Secret Reserve Prices by Uninformed Sellers

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Abstract

This paper proposes a novel explanation for the use of secret reserve prices in auctions. If bidders are better informed than the seller about a common component of auction heterogeneity, the seller can allocate more efficiently by keeping her reserve price secret and revising it after the bids are submitted. We build a model of a first-price auction under unobserved auction heterogeneity (imperfectly observed by the seller) that captures this rationale and derive conditions for identification. The model is estimated using data on French timber auctions, where the government uses an ex-ante secret reserve price (which can be revised down if no bid is above it). Counterfactual analysis shows that: (1) acquiring perfect signals about auction heterogeneity would allow the seller to increase revenue by 9.76% and surplus by 6.81% (first-best); (2) switching from a public to a secret reserve price (with learning) reduces the welfare loss by 65%.

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1 Introduction

In many auction markets, sellers do not reveal their reserve price, i.e., the minimum bid they are willing to accept. This is the case, for instance, in auctions for fine art and wine (Ashenfelter (1989)), online auctions (Bajari and Hortaçsu (2003), Hossein (2008)), and auctions of natural resources such as oil or timber (Hendricks, Porter and Spady (1989), Elyakim, Laffont, Loisel and Vuong (1994)). Given results in traditional models of auctions, the use of secret reserve prices remains an important puzzle. This paper proposes a novel explanation which, in contrast to the previous literature, does not rely on departures from bidders and seller rationality and risk-neutrality.

If bidders are better informed than the seller about the underlying heterogeneity of the auctioned item, the seller can allocate more efficiently by keeping her reserve price secret and adjusting it after bids are submitted. Indeed, bids convey useful information about the seller's reservation value. In contrast, by committing to an ex-ante public reserve price, the seller loses the option value of learning from the bids. The literature on strategic bid skewing provides strong evidence that bidders often possess more precise information about the ex-post realization of quantities in timber auctions (Athey and Levine (2001)) and in procurement of construction projects (Luo and Takahashi (2019), Bolotnyy and Vasserman (2019)) or information about future adaptation costs (Bajari, Houghton, and Tadelis (2014)).

The main contribution of this paper is to provide a rationale for the use of secret rather than public reserve prices. Our explanation complements other rationales proposed in the literature, as we see it applying to environments where the auctioneer cares about efficiency (i.e., government agencies) and faces some uncertainty about their reservation value.

Our approach is guided by two important features of our empirical setting, the French timber industry. First, in timber auctions, the Public Forest Service (Office National des Forêts, ONF) sets an ex-ante secret reserve price which can be revised down if no bid is above it: around 40% of auctioned tracts are sold at a bid under the ex-ante secret reserve price. This feature cannot be accounted for by previous rationales for secret reserve prices. Discussions with ONF officers indicate that revisions are based on the distribution of submitted bids and occur, in particular, if bids reveal that the initial appraisal value was inflated.

Second, bidders possess more precise information than the ONF about tract heterogeneity. Tracts differ with respect to timber volumes, composition, location, harvesting conditions etc. In advance of each sale, the ONF collects tract characteristics and share them via a booklet with prospective bidders. Due to the large number of tracts surveyed and the ONF's limited resources, timber volumes reported in the booklet are purely indicative and often imprecise (the ONF has no contractual obligation vis-a-vis reported volumes).[1] Bidders have, therefore, strong incentives and do conduct their own "cruises" since the winner pays a lump-sum (or fixed-price) amount irrespective of actual timber volumes or quality.

We have access to data on ten sales of standing timber by the ONF that took place in the Lorraine region in the Fall of 2003.[2] We observe information on 2262 tracts auctioned via first-price sealed auctions, including: bids and bidder identities, tract level characteristics reported in the sale booklets (estimates of volume per species, surface, number of trees etc.), and, importantly, the ONF's ex-ante secret reserve price. By combining the latter variable with information about bids received and whether the tract was sold, we are able to identify the instances where the ONF adjusts its initial reserve price down to accept the highest bid. To the best of our knowledge, this is a unique feature of our data.

Reduced form analysis of the ONF's revision rule reveals that: (1) when the highest bid is above the ex-ante secret reserve price, the tract is always sold to the highest bidder; (2) when the highest bid is below the ex-ante secret reserve price, the probability of sale decreases with the distance (in absolute value) between the the bid and ex-ante reserve price and increases with the number of bids received. To further investigate the effect of the ONF's revision rule on auction outcomes (i.e., revenue and surplus), we estimate a structural model that captures the main features of our empirical setting. In particular, the model can be used to compute the value to the ONF of acquiring better signals about the underlying heterogeneity of a tract, and to compare the current policy to alternative reserve price policies.

We develop a stylized model where firms bid in a first-price auction with unobserved auction heterogeneity (e.g., timber quality and volume). The unobserved heterogeneity component enters both bidders and seller's values. While this component is perfectly observed by the bidders as in the canonical model of Krasnokutskaya (2011), it is imperfectly observed by the seller.[3] The seller sets an ex-ante secret reserve price based on her noisy signal of unobserved auction heterogeneity, and can revise this reserve price flexibly after bids are submitted.[4] We show that if the seller's revision rule satisfies a homogeneity assumption

[1] Interviews conducted with bidders in Marty (2012) (in French) show that discrepancies between announced and actual timber volumes are quite common:

> See what the ONF announced for the domanial forest X. We have just finished exploiting it, it's the ONF clerk who cruised it [...] They announced 798 m^3 of oak. [...] We found exactly 500 m^3 and they announce 798 m^3!

[2] This dataset was collected by Costa and Preget (2004).

[3] Correlation in bids could be due to affiliation (i.e., factors that are unobserved to the bidders and the econometrician) or unobserved heterogeneity (i.e., factors that are observable to all bidders but not the econometrician). In the context of timber auctions, the latter appears to be the main source of correlation.

[4] The seller has no commitment power, and therefore sets the ex-post reserve price to their expected reservation value given initial appraisal and submitted bids.

(in bids and ex-ante reserve price), separability of the unobserved heterogeneity component and bidders' idiosyncratic private values carries to the equilibrium bid function as in the canonical model.

Under mutual independence of the unobserved heterogeneity component, bidders' idiosyncratic values, and the seller's noisy signal, the model is identified from information on bids, ex-ante secret reserve prices, and allocation decisions (sold-unsold). The identification proceeds in three steps: first, by using the joint distribution of an arbitrary bid and the corresponding ex-ante secret reserve price, the distribution of unobserved heterogeneity, bidders' individual bid component, and the seller's signal can be identified. Second, the conditional probability of winning is obtained from the unconditional probability of winning and the distribution of unobserved auction heterogeneity. Third, the distribution of bidders' idiosyncratic value component is derived from knowledge of the conditional probability of winning by inverting the first-order condition. The estimation procedure proposed in the paper follows the steps of the constructive identification argument.

Using the estimated model, we conduct several counterfactuals. As a benchmark, we compute the first-best outcome: that is, assuming the auctioneer perfectly observes the auction heterogeneity component and sets a public reserve price equal to her true reservation value (efficient auction). This counterfactual gives an upper bound on total surplus and, importantly, allow us to compute the value to the ONF of acquiring better signals (value of information). Second, we compare the current policy to several alternatives: (a) no reserve price, (b) making the ex-ante secret reserve price public, (c) setting an ex-post secret reserve price equal to a convex combination of average bids and ex-ante secret reserve price. Policy (c) nests both policies (a) and (b) with a weight on bids of one and zero respectively.

Counterfactual simulations show that the current policy comes close to the first-best. Acquiring perfect signals about auction heterogeneity would allow the seller to increase revenue by 9.76% and surplus by 6.81% (first-best). Switching from a public reserve price (policy (b)) to a secret reserve price with efficient learning (policy (c)) reduces the surplus gap by 65%. The overall magnitude of the effects is, however, small: under efficient learning, surplus would increase by 1.2% and revenue would decrease by 3.4% compared to a public reserve price (policy (b)). By learning from the bids, the seller trades-off greater allocative efficiency against lower revenue per auction.

Related Literature. This paper contributes to three strands of the literature. The first strand concerns solutions to the secret reserve price "puzzle." Vincent (1995) develops an example where secret reserve prices can induce greater participation in second-price auctions with interdependent values. Li and Tan (2017) show that secret reserve prices can yield higher revenue in first-price auction with I.P.V. if bidders are sufficiently risk-averse. Horstmann and LaCasse (1997) show that in a common value setting, sellers of high-value items can signal to potential bidders by using secret reserve prices when there are resale opportunities. Rosar (2014) shows that a secret reserve price can be revenue-enhancing if the seller must fix the auction rules (bid space and public or secret reserve price) before knowing their value. Three recent contributions propose explanations based on non-standard/irrational agents: Rosenkranz and Schmitz (2007) study first-price and second-price auctions if agents have reference-based utility. Hossain (2008) studies a dynamic second-price auction where a fraction of bidders are uninformed and learn only whether their private valuation is above a posted price. Jehiel and Lamy (2015) use a competing auction environment with some buyers who do not have rational expectations about the distribution of reserve prices when kept secret. In our particular case, buyers are firms with at least some firm-specific component of value, contract sizes are typically small relative to firm size, and buyers are well-informed about tract heterogeneity, making explanations based on risk-aversion, irrational belief or signalling less appealing.

Second, the paper contributes to the empirical literature on timber auctions. This literature encompasses studies of transaction costs and choice between unit-price and lump-sum format (Leffler and Rucker (1991)), post-auction bargaining between seller and bidders (Elyakim, Laffont, Loisel, and Vuong (1997)), the effect of resale (Haile (2000)), bid skewing in unit-price auctions (Athey and Levin (2001)), collusion (Baldwin, Marshall and Richard (1997)), the presence of risk-aversion (Lu and Perrigne (2008), Campo, Guerre, Perrigne, and Vuong (2011)). The closest papers to our study are: Athey and Levin (2001) who highlight the importance of private information about auction heterogeneity in the U.S. Forest Service timber auctions; and Li and Perrigne (2003) and Perrigne (2003) who uses French timber auction data and analyze the revenue effects of secret versus public reserve prices. The latter paper allow for risk-aversion and shows that secret reserve prices can be revenue-enhancing.

Finally, we build on the literature on unobserved auction heterogeneity. In particular, our identification approach is closest to Krasnokutskaya (2011) who adapts the nonparametric approach of Guerre, Perrigne, and Vuong (2000) to accommodate auction level heterogeneity observed by the bidders but not by the econometrician. Ignoring unobserved heterogeneity can have significant impact on structural estimates as found by Asker (2010), Krasnokutskaya (2011), and Krasnokutskaya and Seim (2011). Haile and Kitamura (2018) provide an excellent survey of approaches to identification developed in recent work on first-price auctions with unobserved heterogeneity. We contribute to this literature by extending the identification results of Krasnokutskaya (2011) to settings with secret reserve prices.

