Endogenized Product Variety, Search Friction and Collusion*

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Abstract

This paper analyzes the relationship between product variety and collusion in the setting of Bertrand duopoly with endogenized, vertically differentiated product variety and multidimensional consumer heterogeneity. We explicitly model firms' strategies in first-stage product variety and second-stage pricing when consumers differ in their preferences and search frictions. We find that in many cases, collusion is easier when firms offer only high-quality products than when firms offer both types of products. In particular, collusion on high-quality products to high-end consumers is easier and more profitable when the fraction of high-end consumers is large and the discount factor is intermediate. Moreover, reducing the quality of baseline products can facilitate collusion on high-quality products to high-end consumers. We also discuss the policy implication of mandatory provision of the low-quality product on consumer welfare.

Key Words: Product variety, search friction, tacit collusion

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1 Introduction

In many businesses firms are facing regulations that while selling high-quality products to consumers, they must also provide a type of baseline products. For example, in US, cable TV systems generally are required to offer a "basic tier" of programming to all subscribers before they purchase any additional programming. Local franchising authorities may review any increases in basic service tier rates to verify that price adjustments are driven by cost increment. Similarly, Canada Bank Act and Access to Basic Banking Services Regulations provide a statutory obligation for banks to open basic retail accounts for consumers. These regulations are possibly driven by the concern that in a Laissez faire environment, firms might wish to offer exclusively high-quality products and cease to offer low-quality products, and such behavior will naturally hurt consumers with low willingness to pay as they will be excluded from the market. A classical interpretation of offering exclusively high value-added products is monopoly's profit-maximizing strategy: by offering a niche product to serve exclusively for price-inelastic, high willingness-to-pay consumers, local monopolies can earn a higher profit than by introducing also a mass product. In many scenarios, however, the local monopoly assumption may be doubted: a few cable TV systems typically compete against each other closely, and a number of banks operate in the same region to run businesses. Thus, to provide a economic rationale for these types of regulations, a model with oligopoly competition is a more desirable option.

As we attempt to rationalize the high-quality only strategy among oligopolies, two questions arise. Why do oligopolies wish not to provide a more affordable product besides the premium one? Do regulations that require mandatory provision of a more affordable product necessarily improve consumer welfare? To provide an answer to these two questions, it is necessary to propose a model that incorporates both consumer heterogeneity and endogenous product selections. The element of endogenous product selections is necessary, since the extra cost of offering an additional low-quality product besides the premium one is often times negligible: all firms need to do is add a line in its menu brochure. The fact that firms resist from doing so is likely an outcome of strategic interaction among oligopolies instead of cost-saving motives. In order to gain an insight of firms' strategic behavior as such, it is also necessary to incorporate some degree of consumer heterogeneity. For example, when customers visit a shopping mall, preference matching and affordability are two main considerations. In the literature of oligopoly competition, it is well known that several factors affect the sustainability of collusion. Among them, factors that are most related to this type of strategies are product differentiation, product variety,

consumer loyalty and discount factor. The existing literature has been successful in providing insights of how each factor alone plays a role in firms' strategic actions: see, for example, Hotelling (1929), Bain (1968), Deneckere (1983), Chang (1991), Ross (1992), Hackener (1994) and Symeonidis (2002) on product differentiation and collusion. However, literature on product variety and collusion has focused on comparative statics with fixed and horizontally differentiated products that are complements to or substitutes for each other, and literature on consumer search and collusion typically assumes fixed and exogenous product selections.

The main purpose of this paper is to investigate how product variety in a vertically differentiated context affects an oligopoly's strategic behaviors. Specifically, we propose a model of oligopoly with (i) endogenous selection of vertically differentiated products and (ii) consumer heterogeneity in preference and search behavior. Two identical firms choose to sell up to two types of products: high-quality ones and a low-quality ones. Products have fixed quality levels and different production costs. Consumers with unit demand differ in willingness to pay and activeness of price search: high valuation consumers are more likely to be deal hunters than are low valuation consumers. First, we fix product variety and study firms' strategies when they perform Bertrand price competition; later on, we endogenize product selections, characterize equilibrium outcomes and derive policy implications.

Our main result answers the first question: collusion is easier when firms only offer high-quality products and do not bring low-quality products to the market. In particular, when the fraction of high-end consumers is large and the discount factor is intermediate, collusion on high-quality products to high-end consumers is sustainable and most profitable. On the other hand, collusion on both types of products, despite possibly higher profit, is harder to sustain. This result contrasts sharply with the conventional wisdom that in a multimarket context, more product variety facilitates collusion. The reason is that in our setting, deviation is more profitable when firms sell both types of products. Comparing across the first scenario when firms offer only high-quality products to high-end consumers, and the second scenario when firms offer both types of products and consumers self-select, a small price undercut will have two effects in deviation profits: price effect and quantity effect. In the first scenario, if consumers are vertically differentiated, firms do not need to pay the information rent for high-end consumers and thus are able to charge a higher collusion price. A small price undercut will lead to higher deviation profit from high-quality products. On the other hand, in the second scenario, an undercutting firm can deviate price-setting for high-quality products and low-quality products simultaneously. This

allows the firm to earn a deviation profit from both high-end consumers and low-end consumers who originally patronize its competitor. These two effect work in opposite directions, and in general the overall effect is ambiguous. However, due to the assumption of negative relationship between consumers' willingness to pay and their sensitivity to price cuts, the second effect dominates because a price undercut from high-quality products alone attracts less than half of consumers than a price undercut from both types of products does. Thus, high-high collusion is easier to sustain than both-both collusion.

The answer to the second question is ambiguous. In some cases, requiring firms to provide the more afforable product does enhance consumer welfare and restore first-best. However, there are also cases that such a regulation may hurt both firms and consumers. We derive a set of sufficient conditions and provide a numerical example to illustrate this possibility. Without regulation, firms choose to offer high-quality products for both types of consumers. Mandatory provision of low-quality products distorts firms' strategic behavior and induces firms to offer low-quality products and cease to offer high-quality ones. Firms earn less profit, because the optimal choice is no longer feasible. Consumers benefit from a more affordable price but at the expense of getting a lower utility. Under certain conditions, the effect of utility loss dominates the effect of cost saving, leading to a reduction of consumer welfare.

Our result could serve as a reference guide for antitrust committees to examine ongoing issues on collusion. For example, in Hong Kong, only premium gasoline rated at 98 RON is sold on the market, whereas in other countries and regions, more varieties of gasoline, such as 98 RON, 95 RON, 91 RON, etc., are provided. Hong Kong Competition Commission has brought attention to this issue and has been wondering whether reintroducing 95 RON to the market would enhance consumer welfare. The Commission suggested that reintroducing 95 RON, which is compatible to the vast majority of cars, would have two potential benefits: (i) consumers would have an option to use the more affordable gasoline and (ii) higher competitive pressure on the gasoline market could possibly lead to a lower price for premium type gasoline as well. Our result suggests that requiring gas stations to reintroduce 95 RON gasoline may either improve or hurt consumer welfare.

Our work is related to several literatures. One is the literature on multiproduct firms and collusion. Symeonidis (2002) discusses cartel stability with firms in a single market and shows that in most cases, less product variety facilitates collusion. A main difference is that the paper takes product variety as given. Under this assumption, other firms cannot introduce more variety as a retaliation to nonco-operating firms. Our work allows for endogenized product varieties, as it is often cost efficient for a

multiproduct firm to add a product within its industry. Another difference is that Symeonidis (2002) assumes horizontal product differentiation and the result derives from a structure of symmetry, whereas our work considers vertical product differentiation and does not require symmetry assumptions at product level. Bernheim and Whinston (1990) discuss multimarket contact and collusion, and their result is that collusion is easier to sustain in the scenario of multimarket contact. Our work examines firms with multiple products competing in a single industry and thus differs. Lancaster (1990) provides a survey of the economics of product variety. The discussion is within the scope of horizontal differentiation.

Another related literature is on consumers' search behavior and collusion. A seminal work is Diamond (1971) on search cost and collusion. Stahl (1989) and Petrikaitė (2016) extends the discussion within this framework. A substantive difference is that we assume searching is costless, and the theoretical prediction is quite different. We shall illustrate this point when we introduce our model. Campbell, Ray and Muhanna (2005) examine search and collusion in electronic markets and conclude that collusion is easier when consumer search is less costly. Pot et. al. (2013) discuss the effect of consumer inertia on dynamic competition and identify that enduring price fluctuation can be an equilibrium path phenomenon. Chen and Rosenthal (1996) consider the effect of discount factor and the stickiness of consumer loyalty on equilibrium strategies and on long-run market structure in the framework of heterogenenous firms.

Our work is also related to the literature on gasoline price collusion. For example, Clark and Houde (2013) shows that delayed price adjustment can be a mechanism to facilitate collusion with heterogeneous consumers and firms. The paper assumes homogeneous and exogenous product offerings whereas our work allows for differentiated and endogenous ones. Besides, our finding in comparative statics that lowering down the quality of baseline products facilitates collusion shares a similar insight as does Ecchia and Lambertini (1997), which shows that setting a quality standard can make collusion harder to sustain.

This paper is organized as follow. Section 2 sets up the model. Section 3 analyzes collusion equilibria with exogenous product variety. Section 4 endogenizes product variety and provides our main result. Section 5 analyzes social welfare and policy implications. Section 6 concludes.

2 The Model

There are two types of consumers, namely, high-end consumers and low-end consumers. The fraction of high-end consumers is θ, and the fraction of low-end consumers is (1-θ). A fraction λ_H of high-end consumers are passive consumers, and the remainder are deal hunters. A fraction λ_L of low-end consumers are passive consumers, and the remainder are deal hunters. A key assumption is that $\lambda_H > \lambda_L$. This is to capture the fact that high-end consumers typically have a high time cost and are less likely to spend time searching for price differences. An empirical evidence can be found in Narasimhan (1984), who finds a negative relationship between time cost and price elasticity through coupon usage. Formally, we define deal hunters as those who immediately identify any price changes and make their purchase decisions accordingly, and we define passive consumers as those who do not search for price differences unless the price quote differs from their expectations. Despite of apparent similarity, this assumption is theoretically different from the standard setting of Diamond (1971). This is because in our setting, a firm slightly raising its prices can no longer be a profitable deviation. Consider, for example, the standard Bertrand duopoly with homogeneous products and a representative consumer. Once a firm starts to deviate by slightly raising its price, no matter how small the price raise is, there will be a departure from the consumer's expected price quote. The consumer will start to search and will patronize the non-deviating firm thereafter. It is easy to verify that any price level between monopoly price and marginal cost can be supported by some belief as an equilibrium outcome. We assume firms will always select an equilibrium that maximizes their profits. In the standard Bertrand setting, for example, firms will set the monopoly price.

