Explicit collusion in oligopoly

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Abstract

This paper studies the enforcement of a cartel with private information about production cost under a static setting. We consider the problem of a cartel authority to implement the ex-post efficient production when facing a non-cooperative threat game (either Bertrand or Cournot).

We first show that, to implement an ex-post efficient allocation, paying a minimum ex-ante subsidy forces the individual rationality constraint to be binding at an interior point under Cournot environment and binding at the lowest point under Bertrand environment. When marginal cost is drawn from a uniform distribution and market demand is large, this minimum ex-ante subsidy is higher in a Cournot environment than in a Bertrand environment.

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1 Introduction

It's well-known that oligopolists have incentives to collude on their production outputs in order to achieve higher industry profit. They can explicitly collude by forming a cartel or implicitly collude in a non-cooperative setting. However when they collude, they also have incentives to cheat for their own interest. No matter whether it's explicit collusion or tacit collusion, standard literature focuses on the environment with complete information and applies the solution concept of Nash equilibrium in a static setting or subgame-perfect Nash equilibrium in a dynamic setting.

However, when firms hold private information about their production technology, the enforcement of collusion agreement becomes more subtle. For instance, if the cartel wants to maximize industry profit, the high cost firm requires sidepayments from the low cost firm as an incentive to not producing. The assignment of quantities produced and sidepayments, on the one hand, constitutes the individual rationality constraint for the agreement; on the other hand, it should be designed so that firms truthfully reveal their types.

We apply the mechanism design approach to study the collusion problem of privately informed oligopolists. There is not much literature on this problem. Cramton & Palfrey (1987) assume continous cost types and finite number of firms. They demonstrate that, for a uniform distribution of cost types, there is no incentive compatible and individual rational sidepayment rule that can implement the monopoly outcome for large cartels. Kihlstrom & Vives (1989) incorporate another kind of individual rationality constraint, which is ex-post individual rationality constraint requiring that all firms will choose to remain in the mechanism after learning the costs of all other firms. They show that to implement the monopoly outcome, it may not be possible to devise sidepayments that simultaneously satisfy both interim and ex-post individual rationality constraints. However, they only consider the situation of two firms with two discrete types.

I follow Cramton & Palfrey (1987) in the sense that the support of marginal costs is an interval and sidepayments are allowed. There are two big differences between mine and the work of Cramton & Palfrey (1987). First, when they consider the non-cooperative threat game under Bertrand competition and the corresponding individual rationality, they model a dynamic auction process with complete information to characterize the equilibrium. This is somehow in contrast with the setting of incomplete information. Instead, we maintain the feature of private information under Bertrand environment and characterize some properties of the Bayes-Nash equilirbium. Even though there is no closed-form solution for the Bayes-Nash equilibrium, these properties help us to characterize the optimal collusion under Bertrand environment.

Second, I do not assume any form of budget balance at the beginning. Instead, for any given output allocation rule $\boldsymbol{q}(\boldsymbol{c})$, I define a function $\gamma^I(\boldsymbol{q}(\boldsymbol{c}))$ to be the minimum expected subsidy (or maximum expected tax) that the cartel authority has to pay to (or can extract from) the cartel under the non-cooperative threat game I. We show that, to implement an ex-post efficient allocation, paying a minimum ex-ante subsidy forces the individual rationality constraint to be binding at an interior point under Cournot competition and binding at the lowest point under Bertrand competition. After the construction of function $\gamma^I(\boldsymbol{q}(\boldsymbol{c}))$, we are still able to check whether a specific allocation rule is implementable under ex-ante budget balance constraint by simply checking the sign of $\gamma^I(\boldsymbol{q}(\boldsymbol{c}))$. We identify the sufficient conditions of the cost structure such that an ex-post efficient allocation is implementable or not subject to a weak ex-ante budget balance constraint.

We further compare the minimum ex-ante subsidy required between the Bertrand environment and the Cournot environment. At first glance, the minimum ex-ante subsidy required under the Cournot environment may be higher since in general, compared to Bertrand competition the market is less competitive and firms generate higher expected profit in Cournot competition and thus require more information rent to sustain in collusion. However, this may not be true because Bertrand competition requires the Individual Rationality to be binding at a lower cost type compared to Cournot competition. Since firms' expected profit is decreasing in production cost, it's not clear that this effect will dominate or be dominated by the effect from the competitiveness of the market. We show that under uniform distribution of cost, when the market demand is large the latter effect dominates the former one thus the minimum ex-ante subsidy required is higher in a Cournot environment than in a Bertrand environment.

The structure of the paper is the following: Section 2 introduces the model setup and the conditions (interim IR, interim IC) that a feasible mechanism needs to satisfy. Section 3 discusses in detail different types of competitive threat games, their corresponding IR constraint and their implications for an ex-post efficient mechanism. Section 4 includes the conclusions. An appendix collects the proofs.

2 Model

There are $n \geq 2$ firms producing a homogenous product. Each firm has constant marginal cost and for each i, firm i's marginal cost, c_i, is firm i's private information. It's common knowledge that $c_i, i = 1, 2...n$ are independently and identically drawn from the same distribution $F(\cdot)$ with support $[\underline{c}, \bar{c}]$.

The firms can choose to collude by some explicit agreement. If the agreement is not reached, they will compete in a non-cooperative way under incomplete information. The agreement allows for side payments between all firms. Suppose there is a cartel authority that controls the implementation of the agreement. By the revelation principle, without loss of generality we can think of a direct mechanism $M = \{(q_i(\boldsymbol{c}), p(\boldsymbol{c}), r_i(\boldsymbol{c}))\}_{i=1,\ldots,n}$ where $q_i(\boldsymbol{c})$ is the quantity firm i will produce in the mechanism, $p(\boldsymbol{c})$ is the price for all products and $r_i(\boldsymbol{c})$ is the revenue assigned to firm i.

We first require every mechanism to be consistent with the market demand such that

$$p(\boldsymbol{c}) = P\left(\sum_{i=1}^{n} q_i(\boldsymbol{c})\right) \tag{1}$$

where $P(.)$ is the inverse demand function that is continuously differentiable.

