Uncertainty and Bargaining:
An application to the German dairy market

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Abstract

This paper empirically analyzes the influence of uncertain disagreement outcomes, on the settlement of new agreements between parties with uneven bargaining power. Including the findings of theoretical literature on the requirements to reach agreements under uncertain disagreement conditions, and by using an assymetric Nash bargaining framework, we are proposing an approach to derive the bargaining solutions that estimates the uncertainty of the bargainers and determines whether a new compromise could be reached under uncertain conditions.

1 Introduction

In some industries, such as the food supply chain, the vertical commercial relationships between two actors involve the exchange of a set of different products (brands) that could be separately bargained. However, the empirical analysis of bargaining outcomes has overlooked the particularities that exist in recurrent multi-product and multi-bargainings commercial relationships, in particular regarding the existence of uncertainty within the disagreement outcome. Recurrent commercial relationships are not friction free, which could make them prone to uncertainty regarding the effects of disagreement in the commercial relationship. A usual assumption in empirical work is the independency among bargainings, but assuming this among negotiations within the same pair of

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bargainers could not reflect the conditions that face bargainers, and may misestimate the surplus division between them. In this paper, we include in the analysis the interdependency of bargainings within the same commercial relationship, and propose an empirical approach that is able to both determine the feasibility to reach a new agreement under uncertain disagreement conditions and estimate the uncertainty of bargainers during the negotiations.

The apparent presence of uncertainty in bargainings in the food supply chain has been brought to the public attention through the last decades, in which the discussion regarding bargaining disparities and unfair trade practices (UTPs)[1] have been linked to the negotiations within this industry, given the increasing concentration of the markets in the different stages of this supply chain [OECD(2013)].

The competition authorities have reacted differently toward this topic. The European Commission, for instance, has been discussing a legislative proposal aiming to prohibit some of the witnessed UTPs between retailers and suppliers, in order *"to grant"* small and middle sized firms *"greater certainty"* and *"to eliminate the "fear factor" in the supply chain"*[2], joining in this way to countries, such as Australia, United Kingdom and Ecuador, that have addressed this matter through legislation [Junta de Regulación de la Ley Orgánica de Regulación y Control del Poder de Mercado(2017), Mills(2003)]. Given this context, the study and estimation of bargaining outcomes gain a particular importance, because it enables us to analyze of the bargaining power distribution among actors of the vertical relationship, becoming a tool in the evaluation of the suitability of this kind of public measures to address this problematic.

Empirical literature on the surplus splitting between bargainers has been recently expanding. The empirical implementation of Nash-in-Nash outcomes to approach the surplus division from negotiations was introduced by Draganska et al. (2010), their specification allowed the estimation of the bargaining power and surplus distribution among actors, and since then has proliferated in the literature the implementation of Nash bargaining solutions through structural econometric models, allowing us to have a better understanding on topics such as the effects of bundling [Crawford

[1] According to the European Commission, UTPs *"are practices that deviate grossly from good commercial conduct, are contrary to good faith and fair dealing and are unilaterally imposed by one trading partner on another"* [European Commission (2016)]

[2] European Commission acts to ban unfair trade practices in the food supply chain (2018, April 12), European Commission, [Retrieved (2018, September 28): `http://europa.eu/rapid/press-release_IP-18-2702_en.htm`].

and Yurukoglu (2012)], price discrimination [Grennan (2013)], health insurance competition [Ho and Lee (2017)], or vertical integrations [Crawford et al. (2018)] on welfare; or the effects of horizontal integration [Gowrisankaran et al. (2015)] or product charateristics [Bonnet and Bouamra-Mechemache (2015)] on bargaining leverage, to cite a few.

However, the bargaining environment considered until now has implicitly assumed smooth and transparent interactions among bargainers, overlooking the potential informational asymmetries that could exist in the commercial relationships, which has been pointed out as a weakness of this bargaining setting[3].

Collard-Wexler et al. (forthcoming), has provided a micro-theoretical foundation for the suitability of Nash-in-Nash bargaining to analyze the surplus division. In their work, they have considered a multiple firms upstream and downstream framework, in which each pair of agents bargain over a single contract (product), addressing the potential *"inter-relationship uncertainty"* (the uncertainty regarding the competitors contracts) by assuming *passive beliefs*[4]. This setting could suit well a single-product and/or bundled kind of negotiations, but could not address fully the multiproduct vertical relationships, in which the negotiations within the relationship could not be completelly independent, existing still room to analyze the potential interdependence of negotiations within the same pair of bargainers, opening the discussion regarding the effects of a potential *"intra-relationship uncertainty"*.

We consider that the inclusion of the *intra-bargaining uncertainty* in the study and estimation of bargaining solutions could be particularly helpful to understand better known complicated bargaining relationships, such as the vertical relationships in the food supply chain, and could allow to assess and interpret better the observed information in this kind of markets.

Given the multi-product and multi-bargaining intra-commercial relationship setting of these kind of industries, assuming *passive beliefs* among bargainings of the same commercial relationship could be an strong assumption, in particular for markets with witnessed complicated relationships. By taking into account this factor, we increase the attention in a key element of the bargaining outcome analysis, the disagreement payoff, given that under the existence of frictions in the vertical

[3]See, Crawford and Yurukoglu (2012) pp. 659.

[4]According to McAfee and Schwartz (1994), *"[u]nder passive beliefs, when a firm receives an offer different from what it expects in the candidate equilibrium, it does not revise its beliefs about the offers made to others"*, [See, McAfee and Schwartz (1994) pp. 219.]

relationship, this element would gather the fear/threats that the bargainers develop due to their asymmetries of information within each vertical relationship. And this is the kind of uncertainty that we are addressing, we are analyzing the influence of uncertain disagreement outcomes on the settlement of new agreements between parties with uneven bargaining power.

Theoretical literature on bargaining has already included uncertain disagreement payoffs in negotiations [e.g. Peters and Van Damme (1991), Chun and Thomson (1990a, b, c), Livne (1988)]. We are basing our empirical approach on the findings of Chun and Thomson (1990a, b, c), who analyzed a situation in which at the moment of the negotiation the bargainers would know what they would get from an agreement but they would be uncertain about the disagreement implications, having to develop beliefs on the scenarios that they expect to happen if nonagreement is reached. In their work, they established an incentive condition that the potential new agreement under uncertainty should fulfilled in order to be prefered by both parties, instead of waiting until the uncertainty disappears to then agree on something. Their findings also suggest, that this incentive condition may not be fulfilled in relationships with uneven bargaining power, in such case, the bargainers would prefer reach no new agreement (i.e. keeping their current surplus splitting) until the uncertainty disappears.

Notice that whereas a new agreement under uncertain conditions could be reach or not, if there is uncertainty in the market, it will be reflected in the observed information, being possible to estimated, allowing us to have an strategic understanding of negotiations. At the same time, and if a new agreement could be reach under uncertain disagreement circumstances, not considering the *intra-relationship uncertainty* in the analysis would misestimate the bargainers' disagreement profits afecting, therefore, the estimation of the surplus distribution.

In this way, by the inclusion of the incentive condition from Chun and Thomson (1990a, b, c) into the assessment of bargaininig outcomes and using an assymetric Nash bargaining framework, we are proposing an structural econometric model that estimates the uncertainty of the bargainers and determines whether a new compromise could be reach given the uncertainty in the market.

Our proposal could be also use in the analysis of further issues of the retailing industry, such as listing and delisting of products. Aside from this, our work could also contribute to the empirical analysis of strategic bargaining, reputation and threats, among other applications due to its flexibility.

To exemplify the implementations of our especification, we are analyzing the negotiation outcomes of dairies (manufacturers) and retailers before the German-dairy-farmers strike took place at the end of may 2008. We study how the possibility of the farmers strike in the upstream market influenced the renegotiations between the retailers and manufacturers in the downstream milk market. Finally, we compare the bargaining results from the before- and after-strike periods to evidence the cost of the strike for both actors.

However, it is important to point out that **the results shown in this version are still preliminary**, given that we are still refining and increasing robustness of our final estimations. This preliminary results allow us to explain the applicability of our model, and despite being not yet the final results, they suggest that the model works as expected.

This paper develops as follows in section 2 the theoretical foundation of our aproach is explained. In section 3 we introduce the case of the *milk-strike* and explain the sources of uncertainty of the bargainers in this market. In section 4 is explained our identification strategy, as well as the structural econometric model, to estimate the bargaining outcomes and uncertainties. Section 5 describes the data used for the results of this version, as well as presents the estimated preliminary results. Finally, section 6 concludes.

2 Uncertain disagreement and bargaining

In some markets prices, and consequently margins, are not exogenous to the bargaining abilites of firms, but rather they result from bilateral negotiations among them. To consider this in the empirical analysis has allowed to have a better understanding on the information available and the implications of the surplus division in vertical relationships.

Recent literature has been addressing the estimation of bargaining outcomes through the implementation of Nash-in-Nash solutions [e.g. Crawford et al. (2018), Ho and Lee (2017), Bonnet and Bouamra-Mechemache (2015), Grennan (2013), Crawford and Yurukoglu (2012), Draganska et al. (2010)]; deriving the empirical specification from the solution of a general Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^m - d_j^m\right)^{\lambda_j^m} \left(\pi_j^r - d_j^r\right)^{\lambda_j^r}$$

in which two bargainers, r and m, engage in a negotiation over w_j, the price of product j, having both agents involved in the negotiation a bargaining position -power- λ_j^i (being $i = r, m$ and

$\sum_i \lambda_j^i = 1$), and taking r and m a decision by comparing the profits from reaching an agreement π_j^i against their profits if nonagreement is reached (disagreement payoff) d_j^i. The solution derived from this problem has been proved to be a viable method to empirically approach the bilateral negotiations subject to some assumptions [Collard-Wexler et al. (forthcoming)].

Among these assumptions is the use of *passive beliefs*, which according to McAfee and Schwartz (1994) *"[u]nder passive beliefs, when a firm receives an offer different from what it expects in the candidate equilibrium, it does not revise its beliefs about the offers made to others"*. By assuming *passive beliefs* is addressed the bargainers' imperfect information during negotiations about the results of the other bargainings that are taking place in the market [Collard-Wexler et al. (forthcoming), Draganska et al. (2010)].

However, this assumption has been theoretically analized in settings in which each vertical relationships bargains over a single product/contract [Collard-Wexler et al. (forthcoming)]; and by stablishing an assumption regarding the beliefs on the negotiations with/of competitors, bargainers would expect only one scenario, the scenario in which just the current negotiation was not succesful; implying that the negotiations are independent from each other.

On the other hand, there are vertical relationships in which a set of different products are bargained separately from each other; for instance, in the retailing industry, both manufacturers and retailers can have different representatives for brands or product lines, and in this way, manufacturers with a wide variety of products could have several negotiations with the same retailer over different products. In this kind of multi-product and multi-bargaining environment, the assumption of *passive beliefs* could address the uncertainty regarding the competitors' bargainings, the *"inter-relationship uncertainty"*; however, assuming *passive beliefs* among the bargainings within the same bilateral relationship, could impose an independency between negotiations that in reality could not always exist, in particular in case of conflict in the vertical relationship, in such cases there would be an *"intra-relationship uncertainty"*, i.e. the lack of certainty regarding how not reaching agreement in one negotiation would affect the others from the same pair of bargainers; having the disagreement outcome, in this way, a key rol when this *intra-relationship uncertainty* is included in the analysis.

As is already known, the disagreement outcome serves as a referent point to evaluate the gains from an offer, the higher the disagreement payoff (outside option) the better the bargaining position of the agent in the negotiation. And a good assessment of the disagrement outcome will translate as well in a better evalutation of the offer. Hence, if there is *intra-relationship uncertainty*, could an offer be evaluated under uncertain disagreement conditions?

Theoretical literature on bargaining has already analyzed bargaining outcomes under disagreement uncertainty [e.g. Peters and Van Damme (1991), Chun and Thomson (1990a, b, c), Livne (1988)]. In particular, Chun and Thomson (1990a, b, c) axiomatically analyzed bargainings when both agents, at the moment of the negotiation, are uncertain about the scenario they would face if nonagreement is reached, an uncertainty that will disappear in some point in the future. In their work, it was stablished an incentive condition in order both bargainers would preferred to reach an agreement under such uncertain conditions, instead of waiting until there is non-uncertainty to then agree onto something.

The idea behind the *(weak) disagreement point concavity* condition is that the solution under disagreement uncertainty should make bargainers at least as good as they expect to be if there would be non-uncertainty. In this way, bargainers would not have the incentive to wait until the uncertainty is gone to reach then an agreement [Chun and Thomson (1990a, b, c)]. However, their findings suggest that the set of non-symmetric Nash bargaining solutions may not fulfilled this condition.

Hence, bargainers with uneven bargaining power may not have the incentive to reach an agreement under uncertain disagreement conditions. However, if a new agreement could be reached under such bargaining circumstances, then assuming *intra-relationship passive beliefs* would misestimate the disagreement profits and, consequently, the surplus division between the agents.

At the same time, and regardless whether the solution under uncertainty will become the new agreement or not, if there is uncertainty in the negotiations, it will be reflected in the results that we are observing. In this way, and by recalling that in recurrent bargaining relationships there will be an preestablished bargaining condition at the moment of the negotiation -*status quo*- (the bargaining power among the agents until that point), through the implementation of the Chun and Thomson (1990a, b, c) incentive condition, the beliefs and potential new solution can be estimated, which can allow us to strategically analyze the negotiations.

Uncertain disagreement conditions: The model

Consider an upstream market consisting of M firms, each upstream firm m offers to the R available downstream firms a set of products J^m, being able each downstream firm r to bargain the price of the available products with the corresponding firm in the upstream market. Let us denote a bargained product between firm m and r firms as j, and bargained price as w_j.

The firms of each stage (downstream or upstream) are competitors among each other, and it will be assumed that all products will be individually bargained in vertical-simultaneous negotiations; given this, we are assuming that the firms will send a *delegated agent*[5] to the negotiation of each of its products with each of its counterparties, i.e. there will not only be concurrent bargainings involving the same firm with different counterparties, but also there will be simultaneous negotiations involving the same pair of firms but regarding different goods[6].

Assume that each *delegated agent* will have *passive beliefs* regarding the outcome of the other firms, i.e. when an unexpected outcome arises in the negotiation, bargainers will not revise their beliefs regarding their competitors' outcomes.

At the same time, it is assumed, that the *delegated agents* know the scenario that their represented would face if an agreement in their negotiations are reached. In this way, let us denote the profits from an agreement between upstream firm m and downstream firm r over product j as $\pi_j^m(w_j)$ and $\pi_j^r(w_j)$ respectively.

