Modeling Horizontal Shareholding with Ownership Dispersion*

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Abstract

The dominant formulation for modeling the objective function of the firm's manager in the presence of horizontal shareholding has been critiqued for producing the result that it may solely reflect the interests of a small number of shareholders even if, collectively, those shareholders do not have full control of the firm. We show that this issue can be avoided with an alternative formulation. This formulation is derived from a probabilistic voting model in which shareholders may differ in their evaluation of the amount of resources the manager will divert from the firm for personal use, which yields the result that the manager maximizes a control-weighted sum of the shareholders' relative (rather than absolute) expected returns.

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1 Introduction

Horizontal shareholding exists when shareholders own partial financial rights in several horizontal competitors in an industry. Such horizontal shareholding can induce a conflict in the firm-specific interests of shareholders, wherein horizontal shareholders in any given firm want that firm to pursue a less competitive strategy than the strategy desired by non-horizontal shareholders.[1] Hence, firm managers must weigh the eventual conflicting (financial) interests of different shareholders according to their relative (control) influence over the decision-making of their firm.

We identify six desirable properties for the weighting scheme (objective function) used by managers. The first five properties follow (a selection of) the proposals in Schmalz (2018a): (*i*) absent horizontal shareholding, managers would not weigh the expected profit of rival firms, which implies that managers would decide the strategy of their firm to maximize own expected profit; (*ii*) with non-infinitesimal horizontal shareholding, managers would weigh the interests of horizontal shareholders by assigning a positive weight to the expected profit of rival firms when those shareholders have control rights in the firm and financial rights in both firms, which implies that managers would internalize the impact of their firm's strategy on the expected profit of rival firms; (*iii*) the weight that managers assign to the expected profit of rival firms would be continuous on the financial and control rights of the shareholders that have financial rights in the firm; (*iv*) the weight that managers assign to the expected profit of rival firms would be one when all the shareholders that have financial rights in the firm are fully diversified across rivals, which implies that managers would maximize the expected profit of the industry; and (*v*) the elements of the weight that managers assign to the expected profit of rival firms would have empirical counterparts, i.e., would be measurable. The final property is motivated by the discussions in Gramlich and Grundl (2017), O'Brien and Waehrer (2017), Crawford *et al.* (2018) and Schmalz (2018a). It addresses the influence of shareholders over managers in the presence of ownership dispersion: (*vi*) managers would not weigh - solely - the interests of horizontal (non-horizontal) shareholders when those shareholders do not have full control, even if the ownership of each non-horizontal (horizontal) shareholder is dispersed among a collection of infinitesimal identical shareholders.

The dominant formulation of the objective function of managers is due to O'Brien and Salop (2000, henceforth O&S) who, incorporating features from both Rotemberg (1984) and Bresnahan and Salop (1986), assume that the interests of each shareholder can be captured

[1] Although non-horizontal shareholders may favor a different firm-specific strategy, that does not mean they are harmed by horizontal shareholding because horizontal shareholding also reduces the competitiveness of rival firms, and non-horizontal shareholders benefit from a reduction of competition between the firm and its rivals (please see Schmalz, 2018b for a formal model).

by the expected return from her financial investments and, as such, *the manager would decide the strategy of the firm to maximize a control-weighted sum of the firm's shareholders expected returns*. Because those returns are a function of the profits of the firms in which shareholders hold financial rights, this implies that the manager *would decide the strategy of the firm to maximize a weighted sum of the expected profits of all the firms in the industry over which controlling shareholders have financial rights in*. Azar (2016, 2017) and Brito *et al.* (2018) show that this formulation can be microfounded through a voting model in which shareholders vote to elect the manager from two potential candidates, an incumbent and a challenger, with conceivably differing strategy proposals to the firm.[2] Shareholders are assumed to care about the expected returns that result from the different strategy proposals and to have a profit-irrelevant bias for (or against) the challenger - since the credibility (or lack of credibility) of the incumbent, being already in office, is known to shareholders, while that of the challenger is not. Voting is probabilistic in the sense that this bias, while known to voters, is unobserved by candidates, who treat it as random.

However, this dominant formulation fails, as Gramlich and Grundl (2017), O'Brien and Waehrer (2017) and Crawford *et al.* (2018) discuss, property (*vi*). As the ownership of each non-horizontal (horizontal) shareholder becomes dispersed (among a collection of infinitesimal identical shareholders), the objective function of managers would weigh solely the interests of horizontal (non-horizontal) shareholders, even if those horizontal (non-horizontal) shareholders *do not* have full control of the firm. In other words, the dominant formulation yields that the objective function of managers may solely reflect the interests of a small number of shareholders even if, collectively, those shareholders do not have full control of the firm.

In this paper, we propose an alternative formulation. In the lines of Azar (2016, 2017) and Brito *et al.* (2018), we also use a probabilistic voting model, in which shareholders are assumed to care about the expected returns that result from the different strategic proposals and to have a bias for (or against) the challenger. However, in contrast with that literature, we assume that the bias is the result of a difference in shareholders expectations regarding their returns, which implies it is profit-relevant. We root this bias in the evaluation of shareholders regarding the amount of resources candidates will, once elected, divert from the firm for personal use.[3] This key, distinctive assumption implies that the relevance of the bias - from the perspective of candidates, who treat it as random - will now be proportional to the financial stakes of each shareholder - since the diversion of resources from the firm

[2] Azar (2017) also considers a probabilistic voting model in which shareholders vote on whether to approve a manager-proposed change in the firm's strategic plan.

[3] Naturally, other microfoundations would be possible. We discuss two alternative microfoundations, below.

affects each shareholder in proportion to her share of financial rights. As a consequence, all the determinants of shareholders voting behavior - the expected profits of firms (gross of diversion) and diversion by managers - will now be proportional to the financial stakes of each shareholder. We show that the equilibrium of this probabilistic voting model can be entirely equivalent to a setting in which the manager *would decide the strategy of the firm to maximize a control-weighted sum of the firm's shareholders* *relative* *expected returns* (with respect to their financial stakes in the firm), which capture the shareholders ideal expected profit weights (regarding the firm).[4,5] This proposed alternative formulation satisfies property (*vi*) in the sense that the objective function of managers would solely weigh the interests of horizontal (non-horizontal) shareholders as the ownership of each non-horizontal (horizontal) shareholder becomes dispersed (among a collection of infinitesimal identical shareholders) if horizontal (non-horizontal) shareholders *do* have full control of the firm. Furthermore, it is similar in nature to the formulation in Crawford *et al.* (2018) who, to address this property, consider normalizing the expected returns of shareholders by their overall financial investments, but we microfound our proposal through a probabilistic voting model.

The remainder of the paper is organized as follows. Section 2 presents the theoretical framework under which the proposed alternative formulation for the objective function of managers is derived, Section 3 discusses extensions to - and competition policy applications of - this theoretical framework and Section 4 concludes.

2 The Theoretical Framework

This section introduces the theoretical framework under which the proposed alternative formulation for the objective function of managers is derived. The general setting combines features from Brito *et al.* (2014), Azar (2016, 2017) and Brito *et al.* (2018) as follows.

First, we consider that there are K shareholders, external to the industry, indexed by k, and N single-product firms, indexed by j, whose total stock is composed of voting stock and non-voting (preferred) stock. Both give the holder the right to a share of the firm's profits, but only the former gives the holder the right to vote in the firm's shareholder assembly. The holdings $\phi_{kj} \in [0, 1]$ of total stock of shareholder k in firm j, regardless of

[4] The ideal weight associated to the own firm's expected profit is normalized to one while the ideal weight associated to a rival firm's expected profit is given by ratio between the shareholder's financial rights in the rival and her financial rights in the firm.

[5] This means that if a shareholder owns a portfolio that is proportional to another shareholder's portfolio, the manager will consider they both have the same relative expected returns. Their control rights will naturally be different, but their relative expected returns will be the same. This makes the smaller shareholder more relevant in the manager's objective function than under the dominant formulation, in which this shareholder would have smaller control rights and also smaller absolute expected returns.

whether it be voting or non-voting stock, capture her *financial rights* to the firm's profits. The holdings $\upsilon_{kj} \in [0,1]$ of voting stock of shareholder k in firm j, capture her *voting rights* in the firm. These voting rights may not necessarily coincide with her *control rights* in the firm, $\gamma_{kj} \in [0,1]$, which refer to her rights to influence the decisions of firm j, to be discussed below.[6]

Second, we consider that the ownership structure is such that a subset of shareholders can hold financial and voting rights in multiple firms of the industry. This horizontal shareholding can induce a conflict in the firm-specific interests of shareholders, which managers must weigh according to the corresponding relative influence over the decision-making of their firm.

Finally, we consider that the profits of each of the different firms in the industry is a function not only of the strategies of all the firms but also of a state of nature. This implies that firms' profits and, consequently, shareholders' returns - since they are a function of the profits of the firms in which they hold financial rights - are random.