Firms voluntarily participate to the mechanism so an interim individual rationality constraint has to be satisfied. Let's denote $\boldsymbol{c} = (c_1, c_2, ..., c_n)$, $\boldsymbol{c_{-i}} = (c_1, c_2, ...c_{i-1}, c_{i+1}, ..., c_n)$, $\boldsymbol{q}(\boldsymbol{c}) = (q_1(\boldsymbol{c}), q_2(\boldsymbol{c}), ..., q_n(\boldsymbol{c}))$ and denote firm i's profit as $\pi_i(\boldsymbol{c}) = r_i(\boldsymbol{c}) - c_i q_i(\boldsymbol{c})$. Firm i's expected quantity, expected revenue and expected profit in the mechanism are respectively denoted as $Q_i(c_i) = E_{\boldsymbol{c_{-i}}}(q_i(\boldsymbol{c}))$, $R_i(c_i) = E_{\boldsymbol{c_{-i}}}(r_i(\boldsymbol{c}))$ and $\Pi_i(c_i) = E_{\boldsymbol{c_{-i}}}(\pi_i(\boldsymbol{c})) = R_i(c_i) - c_i Q_i(c_i)$. Then a mechanism is interim individual rational (IR) if and only if

$$\Pi_i(c_i) \geq \Pi_i^I(c_i) \quad \forall c_i \in [\underline{c}, \bar{c}] \;\; \forall i \tag{2}$$

where $\Pi_i^I(c_i)$ is the expected profit firm i will obtain in the equilibrium outcome of the competitive game I under private information about costs.

Interim incentive compatibility (IC) requires that

$$\Pi_i(c_i) \geq R_i(c_i') - c_i Q_i(c_i') \quad \forall c_i, c_i' \in [\underline{c}, \bar{c}] \tag{3}$$

Using standard arguments from the Envelope Theorem, interim IC implies that

$Q_i(.)$ is a non-increasing function and

$$\Pi_i(c) = \Pi_i(\bar{c}) + \int_c^{\bar{c}} Q_i(t)dt \tag{4}$$

$$R_i(c) = R_i(\bar{c}) - \int_c^{\bar{c}} tdQ_i(t) \tag{5}$$

A simple implication from (4) and (5) is that if the interim quantity allocation rule and the interim profit or revenue of the highest cost firm are specified, then the interim profit or revenue of each type firm are uniquely determined.

There may be other constraints that a mechanism designer wants to impose, for example, an ex-post or ex-ante, weak or strong budget balance (BB) constraint, if the designer is the cartel authority which doesn't have funding from outside resources to sustain the cartel. Cramton & Palfrey (1987) considers an ex-post strong budget balance constraint which requires that the sum of all firms' profit is equal to the total industry profit from production:

$$\sum_{i=1}^{n} \pi_i(\boldsymbol{c}) = \sum_{i=1}^{n} q_i(\boldsymbol{c})(p(\boldsymbol{c}) - c_i) \quad \forall \boldsymbol{c} \in [\underline{c}, \bar{c}]^n \tag{6}$$

Currently, we are not imposing any form of BB constraint. We are not doing so mainly by two reasons. First, the consequence of this constraint is that once it's imposed, interim profit or revenue for any cost type is uniquely determined if an allocation rule is specified.[1] In this situation, the enforcement of a particular mechanism, for example, an ex-post efficient mechanism, is transformed into a problem of checking whether individual rationality is satisfied or not. However, instead of checking whether a mechanism is implementable subject to a specific budget balance constraint, our paper looks at the problem that in which competition environment a mechanism is more

[1]See Theorem 1 of Cramton & Palfrey (1987).

likely to sustain. As we will see in Section 3, we define the minimum amount of ex-ante subsidy as a function of the allocation rule. Even if there is no restriction from a budget balance constraint, the construction of this function allows us to know where the individual rationality constraint is binding under different competition environment.

Second, without assuming any form of budget balance constraint does not mean we are not able to check any of them after we solve the problem. The construction of the minimum ex-ante subsidy just allows us to check whether a specific allocation rule is implementable or not subject to a weak or strong ex-ante budget balance constraint.

3 Non-cooperative threat game and individual rationality

We assume that the cartel authority can not force any firm to quit the market if that firm chooses not to participate in the collusive mechanism. Instead the break down of the cartel will lead to a (symmetric) non-cooperative threat game played by all firms. Thus the reservation utility/profit for firm of any type may not be normalized to 0 and whether firms can achieve good or bad outcomes in the threat game determines the possibilities of the enforcement of the cartel. Two aspects will determine the outcomes of the threat game: equilibrium selection and competition patterns.

Given a certain environment or competition pattern I (Bertrand or Cournot or any case in between), one issue that arises is that there may exist multiple equilibira which specify different outcomes for any type firm due to incomplete information. Thus it's important to select the equilibrium outcome according to some appropriate criterion. For example, if we want to implement a mechanism that is robust to any specific threat game, we can choose the equilibrium outcome that gives a firm highest expected profit from the equilibria set of that threat game. If all firms are actually pessimistic about the possible outcome they may obtain in a threat game, we can construct the maxmin IR by selecting out the maxmin outcome for each cost type firm.