On the other hand, it is also assumed that at the moment of the negotiation the *delegated agents* are not certain about the implications of a disagreement in their negotiations for the other bargainings between the same two firms. We assume that the agents of firms m and r consider two different scenarios in case of disagreement over product j: 1) this disagreement will not affect the other negotiations between these two firms, therefore the disagreement payoff for firms m and r will be d_j^m and d_j^r, which are the profits of both firms when there is nonagreement over product j; 2) a disagreement over product j would mean the break of the commercial relationships between these two firms, i.e. this disagreement would affect the reach of an agreement in the other negotiations

[5]The concept of *delegated agent* has been implicitly or explicitly mentioned before in the literature to address concurrent negotiations of a firm with different counterparties, e.g. Collard-Wexler et al.(forthcoming), Chipty and Snyder (1999).

[6]It is a common practice among firms with a wide variety of products to have different representatives for each product line or brand.

between the same two firms, and therefore it would be no exchange of products between these firms, denoting as J^{mr} the set of products negotiated between firms m and r, then the disagreement payoffs for firms m and r in this scenario will be $d^m_{J^{mr}}$ and $d^r_{J^{mr}}$ respectively, which are their profits when there is non-exchange of products between these firms[7].

Given these two possible scenarios, the *delegated agents* would develop beliefs on facing each of them. Denoting the upstream and downstream firm *delegated agents'* beliefs on the first scenario as δ_j and θ_j respectively (consequently, their beliefs on the second scenario would be $1-\delta_j$ and $1-\theta_j$), where $\delta_j, \theta_j \in [0,1]$, we will have that the generalized Nash bargaining product, given the uncertain disagreement conditions, will be the following:

$$\underset{w_j}{\text{Max}} \left(\pi^m_j - E(d^m_j)\right)^{\lambda^m_j} \left(\pi^r_j - E(d^r_j)\right)^{\lambda^r_j} \tag{1}$$

where λ^m_j $[\lambda^r_j]$ represents the upstream [downstream] firm's bargaining power in that negotiation, being $\lambda^m_j + \lambda^r_j = 1$, and $E(d^r_j)$ $[E(d^m_j)]$ is the upstream [downstream] firm's expected disagreement payoff, i.e. $E(d^r_j) = \theta_j d^r_j + (1-\theta_j) d^r_{J^{mr}}$ $[E(d^m_j) = \delta d^m_j + (1-\delta) d^m_{J^{mr}}]$.

However, according to Chun and Thomson (1990a, b, c), in order to reach an agreement under uncertain disagreement conditions, the feasible solution under uncertainty $(F(\pi_j, E(d_j)) = w^*_j)$, should make both bargainers at least as good as they expect to be if there were non-uncertainty:

$$F(\pi_j, E(d_j)) \geqslant E(F(\pi_j, d_j)) \tag{2}$$

In this way, under uncertainty the agents would evaluate at the moment of the negotiation that the condition in (2) is fulfilled; e.g. in order the upstream firm has the incentive to reach an agreement under uncertainty, this agreement should $F(\pi^m_j, E(d^m_j)) \geqslant \delta F(\pi^m_j, d^m_j) + (1-\delta) F(\pi^m_j, d^m_{J^{mr}})$.

It is important to mention, that according to Chun and Thomson (1990a, b, c) findings, the weighted Nash bargaining solution may not fulfilled this condition, i.e. bargainers could preferred to wait before reaching an agreement under uncertain disagreement circumstances, staying therefore in their current situation (*status quo*) until the uncertainty disappears, to reach then an agreement under certain conditions.

[7]Notice that this setting would portrait the threat of a retaliation when nonagreement is reached over a product, which we consider is a plausible uncertainty in conflicting bilateral multi-product relationships.

We propose to implement this incentive condition, in the analysis of conflicting situations between recurrent bargainers, to be able to measure their beliefs on the potential disagreement scenarios, which could allows us to have an strategic insights of the vertical relationships. In the following section we present a case through which we exemplify the implementation of this condition.

3 Uncertainty and Bargaining: A motivating case

The case we are using to analize uncertainty in negotiations was at the time an unusual episode in the German dairy industry, the 2008 dairy-farmers strike, hereinafter *"milk-strike"* or *"strike"*, which took place at the end of may of 2008, in which dairy farmers stopped the delivery of milk to dairy manufacturers as a sign of protest against the raw-milk prices they were receiving[8].

Despite dairy manufacturers are the direct buyers from farmers, these latter claimed that the raw-milk prices were the result of the consumer-price policy used by retailers that pushed down the consumer-milk price, leaving not enough margin to manufacturers, and consequently to farmers. And at a certain point of the strike, both dairy farmers and manufacturers were in a common front against retailers to increase milk prices, and with this their margins[9]

The *milk-strike* drew attention to the relationship among retailers and suppliers, from both public opinion as well as political actors, due to the moral connotations of farmers' main demand: a *"fair price"*. The *strike* finished after some well-known retailers promised to increase milk prices[10]. After the *milk-strike*, the German antitrust agency also opened a sector investigation focussed on buyer power and unfair practices[11].

As it is already known, strikes appear from unsatisfactory negotiations, and they have been

[8]We would like to mention that this *milk-strike* motivated an unpublished Master Thesis by Anna Popova in which, through a less elaborated discrete choice demand analysis, was discussed the suitability of the logit and conditional logit models to derive the demands of milk for the before- and after-strike periods, elasticities for both periods were also derived for some cases; however, due to the limitations of the study those elasticities were imprecise.

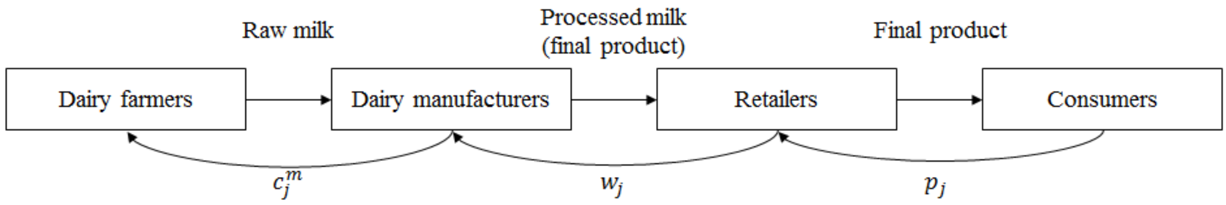
[9]Wütende Milchbauern blockieren Molkereien (2008, June 2), Welt, [Retrieved (2018, September 24): `https://www.welt.de/jahresrueckblick-2008/juni/article2737068/Wuetende-Milchbauern-blockieren-Molkereien.html`].

[10]"Ich fordere Sie auf, ab heute Abend wieder Milch zu liefern" (2008, June 5), Frankfurter Allgemeine, [Retrieved (2018, september 24): `http://www.faz.net/aktuell/wirtschaft/unternehmen/milchbauernverband-fuer-boykottende-ich-fordere-sie-auf-ab-heute-abend-wieder-milch-zu-liefern-1544740.html`].

[11]See, Bundeskartellamt (2009).

more witnessed in wage bargainings between unions and firms, in which context has been studied by the literature [e.g. Gu and Kuhn (1998), Varoufakis(1996), Cramton and Tracy (1992), Card (1990a, b), Hart(1989)]. However, strikes are not a common leverage strategy in the bargaining between actors of the food supply chain, nor are they meant to transcend the negotiations of the other stages of the value chain.

Figure 1: Milk Value Chain



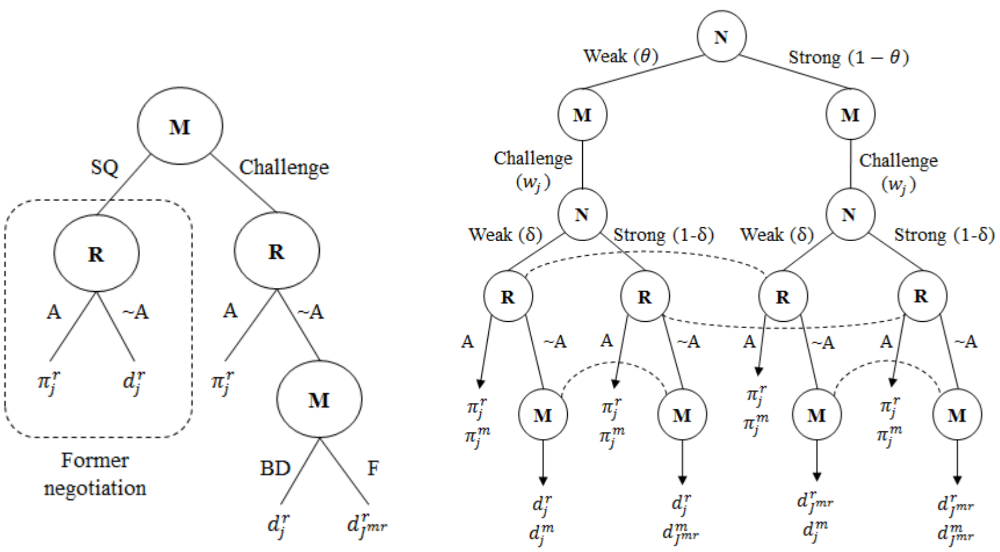
In this way, in order this *strike* had taken place, dairy manufacturers should have not satisfied farmers' demands of a higher raw-milk price (c_j^m) for a while before the *strike* broke. Meanwhile, and given the position of dairy manufacturers in the value chain (as can be observed in figure 1), in order to attend to farmers' demands without compromising their own margins, manufacturers should have also asked retailers for a higher price for their products, which would imply a renegotiation of the wholesale-milk price (w_j) between retailers and manufacturers. And it is in the link of the milk-supply chain, involving manufacturers and retailers, in which we are focusing our analysis, given that the bargaining between these actors will determined whether farmers' needs could have been satisfied or the strike was inevitable.

Notice that due to the apparent unsatisfaction of the farmers in the upstream market, a wholesale-price renegotiation between retailers and manufacturers would have not developed under the same bargaining environment as the former dealings did, given that the threat of a possible farmers strike could have brought noise to the negotiation table, noise in the shape of uncertainty.

In this way, if a manufacturer chose renegotiate the wholesale price, challenging then the current status quo (SQ) of this commercial relationship, this action could lead to two different scenarios when the negotiation is not sucessful from the retailer's perspective: 1) as any other former negotiation, disagreement regarding the wholesale price of product j, means losing j from retailer's shelf, but still the retailer could count with the other products of that manufacturer (d_j); 2) given the pressure of the upstream market, if disagreement over the wholesale price of product j takes place, the manufacturer would take farmers' side, and chooses not to sell any of the other products to the retailer as well ($d_{J^{mr}}$, where J^{mr} is the set of products that manufacturer m usually sells to the retailer r).

Figure 2a, represents the renegotiation from retailer's standpoint above described which, as can be observed, is similar to the classic crisis bargaining game, usually considered in the analysis of wars in international relations literature [e.g. Fey et al. (2013), Fey and Ramsay (2011), Lewis and Schultz (2003), Fearon (1995)]. In crisis bargaining games one of the bargainers chooses either to stay in the status quo or to challenge it by making an offer to the other party, which in case to be rejected could lead to a war scenario, depending the realization of this scenario on the type of the bargainers. The type of the bargainers will be determine by the source of uncertainty in the game, being one of the possible sources the relative strenght of the opponent (either relative strong or weak); and therefore, the parties have to develop beliefs on the type of their opponent through their interactions [Fey and Ramsay (2011)].

Figure 2: Game representation



(a) (b)

In this way, and for the case under analysis, in the before-strike period, if a manufacturer chooses to challenge the status quo, the retailer could be facing either a "strong" manufacturer which, if disagreement over product j takes place, is ready for a "war" by supporting the farmers and not delivering any of the other products to the retailer (then, being the retailer-disagreement payoff d^r_{Jmr}); or the retailer could be dealing with a "weak" manufacturer which, even though there is nonagreement regarding product j, would still sell the other products to the retailer (d^r_j).

Similarly, under manufacturer's perspective, once they challenge the status quo, and given the noise in the upstream market, if disagreement occurs they could be facing either a retailer that is prepared to resist a "war"(strike) scenario: "strong" retailer (then, being the manufacturer-disagreement payoff d^m_{Jrm}); or being in front of a retailer that is not prepared for a "war" scenario and will pursue to keep the other products of the manufacturer in his shelfs: "weak" retailer (d^m_j).

Therefore, and as can be seen in figure 2b, in the before-strike period, regardless their type, if the manufacturer chooses to challenge the status quo, both manufacturer and retailer would be uncertain about their disagreement payoff in a renegotiation of the wholesale price, and consequently they both would develop beliefs on which scenario (type of bargainer) they will face if there is nonagreement.

Consequently, if there were renegotiations of the wholesale price in the before-strike period, these would respond to the kind of dealings presented in section 2. In which, the potential agreement under uncertainty should fulfilled the condition previously described; otherwise, any of the bargainers would prefer not to continue with the renegotiations, i.e. staying in the status quo, until there is nonuncertainty to reach a new agreement, knowing with certainty his disagreement payoff.

Given that the strike occured, we should find that the *(weak) disagreement point concavity* condition should have not been satisfied the months before the strike broke, meaning that the lack of certainty on the disagreement payoff due to the noise from a possible strike in the upstream market and the uneven bargaining power between manufacturers and retailers, resulted in failed wholesale price renegotiations, persisting the unsatisfactory conditions from the status quo, and paving the way to the *strike*, which would disappear the uncertainty to reach new agreements afterwards.

Finally, notice that in the period after the strike the noise from the upstream market is already gone and therefore renegotiations over the wholesale price will reach new agreements, due to the lack of uncertainty in the market.

4 Identification strategy

The idea of our paper is to take the case of the *milk-strike* and compare the distribution of the rent among manufacturers and retailers before and after it, as well as to verify the lack of incentives to reach a new agreement in the before-strike period, due to the uncertainty present in those renegotiations.

For each period we estimate the bargaining outcomes by backward induction. We first estimate the demand of milk, from which we derive the demand elasticities and marginal effects; from which we are able to assess the retailers margins, by assuming a Nash-Bertrand competition among them. Then, by implementing an structural econometric Nash-bargaining model, we analyze the distribution of the rent between retailers and manufacturers. Additionally, we are including to the analysis a test on whether renegotiations between manufacturers and retailers on the wholesale prices in the before-strike period would have been resolved given the uncertainty produced by a possible milk-farmers strike.

Through this analysis we aim to infer whether a possibly *fairness* issue causes a shift of consumer behavior as well as the consequences of the strike on the bargaining power and the surplus division between manufacturers and retailers. Additionally, we are aiming to corroborate Chun and Thomson (1990 a, b, c) theory, by analyzing the bargainers' incentives in the before-strike period to reach a new agreement, given their bargaining power distribution and the uncertainty coming from the upstream market.