2.1 Dominant Formulation

As discussed above, the dominant formulation of the objective function of managers is due to O&S, who assume that the interests of each shareholder can be captured by the expected return from her financial investments and, as such, *the manager would decide the strategy of the firm to maximize a control-weighted sum of the expected returns of the firm's shareholders*, as follows:

$$\max_{x_j} \sum\nolimits_{k\in\Theta} \gamma_{kj}\mathbb{E}\left[R_k\right] = \sum\nolimits_{k\in\Theta_j} \gamma_{kj}\mathbb{E}\left[R_k\right], \tag{1}$$

where Θ denotes the set of existing shareholders, Θ_j denotes the subset of shareholders that hold financial rights in firm j, x_j denotes the strategy of firm j and, finally, R_k denotes the return of shareholder k, which is a function of the profits of all the firms in which shareholder k holds financial rights: $R_k = \sum_{g\in\Im} \phi_{kg}\Pi_g$, with $\Im$ denoting the set of existing firms and Π_g denoting the profit of firm g. This implies that the manager of any firm j would maximize a weighted sum of the expected profits of (potentially) all the firms in the industry:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj}\mathbb{E}\left[R_k\right] = \sum\nolimits_{k\in\Theta_j} \gamma_{kj}\phi_{kj}\mathbb{E}\left[\Pi_j\right] + \sum\nolimits_{g\in\Im, g\neq j}\left(\sum\nolimits_{k\in\Theta_j} \gamma_{kj}\phi_{kg}\right)\mathbb{E}\left[\Pi_g\right], \tag{2}$$

which, normalizing the weight on own expected profit to one, is entirely equivalent to:

$$\max_{x_j} \mathbb{E}\left[\Pi_j\right] + \sum\nolimits_{g\in\Im, g\neq j} \frac{\sum_{k\in\Theta_j} \gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta_j} \gamma_{kj}\phi_{kj}}\mathbb{E}\left[\Pi_g\right]. \tag{3}$$

[6] Short-sales are not allowed and so financial, voting and control rights are non-negative.

As a consequence, the weight w_{jg} that the manager of firm j assigns to the expected profit of rival firm g is, under this dominant formulation, thus, given by $w_{jg} = \frac{\sum_{k\in\Theta_j}\gamma_{kj}\phi_{kg}}{\sum_{k\in\Theta_j}\gamma_{kj}\phi_{kj}} \geq 0$ for any $j, g \in \Im$ and $j \neq g$.[7] Azar (2016, 2017) and Brito *et al.* (2018) microfound this formulation through a voting model in which shareholders vote to elect the manager from two potential candidates with conceivably differing strategy proposals to the firm. They show that if candidates choose strategy proposals to maximize their expected utility from corporate office, control rights would be endogenously measured by Banzhaf (1965)'s power index while if they choose strategy proposal to maximize their expected vote share, control rights would be endogenously measured by voting rights. Proposition 1 establishes the properties of the weighting scheme under this dominant formulation using the results in Azar (2016, 2017) and Brito *et al.* (2018).

Proposition 1 *Using the corporate control measures derived in Azar (*2016, 2017*) and Brito et al. (*2018*), the objective function of managers under the dominant formulation satisfies properties (i) to (v). However, it fails property (vi).*
Proof. See Appendix.

Proposition 1 makes clear, as discussed in Gramlich and Grundl (2017), O'Brien and Waehrer (2017) and Crawford *et al.* (2018), that the dominant formulation fails property (*vi*). The reason is rooted on the fact that, under this formulation, a dispersion of ownership from a large non-horizontal (horizontal) shareholder to a collection of small identical non-horizontal (horizontal) shareholders that is equally large in aggregate decreases the relative influence over the manager of the firm, independently of the impact of the dispersion on the distribution of the firm's control rights. As a consequence, when the ownership of each non-horizontal shareholder becomes dispersed among a collection of infinitesimal identical shareholders, $\sum_{k\in\Theta_j}\gamma_{kj}\phi_{kj}$ tends to reflect solely the interests of the (the non-dispersed) horizontal shareholders as the summation referent to the non-horizontal shareholders approximates zero. As such, the manager weighs solely the interests of the horizontal shareholders, even when their voting rights do not induce full control of the firm. Similarly, when the ownership of each horizontal shareholder becomes dispersed among a collection of infinitesimal identical shareholders (a) $\sum_{k\in\Theta_j}\gamma_{kj}\phi_{kj}$ tends to reflect solely the interests of the non-horizontal shareholders as the summation referent to the horizontal shareholders approximates zero; and (b) $\sum_{k\in\Theta_j}\gamma_{kj}\phi_{kg}$ approximates zero. As such, the manager weighs solely the interests of non-horizontal shareholders, even when their voting rights do not induce full control of the firm.

[7] In order to see why the weights w_{jg} are non-negative, note that $\gamma_{kj} \geq 0$, $\phi_{kj} > 0$ and $\phi_{kg} \geq 0$ for all $k \in \Theta_j$ and all $j, g \in \Im$.

2.2 Proposed Alternative Formulation

We follow Azar (2016, 2017) and Brito *et al.* (2018) in microfounding the proposed alternative formulation through a voting model in which shareholders vote at the firm's shareholder assembly to elect the manager from two potential candidates, an incumbent a_j and a challenger b_j, with the candidate receiving the majority of voting rights being elected manager of the firm. Candidates are assumed to be opportunistic in the sense their only motivation is to hold office. To do so, they compete for the voting rights of shareholders by - simultaneously and noncooperatively - proposing a strategy for the firm, which is assumed binding in line with the literature on electoral competition (Down, 1957; Lindbeck and Weibull, 1987; Polo, 1998; and Persson and Tabellini, 2000). Shareholders are assumed to care about the utility derived from their expected returns and, as such, vote sincerely - simultaneously and noncooperatively - for the candidate whose strategy proposal maximizes their expected returns, randomizing between the two in case of indifference. Shareholders expectation regarding their returns may have, however, a bias for (or against) the challenger. We assume the following regarding this bias.

Assumption 1 *The bias of shareholders for (or against) the challenger is profit-relevant, rooted in the evaluation of shareholders regarding the amount of resources candidates will, once elected, divert from the firm for personal use.*

Assumption 1 constitutes implies that we root the profit-relevant bias in the disloyalty of candidates to shareholders (as in Bebchuk and Jolls, 1999; Pagano and Immordino, 2012; Amess *et al.*, 2015; Noe *et al.*, 2015; and Goshen and Levit, 2019). In particular, we assume that the two candidates may differ in their ability or willingness, once elected, to divert resources from the firm for personal use.[8] This diversion is assumed a permanent trait of the candidates that cannot be credibly modified or communicated to shareholders (as in Lindbeck and Weibull, 1987; and Persson and Tabellini, 2000) and over which shareholders may have different evaluations.

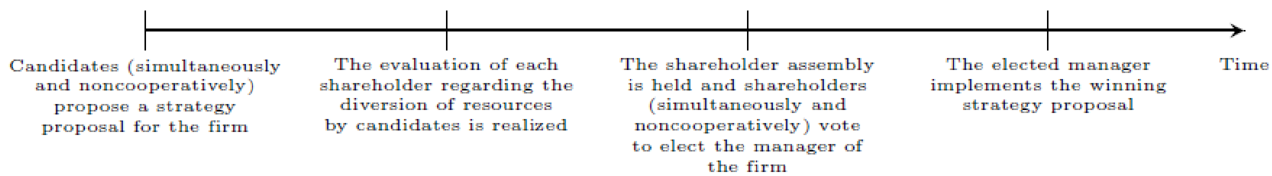
Although we root the bias in the evaluation of shareholders regarding the amount of resources candidates will, once elected, divert from the firm for personal use, other microfoundations would be possible. Assuming that the two candidates may differ in the amount of resources diverted for personal use (disloyalty) is equivalent to assuming that the two candidates may differ in their incompetence or in their cost to exert effort (see, for example, Gomes, 2000; and Goshen and Levit, 2019). Further, assuming that shareholders may differ in their evaluation of the amount of resources candidates will divert, once elected, for

[8] See, for example, Shleifer and Vishny (1997) for a survey of the various mechanisms through which managers may divert resources from the firm.

personal use is equivalent to assuming that firms have a governance mechanism to detect managerial diversion (as in Desai *et al.*, 2007; Pagano and Immordino, 2012; Amess *et al.*, 2015; Noe *et al.*, 2015; and Li and Li, 2018) and shareholders may differ in their evaluation of (and/or may not all be equally informed about) the effectiveness of this mechanism in deterring illicit diversion (and enforce its reimbursement), modelled to depend on the tenure of the candidate within the firm.