An easy way to resolve this issue is to maintain the uniqueness of equibrium by adding some assumptions.[2] For now we assume there is a unique equibrium for a specific competitive environment. Then our problem is to study that, under different competition patterns, what's the minimum subsidy the cartel authority has to pay to implement a specific feasible allocation rule or production plan. We define this expected subsidy to be

$$\begin{aligned} \gamma^I(\boldsymbol{q}(\boldsymbol{c})) = & \min_{\pi_1(\boldsymbol{c}),..,\pi_n(\boldsymbol{c})} E_{\boldsymbol{c}}[\sum_{i=1}^n \pi_i(\boldsymbol{c}) - \sum_{i=1}^n q_i(\boldsymbol{c})(p(\boldsymbol{c}) - c_i)] \\ & s.t. \quad (IR) \quad \Pi_i(c_i) \geq \Pi_i^I(c_i) \quad \forall c_i \in [\underline{c}, \bar{c}] \quad \forall i \\ & \qquad (IC) \quad \Pi_i(c_i) \geq R_i(c_i') - c_i Q_i(c_i') \quad \forall c_i, c_i' \in [\underline{c}, \bar{c}] \quad \forall i \end{aligned} \tag{7}$$

First by (4) we have $\Pi_i(c_i) = \Pi_i(\bar{c}) + \int_{c_i}^{\bar{c}} Q_i(t)dt$, and we can rewrite the expression in (7) as

$$\begin{aligned} E_{\boldsymbol{c}}[\sum_{i=1}^n \pi_i(\boldsymbol{c}) - \sum_{i=1}^n q_i(\boldsymbol{c})(p(\boldsymbol{c}) - c_i)] &= E_{\boldsymbol{c}}[-P(\sum_{i=1}^n q_i(\boldsymbol{c})) \sum_{i=1}^n q_i(\boldsymbol{c})] + \sum_{i=1}^n E_{c_i}[\Pi_i(c_i) + c_i Q_i(c_i)] \\ &= E_{\boldsymbol{c}}[-P(\sum_{i=1}^n q_i(\boldsymbol{c})) \sum_{i=1}^n q_i(\boldsymbol{c})] + \sum_{i=1}^n E_{c_i}[\int_{c_i}^{\bar{c}} Q_i(t)dt + c_i Q_i(c_i)] + \sum_{i=1}^n \Pi_i(\bar{c}) \end{aligned} \tag{8}$$

The first and the second term of (8) are determined by $\boldsymbol{q}(\boldsymbol{c})$. Then the minimization of (8) requires that $\sum_{i=1}^n \Pi_i(\bar{c})$ is minimized when IR and IC in (7) are both satisfied. Again, IC implies $\Pi_i(c_i) = \Pi_i(\bar{c}) + \int_{c_i}^{\bar{c}} Q_i(t)dt$ and this means $\Pi_i(c_i) \geq \Pi_i^I(c_i)$ if and

[2] For example, linear specification for demand function will ensure unique equilibrium outcome under Cournot or Bertrand equilibrium.

only if $\Pi_i(\bar{c}) + \int_{c_i}^{\bar{c}} Q_i(t)dt \geq \Pi_i^I(c_i)$. Thus IR in (7) is satisfied if and only if

$$\Pi_i(\bar{c}) \geq \max_{c_i \in [\underline{c}, \bar{c}]} [\Pi_i^I(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt] \tag{9}$$

Thus the minimization of (8) requires that the equality in (9) is satisfied:

$$\Pi_i(\bar{c}) = \max_{c_i \in [\underline{c}, \bar{c}]} [\Pi_i^I(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt] \tag{10}$$

This also means that if $\hat{c}_i = \arg\max_{c_i \in [\underline{c}, \bar{c}]} [\Pi_i^I(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt]$, then $\hat{c}_i$ is a point at which (IR) is binding. If $\Pi_i^I(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt$ have multiple maximum points, then they are all the points at which (IR) is binding and we just need to choose one of them.

We can first analyze (10) by looking at the first order condition. If we denote $Q_i^I(c_i)$ to be the expected production for firm i with cost c_i, by the envelope theorem, the first order derivative of $\Pi_i^I(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt$ is:

$$\frac{d[\Pi_i^I(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i} = Q_i(c_i) - Q_i^I(c_i) \tag{11}$$

However, the first order condition is not sufficient to get the global maximum. Without specifying the function form of $Q_i^I(c_i)$ and $Q_i(c_i)$, it's not clear what the maximized point is or where the interim IR is binding at. To get some insights, we parameterize the problem by the following assumptions.

First, on the demand side, we assume that the inverse demand function is given by

$$P(Q) = \max\{a - Q, 0\} \tag{12}$$

where Q is the total output of the market.[3]

[3] As we illustrated before, linear specification guarantees the uniqueness of the equilibrium of non-cooperative game. It also simplifies our calculation process.

Second, we look at the ex-post efficient mechanism under different competitive environments. Ex-post efficient mechanism is the first-best mechanism that will generate a monopoly outcome in which only the lowest cost firm produces and all higher costs firms produce nothing. In this situation,[4]

$$q_i(\boldsymbol{c}) = \begin{cases} Q^M(c_i) & \text{if } c_i = \min\{c_1, c_2, ..., c_n\} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

and

$$Q_i(c_i) = Q^M(c_i)(1 - F(c_i))^{n-1} \tag{14}$$

where $Q^M(c_i) = \frac{a-c_i}{2}$ is the monopoly output for a monopolist with cost c_i and $(1 - F(c_i))^{n-1}$ is the probability that all other firms have higher costs than c_i.

In addition, technically we assume that $\underline{c} = 0$, $\bar{c} = 1$, and $a > \frac{n+1}{2} - \frac{n-1}{2}\mu$, where $\mu = \int_0^1 cf(c)dc$. The assumption $a > \frac{n+1}{2} - \frac{n-1}{2}\mu$ guarantees that under both Cournot and Bertrand environment, even the highest cost ($c = \bar{c} = 1$) firm will produce positive output in the equilibrium. This assumption simplifies our exposition but will not affect our conclusion.

Given these assumptions, we are able to charaterize the Bayes-Nash equilibrium and the optimal collusion mechanism under different competitive environments. For simplicity we look at two common cases: Cournot competiton ($I = C$) and Bertrand competiton ($I = B$).

[4]Here we ignore the possibility that there are more than one firm having the lowest cost, since our continous type setting let this situation happen with probability zero.

3.1 Cournot environment

Lemma 1 gives the characterization of the expected quantity and expected profit under Cournot competiton.