Two identical firms sell up to two types of products: high-quality products and low-quality products. The marginal cost of the high-quality product is c_H, and the marginal cost of the low-quality product is c_L. High-end consumers' valuation of the high-quality product is v_H, and the low-quality product βv_L. Low-end consumers' valuation of the high-quality product is αv_H, and a low-quality product v_L. Assume $\alpha < 1$, that is, high-end consumers value high-quality products more than low-end consumers do. Moreover, assume it is socially efficient that firms produce high-quality products for high-end consumers and low-quality products for low-end consumers:

$$v_H - c_H > \beta v_L - c_L > 0 \tag{1}$$

$$v_L - c_L > \alpha v_H - c_H > 0 \tag{2}$$

We do not impose further restrictions on the parameter β except that β must be positive, as implied by (1). When $\beta > 1$, high-end consumers value both high-quality products and low-quality products more than low-end consumers do. This scenario fits into the setting of vertically differentiated consumers. When $\beta < 1$, high-end consumers have a higher valuation of high-quality products, whereas low-end consumers have a higher valuation of low-quality products. This scenario fits into the setting of horizontally differentiated consumers. As we will show later, in both scenarios our main result follows, thus our work allows for a wide applications in business competitions and regulations. In the latter scenario, we still classify consumers into high-end group and low-end group by comparing their valuations of the high-quality product.

We do not assume the production cost of high-quality products is greater than the production cost of low-quality products. This allows our analysis to follow when technology progress significantly drives down the production cost of high-quality products.

3 Collusion Equilibrium for Exogenous Product Selections

In this section, we regard product variety as exogenously given and assume that both firms offer the same type of product variety. In particular, firms might offer only high-quality products, only low-quality products or both types of products. Our focus is on symmetric equilibria, since firms with the same production technology compete in the same single market.

The timing is as follow. Firms observe the product variety and then set prices simultaneously. Consumers with unit demand randomly visit a firm with probability $\frac{1}{2}$. Finally, consumers make their purchase decisions. If a consumer chooses to buy a product, her utility is equal to the valuation of that product minus the cost of that product; if she buys nothing, her utility is equal to the reservation utility. The reservation utility is normalized to be zero.

With Bertrand competition and identical product offerings, when firms don't collude, they will both charge marginal cost and will earn zero profit; when firms collude, they may earn a positive profit if the collusion is sustainable. Now we discuss separately three cases: 1) only high-quality products are sold on the market, 2) both types of products are sold on the market and 3) only low-quality products are sold on the market. We first characterize the condition to sustain collusion and firms' collusive profits

when they offer only high-quality products to high-end consumers, and then consider other possible types of collusion and do pairwise comparison with the one of our interest.

3.1 Only high-quality products are sold

When firms collude on only selling high-quality products, two options are available: (i) selling them to high-end consumers only and (ii) selling them to both types of consumers

3.1.1 High-quality products to high-end consumers only

When firms sell high-quality products to high-end consumers only, they will charge $p_H = v_H$ to maximize joint profits. When neither of the firms deviates, all high-end consumers buy a unit of product, all low-end consumers buy nothing, and each firm earns a collusive profit π_H^1:

$$\pi_H^1 = \frac{\theta(v_H - c_H)}{2(1-\delta)}$$

If one of the firms slightly undercuts its price below v_H, it will attract all the high-end deal hunters and half of the high-end passive consumers. The deviator will lose all future profits as the cooperating firm will set the price at marginal cost thereafter. In this case, the deviator can earn a profit π_D^{11}:

$$\pi_D^{11} = \theta(1 - \frac{\lambda_H}{2})(v_H - c_H)$$

Alternatively, the firm can cut its price to just below αv_H to attract half of the deal hunters among the low-end consumers. In this case, the deviation profit is π_D^{12}:

$$\pi_D^{12} = [\theta(1 - \frac{\lambda_H}{2}) + (1-\theta)(1 - \frac{\lambda_L}{2})](\alpha v_H - c_H)$$

It can be verified that $\pi_D^{11} \geq \pi_D^{12}$ if and only if

$$\theta \geq \frac{(1 - \frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1 - \frac{\lambda_H}{2})(1-\alpha)v_H + (1 - \frac{\lambda_L}{2})(\alpha v_H - c_H)}$$

which is equivalent to

$$\alpha \leq \widehat{\alpha} \equiv \frac{\theta(1 - \frac{\lambda_H}{2})v_H + (1-\theta)(1 - \frac{\lambda_L}{2})c_H}{[\theta(1 - \frac{\lambda_H}{2}) + (1-\theta)(1 - \frac{\lambda_L}{2})]v_H}$$

When $\alpha > \widehat{\alpha}$, low-end consumers' willingness-to-pay for high-quality products is large enough and is similar to high-end consumers' willingness-to-pay for high-quality products. In this case, a price reduction to attract low-end consumers is more than compensated for a larger sale. Conversely, when $\alpha \leq \widehat{\alpha}$, the additional profit generated from a large price deviation does not cover the profit reduction from high-end consumers, and thus a tiny price deviation is more profitable.

Typically, the fraction of high-end consumers in a market is not too low. To capture this, assume $\alpha \leq \widehat{\alpha}$. The condition for sustaining the collusion is thus:

$$\frac{\theta(v_H - c_H)}{2(1-\delta)} \geq \theta(1 - \frac{\lambda_H}{2})(v_H - c_H)$$

$$\delta \geq \delta^1 \equiv \frac{1-\lambda_H}{2-\lambda_H}$$

We can tell from the equation above that the higher the λ_H, the smaller the value on the right hand side. That is to say, a larger fraction of inactive searchers among high-end consumers can make the collusion more sustainable.

Suppose instead that $\alpha > \widehat{\alpha}$. Then the condition for sustaining the collusion is:

$$\delta \geq \widetilde{\delta^1} = 1 - \frac{\theta(v_H - c_H)}{(2 - \theta\lambda_H - (1-\theta)\lambda_L)(\alpha v_H - c_H)}$$

When $\alpha \to 1, \widetilde{\delta^1} \to 1 - \frac{\theta}{2-\theta\lambda_H-(1-\theta)\lambda_L} = \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)+1-\theta}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)} > \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$. As we will show immediately, the last expression is the threshold discount factor to sustain collusion on both types of consumers. Therefore, when α is sufficiently large, collusion on high-quality products to high-end consumers is harder to sustain than collusion on high-quality products to both types of consumers.

In the subsequent analysis, we will maintain the assumption $\alpha \leq \widehat{\alpha}$.

3.1.2 High-quality products to both types of consumers

When firms sell high-quality products to both types of consumers, firms will charge $p_H = \alpha v_H$ to maximize joint profit. When neither of the firms deviates, each firm earns the collusive profit π_H^2:

$$\pi_H^2 = \frac{\alpha v_H - c_H}{2(1-\delta)}$$

If one of the firms slightly undercuts its price, it will attract all the deal hunters and half of the passive consumers. The deviator can get profit π_D^2:

$$\pi_D^2 = [(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})](\alpha v_H - c_H)$$

Thus, the condition for sustaining the collusion is $\pi_H^2 \geq \pi_D^2$:

$$\frac{\alpha v_H - c_H}{2(1-\delta)} \geq [(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})](\alpha v_H - c_H)$$

$$\delta \geq \delta^2 \equiv \frac{\theta(1-\lambda_H) + (1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H) + (1-\theta)(2-\lambda_L)}$$

When $\theta \to 0$, $\delta^2 \to \frac{1-\lambda_L}{2-\lambda_L}$. When $\theta \to 1$, $\delta^2 \to \frac{1-\lambda_H}{2-\lambda_H} = \delta^1$. Our assumption $\lambda_H > \lambda_L$ implies $\frac{1-\lambda_L}{2-\lambda_L} > \frac{1-\lambda_H}{2-\lambda_H}$. Since $\frac{\partial \delta^2}{\partial \theta} < 0$, we have $\delta^2 > \delta^1$ for all $\theta \in (0,1)$. Therefore, collusion is easier by selling high-quality products to high-end consumers only than by selling high-quality products to both types of consumers.

For δ being sufficiently large, both types of collusion can be sustained. Now we compare firms' collusion profits:

$$\pi_H^1 \geq \pi_H^2 \Longleftrightarrow \frac{\theta(v_H - c_H)}{2(1-\delta)} \geq \frac{v_L - c_H}{2(1-\delta)} \Longleftrightarrow \theta \geq \widetilde{\theta} \equiv \frac{v_L - c_H}{v_H - c_H}$$

That is, when the fraction of high-end consumers is large enough, collusion profit is higher by selling high-quality products to high-end consumers only.

Intuitively, a small price undercuts attracts only high-end consumers from competitor whereas a large price undercuts attracts both high-end and low-end consumers from competitor. When a small price undercut is more profitable, the fraction of high-end consumers must be above some threshold, which implies the price effect of being able to charge a higher deviaton price dominates the quantity effect of attracting additional consumers from the low-end pool. On the other hand, when a large price undercut is more profitable, it will attract a lot more consumers than a small undercut does. In the limiting case when $\alpha \to 1$, the quantity effect of attracting more consumers outweights the price effect; because low-end pool consists of a larger fraction of deal-hunters, collusion is harder to sustain.

We summarize our findings above in Proposition 1.