The paper is organized as follows. Section 2 provides a simple example that illustrates the main intuition of the paper. Section 3 gives background information about the ONF timber

sale program. Section 4 describes the data and present reduced form evidence on the ONF's secret reserve policy rule. Section 5 presents the model. Section 6 shows the identification and estimation result of the structural model. Section 7 presents the counterfactual analysis. Section 8 concludes.
All tables and figures are located after the main text.

2 A Simple Example

Before presenting the model, it is instructive to consider an example. In this setting, the information structure is simple enough so that the seller learns perfectly from the bids. The example highlights the main intuition and key features entering the general model.

A seller (she) offers a single object for sale to n bidders via a first-price auction. (The results can be extended to other auction formats such as second-price and English auctions.) The seller's reservation value, denoted Y, can take values in $\{\frac{1}{2}, \frac{3}{2}\}$. Bidder i's value is the sum of two components: the common component Y and a bidder-specific private value $X_i \sim \mathcal{U}[-\frac{1}{2}, \frac{1}{2}]$, which is independent of Y and across bidders. Therefore, if Y equals $\frac{1}{2}$, bidder values are distributed $\mathcal{U}[0, 1]$; whereas if Y equals $\frac{3}{2}$, bidder values are distributed $\mathcal{U}[1, 2]$, as shown in Figure 1.

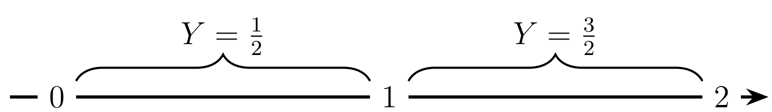

Figure 1

In addition to knowing his private value X_i, bidder i perfectly observe the common component Y. The seller, however, does not. Let the seller's prior belief about Y be uniform $\left(\frac{1}{2}, \frac{1}{2}\right)$. The seller acts as a social planner and aims to maximize total surplus.

In the first-best full information case, the seller perfectly observes Y and maximizes total surplus by setting an ex-post efficient public reserve price equal to Y. Ex-post surplus is equal to $\max\{Y, \max_i Y + X_i\}$.

Next, we compare surplus when the seller holds beliefs $\left(\frac{1}{2}, \frac{1}{2}\right)$ about Y and use a public or secret reserve price. Under a public reserve price, expected surplus is maximized with a reserve price equal to $\frac{1}{2}$. If $Y = \frac{1}{2}$, allocation is efficient; if $Y = \frac{3}{2}$, however, the item is always sold, which is inefficient if the highest bidder's value is less than $\frac{3}{2}$.[5]

[5] By setting a reserve price of $\frac{1}{2}$, the seller allocates efficiently when $Y = \frac{1}{2}$ but misallocates when $Y = \frac{3}{2}$. The benefits (relative to a reserve price of $\frac{3}{2}$) in terms of expected surplus is $\int_{\frac{1}{2}}^{1} \left(u - \frac{1}{2}\right) dH(u)$, where

With a secret reserve price, we argue that the seller can perfectly learn the value of Y from the bids and reach the first-best level of surplus. Consider the following equilibrium of the two-stage game in which bidders submit sealed bids, and given the bids and her prior belief about Y, the seller chooses an ex-post reserve price: In stage 2, if all bids are below 1, the seller sets the ex-post secret reserve price equal to $\frac{1}{2}$. Otherwise, the seller sets a reserve price equal to $\frac{3}{2}$. In the first stage, if $Y = \frac{1}{2}$, bidders bid as in a first-price auction with reserve price of $\frac{1}{2}$. if $Y = \frac{3}{2}$, bidders bid as in a first-price auction with reserve price of $\frac{3}{2}$.

In this example, not committing to a public reserve price allows the seller to delay her allocation decision until perfectly learning her true value Y from the bids. The allocation is ex-post efficient. In the general model of Section 5, we investigate the seller and bidders' behavior when bids do not perfectly reveal the common component of value. Before doing so, the next section provides background information about the industry and the data that motivate features of the general model.

3 Industry Background

Our empirical work will focus on timber auctions conducted by the French national Public Forest Service (Office National des Forêts, ONF hereafter). This government agency is in charge of the management of France's approximately 11 million hectares of public forests and the sale of standing timber to mills and logging companies. Competitive bidding is the main mechanism chosen by the ONF for its timber sales (about 85% of total sales). We focus here on sales via (lump-sum) first-price sealed bid auctions, the most common auction format used by the ONF.[6]

Each administrative region in France has its own ONF local office. The data analyzed in this paper comes from the Grand Est (previously Lorraine) region (Eastern France). Local offices are in charge of the management of the public forestry on their own territory and are responsible for organizing auctions. Each regional office uses the profits from these sales to cover their operating costs. As auctioneer, the ONF's objective is to secure timber supply to the local timber industry at a price that allows them to remain competitive. Therefore, we interpret the ONF's objective as the maximization of the timber industry surplus subject to financial constraints (budget balance).

In advance of each sale, the ONF organizes a cruise of the various tracts (around 100 per

$H(u) = u^n$ is the distribution of the first-order statistic of bidder values ($Y = \frac{1}{2}$). The costs in terms of expected surplus is $\int_1^{\frac{3}{2}} \left(\frac{3}{2} - u\right) d\widetilde{H}(u)$, where $\widetilde{H}(u) = (u-1)^n$ is the distribution of the first-order statistic of bidder values ($Y = \frac{3}{2}$). The benefits term dominates under a uniform prior.

[6] Contrary to North American timber auctions, unit-price auctions where bidders submit a bid per species are less common.

sale) and publicly announces the findings in a booklet available to potential bidders. The booklet describes the characteristics of each auctioned tract such as the volume of the species in the tract, the surface of the tract, any visible damage etc. Due to the large number of tracts surveyed and the ONF's limited resources, timber volumes reported in the booklet are purely indicative and often imprecise (the ONF has no contractual obligation vis-a-vis reported volumes).

Potential bidders are private firms, typically local sawmills. As the location of all auctioned tracts is given in the sale booklet, bidders have the opportunity to cruise the tracts and form their own estimates of tract characteristics. These cruises allow the bidder to gather two sorts of information: first, additional and/or more precise information about tract characteristics common to all firms (quantity, quality etc.); second, information about their private value for the tract, which depends on firm-specific harvesting costs and the type of final product they will be able to sell using this timber. Bidders' private values vary due to their diversity of operations: logging enterprises, sawmills, paper mills, board factories, etc.[7] Bidders usually cruise the tract they intend to bid on. (in our dataset, bidders participate on average in 13 auctions per sale)

The ONF also computes an appraisal value for each tract which is not disclosed to the bidders, and based on this value, sets an (ex-ante) reserve price. The reserve price is kept secret at the time of the auction. On the day of the sale, the ONF director collects sealed bids for each tract, opens the bids and ranks them. If the highest bid is above the ex-ante secret reserve price, the ONF sells the tract at the bid price. If the highest bid is below the ex-ante secret reserve price, the ONF may still decide to sell the tract to the highest bidder. The main criteria leading to a sale decision are: the number of bids received and their distribution, the difference between the highest bid and the ex-ante secret reserve price, and revenue constraints. About 40% of auctioned tracts are sold at a bid under the ONF's ex-ante reserve price. Whenever a tract is sold, the winner's identity and bid are publicly announced. If a tract goes unsold, the ex-ante secret reserve price is announced.

Discussions with ONF officers reveal that the ONF does not commit to any public reserve price because they do not perfectly know their reservation value. This value corresponds to the expected outcome in a future sale, which depends on the tract characteristics, how market participants value each characteristics, and future timber market conditions. Bidders are more likely to have better information about tract characteristics, thanks to their private cruises. Therefore, the ONF may adjust its reservation value after observing the bids.

[7] In 2018, of the 15 millions m^3 of timber sold, 4 were destined for construction, 3 for furniture manufacturing, 4 for the paper and cardboard industry, and 4 for energy.

4 Data and Reduced-Form Analysis

This section describes the data used in our empirical application. We use data on ten sales of standing timber by the ONF that took place in the Lorraine region in the Fall of 2003. This dataset was collected in the context of a report commissioned by the ONF (Costa and Preget (2004)). The data contains information on 2262 tracts auctioned via first-price sealed auctions, including: bids and bidder identities, tract level characteristics reported in the sale booklets (estimates of volume per species, surface, number of trees etc.), the ONF's initial appraisal value for each tract, and the ONF's ex-ante secret reserve price.

The data contains a array of tract characteristics, which help control for auction heterogeneity. These tract characteristics are disclosed in the sale booklet to all prospective bidders. Descriptive statistics for the continuous and categorical variables are presented in Table 1 and Table 2 respectively. To capture tract level heterogeneity in volume per species, we construct a Herfindahl index of tract heterogeneity. Figure 2 shows histogram of the Herfindahl index and the number of bidders per auction.

We analyze the main determinants of the bids, reserve price, and number of bidders in Table 3, Table 4, and Table 5. The volume of timber is positively correlated with these outcomes variables, whereas the surface of the tract is negatively correlated with these outcome variables. The Herfindahl index is positively correlated with tract value. Tract quality (as controlled for by the categorical variables) has an expected sign: e.g., tracts with strong grapeshot damages from WWI have lower value and attract fewer bidders.

A unique feature of the data is that we observe the seller's ex-ante secret reserve price as well as the allocation decision after bids are submitted. By combining the ex-ante secret reserve price with information about bids received and whether the tract was sold, we are able to identify the instances where the ONF adjusts its initial reserve price down to accept the highest bid. In particular, Figure 3 shows that 50% of tracts in which the highest bid is below the ex-ante reserve price, end up being sold.

Preliminary analysis indicates that revisions to the ex-ante reserve price are based on the highest bid and number of bids received. Figure 4 shows a scatter-plot of auctioned tract sale status (i.e., sold or unsold) as a function of the highest bid (y-axis) and reserve price (x-axis) normalized by the appraisal value (seller's estimate). The figure shows that tracts are always sold when the highest bid is above the ex-ante secret reserve price. Otherwise, the further the highest bid is from the reserve price, the lower the likelihood of the tract being sold.

In Table 6, we present the averages of the reserve price, highest bid, estimate, as well as the fraction of revisions among the tracts that were eventually sold, grouped by the number of bidders. The fraction of tracts sold after the revise price was revised down increases with the number of bids received: if the highest bid is below the ex-ante secret reserve price, the tract is more likely to be sold when it attracted more bids.

Finally, we estimate a linear probability model for the revision decision and report it in Table 7. The strongest predictor of the revision decision is the proximity of the highest bid to the ex-ante reserve price. Tracts that are state-owned and tracts with more bidders have a higher chance of revision. A higher Herfindahl index is negatively associated with the revision probability.