Lemma 1. *Under Cournot competition, given linear demand there exists a unique (symmetric) equilibrium. The expected quantity and the expected profit of firm i with cost c_i are respectively given by*

$$Q_i^C(c_i) = \frac{2a + (n-1)\mu}{2(n+1)} - \frac{c_i}{2} \tag{15}$$

$$\Pi_i^C(c_i) = \left(\frac{2a + (n-1)\mu}{2(n+1)} - \frac{c_i}{2}\right)^2 \tag{16}$$

According to Lemma 1, we have the following proposition that indicates where the interim IR is possibly binding under Cournot competition.

Proposition 1. *Under Cournot environment and linear demand, to implement an ex-post efficient allocation rule, if the cartel authority pays a minimum amount of ex-ante subsidy, then the interim IR is binding at an interior point $\hat{c}_i$ where $\hat{c}_i$ is a solution to the equation $\frac{2a+(n-1)\mu}{2(n+1)} - \frac{c_i}{2} = \frac{(a-c_i)(1-F(c_i))^{n-1}}{2}$.*

Proposition 1 does not tell the exact position at which the interim IR is binding since the equation in Proposition 1 may have multiple solutions. To capture more precise insights, we can further assume that the cost structure is uniformly distributed, i.e., $F(c_i) = c_i \ \forall c_i \in [0, 1]$. In this case, the equation in Proposition 1 has a unique solution and we have the following proposition.

Proposition 2. *Under Cournot environment, linear demand and uniform distribution of cost structure, to implement an ex-post efficient allocation rule, the cartel authority at least needs to pay an ex-ante subsidy such that $\Pi_i(c_i) = \Pi_i^C(\hat{c}_i) + \int_{c_1}^{\hat{c}_i} Q_i(t)dt$, where*

$\hat{c}_i$ *is the point that the interim IR is binding at and it's the unique solution to the equation* $\frac{(a-c_i)(1-c_i)^{n-1}}{2} = \frac{4a+n-1}{4(n+1)} - \frac{c_i}{2}$.

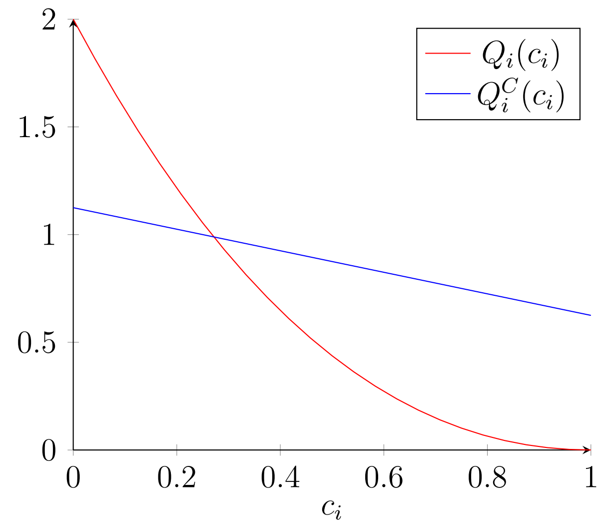

Figure 1: $n = 3, a = 4$ and $F(c_i) = c_i$

Figure 1 shows the situation in which $n = 3, a = 4$. [5] It shows that there exists a unique $\hat{c}_i \in (0, 1)$ such that $Q_i^C(\hat{c}_i) = Q_i(\hat{c}_i)$ and $Q_i^C(c_i) < Q_i(c_i)$ if $c_i < \hat{c}_i$ and $Q_i^C(c_i) < Q_i(c_i)$ if $c_i > \hat{c}_i$. This implies that $\frac{d[\Pi_i^C(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i} > 0$ for $c_i \in [0, \hat{c}_i)$ and $\frac{d[\Pi_i^C(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i} < 0$ for $c_i \in (\hat{c}_i, 1]$. Thus we have $\arg\max_{c_i \in [\underline{c}, \bar{c}]} [\Pi_i^C(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt] = \hat{c}_i$.

3.2 Bertrand environment

Under Bertrand competition environment with private information, in general we can't obtain a closed-form solution for the Bayes-Nash equilibrium. However, Lemma 2 and Lemma 3 provide properties that help us to characterize the optimal collusion mechanism.[6]

[5]Our conclusions still hold If a and n take other values.

[6]Lemma 2 and Lemma 3 are actually modified versions of Proposition 1 and Proposition 2 from Spulber (1995). In Spulber (1995), all conclusions hold for differentiable decreasing demand and

Lemma 2. *Each firm's equilibrium pricing strategy is always less than its monopoly price, i.e.,* $p_i^B(c_i) < p^M(c_i)$ $\forall c_i \in [0,1]$ $\forall i$.

Lemma 3. *Under Bertrand competition, given linear demand there exists a unique (symmetric) equilibrium pricing strategy* $p_i^B(.)$ *obtained solving the differential equation (17) and boundary equation (18) below*

$$p_i^{B\prime}(c_i) = (n-1)\frac{f(c_i)}{1-F(c_i)}\frac{(p_i^B(c_i)-c_i)(a-p_i^B(c_i))}{a+c_i-2p_i^B(c_i)} \tag{17}$$

$$p_i^B(1) = 1 \tag{18}$$

The expected quantity and the expected profit of firm i *with cost* c_i *are respectively given by*

$$Q_i^B(c_i) = (a-p_i^B(c_i))(1-F(c_i))^{n-1} \tag{19}$$

$$\Pi_i^B(c_i) = (p_i^B(c_i)-c_i)(a-p_i^B(c_i))(1-F(c_i))^{n-1} = \int_{c_i}^{1}(a-p_i^B(t))(1-F(t))^{n-1}dt \tag{20}$$

According to Lemma 2, we are able to know where the interim IR is binding at under Bertrand competition.