4.1 Demand model

For each period (before- and after-strike), we derive the demand by implementing a random-coefficient logit model. We define the utility of a consumer i derived from product j at time t as follows:

$$U_{ijt} = x_j^{'}\boldsymbol{\beta} + \alpha_i p_{jt} + \varepsilon_{ijt} \tag{3}$$

where $\boldsymbol{\beta}$ is a vector of individual-specific coefficients capturing the time invariant effect on the utility of product atributes in x_j, such as brand, the retailer, and the level of fat. p_{jt} represents the price of product j at time t, while the random coefficient is denoted by α_i representing the marginal disutility of the price that varies across consumers, and which distributes by $\alpha_i = \alpha + \sigma_\alpha \vartheta_{\alpha_i}$, where

ϑ_{α_i} are the unobserved consumer characteristics, and σ_α measures the unobserved heterogeneity of consumers. We denote as F_w the assumed independently and normally distribution of ϑ_{α_i}, such that $\vartheta_{\alpha_i} \sim N(\alpha, \sigma_\alpha)$. Finally, ε_{ijt} denotes a random shock on the utility.

We included an outside option to account for a substitute to the J alternatives available in the market, being normalized the utility of the outside option to zero, implying that ε_{ijt} is the consumer indirect utility when choosing this alternative.

As proposed by Petrin and Train (2010), we implemented a control funtion in order to control for endogeneity in specification (3) coming from ommitted variables. In this way, the estimation of the market demand will consist of two stages. In the first stage a regression of the mean monthly milk prices will be perfomed, which follows the specification below:

$$\overline{p}_{jt} = x_j^{'}\boldsymbol{\tau} + cs_{jt}^{'}\boldsymbol{\psi} + v_{jt} \tag{4}$$

where $\boldsymbol{\tau}$ captures the time invariant effect of variables included in x_j, such as brand and retailer, on the mean monthly price $\overline{p}_{jt}$; while $\boldsymbol{\psi}$ captures the effect of some input prices included in cs_{jt}. Finally, v_{jt} represents the random shock of the regression.

Afterwards, the estimated $\hat{v}_{jt}$ will be included in the consumer utility especification (3) as follows:

$$U_{ijt} = x_j^{'}\boldsymbol{\beta} + \alpha_i p_{jt} + \rho\hat{v}_{jt} + \mu_{ijt} \tag{5}$$

where $\mu_{ijt} = \varepsilon_{ijt} - \rho\hat{v}_{jt}$, which is assumed to be orthogonal to p_{jt} and distributed IID extreme value type 1.

In this way, the probability of consumer i buying alternative j on time t conditional to α_i will be represented by:

$$L_{ijt}(\alpha_i) = \frac{exp(x_j^{'}\boldsymbol{\beta}+\alpha_i p_{jt}+\rho\hat{v}_{jt})}{1+\sum_{k=1}^{J} exp(x_k^{'}\boldsymbol{\beta}+\alpha_i p_{kt}+\rho\hat{v}_{kt})}$$

and consequently, the market share of product j in time t will be given by integrating the consumer-level choice:

$$s_{jt} = \int \frac{exp(x_j^{'}\boldsymbol{\beta}+\alpha_i p_{jt}+\rho\hat{v}_{jt})}{1+\sum_{k=1}^{J} exp(x_k^{'}\boldsymbol{\beta}+\alpha_i p_{kt}+\rho\hat{v}_{kt})} dF_w$$

Finally, the own- and cross- price elasticities will be computed according to the following expressions:

$$\epsilon_{kjt} = \frac{\partial s_{kt}}{\partial p_{jt}}\frac{p_{jt}}{s_{kt}} = \begin{cases} -\frac{p_{jt}}{s_{jt}} \int \alpha_i L_{ijt}(1 - L_{ijt}) f_w d_w & \text{if } k = j \\ \frac{p_{jt}}{s_{kt}} \int \alpha_i L_{ijt} L_{ikt} f_w d_w & \text{if } k \neq j \end{cases}$$

4.2 Supply model: Retailers margin

Once the demands for each period have been estimated, then we can continue with the backward induction by estimating the retailers' margins for each period.

Notice that at this stage, the manufacturers' reactions towards disagreement is already known, therefore there is non-uncertainty for the retailers at this point, and the wholesale price is considered as given.

We assume Bertrand-Nash competition among retailers to set the price of product j, which is defined as a retailer-brand combination [Draganska et al. (2010) and Bonnet and Bouamra-Mechemache (2015)].

In this context, the retailers maximization problem will be given by:

$$\underset{p_j}{\text{Max}}\, \pi^r = \sum_{j \in J^r} \left[p_j - w_j - c_j^r\right] s_j(p) M \tag{6}$$

where J^r denotes the set of products sold by retailer r (being $\sum_r J^r = J$), p_j and c_j^r are the retailers price and marginal cost of product j respectively, while the wholesale price of this product is represented by w_j. The total market size is denoted by M, and s_j represents the market share of product j.

Thence, and defining product j retailers margin $(p_j - w_j - c_j^r)$ as γ_j, the subgame Nash equilibrium prices for the J^r products of retailer r are derived from the first-order condition (6):

$$s_j(p) + \sum_{k \in J^r} \gamma_k \frac{\partial s_k(p)}{\partial p_j} = 0, \tag{7}$$

Which matrix notation would be $(T^r * \Delta)\boldsymbol{\gamma} + \boldsymbol{s(p)} = 0$, in which $*$ represents the Hadamard product operator, T^r is the retailers ownership matrix, which element $T^r(k, j) = 1$ if both products k and j are sold by the same retailer and $T^r[j, k] = 0$ otherwise, Δ is a matrix of the marginal effects of the price on the market shares, which general element $\Delta[j, k] = \frac{\partial s_k(p)}{\partial p_j}$, $\boldsymbol{s(p)}$ is the vector of market shares, and $\boldsymbol{\gamma}$ is the vector of retailers margins, [Draganska et al. (2010)].

In this way, we have that the retailers margins can be expressed as follows:

$$\boldsymbol{\gamma} = -\left(T^r * \Delta\right)^{\dagger} \boldsymbol{s}(\boldsymbol{p}) \tag{8}$$

where, $(T^r * \Delta)^{\dagger}$ is the Moore-Penrose inverse matrix of $(T^r * \Delta)$.

4.3 Supply model: Manufacturers margin

Once the retailer's margins were computed in each period following (8), we proceed to analyze the bargaining between retailers and manufacturers over the whosale margin (w_j), and derive an estimation of the manufacturers' margins.

As mention in section 3, in the before-strike period, before the farmers' demands appeared manufacturers and retailers were already in a commercial relationship, meaning they had already an agreement regarding the wholesale price (*status quo*), agreement that should be challenged in a renegotiation. Given that there has been negotiations before, manufacturers and retailers have already a pre-set bargaining power, the one derived from the *status quo*. In this way, it is first needed to assess their preestablished bargaining power distribution, which is the one that would rule their renegotiations. And given that the negotiations that derived the *status quo* were held in an environment without the noise coming from the upstream market, then there was no sign of a possible conflict in that negotiation, i.e. bargainers would not anticipate a possible retaliation in case of disagreement in the negotiation of one product. In this way, we consider plausible to assume *intra-relationship passive beliefs* for deriving the *status quo*.

However, and as explained in section 3, due to a possible farmers' strike the renegotiations would not develop in an *intra-relationship uncertainty* free environment, having both bargainers to develop beliefs on the possible scenarios that they expect to face in case of disagreement. Therefore, having they to renegotiate considering an expected disagreement outcome, but with a presstablished bargaining power.

As mention before, the potential new solution under uncertain disagreement outcomes should fulfilled the incentive condition in (2). In this way, the new wholesale price under uncertainty (w_j^u) requires to be:

$$w_j^u \geqslant \zeta w_j^{SQ} + (1-\zeta) w_j^{war} \tag{9}$$

in which w_j^{SQ} is the wholesale price when the bargainers believe that a disagreement on j would not have an effect on the other negotiations of the bilateral relationship (like in the *status quo*), w_j^{war} is the wholesale price when they believe that a disagreement on j would translate into the breaking off the commercial relationship, and ζ is the bargainer' belief on facing a *status quo* type of outcome.

Notice that while bargaining, the manufacturers would also be uncertain regarding their marginal costs, given that it depends on the actions that the farmers take, given manufacturers' negotiations. In this way, and defining the manufacturers' margin under uncertainty as: $\Gamma_j^{m,u} = w_j^u - (\delta c_j^{m,SQ} + (1-\delta)c_j^{m,war})$, being $c_j^{m,SQ}$ and $c_j^{m,war}$ the manufacturers' marginal costs in a *status quo* and in the scenario in which commercial relationships are broken due to the milk-farmers strike respectively. In this way, expression (9) is equivalent to the following:

$$\Gamma^u(w_j^u, \delta c_j^{m,SQ} + (1-\delta)c_j^{m,war}) \geq \delta\Gamma^{SQ}(w_j^{SQ}, c_j^{m,SQ}) + (1-\delta)\Gamma^{war}(w_j^{war}, c_j^{m,war}) \qquad (10)$$

Therefore, through expression (10) is possible to test the fulfillment of condition (2). Notice that given that retailers prices are set in a stage after the bargaining with manufacturers, they will set their prices with full information on the result of the bargainings, for this reason, we will analyze the fulfillment of the incentive condition on manufacturers, which margins depend on the negotiation with retailers.

In this way, in before-strike period it is needed to derive the manufacturers' margins in the *status quo*, in the scenario in which the commmercial relationships break off ("war") and the potential manufacturers' margin under uncertainty. Given that the strike took place, we expect to find that in the months before the strike that the condition (10) was not fulfilled, which would implied that the renegotiations were not succesful and bargainers kept the surplus division already agreed (*status quo*), which were not satisfying farmers' needs in the upstream market.

On the other hand, and as mentioned before, in the after-strike period the uncertainty regarding the strike disappears from the negotiations, in this way we can derive manufacturers' margins by considering a *intra-relationship uncertainty* free scenario [Draganska et al. (2010)].

In this way, in the following subsections we present the expression derived to compute the manufacturers' margins before and after the strike.

4.3.1 Manufacturers margin under certainty (Status Quo)

As we have mentioned, in the before-strike period manufacturers should challenge the agreement regarding the wholesale price they already have with retailers - an agreement that was reached in an environment without uncertainty (*status quo*) - in order to satisfy farmers' demands without compromising their own margin.

Similarly, and given that the strike would not longer be a threat in the after-strike period, in any renegotiations that took place after the strike the bargainers would not be uncertain rearding the intentions of their counterparty if there is nonagreement regarding a product.

Therefore, for both cases we are considering an *intra-relationship uncertainty* free sceanrio, which has been already applied in the literature, assuming that manufacturers bargain with retailers each product separately and where they believe that disagreement on product j would not affect the others' negotiations [Draganska et al. (2010)], then the bargaining over the wholesale price of j can be expressed by the following Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^r - d_j^r\right)^{\lambda_j} \left(\pi_j^m - d_j^m\right)^{(1-\lambda_j)}$$

in which π_j^r and π_j^m represent respectively the retailers and manufacturers profits from selling product j (agreement payoff), while d_j^r and d_j^m represent respectively the retailer and manufacturer profits when product j is delisted from retailer r (disagreement payoffs):

	Agreements	Disagreement
Manufacturer	$\pi_j^m = \Gamma_j s_j(p)M + \sum_{\substack{k \in J^m \\ k \neq j}} \Gamma_k s_k(p)M$	$d_j^m = \sum_{\substack{k \in J^m \\ k \neq j}} \Gamma_k s_k^{-j}(p)M$
Retailer	$\pi_j^r = \gamma_j s_j(p)M + \sum_{\substack{k \in J^r \\ k \neq j}} \gamma_k s_k(p)M$	$d_j^r = \sum_{\substack{k \in J^r \\ k \neq j}} \gamma_k s_k^{-j}(p)M$

where Γ_j denotes the manufacturer margin of product j ($\Gamma_j = w_j - c_j^m$, being c_j^m the manufacturer marginal cost of producing product j), $s_k^{-j}(p)$ represents the market share of product k when there was nonagreement on product j. And J^m represents the set of products of manufacturer m, being $J = \sum_m J^m$.

In this way, from the maximization of the Nash product above presented, we will get the following expression:

$$(\pi_j^m - d_j^m)\frac{\partial \pi_j^r}{\partial w_j} = -\frac{(1-\lambda_j)}{\lambda_j}(\pi_j^r - d_j^r)\frac{\partial \pi_j^m}{\partial w_j}$$

Given that $\frac{\partial \pi_j^r}{\partial w_j} = -s_j(p)M$ and $\frac{\partial \pi_j^m}{\partial w_j} = s_j(p)M$, and applying the agreements and disagreements payoffs we get the following:

$$\left(\Gamma_j s_j(p) - \sum_{\substack{k \in J^m \\ k \neq j}} \Gamma_k \Delta s_k^{-j} \right) = \frac{(1-\lambda_j)}{\lambda_j} \left(\gamma_j s_j(p) - \sum_{\substack{k \in J^r \\ k \neq j}} \gamma_k \Delta s_k^{-j} \right) \tag{11}$$

where Δs_k^{-j} represents the change in the market share of product k when product is not longer available in the market, i.e. $\Delta s_k^{-j} = s_k(p)^{-j} - s_k(p)$.

Denoting $\mathbf{\Gamma}$ as the vector of manufacturer margins, and defining T^m as the manufacturers ownership matrix which element $T^m[j,k] = 1$ if both products k and j are produced by the same manufacturer and $T^m[j,k] = 0$ otherwise, and dfining D^j as the matrix of shares and share variations, which $D^j[j,j] = s_j$ and $D^j[j,k] = -\Delta s_k^{-j}$, then the matrix notation of the system of equations from expression 11:

$$(T^m * D^j)\mathbf{\Gamma} = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma} \right] \tag{12}$$

where $*$ and T^r represent the same as before, $\boldsymbol{\gamma}$ is the retailer margin vector obtained through (8), and $\tilde{\boldsymbol{\lambda}}$ is the vector of the bargaining parameters ratio[12].