5 The Model

This section presents the first-price auction model under secret reserve prices and unobserved auction heterogeneity. As in the model of Krasnokutskaya (2011), bidders perfectly observe the component component of auction heterogeneity. The seller, however, only receives a noisy signal of the common component. We derive properties of the equilibrium bidding strategy in this context.

Random variables are denoted with upper case letters. Lower case letters denote realizations of random variables. Vectors are denoted in bold.

The seller (she) offers a single object for sale to n bidders in a first-price sealed bid auction. The object is sold under unobserved auction heterogeneity. That is, bidder i's valuation is equal to the product of two components: one is common and known to all bidders; the other is individual and the private information of bidder i. Both the common and the individual valuation components are random variables, and they are denoted by Y and X_i, respectively.

The seller's reservation value is her opportunity cost of selling the object. If a tract goes unsold, the seller can re-auction it the following year. Therefore, the seller's reservation value depends on the common component Y.

In our setting, the seller is imperfectly informed about the realization of Y. We assume that the seller observes only a noisy signal of Y, denoted $\tilde{Y} = Y \times S$, for some random variable S. In our empirical application, the variable $\tilde{Y}$ corresponds to the seller's private appraisal value (after controlling for observable tract characteristics disclosed in the booklet).

Information sets: The information set of bidder i is $\{x_i, y\}$. The seller's information set is $\{\tilde{y}\}$.

Primitives of the model: We assume that bidders' private values are symmetric and independent. All random variables Y, X_i's, and the seller's signal S are assumed mutually independent. The primitives are the marginal distribution of Y, X_i's, and the seller's signal S, denoted F_X, F_Y, and F_S respectively. Denote by $[\underline{x}, \overline{x}]$, $[\underline{y}, \overline{y}]$, and $[\underline{s}, \overline{s}]$ the supports of these random variables. The lower bounds satisfy: $\underline{x} > 0$, $\underline{y} > 0$, and $\underline{s} > 0$.

Reserve price: Based on the realization of $\tilde{Y}$, the seller sets an ex-ante secret reserve price, denoted R_0. We impose the restriction that R_0 is a linear function of $\tilde{Y}$, i.e., the seller's reserve price is set to a fraction of the appraisal value. This implies, in particular, that R_0 is linear in the latent variable Y. After sealed bids are submitted, the seller may revise her secret reserve price. Let $R_1(\mathbf{b}, r_0)$ denote the ex-post secret reserve price as a function of the vector of bids received $\mathbf{b} = (b_1, ..., b_n)$ and the ex-ante reserve price. The object is allocated to the highest bidder if his bid exceeds R_1. Denote by $D(\mathbf{b}, r_0)$ the dummy variable that equals one if $\max_i B_i \geq R_1$ and equals zero otherwise.[8]

Strategy and payoffs: Given the realization of the common component $y \in [\underline{y}, \overline{y}]$, a bidding strategy is a real-valued function defined on $[\underline{x}, \overline{x}]$:

$$\beta_y : [\underline{x}, \overline{x}] \to [0, \infty)$$

The profit realization of bidder i, $\pi(x_i, y; b_i)$, equals $(x_i y - b_i)$ if bidder i wins the item with a bid b_i and zero if he loses. At the time of bidding, bidder i knows y and x_i but not his opponents's bids $\mathbf{b}_{-i} = \{b_j\}_{j \neq i}$. The interim expected profit of bidder i is given by

$$\pi(x_i, y; b_i) = (x_i y - b_i) P(b_i \geq R_1(b_i, \mathbf{B}_{-i}, R_0) \cap b_i \geq B_j, j \neq i \mid Y = y)$$

To win the item, bidder i must not only outbid his opponents but also the ex-post secret reserve price chosen by the seller. There are two sources of randomness to the ex-post secret reserve price: first, R_1 depends on the seller's ex-ante valuation for the object $\tilde{Y}$ (or equivalently, on R_0) which differs from the known realization of the common component Y; second, R_1 depends on bids submitted by bidder i's opponents.

Equilibrium: A symmetric Bayesian Nash equilibrium is characterized by a function $\beta_y(.)$ such that $\pi(x_i, y; b_i)$ is maximized when $b_i = \beta_y(x_i)$ and $b_j = \beta_y(x_j)$ for $j \neq i$, for every $i \in \{1, ..., n\}$ and every realization of X_i. It is assumed that there is such an equilibrium in which each bidder follows a strategy that is increasing in x_i and y and differentiable.

Multiplicative separability: We extends a property of equilibrium bidding strategies under unobserved auction heterogeneity shown in Krasnokutskaya (2011) to the case with secret reserve prices.

Assumption 1 (Homogeneity of degree one)**.** *The ex-post reserve price is homogeneous of degree one in the bids and ex-ante reserve price. That is, for a bid vector* $\mathbf{b} = (b_1, ..., b_n)$ *and*

[8] More generally, if the revision rule is stochastic, $D(\mathbf{b}, r_0)$ equals the probability that the item is allocated.

ex-ante reserve price R_0

$$R_1(k\mathbf{b}, kr_0) = kR_1(\mathbf{b}, r_0), \quad \forall k \in \mathbb{R}_+^*$$

As shown in Section 4, this property is satisfied in our empirical application.

Proposition 1. *Under Assumption 1, if* $\alpha(.)$ *is an equilibrium bidding strategy of the game indexed by* $y = 1$*, then an equilibrium bidding strategy in the game indexed by* y*, with* $y \in [\underline{y}, \overline{y}]$ *given by* $\beta_y(.)$ *is such that* $\beta_y(x_i) = y\alpha(x_i)$*, for all* i*.*

Proof Let $\alpha(.)$ be an equilibrium of the game indexed by $Y = 1$. Then the interim expected profits of bidder i, given a bid a_i, is

$$\pi(x_i, 1; a_i) = (x_i - a_i)P(a_i \geq R_1(a_i, \mathbf{A}_{-i}, R_0) \cap a_i \geq A_j, j \neq i \mid Y = y)$$

$\pi(x_i, 1; a_i)$ is maximized when $a_i = \alpha(x_i)$ and bidder i's opponents follow strategy $\alpha(.)$. Define the bidding function $\widetilde{\beta}_y(x) = y \cdot \alpha(x)$ for $y \in [\underline{y}, \overline{y}]$ and $x \in [\underline{x}, \overline{x}]$. We have that

$$\frac{\pi(x_i, y; b_i)}{y} = \left(x_i - \frac{b_i}{y}\right) P\left(\frac{b_i}{y} \geq R_1\left(\frac{b_i}{y}, \frac{\mathbf{B}_{-i}}{y}, \frac{R_0}{y}\right) \cap \frac{b_i}{y} \geq \frac{B_j}{y}, j \neq i \mid Y = y\right)$$

Assume bidder i's opponents follow strategy $\widetilde{\beta}_y(.)$. Then, $\frac{\mathbf{B}_{-i}}{y}$ is distributed according to $F_X \circ \alpha^{-1}$ and, therefore, is independent of Y. $\frac{R_0}{y}$ is also independent of Y. We can drop the conditioning on $Y = y$. Defining $\frac{b_i}{y} = \widetilde{b}_i$ (similarly for R_0 and $\mathbf{B}_{-i}$) we have

$$\frac{\pi(x_i, y; b_i)}{y} = \left(x_i - \widetilde{b}_i\right) P\left(\widetilde{b}_i \geq R_1\left(\widetilde{b}_i, \widetilde{\mathbf{B}}_{-i}, \widetilde{R_0}\right) \cap \widetilde{b}_i \geq \widetilde{\mathbf{B}}_{-i}, j \neq i\right)$$

The right-hand side coincides with interim expected profits in the game indexed by $Y = 1$. These profits are maximized when $\widetilde{b}_i = \alpha(x_i)$, or equivalently, $b_i = y \cdot \alpha(x_i) = \widetilde{\beta}_y(x_i)$. Hence, $\widetilde{\beta}_y(x_i)$ is a best-response if bidder $j \neq i$ follow strategy $\widetilde{\beta}_y(.)$. □

6 Estimation

This section presents the identification of the model, the estimation approach, and the results.

6.1 Identification

In our setting, the econometrician has access to: bid data $(B_1, ..., B_n)$, the seller's ex-ante estimate (appraisal value) $\tilde{Y}$ and secret reserve price R_0, and auction outcomes. The data is based on N independent draws from the distribution of $(Y, \{X_i\}_{i=1...n}, S)$. We derive properties of the available data such that the model primitives are identified.

Denote by B_i the random variable corresponding to the bid of bidder i with distribution $G_B(.)$ and density $g_B(.)$; and let b_{ij} denote its realization in auction j. The econometrician observes the joint distribution of bids $(B_1, ..., B_n)$. For the rest of this section, the number of bidders is fixed to n.

Proposition 1 establishes that $b_{ij} = ya_{ij}$, where a_{ij} is a hypothetical bid that would have been submitted by bidder i if y were equal to one. We use A_i to denote the random variable with realizations equal to a_{ij}. The associated distribution function is denoted by $G_A(.)$ with the probability density function $g_A()$. Note that y and a_{ij} are not observed by the econometrician. The distribution of A_i is latent.

The identification result is established in three steps. First, it is shown that the probability density function of Y, A and S can be uniquely determined from the joint distribution of a bid and the seller's ex-ante estimate (appraisal value). Second, the winning probability conditional on $Y = 1$ is identified from knowledge of the unconditional winning probability and the distribution of the common component Y. Third, monotonicity of the inverse bid function is used to identify the distribution of X from the distribution of A.

Proposition 2. *The probability density functions f_Y, f_S, and f_X are identified from the distributions of bids, ex-ante estimate (appraisal value), and allocation rule.*

The proof of this proposition consists of three steps. The first and third steps follow the arguments of Li and Vuong (1998) and Krasnokutskaya (2011). Our setting differs from the standard model of first-price auction, however, due to the seller's revision rule. Step 2 accounts for the seller's revision rule when expressing a bidder's conditional probability of winning. This probability is used in step 3 to invert the first-order condition and recover the distribution of bidder specific value.