Proposition 1 *When $\alpha \leq \widehat{\alpha}$, selling high-quality products exclusively to high-end consumers makes collusion easier. Moreover, when both the discount factor δ and the fraction of high-end consumers θ are sufficiently large, selling high-quality products exclusively to high-end consumers is more profitable than selling high-quality products to both types of consumers. When α is sufficiently close to 1, selling high-quality products to both types of consumers makes collusion easier.*

3.2 Both types of products are sold

When firms sell both types of products, to maximize joint profit, they solve the following optimization problem:

$$
\begin{aligned}
& \max \pi_B = \theta(p_H - c_H) + (1-\theta)(p_L - c_L) \\
& p_H, p_L \\
s.t. v_H - p_H &\geq \beta v_L - p_L \\
v_L - p_L &\geq \alpha v_H - p_H \\
v_H - p_H &\geq 0 \\
v_L - p_L &\geq 0 \\
\textit{Under the assumptions} &: \\
v_H - c_H &> \beta v_L - c_L > 0 \\
v_L - c_L &> \alpha v_H - c_H > 0
\end{aligned}
$$

Solving the optimization problem yields:

$$
\begin{aligned}
\text{If } \beta &\leq 1, \text{ then } (p_H, p_L) = (v_H, v_L) \\
\text{If } \beta &> 1, \text{ then } (p_H, p_L) = (v_H - (\beta - 1)v_L, v_L)
\end{aligned}
$$

When consumers are horizontally differentiated, firms can fully extract consumer welfare by charging the price of high-quality products equal to high-end consumers' valuation of high-quality products, and charging the price of low-quality products equal to low-end consumers' valuation of low-quality

products. Consumers will self-select their preferred product and get zero utility. On the other hand, when consumers are vertically differentiated, the IC constraint for high-end consumers is binding. Firms must lower down the price of high-quality products a bit to attract high-end consumers. High-end consumers enjoy an information rent and get strictly positive utility, whereas low-end consumers get zero utility.

When companies collude, each of them earns a profit π_B.

$$\pi_B = \frac{\theta(p_H - c_H) + (1-\theta)(p_L - c_L)}{2(1-\delta)}$$

Suppose a firm slightly undercuts the prices of both types of products. It will attract all the deal hunters and half of the inactive consumers. The deviation profit is denoted as π_D^3:

$$\pi_D^3 = \theta(1 - \frac{\lambda_H}{2})(p_H - c_H) + (1-\theta)(1 - \frac{\lambda_L}{2})(p_L - c_L)$$

In this case, the condition to sustain collusion is:

$$\frac{\theta(p_H - c_H) + (1-\theta)(p_L - c_L)}{2(1-\delta)} \geq \theta(1 - \frac{\lambda_H}{2})(p_H - c_H) + (1-\theta)(1 - \frac{\lambda_L}{2})(p_L - c_L)$$

$$\delta \geq \delta^3 \equiv \frac{\theta(1-\lambda_H)(p_H - c_H) + (1-\theta)(1-\lambda_L)(p_L - c_L)}{\theta(2-\lambda_H)(p_H - c_H) + (1-\theta)(2-\lambda_L)(p_L - c_L)} \tag{3}$$

Collusion is sustainable if and only if the solution (p_H, p_L) to the linear programming satisfies condition (4).

When $\theta \to 0$, $\delta^3 \to \frac{1-\lambda_L}{2-\lambda_L}$; when $\theta \to 1$, $\delta^3 \to \frac{1-\lambda_H}{2-\lambda_H}$. Since $\frac{\partial \delta^3}{\partial \theta} < 0$, by monotonicity, we have $\frac{1-\lambda_H}{2-\lambda_H} < \delta^3 < \frac{1-\lambda_L}{2-\lambda_L}$ for all $\theta \in (0,1)$.

Recall that when only high-quality products are sold to high-end consumers, the condition to sustain collusion is $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$. When both types of products are sold to both types of consumers, the condition is $\delta \geq \delta^3$. Simple algebra shows that $\frac{1-\lambda_H}{2-\lambda_H} < \delta < \frac{1-\lambda_L}{2-\lambda_L}$. Therefore, collusion is easier to sustain when firms only sell high-quality products to high-end consumers.

When consumers are horizontally differentiated, firms do not need to pay information rent for high-quality products. Deviation is more attempting because there is an additional gain from low-end consumers, which deters collusion. When consumers are vertically differentiated, firms in collusion

cannot extract all surplus from high-end consumers. Compared with the deviation profit when firms are offering only high-quality products, there is a positive effect from low-end consumers and a negative effect from high-end consumers. Because a deal hunter is more likely from the low-end pool, the positive effect dominates the negative effect. Therefore, deviation is more attemping and collusion is harder to sustain.

When δ is sufficiently large, both types of collusion can be sustained. We now compare firms' collusion profits.

The comparison when $(p_H, p_L) = (v_H, v_L)$ is straightforward, since $\pi_B = \frac{\theta(v_H - c_H) + (1-\theta)(v_L - c_L)}{2(1-\delta)} > \frac{\theta(v_H - c_H)}{2(1-\delta)} = \pi_H^1$. In this case, offering both types of products is more profitable.

Suppose $\beta > 1$, we have $(p_H, p_L) = (v_H - (\beta - 1)v_L, v_L)$. Then:

$$\pi_B - \pi_H^1 = \frac{\theta(1-\beta)v_L + (1-\theta)(v_L - c_L)}{2(1-\delta)}$$

When $\theta \to 1$, the nominator goes to $(1-\beta)v_L < 0$. When $\theta \to 0$, the nominator goes to $(v_L - c_L) > 0$. Since $\frac{\partial(\pi_B - \pi_H^1)}{\partial \theta} < 0$, by monotonicity, there exists $\theta^* \in (0,1)$ such that $\pi_B - \pi_H^1 < 0$ when $\theta > \theta^*$. This result follows exactly from the classical theory of monopoly's pricing strategy with heterogeneous consumers. Intuitively, when the fraction of high-end consumers is sufficiently large and firms have to pay an information rent when they sell both products, it is more profitable to serve exclusively for high-end consumers since firms will be able to earn a higher markup from them without much reduction in total quantity.

We summarize our findings in Proposition 2.

Proposition 2 *Suppose $\alpha \leq \widehat{\alpha}$. When firms only sell high-quality products to high-end consumers, each firm earns a collusion profit equal to $\frac{\theta(v_H - c_H)}{2(1-\delta)}$ for all $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$. When firms sell both high-quality and low-quality products, there exists $\delta^3 > \frac{1-\lambda_H}{2-\lambda_H}$ such that collusion cannot be sustained for $\delta \in [\frac{1-\lambda_H}{2-\lambda_H}, \delta^3]$. For this intermediate range of discount factor, offering high-end products to high-end consumers is also necessarily more profitable. When δ is sufficiently large, selling high-quality products exclusively to high-end consumers can still be more profitable, provided that $\beta > 1$ and the fraction of high-end consumers θ is large enough.*

3.3 Only low-quality products are sold

When firms collude on selling low-quality products only, two options are available, depending on high-end consumers' valuation of low-quality products.We abstract from the discusison of $\beta = 1$ since the probability of $\beta = 1$ in a continuous support is zero.

Our discussion of this case is rendered in Appendix. Proposition 3 summarizes our discussion in this section.

Proposition 3 *When $\alpha \leq \widehat{\alpha}$, if at least one of the following two conditions is satisfied: (i) β is close to 1; (ii) the fraction of high-end consumers θ is sufficiently large, then collusion on high-quality products to high-end consumers is easier to sustain than collusion on low-quality products. Moreover, when the discount factor is sufficiently large such that these types of collusion can all be sustained, if (ii) is satisfied, selling high-quality products to high-end consumers is more profitable than selling low-quality products to consumers.*

4 Collusion Equilibrium for Endogenous Product Selections

Now we relax our restriction on product variety and allow firms to choose freely the types of products they would like to offer before they start price competition. Firms could offer high-quality products only, low-quality products only or both types of products. We restrict our discussions on symmetric equilibria since firms are identical, but allows for asymmetry in both product varieties and prices off equilibrium path. In section 4.1 and 4.2, we regard product variety as a long-run decision and prices as short-run decisions: once firms decide on product variety, their decisions are fixed and perfectly observed, and they will provide the selected product offering forever. Then, as a robustness check, in section 4.3 we consider an alternative punishment strategy that allows firms to flexibly switch between choices of product offerings.

4.1 High-quality products to high-end consumers

Suppose firms collude on high-quality products to high-end consumers. Propositions 1-3 have illustrated conditions under which firms will sustain this type of collusion with exogenous product varieties. Now we further endogenize product varieties and discuss whether firms will have an incentive to deviate in the first-stage product choices.

There are two possible ways to deviate in product offerings: (i) deviating to products of both qualities and (ii) deviating to low-quality products only.

Table 1 summarizes our analysis for high-high type of collusion. The left column denotes for firm 1's action: cooperating, deviating to both qualities or deviating to low quality. Similarly firm 2 may cooperate, deviate to both qualities or deviate to low quality. In each pair of profits, the first one denotes for firm 1's profit and the second one denotes for firm 2's profit. We start with the case of high-quality products to high-end consumers. The analysis for other types of collusion can be characterized in a similar way.

Table 1: Firms' Profits with Endogenous Product Selections

		Firm 2		
		High quality only	Both qualities	Low quality only
Firm 1	High quality only	$(\frac{\theta(v_H-c_H)}{2(1-\delta)}, \frac{\theta(v_H-c_H)}{2(1-\delta)})$	$(0, \frac{(1-\theta)(c_H-\alpha v_H+v_L-c_L)}{1-\delta})$	$(0, \frac{(1-\theta)(c_H-\alpha v_H+v_L-c_L)}{1-\delta})$
	Both qualities	$(\frac{(1-\theta)(c_H-\alpha v_H+v_L-c_L)}{1-\delta}, 0)$		
	Low quality only	$(\frac{(1-\theta)(c_H-\alpha v_H+v_L-c_L)}{1-\delta}, 0)$		

4.1.1 Deviating to both types of products

Any deviation will trigger a price war, since the cooperator will set the price of high-quality products equal to the marginal cost starting from the period when deviation occurs. Therefore, the deviating firm can only make a profit from low-quality products. If the deviating firm is able to attract high-end consumers to buy low-quality products and earns a positive profit from them, there must exist some $p_L > c_L$ such that $\beta v_L - p_L \geq v_H - c_H$. Assumption (1) rules out the case. Thus, it suffices to consider whether the deviating firm can attract low-end consumers to buy low-quality products, which requires $v_L - p_L \geq \alpha v_H - c_H$ for some $p_L > c_L$. It is easy to verify that the deviator will charge $p_L = c_H - \alpha v_H + v_L$ and earn a deviation profit equal to $\frac{(1-\theta)(c_H-av_H+v_L-c_L)}{1-\delta}$.

