with $q_i^X \geq q_i^Y$ provides the equilibrium. In addition, since $p_i^X = \alpha_i(Q_i) \cdot q_j^X$ we have that $\frac{\partial p_i^X}{\partial q_i^X} = \frac{\partial p_i^X}{\partial Q_i} = \alpha_i'(Q_i) \cdot q_j^X$. Substituting each into the above first-order condition implies

$$p_i^X = c_i - \frac{\partial p_i^X}{\partial Q_i^G} \cdot q_i - \alpha_j(Q_j^G) \cdot q_j.$$

Given that a ϕ_1 portion of consumers single-home and a ϕ_2 portion of sellers single-home, we have that $Q_i^G = 1 + (N-1)\phi_i q_i$, and so

$$p_i^X = p_i^Y = p_i^G = c_i + \frac{1}{1+(N-1)\phi_i} \cdot \frac{p_i}{-\epsilon_i} - \frac{1}{1+(N-1)\phi_j} \cdot \alpha_j(Q_j^G)Q_j^G.$$

□

Proof of Lemma 1: Equation (19), with $c_i = 0$, and given that consumers single-home and sellers multi-home ($Q_1^{CB} = \sum_{X=1}^{N} q_1^X$ and $Q_2^{CB} = q_2^M = q_2^X$ for $X = 1, 2, ..., N$), the first-order conditions for consumers and sellers reduce to

$$0 = \alpha_1'(Q_1^{CB}) \cdot \frac{Q_1^{CB}}{N} + \alpha_1(Q_1^{CB}) + \alpha_2(Q_2^{CB}),$$

$$0 = \alpha_2'(Q_2^{CB}) \cdot Q_2^{CB} + \alpha_2(Q_2^{CB}) + \alpha_1(Q_1^{CB}).$$

Totally differentiating each equation when the $\alpha_i''(\cdot) = 0$ implies that

$$0 = \left(1 + \frac{1}{N}\right)\alpha_1'(Q_1^{CB})\partial Q_1^{CB} + \alpha_2'(Q_2^{CB})\partial Q_2^{CB} - \alpha_1'(Q_1^{CB}) \cdot \frac{Q_1^{CB}}{N^2}\partial N,$$

$$0 = 2\alpha_2'(Q_2^{CB})\partial Q_2^{CB} + \alpha_1'(Q_1^{CB})\partial Q_1^{CB}.$$

After some algebra, we have that $\frac{\partial Q_1^{CB}}{\partial N} = \frac{2Q_1^{CB}}{N(N+2)} > 0$ and $\frac{\partial Q_2^{CB}}{\partial N} = -\frac{\alpha_1'(Q_1^{CB})}{\alpha_2'(Q_2^{CB})}\frac{Q_1^{CB}}{N(N+2)} < 0$.

In the competitive bottleneck equilibrium, we have that $p_1^{CB} = \alpha_1(Q_1^{CB}) \cdot Q_2^{CB}$ and $p_2^{CB} = \alpha_2(Q_2^{CB}) \cdot \frac{Q_1^{CB}}{N}$. Differentiating with respect to N implies that

$$\frac{\partial p_1^{CB}}{\partial N} = \alpha_1'(Q_1^{CB})Q_2^{CB}\frac{\partial Q_1^{CB}}{\partial N} + \alpha_1(Q_1^{CB})\frac{\partial Q_2^{CB}}{\partial N},$$

$$\frac{\partial p_2^{CB}}{\partial N} = \alpha_2'(Q_2^{CB})\frac{Q_1^{CB}}{N}\frac{\partial Q_2^{CB}}{\partial N} + \alpha_2(Q_2^{CB})\frac{1}{N}\frac{\partial Q_1^{CB}}{\partial N} - \alpha_2(Q_2^{CB})\frac{Q_1^{CB}}{N^2}.$$

The first equation is less than zero since $\alpha_1'(\cdot)$, $\frac{\partial Q_2^{CB}}{\partial N} < 0$ and $\frac{\partial Q_1^{CB}}{\partial N} > 0$. The second equation reduces so that $\frac{\partial p_2^{CB}}{\partial N} \lesseqqgtr 0$ if and only if $-\alpha_1'(Q_1^{CB})Q_1^{CB} - \alpha_2(Q_2^{CB})N \lesseqqgtr 0$.

Here, however, the consideration for interior solutions might play a role. To see this, note that with differentiable $\alpha_i^(\cdot)$, the assumption that $\alpha_i''(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear. Thus, we impose a linear structure so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. This linearization implies that Equation (19) results in first-order conditions for consumers and sellers that reduce to:

$$0 = a_1 - \left(1 + \frac{1}{N}\right) b_1 \cdot Q_1^{CB} + a_2 - b_2 \cdot Q_2^{CB},$$

$$0 = a_2 - 2b_2 \cdot Q_2^{CB} + a_1 - b_1 \cdot Q_1^{CB}.$$

As a result, we have that

$$Q_1^{CB} = \frac{(a_1 + a_2)N}{b_1(N+2)} \ \& \ Q_2^{CB} = \frac{a_1 + a_2}{b_2(N+2)}.$$