Step 1: Identification of the probability density functions of Y, A_i, and S.
We apply the statistical result from Kotlarski (1966) to the log transformed random variables $B_i = A_i \times Y$ and $\widetilde{Y} = S \times Y$

$$\log(B_i) = \log(A_i) + \log(Y)$$

$$\log(\widetilde{Y}) = \log(S) + \log(Y)$$

Kotlarski's result uses the fact that the characteristic function of the sum of two independent random variables is equal to the product of the characteristic functions of these variables. This property allows us to find the characteristic functions of $\log(Y)$, $\log(A_i)$, and $\log(S)$ from the joint characteristic function of $\log(B_i)$ and $\log(\widetilde{Y})$. Let $\Psi(.,.)$ and $\Psi_1(.,.)$ denote the joint characteristic function of $(\log(\widetilde{Y}), \log(B_i))$ and the partial derivative of this characteristic function with respect to the first component, respectively. Also, let $\Phi_{\log(Y)}(.)$, $\Phi_{\log(A_i)}(.)$, and $\Phi_{\log(S)}(.)$ denote the characteristic functions of $\log(Y)$, $\log(A_i)$, and $\log(S)$. Then,

$$\Phi_{\log(Y)}(t) = \exp\left(\int_0^t \frac{\Psi_1(0,u_2)}{\Psi(0,u_2)} du_2 - itE[\log(S)]\right)$$

$$\Phi_{\log(S)}(t) = \frac{\Psi(t,0)}{\Phi_{\log(Y)}(t)}$$

$$\Phi_{\log(A)}(t) = \frac{\Psi(0,t)}{\Phi_{\log(Y)}(t)}$$

From the knowledge of the characteristic functions, we can derive the probability density functions of Y, S, and A, given the normalization $E[\log(S)] = 0$.

Step 2: Identification of the winning probability conditional on $Y = 1$. The unconditionnal winning probability given a bid b_i can be expressed as follows

$$\begin{aligned} P(D(b_i, \mathbf{B}_{-i}, R_0) = 1 \cap b_i \geq B_j, j \neq i) &= \int P(D(b_i, \mathbf{B}_{-i}, R_0) = 1 \ \cap \ b_i \geq B_j, j \neq i | Y = y) dF_Y(y) \\ &= \int P\left(D\left(\frac{b_i}{y}, \frac{\mathbf{B}_{-i}}{y}, \frac{R_0}{y}\right) = 1 \ \cap \ \frac{b_i}{y} \geq \frac{B_j}{y}, j \neq i\right) dF_Y(y) \end{aligned} \tag{1}$$

Equation (1) can be rewritten as $F_L(b_i) = \int F_M(\frac{b_i}{y}) dF_Y(y)$ for some random variables L and M. These random variables satisfy $L = MY$, or $\log(L) = \log(M) + \log(Y)$. The characteristic functions of L and Y are known (since L is the *unconditional* probability of winning given a bid b and Y is identified in the previous step). By independence of Y and L, we can recover the characteristic function of $\log(M)$ from the characteristic functions of $\log(L)$ and $\log(Y)$

$$\Phi_{\log(M)}(t) = \frac{\Phi_{\log(L)}(t)}{\Phi_{\log(Y)}(t)}$$

The characteristic function of M and its probability density function can be subsequently recovered from knowledge of the characteristic function of $\log(M)$. The cumulative distribution of M, denoted $F_M(a)$, gives the probability of winning given a bid a and conditional on Y equal to one.

Step 3: Identification of the probability density functions of X

We apply the result from Laffont and Vuong (1996) based on the first-order condition: Having recovered the probability of winning conditional on $Y = 1$ ($F_M(a)$), we can solve bidders' optimization problem and find the equilibrium inverse bidding strategy. The inverse bid function is combined with the distribution of normalized bids G_A (obtained in step 1) to back out the distribution of individual valuations X_i.

6.2 Estimation

In our empirical application, tracts differ in observed dimensions (available to all bidders in the sale booklet). We control for this observed common component of heterogeneity in an initial step. The rest of the estimation approach follows the steps of the identification.

1. Account for observed auction heterogeneity.
   The estimation procedure assumes that the data available is from auctions of ex-ante identical tracts. This assumption is not valid in our setting, because tracts differ in dimensions which are public information and observed by the bidders before submitting their bids (i.e., available in the sale booklet). This public information will enter not only a bidder's private value of winning the tract but also his belief about other bidders' values.

   We follow the approach of Haile, Hong, and Shum (2006) to account for auction-specific observed heterogeneity. Their approach leverages the separability of common observable component from the bidder-specific and common unobserved heterogeneity components of bids. Let the seller's ex-ante estimate of tract value in auction k be

   $$\widetilde{y}_k = \Gamma(\mathbf{x}_k)\hat{y}_j$$

   Similarly, let the value of bidder i in auction k be

   $$v_{ik} = \Gamma(\mathbf{x}_k)\hat{v}_{ij}$$

   By multiplicative separability (Proposition 1), the corresponding bid of bidder i in auction j satisfies

   $$b_{ik} = \Gamma(\mathbf{x}_k)\hat{b}_{ij}$$

   where $\Gamma(\mathbf{x}_k)$ is a function of the vector of observed auction characteristics $\mathbf{x}_k$ for auction k reported in the sale booklet.

   Assumption 2 (Common observed heterogeneity)**.** *The observed common auction heterogeneity component enters identically into bidders and seller's values.*

Assume the following parametric specification: $\Gamma(\mathbf{x}_k) = \exp(\mathbf{x}_k'\delta)$. We run a pooled first-stage regression of the dependent variables $z_{ik} \in \{b_{ik}, \widetilde{y}_k\}$ on observed tract characteristics

$$\log z_{ik} = (\mathbf{x}_k, n)'\delta + \sigma_{ij} \tag{2}$$

where z_{idt} denotes the bid of bidder i in auction k and the seller's ex-ante estimate, and n is the number of bidders participating in auction j, σ_{iju} is the error term. $\mathbf{x}_k$ include variables for tract surface, number of trees, number of poles, volumes per species, herfindhal index, sale dummy, order of the tract within the sale, and categorical variables (type of forest, type of cut, grapeshot damage, owner, type of landing area). All continuous variables are in logarithm. We recover the residuals $\log(\hat{b}_{ik}) = \log(b_{ik}) - (\mathbf{x}_k)'\widehat{\delta}$ (for the bidders) and $\log(\hat{y}_k) = \log(\tilde{y}_k) - (\mathbf{x}_k)'\widehat{\delta}$ (for the seller). For the rest of the estimation, the number of bidders is fixed to n. We refer to the residuals $(\hat{b}_{ik}, \hat{y}_k)$ as homogenized bids and estimate respectively.

2. Separate the unobserved heterogeneity component from the bidder-specific component and seller's signal. We use the fact that homogenized bids and estimates obtained from the previous step are multiplicatively separable in the common unobserved component Y.

$$\log(\hat{b}_{ij}) = \log y_j + \log \beta(x_{ij}) \quad \text{and} \quad \log(\hat{y}_j) = \log y_j + \log s_j$$

where $\beta(x_{ij})$ is the idiosyncratic component of bids attributable to variation in bidder's private valuations. The joint characteristic function of an arbitrary bid and ex-ante estimate (in logs) can be estimated as

$$\widehat{\Psi}(t_1, t_2) = \frac{1}{n \times N} \sum_{i,j} \exp(it_1 \log(\hat{y}_j) + it_2 \log(\hat{b}_{ij}))$$

Next, the characteristic functions of the marginal distributions $(\log(Y), \log(S), \log(A))$ can be recovered as

$$\widehat{\Phi}_{\log(Y)}(t) = \exp\left(\int_0^t \frac{\widehat{\Psi}_1(0, u_2)}{\widehat{\Psi}(0, u_2)} du_2 - itE[\log(S)]\right)$$

$$\widehat{\Phi}_{\log(S)}(t) = \frac{\widehat{\Psi}(t, 0)}{\widehat{\Phi}_{\log(Y)}(t)} \quad \text{and} \quad \widehat{\Phi}_{\log(A)}(t) = \frac{\widehat{\Psi}(0, t)}{\widehat{\Phi}_{\log(Y)}(t)}$$

where $\widehat{\Psi}_1$ is the derivative of the joint characteristic function with respect to its first argument. The normalization $E[\log(S)] = 0$ is imposed.

Densities are recovered using the inverse Fourier transform

$$\widehat{f}_{\log(Z)}(z) = \frac{1}{2\pi} \int_{-T}^{T} d(t) \exp(-itz) \widehat{\Phi}_{\log(Z)}(t) dt \tag{3}$$

where $Z \in \{A, Y, S\}$, T is a smoothing parameter, and $d(t)$ is a damping function (for the choice of T and $d(t)$, see the discussion of "Practical issues" below).

Finally, the densities of de-logged variables Z are recovered as

$$\widehat{f}_Z(z) = \frac{\widehat{f}_{\log(Z)}(\log(z))}{z}$$

where $Z \in \{A, Y, S\}$.

3. Estimate the probability of winning conditional on $Y = 1$. From equation (Equation (1)), the probability of winning conditional on $Y = 1$ can be estimated from the unconditional probability of winning and the distribution of Y. The latter has been estimated in the previous step. The unconditional probability is estimated as follows.

   Given a homogenized bid b_i (for simplicity, we drop hats in the expression $\hat{b_i}$)

$$P(D(b_i, \mathbf{B}_{-i}, R_0) = 1 \cap b \geq \mathbf{B}_{-i}) = P(D(b_i, \mathbf{B}_{-i}, R_0) = 1 | b_i \geq \mathbf{B}_{-i}) P(b_i \geq \mathbf{B}_{-i})$$

   The first element on the right hand side is the probability of out-bidding the ex-post secret reserve price, given that all opponents have submitted a lower bid. The second element in the product is the probability of out-bidding all other bidders.

   With n bidders, the probability of submitting the highest bid is estimated as $G_B(b)^{n-1}$, where $G_B(.)$ is the empirical CDF of homogenized bids (obtained after step 1).

   The first probability $P(D(b_i, \mathbf{B}_{-i}, R_0) = 1 | b_i \geq \mathbf{B}_{-i})$ is estimated using the sub-sample formed of the maximum bid in each auction. A non-parametric local regression method is used

$$\widehat{P}(D(b, \mathbf{B}_{-i}, R_0) = 1 | b \geq \mathbf{B}_{-i}) = \frac{\sum_{ik} D_{ik} K(b_{ik} - b)}{\sum_{ik} K(b_{ik} - b)}$$

The unconditional probability of winning is estimated by combining the previous two estimates

$$\widehat{P}(D(b, \mathbf{B}_{-i}, R_0) = \widehat{P}(D(b, \mathbf{B}_{-i}, R_0) = 1 | b \geq \mathbf{B}_{-i}) \widehat{P}(b \geq \mathbf{B}_{-i})$$

This probability satisfies the properties of a cumulative distribution function. Denote by L the corresponding random variable, with c.d.f $F_L(b) \equiv \widehat{P}(D(b, \mathbf{B}_{-i}, R_0)$, and $Q_{\log(L)}(p)$ the quantile function of the random variable $\log(L)$. The characteristic function of $\log(L)$ is estimated as

$$\widehat{\Phi}_{\log(L)}(t) = \int_0^1 \exp(itQ_{\log(L)}(p))dp$$

Next, from $\log(L) = \log(M) + \log(Y)$, the characteristic function of $\log(M)$ is estimated from knowledge of the characteristic functions of $\log(L)$ and $\log(Y)$:

$$\widehat{\Phi}_{\log(M)}(t) = \frac{\widehat{\Phi}_{\log(L)}(t)}{\widehat{\Phi}_{\log(Y)}(t)}$$

The density and cumulative distributions of M are recovered from $\widehat{\Phi}_{\log(M)}(t)$ (by the inversion formula, as in step 2). The estimated cumulative distribution function of M, denoted $\widehat{F}_M(.)$ corresponds to the probability of winning conditional on $Y = 1$.