Proposition 3. *Under Bertrand environment and linear demand, to implement an ex-post efficient allocation rule, the cartel authority at least needs to pay an ex-ante subsidy such that* $\Pi_i(c_i) = \Pi_i^B(0) - \int_0^{c_i} Q_i(t)dt$*, where* 0 *(the lowest cost type) is the point at which the IR is binding.*

Figure 2 shows that $Q_i^B(c_i) > Q_i(c_i)$ $\forall c_i \in [0,1)$. Thus $\frac{d[\Pi_i^B(c_i)-\int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i} < 0$ $\forall c_i \in [0,1)$ and $\arg\max_{c_i\in[\underline{c},\bar{c}]}[\Pi_i^B(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt] = \underline{c} = 0$.

weakly convex cost function. Our specification of linear demand and constant marginal cost obviously satisfies the requirement.

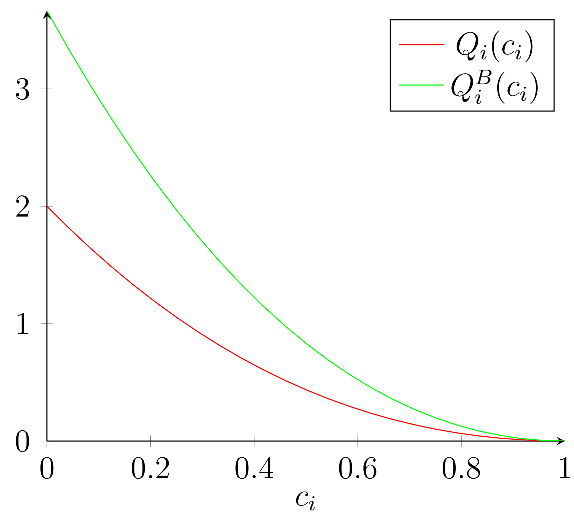

Figure 2: $n = 3, a = 4$ and $F(c_i) = c_i$

3.3 Comparison between Cournot and Bertrand environment

One observation from Proposition 1 and Proposition 3 is that Bertrand environment requires the interim IR to be binding at a lower cost type compared to Cournot environment. This observation actually makes the following question more interesting: Does Cournot competition environment require more subsidy to sustain the collusion than Bertrand competition envirionment? At the first glance, this may be true since in general, compared to Bertrand competition the market is less competitive and firms generate higher expected profit in Cournot competition and thus require more information rent to sustain in collusion. However, the observation above indicates that this may not be true. Since firms' expected profit is decreasing in production cost, the IR binding at a lower type may lead the collusion to require more subsidy in Bertrand competition environment than Cournot competition environment. This effect is due to the interaction by the Incentive Compatibility constraint and the Individual Rationality Constraint.

We denote $\Pi_{i,B}(c_i)$ and $\Pi_{i,C}(c_i)$ respectively as the solution of program (7), i.e., the

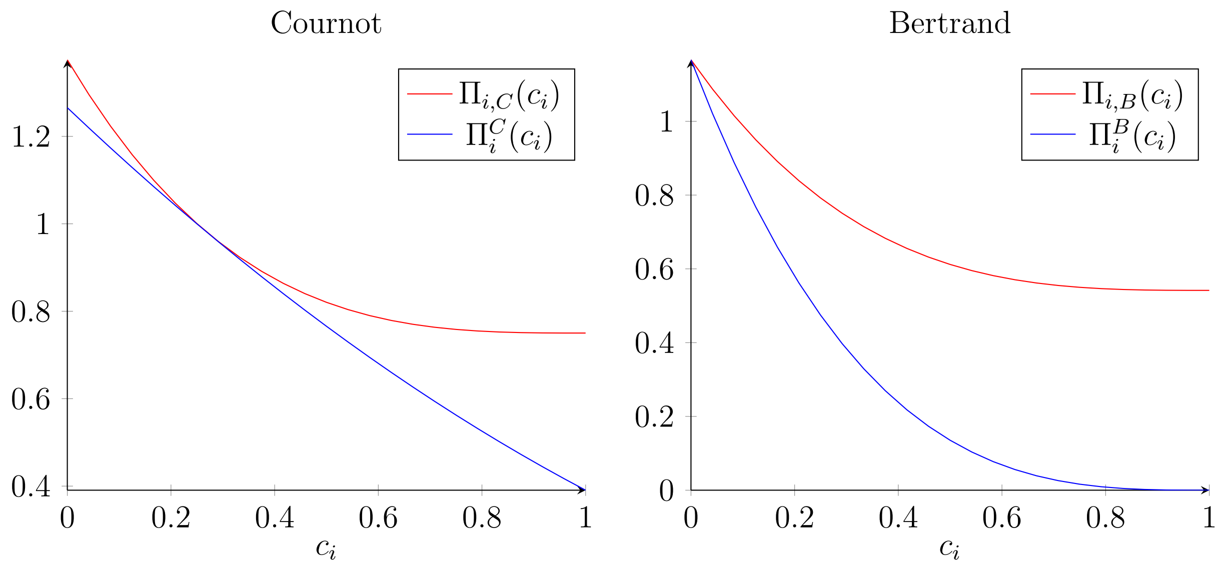

Figure 3: $n = 3, a = 4$ and $F(c_i) = c_i$

expected profit for a firm with cost c_i within the collusive mechanism under Bertrand environment and Cournot environment. Figure 3 illustrates a possible comparison between the expected profit with the mechanism and the expected profit without the mechanism under Cournot or Bertrand environment. It shows the binding position for Cournot environment is an interior type and the binding position for Bertrand environment is the lowest type.