Thus, the condition that firms will not deviate in product variety is:

$$\frac{\theta(v_H - c_H)}{2(1-\delta)} \geq \frac{(1-\theta)(c_H - av_H + v_L - c_L)}{1-\delta}$$

which is equivalent to

$$\theta \geq \frac{2(c_H - \alpha v_H + v_L - c_L)}{v_H - c_H + 2(c_H - \alpha v_H + v_L - c_L)} \tag{4}$$

When the fraction of high-end consumers is large, firms will be able to sustain collusion on high-quality products. The reason is that the deviator can only attract low-end consumers to buy low-quality products in the period of deviation. When the fraction of low-end consumers is small, the deviation profit from low-end consumers is small, firms are less likely to deviate, and collusion is easier to facilitate.

4.1.2 Deviating to low-quality products only

Once a firm deviates to providing low-quality products only, the competitor will charge $p_H = c_H$ starting from the period of deviation. In order to attract high-end consumers to buy low-quality products, the incentive compatibility constraint for high-end consumers must be satisfied:

$$v_H - c_H \leq \beta v_L - p_L$$

which is equivalent to:

$$p_L \leq \beta v_L + c_H - v_H$$

But, to be profitable, the price must be higher than the marginal cost:

$$p_L > c_L$$

Combining these two inequalities, we have:

$$\beta v_L - c_L > \beta v_L - p_L \geq v_H - c_H$$

contradiction to assumption (1).

Thus, the deviating firm can only attract low-end consumers to gain additional profit. The incentive compatibility constraint for low-end consumers must be satisfied:

$$\alpha v_H - c_H \leq v_L - p_L$$

Let $p_L = v_L + c_H - \alpha v_H$. This is the highest price that the deviating firm can charge to satisfy the incentive compatibility constraint. Under our assumptions, this price can always yield a profit for the deviating firm, since

$$p_L - c_L = v_L - c_L - (\alpha v_H - c_H) > 0$$

Note that the deviating firm will charge the same price for the low-quality products when it either deviates to providing low-quality products only or when it deviates to providing both types of products. Given that both firms will charge the marginal cost for high-quality products, deviator will not earn any profit from high-quality products. Thus, both types of deviation in product configurations generates the same amount of profit, and the condition that a firm will not deviate in product configurations is exactly inequality (5).

4.2 Other equilibria

Firms might be able to sustain other equilibria besides the one of our most interest. For example, consider the case of high-quality products to both types of consumers. If firms are allowed to freely choose product offerings, there are two possible types of deviations: (i) deviating to products of both qualities and (ii) deviating to low-quality products only. Using the same rationale as above, we calculate that for both types of deviation, the deviation profit is equal to $\frac{(1-\theta)(c_H - av_H + v_L) - c_L}{1-\delta}$.Thus, the condition that each firm does not deviate to offering both qualities is:

$$\frac{\alpha v_H - c_H}{2(1-\delta)} \geq \frac{(1-\theta)(c_H - av_H + v_L - c_L)}{1-\delta}$$

which is equivalent to:

$$\theta \geq 1 - \frac{1}{2}\frac{\alpha v_H - c_H}{(v_L - c_L) - (\alpha v_H - c_H)}$$

We also derive conditions under which other types of collusion can be sustained.

Suppose firms are offering low-quality products. A deviator can switch to high-quality products or switch to both types of products. Starting from the period of deviation, the cooperator will set $p_L = c_L$, and the deviator will set $p_H = v_H - \beta v_L + c_L$ to attract high-end consumers and earn a profit $\frac{\theta[(v_H - c_H) - (\beta v_L - c_L)]}{1-\delta}$.

Thus, the conditions to sustain collusion are as follow:

(i) If $\beta > 1$, the condition to sustain collusion on high-end consumers is $\beta v_L - c_L > \frac{2}{3}(v_H - c_H)$, and the condition to sustain collusion on both types of consumers is $\theta < \frac{v_L - c_L}{2[(v_H - c_H) - (\beta v_L - c_L)]}$.

(ii) If $\beta < 1$, the condition to sustain collusion on low-end consumers is $\theta < \frac{v_L - c_L}{2(v_H - c_H - (\beta v_L - c_L)) + v_L - c_L}$, and the condition to sustain collusion on both types of consumers is $\theta < \frac{\beta v_L - c_L}{2[(v_H - c_H) - (\beta v_L - c_L)]}$.

Suppose firms are offering both types of products to both types of consumers. Deviating to offering any single type of products is not profitable because the competitor will charge marginal costs for both products. The deviator will earn zero profit.

Table 2 summarizes firms' decisions when $\beta > 1$.

Table 2: Firms' Decisions when $\beta > 1$

	Cutoff δ to sustain collusion	Collusion profit	Other conditions to sustain collusion
HQ for HE	$\frac{1-\lambda_H}{2-\lambda_H}$	$\frac{\theta(v_H - c_H)}{2(1-\delta)}$	$\theta \geq \frac{(1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H + (1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}$ $\theta \geq \frac{2(c_H - \alpha v_H + v_L - c_L)}{v_H - c_H + 2(c_H - \alpha v_H + v_L - c_L)}$
HQ for Both	$\frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$	$\frac{\alpha v_H - c_H}{2(1-\delta)}$	$\theta \geq 1 - \frac{1}{2}\frac{\alpha v_H - c_H}{(v_L - c_L) - (\alpha v_H - c_H)}$
LQ for HE	$\frac{1-\lambda_H}{2-\lambda_H}$	$\frac{\theta(\beta v_L - c_L)}{2(1-\delta)}$	$\{ \beta \geq 1 + \frac{(1-\theta)(1-\frac{\lambda_L}{2})(v_L - c_L)}{\theta(1-\frac{\lambda_H}{2})v_L} \}$ or $\theta \to 1$ $\beta v_L - c_L > \frac{2}{3}(v_H - c_H)$
LQ for Both	$\frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$	$\frac{v_L - c_L}{2(1-\delta)}$	$\theta < \frac{v_L - c_L}{2[(v_H - c_H) - (\beta v_L - c_L)]}$
Both for Both	$\frac{\theta(1-\lambda_H)(p_H - c_H)+(1-\theta)(1-\lambda_L)(p_L - c_L)}{\theta(2-\lambda_H)(p_H - c_H)+(1-\theta)(2-\lambda_L)(p_L - c_L)}$	$\frac{\theta(p_H - c_H)+(1-\theta)(p_L - c_L)}{2(1-\delta)}$	-

HQ, HE: High-quality products, high-end consumers

HQ, Both: High-quality products, both types of consumers

LQ, HE: Low-quality, high-end consumers

LQ, Both: Low-quality, both types of consumers

Both, Both: Both qualities, both types of consumers

When will firms choose to provide only high-quality products? Suppose consumers are predominantly from the high-end pool such that $\theta \geq \max\{\frac{(1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H + (1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}, \frac{2(c_H - \alpha v_H + v_L - c_L)}{v_H - c_H + 2(c_H - \alpha v_H + v_L - c_L)}\}$ is satisfied. Since $\frac{\theta(v_H - c_H)}{2(1-\delta)} > \frac{\theta(\beta v_L - c_L)}{2(1-\delta)}, \frac{1-\lambda_H}{2-\lambda_H} < \frac{\theta(1-\lambda_H)(p_H - c_H)+(1-\theta)(1-\lambda_L)(p_L - c_L)}{\theta(2-\lambda_H)(p_H - c_H)+(1-\theta)(2-\lambda_L)(p_L - c_L)}$ and $\frac{1-\lambda_H}{2-\lambda_H} < \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$, when $\frac{1-\lambda_H}{2-\lambda_H} < \delta < \min\{\frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}, \frac{\theta(1-\lambda_H)(p_H - c_H)+(1-\theta)(1-\lambda_L)(p_L - c_L)}{\theta(2-\lambda_H)(p_H - c_H)+(1-\theta)(2-\lambda_L)(p_L - c_L)}\}$, only collusion on high-quality products to high-end consumers and collusion on low-quality products to high-end consumers can be sustained. Among the two types of collusion, the one on high-quality products is more profitable. Thus, when the fraction of high-end consumers is large and the discount factor is intermediate, firms will choose to offer high-quality products to high-end consumers and earn a profit $\frac{\theta(v_H - c_H)}{2(1-\delta)}$. This result generalizes our findings in section 3.

Table 3 summarizes firms' decisions when $\beta < 1$.