FIGURE 1

Timing



In our framework, the time of events for each firm j, depicted in Figure 1, is as follows. First, the two candidates propose a strategy for the firm. Let $x_{a_j} \in \Omega_j$ and $x_{b_j} \in \Omega_j$ denote the strategy proposals of the incumbent and the challenger, respectively, and Ω_j denote the strategy space available to the candidates, which can refer to any decision variable(s) - e.g., quantity, price, R&D investment, etc. - of the firm. Candidates know the distribution from which the evaluation of shareholders regarding managerial diversion is drawn, but not the realized values. Second, the actual evaluation of each shareholder regarding the diversion of resources by candidates is realized. Let $\xi_{kj} = \xi_{kj}^a - \xi_{kj}^b \lesseqgtr 0$ denote the realized evaluation of shareholder k regarding the difference in the amount of resources to be diverted by the incumbent and the challenger.[9] We follow the literature on electoral competition (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000; Ponzetto, 2011; Matějka and Tabellini, 2017) in allowing this realized evaluation to be disaggregated into two independently drawn components: $\xi_{kj} = \tilde{\xi}_j + \tilde{\xi}_{kj}$, where $\tilde{\xi}_j$ denotes a component common to all shareholders of firm j and $\tilde{\xi}_{kj}$ denotes a component specific to shareholder k, which is independent and identically drawn across the shareholders of firm j. Let $H_j(\cdot)$ and $G_j(\cdot)$ denote the cumulative distribution functions from which these common and specific components, respectively, are drawn. Third, the shareholder assembly is held and the candidate that receives the majority of the firm's voting rights is elected manager. Let $m_j \equiv \{a_j, b_j\}$ denote the identity of the

[9]The assumption that the amount of resources diverted for personal use is fixed, in the sense it does not depend on the profit of the firm (and consequently on the strategy proposal of the candidates), while contrasting with some literature that has assumed this amount to be a proportion of the profits (or cash-flows) of the firm (see, for example, Gomes, 2000; Desai *et al.,* 2007; Li and Li, 2018; Iacopetta *et al.,* 2019), it is far from uncommon (see, for example, Bebchuk and Jolls, 1999; Pagano and Immordino, 2012; Amess *et al.,* 2015; Noe *et al.,* 2015; Goshen and Levit, 2019).

elected manager. Finally, the elected manager implements the winning strategy proposal $x_j \equiv \{x_{aj}, x_{bj}\}$.

2.2.1 Shareholders Voting

We begin by addressing the equilibrium regarding the voting behavior of shareholders. As discussed above, shareholders are assumed to care about the utility derived from their expected returns. We follow O&S in assuming that the utility u_k of each shareholder k is a linear function of the expected return from her financial rights, which - in the above framework - will be a function of - both - the winning strategy proposals in all the firms of the industry and the identity of the corresponding elected managers:

$$u_k(\mathbf{x}, \mathbf{m}) = \mathbb{E}_k[R_k(\mathbf{x}, \mathbf{m})] = \sum\nolimits_{g \in \Im} \phi_{kg} \mathbb{E}_k[\Pi_g(\mathbf{x}, m_g)], \tag{4}$$

where $\mathbf{m} = (m_1, \ldots, m_j, \ldots, m_N)^\top$ denotes the $N \times 1$ vector of elected managers for all the firms in the industry and $\mathbf{x} = (x_1, \ldots, x_j, \ldots, x_N)^\top$ denotes the corresponding $N \times 1$ vector of winning strategy proposals.

Assumption 1 implies that the expected profit of each firm g for shareholder k can be written as the sum of two components: (a) the firm's expected gross profit, $\mathbb{E}[\pi_g(\mathbf{x})]$, which captures the expected profit before manager diversion, which is a function of the winning strategies in all the firms of the industry and assumed to be publicly generated - by, for example, the documentation distributed and discussed in the shareholder assemblies of the different firms - so that every shareholder has, conditional on the candidates proposals, the same expectation; and (b) the realized evaluation of shareholder k regarding the amount of resources to be diverted from the firm by the incumbent, ξ^a_{kg}, and the challenger, ξ^b_{kg}, which is a function of the elected manager. As a consequence, we can write the utility u_k of each shareholder k as follows:

$$\begin{aligned} u_k(\mathbf{x}, \mathbf{m}) &= \sum\nolimits_{g \in \Im} \phi_{kg} \left(\mathbb{E}[\pi_g(\mathbf{x})] - 1(m_g = a_g)\, \xi^a_{kg} - 1(m_g = b_g)\, \xi^b_{kg}\right) \\ &= \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right] - D_k(\mathbf{m}), \end{aligned} \tag{5}$$

where $\tilde{R}_k(\mathbf{x}) = \sum_{g \in \Im} \phi_{kg} \mathbb{E}[\pi_g(\mathbf{x})]$ denotes the expected gross return of shareholder k, and $D_k(\mathbf{m}) = \sum_{g \in \Im} \phi_{kg}(1(m_g = a_g)\xi^a_{kg} + 1(m_g = b_g)\xi^b_{kg})$ denotes the evaluation of shareholder k regarding the amount of resources to be diverted.[10]

As discussed above, we assume that shareholders vote sincerely, in each firm's shareholder

[10] $1(m_g = a_g)$ and $1(m_g = b_g)$ denote a dummy variable that takes the value 1 if the incumbent and the challenger, respectively, is elected manager of firm g.

assembly, for the candidate whose strategy proposal, given their evaluation of managerial diversion, maximizes their utilities, randomizing between the two in case of indifference. Following Alesina and Rosenthal (1995), Azar (2016) and Brito *et al.* (2018), we also assume the following regarding this voting behavior.

Assumption 2 *Shareholders are conditionally sincere.*

Assumption 2 implies that the vote of shareholders is, conditional on the equilibrium strategy proposals of the candidates to the remaining firms, deterministic, as follows: shareholder k will vote for firm j's incumbent with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)$, will vote for firm j's challenger with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) < u_k(\mathbf{x}_b, \mathbf{m}_b)$, and will randomize between the two candidates with equal probability if $u_k(\mathbf{x}_a, \mathbf{m}_a) = u_k(\mathbf{x}_b, \mathbf{m}_b)$, where $\mathbf{x}_a = (x_1, \ldots, x_{aj}, \ldots, x_N)^\top$, $\mathbf{x}_b = (x_1, \ldots, x_{bj}, \ldots, x_N)^\top$, $\mathbf{m}_a = (m_1, \ldots, a_j, \ldots, m_N)^\top$ and $\mathbf{m}_b = (m_1, \ldots, b_j, \ldots, m_N)^\top$ condition on the equilibrium strategy proposals of the candidates to the remaining firms.

2.2.2 Candidates Strategy Proposals

We now address the choice of strategy proposals by candidates. As discussed above, candidates are opportunistic in the sense their only motivation is to hold office. Since at this stage, candidates know the distribution of the evaluation of shareholders regarding managerial diversion, but not the realized values, from their perspective, voting by shareholders is probabilistic. We follow Azar (2016, 2017) and Brito *et al.* (2018) in considering candidates choose their strategy proposals under two alternative assumptions.

Assumption 3a *Candidates choose strategy proposals to maximize their expected utility from corporate office.*

Assumption 3b *Candidates choose strategy proposals to maximize their expected vote share.*

We begin by addressing the choice of strategy proposals by candidates under Assumption 3a. In this setting, candidates choose strategy proposals to maximize the product of the probability that they are elected and the utility obtained from the rent associated with corporate office they expect to accrue conditional upon being elected. Let $\Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)$ and $\Pr(m_j = b_j|\mathbf{x}_a, \mathbf{x}_b)$ denote the probability that the incumbent and the challenger, respectively, are elected and let Ξ_{a_j} and Ξ_{b_j} denote the utility that the incumbent and the challenger, respectively, expect to accrue conditional upon being elected (and includes the resources each expects - in effect - to divert for personal use). As such, the incumbent chooses

x_{a_j} so to solve:

$$\max_{x_{a_j}} \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) \Xi_{a_j}, \tag{6}$$

while the challenger chooses x_{b_j} so to solve:

$$\max_{x_{b_j}} \Pr\left(m_j = b_j | \mathbf{x}_a, \mathbf{x}_b\right) \Xi_{b_j}. \tag{7}$$

Since $\Pr(m_j = b_j|\mathbf{x}_a, \mathbf{x}_b) = 1 - \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)$, it is straightforward to see that the solution to the maximization problem of the two candidates to firm j is symmetric. As such, we solve - for simplicity of exposition - solely the incumbent's problem. To do so, we must beforehand derive the probability that, in the candidates' perspective and given the common component $\tilde{\xi}_j$, each shareholder k votes for the incumbent, which is given by:

$$\begin{aligned}
\Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right) &= \Pr\left(u_k\left(\mathbf{x}_a, \mathbf{m}_a\right) > u_k\left(\mathbf{x}_b, \mathbf{m}_b\right)\right) \\
&= \Pr\left(\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] > \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right] + \phi_{kj}\left(\xi^a_{kj} - \xi^b_{kj}\right)\right) \\
&= \Pr\left(\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] > \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right] + \phi_{kj}\xi_{kj}\right) \\
&= \Pr\left(\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] > \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right] + \phi_{kj}\left(\tilde{\xi}_j + \tilde{\xi}_{kj}\right)\right) \\
&= \Pr\left(\tilde{\xi}_{kj} < \frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}} - \tilde{\xi}_j\right) \\
&= G_j\left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}} - \tilde{\xi}_j\right).
\end{aligned} \tag{8}$$