Therefore, the manufacturer's margin can be expressed by: $\mathbf{\Gamma}^{\boldsymbol{SQ}} = (T^m * D^j)^\dagger \left(\tilde{\boldsymbol{\lambda}} * \left((T^r * D^j)\boldsymbol{\gamma} \right) \right)$, where $(T^m * D^j)^\dagger$ represents the Moore-Penrose inverse of matrix $(T^m * D^j)$. By denoting the general element of the vector $(T^r * D^j)\boldsymbol{\gamma}[i,1] = b_i$, then $\mathbf{\Gamma}^{\boldsymbol{SQ}}$ can be expressed as $\mathbf{\Gamma}^{\boldsymbol{SQ}} = C\tilde{\boldsymbol{\lambda}}$, where C is an square matrix of dimension J which general element is $C[i,k] = (T^m * D^j)^\dagger[i,k]b_k$.[13]

Then, recalling that $\mathbf{\Gamma} = \boldsymbol{w} - \boldsymbol{c^m}$ and $\boldsymbol{\gamma} = \boldsymbol{p} - \boldsymbol{w} - \boldsymbol{c^r}$, we have that $\boldsymbol{c^r} + \boldsymbol{c^m} = \boldsymbol{p} - \boldsymbol{\gamma} - \mathbf{\Gamma}$; therefore, $\boldsymbol{c^r} + \boldsymbol{c^m} = \boldsymbol{p} - \boldsymbol{\gamma} - C\tilde{\boldsymbol{\lambda}}$, and due to unobservability of the marginal costs we will assume $\boldsymbol{c^r} + \boldsymbol{c^m} = IP\boldsymbol{\kappa} + \boldsymbol{\eta}$, where $\boldsymbol{\kappa}$ is the vector of the coefficients that capture the effect of the Z cost shifters (inputs), considered in matrix IP, on the total marginal cost, while $\boldsymbol{\eta}$ is an error term [Draganska et al (2010), Bonnet and Bouamra-Mechemache (2016)]. Therefore, $\tilde{\boldsymbol{\lambda}}$ can be estimated

[12] $\tilde{\boldsymbol{\lambda}} = \begin{pmatrix} \tilde{\lambda}_1 \\ \tilde{\lambda}_2 \\ \vdots \\ \tilde{\lambda}_J \end{pmatrix} = \begin{pmatrix} \frac{1-\lambda_1}{\lambda_1} \\ \frac{1-\lambda_2}{\lambda_2} \\ \vdots \\ \frac{1-\lambda_J}{\lambda_J} \end{pmatrix}$

[13] See Appendix A to more details on matrix C.

by the following specification:

$$\boldsymbol{p} - \boldsymbol{\gamma} = C\tilde{\boldsymbol{\lambda}} + IP\boldsymbol{\kappa} + \boldsymbol{\eta} \tag{13}$$

From the above specification we will estimate $\tilde{\boldsymbol{\lambda}}$ by applying a non-linear least squares. Once the vector $\tilde{\boldsymbol{\lambda}}$ (and consequently $\boldsymbol{\lambda}$) was estimated, the vector of manufacturers margins can be recover from (12).

A similar process can be used to recover the manufacturers' margin from the contingent scenario of a strike, evaluated at the moment of the uncertainty, more details on this process can be found in Appendix B

4.3.2 Manufacturers margin under uncertainty

As mention in section 3, in the before-strike period a renegotiation of the wholesale price between manufacturers and retailers would develop under a different environment as the former negotiation (status quo) did, this new bargaining would suffer from asymmetric information regarding the possible reactions or expectations ("type") of the counterparty regarding their profits when there is nonagreement, having both retailer and manufacturer to develop beliefs on the possible scenarios they would be facing, and therefore having to bargain considering expected disagreement payoff.

In this way, a renegotiation over the wholesale price of product j (w_j) will be the result of the following Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^r - E(d_j^r)\right)^{\lambda_j} \left(\pi_j^m - E(d_j^m)\right)^{(1-\lambda_j)} \tag{14}$$

just as in (14) π_j^r and π_j^m represent respectively the retailers and manufacturers profits from selling product j, which expressions were presented in section 4.3.1.

On the other hand, $E(d_j^r)$ in (14) is the retailer's expected disagreement payoff, where $E(d_j^r) = \theta d_j^r + (1-\theta)d_{Jmr}^r$, in which d_j^r represents the retailer's disagreement payoff of not facing "war/strike" scenario, being θ the retailer's belief on this scenario, and where $\theta \in [0,1]$; while d_{Jmr}^r is the retailer's disagreement payoff when facing a manufacturer that will retaliate if they do not reach an new agreement, which retailer's belief on this scenario is $1-\theta$.

Similarly, $E(d_j^m)$ in (14) represents the manufacturer's expected disagreement payoff, where $E(d_j^m) = \delta d_j^m + (1-\delta)d_{Jmr}^m$, in which d_j^m and d_{Jmr}^m represent the manufacturer's disagreement payoff of facing a "status quo" or "war/strike" scenario respectively, and δ denotes the manufacturer's belief on not having to count for an imminent strike, where $\delta \in [0,1]$. In this way, the disagreement payoffs for retailers and manufacturers will be given by:

	Peaceful/Status Quo Scenario	War/Strike Scenario
Manufacturer	$d_j^m = \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k^{-j}(p)M$	$d_{J^{mr}}^m = \sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k s_k^{-J^{mr}}(p)M$
Retailer	$d_j^r = \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k^{-j}(p)M$	$d_{J^{mr}}^r = \sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k s_k^{-J^{mr}}(p)M$

where J^m, J^r and $s_k^{-j}(p)$ represent the same as before, while $s_k^{-J^{mr}}(p)$ is the market share of product k when there is nonagreement on product j resulting in none transaction between that manufacturer and retailer, meaning that all products that were traded between these two agents (we called this set of products J^{mr}) are not longer available in the market.

In this way, and given that $\frac{\partial \pi_j^r}{\partial w_j} = -s_j(p)M$ and $\frac{\partial \pi_j^m}{\partial w_j} = s_j(p)M$, from the first order condition of (14) we get the following expression:

$$\pi_j^m - E(d_j^m) = \tilde{\lambda}_j(\pi_j^r - E(d_j^r)) \tag{15}$$

where again $\tilde{\lambda}_j = \frac{(1-\lambda_j)}{\lambda_j}$. After including the agreements and disagreements payoffs in the previous expression, we get the following[14]:

$$\Gamma_j s_j - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k \Delta s_k^{-j} - \tilde{\delta}_j \left(\sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k s_k^{-J^{mr}} - \sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k s_k^{-j}\right) = \tilde{\lambda}_j \left[\gamma_j s_j - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k \Delta s_k^{-j} - \tilde{\theta}_j \left(\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k s_k^{-J^{mr}} - \sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k s_k^{-j}\right)\right] \tag{16}$$

where $\tilde{\theta} = 1-\theta$ and $\tilde{\delta} = 1-\delta$. Notice that the expression presented by Draganska et al. (2010) would be the case when $\tilde{\theta}_j = \tilde{\delta}_j = 0$ (i.e. $\theta_j = \delta_j = 1$).

By defining S^j is the matrix of shares which element $S^j[j,j] = 0$ and $S^j[j,k] = s_k^{-j}(p)$ otherwise. Similarly $S^{J^{mr}}$ is the matrix of shares which element $S^{J^{mr}}[j,k] = 0$ if j belong to the same retailer-manufacturer (J^{mr}) that the negotiated product k and $S^{J^{mr}}[j,k] = s_k^{-J^{mr}}(p)$ otherwise, where $s_k^{-J^{mr}}(p)$ is the share of product k when product j and the other products belonging to the same J^{mr} are not longer available in the market. In this way, the matrix notation of equation (16 will be the following:

[14] See Appendix C to more details on this result.

$$(T^m * D^j)\mathbf{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\mathbf{\Gamma}\right] = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma} - \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] \quad (17)$$

where $*$, T^r, T^m, $\mathbf{\Gamma}$, $\tilde{\boldsymbol{\lambda}}$, and D^j represent the same as before.

Given that this renegotiation will take place under the bargaining conditions present at that moment, which includes the bargainers' bargaining position (power) at that moment, then $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{\boldsymbol{SQ}}$ and, as seen in section 4.3.1, $\mathbf{\Gamma}$ can be expressed as $\mathbf{\Gamma} = \boldsymbol{p} - \boldsymbol{\gamma} - (IP\boldsymbol{\kappa} + \boldsymbol{\eta})$, then (17) becomes[15]:

$$\boldsymbol{p} - \boldsymbol{\gamma} - \left(T^m * D^j\right)^{\dagger} \left[\tilde{\boldsymbol{\lambda}}^{\boldsymbol{SQ}} * \left[\left(T^r * D^j\right)\boldsymbol{\gamma}\right)\right] = IP\boldsymbol{\kappa} + E\tilde{\boldsymbol{\delta}} + H\tilde{\boldsymbol{\theta}} + \sum_{z=1}^{Z} F_z \tilde{\boldsymbol{\delta}} \kappa_z + [G + I]\boldsymbol{\eta} \quad (18)$$

where $(T^m * D^j)^{\dagger}$, as before is the Moore-Penrose inverse of matrix $(T^m * D^j)$, and E is an square matrix of dimension J, which general element is $E[i,j] = (T^m * D^j)^{\dagger}[i,j] \left(\sum_{k=1}^{J} ts^m_{jk}(p_k - \gamma^r_k)\right)$, in which again $ts^m_{jk} = (T^m * (S^{J^{mr}} - S^j))[j,k]$.

While H is an square matrix of the same dimension as E, which general element is $H[i,j] = -(T^m * D^j)^{\dagger}[i,j]d_j$, where d_j is the element in position j of the vector $\left(\tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right)$.

Additionally, matrix F_z is an square matrix of dimension J which general element is $F_z[i,j] = -(T^m * D^j)^{\dagger}[i,j] \left(\sum_{k=1}^{J} ts^m_{jk} IP[k,z]\right)$, where ts^m_{jk} is the same as before.

Finally, I is an identity matrix, and G is a matrix of dimension J, which general element is $G[i,j] = -\sum_{k=1}^{J}(T^m * D^j)^{\dagger}[j,k]\tilde{\delta}_k ts^w_{kj}$.

Then, from (18) the beliefs of both retailers and manufacturers can be computed, afterwards the manufacturers' margins of a possible agreement under uncertainty can be recover from equation (17), as can be seen in equation (21) in Appendix (D). Finally, these are the manufacturers' margins resulted from the possible new agreement, which should fulfill the *(weak) disagreement point concavity* condition, presented in (10).

5 Data and Results

*[**Disclaimer:** In this section we present some preliminary results to give a glance on the implementation of our approach, and show that, although been in its early stages, seems to perform as*

[15] For more details on the following matrices E, F_z, G and H, see Appendix D.

expected. In this way, in the first part of the section it is briefly presented the data used to exemplified the application of the model, afterwards the results from this trial application are shown, which seem to be in line with our expectations.

Despite this results, we considered necessary to implement some changes to improve the robustness of the estimations, such as: enlarge the time frame of the analysis, the choice set of the demand, among others changes, that until now seem to perform similar to the former ones. Even though we do not have the final results, in appendix (E) can be found some information on the steps that were taken until now, and some tables with the results of these steps.]

5.1 Data

The data of this work came from the Consumer panel (Verbraucherpanel) collected by GFK Panelservice SE, which is a household-scan data set from german representative consumer panel data. For this trial version, we used the milk dataset for year 2008, consisting of 525, 191 observations, this dataset provided information on date, number mililiters, number of packages, amount spent on milk, level of fat of the milk, brand, retailer and manufacturer per purchase.

Regarding the retailers, all outlets in the dataset were kept except wholesalers, which represented 0.46% of the milk observations in year 2008.

The dataset allowed us to distinguish between conventional brands and private labels. Given the fact that private labels are specific of a retailer, and because of this the consumer will not find all private labels in each store but it is highly likely to find a private label where the purchasing choice is taking place; then, we grouped all private labels as a single brand. And as mentioned before, we assume that a product is defined as the retailer-brand combination.

The outside option included the alternatives from the small manufacturers within retailer.[16]

Given that manufacturers bargain with retailers over their whole portafolio of products including both their well-known brands and small brands, and in order to keep the interactions between retailers and manufacturers over all kind of alternatives, the well-known brands were kept as they were while the small brands were grouped within manufacturer and consider as a single brand of that manufacturer.[17]

[16]If a manufacturer could not sell at least 1440 products in a retailer - around 5 products per day, regardless the brand-, then it was considered a small manufacturer in that retailer and those products were part the outside option.

[17]Brands from well-known manufacturers which repeated less than 1440 times in the database - they were sold less

In this way, this study considered 13 brands[18], 19 retailers, which resulted in 39 differentiated alternatives aside from the outside option.

Regarding the fat-level variable to be implemented in the demands estimations, it was defined two levels of fat, one from 0 to 1.5% and other from 1.6% to 3.9%.

The final data set consists of 522,759 observations, 44.09% from them corresponds to the period January - May (before period) and 55.91% to the period June - December (after period) of 2008. The outside option represents 14.55% of the observations. Conventional brands constitutes 6.55% of the observations while private labels 78.9%. Excluding the outside option, the mean price in cents per mililiter of milk in 2008 was 0.065 with an standard devitation of 0.012; the mean price(standard deviation) for conventional brands and private labels correspond to 0.081(0.021) and 0.064(0.010) respectively. Similarly, the mean of the monthly market share(standard deviation) of conventional brands in 2008 was 7.56%(0.009), while for private labels was 92.44%(0.009). Further descriptive statistics for the periods before and after the strike can be found in Tables 1 and 2.

Table 1: Descriptive statistics

	Before			After		
		Mean			Mean	
	Freq.	Monthly Market Share	Price[1]	Freq.	Monthly Market Share	Price[1]
Outside Option	14.28%			14.76%		
Private Labels	79.03%	1.017%(0.0134)	0.067(0.009)	78.79%	0.74%(0.0097)	0.062(0.011)
Conventional Brands	6.68%	0.08%(0.0011)	0.083(0.021)	6.45%	0.05%(0.0005)	0.080(0.022)

1) Price in cents per mililiter.

Standard Deviations are in parenthesis.

than 5 time per day aprox. in whole Germany-, were considered as a small brand of that manufacturer.

[18]Private labels counted as one brand.

Table 2: Monthly Quantity and Purchases

Before[1]			After[1]		
Month	Quantity[2]	Purchases	Month	Quantity[2]	Purchases
Jan	129, 724.2	38, 328	Jun	108, 849.1	31, 299
Feb	129, 237.9	39, 574	Jul	110, 740.8	33, 989
Mar	129, 567.8	39, 894	Aug	120, 768.7	35, 864
Apr	138, 113.6	41, 030	Sep	122, 266	36, 467
May	149, 497	38, 743	Oct	130, 506.3	39, 073
			Nov	130, 364.8	36, 104
			Dec	123, 056.4	36, 348
Mean	135, 228.1	39, 513.8	Mean	120, 936.01	35, 592

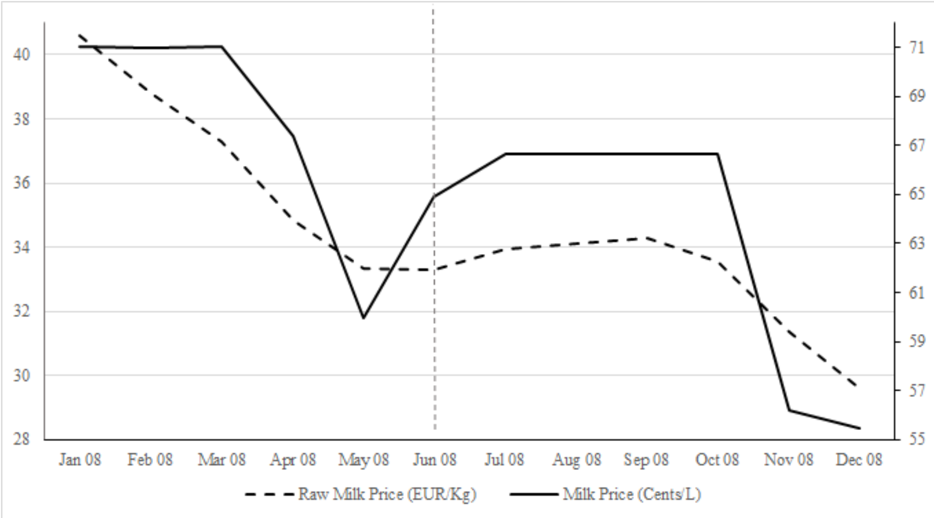
1) Excluding outside option observations. 2) Liters of milk

Aside from the GFK database, data on monthly raw milk price (*Preise für angelieferte Vollmilch ab Hof bei tatsächlichem Fett- und Eiweissgehalt*) coming from the German Federal Ministry of Food and Agriculture[19] (*Bundesministerium für Ernärung, Landwirtschaft und Verbraucherschutz*) was used as supply cost shifter of this industry. Figure 3 presents the performance of this variable as well as the monthly average milk prices[20] throughout 2008.