Table 3: Firms' decisions when $\beta < 1$

	Cutoff δ to sustain collusion	Collusion profit	Other conditions to sustain collusion
HQ for HE	$\frac{1-\lambda_H}{2-\lambda_H}$	$\frac{\theta(v_H-c_H)}{2(1-\delta)}$	$\theta \geq \frac{(1-\frac{\lambda_L}{2})(\alpha v_H-c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H+(1-\frac{\lambda_L}{2})(\alpha v_H-c_H)}$ $\theta \geq \frac{2(c_H-\alpha v_H+v_L-c_L)}{v_H-c_H+2(c_H-\alpha v_H+v_L-c_L)}$
HQ for Both	$\frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$	$\frac{\alpha v_H-c_H}{2(1-\delta)}$	$\theta \geq 1-\frac{1}{2}\frac{\alpha v_H-c_H}{(v_L-c_L)-(\alpha v_H-c_H)}$
LQ for LE	$\frac{1-\lambda_L}{2-\lambda_L}$	$\frac{(1-\theta)(v_L-c_L)}{2(1-\delta)}$	$\{\beta \leq \frac{(1-\theta)(1-\frac{\lambda_L}{2})v_L+\theta(1-\frac{\lambda_H}{2})c_L}{[(1-\theta)(1-\frac{\lambda_L}{2})+\theta(1-\frac{\lambda_H}{2})]v_L}\}$ or $\theta \to 1$ $\theta < \frac{v_L-c_L}{2[v_H-c_H-(\beta v_L-c_L)]+v_L-c_L}$
LQ for Both	$\frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$	$\frac{\beta v_L-c_L}{2(1-\delta)}$	$\theta < \frac{\beta v_L-c_L}{2[(v_H-c_H)-(\beta v_L-c_L)]}$
Both for Both	$\frac{\theta(1-\lambda_H)(v_H-c_H)+(1-\theta)(1-\lambda_L)(v_L-c_L)}{\theta(2-\lambda_H)(v_H-c_H)+(1-\theta)(2-\lambda_L)(v_L-c_L)}$	$\frac{\theta(v_H-c_H)+(1-\theta)(v_L-c_L)}{2(1-\delta)}$	--

HQ, HE: High-quality products, high-end consumers

HQ, Both: High-quality products, both types of consumers

LQ, LE: Low-quality, low-end consumers

LQ, Both: Low-quality, both types of consumers

Both, Both: Both qualities, both types of consumers

Suppose θ is large enough. For $\delta \in (\frac{1-\lambda_H}{2-\lambda_H}, \min\{\frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}, \frac{\theta(1-\lambda_H)(v_H-c_H)+(1-\theta)(1-\lambda_L)(v_L-c_L)}{\theta(2-\lambda_H)(v_H-c_H)+(1-\theta)(2-\lambda_L)(v_L-c_L)}\})$, since $\frac{1-\lambda_H}{2-\lambda_H} < \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)} < \frac{1-\lambda_L}{2-\lambda_L}$ and $\frac{1-\lambda_H}{2-\lambda_H} < \frac{\theta(1-\lambda_H)(v_H-c_H)+(1-\theta)(1-\lambda_L)(v_L-c_L)}{\theta(2-\lambda_H)(v_H-c_H)+(1-\theta)(2-\lambda_L)(v_L-c_L)}$, only high-high collusion can be sustained. Firms will offer high-quality products to high-end consumers and earn a profit $\frac{\theta(v_H-c_H)}{2(1-\delta)}$.

When the discount factor is sufficiently large, all types of collusions can be sustained. Firms may still choose to provide high-quality products to high-end consumers. For example, suppose $\beta > 1$ and $\theta \geq \max\{\frac{\alpha v_H-c_H}{v_H-c_H}, \frac{v_L-c_L}{v_H-c_H}, \frac{(1-\frac{\lambda_L}{2})(\alpha v_H-c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H+(1-\frac{\lambda_L}{2})(\alpha v_H-c_H)}, \frac{2(c_H-\alpha v_H+v_L-c_L)}{v_H-c_H+2(c_H-\alpha v_H+v_L-c_L)}\}$, collusion profit on high-quality for high-end consumers is the highest among all the possibilities.

We summarize the result in Proposition 4.

Proposition 4 *Suppose $\alpha \leq \widehat{\alpha}$. There exists $\widehat{\theta} \in (0,1)$ such that for $\theta \geq \widehat{\theta}$ and $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$, it is an equilibrium outcome that each firm chooses to sell only high-quality products, set $p_H = v_H$ and earn a profit $\frac{\theta(v_H-c_H)}{2(1-\delta)}$. Moreover, there exists $\widetilde{\delta} \in (0,1)$ such that for $\delta \in (\frac{1-\lambda_H}{2-\lambda_H}, \widetilde{\delta})$, this equilibrium is the most profitable one. For $\delta > \widetilde{\delta}$, this equilibrium can still be the most profitable one,*

provided that $\theta \geq \max\{\frac{\alpha v_H - c_H}{v_H - c_H}, \frac{v_L - c_L}{v_H - c_H}, \frac{(1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H + (1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}, \frac{2(c_H - \alpha v_H + v_L - c_L)}{v_H - c_H + 2(c_H - \alpha v_H + v_L - c_L)}\}$ *and* $\beta > 1$. *Thus, when* $\{\alpha \leq \widehat{\alpha},\ \theta \geq \widehat{\theta}$ *and* $\delta \in (\frac{1-\lambda_H}{2-\lambda_H}, \widetilde{\delta})\}$ *or* $\{\alpha \leq \widehat{\alpha},\ \theta \geq \overline{\theta}$ *and* $\delta > \widetilde{\delta}\}$, *where* $\overline{\theta} = \max\{\widehat{\theta}, \frac{\alpha v_H - c_H}{v_H - c_H}, \frac{v_L - c_L}{v_H - c_H}, \frac{(1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H + (1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}, \frac{2(c_H - \alpha v_H + v_L - c_L)}{v_H - c_H + 2(c_H - \alpha v_H + v_L - c_L)}\}$, *the unique prediction is that firms will offer exclusively high-quality products to high-end consumers.*

Sometimes, firms are under regulations that prevent them from ceasing to offer baseline products. Nevertheless, firms might be tempting to lower down the quality of baseline products. It is well-known that such behavior can be interpreted as a price-discrimination strategy in the setting of monopoly offering vertically differentiated products. Here we point out another possibility: lowering down the quality of baseline products can facilitate collusion on high-quality products.

Note that $\frac{\partial}{\partial v_L}(\frac{2(c_H - \alpha v_H + v_L - c_L)}{v_H - c_H + 2(c_H - \alpha v_H + v_L - c_L)}) > 0$, we have the following proposition:

Proposition 5 *Decreasing the quality of low-quality product can facilitate firms" collusion on high-quality products, in the sense that the threshold of the fraction of high-end consumers to facilitate this type of collusion will be lower.*

This result follows from the fact that, a lower quality level of baseline products does not affect firms' collusion profit of high-quality products on high-end consumers, but it reduces firms' deviation profit to other types of product configurations. Therefore, lowering down the quality level of baseline products can reduce firms' incentive to deviate in product variety and thus facilitate high-high type of collusion.

This finding is directly applicable to business practices. For example, it is reported that in the past year in China, the speed of 4G networks has become lower and lower, and sometimes it hinders 4G network users from using their smartphone applications normally. One explanation given by network provisioners is that they have been allocating more resources to prepare for 5G network, which is deemed as a high-quality product. Thus, due to limited resources allocated to 4G networks, which is deemed as a low-quality product, the speed of 4G networks unevitably went down. Our model illustrates the possibility that firms doing so might be attempting to facilitate collusion on 5G network services.

4.3 Alternative path of deviation

In practice, if the cost of switching between product varieties is small, firms may adopt an alternative path of deviation. In each period firms decide simultaneously on variety and then they set prices. On

the equilibrium path, both firms choose some particular product variety and then collude. If a firm deviates in product variety, in the current period, the cooperating firm charges marginal cost for the product it is offering; starting from the next period, both firms produce two varieties and indefinitely earn zero profits. If a firm does not deviate by changing variety, but only deviates in price, then the cooperating firm will charge marginal cost for the product; starting from the next period, both firms introduce both varieties and earn zero profit thereafter. This trigger strategy is more harsh than the one discussed in section 4.1. As a robustness check, we will analyze firms' strategic behavior with the new trigger strategy in this section.

We consider the case when θ is sufficiently close to 1, that is, when the fraction of high-end consumers is sufficiently large. This is to explain firms' strategic behaviors in businesses where consumers are predominantly from high-end group. If firms are free to choose the product variety, firms will bring high-quality products into the portfolio since collusion on a product portfolio that contains the high-quality ones is both easier and more profitable. Whether firms will also offer low-quality products depends on how consumers are differentiated. If consumers are vertically differentiated, then firms will not offer low-quality products since they will otherwise have to sacrifice an information rent for almost all of their sales; if consumers are horizontally differentiated, then firms will be indifferent between offering only high-quality products and offering both types of products. This part of analysis is analogous to the analysis of a multiproduct monopoly in a single market.

We render the details in appendix and summarize our findings in proposition 6:

Proposition 6 *In the setting of alternative path of deviation, suppose the fraction of high-end consumers is sufficiently close to 1. (i) If* $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ *and* $\beta > 1$*, firms collude on offering high-quality products exclusively to high-end consumers; (ii) if* $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ *and* $\beta < 1$*, firms either collude on offering high-quality products to high-end consumers only, or collude on offering high-quality products to high-end consumers and low-quality products to low-end consumers. (iii) If* $\delta < \frac{1-\lambda_H}{2-\lambda_H}$*, no collusion can be sustained.*

5 Welfare Analysis and Policy Implications

Our welfare analysis follows immediately from the equilibrium characterization discussed in Section 4. As firms' profit has been calculated, the remaining task is to calculate consumer welfare. In this section

we are back to the setting of section 4.1: once firms decide on product variety, their decisions are fixed and perfectly observed and they will provide the selected product offering forever.

(i) High-quality products to high-end consumers only:

In this case, firms set price $p_H = v_H$. Only high-end consumers make purchases and are indifferent between purchasing and not purchasing the product. Thus consumer welfare is zero.

(ii) High-quality products to both types of consumers:

In this case, firms set price $p_H = \alpha v_H$. Low-end consumers get zero utility, high-end consumers get utility $(1-\alpha)v_H$ and consumer welfare is $\theta(1-\alpha)v_H$.

(iii) High-quality products to high-end consumers and low-quality products to low-end consumers:

In this case, if condition (3) is satisfied, firms set price $(p_H, p_L) = (v_{H,}v_L)$ if $\beta \leq 1$ and set price $(p_H, p_L) = (v_H - (\beta-1)v_L, v_L)$ if $\beta > 1$. In the former case, consumer surplus is zero; in the latter case, consumer surplus is $\theta(\beta-1)v_L$.If condition (3) is not satisfied, collusion cannot be sustained.

(iv) Low-quality products to high-end/low-end consumers only:

The former case might happen when $\beta > 1$. Firms will set price $p_L = \beta v_L$ and consumer welfare is zero. The latter case might happen when $\beta < 1$. Firms will set price $p_L = v_L$ and consumer welfare is zero.

(v) Low-quality products to both types of consumers:

In this case, firms set price $p_L = v_L$ if $\beta > 1$, and set price $p_L = \beta v_L$ if $\beta < 1$. In the former case, consumer surplus is $\theta(\beta-1)v_L$; in the latter case, consumer surplus is $(1-\theta)(1-\beta)v_L$.