Figure 3 also shows a situation in which Cournot competition requires more subsidy than Bertrand competition to sustain the collusion. However, this conclusion does not necessarily hold for all different values of n and a. To analyze this, we just need to compare $\Pi_{i,B}(c_i)$ and $\Pi_{i,C}(c_i)$ for a specific type.[7] We can choose $c_i = \hat{c}_i$ (the point where the interim IR is binding under Counot competiton) and decompose the

[7] Since the same allocation rule is implemented in both environments, to compare the sustainability of the cartel in Bertrand and in Cournot (thus to compare $\gamma^B(\boldsymbol{q}(\boldsymbol{c}))$ and $\gamma^C(\boldsymbol{q}(\boldsymbol{c})))$ is essential to compare $\Pi_{i,B}(c_i)$ and $\Pi_{i,C}(c_i)$.

difference between $\Pi_{i,B}(\hat{c}_i)$ and $\Pi_{i,C}(\hat{c}_i)$ as follows:

$$\begin{aligned}
\Pi_{i,B}(\hat{c}_i) - \Pi_{i,C}(\hat{c}_i) &= \Pi_{i,B}(0) - \int_0^{\hat{c}_i} Q_i(t)dt - \Pi_{i,C}(\hat{c}_i) \\
&= \Pi_i^B(0) - \int_0^{\hat{c}_i} Q_i(t)dt - \Pi_i^C(\hat{c}_i) \\
&= [\Pi_i^B(\hat{c}_i) - \Pi_i^C(\hat{c}_i)] + [\Pi_i^B(0) - \Pi_i^B(\hat{c}_i) - \int_0^{\hat{c}_i} Q_i(t)dt]
\end{aligned}$$

The first term $\Pi_i^B(\hat{c}_i) - \Pi_i^C(\hat{c}_i)$ captures the difference of the expected profit of a specific type firm between Cournot competition and Bertrand competition without the mechanism. It's in general negative since the market is more competitive under the Bertrand environment compared to Counrnot environment. The second term $\Pi_i^B(0) - \Pi_i^B(\hat{c}_i) - \int_0^{\hat{c}_i} Q_i(t)dt$ captures the interaction effect of IR and IC due to different binding point under Bertrand environment and under Cournot environment. This term is positive, since by Lemma 2,

$$\begin{aligned}
&\Pi_i^B(0) - \Pi_i^B(\hat{c}_i) - \int_0^{\hat{c}_i} Q_i(t)dt \\
=& \int_0^{\hat{c}_i} (a - p_i^B(t))(1 - F(t))^{n-1}dt - \int_0^{\hat{c}_i} (a - p^M(t))(1 - F(t))^{n-1}dt \\
=& \int_0^{\hat{c}_i} (p^M(t) - p_i^B(t))(1 - F(t))^{n-1}dt > 0
\end{aligned}$$

But it's not clear that this effect will dominate or be dominated by the effect of the competitiveness of the market.

Two aspects make it difficult to get an exact conclusion of the comparision. First, for a general distribution of cost structure, it's not clear at which position the interim IR is exactly binding under Cournot competition. This is because the solutions of the equation in Proposition 1 only give the local minimum or local maximum instead of the global maximum of the expression $\Pi_i^C(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt$. Thus we can't obtain a closed-form expression for the expected profit $\Pi_i^C(c_i)$. Second, even under the assumption of a specific distribution for cost structure (for example, a uniform distribution), the closed-form solution for the Bayes-Nash equilibrium under Bertrand environment can't be obtained.

Even though the problem is restricted by the reasons above, we are able to identify a sufficient condition of the parameters for the required ex-ante subsidy under Cournot competition to be higher than under Bertrand competition. This is given by the following proposition.

Proposition 4. *Given linear demand, uniform distribution of cost structure and a condition of the parameters* $a > \frac{9n+11}{4}$*, if the cartel authority implements an ex-post efficient allocation rule and pays a minimum ex-ante subsidy, then*

(1) $\Pi_{i,B}(c_i) < \Pi_{i,C}(c_i)\ \forall c_i \in [0,1]$*;*

(2) $\gamma^B(\boldsymbol{q}(\boldsymbol{c})) < \gamma^C(\boldsymbol{q}(\boldsymbol{c}))$*.*

Proposition 4 indicates that given uniform distribution of cost structure, Cournot competition requires more subsidy than Bertrand competition to sustain collusion if the market demand is large enough (represented by $a > \frac{9n+11}{4}$). This is mainly because when the market demand is large, the effect of the competitiveness of the market is dominating the interaction effect between the IR constraint and the IC constraint.

A sufficient conditon for the opposite to be true is not easy to find, but an example with discrete cost types can be constructed as follows. Suppose $n = 2$, $a = 2$ and $c_i \in \{0.5, 1\}$ with $Pr(c_i = 0.5) = 0.5$ for $i = 1, 2$. In the equilibirum of Cournot competition with incomplete information, firm $i's$ expected output and expected profit when it has a cost equal to 0.5 are respectively $Q_i^C(0.5) = \frac{13}{24}$ and $\Pi_i^C(0.5) = \frac{169}{576}$. If the firm has a cost equal to 1, its expected output and expected profit are $Q_i^C(1) = \frac{7}{24}$ and $\Pi_i^C(1) = \frac{49}{576}$. In the equilibirum of Bertrand competition with incomplete information, firm $i's$ pricing strategy and expected profit when it has a cost 0.5 are respectively

$p_i^B(0.5) = \frac{5}{4}$ and $\Pi_i^B(0.5) = \frac{27}{64}$. If the firm has a cost equal to 1, its pricing strategy and expected profit are $p_i^B(1) = \frac{3}{2}$ and $\Pi_i^B(1) = \frac{1}{16}$. Now if the two firms collude to implement the ex-post efficient allocation, it is easy to verify that, in order to pay minimum ex-ante subsidy, the individual rationality constraint is binding at the cost type equal to 1 under Cournot environment and binding at the cost type equal to 0.5 under Bertrand environment. It turns out that the expected profit of a firm with cost equal to 1 within the cartel under Cournot environment and Bertrand environment are repectively $\Pi_{i,C}(1) = \frac{49}{576}$ and $\Pi_{i,B}(1) = \frac{23}{64}$. This shows that although $\Pi_i^C(1) > \Pi_i^B(1)$, which means that a firm with the cost equal to 1 has more expected profit under Cournot competiiton than under Bertrand competition without the collusive mechanism, we have $\Pi_{i,C}(1) < \Pi_{i,B}(1)$ indicating that it is actually harder to enforce this firm to stay in the cartel under Bertrand environment rather than under Cournot environment.