4. Recover the distribution of idiosyncratic values X_i and equilibrium bid function. Conditional on $Y = 1$, an estimate of the inverse bid function is obtained from the first-order condition

$$x = \widehat{\xi}(b) = b + \frac{\widehat{F}_M(b)}{\widehat{f}_M(b)}$$

Denote by $\widehat{\beta} = \widehat{\xi}^{-1}$ the corresponding estimate of the equilibrium bid function.

Finally, the distribution of private values is estimated by applying the distribution of bids (conditional on $Y = 1$), obtained in step 2, to the equilibrium bid function.

$$\widehat{F}_X(x) = \widehat{G}_A(\beta(x))$$

The estimation procedure is sufficiently quick that confidence intervals can be computed for all inferred values by bootstrap sampling at the auction level.

Practical considerations. A number of practical issues need to be addressed to perform the previous estimation. To implement the inverse Fourier transform (equation (Equation (3))), we use a damping function to control fluctuations in the tail of the characteristic functions. Following Diggle and Hall (1993), we use the function

$$d(t) = \max\left(0, 1 - \frac{|t|}{T}\right)$$

For each random variable in $\{A, Y, S\}$, the smoothing parameter T is chosen to match empirical moments of these variables. We use the first and second moments:

$$\widehat{\mu}_{LS} = 0 \ \ , \widehat{\mu}_{LY} = \overline{\log(\hat{y}_k)} \ \ , \widehat{\mu}_{LA} = \overline{\log(\widehat{b}_{ik})} - \widehat{\mu}_{LY}$$

$$\widehat{\sigma}^2_{LA} = \widehat{\sigma}^2_{LB} - \widehat{\sigma}^2_{LY} \ \ , \widehat{\sigma}^2_{LS} = \widehat{\sigma}^2_{L\widetilde{Y}} - \widehat{\sigma}^2_{LY} \ \ , \widehat{\sigma}^2_{LY} = \frac{\widehat{\sigma}^2_{LB_i} + \widehat{\sigma}^2_{LB_j} - \widehat{\sigma}^2_{LB_i - LB_j}}{2}$$

For each random variable $Z \in \{A, Y, S\}$, T is chosen to minimize $\frac{(\widehat{\mu}_{LZ} - \widetilde{\mu}_{LZ})^2 + (\widehat{\sigma}^2_{LZ} - \widetilde{\sigma}^2_{LZ})^2}{\widehat{\sigma}^2_{LZ}}$. In practice, we obtain values of T for $\{A, Y, S\}$ equal to 11, 8, and 11 respectively.

Density estimates from the procedure in Step 2 suffer from being imprecise in the tails in finite samples. This leads to small positive densities being inferred over a very wide support. This problem is dealt with as follows: the support boundaries of the random variables obtained in step 2 ($[\underline{a}, \overline{a}]$, $[\underline{y}, \overline{y}]$, $[\underline{s}, \overline{s}]$) are estimated by combining the support of variables observed in the data and restrictions imposed by the model. In particular, we use the following restrictions

$$\begin{cases} \log(\widehat{\underline{b}}) = \log(\underline{a}) + \log(\underline{y}) \quad \text{and} \quad \log(\widehat{\overline{b}}) = \log(\overline{a}) + \log(\overline{y}) \\ \log(\widehat{\underline{y}}) = \log(\underline{s}) + \log(\underline{y}) \quad \text{and} \quad \log(\widehat{\overline{y}}) = \log(\overline{s}) + \log(\overline{y}) \\ \max\limits_{i,j,k}\{\log(\widehat{b}_{ik}) - \log(\widehat{b}_{jk})\} = \log(\overline{a}) - \log(\underline{a}) \\ \int_{\log(\underline{s})}^{\log(\overline{s})} y \cdot f_{\log(S)}(y) dy = 0 \quad (E[\log(S)] = 0) \end{cases} \tag{4}$$

where $(\widehat{\underline{b}}, \widehat{\overline{b}})$ and $(\widehat{\underline{y}}, \widehat{\overline{y}})$ are estimates of the support boundaries of homogenized bids and seller's ex-ante estimates. This system of equations uniquely determines the unknown support boundaries $[\underline{a}, \overline{a}]$, $[\underline{y}, \overline{y}]$, and $[\underline{s}, \overline{s}]$ (see Appendix A.2 in Krasnokutskaya (2011)). Estimates of the support boundaries of the normalized bids and ex-ante estimates and the normalization $E[\log(S)] = 0$ allow us to recover these unknowns.

6.3 Estimation results

The results presented below correspond to auctions that attracted three bidders (283 tracts in total). The results for different values of the number of bidders are qualitatively similar.

The densities of unobserved auction heterogeneity, seller's signal, and individual bid component are represented in Figure 5. The distribution of unobserved heterogeneity has a mean of 1.27 and a standard deviation of 0.39. After incorporating observed auction heterogeneity ($\Gamma(\mathbf{x}_k)$), the mean and standard deviation of the common component are equal to $15,801$€ and $26,144$ €, respectively. The recovered distribution for the seller's signal indicates that the seller gets a relatively unbiased estimate of unobserved auction heterogeneity (the mean is 1.03 and the standard deviation equals 0.25).

The variance of bidders' values ($X_i \times Y$) can be decomposed into the variance due to the unobserved auction heterogeneity and the variance due to idiosyncratic private value.

$$\text{Var}(XY) \approx E[X^2]\text{Var}(Y) + E[Y^2]\text{Var}(X)$$

Unobserved auction heterogeneity explains 39% of the variance in bidder values. Failing to control for this unobserved common component would have resulted in over-estimates of the variance of idiosyncratic private values.

The unconditional probability of winning, i.e. the probability of being the highest bidder and outbidding the seller's ex-post secret reserve price, is 35% lower than the probability of being the highest bidder. Figure 6 shows the probability of winning conditional on $Y = 1$ and the distribution of individual private valuation. The estimated bid function is used to compute mark-downs. Bidders shade their bids by 18% on average below their value, and up to 45% for the highest bidder-type.

6.4 Specification tests

The model with unobserved heterogeneity implies a number of testable implications. We perform these specification tests here.

Test against APV models. Both the model with unobserved heterogeneity and the APV model imply correlation in bids. However, the unobserved heterogeneity model implies that bids are conditionally independent, whereas most APV models imply that bids are affiliated (see Kranokutskaya (2011)).

To distinguish the model with unobserved heterogeneity from an APV model, we test for the independence of bid ratios formed from a quadruple of bids submitted in the same auction. Under unobserved heterogeneity, the pairwise ratios should be independent. This property does not hold for a large class of APV models.[9] Figure 8 shows density estimates as well the correlation between pairwise bid ratios. The Pearson test shows a very small and statistically significant correlation of 0.071. We interpret this finding as a strong evidence in favor of the model with unobserved heterogeneity.

7 Counterfactual analysis

In this section, the estimated distributions of values, seller information, observed and unobserved heterogeneity are combined to simulate a set of auctions under counterfactual informational structures and alternative reserve price policies. The outcomes of interest are the expected surplus and revenue per auction.

7.1 Simulation of counterfactuals

First-best outcome. We start by computing surplus and revenue under the assumption that the seller has perfect information about the unobserved auction heterogeneity component Y and announces a public reserve price equal to their true reservation value. Let R_0^{FB} denote the reserve price, when $S = 1$ (no noise in the seller's appraisal value). Under symmetry of bidders' private values, the equilibrium bid function if $Y = 1$ has a simple closed-form expression (first-price auction with a public reserve price)

$$\beta(x) = x - \frac{1}{F_X(x)^{n-1}} \int_{r_0^{FB}}^{x} F_X(u)^{n-1} du$$

This benchmark gives an upper bound on attainable surplus, because the auction is ex-post efficient. Additionally, the benchmark allow us to determine the benefit for the ONF of collecting more precise signal about unobserved tract heterogeneity.

Alternative reserve price policies. We compare revenue and surplus under the current policy to alternative reserve price rules, namely: (a) no reserve price, (b) a public reserve price, (c) secret reserve price, revised based on a convex combination of bids and ex-ante estimate. Policy (c) nests both policies (a) and (b) with a weight on bids of one and zero respectively.

Under symmetry of bidders' private values, the equilibrium bid functions with no reserve and public reserve prices have standard closed-form expressions. [10]

[9] Because the set of affiliated distribution includes the set of conditionally independent distributions, this test has no power against certain APV models, as noted by Kranokutskaya (2011)

[10] Multiplicative separabilty of the bid functions holds because the public reserve price is linear in Y.

Under policy (c), an equilibrium is derived as follows. Let ϕ be a strictly increasing function from $(0, \infty)$ to $(0, \infty)$. We define the following rule setting the ex-post reserve price

$$R_1(\mathbf{b}, R_0; \phi) = \phi(\alpha \widetilde{k} \overline{\mathbf{b}} + (1-\alpha) R_0)$$

where $\widetilde{k}$ is normalization to control for the expected individual bid component, $\overline{\mathbf{b}}$ is the mean bid and $\alpha \in [0, 1]$. The interim expected profit of bidder i is given by

$$\pi(x_i, y; b_i) = (x_i y - b) P(b_i \geq R_1(b_i, \mathbf{B}_{-i}, R_0; \phi) \cap b_i \geq B_j, j \neq i \mid Y = y)$$

A symmetric Bayesian Nash equilibrium is characterized by a function $\beta_y(.)$ such that $\pi(x_i, y; b_i)$ is maximized when $b_i = \beta_y(x_i)$, $b_j = \beta_y(x_j)$ for $j \neq i$, and $\phi = \beta_y$ (for every $i \in \{1, ..., n\}$ and every realization of X_i). An equilibrium is found numerically by best-response iteration.

7.2 Counterfactual results.

Figure 7 compares the equilibrium bid function under the current reserve price policy to equilibrium bid functions under policies (a), (b), and (c). In the latter, the cases with $\alpha = 0$ and $\alpha = 1$ are plotted. These extreme cases provide upper and lower bounds for the equilibrium bid function under intermediate values of α ($\alpha \in (0, 1)$). The left panel shows that the equilibrium bid function under the current policy (secret reserve price, revised down) is close to the bid function with no reserve price, indicating that revisions relax any ex-ante commitment of the seller. The right panel shows that as α goes to 1, bidders bid less aggressively. This is consistent with the fact that as more weight is put on the average bid (in setting the ex-post reserve price), bidders face a lower reserve price. In the limit where $\alpha = 1$, the bid function converges to the case with no reserve price.