Now we are ready to compare social welfare across different settings. Consider first the unregulated market and now suppose $\beta > 1$. Section 4 has shown that when the fraction of high-end consumers is high and the discount factor is intermediate, firms will sell high-quality products to high-end consumers. Suppose instead that government impose a regulation that firms must offer low-quality products. Then for intermediate discount factors, collusion could no longer be sustained and Bertrant-type competition would drive down prices equal to marginal costs. Producer surplus would decrease and consumer surplus would increase. In this case, the first-best outcome would be restored, and consumers would benefit from the government regulation.

Proposition 7 *When the fraction of high-end consumers is sufficiently large and the discount factor is intermediate, government regulation on reintroducing low-quality products will benefit consumers and restore the first-best outcome.*

In the case of our interest, mandatory provision of low-quality products will improve consumer welfare. In other cases, such a regulation can nevertheless hurt consumers and firms altogether. The reason is that with such a regulation, firms can cease to provide high-quality products and switch to low-quality products. Consumers pay less for low-quality products but also enjoy less benefit from them, and the overall welfare effect can be negative.

Formally, this will happen if the following inequalities are satisfied:

$$\beta < 1$$

$$\alpha v_H - c_H > \beta v_L - c_L \tag{5}$$

$$1 - \frac{1}{2}\frac{\alpha v_H - c_H}{(v_L - c_L) - (\alpha v_H - c_H)} \leq \theta < \frac{\alpha v_H - c_H}{v_H - c_H} \tag{6}$$

$$1 - \frac{1}{2}\frac{1}{(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})} < \frac{\theta(1-\lambda_H)(v_H - c_H) + (1-\theta)(1-\lambda_L)(v_L - c_L)}{\theta(2-\lambda_H)(v_H - c_H) + (1-\theta)(2-\lambda_L)(v_L - c_L)} \tag{7}$$

$$\theta < \frac{\beta v_L - c_L}{2[(v_H - c_H) - (\beta v_L - c_L)]} \tag{8}$$

$$\theta < \frac{\beta v_L - c_L}{v_H - c_H} \tag{9}$$

$$\theta(1-\alpha)v_H > (1-\theta)(1-\beta)v_L \tag{10}$$

Inequalities (6) and (7) together imply that collusion on high-quality products is sustainable and more profitable than other types of collusion. Inequality (8) ensures that there exist intermediate discount factors such that collusion on high-quality products is sustainable, whereas collusion on both types of products cannot be sustained. Inequalities (9) and (10) imply that once firms operate under the regulation, providing low-quality products only will generate a higher profit than other types of operations. Inequality (11) implies that the change of product variety will lead to a reduction in consumer welfare.

As an illustration, Table 8 displays a set of parameter values that satisfy inequality (6)-(11).

Table 8: A Set of Parameter Values

Parameter	α	β	θ	δ	v_H	c_H	v_L	c_L	λ_H	λ_L
Value	0.9	0.7	0.8	0.3	0.7	0.3	0.6	0.1	0.7	0.1

When the government does not require firms to introduce low-quality products to the market, firms offer high-quality products for both types of consumers, each firm earns a profit equal to 0.30, and consumer welfare is equal to 0.06. Suppose instead that government regulation is present, firms will offer low-quality products for both types of consumers, each firm will earn a profit equal to 0.29, and consumer welfare will be equal to 0.04.

Proposition 8 *It is possible that government regulation on reintroducing baseline products can hurt both firms and consumers.*

How can an antitrust committee anticipates the policy outcome on such a regulation, i.e., whether such a policy will facilitate competition and help consumers? The challenge is that observing only high-quality products on the market is insufficient to determine the policy outcome. Nevertheless, looking at data from consumer side can be helpful. Suppose only high-end consumers are purchasing products and consumers are vertically differentiated, then mandatory provision of low-quality products will induce firms to provide both types of products and result in higher consumer welfare. On the other hand, suppose both types of consumers are purchasing products, then such a policy can possibly hurt consumers.

6 Conclusion

This paper studies the relationship between product variety and cartel stability in the framework of Bertrand duopoly with endogenized, vertically differentiated product space and heterogeneity in consumer valuation and search friction. Our main result is that when the value difference of high-quality product between the two groups is large, collusion is easier and more profitable when both firms choose to provide only high-quality products. In particular, from a firm's perspective, offering high-quality products only is the most desirable option when the discount factor is intermediate, or when the discount factor is large, the fraction of high-end consumers is large and the consumer preference satisfies a

set of conditions. Our key assumption is that a deal hunter is more likely from the high-end consumer pool. This assumption ensures the existence of a range of intermediate discount factors, for which collusion on high-quality products can be sustained whereas collusion on both types of products cannot be sustained.

Our analysis can be applied to many businesses. For example, in Hong Kong, gas stations offer only 98 RON gas to drivers instead of introducing petrol at lower qualities as a substitute. Offering only high-quality petrol can help gas stations sustain collusion by exclusively serving for high-end consumers. Drivers in Hong Kong are typically high-end consumers, and many of them are not sensitive to petrol price. Therefore, our model fits well into this setting.

Our model is also applicable to justify policies that require mandatory provision of baseline products. Our model suggests that a regulation that requires mandatory provision of more affordable products will deter collusion of high-quality products on high-end consumers. It may well be necessary to impose regulations on minimum quality standard, since reducing the quality of baseline product can facilitate this type of collusion.

7 Appendix

7.1 Collusion equilibrium for exogenous product selections when only low-quality products are sold

7.1.1 Case 1: $\beta > 1$

When $\beta > 1$, consumers are vertically differentiated. Firms may collude by (i) selling low-quality products to high-end consumers only, or (ii) selling low-quality products to both types of consumers.

Low-quality products to high-end consumers only In this case, firms will charge $p_L = \beta v_L$ to maximize joint profit.

When neither of the firms deviates, each firm earns the collusive profit π_L^1:

$$\pi_L^1 = \frac{\theta(\beta v_L - c_L)}{2(1-\delta)}$$

If one of the firms slightly undercuts its price, it will steal all the high-end deal hunters and half of the

high-end passive consumers in the market. The remaining half of high-end passive consumers will stay with the competitor. In this case, the deviator can earn a profit equal to π_D^{31}:

$$\pi_D^{31} = \theta(1 - \frac{\lambda_H}{2})(\beta v_L - c_L)$$

Alternatively, the firm can cut the price of high-quality products to just below v_L to attract all the deal hunters and half of the passive consumers. The deviation profit is denoted as π_D^{32}:

$$\pi_D^{32} = [\theta(1 - \frac{\lambda_H}{2}) + (1-\theta)(1 - \frac{\lambda_L}{2})](v_L - c_L)$$

It can be verified that $\pi_D^{31} \geq \pi_D^{32}$ if and only if:

$$\beta \geq \widehat{\beta} \equiv 1 + \frac{v_L - c_L}{v_L} \frac{(1-\theta)(1-\frac{\lambda_L}{2})}{\theta(1-\frac{\lambda_H}{2})}$$

First we consider the case $\beta \geq \widehat{\beta}$. This happens when high-end consumers' willingness to pay for low-quality products is high enough. In this case, slightly cutting down the price and getting deviation profit π_D^{31} is more profitable. The condition to sustain the collusion is $\pi_L^1 \geq \pi_D^{31}$:

$$\frac{\theta(\beta v_L - c_L)}{2(1-\delta)} \geq \theta(1 - \frac{\lambda_H}{2})(\beta v_L - c_L)$$

$$\delta \geq \delta^1 \equiv \frac{1-\lambda_H}{2-\lambda_H}$$

Note that the threshold discount factor, δ^1, is the same as the threshold discount factor to sustain collusion when firms sell high-quality products to high-end consumers. Our assumption $v_H - c_H > \beta v_L - c_L$ ensures $\pi_L^1 < \pi_H^1$. Thus, offering high-quality products to high-end consumers is always preferred to offering low-quality products to high-end consumers. When selling to a single type of consumers, firms can extract all consumer surplus. Thus, the markup of selling high-quality products is equal to high-end consumers' valuation of high-quality products minus the production cost of high-quality products, and the markup of selling low-quality products is equal to high-end consumers' valuation of low-quality products minus the production cost of low-quality products. Assumption (1) implies that the markup of high-quality products is higher than the markup of low-quality products, thus it is more profitable to sell high-quality products.

Next we consider the case $\beta < \widehat{\beta}$. The condition to sustain the collusion is then:

$$\delta \geq 1 - \frac{\theta(\beta v_L - c_L)}{(2 - \theta\lambda_H - (1-\theta)\lambda_L)(v_L - c_L)}$$

When $\beta \to 1, 1 - \frac{\theta(\beta v_L - c_L)}{(2-\theta\lambda_H-(1-\theta)\lambda_L)(v_L-c_L)} \to 1 - \frac{\theta}{(2-\theta\lambda_H-(1-\theta)\lambda_L)} = \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)+1-\theta}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)} > \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)} = \delta^2 > \delta^1$. That is, the threshold discount factor to sustain collusion on low-quality products to high-end consumers is higher than the threshold discount factor to sustain collusion on high-quality products to high-end consumers. Thus, collusion on high-quality products is easier than collusion on low-quality products.

When β increases, the threshold discount factor will decrease, making collusion on low-quality products easier. However, as long as θ is sufficiently close to 1, we have $1 - \frac{\theta(\beta v_L - c_L)}{(2-\theta\lambda_H-(1-\theta)\lambda_L)(v_L-c_L)} \to 1 - \frac{\beta v_L - c_L}{v_L - c_L}\frac{1}{2-\lambda_H} > 1 - \frac{1}{2-\lambda_H} = \frac{1-\lambda_H}{2-\lambda_H}$. Thus, as long as the fraction of high-end consumers is large enough, regardless of the value β, our prediction that firms will choose to offer high-quality products to high-end consumers remains to be true.

The intuition is as follow. When β is less than the threshold, the difference between the deviation price to attract low-end consumers and the deviation price to just attract high-end consumers is small. The quantity effect of attracting more consumers dominates the price effect of having a lower markup. When β decreases, deviation becomes more attempting because the collusion profit decreases and the deviation profit remains the same. As a result, collusion is harder to sustain. In the limiting case when β is sufficiently close to 1, a firm's small price undercut is enough to attract all deal hunters from its competitor, and deviation becomes very profitable. Collusion on high-quality products is then easier than collusion on low-quality products.