3.4 (Weak) ex-ante budget balance

Up to now we didn't assume any form of budget balance constraint, but the construction of $\gamma^I(\boldsymbol{q}(\boldsymbol{c}))$ allows us to check whether a specific allocation rule $\boldsymbol{q}(\boldsymbol{c})$ is implementable or not subject to an ex-ante budget balance constraint. This is done simply by checking the sign of $\gamma^I(\boldsymbol{q}(\boldsymbol{c}))$. The following proposition identifies a sufficient condition of the cost structure that an ex-post efficient allocation rule is implementable under Bertrand competition environment.

Proposition 5. *Given linear demand, an ex-post efficient allocation is implementable under Bertrand competition environment subject to interim IR, interim IC and (weak) ex-ante BB if* $(a - c)f(c) \geq 2 + 2F(c)\ \forall c \in [0, 1]$.

We are also able to identify a sufficient condition of the cost structure such that an ex-post efficient allocation rule is not implementable under Bertrand competition environment.

Proposition 6. *Given linear demand, an ex-post efficient allocation is not implementable under Bertrand competition environment subject to interim IR, interim IC and (weak) ex-ante BB if* $(a-c)f(c) \leq 2F(c)\ \forall c \in [0,1]$.

4 Conclusions

We consider a cartel authority who maximizes industry profit when facing a non-cooperative threat game. We show that, to implement an ex-post efficient allocation, paying a minimum ex-ante subsidy forces the Individual rationality constraint to be binding at an interior point under Cournot competition and binding at the lowest point under Bertrand competition. We further show that under uniform cost structure, when market demand is large enough, this minimum ex-ante subsidy is higher in a Cournot environment than in a Bertrand environment.

Appendix

Proof of Lemma 1.

We denote $E_{c_i}(Q_i^C(c_i)) = \bar{Q}_i\ \forall i$. With cost c_i, the firm i's problem is

$$\max_{q_i} E_{c_{-i}}(a - \sum_{i=1}^{n} q_i - c_i)q_i = (a - c_i - \sum_{j\neq i} \bar{Q}_j - q_i)q_i$$

The first order condition gives $Q_i^C(c_i) = \frac{a-c_i-\sum_{j\neq i}\bar{Q}_j}{2}$. Thus $\bar{Q}_i = E_{c_i}(Q_i^C(c_i)) = \frac{a-\mu-\sum_{j\neq i}\bar{Q}_j}{2}$ and we have $\sum \bar{Q}_j + \bar{Q}_i = a - \mu$. This indicates that $\bar{Q}_i = \bar{Q}_j\ \forall i \neq j$ and $\bar{Q}_i = \frac{a-\mu}{n+1}$. Pluging back, we get $Q_i^C(c_i) = \frac{2a+(n-1)\mu}{2(n+1)} - \frac{c_i}{2}$ and $\Pi_i^C(c_i) = (\frac{2a+(n-1)\mu}{2(n+1)} - \frac{c_i}{2})^2$.

Proof of Proposition 1.

Let's denote $D(c_i) = \frac{(a-c_i)(1-F(c_i))^{n-1}}{2} - [\frac{2a+(n-1)\mu}{2(n+1)} - \frac{c_i}{2}]$. Then $D(0) = \frac{(n-1)(a-\mu)}{2(n+1)} > 0$ and $D(1) = \frac{\frac{n+1}{2} - \frac{n-1}{2}\mu - a}{n+1} < 0$. This implies that $Q_i(0) > Q_i^C(0)$ and $Q_i(1) < Q_i^C(1)$, which also means that $\frac{d[\Pi_i^C(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i}|_{c_i=0} > 0$ and $\frac{d[\Pi_i^C(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i}|_{c_i=1} < 0$. Thus $c_i = \underline{c} = 0$ and $c_i = \bar{c} = 1$ can't be the point that the interim IR is binding at. Then the interim IR is binding at an interior point $\hat{c}_i$ and $\hat{c}_i$ is a solution of the first order condition, i.e, $\frac{2a+(n-1)\mu}{2(n+1)} - \frac{c_i}{2} = \frac{(a-c_i)(1-F(c_i))^{n-1}}{2}$.

Proof of Proposition 2.

The proof is the same with Proposition 1 except that we need to verify that under Cournot environment, there exist a unique $\hat{c}_i \in (0,1)$ such that $Q_i^C(\hat{c}_i) = Q_i(\hat{c}_i)$, i.e., $\frac{(a-c_i)(1-c_i)^{n-1}}{2} = \frac{4a+n-1}{4(n+1)} - \frac{c_i}{2}$ has a unique solution in $(0,1)$. Denote $D(c_i) = \frac{(a-c_i)(1-c_i)^{n-1}}{2} - [\frac{4a+n-1}{4(n+1)} - \frac{c_i}{2}]$. Then $D(0) = \frac{(2a-1)(n-1)}{4(n+1)} > 0$ and we have $D'(c_i) = \frac{1-(1-c_i)^{n-2}[(n-1)(a-c_i)+(1-c_i)]}{2} < \frac{1-n(1-c_i)^{n-1}}{2} \leqslant 0$ for $c_i \leqslant 1 - \frac{1}{n^{\frac{1}{n-1}}}$. Since $D(1 - \frac{1}{n^{\frac{1}{n-1}}}) = \frac{n-1}{2n}[1 - \frac{1}{n^{\frac{1}{n-1}}} - \frac{2a+n}{2(n+1)}] < \frac{n-1}{2n}[1 - \frac{1}{n^{\frac{1}{n-1}}} - \frac{3}{4}] < 0$. Then by the Intermediate Value theorem , there is a unique $\hat{c}_i \in (0,1)$ such that $\frac{(a-\hat{c}_i)(1-\hat{c}_i)^{n-1}}{2} = \frac{4a+n-1}{4(n+1)} - \frac{\hat{c}_i}{2}$. Thus $\hat{c}_i$ is the point that the IR is binding at and $\Pi_i(c_i) = \Pi_i^C(\hat{c}_i) + \int_{c_1}^{\hat{c}_i} Q_i(t)dt$.