This result makes use of the fact that $\sum_{g\in\Im, g\neq j} \phi_{kg}(1(m_g = a_g)\xi^a_{kj} + 1(m_g = b_g)\xi^b_{kj})$ enters the utility obtained from both candidates and that, as discussed above, $\xi_{kj} = \xi^a_{kj} - \xi^b_{kj}$ and $\xi_{kj} = \tilde{\xi}_j + \tilde{\xi}_{kj}$. This implies that, given the common component $\tilde{\xi}_j$, the vote share of the incumbent is given by the sum, across all shareholders, of the product of the probability that each shareholder votes for them and the corresponding shareholder's voting rights: $\sum_{k\in\Theta_j} \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j)\upsilon_{kj} = \sum_{k\in\Theta_j} G_j(\frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)] - \mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\phi_{kj}} - \tilde{\xi}_j)\upsilon_{kj}$. As this vote share depends on the realized value of $\tilde{\xi}_j$, it is, from the perspective of candidates, probabilistic. In turn, this implies that the probability $\Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b\right)$ with which the incumbent is elected manager of the firm is given by:

$$\begin{aligned}\Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) &= \Pr\left(\sum\nolimits_{k\in\Theta_j} \Pr\nolimits_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right) \upsilon_{kj} \geq \frac{1}{2}\right) \\ &= \Pr\left(\sum\nolimits_{k\in\Theta_j} G_j\left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}} - \tilde{\xi}_j\right) \upsilon_{kj} \geq \frac{1}{2}\right),\end{aligned} \tag{9}$$

which captures the probability with which she receives the majority of voting rights. As a consequence, the incumbent chooses x_{a_j} so to solve:

$$\max_{x_{a_j}} \Pr\left(\sum\nolimits_{k\in\Theta_j} G_j\left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}} - \tilde{\xi}_j\right) \upsilon_{kj} \geq \frac{1}{2}\right) \Xi_{a_j}. \tag{10}$$

We now address the choice of strategy proposals by candidates under Assumption 3b. In this setting, candidates choose strategy proposals to maximize the expected sum (since it depends on the realized value of $\tilde{\xi}_j$), across all shareholders, of the product of the probability that each shareholder votes for them and the corresponding shareholder's voting rights. As such, the incumbent chooses x_{a_j} so to solve:

$$\max_{x_{a_j}} \mathbb{E}\left[\sum\nolimits_{k\in\Theta_j} \Pr\nolimits_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right) \upsilon_{kj}\right], \tag{11}$$

while the challenger chooses x_{b_j} so to solve:

$$\max_{x_{b_j}} \mathbb{E}\left[\sum\nolimits_{k\in\Theta_j} \Pr\nolimits_{kb_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j\right) \upsilon_{kj}\right], \tag{12}$$

where the first expected value denotes the expected value with respect to the common component $\tilde{\xi}_j$. Since $\Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j) = 1 - \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b, \tilde{\xi}_j)$ for each shareholder k, it is straightforward to see that the solution to the maximization problem of the two candidates to firm j is symmetric. As such, we solve - for simplicity of exposition - solely the incumbent's problem, who chooses x_{a_j} so to solve:

$$\max_{x_{a_j}} \sum\nolimits_{k\in\Theta_j} \mathbb{E}\left[G_j\left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}} - \tilde{\xi}_j\right)\right] \upsilon_{kj}. \tag{13}$$

The objective functions (10) and (13), constructed under Assumptions 3a and 3b, respectively, make clear that candidates, when designing strategy proposals, pay attention not to the absolute expected (gross) returns of each shareholder k, $\mathbb{E}[\tilde{R}_k(\cdot)]$, but to her *relative* expected (gross) returns, $\frac{\mathbb{E}[\tilde{R}_k(\cdot)]}{\phi_{kj}} = \mathbb{E}[\pi_g(\mathbf{x})] + \sum_{g\in\Im, g\neq j} \frac{\phi_{kg}}{\phi_{kj}} \mathbb{E}[\pi_g(\mathbf{x})]$, which establish the weights shareholder k would *desire* the manager to associate to the expected (gross) profit of each firm in the industry - relative to the expected (gross) profit of the firm.

2.2.3 Nash-Equilibrium

Having described the maximization problem of the candidates, we now address the pure-strategy Nash equilibrium for the candidates strategy proposals' game. To do so, we follow Azar (2016, 2017) and Brito *et al.* (2018) in making the following technical assumptions regarding the strategy space Ω_j available to the candidates of each firm j, the expected gross return $\mathbb{E}[\tilde{R}_k(\mathbf{x})]$ of each shareholder k and the cumulative distribution functions $H_j(\cdot)$ and $G_j(\cdot)$.

Assumption 4 *The strategy space Ω_j available to the candidates of each firm j is a nonempty compact subset of $\Re$.*

Assumption 5 *The expected gross return $\mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]$ of shareholder k is (a) continuous and twice differentiable in $\mathbf{x}$, with continuous second derivatives; and (b) strictly concave in firm j's strategy $x_j \in \{x_{a_j}, x_{b_j}\}$, conditional on the strategies of the remaining firms.*

Assumption 6 *$H_j(\cdot)$ and $G_j(\cdot)$ are the cumulative distribution functions of uniform distributions over the range $\left[-\frac{1}{2}\varphi_j, \frac{1}{2}\varphi_j\right]$ and $\left[-\frac{1}{2}\tau_j, \frac{1}{2}\tau_j\right]$, respectively, with φ_j, τ_j sufficiently large for each firm j.*[11,12]

Assumptions 1 to 6 ensure the existence of a pure-strategy Nash equilibrium for the candidates strategy proposals' game $(x_{a_1}, x_{b_1}, \ldots x_{a_j}, x_{b_j}, \ldots, x_{a_N}, x_{b_N})$, characterized as follows.

Proposition 2 *Under Assumptions* 1, 2, 3a *or* 3b*,* 4, 5 *and* 6*, there exists a pure-strategy Nash equilibrium for the candidates strategy proposals' game that is entirely equivalent to the pure-strategy Nash-equilibrium from the case in which each candidate maximizes the following*

[11] Assumption 6 requires the support of $H_j(\cdot)$ and $G_j(\cdot)$ to be sufficiently large (relative to their argument - not to gross profits) so to rule out corner solutions for probabilities (8) and (9). This technical assumption is standard and explicit in the electoral competition literature (Persson and Tabellini, 2000; Ponzetto, 2011; Matějka and Tabellini, 2017) so that the behavior underlined by probabilities (8) and (9) is not perfectly predictable on the basis of strategy proposals. It is also standard - although implicitly - in the literature that microfounds the O&S formulation (Azar, 2016, 2017; Brito *et al.*, 2018).

[12] The requirements regarding the cumulative distribution function $H_j(\cdot)$ can be relaxed under Assumption 3b. In this scenario the only requirement regarding the cumulative distribution function $H_j(\cdot)$ is that $\mathbb{E}(\tilde{\xi}_j) = 0$ for each firm j.

objective function:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[R_k\right]}{\phi_{kj}}, \tag{14}$$

where $\frac{\mathbb{E}[R_k]}{\phi_{kj}}$ *denotes the relative expected return of shareholder* k *and* γ_{kj} *is measured by the voting rights of shareholder* k *in firm* j*:* $\gamma_{kj} = \upsilon_{kj}$.

Proof. See Appendix.

Proposition 2 establishes that the two candidates would choose the same strategy proposal for each firm j, conditional on the strategies of the candidates to the remaining firms. Further, it establishes also, in contrast with O&S, that candidates (and thus the elected manager) would decide the strategy of the firm to maximize a weighted sum of the firm's shareholders *relative* expected returns, which establish their *ideal* expected profit weights. Furthermore, it establishes that the weights γ_{kj}, which capture the control of shareholders over the decision-making of the firm, would be endogenously measured by their voting rights. Finally, it implies that the manager of each firm j would still maximize a weighted sum of the expected total profits of (potentially) all the firms in the industry:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[R_k\right]}{\phi_{kj}} = \mathbb{E}\left[\Pi_j\right] + \sum\nolimits_{g\in\Im, g\neq j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\phi_{kg}}{\phi_{kj}} \mathbb{E}\left[\Pi_g\right], \tag{15}$$

where the weight that the manager of firm j assigns to the expected profit of rival firm g is, under this proposed new, alternative formulation of the objective function of managers, thus, given by $w_{jg} = \sum_{k\in\Theta_j} \gamma_{kj} \frac{\phi_{kg}}{\phi_{kj}} \geq 0$ for any $j, g \in \Im$ and $j \neq g$.[13] The key, distinctive assumption driving this alternative formulation is Assumption 1.[14] A profit-relevant bias implies that all the determinants of shareholders voting behavior - the expected profits of firms (gross of diversion) and diversion by managers - will be proportional to the financial stakes of each shareholder and, in turn, that candidates (and thus the elected manager) would decide the strategy of the firm to maximize a weighted sum of the shareholders *relative* expected returns.

Proposition 3 establishes the properties of the weighting scheme under this proposed alternative formulation.

Proposition 3 *Using the corporate control measures derived in Proposition* 2*, the objective function of managers under the proposed alternative formulation satisfies properties (i) to*

[13] The weights w_{jg} are non-negative by the same arguments as were used in footnote 7.