Low-quality products to both types of consumers In this case, firms will charge $p_L = v_L$ to maximize joint profit.

When neither of the firms deviates, each firm earns the collusive profit π_L^2:

$$\pi_L^2 = \frac{v_L - c_L}{2(1-\delta)}$$

If one of the firms slightly undercuts its price, it will attract all the deal hunters and half of the passive consumers. In this case, the deviator will earn a profit π_D^4.

$$\pi_D^4 = [(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})](v_L - c_L)$$

The condition for sustaining this type of collusion is $\pi_L^2 \geq \pi_D^4$:

$$\frac{v_L - c_L}{2(1-\delta)} \geq [(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})](v_L - c_L)$$

$$\delta \geq \delta^2 \equiv \frac{\theta(1-\lambda_H) + (1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H) + (1-\theta)(2-\lambda_L)}$$

The threshold discount factor δ^2 is the same as the minimal discount factor to sustain collusion when firms sell high-quality products to both types of consumers, and is larger than the minimal discount factor to sustain collusion when firms sell high-quality products to high-end consumers.

For δ being sufficiently large, both types of collusion can be sustained. Now we compare firms' collusion profits:

$$\pi_H^1 > \pi_L^2 \Longleftrightarrow \theta > \frac{v_L - c_L}{v_H - c_H}$$

The above comparison implies that when the fraction of high-end consumers is larger than the ratio $\frac{v_L - c_L}{v_H - c_H}$, collusion on high-quality products is more profitable than collusion on low-quality products. The reason is as follow. Firms earn a markup $v_H - c_H$ by selling high-quality products to high-end consumers and earn a markup $v_L - c_L$ by selling low-quality products to both types of consumers. The profitablility in the former case is equal to the markup times the fraction of high-end consumers, and the profitablility in the latter case is equal to the markup times the mass of all consumers.

7.1.2 Case 2: $\beta < 1$

When $\beta < 1$, consumers are horizontally differentiated. High-end consumers possess a higher willingness-to-pay for high-quality products, whereas low-end consumers possess a higher willingness-to-pay for low-quality products. Firms may collude by (i) selling low-quality products to low-end consumers only, or (ii) selling low-quality products to both types of consumers. The analysis is similar to the case of $\beta > 1$.

Low-quality products to low-end consumers only In this scenario, firms will charge $p_L = v_L$ to maximize joint profit. When neither of the firms deviates, each firm earns a collusive profit denoted as π_L^3:

$$\pi_L^3 = \frac{(1-\theta)(v_L - c_L)}{2(1-\delta)}$$

If one of the firms slightly undercuts its price, it will attract all the low-end deal hunters and half of the low-end passive consumers. The deviator can earn a profit π_D^{51}:

$$\pi_D^{51} = (1-\theta)(1-\frac{\lambda_L}{2})(v_L - c_L)$$

Alternatively, the firm can cut the price of high-quality products to just below βv_L to attract all the deal hunters and half of the passive consumers. The deviation profit is π_D^{52}:

$$\pi_D^{52} = [\theta(1-\frac{\lambda_H}{2}) + (1-\theta)(1-\frac{\lambda_L}{2})](\beta v_L - c_L)$$

It can be verified that $\pi_D^{51} \geq \pi_D^{52}$ if and only if:

$$\beta \leq \widetilde{\beta} \equiv \frac{1}{v_L}[\frac{(1-\theta)(1-\frac{\lambda_L}{2})(v_L - c_L)}{(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})} + c_L]$$

First we discuss the case $\beta \leq \widetilde{\beta}$. This happens when high-end consumers' willingness-to-pay for low-quality products is low. In this case, it is more profitable for the firm to just cut down the price to $p_L = v_L$ and earn a deviation profit π_D^{51}. Then the condition for sustaining the collusion is:

$$\frac{(1-\theta)(v_L - c_L)}{2(1-\delta)} \geq (1-\theta)(1-\frac{\lambda_L}{2})(v_L - c_L)$$

$$\delta \geq \delta^4 \equiv \frac{1-\lambda_L}{2-\lambda_L}$$

Our assumption $\lambda_H > \lambda_L$ implies $\frac{1-\lambda_L}{2-\lambda_L} > \frac{1-\lambda_H}{2-\lambda_H}$. Thus, the threshold discount factor δ^4 is larger than the threshold discount factor to sustain collusion when firms sell only high-quality products to high-end consumers. In other words, collusion on high-qualty products to high-end consumers is easier to sustain among the two. This result follows directly from the assumption that low-end consumers are more likely to search. When firms collude on low-quality products to low-end consumers, a firm's

small price undercut is more attractive because it can attract a larger fraction of consumers from its competitor.

Next we discuss the case $\beta \geq \widetilde{\beta}$. The condition to sustain the collusion is:

$$\frac{(1-\theta)(v_L - c_L)}{2(1-\delta)} \geq [\theta(1-\frac{\lambda_H}{2}) + (1-\theta)(1-\frac{\lambda_L}{2})](\beta v_L - c_L)$$

$$\delta \geq \delta^5 \equiv 1 - \frac{(1-\theta)(v_L - c_L)}{2[\theta(1-\frac{\lambda_H}{2}) + (1-\theta)(1-\frac{\lambda_L}{2})](\beta v_L - c_L)}$$

When $\beta \to 1, \delta^5 \to 1 - \frac{1-\theta}{2[\theta(1-\frac{\lambda_H}{2})+(1-\theta)(1-\frac{\lambda_L}{2})]} = \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)+\theta}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)} > \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)} = \delta^2 > \delta^1$. The reason is that, when β is less than 1 and close to 1, deviation from low-low collusion is more profitable because a deviating firm is able to attract all deal hunters from its competitor by a small amount of price undercut. In other words, the quantity effect dominates the price effect. Therefore, low-low collusion is harder to sustain than high-high collusion. When β decreases, the threshold discount factor will decrease and collusion on low-quality products becomes easier. However, as long as θ is sufficiently large, the right hand side of the above inequality will be close to 1, which is clearly greater than δ^1. This is because when consumers are predominantly high-end, a price undercut to attract purchases from high-end consumers becomes very appealing. Collusion on low-quality products for low-end consumers, therefore, becomes harder to sustain. In sum, when either β is close to 1 or θ is close to 1, collusion on low-quality products to low-end consumers is harder to sustain than is collusion on high-quality products to high-end consumers.

Suppose δ is sufficiently large (in particular, $\delta \geq \frac{1-\lambda_L}{2-\lambda_L}$), then both types of collusion can be sustained. We compare collusion profits in the two cases.

$$\pi_H^1 \geq \pi_L^3 \Longleftrightarrow \theta \geq \theta^2 \equiv \frac{v_L - c_L}{v_H - c_H + v_L - c_L}$$

Thus, It is more profitable to offer high-quality products to high-end consumers when the fraction of high-end consumers is larger than θ^2. The result follows from comparing markup and consumer base between the two types of collusion.

Low-quality products to both types of consumers In this scenario, firms will charge $p_L = \beta v_L$ to maximize joint profit.

When neither of the firms deviates, each firm earns the collusive profit π_L^2:

$$\pi_L^4 = \frac{\beta v_L - c_L}{2(1-\delta)}$$

If one of the firms slightly undercuts its price, it will steal all the deal hunters from its competitor while retaining half of the passive searchers. In this case, the deviator can earn a profit π_D^6:

$$\pi_D^6 = [(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})](\beta v_L - c_L)$$

Thus, the condition for sustaining the collusion is $\pi_L^4 \geq \pi_D^6$:

$$\frac{\beta v_L - c_L}{2(1-\delta)} \geq [(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})](\beta v_L - c_L)$$

$$\delta \geq \delta^2 \equiv 1 - \frac{1}{2}\frac{1}{(1-\theta)(1-\frac{\lambda_L}{2}) + \theta(1-\frac{\lambda_H}{2})}$$

Note that the threshold discount factor here is the same as the threshold discount factor to sustain collusion on high-quality products to both types of consumers, and is larger than the threshold discount factor to sustain collusion on high-quality products to high-end consumers only. The result follows from the assumption that high-end consumers are less likely to search for price differences.

For δ being sufficiently large, both types of collusion can be sustained. Now we compare firms' collusion profits:

$$\pi_H^1 > \pi_L^4 \Longleftrightarrow \theta > \frac{\beta v_L - c_L}{v_H - c_H}$$

Our assumption $v_H - c_H > \beta v_L - c_L > 0$ ensures that the right hand side is between 0 and 1. Thus, when the fraction of high-end consumers is large enough, collusion on high-quality products is more profitable than collusion on low-quality products.

Case 1 and Case 2 together imply that when β is close to 1, high-end consumers' valuation of low-quality products is similar to low-end consumers' valuation of low-quality products. When firms collude on low-quality products, a small price undercut is enough to attract all deal hunters. The quantity effect in deviation dominates the price effect, making deviation more appealing. Therefore, collusion on low-quality products is more difficult than collusion on high-quality products.

7.2 Alternative path of deviation

The condition to sustain collusion on high-quality products to high-end consumers now becomes:

$$\frac{\theta(v_H - c_H)}{2(1-\delta)} \geq (1-\theta)(c_H - av_H + v_L - c_L)$$

$$\delta \geq 1 - \frac{\theta(v_H - c_H)}{2(1-\theta)(c_H - \alpha v_H + v_L - c_L)}$$

or equivalently:

$$\theta \geq \frac{2(1-\delta)(c_H - \alpha v_H + v_L - c_L)}{(v_H - c_H) + 2(1-\delta)(c_H - \alpha v_H + v_L - c_L)}$$

Since $\delta \leq \frac{1-\lambda_H}{2-\lambda_H} < \frac{1}{2} < 1$, we have $\frac{2(1-\delta)(c_H-\alpha v_H+v_L-c_L)}{(v_H-c_H)+2(1-\delta)(c_H-\alpha v_H+v_L-c_L)} < \frac{2(c_H-\alpha v_H+v_L-c_L)}{v_H-c_H+2(c_H-\alpha v_H+v_L-c_L)}$. Thus, collusion on high-quality products to high-end consumers is easier to sustain in the alternative setting. But meanwhile, other types of single-product collusion are also easier to sustain. Similar to the analysis of the original fixed-variety setting, we calculate the conditions to sustain different types of collusions as follow.