[19]Statistical monthly report 03-2009: Table MBT-0301431-0000

[20]Excluding the prices of the outside option.

Figure 3: Average Milk Prices



5.2 Preliminary results

For the estimation of demand, we took into account the potential endogeneity issues coming from the supply side, to control for this, we use a two-step estimation, in which in the first step we implement a control function, as proposed by Petrin and Train(2010), following the specification in (4), in which the farmers milk price was used as a cost shifter. Our estimation provides us with a positive strongly significant impact, together with a high R-square. Moreover, the F-value is well above the value 10 known as the threshold for weak instruments suggested by Staiger and Stock(1997)[21]. Therefore, we are confident that we can sufficiently predict the price. Table 3 shows the results from the first-stage of the demand estimation.

[21] F-value = 1361.83

Table 3: First-stage estimation: Control function

Dep. Var: $\overline{p}_{jt}$	Coefficient
Raw milk price	0.002***(0.00006)
Retailers fixed effects	X
Brand fixed effects	X
R^2	0.9929
Number of observations	468

***, **, * denote 1%, 5% and 10% level of significance respectively.

Standard Errors are in parenthesis.

In the second step, we follow the demand model introduced in section 4.1, in which the milk demand for each period was estimated using the dataset described in section 5.1, from which a random sample of $200,000$ observations was selected, which consisted of $88,140$ observations from period January - May and $111,860$ from June - December 2008. And based on Revelt and Train (1998) and Train (2003), a random coefficient logit model was estimated by maximizing the simulated log-likelihood for each period according to the following specification:

$$SLL = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T} d_{ijt}\ \ln(\check{L}_{ijt}), \text{ where } \check{L}_{ijt} = \tfrac{1}{R}\sum_{r=1}^{R} L_{ijt}$$

where d_{ijt} is equal to 1 if consumer i chooses product j at period t and 0 otherwise, $\check{L}_{ijt}$ is the individual simulated market share of product j at period t, and R is the total number of Halton draws for each consumer i. In table 4 are the results from the demand estimation for both period, the pre-treatment (column 1) and the post treatment period column (2). As can be seen the estimated price coefficients (α) are, as expected, both negative and significant at the 1% level. Also, the standard deviations of the price (σ_α) are significant, indicating in both periods that the impact of the price on the demand is heterogeneous. In addition, it becomes evident that both specifications are rather similar in terms of the coefficient. Still, the impact of the price seems to be lower after the strike took place. This is consistent with the elasticities provided in Table 5.

Table 4: Demand results

	Before	After
Price (α)	$-165.11^{***}(1.831)$	$-131.82^{***}(1.58)$
Price (σ_α)	$60.42^{***}(0.71)$	$57.95^{***}(0.58)$
CF (ρ)	$13.04^{***}(2.77)$	$24.42^{***}(2.79)$
Fat level fixed effects	X	X
Retailers fixed effects	X	X
Brand fixed effects	X	X
Log likelihood	-154013.84	-204642.09

***, **, * denote 1%, 5% and 10% level of significance respectively.

Standard Errors are in parenthesis.

The demand elasticities were computed using the expresions available on section 4.1 and following the simulation process suggested by Cameron and Trivedi (2010)[22], to get the mixed logit marginal effects.

The elasticities derived from the demand estimations have the expected negative sign, as can e observed in Table 5. However, there is a decrease (in absolute terms) after the strike, indicating that consumers are less sensitive to price changes. According to this results, while the demand of private labels became 27.58% less sensitive to prices changes after the strike, the demand of conventional brands became in average 35.51% less sensitive.

This results could reflect a behavioral response of the demand, that could be related to the Pay-What-You-Want (PWYW) literature, in which individual derive utility from the image they portrait and they are willing to signal their prosocial views through their consumptions (*self-signaling*) [e.g. Greif and Egbert (2016), Gneezy et al. (2012)].

In this way, and reading the results under PWYW perspective, in the before-strike period consumers were not aware about the *"unfair"* conditions for milk farmers derived from the price policy of retailers, and once they became aware of the issue, a portion of consumers chose to leave the market, as a sign of protest against the *unfair* conditions, as can be observed the drop in the

[22]See Cameron and Trivedi (2010), p. 353.

purchases and quantities boughts from the before- to the after-strike period in Table 2. The decrease of the demanded quantity, increased the incentives in the retailers to decrease prices to attract more consumers, but this also increase the presure to bargain harder with manufacturers to decrease the wholesale prices (w_j) and therefore retailers' costs, which would affect eventually to the farmers, as can be observed in Figure 3, both the milk prices' and raw milk prices' performance increased right after the strike, but both drop by the end of the period, supporting our hypothesis. Notice as well that, even with the drop of the milk prices at the end of after-strike period, the purchases did not recover to the levels before the strike, which would be in line with the hypothesis of a behavioral response of the demand. Also notice that, the decrease of the raw milk prices after the strike hints as well, that the bargining process resulted in favor of the retailers, which will be corroborated below.

Still, we should take this results from the demand response carefully, given the fact that the average milk price after the strike was lower, as can be observed in Figure 3, which could be suggest as well a possible move along the demand curve and therefore lower elasticicities (in absolute terms).

Table 5: Own-price elasticities (OPE) and Retailer Margins

Brand	OPE		Retailer Margins	
	Before- strike Period	After-strike Period	Before-strike Period	After-strike Period
PL	−9.76	−7.06	0.01	0.01
B1	−6.02	−3.31	0.02	0.04
B2	−7.65	−4.50	0.02	0.02
B3	−8.77	−4.46	0.01	0.02
B4	−10.18	−7.37	0.01	0.01
B5	−9.01	−5.81	0.01	0.02
B6	−10.02	−7.35	0.01	0.01
B7	−10.22	−7.41	0.01	0.01
B8	−9.67	−6.79	0.01	0.01
B9	−10.07	−7.38	0.01	0.01
B10	−8.09	−4.88	0.01	0.02
B11	−8.72	−5.44	0.01	0.02
B12	−7.27	−4.61	0.02	0.02
Mean	−8.89	−6.08	0.01	0.02

The name of the brands cannot be provided due to confidentiality agreement with GFK Panelservice SE.

Afterwards, the retailers' margins were computed for both periods, which results can be found in Table 5, and as can be observed in most of the cases the retailers' margins after the strike were larger than before, which would be in line with the hypothesis of a tougher bargaining position of retailers after the strike period.

Finally the manufacturers' margins were computed. As mention before, for the before- and after-strike periods the specification presented in 13 will be used, in order to get the *status quo* results (Γ_j^{SQ}) and retailers' bargaining power (λ_j^{SQ}) in the before-strike period, and the manufacturers' margins (Γ_j) and retailers bargaining power in the after-strike period. Additionally, for the before-strike period, and once λ_j^{SQ} was obtained, the beliefs ($1-\theta$ and $1-\delta$) of the bargainers were computed by following specification in expression (18), and the manufacturers' margins of the "war" contingent scenario (Γ_j^{war}) following the process presented in Appendix (B). Notice that the contingent scenario of the *status-quo* kind of negotiation would give the same results from the *status-quo*. Table 6 presents the described results.

Due these are early results, we still have to compute the manufacturers' margins under uncertainty (Γ_j^u); however, as can be observed in Table 6 the beliefs of the bargainers to face a "war" type of scenario were almost 1; hence, the $\Gamma_j^u \approx \Gamma_j^{war}$. Therefore, by comparing Γ_j^{war} with $\delta\Gamma_j^{SQ} + (1-\delta)\Gamma_j^{war}$ we can corroborate that the incentive condition (10) is not fulfilled, i.e. manufacturers would not have the incentive to reach a new agreement under such uncertain conditions and would have preferred to stay with the surplus division that they already agreed with retailers (Γ_j^{SQ}), surplus division that would not satisfied the demands of farmers, being the strike an imminent event.

In this way, to evaluate the effects on the surplus division of the strike, the results of the *status-quo* of the before-strike period are the one to compare against the after-strike results. And as can be seen, in most of the cases the manufacturers margins after the strike were lower than before, and the bargaining power of retailers (λ_j) were higher after the strike, which are in line with the retailers margins results, which increased in most cases in the after-strike period.

Table 6: Manufacturers Margins and Bargaining Parameter

Brand	Before-strike Period			After-strike Period	
	Margin				
	Γ^{war} $1-\delta=0.997$ $1-\theta=0.998$	Γ^{SQ}	λ_j^{SQ}	Γ	λ_j
PL	0.0062	0.0025	0.8088	0.0026	0.8292
B1	0.0136	0.0187	0.5422	0.0080	0.8221
B2	0.0181	0.0183	0.5398	0.0115	0.6734
B3	0.0072	0.0141	0.4467	0.0033	0.8752
B4	0.0085	0.0046	0.7003	0.0004	0.9595
B5	0.0060	0.0116	0.5219	0.0036	0.8233
B6	0.0023	0.0020	0.8154	0.0031	0.7785
B7	0.0078	0.0051	0.6178	0.0046	0.6675
B8	0.0136	0.0046	0.7626	0.0072	0.7544
B9	0.0062	0.0010	0.8916	0.0047	0.6812
B10	0.0060	0.0187	0.4129	0.0031	0.8645
B11	0.0072	0.0152	0.4285	0.0032	0.8429
B12	0.0023	0.0337	0.3276	0.0285	0.4342
Mean	0.0070	0.0088	0.6667	0.0049	0.7994

The name of the brands cannot be provided due to confidentiality agreement with GFK Panelservice SE.

6 Conclusions

Through this work we have presented an estimation proposal that can allow the strategic analysis of bargainings in recurrent multi-product and multi-bargaining commercial relationships, in which agents involved in the negotiation have uneven bargaining power.

Even though it is still a work in process, we aimed to show that the implementation is possible, and through it to deepen the analysis of bargaining solutions and the influence of uncertainty in the outcome we observed in markets; being able to have a better understanding of the incentives that agents have, which can be used as a tool to anticipate the potential results of strategies that are under consideration.

We consider that this work could be use in the study of other issues in this and other industries, such as listing and/or delisting of products. At the same time, we find that this work could be a contribution for the empirical study of reputation and threats.

A Matrix C in section 4.3.1

From expression (12) we can express the manufacturers' margin vector as:

$$\mathbf{\Gamma^{SQ}} = (T^m * D^j)^\dagger \left(\tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\gamma\right]\right)$$

where $(T^m * D^j)^\dagger$ represents the Moore-Penrose inverse of matrix $(T^m * D^j)$. By denoting the vector $(T^r * D^j)\boldsymbol{\gamma}$ as $\boldsymbol{b}$, and its general element as $\boldsymbol{b}[i,1] = b_i$. Similarly, denoting the general element of matrix $(T^m * D^j)^\dagger[i,k] = a_{ik}$, we will have:

$$\begin{aligned}
\mathbf{\Gamma^{SQ}} = &\ (T^m * D^j)^\dagger(\tilde{\boldsymbol{\lambda}} * \boldsymbol{b}) \\
\mathbf{\Gamma^{SQ}} = &\ \begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{JJ} \end{pmatrix} \begin{pmatrix} \tilde{\lambda}_1 b_1 \\ \vdots \\ \tilde{\lambda}_J b_J \end{pmatrix} \\
\mathbf{\Gamma^{SQ}} = &\ \begin{pmatrix} a_{11} b_1 \tilde{\lambda}_1 + \cdots + a_{1J} b_J \tilde{\lambda}_J \\ \vdots \\ a_{J1} b_1 \tilde{\lambda}_1 + \cdots + a_{JJ} b_J \tilde{\lambda}_J \end{pmatrix} \\
\mathbf{\Gamma^{SQ}} = &\ \begin{pmatrix} a_{11} b_1 & \cdots & a_{1J} b_J \\ \vdots & \ddots & \vdots \\ a_{J1} b_1 & \cdots & a_{JJ} b_J \end{pmatrix} \begin{pmatrix} \tilde{\lambda}_1 \\ \vdots \\ \tilde{\lambda}_J \end{pmatrix} \\
\mathbf{\Gamma^{SQ}} = &\ C\tilde{\boldsymbol{\lambda}}
\end{aligned}$$

In this way, the vector of manufacturers' margins can be expressed as $\mathbf{\Gamma^{SQ}} = C\tilde{\boldsymbol{\lambda}}$, where C is an square matrix of dimension J which general element is $C[i,k] = (T^m * D^j)^\dagger[i,k]b_k$, where b_k is the element at position k of vector $(T^r * D^j)\boldsymbol{\gamma}$.

B Manufacturers' margins in the contingent scenario of strike/war

The bargaining knowing with certainty that the scenario to be face when disagreement on one product is the losing of the whole commerical relationship, can be resolved in a similar fashion that the one presented in section 4.3.1 [Draganska et al. (2010)]. This would be a bargaining in which manufacturers negotiate with retailers each product separately, but both will be certain of a retaliation if disagreement occurs, then the negotiation over the whosale price of product j will be given by the following Nash product:

$$\underset{w_j}{\text{Max}} \left(\pi_j^r - d_{J^{mr}}^r\right)^{\lambda_j} \left(\pi_j^m - d_{J^{mr}}^m\right)^{(1-\lambda_j)} \tag{19}$$

in expression (19) π_j^r and π_j^m represent respectively the retailers and manufacturers profits from selling product j (agreement payoff), where J^{mr} represent the set of all products traded between the retailer and manufacturer engage in the bargaining, where $j \in J^{mr}$, and $d_{J^{mr}}^r$ and $d_{J^{mr}}^m$ represent respectively the retailer and manufacturer profits when none of the products belonging to J^{mr} are not longer available in retailer r (disagreement payoffs):

	Agreements	Disagreement
Manufacturer	$\pi_j^m = \Gamma_j s_j(p)M + \sum_{\substack{k \in J^m \\ k \neq j}} \Gamma_k s_k(p)M$	$d_{J^{mr}}^m = \sum_{\substack{k \in J^m \\ k \notin J^{mr}}} \Gamma_k s_k^{-J^{mr}}(p)M$
Retailer	$\pi_j^r = \gamma_j s_j(p)M + \sum_{\substack{k \in J^r \\ k \neq j}} \gamma_k s_k(p)M$	$d_{J^{mr}}^r = \sum_{\substack{k \in J^r \\ k \notin J^{mr}}} \gamma_k s_k^{-J^{mr}}(p)M$

where as before Γ_j denotes the manufacturer's margin for product j; $s_k^{-J^{mr}}(p)$ represents the market share of product k when there was nonagreement on product j and therefore none of the products belonging to J^{mr} are available in the market.