[14] Assumptions 2 to 5 are similar to the literature that microfounds the O&S formulation (Azar, 2016, 2017; Brito *et al.*,2018). Although Assumption 6, in contrast to that literature, allows for a correlation in the bias of shareholders, we show below that it is not this correlation that is driving the proposed alternative formulation. This makes clear that the key, distinctive assumption is Assumption 1.

(vi).

Proof. See Appendix.

Proposition 3 makes clear that the proposed alternative formulation satisfies property (*vi*), addressing the critique in Gramlich and Grundl (2017), O'Brien and Waehrer (2017) and Crawford *et al.* (2018) regarding the dominant formulation. The reason is rooted on the fact that, under this proposed alternative formulation, a dispersion of ownership from a large non-horizontal (horizontal) shareholder to a collection of small identical non-horizontal (horizontal) shareholders that is equally large in aggregate does not impact the relative influence over the manager of the firm,[15] unless such dispersion impacts the distribution of the firm's control rights.[16] As a consequence, the interests of horizontal (non-horizontal) shareholders will only disappear from the objective function of the manager when the voting rights of the non-horizontal (horizontal) shareholders do induce full control of the firm.

2.2.4 Shareholders Independent Evaluation

We have assumed a framework in which the bias of shareholders for or against the challenger - rooted in their evaluation regarding managerial diversion - may be correlated across shareholders. This contrasts with the literature that microfounds the dominant formulation of the objective function of managers (Azar, 2016, 2017; Brito *et al.*, 2018), which assumes this bias to be independent across shareholders. In this section, we derive our proposed alternative formulation for the setting in which $\tilde{\xi}_j = 0$ for each firm j. This implies that the evaluation of shareholders regarding managerial diversion in each firm coincides with the idiosyncratic component, which is independently drawn across shareholders. In this setting, shareholder k votes for the incumbent with probability $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$, which is given by:

$$\begin{aligned} \Pr_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) &= \Pr\left(\tilde{\xi}_{kj} < \frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}}\right) \\ &= G_j\left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}}\right). \end{aligned} \tag{16}$$

We can then use this result to derive the probability $\Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)$ with which the incumbent is elected manager of the firm. To do so, let ℓ_j denote the number of shareholders

[15] Since the dispersion is among identical shareholders, it does not impact relative returns.

[16] The weight w_{jg} is - for a given distribution of control rights - constant with respect to the dispersion of the ownership of each shareholder (among a collection of infinitesimal identical shareholders). However, that does not imply that such dispersion does not have an impact on w_{jg}. It does, but solely via the control rights which are induced by the distribution of the firm's voting rights.

with voting rights in firm j, $\wp_j$ denote all the $2^{\ell_j - 1}$ possible subsets of those shareholders that can award the majority of votes to a candidate and $\Theta_j^\imath \in \wp_j$ denote a particular subset of those shareholders. Given that the election of the incumbent is ensured with the votes of the shareholders in each subset in $\wp_j$, we have that $\Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)$ just sums the probabilities with which she is elected in each subset $\Theta_j^\imath$, $\Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath)$, as follows:

$$\Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b\right) = \sum\nolimits_{\Theta_j^\imath \in \wp_j} \Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right). \tag{17}$$

The independence assumption implies that $\Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right)$ is just the product of the voting probabilities of the corresponding shareholders:

$$\Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right) = \prod\nolimits_{k \in \Theta_j^\imath} \Pr\nolimits_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \prod\nolimits_{k \notin \Theta_j^\imath} \left(1 - \Pr\nolimits_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)\right). \tag{18}$$

As such, the probability with which the incumbent is elected manager of the firm is given by:

$$\begin{aligned}
\Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b\right) &= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b, \Theta_j^\imath\right) \\
&= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k \in \Theta_j^\imath} \Pr\nolimits_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right) \prod\nolimits_{k \notin \Theta_j^\imath} \left(1 - \Pr\nolimits_{ka_j}\left(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b\right)\right) \\
&= \sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k=1}^{\ell_j} \left(\left(1 - d_k\right) + \left(2d_k - 1\right) G_j \left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}}\right)\right),
\end{aligned} \tag{19}$$

where the last equality makes use of probability (16), with d_k taking the value one if shareholder $k \in \Theta_j^\imath$ and takes the value zero otherwise. This implies that the incumbent's problem can be rewritten as follows. Under Assumption 3a, we have that:

$$\max_{x_{a_j}} \sum\nolimits_{\Theta_j^\imath \in \wp_j} \prod\nolimits_{k=1}^{\ell_j} \left(\left(1 - d_k\right) + \left(2d_k - 1\right) G_j \left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}}\right)\right) \Xi_{a_j}, \tag{20}$$

while, under Assumption 3b, we have that:

$$\max_{x_{a_j}} \sum\nolimits_{k \in \Theta_j} G_j \left(\frac{\mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_a\right)\right] - \mathbb{E}\left[\tilde{R}_k\left(\mathbf{x}_b\right)\right]}{\phi_{kj}}\right) \upsilon_{kj}. \tag{21}$$

The pure-strategy Nash equilibrium $(x_{a_1}, x_{b_1}, \ldots x_{a_j}, x_{b_j}, \ldots, x_{a_N}, x_{b_N})$ for the candidates strategy proposals' game is, then, characterized as follows.

Proposition 4 *If* $\tilde{\xi}_j = 0$ *for each firm* j*, under Assumptions* 1, 2, 3a *or* 3b*,* 4*,* 5 *and* 6*, there exists a pure-strategy Nash equilibrium for the candidates strategy proposals' game that is entirely equivalent to the pure-strategy Nash-equilibrium from the case in which each candidate maximizes the following objective:*

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[R_k\right]}{\phi_{kj}}, \tag{22}$$

where $\frac{\mathbb{E}[R_k]}{\phi_{kj}}$ *denotes the relative expected return of shareholder* k*. Under Assumption* 3a*,* γ_{kj} *is measured by the normalized Banzhaf (*1965*) power index of shareholder* k *in firm* j*:* $\gamma_{kj} = \frac{\lambda^p_{kj}}{\sum_{h\in\Theta_j}\lambda^p_{hj}}$*, where* λ^p_{kj} *denotes the number of subsets of* Θ_j *that can award victory to a candidate in which shareholder* k *is pivotal. Under assumption* 3b*,* γ_{kj} *is measured by the voting rights of shareholder* k *in firm* j*:* $\gamma_{kj} = \upsilon_{kj}$*.*

Proof. See Appendix.

Proposition 4 establishes that it is not the correlation in the bias of shareholders that is driving the result that candidates (and thus the elected manager) would decide the strategy of the firm to maximize a weighted sum of the firm's shareholders relative expected returns. The exact same formulation is obtained under the independence assumption. Moreover, Proposition 4 also establishes that the independence assumption is, as expected, instrumental in the derivation (by the literature that microfounds the dominant formulation of the objective function of managers) of the Banzhaf (1965)'s power index as an endogenous measure of corporate control.

3 Extensions and Applications

In this section, we introduce and discuss extensions to the framework discussed above as well as competition policy applications of the proposed alternative formulation for the objective function of managers.

3.1 Shareholders Inattention

We have assumed a framework in which shareholders are fully attentive to the strategy proposals of candidates. In this section, we examine the robustness of the proposed alternative formulation by discussing an extension framework in which shareholders can either be attentive or inattentive to those proposals. In particular, we follow Gilje *et al.* (2019) in considering that each shareholder k is attentive to the strategy proposals of firm j's candidates with probability δ_{kj} and inattentive with probability $1-\delta_{kj}$. If attentive, as discussed above, shareholder k will vote for the incumbent with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) > u_k(\mathbf{x}_b, \mathbf{m}_b)$, will vote for the challenger with probability 1 if $u_k(\mathbf{x}_a, \mathbf{m}_a) < u_k(\mathbf{x}_b, \mathbf{m}_b)$, and will randomize between the two candidates with equal probability if $u_k(\mathbf{x}_a, \mathbf{m}_a) = u_k(\mathbf{x}_b, \mathbf{m}_b)$. If inattentive, shareholder k will, irrespective of the strategy proposals of the candidates, vote for the incumbent with probability λ_k and will vote for the challenger with probability $1-\lambda_k$.

In this setting, it is relatively straightforward to show that the weights of the proposed alternative formulation would be given by $w_{jg} = \sum_{k\in\Theta_j} \gamma_{kj}\delta_{kj}\frac{\phi_{kg}}{\phi_{kj}} \geq 0$ for any $j, g \in \Im$ and $j \neq g$,[17] a weight that is qualitatively similar to the measure proposed by Gilje *et al.* (2019) to capture the impact of common ownership on managerial incentives. The attention probabilities δ_{kj} can be modelled to be a function of a multitude of observed firm and shareholder factors (for example, the importance of firm j in shareholder k's investment portfolio) and estimated using voting data (see Gilje *et al.,* 2019 for an illustrative example and the references therein). The extension of this framework to endogeneize the determinants of the attention probabilities seems to be a very interesting avenue for future research.