Table 8 summarizes the results of the counterfactual analysis. The table records the expected surplus and revenue per auction under the current and counterfactual reserve price policies. Average revenue and surplus per auction under the current policy are 12,605€ and 16,594€. Learning the realization of unobserved auction heterogeneity would allow the seller to increase revenue by 9.76% and surplus by 6.81% (first-best outcome). Interestingly, announcing a public reserve price (equal to the seller's ex-ante secret reserve price) increases revenue by 8.56% and surplus by 3.96%. The fraction of tracts sold is similar across the current policy, public reserve prices, and the first-best.

Figure 9 shows (percentage) change in surplus and revenue relative to the baseline for the counterfactual reserve price policies. In particular, we plot the change in outcomes from the adoption of the ex-post reserve price given by policy (c) for different values of $\alpha \in [0, 1]$.

There is an interior value of the parameter ($\alpha = 0.53$) that maximizes surplus. Under this value, the seller optimally combines information from their ex-ante estimate and the bids (within the class of revision rules given by (c)).

The surplus gap between the public reserve price and the first-best is reduced by 65% by the adoption of a secret reserve price policy with efficient learning ($\alpha = 0.53$). The magnitude of the effects is, however, small: under efficient learning, surplus increases by 1.2% but revenue decreases by 3.4% compared to a public reserve price. The seller trades off greater allocative efficiency against lower revenue per auction.

8 Conclusion

This paper provides a novel rationale for the use of secret reserve prices in auctions. If the seller is less informed than the bidders about the underlying heterogeneity of the auctioned item, she may learn from the bids and adjust her initial appraisal value. Doing so allows the seller to allocate the item more efficiently, albeit at a cost of lower revenue.

The French timber industry provides an empirical setting where this type of information asymmetry between bidders and seller is important. Additionally, the ONF uses a secret reserve price which can be revised down if no bid is above it. We build a model of bidding in first-price auctions that captures these two features and show that the model is identified from data on bids, allocation rule, and the ONF's ex-ante reservation value (i.e., before the bids are submitted).

Using the estimated model, we conduct counterfactual analysis of alternative reserve price policies. The results show that (1) acquiring perfect signals about auction heterogeneity would allow the seller to increase revenue by 9.76% and surplus by 6.81% (first-best); (2) switching from a public to a secret reserve price (with learning) reduces the welfare loss by 65%.

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Tables

Variable	mean	std	min	max	correlation
Surface (ha)	11.34	12.21	0.2	299.0	-0.01
Trees (number)	240.11	204.49	18	2259	0.02
Poles (number)	195.28	529.19	0	11366	-0.05
Herfindahl index	62.89	20.72	20.7	100.0	0.0
Volumes (in m^3)					
Crown	121.5	133.96	0.0	1196.47	-0.02
Stump	0.23	4.73	0.0	153.83	0.01
Stem oak	58.96	101.24	0.0	859.98	0.01
Stem spruce	28.73	80.46	0.0	810.93	0.06
Stem beech	96.68	146.21	0.0	1365.8	-0.01
Stem pine	13.83	60.35	0.0	788.52	0.01
Stem fir	89.42	170.48	0.0	1240.98	0.02
reserve price (in €)	10886.48	10276.99	100	112000	0.02

Table 1: Descriptive statistics for continuous variables. The last column shows the correlation with the seller's decision to revise its ex-ante reserve price down.

Number of bids	0	1	2	3	4-5	6+
Number of tracts	491	511	386	286	326	262
Percentage of tracts	0.22	0.23	0.17	0.13	0.14	0.12
Fraction revised down	nan	0.32	0.51	0.58	0.65	0.7
Avg. log estimate	8.77	8.9	9.07	9.24	9.46	9.72
Avg. log reserve price	8.48	8.66	8.86	9.08	9.32	9.61
Avg. log max. bid	nan	8.48	8.81	9.04	9.36	9.68

Table 6: Descriptive statistics by number of bids submitted

categorial	code	Definition	# tracts	% tracts	correlation
stand					
	F	High forest	1202	53.14	55.63
	C	Conversion of a stand	804	35.54	38.87
	T	Coppice	149	6.59	45.16
	TSF	Coppice with standards	107	4.73	50.0
cut					
	A	Arranged Cutting	1223	54.07	51.57
	R	Regeneration Cutting	757	33.47	46.92
	J	Selection Cutting	165	7.29	41.03
	Other	Other Cutting	71	3.14	54.55
	PA	Accidennal Products	46	2.03	41.67
grapeshot					
	M0	No mitraille	1652	73.03	52.14
	M1	Light mitraille	373	16.49	36.71
	M2	Average mitraille	150	6.63	50.0
	M3	Heavy mitraille	57	2.52	15.38
owner					
	AS	Community-owned forest	1637	72.37	43.63
	DO	State-owned forest	621	27.45	62.89
	PR	Privately-owned forest	4	0.18	nan
land area					
	NArr	Unarranged	1918	84.79	50.8
	A	Arranged	277	12.25	42.65
	N	None	67	2.96	25.81
quality					
	M	Normal	931	41.16	47.93
	B	Good	909	40.19	51.25
	MD	Mediocre	240	10.61	40.66
	TB	Very good	103	4.55	52.46
	MA	Bad	49	2.17	45.45
conditions					
	ADN	Normal logging and extraction	1383	61.14	50.46
	ADF	Easy log. and ext.	492	21.75	44.13
	ADD	Difficult log. and ext.	227	10.04	50.5
	DD	Difficult extraction	69	3.05	56.76
	ADTD	Very Difficult logg. and ext.	63	2.79	43.48

Table 2: Descriptive statistics for categorical variables. The last column shows the correlation with the seller's decision to revise its ex-ante reserve price down.

	Dependent variable:				
	ln mean bid				
	(1)	(2)	(3)	(4)	(5)
appraisal value	0.92*** (0.01)	0.93*** (0.01)	0.85*** (0.01)	0.83*** (0.01)	0.81*** (0.01)
Owner (DO)	0.01 (0.02)	0.01 (0.02)	0.01 (0.02)	−0.01 (0.02)	−0.01 (0.02)
Owner (PR)	0.05 (0.18)	0.10 (0.17)	0.03 (0.17)	0.11 (0.20)	0.08 (0.20)
Herfindahl index	−0.01 (0.02)	0.03 (0.02)	0.16*** (0.04)	0.16*** (0.04)	0.17*** (0.04)
tract order		0.03*** (0.01)	0.03*** (0.01)	0.04*** (0.01)	0.04*** (0.01)
Surface			−0.02 (0.01)	0.01 (0.02)	0.02 (0.02)
Number of trees			0.10*** (0.02)	0.10*** (0.02)	0.11*** (0.02)
Constant	0.42*** (0.12)	0.11 (0.13)	−0.34** (0.16)	−0.28* (0.16)	−0.21 (0.17)
controls	-	session	volume info, session	volume info, session, categorial	volume info, session, categorial, conditions, quality
Observations	1,771	1,771	1,771	1,747	1,740
R^2	0.88	0.89	0.89	0.90	0.90
Adjusted R^2	0.88	0.89	0.89	0.90	0.90
Residual Std. Error	0.31	0.29	0.29	0.28	0.28
F Statistic	3,196.74***	989.23***	548.55***	342.91***	294.08***

Note: Continuous variables are in log — *p<0.1; **p<0.05; ***p<0.01

Table 3: Determinants of the (log) mean bid

	Dependent variable:				
	ln res i				
	(1)	(2)	(3)	(4)	(5)
ln estim	0.99*** (0.01)	1.00*** (0.01)	0.97*** (0.01)	0.95*** (0.01)	0.93*** (0.01)
ownerDO	−0.04*** (0.01)	−0.04*** (0.01)	−0.04*** (0.01)	−0.04*** (0.01)	−0.04*** (0.01)
ownerPR	0.08 (0.13)	0.14 (0.12)	0.13 (0.12)	0.13 (0.14)	0.10 (0.14)
ln herf100	−0.03** (0.02)	0.01 (0.02)	0.02 (0.02)	0.02 (0.02)	0.02 (0.02)
ln lot order		−0.004 (0.01)	−0.003 (0.01)	−0.001 (0.01)	0.0004 (0.01)
ln surf			−0.04*** (0.01)	−0.03*** (0.01)	−0.03*** (0.01)
ln n trees			0.01 (0.02)	0.01 (0.02)	0.01 (0.02)
Constant	0.003 (0.09)	−0.51*** (0.09)	−0.53*** (0.10)	−0.43*** (0.10)	−0.28*** (0.10)
controls	-	session	volume info, session	volume info, session, categorial	volume info, session, categorial, conditions, quality
Observations	2,262	2,262	2,262	2,232	2,223
R^2	0.92	0.94	0.94	0.94	0.94
Adjusted R^2	0.92	0.94	0.94	0.94	0.94
Residual Std. Error	0.26	0.24	0.24	0.24	0.23
F Statistic	6,532.89***	2,326.65***	1,648.64***	881.92***	752.58***

Note: *p<0.1; **p<0.05; ***p<0.01

Table 4: Determinants of (log) tract reserve price

	Dependent variable:				
	n bidders				
	(1)	(2)	(3)	(4)	(5)
ln estim	0.86*** (0.05)	0.87*** (0.05)	1.28*** (0.08)	1.24*** (0.08)	1.02*** (0.09)
ownerDO	0.37*** (0.11)	0.57*** (0.11)	0.74*** (0.11)	0.59*** (0.11)	0.55*** (0.11)
ownerPR	−0.14 (1.12)	0.47 (1.01)	−0.38 (0.98)	−0.82 (1.12)	−0.87 (1.09)
ln herf100	0.05 (0.14)	1.03*** (0.14)	0.82*** (0.21)	0.76*** (0.21)	0.80*** (0.21)
ln lot order		0.13*** (0.05)	0.09** (0.05)	0.11** (0.05)	0.14*** (0.05)
ln surf			−0.44*** (0.08)	−0.39*** (0.09)	−0.36*** (0.09)
ln n trees			−0.17 (0.11)	−0.16 (0.11)	−0.07 (0.11)
Constant	−5.65*** (0.75)	−11.05*** (0.74)	−11.69*** (0.94)	−11.15*** (0.95)	−10.41*** (0.97)
controls	-	session	volume info, session	volume info, session, categorial	volume info, session, categorial, conditions, quality
Observations	2,262	2,262	2,262	2,232	2,223
R^2	0.12	0.30	0.35	0.38	0.41
Adjusted R^2	0.12	0.30	0.34	0.37	0.40
Residual Std. Error	2.24	2.00	1.93	1.90	1.86
F Statistic	76.61***	68.97***	45.04***	31.10***	29.51***