Following the same proceding presented in section 4.3.1 we get the following expression:

$$\left(\sum_{\substack{j \in J^m \\ j \in J^{mr}}} \Gamma_j s_j(p) - \sum_{\substack{k \in J^m \\ k \notin J^{mr}}} \Gamma_k \Delta s_k^{-J^{mr}} \right) = \frac{(1-\lambda_j)}{\lambda_j} \left(\sum_{\substack{j \in J^m \\ j \in J^{mr}}} \gamma_j s_j(p) - \sum_{\substack{k \in J^r \\ k \notin J^{mr}}} \gamma_k \Delta s_k^{-J^{mr}} \right)$$

where $\Delta s_k^{-J^{mr}}$ represents the change in the market share of product k when the products belonging to J^{mr} are not longer available in the market, i.e. $\Delta s_k^{-J^{mr}} = s_k(p)^{-J^{mr}} - s_k(p)$.

As before, $\mathbf{\Gamma}$ is the vector of manufacturer margins, T^m is the manufacturers ownership matrix which element $T^m[j,k] = 1$ if both products k and j are produced by the same manufacturer and $T^m[j,k] = 0$ otherwise. And defining $D^{J^{mr}}$ as the matrix of shares and share variations, which $D^{J^{mr}}[j,k] = s_k$ if k and j belong to J^{mr}, while $D^{J^{mr}}[j,k] = -\Delta s_k^{-J^{mr}}$ otherwise.

$$(T^m * D^{J^{mr}})\mathbf{\Gamma} = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^{J^{mr}})\boldsymbol{\gamma}\right] \tag{20}$$

where $*$, T^r and $\tilde{\boldsymbol{\lambda}}$ represent the same as before, $\boldsymbol{\gamma}$ is the retailer margin vector obtained through (8).

Notice that this contingent solution will be evaluated by the agent at the renegotiation, therefore under the preestablished bargaining conditions, which include the bargaining power, ie. $\boldsymbol{\lambda} = \boldsymbol{\lambda^{SQ}}$.

Therefore, this contingent manufacturer's margin can be computed by:

$$\boldsymbol{\Gamma^{war}} = (T^m * D^{J^{mr}})^{\dagger} \left(\boldsymbol{\tilde{\lambda}^{SQ}} * \left((T^r * D^{J^{mr}})\gamma\right)\right)$$

where $(T^m * D^{J^{mr}})^{\dagger}$ represents the Moore-Penrose inverse of matrix $(T^m * D^{J^{mr}})$.

C $\pi_j^r - E(d_j^r)$ and $\pi_j^w - E(d_j^w)$ in expression (15) section 4.3.2

Recalling expression (15) introduced in section 4.3.2:

$$\pi_j^w - E(d_j^m) = \tilde{\lambda}_j(\pi_j^r - E(d_j^r))$$

Notice that $\pi_j^r - E(d_j^r)$ and $\pi_j^m - E(d_j^m)$ are similar expression, therefore the process to derive the solution of one apply to the other. Considering first the case of $\pi_j^r - E(d_j^r)$, we will have:

$$\begin{aligned}
\pi_j^r - E(d_j^r) &= \pi_j^r - \theta_j d_j^r - (1-\theta_j) d_{J^{mr}}^r \\
&= \gamma_j M s_j + \textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M s_k - \theta_j (\textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M s_k^{-j}) - (1-\theta_j)(\textstyle\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k M s_k^{-J^{mr}}) \\
&= \gamma_j M s_j + \textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M (s_k - \theta_j s_k^{-j}) - (1-\theta_j)(\textstyle\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k M s_k^{-J^{mr}})
\end{aligned}$$

Defining $\Delta s_k^{-j} = s_k^{-j} - s_k$, then $\theta_j s_k^{-j} - s_k = \Delta s_k^{-j} - s_k^{-j} + \theta_j s_k^{-j}$.

$$\pi_j^r - E(d_j^r) = \gamma_j M s_j - \textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M \Delta s_k^{-j} - \tilde{\theta_j}\left[\textstyle\sum_{\substack{k\in J^r \\ k\notin J^{mr}}} \gamma_k M s_k^{-J^{mr}} - \textstyle\sum_{\substack{k\in J^r \\ k\neq j}} \gamma_k M s_k^{-j}\right]$$

where $\tilde{\theta_j} = (1-\theta_j)$.

Following the same procedure we get that the expression for $\pi_j^m - E(d_j^m)$, and expressing $1-\delta_j$ as $\tilde{\delta_j}$, then we have the following result:

$$\pi_j^m - E(d_j^m) = \Gamma_j M s_j - \textstyle\sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k M \Delta s_k^{-j} - \tilde{\delta_j}\left[\textstyle\sum_{\substack{k\in J^m \\ k\notin J^{mr}}} \Gamma_k M s_k^{-J^{mr}} - \textstyle\sum_{\substack{k\in J^m \\ k\neq j}} \Gamma_k M s_k^{-j}\right]$$

D Matrices on Section 4

D.1 Manufacturers' margins under uncertainty of section 4.3.2

Recalling (17) introduced in section 4.3.2:

$$(T^m * D^j)\mathbf{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\mathbf{\Gamma}\right] = \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma} - \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right]$$

Focusing just at the left-hand side of this equation, which we will called "LHS":

$$LHS = (T^m * D^j)\mathbf{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\mathbf{\Gamma}\right]$$

Denoting the general element of matrix $(T^m - D^j)[i,j]$ as td^m_{ij}; while calling the general element of matrix $(T^m * (S^{J^{mr}} - S^j))$ as ts^m_{ij}, then we will have:

$$\begin{aligned}
LHS = &\ (T^m * D^j)\mathbf{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\mathbf{\Gamma}\right] \\
LHS = &\begin{pmatrix} td^m_{11} & \cdots & td^m_{1J} \\ \vdots & \ddots & \vdots \\ td^m_{J1} & \cdots & td^m_{JJ} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} - \begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts^m_{11} & \cdots & ts^m_{1J} \\ \vdots & \ddots & \vdots \\ ts^m_{J1} & \cdots & ts^m_{JJ} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} \\
LHS = &\begin{pmatrix} td^m_{11} & \cdots & td^m_{1J} \\ \vdots & \ddots & \vdots \\ td^m_{J1} & \cdots & td^m_{JJ} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} - \begin{pmatrix} \tilde{\delta}_1 (ts^m_{11}\Gamma_1 + \cdots + ts^m_{1J}\Gamma_J) \\ \vdots \\ \tilde{\delta}_n (ts^m_{J1}\Gamma_1 + \cdots + ts^m_{JJ}\Gamma_J) \end{pmatrix} \\
LHS = &\begin{pmatrix} td^m_{11} & \cdots & td^m_{1J} \\ \vdots & \ddots & \vdots \\ td^m_{J1} & \cdots & td^m_{JJ} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} - \begin{pmatrix} \tilde{\delta}_1 ts^m_{11} & \cdots & \tilde{\delta}_1 ts^m_{1J} \\ \vdots & \ddots & \vdots \\ \tilde{\delta}_J ts^m_{J1} & \cdots & \tilde{\delta}_J ts^m_{JJ} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} \\
LHS = &\begin{pmatrix} td^m_{11} - \tilde{\delta}_1 ts^m_{11} & \cdots & td^m_{1J} - \tilde{\delta}_1 ts^m_{1J} \\ \vdots & \ddots & \vdots \\ td^m_{J1} - \tilde{\delta}_J ts^m_{J1} & \cdots & td^m_{JJ} - \tilde{\delta}_J ts^m_{JJ} \end{pmatrix} \begin{pmatrix} \Gamma_1 \\ \vdots \\ \Gamma_J \end{pmatrix} \\
LHS = &\ A\mathbf{\Gamma}
\end{aligned}$$

Therefore, matrix A is an square matrix of dimension J which general element $A[i,j] = (T^m * D^j)[i,j] - \tilde{\delta}_i (T^m * (S^{J^{mr}} - S^j))[i,j]$.

Notice that the right-hand side (RHS) of expression (17) is similar to the LHS; therefore, denoting the general elements of matrices $(T^r * D^j)[i,j]$ and $(T^r * (S^{J^{mr}} - S^j))[i,j]$ as td^r_{ij} and ts^r_{ij} respectively, and following the above process we get:

$$
\begin{aligned}
RHS = & \begin{pmatrix} \tilde{\lambda}_1 \\ \vdots \\ \tilde{\lambda}_J \end{pmatrix} * \left[\begin{pmatrix} td^r_{11} - \tilde{\theta}_1 ts^r_{11} & \cdots & td^r_{1J} - \tilde{\theta}_1 ts^r_{1J} \\ \vdots & \ddots & \vdots \\ td^r_{J1} - \tilde{\theta}_J ts^r_{J1} & \cdots & td^r_{JJ} - \tilde{\theta}_J ts^r_{JJ} \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_J \end{pmatrix} \right] \\
RHS = & \ \tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma}
\end{aligned}
$$

Being B also an square matrix of dimension J which general element $B[i,j] = (T^r * D^j)[i,j] - \tilde{\theta}_i(T^r * (S^{J^{mr}} - S^j))[i,j]$.

Then expression (17) becomes in $A\boldsymbol{\Gamma} = \tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma}$, therefore the manufacturers' margins under uncertainty can be expressed as:

$$
\boldsymbol{\Gamma}^u = A^{\dagger}(\tilde{\boldsymbol{\lambda}} * B\boldsymbol{\gamma}) \tag{21}
$$

D.2 Matrices E, F, G and H

Recalling (17) introduced in section 4.3.2:

$$
\begin{aligned}
(T^m * D^j)\boldsymbol{\Gamma} - \tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\boldsymbol{\Gamma}\right] = & \ \tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma} - \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] \\
\boldsymbol{\Gamma} - (T^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\lambda}} * (T^r * D^j)\boldsymbol{\gamma}\right] = & \ (T^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\delta}} * \left[(T^m * (S^{J^{mr}} - S^j))\boldsymbol{\Gamma}\right]\right] - (T^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\lambda}} * \tilde{\boldsymbol{\theta}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right]
\end{aligned}
$$

Given that $\boldsymbol{\Gamma} = \boldsymbol{p} - \boldsymbol{\gamma} - (IP\boldsymbol{\kappa} + \boldsymbol{\eta})$, then we have:

$$
\begin{aligned}
\boldsymbol{p} - \boldsymbol{\gamma} - (T^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\lambda}} * \left[(T^r * D^j)\boldsymbol{\gamma}\right]\right] = & \ \mathrm{IP}\boldsymbol{\kappa} + (T^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))(\boldsymbol{p} - \boldsymbol{\gamma}))\right] \\
& \text{-} \ (\mathrm{T}^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))IP\boldsymbol{\kappa})\right] \\
& \text{-} \ (\mathrm{T}^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\theta}} * \tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] \\
& \text{-} \ (\mathrm{T}^m * D^j)^{\dagger} \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))\boldsymbol{\eta})\right] + \boldsymbol{\eta}
\end{aligned} \tag{22}
$$

Then, denoting again the general element of matrix $(T^m * D^j)^{\dagger}$ as a_{ij} and the general element of matrix $(T^m * (S^{J^{mr}} - S^j))$ as ts^m_{ij}, then the second term of the right-hand side is:

$$
\begin{aligned}
(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))(\boldsymbol{p} - \boldsymbol{\gamma}))\right] &= \begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts^m_{11} & \cdots & ts^m_{1J} \\ \vdots & \ddots & \vdots \\ ts^m_{1J} & \cdots & ts^m_{JJ} \end{pmatrix} \begin{pmatrix} p_1 - \gamma_1 \\ \vdots \\ p_J - \gamma_J \end{pmatrix} \right] \\
&= \begin{pmatrix} a_{11}\tilde{\delta}_1(ts^m_{11}(p_1-\gamma_1) + \cdots + ts^m_{1J}(p_J - \gamma_J)) + \cdots + a_{1J}\tilde{\delta}_J(ts^m_{J1}(p_1-\gamma_1) + \cdots + ts^m_{JJ}(p_J-\gamma_J)) \\ \vdots \\ a_{J1}\tilde{\delta}_1(ts^m_{11}(p_1-\gamma_1) + \cdots + ts^m_{1J}(p_J - \gamma_J)) + \cdots + a_{JJ}\tilde{\delta}_J(ts^m_{J1}(p_1-\gamma_1) + \cdots + ts^m_{JJ}(p_J-\gamma_J)) \end{pmatrix} \\
&= \begin{pmatrix} a_{11}(ts^m_{11}(p_1-\gamma_1) + \cdots + ts^m_{1J}(p_J-\gamma_J)) & \cdots & a_{1J}(ts^m_{J1}(p_1-\gamma_1) + \cdots + ts^m_{JJ}(p_J-\gamma_J)) \\ \vdots & \ddots & \vdots \\ a_{J1}(ts^m_{11}(p_1-\gamma_1) + \cdots + ts^m_{1J}(p_J-\gamma_J)) & \cdots & a_{JJ}(ts^m_{J1}(p_1-\gamma_1) + \cdots + ts^m_{JJ}(p_J-\gamma_J)) \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} \\
&= \begin{pmatrix} a_{11}\sum_{k=1}^J ts^m_{1k}(p_k-\gamma_k) & \cdots & a_{1J}\sum_{k=1}^J ts^m_{Jk}(p_k-\gamma_k) \\ \vdots & \ddots & \vdots \\ a_{J1}\sum_{k=1}^J ts^m_{1k}(p_k-\gamma_k) & \cdots & a_{JJ}\sum_{k=1}^J ts^m_{Jk}(p_k-\gamma_k) \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} \\
&= E\tilde{\boldsymbol{\delta}}
\end{aligned}
$$

Then, the second term of the right-hand side of (22) can be expressed as $E\tilde{\boldsymbol{\delta}}$, where E is an square matrix of dimension J, which general element is $E[i,j] = (T^m * D^j)^\dagger[i,j]\left(\sum_{k=1}^J ts^m_{jk}(p_k - \gamma^r_k)\right)$, in which again $ts^m_{jk} = (T^m * (S^{J^{mr}} - S^j))[j,k]$.