3.2 Cross-Ownership Structures

We have assumed a framework in which the ownership structure is such that shareholders are external to the industry. In this section, we examine the robustness of the proposed alternative formulation by discussing an extension framework in which shareholders can include rival firms of the industry. In particular, consider that there are K shareholders, indexed by $k \in \Theta \equiv \{1, \ldots, N, \ldots K\}$, who may include not just shareholders that are *external* to the industry (and can engage in common-ownership), but also shareholders from the subset of firms that are *internal* to the industry (and can engage in cross-ownership), both of which can hold financial and voting rights in multiple firms of the industry.

In this setting, it is relatively straightforward to show that the weights of the proposed alternative formulation would be given by $w_{jg} = \sum_{k\in\Theta_j} \gamma^u_{kj}\frac{\phi^u_{kg}}{\phi^u_{kj}} \geq 0$ for any $j, g \in \Im$ and $j \neq g$, where ϕ^u_{kj} and γ^u_{kj} denote the *ultimate* financial and control rights, respectively, of *external* shareholder k in firm j, which can be computed following the algorithm in Brito *et al.* (2018).

[17] This result can be derived within the framework in which the bias of shareholders for or against the challenger - rooted in their evaluation regarding managerial diversion - may be correlated across shareholders under Assumptions 3a and 3b. It can also be derived within the framework in which the bias of shareholders for or against the challenger is independent across shareholders under Assumptions 3a (if inattentive shareholders randomize between the two candidates with equal probability) and 3b.

3.3 Measuring Anti-Competitive Effects

We now discuss competition policy applications of the proposed alternative formulation. We highlight two, related to the quantification of the unilateral anti-competitive effects associated to partial horizontal acquisitions. This alternative formulation can be straightforwardly incorporated into (a) the generalized HHI and GUPPI indicators proposed in Brito *et al.* (2018); and (b) the structural empirical methodology proposed in Brito *et al.* (2014).

4 Conclusions

We propose an alternative formulation to model the objective function of managers in the presence of horizontal shareholding. In this alternative formulation, managers would decide the strategy of the firm by maximizing a weighted sum of the firm's shareholders relative (rather than absolute) expected returns. We do not claim it to be preferred to O&S's formulation. We solely propose it as a microfounded alternative which avoids an allegedly unattractive feature of the O&S's formulation: that it may solely reflect the interests of a small number of shareholders even if, collectively, those shareholders do not have full control of the firm. Future empirical testing might help establish which formulation more accurately predicts firm behavior.

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Mathematical Appendix

In this mathematical appendix, we present the proofs of Propositions 1 to 4.

Proof of Proposition 1

First, absent horizontal shareholding, the manager of each firm j would maximize the expected own-profit $\mathbb{E}[\Pi_j]$, since $\phi_{kg} = 0$ for the subset of shareholders k who hold financial rights in firm j and all $j, g \neq j$. This implies $w_{jg} = 0$ for all j,$g \neq j$ and, thus, that property (i) holds.

Second, with non-infinitesimal horizontal shareholding, the manager of each firm j would internalize the impact of her firm's strategy on the expected profit of rival firm g when the shareholders that have financial rights in both firms have also control rights in the firm, since if $\gamma_{kj} \neq 0$ and $\phi_{kg} >> 0$ for at least one shareholder k, we have that $w_{jg} \neq 0$ for all j,$g \neq j$. This implies that the manager of each firm j would maximize $\mathbb{E}[\Pi_j] + w_{jg}\mathbb{E}[\Pi_g]$ and, thus, that property (ii) holds.

Third, the weight w_{jg} that the manager of each firm j assigns to the expected profit of rival firm g is continuous in ϕ_{kj}, γ_{kj} and ϕ_{kg} for the subset of shareholders k with financial rights in firm j, since the product, sum and quotient, respectively, of continuous functions is continuous. This implies that property (iii) holds.

Fourth, the manager of each firm j would maximize the expected profit of the industry when all shareholders that have financial rights in the firm are fully diversified across rivals, since if those shareholders are fully diversified, for the subset of shareholders k who hold financial rights in firm j, we have $\phi_{kj} = \phi_{kg} = \phi_k$ and $\gamma_{kj} = \gamma_{kg} = \gamma_k$ for all j,$g \neq j$. This implies $w_{jg} = \frac{\sum_{k \in \Theta_j} \gamma_k \phi_k}{\sum_{k \in \Theta_j} \gamma_k \phi_k} = 1$ for all j,$g \neq j$ and that the manager of each firm j would maximize $\mathbb{E}[\Pi_j] + \sum_{g \in \Im, g \neq j} \mathbb{E}[\Pi_g]$. As such, property (iv) holds.

Fifth, the weight w_{jg} that the manager of each firm j assigns to the expected profit of rival firm g can be computed with the control rights of the shareholders in the firm and their financial rights in the two firms. The financial rights have clear empirical counterparts and can thus be measurable. The control rights have less clear empirical counterparts, but Azar (2016, 2017) and Brito *et al.* (2018) show that they can be endogenously measured (depending on the particular assumptions) by the shareholders Banzhaf (1965)'s power index or by their voting rights, both of which have clear empirical counterparts and can thus be measurable.

Finally, the objective function of the manager of firm j will approximate a weighted sum of (solely) the interests of the firm's horizontal (non-horizontal) shareholders when non-horizontal (horizontal) shareholders are highly dispersed, for any given value of the control rights of the horizontal (non-horizontal) shareholders. In order to see why, let the subset of shareholders that hold financial rights in firm j, Θ_j, be divided in two smaller subsets: the subset of horizontal shareholders, Θ_j^h, and the subset of non-horizontal shareholders, Θ_j^{nh}. This implies that the objective function of the manager of firm j can be written as follows:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j^h} \gamma_{kj}\phi_{kj}\mathbb{E}\left[\Pi_j\right] + \sum\nolimits_{k\in\Theta_j^{nh}} \gamma_{kj}\phi_{kj}\mathbb{E}\left[\Pi_j\right] + \sum\nolimits_{g\in\Im.g\neq j} \sum\nolimits_{k\in\Theta_j^h} \gamma_{kj}\phi_{kg}\mathbb{E}\left[\Pi_g\right]. \tag{23}$$

As the ownership of each non-horizontal shareholder becomes dispersed among a collection of infinitesimal identical shareholders, we have that $\sum_{k\in\Theta_j^{nh}} \gamma_{kj}\phi_{kj} \to 0$. As such, the objective function of the manager will weigh solely the interests of horizontal shareholders, even when the voting rights of those horizontal shareholders do not induce full control of the firm:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j^h} \gamma_{kj}\phi_{kj}\mathbb{E}\left[\Pi_j\right] + \sum\nolimits_{g\in\Im.g\neq j} \sum\nolimits_{k\in\Theta_j^h} \gamma_{kj}\phi_{kg}\mathbb{E}\left[\Pi_g\right]. \tag{24}$$

Similarly, as the ownership of each horizontal shareholder becomes dispersed among a collection of infinitesimal identical shareholders, we have that $\sum_{k\in\Theta_j^h} \gamma_{kj}\phi_{kj} \to 0$ and $\sum_{k\in\Theta_j^h} \gamma_{kj}\phi_{kg} \to 0$. As such, the objective function of the manager will weigh solely the interests of non-horizontal shareholders (yielding an objective function proportional to the expected own-profit), even when the voting rights of those non-horizontal shareholders do not induce full control of the firm:

$$\max_{x_j} \sum\nolimits_{k\in\Theta_j^{nh}} \gamma_{kj}\phi_{kj}\mathbb{E}\left[\Pi_j\right]. \tag{25}$$

This implies a failure of property (vi).

Proof of Proposition 2

The structure of this proof follows three steps.

First, we show that the objective function of the incumbent is strictly concave conditional on *the strategy proposal of the challenger to the firm* and on *the strategy proposals of the candidates to the rival firms*. Given that strategy proposals are, under Assumption 4, defined in a convex set, this implies that the incumbent's maximization problem has a unique maximum conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms. Given the symmetry of the solution to the maximization problem of the two candidates to the firm, we have that they will choose best-response functions that are, conditional on the strategy proposals of the candidates to the remaining firms, symmetric with respect to the strategy proposal of the opponent candidate. This implies that the two candidates will choose the same strategy proposal for the firm, conditional on the strategies proposals of the candidates to the rival firms, i.e., they will choose the same best-response function to the strategy proposals of the candidates to the rival firms. Since this *common* best-response function achieves, conditional on the strategies proposals of the candidates to the rival firms, the unique maximum of the objective functions of the two candidates to the firm, there are no unilateral incentives to deviation.

Second, we show that this common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms.

Finally, given that the strategy proposal of each candidate to the different firms is, under Assumption 4, defined in a convex set and the expected gross returns (and since managerial diversion is fixed, also the expected returns) of the firm's shareholders is, under Assumption 5, continuous, the best-response functions of the candidates to the different firms are guaranteed to be upper-hemicontinuous, which implies that we can apply Kakutani's fixed point theorem to ensure that the Nash equilibrium exists.

We now address the sub-proof of the remaining points: (a) that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms; and (b) that the common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms. We first present these sub-proofs under Assumption 3a and then under Assumption 3b.

Under Assumption 3a

We begin by showing that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms.