In the end, an interior solution requires that $Q_i^{CB} < \overline{Q}_i := \frac{a_i}{b_i}$. Given the Q_1^{CB} and Q_2^{CB} above, we require that $2a_1 > Na_2$ and $(N+1)a_1 > a_2$ (which is satisfied to by the former). Returning to the $\frac{\partial p_2^{CB}}{\partial N}$ which is greater than zero if and only if $-\alpha_1'(Q_1^{CB})Q_1^{CB} - \alpha_2(Q_2^{CB})N > 0$. Imposing our linearity specification implies that $-\alpha_1'(Q_1^{CB})Q_1^{CB} - \alpha_2(Q_2^{CB})N > 0$ only if $2a_1 > Na_2$ which we assumed for an interior solution. Thus, $\frac{\partial p_2^{CB}}{\partial N} > 0$. □

Proof of Lemma 2: As noted at the start of Section 6, we assume that the $\alpha_i''(\cdot) = 0$ and that the $c_i = 0$ for $i = 1, 2$. Given that the $\alpha_i^(\cdot)$ are differentiable, $\alpha_i''(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear. Thus, as discussed in the Proof of Lemma 1, we impose a linear structure so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. From the Proof of Lemma 1 we have that

$$Q_1^{CB} = \frac{(a_1 + a_2)N}{b_1(N+2)} \ \& \ Q_2^{CB} = \frac{a_1 + a_2}{b_2(N+2)}.$$

Differentiating implies that

$$\frac{\partial Q_1^{CB}}{\partial N} = \frac{2(a_1 + a_2)}{b_1(N+2)^2} \ \& \ \frac{\partial Q_2^{CB}}{\partial N} = -\frac{a_1 + a_2}{b_2(N+2)^2}.$$

Applying Leibniz integral rule to Equation (18) implies that

$$\frac{\partial W^{CB}}{\partial N} = \alpha_1(Q_1^{CB})Q_2^{CB}\cdot\frac{\partial Q_1^{CB}}{\partial N}+\int_0^{Q_1^{CB}} \alpha_1(\tau_1)\frac{\partial Q_2^{CB}}{\partial N}\ d\tau_1+\alpha_2(Q_2^{CB})Q_1^{CB}\cdot\frac{\partial Q_2^{CB}}{\partial N}+\int_0^{Q_2^{CB}} \alpha_2(\tau_2)\frac{\partial Q_1^{CB}}{\partial N}\ d\tau_2.$$

Substituting for the Q_i^{CB}, $\frac{\partial Q_i^{CB}}{\partial N}$, and $\alpha_i(\cdot)$, and after some algebra, we have that $\frac{\partial W^{CB}}{\partial N} > 0$ if and only if $-N^2 - 2N + 6 > 0$ which never holds for $N \geq 2$. □

Proof of Lemma 3: From the first-order conditions from Proposition 2, we have that

$$0 = \alpha_i(Q_i^G)q_j^G - c_i + \alpha_i'(Q_i^G)q_j^G q_i^G + \alpha_j(Q_j^G)q_j^G,$$

for $i, j = 1, 2$ and $i \neq j$. Applying platform symmetry, so that $q_i^G = \frac{Q_i^G}{1+(N-1)\phi_i}$, and utilizing the assumptions noted at the start of Section 6 ($\alpha_i^{''}(\cdot) = 0$ and $c_i = 0$ for $i = 1, 2$), we have that

$$0 = \alpha_i(Q_i^G) + \alpha_i'(Q_i^G)\frac{Q_i^G}{1 + (N-1)\phi_i} + \alpha_j(Q_j^G),$$

for $i, j = 1, 2$ and $i \neq j$.

To determine an explicit expression for the $\frac{\partial Q_i^G}{\partial N}$, note that differentiable $\alpha_i^(\cdot)$ and the assumption that $\alpha_i^{''}(\cdot) = 0$ implies that the $\alpha_i(\cdot)$ are linear so that $\alpha_i(\tau_i) = a_i - b_i\tau_i$ for $i = 1, 2$. This linearization implies that:

$$0 = a_i - b_iQ_i^G - \frac{b_iQ_i^G}{1 + (N-1)\phi_i} + a_2 - b_2Q_2^G,$$

for $i, j = 1, 2$ and $i \neq j$. Solving the system of equations for Q_1^G and Q_2^G we have that

$$Q_i^G = \frac{(a_1 + a_2)[1 + (N-1)\phi_i]}{b_i[3 + (N-1)\phi_1 + (N-1)\phi_2]},$$

for $i = 1, 2$. Differentiating implies that

$$\frac{\partial Q_i^G}{\partial N} = \frac{(a_1 + a_2)}{b_i[3 + (N-1)\phi_1 + (N-1)\phi_2]^2}\cdot[\phi_i[3+(N-1)\phi_1+(N-1)\phi_2]-(\phi_1+\phi_2)[1+(N-1)\phi_i]],$$

for $i = 1, 2$. This implies that $\frac{\partial Q_i^G}{\partial N} > 0$ if and only if $2\phi_i > \phi_j$ for $i, j = 1, 2$ and $i \neq j$.

$\square$