Note: Continuous variables are in log *p<0.1; **p<0.05; ***p<0.01

Table 5: Determinants of bidder participation

	Dependent variable:				
	revise down (dummy)				
	(1)	(2)	(3)	(4)	(5)
distance reserve price to highest bid	−1.49*** (0.09)	−1.43*** (0.10)	−1.43*** (0.10)	−1.45*** (0.10)	−1.47*** (0.10)
number of bidders	0.03*** (0.01)	0.04*** (0.01)	0.04*** (0.01)	0.05*** (0.01)	0.05*** (0.01)
Owner (DO)	0.15*** (0.03)	0.18*** (0.03)	0.19*** (0.03)	0.19*** (0.04)	0.19*** (0.04)
Herfindahl index	−0.07* (0.04)	−0.12*** (0.04)	−0.11** (0.05)	−0.13*** (0.05)	−0.12** (0.05)
lot order		−0.01 (0.01)	−0.004 (0.01)	−0.01 (0.01)	−0.01 (0.01)
surface			−0.003 (0.03)	−0.02 (0.03)	−0.02 (0.03)
number of trees			0.08* (0.05)	0.08 (0.05)	0.07 (0.05)
Constant	0.93*** (0.17)	1.26*** (0.20)	1.51*** (0.24)	1.52*** (0.25)	1.53*** (0.27)
controls	-	session	volume info, session	volume info, session, categorial	volume info, session, categorial, conditions, quality
Observations	1,045	1,045	1,045	1,029	1,025
R^2	0.25	0.29	0.30	0.32	0.33
Adjusted R^2	0.25	0.28	0.28	0.29	0.30
Residual Std. Error	0.43	0.43	0.42	0.42	0.42
F Statistic	87.59***	29.66***	21.55***	12.62***	10.55***
Note: Continuous variables are in log					*p<0.1; **p<0.05; ***p<0.01

Table 7: Linear probability model for the revision decision.

	Baseline	Public Reserve Price	No Reserve Price	Public Reserve No Uncertainty
Revenue per auction (mean, in 2003 €)	12605.3	13703	12702.4	13750.5
95 % CI	[11393,13796.2]	[12400.2,15009.7]	[11097,13762.7]	[12520.5,15050.7]
Percent change in revenue relative to baseline	0	8.56	-1.14	9.76
95 % CI	[0,0]	[6.4,10.5]	[-8.1,-0.8]	[7.8,11.6]
Surplus per auction (mean, in 2003 €)	16594.2	17332.1	17445.9	17640.3
95 % CI	[14145.8,18932.2]	[14871.4,19796.2]	[14966.3,19939.7]	[15207.1,20041.8]
Percent change in surplus relative to baseline	0	3.96	4.46	6.81
95 % CI	[0,0]	[2.8,5.1]	[2.4,6.7]	[5.3,8.8]
Fraction of lots sold (mean)	0.72	0.79	1	0.82
95 % CI	[0.64,0.77]	[0.69,0.84]	[1,1]	[0.71,0.87]

Table 8: Counterfactual outcomes

Figures

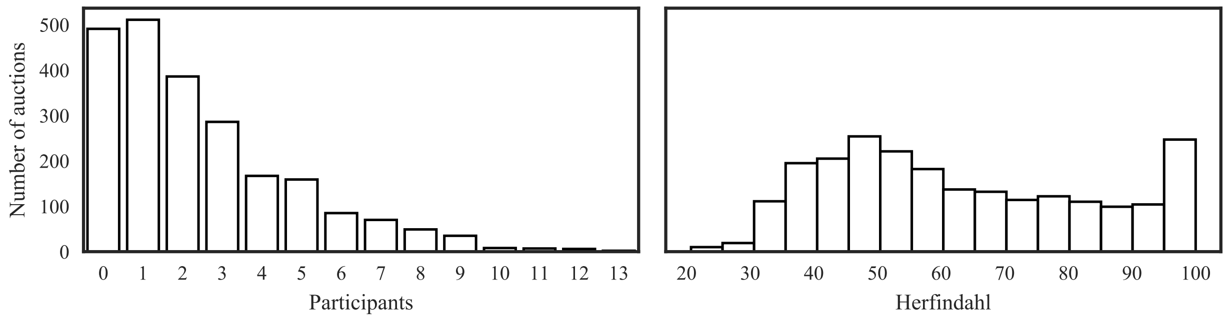

Figure 2: Distribution of number of participants and the Herfindahl index

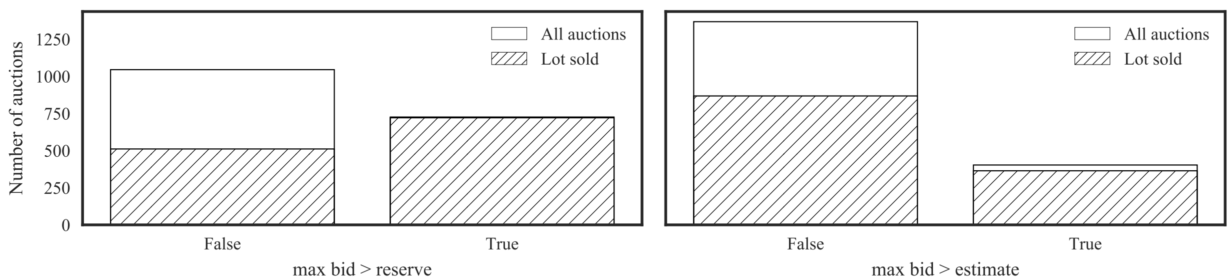

Figure 3: Proportion of tracts sold, depending on whether the maximum bid is above the ex-ante reserve price, or the estimate

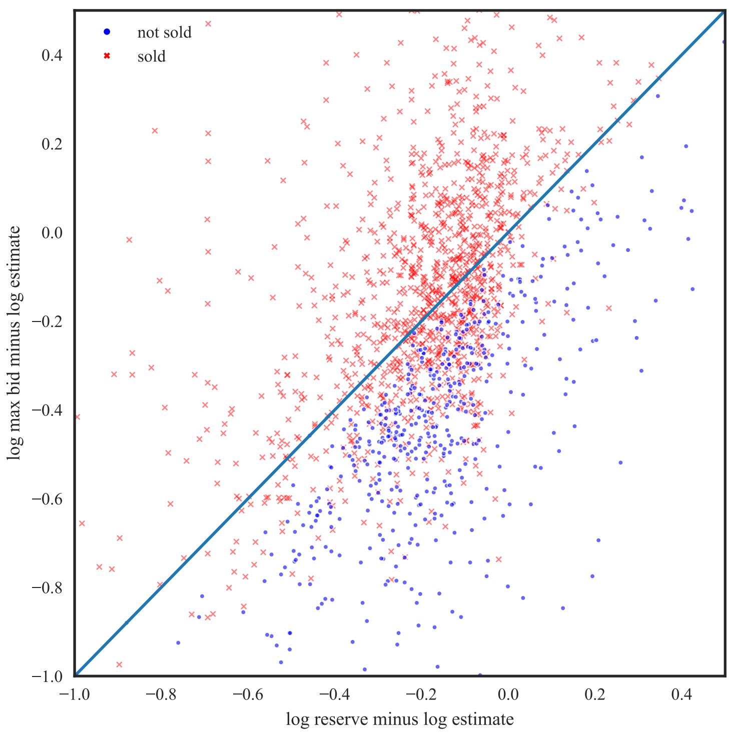

Figure 4: Scatter-plot of tract status (sold in red, unsold in blue) as a function of the (log) ratio of the maximum bid to reserve price and ratio of reserve price to appraisal value (estimate). The 45° line corresponds to tracts where the maximum bid equals the reserve price.

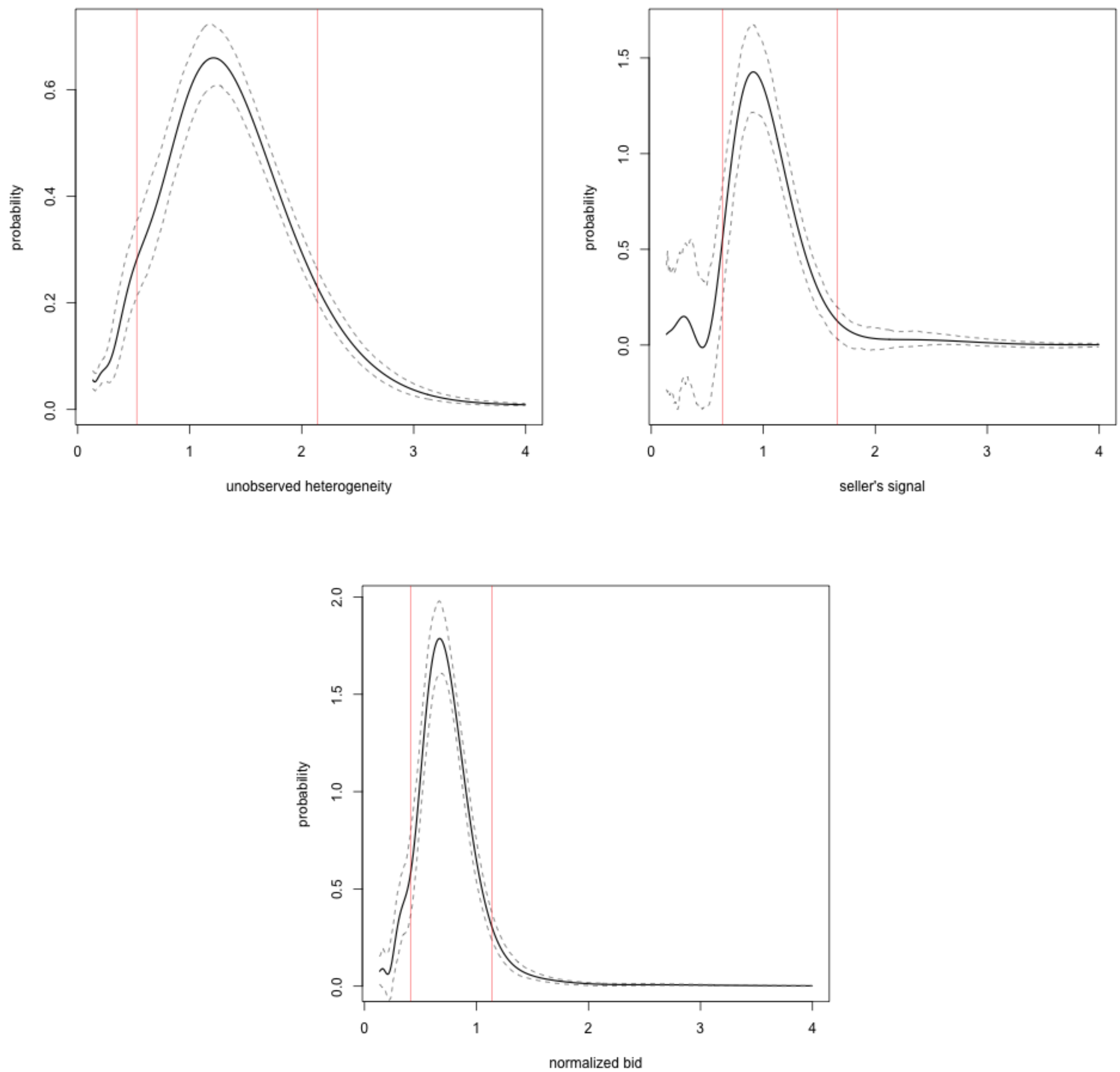

Figure 5: This figure depicts the estimated densities of the unobserved auction heterogeneity component, the seller's ex-ante signal, and the individual bid component. The dotted lines show pointwise 95% confidence intervals estimated through a bootstrap procedure. The red vertical lines represents the estimates of support boundaries.