Let $\varpi_{aj} = \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)\,\Xi_{a_j} = \Pr\left(\sum_{k\in\Theta_j} G_j\left(\frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)]-\mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\phi_{kj}} - \tilde{\xi}_j\right)\upsilon_{kj} \geq \frac{1}{2}\right)\Xi_{a_j}$ denote the objective function of the incumbent under Assumption 3a. Under Assumption 2, shareholders are conditionally sincere, which implies that the incumbent of firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. The first order condition of this problem is, thus, given by:

$$\frac{\partial\varpi_{aj}}{\partial x_{a_j}} = \sum\nolimits_{k\in\Theta_j}\frac{1}{\varphi_j\phi_{kj}}\frac{\partial\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]}{\partial x_{a_j}}\upsilon_{kj}\Xi_{a_j}, \tag{26}$$

which makes use of the fact that, under Assumption 6:

$$\begin{aligned}
\varpi_{aj} &= \Pr\left(\sum\nolimits_{k\in\Theta_j} G_j\left(\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\phi_{kj}} - \tilde{\xi}_j\right)\upsilon_{kj} \geq \frac{1}{2}\right)\Xi_{a_j} \qquad (27)\\
&= \Pr\left(\sum\nolimits_{k\in\Theta_j}\left(\frac{1}{2}+\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\tau_j\phi_{kj}} - \frac{\tilde{\xi}_j}{\tau_j}\right)\upsilon_{kj} \geq \frac{1}{2}\right)\Xi_{a_j}\\
&= \Pr\left(\frac{1}{2}+\sum\nolimits_{k\in\Theta_j}\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\tau_j\phi_{kj}}\upsilon_{kj} - \frac{\tilde{\xi}_j}{\tau_j} \geq \frac{1}{2}\right)\Xi_{a_j}\\
&= \Pr\left(\frac{\tilde{\xi}_j}{\tau_j} \leq \sum\nolimits_{k\in\Theta_j}\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\tau_j\phi_{kj}}\upsilon_{kj}\right)\Xi_{a_j}\\
&= \Pr\left(\tilde{\xi}_j \leq \sum\nolimits_{k\in\Theta_j}\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\phi_{kj}}\upsilon_{kj}\right)\Xi_{a_j}\\
&= H_j\left(\sum\nolimits_{k\in\Theta_j}\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\phi_{kj}}\upsilon_{kj}\right)\Xi_{a_j}\\
&= \left(\frac{1}{2}+\sum\nolimits_{k\in\Theta_j}\frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]-\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right]}{\varphi_j\phi_{kj}}\upsilon_{kj}\right)\Xi_{a_j}.
\end{aligned}$$

In turn, the second order condition is given by:

$$\frac{\partial^2 \varpi_{aj}}{\partial x_{a_j}^2} = \sum\nolimits_{k\in\Theta_j} \frac{1}{\varphi_j \phi_{kj}} \frac{\partial^2 \mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]}{\partial x_{a_j}^2} \upsilon_{kj} \Xi_{a_j}, \tag{28}$$

which implies, given Assumption 5, that the objective function of the manager is strictly concave in x_{a_j}, conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms.

We now show that the common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms, with voting rights as weights. This is established by the first order condition above:

$$\max_{x_j} \Pr(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b)\, \Xi_{a_j} \propto \max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\phi_{kj}} \propto \max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[R_k(\mathbf{x})\right]}{\phi_{kj}}, \tag{29}$$

where $\gamma_{kj} = \upsilon_{kj}$ denotes the weight assigned by firm j's manager to the relative expected return of shareholder k, measured by the voting rights of shareholder k in firm j.

Under Assumption 3b

We again begin by showing that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms.

Let $\varpi_{aj} = \mathbb{E}\left[\sum_{k\in\Theta_j} \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\, \upsilon_{kj}\right] = \sum_{k\in\Theta_j} \mathbb{E}\left[G_j\left(\frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)] - \mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\phi_{kj}} - \tilde{\xi}_j\right)\right] \upsilon_{kj}$ denote the objective function of the incumbent under Assumption 3b. Under Assumption 2, shareholders are conditionally sincere, which implies that the incumbent of firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. The first order condition of this problem is, thus, given by:

$$\frac{\partial \varpi_{aj}}{\partial x_{a_j}} = \sum\nolimits_{k\in\Theta_j} \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]}{\partial x_{a_j}} \upsilon_{kj}, \tag{30}$$

which makes use of the fact that, under Assumption 6, $\mathbb{E}\left[G_j\left(\frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)] - \mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\phi_{kj}} - \tilde{\xi}_j\right)\right] = \frac{1}{2} + \frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)] - \mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\tau_j \phi_{kj}}$ since $\mathbb{E}\left[\frac{\tilde{\xi}_j}{\tau_j}\right] = 0$ (note that this makes clear that, under Assumption 3b, we can relax Assumption 6 regarding the cumulative distribution function $H_j(\cdot)$: the only requirement is that $\mathbb{E}\left(\tilde{\xi}_j\right) = 0$ for each firm j). In turn, the second order condition is given by:

$$\frac{\partial^2 \varpi_{aj}}{\partial x_{a_j}^2} = \sum\nolimits_{k\in\Theta_j} \frac{1}{\tau_j \phi_{kj}} \frac{\partial^2 \mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right]}{\partial x_{a_j}^2} \upsilon_{kj}, \tag{31}$$

which implies, given Assumption 5, that the objective function of the manager is strictly concave in x_{a_j}, conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms.

We now show that the common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms, with voting rights as weights. This is established by the first order condition above:

$$\max_{x_j} \mathbb{E}\left[\sum\nolimits_{k\in\Theta_j} \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\, \upsilon_{kj}\right] \propto \max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\phi_{kj}} \propto \max_{x_j} \sum\nolimits_{k\in\Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[R_k(\mathbf{x})\right]}{\phi_{kj}}, \tag{32}$$

where $\gamma_{kj} = \upsilon_{kj}$ denotes the weight assigned by firm j's manager to the relative expected return of shareholder k, measured by the voting rights of shareholder k in firm j.

Proof of Proposition 3

First, properties (i) to (iii) hold by the same arguments as were used in the proof presented for Proposition 1.

Second, the manager of each firm j would maximize the expected profit of the industry when all shareholders that have financial rights in the firm are fully diversified across rivals, since if those shareholders are fully diversified, for the subset of shareholders k who hold financial rights in firm j, we have $\phi_{kj} = \phi_{kg} = \phi_k$ and $\gamma_{kj} = \gamma_{kg} = \gamma_k$ for all j,$g \neq j$. This implies $w_{jg} = \sum_{k \in \Theta_j} \gamma_k \frac{\phi_k}{\phi_k} = \sum_{k \in \Theta_j} \gamma_k = 1$ for all j,$g \neq j$ and that the manager of each firm j would maximize $\mathbb{E}[\Pi_j] + \sum_{g \in \Im, g \neq j} \mathbb{E}[\Pi_g]$. As such, property (iv) holds.

Third, the weight w_{jg} that the manager of each firm j assigns to the expected profit of rival firm g can be computed with the control rights of the shareholders in the firm and their financial rights in the two firms. The financial rights have clear empirical counterparts and can thus be measurable. Proposition 2 shows that the control rights can be endogenously measured by the shareholders voting rights, which have clear empirical counterparts and can thus be measurable.

Finally, the objective function of the manager of firm j will weigh solely the interests of the firm's horizontal (non-horizontal) shareholders as the ownership of each non-horizontal (horizontal) shareholder becomes dispersed (among a collection of infinitesimal identical shareholders) when the voting rights of the horizontal (non-horizontal) shareholders do induce full control. In order to see why, let the subset of shareholders that hold financial rights in firm j, Θ_j, be divided in two smaller subsets: the subset of horizontal shareholders, Θ_j^h, and the subset of non-horizontal shareholders, Θ_j^{nh}. This implies that the objective function of the manager of firm j can be written as follows:

$$\max_{x_j} \sum_{k \in \Theta_j^h} \gamma_{kj} \mathbb{E}[\Pi_j] + \sum_{k \in \Theta_j^{nh}} \gamma_{kj} \mathbb{E}[\Pi_j] + \sum_{g \in \Im . g \neq j} \sum_{k \in \Theta_j^h} \gamma_{kj} \frac{\phi_{kg}}{\phi_{kj}} \mathbb{E}[\Pi_g] . \tag{33}$$

As the ownership of each non-horizontal shareholder becomes dispersed among a collection of infinitesimal identical shareholders, the objective function of the manager will weigh solely the interests of horizontal shareholders when the voting rights of those horizontal shareholders do induce full control of the firm, i.e., when $\sum_{k \in \Theta_j^{nh}} \gamma_{kj} = 0$. Similarly, as the ownership of each horizontal shareholder becomes dispersed among a collection of infinitesimal identical shareholders, the number of shareholders in Θ_j^h increases, but the ratio $\frac{\phi_{kg}}{\phi_{kj}}$ of each new infinitesimal identical shareholder will be identical to the corresponding ratio of the previous (non-dispersed) shareholder. As a consequence, the objective function of the manager will weigh solely the interests of non-horizontal shareholders when the voting rights of those non-horizontal shareholders do induce full control of the firm, i.e., when $\sum_{k \in \Theta_j^h} \gamma_{kj} = 0$. As such, property (vi) is satisfied.