Similarly, and denoting the general element of matrix IP as Ip_{ij}, we have that the third term of the right-hand side of (22) is the following:

$$
\begin{aligned}
-(T^m * D^j)^\dagger \left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))IP\boldsymbol{\kappa})\right] &= -\begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts^m_{11} & \cdots & ts^m_{1J} \\ \vdots & \ddots & \vdots \\ ts^m_{J1} & \cdots & ts^m_{JJ} \end{pmatrix} \begin{pmatrix} Ip_{11} & \cdots & Ip_{1Z} \\ \vdots & \ddots & \vdots \\ Ip_{J1} & \cdots & Ip_{JZ} \end{pmatrix} \begin{pmatrix} \kappa_1 \\ \vdots \\ \kappa_Z \end{pmatrix} \right] \\
&= \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} + \cdots + \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} \\ \vdots \\ \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} + \cdots + \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \right] \\
&= \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \left(\begin{pmatrix} \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} \\ \vdots \\ \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} + \cdots + \begin{pmatrix} \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} \\ \vdots \\ \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \right) \right] \\
&= \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} \kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} \\ \vdots \\ \kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} \right] + \cdots \\
&\quad + \begin{pmatrix} -a_{11} & \cdots & -a_{1J} \\ \vdots & \ddots & \vdots \\ -a_{J1} & \cdots & -a_{JJ} \end{pmatrix} \left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} \kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} \\ \vdots \\ \kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \right] \\
&= \begin{pmatrix} -a_{11}\tilde{\delta}_1\kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} - \cdots - a_{1J}\tilde{\delta}_J\kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \\ \vdots \\ -a_{J1}\tilde{\delta}_1\kappa_1 \sum_{k=1}^J ts^m_{1k} Ip_{k1} - \cdots - a_{JJ}\tilde{\delta}_J\kappa_1 \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} + \cdots \\
&\quad + \begin{pmatrix} -a_{11}\tilde{\delta}_1\kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} - \cdots - a_{1J}\tilde{\delta}_J\kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \\ \vdots \\ -a_{J1}\tilde{\delta}_1\kappa_Z \sum_{k=1}^J ts^m_{1k} Ip_{kZ} - \cdots - a_{JJ}\tilde{\delta}_J\kappa_Z \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \\
&= \begin{pmatrix} -a_{11} \sum_{k=1}^J ts^m_{1k} Ip_{k1} & \dots & -a_{1J} \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \\ \vdots & \ddots & \vdots \\ -a_{J1} \sum_{k=1}^J ts^m_{1k} Ip_{k1} & \dots & -a_{JJ} \sum_{k=1}^J ts^m_{Jk} Ip_{k1} \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1\kappa_1 \\ \vdots \\ \tilde{\delta}_J\kappa_1 \end{pmatrix} + \cdots \\
&\quad + \begin{pmatrix} -a_{11} \sum_{k=1}^J ts^m_{1k} Ip_{kZ} & \dots & -a_{1J} \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \\ \vdots & \ddots & \vdots \\ -a_{J1} \sum_{k=1}^J ts^m_{1k} Ip_{kZ} & \dots & -a_{JJ} \sum_{k=1}^J ts^m_{Jk} Ip_{kZ} \end{pmatrix} \begin{pmatrix} \tilde{\delta}_1\kappa_Z \\ \vdots \\ \tilde{\delta}_J\kappa_Z \end{pmatrix} \\
&= \sum_{z=1}^Z F_z \tilde{\boldsymbol{\delta}} \kappa_z
\end{aligned}
$$

Therefore, the third term of equation (22) will be $\sum_{z=1}^Z F_z \tilde{\boldsymbol{\delta}} \kappa_z$, where F_z is an square matrix of dimension J, which general element of matrix $F_z[i,j] = -(T^m * D^j)^\dagger[i,j] \left(\sum_{k=1}^J ts^m_{jk} IP[k,z] \right)$.

Additionally, and by denoting the general element of vector $\left(\tilde{\boldsymbol{\lambda}} * \left((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma} \right) \right)[i,1] = d_i$, then the fourth term of the right-hand side of equation (22) is:

$$
\begin{aligned}
-(T^m * \Delta^j)^{\dagger}\left[\tilde{\boldsymbol{\theta}} * \tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right] &= -\begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix}\left[\begin{pmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_J \end{pmatrix} * \begin{pmatrix} d_1 \\ \vdots \\ d_J \end{pmatrix}\right] \\
&= \begin{pmatrix} -a_{11}\tilde{\theta}_1 d_1 - \cdots - a_{1J}\tilde{\theta}_J d_J \\ \vdots \\ -a_{J1}\tilde{\theta}_1 d_1 - \cdots - a_{JJ}\tilde{\theta}_J d_J \end{pmatrix} \\
&= \begin{pmatrix} -a_{11}d_1 & \cdots & -a_{1J}d_J \\ \vdots & \ddots & \vdots \\ -a_{J1}d_1 & \cdots & -a_{JJ}d_J \end{pmatrix}\begin{pmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_J \end{pmatrix} \\
&= H\tilde{\boldsymbol{\theta}}
\end{aligned}
$$

Then, the fourth term of equation (22) can be expressed as $H\tilde{\boldsymbol{\theta}}$, in which matrix H is an square matrix of dimension J, which general element of matrix is $H[i,j] = -(T^m * D^j)^{\dagger}[i,j]d_j$, where d_j is the element in position j of the vector $\left(\tilde{\boldsymbol{\lambda}} * ((T^r * (S^{J^{mr}} - S^j))\boldsymbol{\gamma})\right)$.

Finally, solving the last two terms of the right-hand side of the equation (22), we get:

$$
\begin{aligned}
-(T^m * \Delta^j)^{\dagger}\left[\tilde{\boldsymbol{\delta}} * ((T^m * (S^{J^{mr}} - S^j))\boldsymbol{\eta})\right] + \boldsymbol{\eta} &= -\begin{pmatrix} a_{11} & \cdots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{J1} & \cdots & a_{JJ} \end{pmatrix}\left[\begin{pmatrix} \tilde{\delta}_1 \\ \vdots \\ \tilde{\delta}_J \end{pmatrix} * \begin{pmatrix} ts^m_{11} & \cdots & ts^m_{1J} \\ \vdots & \ddots & \vdots \\ ts^m_{1J} & \cdots & ts^m_{JJ} \end{pmatrix}\begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix}\right] + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -a_{11}\tilde{\delta}_1(ts^m_{11}\eta_1 + \cdots + ts^m_{1J}\eta_J) - \cdots - a_{1J}\tilde{\delta}_J(ts^m_{J1}\eta_1 + \cdots + ts^m_{JJ}\eta_J) \\ \vdots \\ -a_{J1}\tilde{\delta}_1(ts^m_{11}\eta_1 + \cdots + ts^m_{1J}\eta_J) - \cdots - a_{JJ}\tilde{\delta}_J(ts^m_{J1}\eta_1 + \cdots + ts^m_{JJ}\eta_J) \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -\eta_1(a_{11}\tilde{\delta}_1 ts^m_{11} + \cdots + a_{1J}\tilde{\delta}_J ts^m_{J1}) - \cdots - \eta_J(a_{11}\tilde{\delta}_1 ts^m_{1J} + \cdots + a_{1J}\tilde{\delta}_J ts^m_{JJ}) \\ \vdots \\ -\eta_1(a_{J1}\tilde{\delta}_1 ts^m_{11} + \cdots + a_{JJ}\tilde{\delta}_J ts^m_{J1}) - \cdots - \eta_J(a_{J1}\tilde{\delta}_1 ts^m_{1J} + \cdots + a_{JJ}\tilde{\delta}_J ts^m_{JJ}) \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -(a_{11}\tilde{\delta}_1 ts^m_{11} + \cdots + a_{1J}\tilde{\delta}_J ts^m_{J1}) & \ldots & -(a_{11}\tilde{\delta}_1 ts^m_{1J} + \cdots + a_{1J}\tilde{\delta}_J ts^m_{JJ}) \\ \vdots & \ddots & \vdots \\ -(a_{J1}\tilde{\delta}_1 ts^m_{11} + \cdots + a_{JJ}\tilde{\delta}_J ts^m_{J1}) & \ldots & -(a_{J1}\tilde{\delta}_1 ts^m_{1J} + \cdots + a_{JJ}\tilde{\delta}_J ts^m_{JJ}) \end{pmatrix}\begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= \begin{pmatrix} -\sum_{k=1}^{J} a_{1k}\tilde{\delta}_k ts^m_{k1} & \ldots & -\sum_{k=1}^{J} a_{1k}\tilde{\delta}_k ts^m_{kJ} \\ \vdots & \ddots & \vdots \\ -\sum_{k=1}^{J} a_{Jk}\tilde{\delta}_k ts^m_{k1} & \ldots & -\sum_{k=1}^{J} a_{Jk}\tilde{\delta}_k ts^m_{kJ} \end{pmatrix}\begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_J \end{pmatrix} \\
&= G\boldsymbol{\eta} + \boldsymbol{\eta} \\
&= (G + I)\boldsymbol{\eta}
\end{aligned}
$$

In this way, the last two terms of expression (22) will become in the following in $(G + I)\boldsymbol{\eta}$, where I is an identity matrix and G is an square matrix of dimension J which general element is $G[i,j] = -\sum_{k=1}^{J}(T^m * D^j)^{\dagger}[j,k]\tilde{\delta}_k ts^m_{kj}$.

Therefore, expression (17) becomes the following:

$$\boldsymbol{p} - \boldsymbol{\gamma} - \left(T^m * \Delta^j\right)^\dagger \left[\tilde{\boldsymbol{\lambda}} * \left[\left(T^r * \Delta^j\right)\boldsymbol{\gamma}\right)\right] = IP\boldsymbol{\kappa} + E\tilde{\boldsymbol{\delta}} + H\tilde{\boldsymbol{\theta}} + \sum_{z=1}^{Z} F_z\tilde{\boldsymbol{\delta}}\kappa_z + [G+I]\boldsymbol{\eta}$$

E Further Steps

Once the trial estimation showed us that our specification were giving us the expected direction of the results, we consider to take measure to ensure robustness of the final estimations. Mainly, we are enlarging the time frame given the number of parameters to be estimated, in particular for the case of the before-strike period. At the same time, we removed the months of May and June of 2008, in order to avoid the noise in the demand estimations of the escalation period of the strike. Additionally, we increased the choice set in order to have more demand information. Even though, the final results are still in working process, we present in this section some of the results about the steps already taken. In the following subsections we decribe the dataset we are working with, as well as the results from the control function.

E.1 Data

To improve the robustness of the results we enlarged the data frame; in this way, for the before- and after-strike periods, we are considering from May 2007 to April 2008 and from July 2008 to June 2009 respectively. We are excluding the months of May and June 2008 from the analysis, in which the strike took place[23]. The dataset consisted of $543,368$ purchased-milk observations and $621,665$ for the after-strike period. This dataset provided information on date, number mililiters, number of packages, paid amounts, fat level, brand, retailer and manufacturer per purchase.

Regarding the retailers, all outlets in the dataset were kept except wholesalers, which represented 0.51% of the milk observations in the before-strike period and 0.45% in the after-strike.

We continue grouping the private labels as a single brand. On the other hand, the outside option included the manufacturers and retailers that in the whole dataset did not have at least 10000 observations, which were considered as small. Similarly, the small brands whithin retailer

[23]According to the reports the strike took place from May 27, 2008 to June 5, 2008 approx.; therefore, we are excluding the months of May and June, in order to avoid any noise from the strike in the demand estimations.

were also included in the outside option[24]. In this way, this study considered 13 brands[25], 16 retailers, which resulted in 62 differentiated alternatives aside from the outside option.

In order to control for the milk fat level on milk prices, a level of fat variable was defined, which considered two levels: a) 0 - 1.5% and, b) 1.6% - 3.9%.

The final dataset consists of 1,159,514 observations, 46.62% from them corresponds to the before-strike period and 53.38% to the after-strike period. The outside option represents 13.87% of the observations. Conventional brands constitutes 8.44% of the observations while private labels 77.69%. Excluding the outside option, the mean price in cents per liter of milk for the before-strike period was 66.63 with an standard devitation of 12.541, while the mean(standard deviation) for the after-strike period was 58.07(14.896). Further descriptive statistics for the before- and after-strike periods can be found in Table 7, while Figures 4a and 4b present the evolution of purchased milk quantities. quantified by the number of purchases and the amount of liters bought respectively.

Table 7: Descriptive statistics

	Before-strike Period			**After-strike Period**		
		Mean			**Mean**	
	Freq.	Monthly Market Share[1]	Price[2]	Freq.	Monthly Market Share[1]	Price[2]
Outside Option	14.73%			13.12%		
Private Labels (PL)	76.77%	0.54%(0.00578)	64.77(9.85)	78.49%	0.54%(0.00597)	55.70(11.67)
Conventional Brands (CB)	8.50%	0.02%(0.00026)	83.41(19.72)	8.39%	0.02%(0.00027)	80.20(21.96)

1) Price in cents per liter.

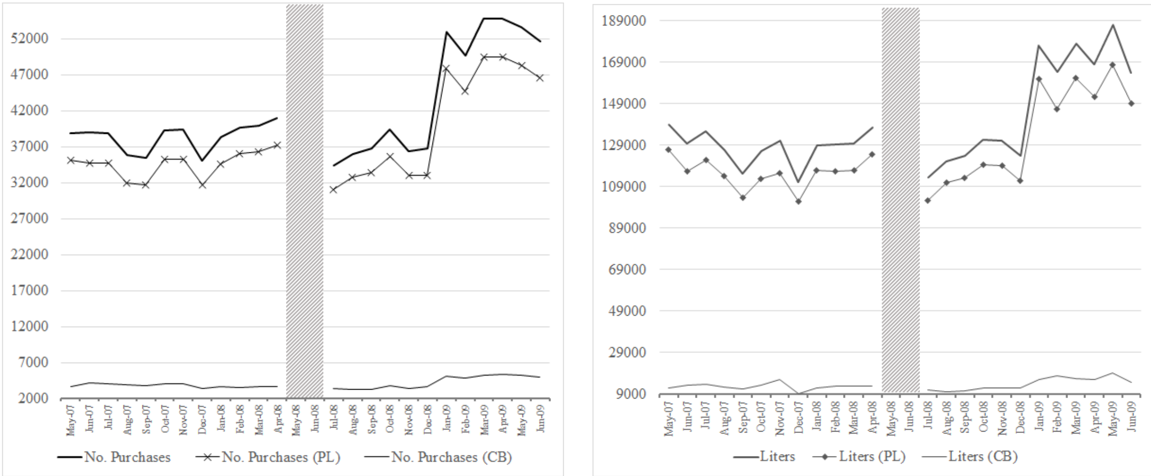
2) Market shares computed from the purchased liters of milk.