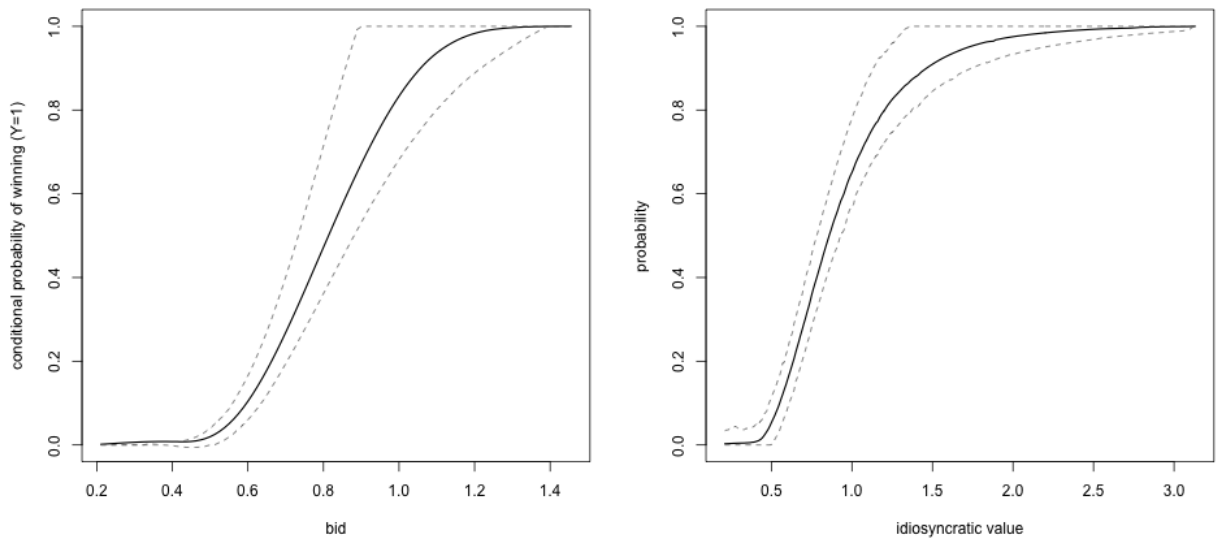

Figure 6: This figure depicts the estimated probability of winning conditional on $Y = 1$ (left) and the cumulative distribution of individual private values. The dotted lines show pointwise 95% confidence intervals estimated through a bootstrap procedure.

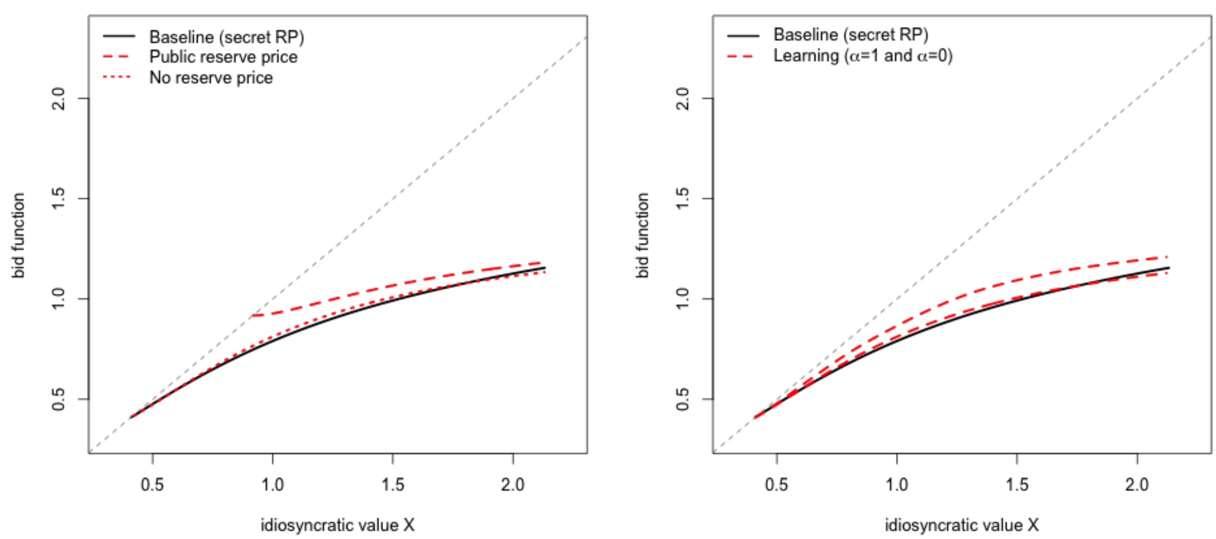

Figure 7: This figure depicts the equilibrium bid functions conditional on $Y = 1$ under the baseline and counterfactual reserve price policies. On the left panel, the public reserve price used is the mean reserve price in the simulated data. On the right panel, the topmost dashed function corresponds to $\alpha = 0$, while the bottom dashed function corresponds to $\alpha = 1$.

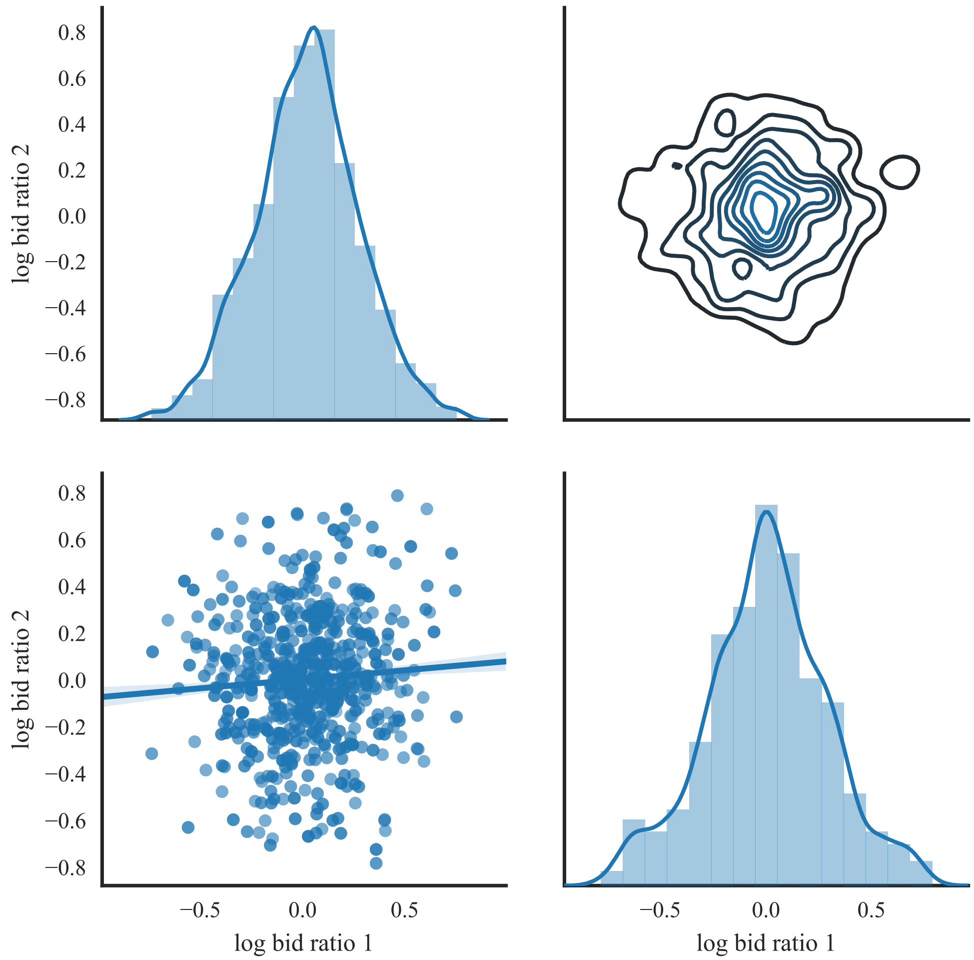

Figure 8: Diagonal figures show histograms of (log) bid ratios formed using a quadruple of bids submitted in the same auction. The bottom left figure depicts a scatter-plot of the two (log) bid ratios, whereas the top right figure depicts the corresponding bi-variate kernel density estimate.

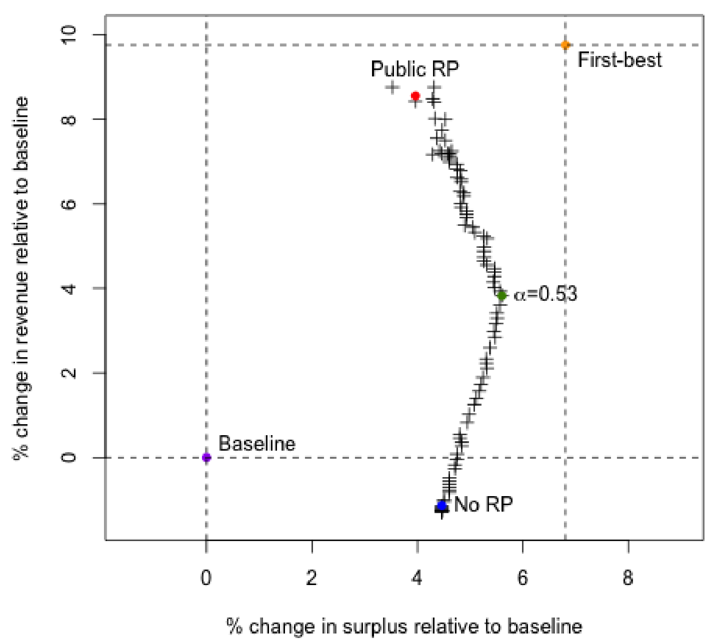

Figure 9: This figure depicts the counterfactual change in expected surplus and revenue per auction from adoption of alternative reserve price policies. The first-best corresponds to the full information case. Crosses corresponds to an ex-post reserve price $R_1(\mathbf{b}, R_0) = \beta(\alpha\overline{\mathbf{b}} + (1-\alpha)R_0)$ where α in $[0, 1]$. As α goes to one, revenue decreases.