Proposition 4

The structure of this proof follows the same three steps as the proof presented for Proposition 2. As such, we just have to address the sub-proof of the remaining points: (a) that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger candidate to the firm and on the strategy proposals of the candidates to the rival firms; and (b) that the common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms. We first present these sub-proofs under Assumption 3a and then under Assumption 3b.

Under Assumption 3a

We begin by showing that the objective function of the incumbent is strictly concave conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms.

Let $\varpi_{aj} = \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)\, \Xi_{a_j}$ denote the objective function of the incumbent under Assumption 3a. Under Assumption 2, shareholders are conditionally sincere, which implies that the incumbent of firm j can choose her strategy proposal taking the strategies of the candidates to the remaining firms as given. The first order condition of this problem is, thus, given by:

$$\frac{\partial \varpi_{aj}}{\partial x_{a_j}} = \sum\nolimits_{k\in\Theta_j} \frac{\partial \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)} \frac{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}} \Xi_{a_j}, \tag{34}$$

where, using probability (18), which makes use of the independence assumption, we have that:

$$\begin{aligned} \frac{\partial \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)} &= \sum\nolimits_{\Theta_j^\imath\in\wp_j, k\in\Theta_j^\imath} \prod\nolimits_{h\in\Theta_j^\imath, h\neq k} \Pr_{ha_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) \prod\nolimits_{h\notin\Theta_j^\imath} \Big(1 - \Pr_{ha_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\Big) \\ &\quad - \sum\nolimits_{\Theta_j^\imath\in\wp_j, k\notin\Theta_j^\imath} \prod\nolimits_{h\in\Theta_j^\imath} \Pr_{ha_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) \prod\nolimits_{h\notin\Theta_j^\imath, h\neq k} \Big(1 - \Pr_{ha_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\Big), \end{aligned} \tag{35}$$

which (a) is, by definition, non-negative since increasing $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ for any k can not have a negative impact on $\Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)$; and (b) does not depend on $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ since $\Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)$ is linear in $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$ for any k taking the corresponding probabilities of the remaining shareholders as given. The second order condition of the incumbent's problem, in turn, is given by:

$$\begin{aligned} \frac{\partial^2 \varpi_{aj}}{\partial x_{a_j}^2} &= \sum\nolimits_{k\in\Theta_j} \frac{\partial^2 \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)\, \partial x_{aj}} \frac{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}} \\ &\quad + \sum\nolimits_{k\in\Theta_j} \frac{\partial \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)} \frac{\partial^2 \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}^2} \\ &= \sum\nolimits_{k\in\Theta_j} \frac{\partial^2 \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)^2} \left(\frac{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}}\right)^2 \\ &\quad + \sum\nolimits_{k\in\Theta_j} \frac{\partial \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)} \frac{\partial^2 \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}^2} \\ &= \sum\nolimits_{k\in\Theta_j} \frac{\partial \Pr(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)} \frac{\partial^2 \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}^2}, \end{aligned} \tag{36}$$

where the last equality makes use of the fact that $\frac{\partial^2 \Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)^2} = 0$ for all k, since it does not depend on $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)$.

Using probability (19), we have that the objective function of the manager is strictly concave in x_{aj}, conditional on the strategy proposal of the challenger to the firm and on the strategy proposals of the candidates to the rival firms, since (a) under Assumption 6 we have that $G_j(\frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)] - \mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\phi_{kj}}) = \frac{1}{2} + \frac{\mathbb{E}[\tilde{R}_k(\mathbf{x}_a)] - \mathbb{E}[\tilde{R}_k(\mathbf{x}_b)]}{\tau_j \phi_{kj}}$ and, thus, $\frac{\partial^2 \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}^2} = \frac{1}{\tau_j \phi_{kj}} \frac{\partial^2 \mathbb{E}[\tilde{R}_k(\mathbf{x}_a)]}{\partial x_{a_j}^2}$, which is negative under Assumption 5; and (b) $\frac{\partial \Pr\left(m_j = a_j|\mathbf{x}_a, \mathbf{x}_b\right)}{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)} > 0$ for at least a shareholder k.

We now show that the common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms, with normalized Banzhaf power indices as weights. To do so, note that since the two candidates will choose the same best-response function, in equilibrium, we have $\mathbb{E}\left[\tilde{R}_k(\mathbf{x}_a)\right] = \mathbb{E}\left[\tilde{R}_k(\mathbf{x}_b)\right] = \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]$ for all k. This implies that the first-order condition reduces to:

$$\frac{1}{2^{\ell_j - 1}} \sum\nolimits_{\Theta_j^\imath\in\wp_j} \sum\nolimits_{k\in\Theta_j^\imath} \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\partial x_j} \Xi_{a_j} - \frac{1}{2^{\ell_j - 1}} \sum\nolimits_{\Theta_j^\imath\in\wp_j} \sum\nolimits_{k\notin\Theta_j^\imath} \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\partial x_j} \Xi_{a_j} \lesseqgtr 0, \tag{37}$$

which makes use of the fact that $\frac{\partial \Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b)}{\partial x_{a_j}} = \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}[\tilde{R}_k(\mathbf{x}_a)]}{\partial x_{a_j}}$ and $\Pr_{ka_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) = \Pr_{kb_j}(\mathbf{x}_a, \mathbf{m}_a, \mathbf{x}_b, \mathbf{m}_b) =$

$\frac{1}{2}$ when $\mathbf{x}_a = \mathbf{x}_b$, both for all k. This first-order condition can, in turn, be rewritten as:

$$\frac{1}{2^{\ell_j - 1}} \sum_{k \in \Theta_j} \left(\lambda_{jk} \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\partial x_j} - \left(2^{\ell_j - 1} - \lambda_{jk}\right) \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\partial x_j} \right) \Xi_{a_j} \lesseqgtr 0, \tag{38}$$

where λ_{jk} denotes the number of subsets in $\wp_j$ in which shareholder k enters and $\left(2^{\ell_j - 1} - \lambda_{jk}\right)$ denotes the number of subsets in $\wp_j$ in which shareholder k does not enter. Finally, consider that λ_{jk} can be divided in two terms: the number of subsets in $\wp_j$ in which shareholder k enters and is pivotal, λ_{jk}^{p}, and the number of subsets in $\wp_j$ in which shareholder k enters and is not pivotal, $\lambda_{jk}^{\tilde{p}}$, where shareholder k is pivotal if for some subset $\Theta_j^{\imath}$ which does not include shareholder k, we have $\sum_{h \in \Theta_j^{\imath}, h \neq k} \upsilon_{hj} \leq 0.5$, but if we include shareholder k, $\sum_{h \in \Theta_j^{\imath}} \upsilon_{hj} > 0.5$. The number of subsets in $\wp_j$ in which shareholder k enters and is not pivotal is, by construction, equal to the number of subsets in $\wp_j$ in which shareholder k does not enter. This implies that $\lambda_{jk}^{\tilde{p}} = \left(2^{\ell_j - 1} - \lambda_{jk}\right)$ and that the first-order condition can be rewritten as:

$$\sum_{k \in \Theta_j} \left(\frac{\lambda_{jk}^{p}}{2^{\ell_j - 1}} \right) \frac{1}{\tau_j \phi_{kj}} \frac{\partial \mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\partial x_j} \Xi_{a_j} \lesseqgtr 0, \tag{39}$$

where $\frac{\lambda_{jk}^{p}}{2^{\ell_j - 1}}$ denotes the Banzhaf power index associated to shareholder k in firm j. This establishes that the common best-response function is the same as the best-response function that would arise from maximizing a weighted average of the relative expected returns of the firm's shareholders conditional on the strategy proposals of the candidates to the rival firms:

$$\max_{x_j} \Pr\left(m_j = a_j | \mathbf{x}_a, \mathbf{x}_b\right) \Xi_{a_j} \propto \max_{x_j} \sum_{k \in \Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[\tilde{R}_k(\mathbf{x})\right]}{\phi_{kj}} \propto \max_{x_j} \sum_{k \in \Theta_j} \gamma_{kj} \frac{\mathbb{E}\left[R_k(\mathbf{x})\right]}{\phi_{kj}}, \tag{40}$$

where $\gamma_{kj} = \frac{\lambda_{jk}^{p}/2^{\ell_j - 1}}{\sum_{h \in \Theta_j} \left(\lambda_{jh}^{p}/2^{\ell_j - 1}\right)} = \frac{\lambda_{jk}^{p}}{\sum_{h \in \Theta_j} \lambda_{jh}^{p}}$ denotes the weight assigned by firm j's manager to the relative expected return of shareholder k, measured by the normalized Banzhaf power index of shareholder k in firm j.

Under Assumption 3b

Proposition 2 holds in this case. As discussed in the proof above, under Assumption 3b, the only requirement regarding the cumulative distribution function $H_j(\cdot)$ is that $\mathbb{E}\left(\tilde{\xi}_j\right) = 0$ for each firm j, which is satisfied if $\tilde{\xi}_j = 0$ for each firm j, establishing the result.