Standard Deviations are in parenthesis

[24] Brands that did not reached at least 624 observations in the whole dataset.

[25] Private labels counted as one brand.

Figure 4: Purchased Milk Quantity



(a) (b)

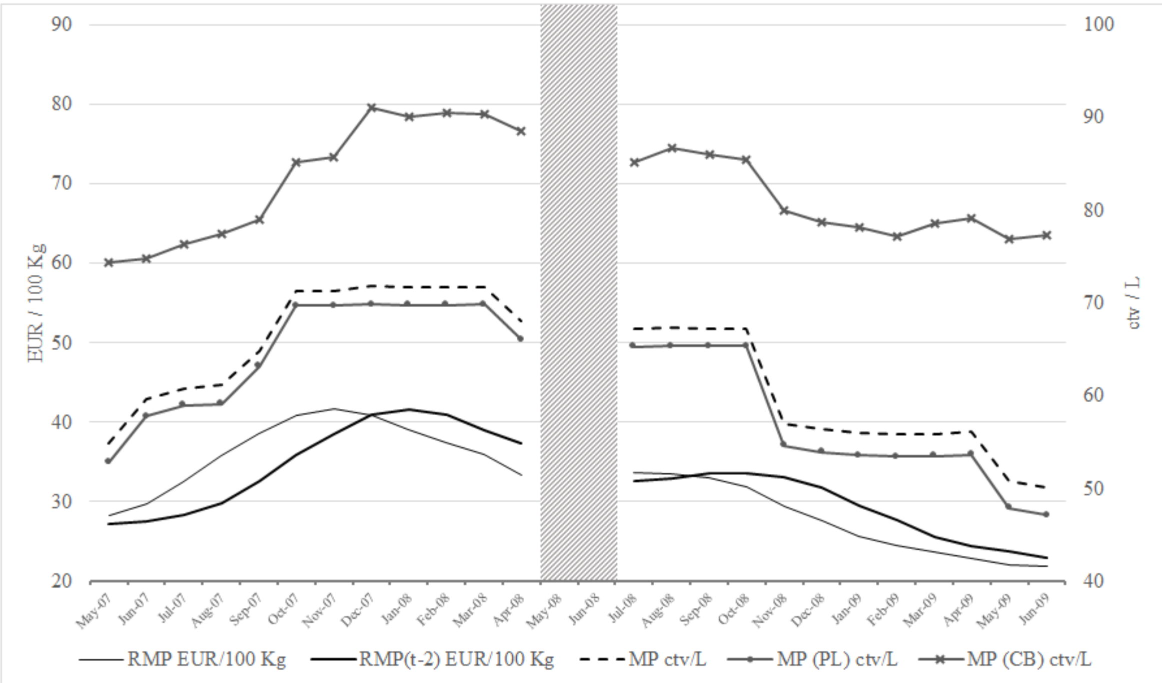
Aside from the GFK database, and to control for the input cost, we used as the monthly raw milk prices, the farmers' whole-milk price (3.7% fat and 3.4% protein)[26] coming from the German Federal Ministry of Food and Agriculture(*Bundesministerium für Ernärung, Landwirtschaft und Verbraucherschutz*)[27]. Figure 5 presents the evolution of the monthly raw-milk prices (RMP) and the monthly average milk-prices (MP)[28] coming from the GFK dataset for both periods.

[26]Preise Vollmilch ab Hof bei 3,7% Fettgehalt und 3,4% Eiweissgehalt.

[27]Statistical monthly reports from 03-2008, 03-2009, 03-2010: Table MBT-0301431-0000.

[28]Excluding the prices of the outside option.

Figure 5: Monthly Average Milk Prices



E.2 Control Function Results

As mention before, to contorl for the possible endogeneity in the demand estimations coming from the input costs, just as mentioned before, we are implementing a control function, as proposed by Petrin and Train(2010), following the specification in (4). We are considering the national milk price as the cost shifter, to this purpouse we are using the farmers' monthly whole-milk price (with 3.7% fat and 3.4% protein) lagged two periods and interacted with the fat-level dummies. Our estimation provides us with a positive strongly significant impact of the two instruments, together with a high R-square for both periods. Moreover, the F-value is well above the value 10 known as the threshold for weak instruments suggested by Staiger and Stock(1997), the results can be found in Table 8. Addittionally, in Appendix (F) are presented the different control function specifications that have been considered, choosing the presented in table (8) as the one that best suited for both periods.

Table 8: First-Stage: Control Function

Variables	**Before-strike**	**After-strike**
RMP_{t-2} x fat level 1	0.0017***	0.0017***
	(0.00004)	(0.00006)
RMP_{t-2} x fat level 2	0.0019***	0.0020***
	(0.00004)	(0.00006)
Brands	X	X
Retailers	X	X
R^2	0.9931	0.9889
Adj. R^2	0.9928	0.9884
F-Test(Instrs. = 0)	1179.13	520.42
Observations	744	744

Raw-milk Price (RMP): Farmers' whole-milk price (3.7% fat and 3.4% protein)

Dummies: fat level 1 = $\mathbf{1}$(0 - 1.5% fat); fat level 2 = $\mathbf{1}$(1.6 - 3.5% fat)

***, **, * denote 1%, 5% and 10% level of significance respectively

Standard-Errors are in parenthesis

F Control function

Table 9: Tested Control Functions

Before-strike Period								
Variables	**Instrument(s) in period**							
	t	**t**	**t - 1**	**t - 1**	**t - 2**	**t - 2**	**t - 3**	**t - 3**
RMP	0.0019***		0.0019***		0.0018***		0.0017***	
	(0.00005)		(0.00004)		(0.00004)		(0.00004)	
RMP x fat level 1		0.0018***		0.0018***		0.0017***		0.0016***
		(0.00005)		(0.00004)		(0.00004)		(0.00004)
RMP x fat level 2		0.0019***		0.0020***		0.0019***		0.0018***
		(0.00005)		(0.00004)		(0.00004)		(0.00004)
Brands	X	X	X	X	X	X	X	X
Retailers	X	X	X	X	X	X	X	X
R^2	0.9893	0.9897	0.9920	0.9926	0.9923	0.9931	0.9911	0.9921
Adj. R^2	0.9889	0.9893	0.9917	0.9923	0.9920	0.9928	0.9908	0.9918
F-Test(Inst(s). = 0)	1276.20	675.48	1965.77	1081.41	2054.64	1179.13	1679.00	986.13
Obs.	744	744	744	744	744	744	744	744

After-strike Period								
Variables	**Instrument(s) in period**							
	t	**t**	**t - 1**	**t - 1**	**t - 2**	**t - 2**	**t - 3**	**t - 3**
RMP	0.0018***		0.0018***		0.0019***		0.0020***	
	(0.00006)		(0.00006)		(0.00006)		(0.00006)	
RMP x fat level 1		0.0016***		0.0016***		0.0017***		0.0018***
		(0.00006)		(0.00006)		(0.00006)		(0.00007)
RMP x fat level 2		0.0019***		0.0019***		0.0020***		0.0020***
		(0.00006)		(0.00006)		(0.00006)		(0.00006)
Brands	X	X	X	X	X	X	X	X
Retailers	X	X	X	X	X	X	X	X
R^2	0.9874	0.9885	0.9877	0.9887	0.9881	0.9889	0.9885	0.9892
Adj. R^2	0.9869	0.9880	0.9872	0.9882	0.9876	0.9884	0.9881	0.9888
F-Test(Inst(s). = 0)	835.57	489.40	873.49	502.45	924.61	520.42	988.12	546.83
Obs.	744	744	744	744	744	744	744	744

Raw-milk Price (RMP): Farmers' whole-milk price (3.7% fat and 3.4% protein)

Dummies: fat level 1 = $\mathbf{1}$(0 - 1.5% fat); fat level 2 = $\mathbf{1}$(1.6 - 3.5% fat)

***, **, * denote 1%, 5% and 10% level of significance respectively

Standard-Errors are in parenthesis

References

[1] Bonnet, C. and Bouamra-Mechemache, Z., *Organic label, bargaining power, and profit-sharing in the French fluid milk market*, American Journal of Agricultural Economics, 98, 1, 113-133, 2015.

[2] Bundeskartellamt, *Sektoruntersuchung Milch*, Zwischenbericht gemäß§32e GWB, 2009. [Retrieved (2018, September 24):`https://www.bundeskartellamt.de/SharedDocs/Publikation/DE/Sektoruntersuchungen/Sektoruntersuchung\%20Milch\%20-\%20Zwischenbericht.html?nn=3591074`].

[3] Cameron, A. and Trivedi, P., *Microeconometrics using stata*, Stata Press, 2010.

[4] Card, D., *Strikes and bargaining: A survey of recent empirical literature*, The American Economic Review, 80, 2, Papers and Proceedings of the Hundred and Second Annual Meeting of the American Economic Association, 410 - 415, 1990.

[5] Card, D., *Strikes and wages: A test of an asymmetric information model*, The Quarterly Journal of Economics, 105, 3, 625 - 659, 1990.

[6] Chipty, T. and Snyder, C., *The role of the firm size in bilateral bargaining: A study of the cable industry*, The Review of Economics and Statistics, 81, 2, 326 – 340, 1999.

[7] Chun, Y. and Thomson, W., *Bargaining problems with uncertain disagreement points*, Econometrica, 58, 4, 951 – 959, 1990.

[8] Chun, Y. and Thomson, W., *Egalitarian solution and uncertain disagreement points*, Economics Letters, 33, 29 – 33, 1990.

[9] Chun, Y. and Thomson, W., *Nash solution and uncertain disagreement point*, Games and Economic Behavior, 2, 213 – 223, 1990.

[10] Collard-Wexler, A., Gowrisankaran, G. and Lee, R., *-in-Nash" Bargaining: A Microfoundation for Applied Work*, Journal of Political Economy, forthcoming.

[11] Cramton, P. and Tracy, J., *Strikes and holdouts in wage bargaining: Theory and data*, The American Economic Review, 82, 1, 100 - 121, 1992.

[12] Crawford, G. and Yurukoglu, A., *The welfare effects of bundling in multichannel television markets*, The American Economic Review, 102, 2, 643 - 685, 2012.

[13] Crawford, G., Lee, R., Whinston, M. and Yurukoglu, A., *The welfare effects of vertical integration in multichannel television markets*, Econometrica, 86, 3, 891 – 954, 2018.

[14] Draganska, M., Klapper, D. and Villas-Boas, S., *A larger slice or a larger pie? An empirical investigation of bargaining power in the distribution channel*, Marketing Science, 29, 1, 57-74, 2010.

[15] European Commission, *Report from the Commission to the European Parliament and the Council - On unfair business-to-business trading practices in the food supply chain*, COM(2016) 32 final, 2016. [Retrieved (2018, September 18): `https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=COM\%3A2016\%3A32\%3AFIN`].

[16] Falkowski, J., Ménard, C., Sexton, R., Swinnen, J. and Vandevelde, S., *Unfair trading practices in the food supply chain: A literature review on methodologies, impacts and regulatory aspects*, Joint Research Centre (JRC)- European Commission, JRC108394, 2017.

[17] Fearon, J., *Rationalist explanations of war*, International Organization, 49, 3, 379 - 414, 1995.

[18] Fey, M. and Ramsay, K., *Uncertainty and incentives in crisis bargaining: Game-free analysis of international conflict*, American Journal of Political Science, 55, 1, 149 - 169, 2011.

[19] Fey, M., Meirowitz, A. and Ramsay, K., *Credibility and commitment in crisis bargaining*, Political Science Research and Methods, 1, 1, 27 - 52, 2013.

[20] Gneezy, A., Gneezy, U., Riener, G., and Nelson, L., *Pay-What-You-Want. identity, and self-signaling in markets*, Proceedings of the Royal Academy of Science, 109, 19, 7236-7240, 2012.

[21] Greiff, M., and Egbert, H., *A survey of the empirical evidence on PWYW pricing.*, MAGKS Papers, 05, 2016.

[22] Grennan, M., *Price discrimination and bargaining: Empirical evidence from medical devices*, The American Economic Review, 103, 1, 145 - 177, 2013.

[23] Gowrisankaran, G., Nevo, A. and Town, R., *Mergers when prices are negotiated: Evidence from the hospital industry*, The American Economic Review, 105, 1, 172 - 203, 2015.

[24] Gu, W. and Kuhn, P., *A theory of holdouts in wage bargaining*, The American Economic Review, 88, 3, 428 - 449, 1998.

[25] Hart, O., *Bargaining and strikes*, The Quarterly Journal of Economics, 104, 1, 25 - 43, 1989.

[26] Ho, K. and Lee, R., *Insurer competition in health care markets*, Econometrica, 85, 2, 379 - 417, 2017.

[27] Junta de Regulación de la Ley Orgánica de Regulación y Control del Poder de Mercado, *Sustituir Resolución No. 008 y expedir las normas regulatorias para las cadenas de supermercados y sus proveedores*, Resolución No. 014, 2017. [Retrieved (2018, September 18): `http://www.planificacion.gob.ec/resoluciones/`].

[28] Lewis, J. and Schultz, K., *Revealing preferences: Empirical estimation of a crisis bargaining game with incomplete information*, Political Analysis, 11, 345 - 367, 2003.

[29] Livne, Z., *The bargaining problem with an uncertain conflict outcome*, Mathematical Social Sciences, 15, 287 – 302, 1988.

[30] McAfee, R. and Schwartz, M., *Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, andUniformity*, The American Economic Review, 84, 1, 210 - 230, 1994.

[31] Mills, G., *Buyer power of supermarkets*, Agenda: A journal of policy analysis and reform, 10, 2, 145 - 162, 2003.

[32] Petrin, A. and Train, K., *A control function approach to endogeneity in consumer choice models*, Journal of Marketing Research, 47, 1, 3-13, 2010.

[33] Peters, H. and Van Damme, E., *Characterizing the Nash and Raiffa bargaining solutions by disagreement points axioms*, Mathematics of Operations Research, 16, 3, 447 – 461, 1991.

[34] Revelt, D. and Train, K., *Mixed logit with repeated choices: households' choices of appliance efficiency level*, Review of Economics and Statistics, 80, 4, 647-657, 1998.

[35] Staiger, D. and Stock, J., *Instrumental Variables Regression with Weak Instruments*, Econometrica, 65, 3, 557-586, 1997.

[36] Thomson, W., *Cooperatice models of bargaining*, Handbook of Game Theory with Economic Applications Vol. 2, Chapter 35, pp. 1237 - 1284, 1994.

[37] Train, K., *Discrete Choice Methods with Simulation*, Cambridge University Press, 2003.

[38] Varoufakis, Y., *Bargainings and strikes: Towards an evolutionary framework*, Labor Economics, 3, 385 – 398, 1996.