High-quality products, both types of consumers: $\frac{\alpha v_H - c_H}{2(1-\delta)} \geq (1-\theta)(c_H - av_H + v_L - c_L) \Leftrightarrow \delta \geq 1 - \frac{\alpha v_H - c_H}{2(1-\theta)(c_H - \alpha v_H + v_L - c_L)}$;

$\beta > 1$: Low-quality products, high-end consumers: $\delta \geq 1 - \frac{\beta v_L - c_L}{2[v_H - c_H - (\beta v_L - c_L)]}$; low-quality products, both types of consumers: $\delta \geq 1 - \frac{v_L - c_L}{2\theta[(v_H - c_H) - (\beta v_L - c_L)]}$;

$\beta < 1$: Low-quality products, low-end consumers: $\delta \geq 1 - \frac{(1-\theta)(v_L - c_L)}{2\theta(v_H - c_H - (\beta v_L - c_L))}$; low-quality products, both types of consumers: $\delta \geq 1 - \frac{\beta v_L - c_L}{2\theta[v_H - c_H - (\beta v_L - c_L)]}$;

Both types of products, both types of consumers: deviation is not profitable because the competitor will charge marginal cost for both types of products starting from the period of deviation.

Table 4 summarizes firms' decisions when $\beta > 1$

Table 4: Firms' Decisions when $\beta > 1$

	Cutoff δ to sustain collusion	Collusion profit	Other conditions to sustain collusion
HQ, HE	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ $\delta \geq 1 - \frac{\theta(v_H - c_H)}{2(1-\theta)(c_H - \alpha v_H + v_L - c_L)}$	$\frac{\theta(v_H - c_H)}{2(1-\delta)}$	$\theta \geq \frac{(1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H + (1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}$
HQ, Both	$\delta \geq \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$ $\delta \geq 1 - \frac{\alpha v_H - c_H}{2(1-\theta)(c_H - \alpha v_H + v_L - c_L)}$	$\frac{\alpha v_H - c_H}{2(1-\delta)}$	-
LQ, HE	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ $\delta \geq 1 - \frac{\beta v_L - c_L}{2[v_H - c_H - (\beta v_L - c_L)]}$	$\frac{\theta(\beta v_L - c_L)}{2(1-\delta)}$	$\beta \geq 1 + \frac{v_L - c_L}{v_L} \frac{(1-\theta)(1-\frac{\lambda_L}{2})}{\theta(1-\frac{\lambda_H}{2})}$ or $\theta \to 1$
LQ, Both	$\delta \geq \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$ $\delta \geq 1 - \frac{v_L - c_L}{2\theta[(v_H - c_H) - (\beta v_L - c_L)]}$	$\frac{v_L - c_L}{2(1-\delta)}$	-
Both, Both	$\delta \geq \frac{\theta(1-\lambda_H)(p_H - c_H)+(1-\theta)(1-\lambda_L)(p_L - c_L)}{\theta(2-\lambda_H)(p_H - c_H)+(1-\theta)(2-\lambda_L)(p_L - c_L)}$	$\frac{\theta(p_H - c_H)+(1-\theta)(p_L - c_L)}{2(1-\delta)}$	-

HQ, HE: High-quality products, high-end consumers

HQ, Both: High-quality products, both types of consumers

LQ, HE: Low-quality, high-end consumers

LQ, Both: Low-quality, both types of consumers

Both, Both: Both qualities, both types of consumers

Table 5 summarizes firms' decisions when $\beta < 1$.

Table 5: Firms' decisions when $\beta < 1$

	Cutoff δ to sustain collusion	Collusion Profit	Other conditions
HQ, HE	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ $\delta \geq 1 - \frac{\theta(v_H - c_H)}{2(1-\theta)(c_H - \alpha v_H + v_L - c_L)}$	$\frac{\theta(v_H - c_H)}{2(1-\delta)}$	$\theta \geq \frac{(1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}{(1-\frac{\lambda_H}{2})(1-\alpha)v_H + (1-\frac{\lambda_L}{2})(\alpha v_H - c_H)}$
HQ, Both	$\delta \geq \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$ $\delta \geq 1 - \frac{\alpha v_H - c_H}{2(1-\theta)(c_H - \alpha v_H + v_L - c_L)}$	$\frac{\alpha v_H - c_H}{2(1-\delta)}$	-
LQ, LE	$\delta \geq \frac{1-\lambda_L}{2-\lambda_L}$ $\delta \geq 1 - \frac{(1-\theta)(v_L - c_L)}{2\theta(v_H - c_H - (\beta v_L - c_L))}$	$\frac{(1-\theta)(v_L - c_L)}{2(1-\delta)}$	$\beta \leq \frac{1}{v_L}[\frac{(1-\theta)(1-\frac{\lambda_L}{2})(v_L - c_L)}{(1-\theta)(1-\frac{\lambda_L}{2})+\theta(1-\frac{\lambda_H}{2})} + c_L]$ or $\theta \to 1$
LQ, Both	$\delta \geq \frac{\theta(1-\lambda_H)+(1-\theta)(1-\lambda_L)}{\theta(2-\lambda_H)+(1-\theta)(2-\lambda_L)}$ $\delta \geq 1 - \frac{\beta v_L - c_L}{2\theta[v_H - c_H - (\beta v_L - c_L)]}$	$\frac{\beta v_L - c_L}{2(1-\delta)}$	-
Both, Both	$\delta \geq \frac{\theta(1-\lambda_H)(v_H - c_H)+(1-\theta)(1-\lambda_L)(v_L - c_L)}{\theta(2-\lambda_H)(v_H - c_H)+(1-\theta)(2-\lambda_L)(v_L - c_L)}$	$\frac{\theta(v_H - c_H)+(1-\theta)(v_L - c_L)}{2(1-\delta)}$	-

HQ, HE: High-quality products, high-end consumers

HQ, Both: High-quality products, both types of consumers

LQ, LE: Low-quality, low-end consumers

LQ, Both: Low-quality, both types of consumers

Both, Both: Both qualities, both types of consumers

Now we consider the case when θ is close to 1 (that is, the fraction of high-end consumers is sufficiently large). This is to explain firms' strategic behaviors in businesses where consumers are predominantly from high-end group. If firms are free to choose the product variety, firms will clearly bring high-quality products into the portfolio because, as we will show, collusion on a product portfolio that contains the high-quality ones is both easier and more profitable. Answering the question whether firms will offer only high-quality products or will offer both types of products requires more detailed analysis, as we will show in Table 6 and Table 7.

If $\beta > 1$, conditions to sustain collusion and corresponding collusion profits can be summarized as Table 6.

Table 6: Firms' Decisions when $\beta > 1$ and $\theta \to 1$

	Cutoff δ to sustain collusion	Collusion Profit
HQ, HE	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$	$\frac{v_H - c_H}{2(1-\delta)}$
HQ, Both	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$	$\frac{\alpha v_H - c_H}{2(1-\delta)}$
LQ, HE	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ $\delta \geq 1 - \frac{\beta v_L - c_L}{2[v_H - c_H - (\beta v_L - c_L)]}$	$\frac{\beta v_L - c_L}{2(1-\delta)}$
LQ, Both	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ $\delta \geq 1 - \frac{v_L - c_L}{2[(v_H - c_H) - (\beta v_L - c_L)]}$	$\frac{v_L - c_L}{2(1-\delta)}$
Both, Both	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$	$\frac{p_H - c_H}{2(1-\delta)}$

Suppose we have $\delta \geq 1 - \frac{v_L - c_L}{2[v_H - c_H - (\beta v_L - c_L)]}$, then as long as $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$, all types of collusions can be sustained. Now we compare collusion profits. Our assumption $\alpha < 1$ implies $\frac{v_H - c_H}{2(1-\delta)} > \frac{\alpha v_H - c_H}{2(1-\delta)}$. Assumption (1) implies $\frac{v_H - c_H}{2(1-\delta)} > \frac{\beta v_L - c_L}{2(1-\delta)}$. Since $\beta > 1$, we further have $\frac{v_H - c_H}{2(1-\delta)} > \frac{v_L - c_L}{2(1-\delta)}$. Since $p_H \leq v_H$, we have $\frac{v_H - c_H}{2(1-\delta)} \geq \frac{p_H - c_H}{2(1-\delta)}$. Thus, in this case, collusion on high-quality products to high-end consumers is one of the most profitable types of collusions. If the condition $(1-\alpha)v_H \geq (\beta - 1)v_L$ is not satisfied, or $v_H \leq \beta v_L$, then collusion on high-quality products to high-end consumers is *the* most profitable type of collusion. Our unique prediction is thus firms will offer exclusively high-quality products to high-end consumers.

If $\beta < 1$, conditions to sustain collusion and corresponding collusion profits can be summarized as

Table 7.

Table 7: Firms' decisions when $\beta < 1$ and $\theta \to 1$

	Cutoff δ to sustain collusion	Collusion Profit
HQ, HE	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$	$\frac{v_H - c_H}{2(1-\delta)}$
HQ, Both	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$	$\frac{\alpha v_H - c_H}{2(1-\delta)}$
LQ, LE	cannot be sustained	-
LQ, Both	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$ $\delta \geq 1 - \frac{\beta v_L - c_L}{2[v_H - c_H - (\beta v_L - c_L)]}$	$\frac{\beta v_L - c_L}{2(1-\delta)}$
Both, Both	$\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$	$\frac{v_H - c_H}{2(1-\delta)}$

Similar to the discussion of the first case, we have $\frac{v_H - c_H}{2(1-\delta)} > \frac{\alpha v_H - c_H}{2(1-\delta)}$ and $\frac{v_H - c_H}{2(1-\delta)} > \frac{\beta v_L - c_L}{2(1-\delta)}$. In this case, as long as $\delta \geq \frac{1-\lambda_H}{2-\lambda_H}$, firms will either choose to offer exclusively high-quality products to high-end consumers, or choose to offer high-quality products to high-end consumers and low-quality products to low-end consumers. Since the conditions to sustain these two types of collusions are the same, and the corresponding collusion profits are equal, we do not have a unique prediction among the two.

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