Proof of Lemma 2.

Under Bertrand competition, the problem for firm i with cost c_i is

$$\max_{p_i} \ (p_i - c_i)(a - p_i)Pr(\max\{p_1(c_1), ..., p_{i-1}(c_{i-1}), p_{i+1}(c_{i+1}), ..., p_n(c_n)\} > p_i)$$

where $(p_i - c_i)(a - p_i)$ is firm i's profit conditional on winning the market and $Pr(\max\{p_1(c_1), ..., p_{i-1}(c_{i-1}), p_{i+1}(c_{i+1}), ..., p_n(c_n)\} > p_i)$ is firm i's probability of winning the market.

First, we can get that $p_i^B(c_i) \leq p^M(c_i)$ $\forall c_i \in [0, 1]$ $\forall i$ since pricing above monopoly price not only reduces the profit conditional on winning the market but also reduces the probability of winning. Then it remains to check that there does not exist a $c_i \in [0, 1]$ such that $p_i^B(c_i) = p^M(c_i)$. Suppose such a c_i exists. We denote $M_i(p_i, c_i) = (p_i - c_i)(a - p_i)$ and $N(p_i) = Pr(\max\{p_1(c_1), ..., p_{i-1}(c_{i-1}), p_{i+1}(c_{i+1}), ..., p_n(c_n)\} > p_i)$, then firm i's expected profit is $M_i(p_i, c_i)N(p_i)$ and it's first order derivative is

$$M_i(p_i, c_i)\frac{dN(p_i)}{dp_i} + \frac{\partial M_i(p_i, c_i)}{\partial p_i}N(p_i)$$

When $p_i^B(c_i) = p^M(c_i)$, this is negative since $\frac{dN(p_i)}{dp_i} < 0$ and $\frac{\partial M_i(p_i,c_i)}{\partial p_i}|_{p_i=p^M(c_i)} = 0$. Thus $p_i^B(c_i) < p^M(c_i)$ $\forall c_i \in [0, 1]$ $\forall i$.

Proof of Lemma 3.

Since each firm's cost structure is symmetric and has a bounded support, the symmetric equilibrium is the unique equilibrium (see Spulber(1995) p.4). We can denote each firm's pricing strategy as $p^B(.)$ and we assume that $p^B(.)$ is nondecreasing and differentiable. Then the problem for a firm i with cost c_i under Bertrand competition environment is

$$\max_{p_i} \ (p_i - c_i)(a - p_i)[1 - F(p^{B-1}(p_i))]^{n-1}$$

The first order condition gives

$$(a+c_i-2p_i)[1-F(p^{B-1}(p_i))]^{n-1}+(p_i-c_i)(a-p_i)(n-1)[1-F(p^{B-1}(p_i))]^{n-2}(-f(p^{B-1}(p_i))\frac{dp^{B-1}(p_i)}{dp_i}=0$$

This is satisfied when $p_i = p^B(c_i)$ and we rearrange it to get

$$p_i^{B\prime}(c_i) = (n-1)\frac{f(c_i)}{1-F(c_i)}\frac{(p_i^B(c_i)-c_i)(a-p_i^B(c_i))}{a+c_i-2p_i^B(c_i)}$$

We can verify that $p^B(.)$ is indeed nondecreasing. This is because $c_i \le p_i^B(c_i) \le p^M(c_i) = \frac{a+c_i}{2}$ by Lemma 2 and we have $p_i^{B\prime}(c_i) \ge 0$.

For the boundary condition, first we have $p_i^B(1) \ge 1$ since $p_i^B(1) < 1$ will generate negative expected profit. Suppose $p_i^B(1) = \hat{p} > 1$, then all firms charging at $\hat{p}$ will get expected profit of 0. But if a firm deviates to charge a price $\bar{p}$ with $1 < \bar{p} < \hat{p}$. Then this firm will get positive expected profit. Thus the only equilirbium strategy when $c_i = 1$ is $p_i^B(1) = 1$.

Proof of Proposition 3.

From Lemma 3 we have $p_i^B(c_i) < p^M(c_i)\ \forall c_i \in [0,1]$. Thus $Q_i^B(c_i) > Q_i(c_i)\ \forall c_i \in [0,1)$ and $\frac{d[\Pi_i^B(c_i)-\int_{c_i}^{\bar{c}} Q_i(t)dt]}{dc_i} < 0\ \forall c_i \in [0,1)$. Then $\arg\max_{c_i \in [\underline{c},\bar{c}]}[\Pi_i^B(c_i) - \int_{c_i}^{\bar{c}} Q_i(t)dt] = \underline{c} = 0$ and 0 (the lowest cost type) is the point that the IR is binding at.

Proof of Proposition 4.

First, (1) is equivalent to (2) since $\boldsymbol{q}(\boldsymbol{c})$ is fixed. Thus we just need to prove (1). What's more, it's sufficient to prove that $\Pi_{i,B}(\hat{c}_i) < \Pi_{i,C}(\hat{c}_i)$ where $\hat{c}_i$ is the point at which the interim IR is binding under Cournot environment.

Under Cournot competition, we have $\Pi_{i,C}(\hat{c}_i) = \Pi_i^C(\hat{c}_i) = (\frac{4a+n-1}{4(n+1)} - \frac{\hat{c}_i}{2})^2 = (\frac{4a+n-1}{4(n+1)} - \frac{\hat{c}_i}{2})(\frac{(a-\hat{c}_i)(1-F(\hat{c}_i))^n}{2})$, where $\frac{4a+n-1}{4(n+1)} - \frac{\hat{c}_i}{2} = \frac{(a-\hat{c}_i)(1-F(\hat{c}_i))^n}{2}$. According to Proposition 3, under Bertrand competition we have $\Pi_{i,B}(\hat{c}_i) = \int_{\hat{c}_i}^1 (a - p_i^B(t))(1-F(t))^{n-1}dt$. Then, to prove $\Pi_{i,B}(\hat{c}_i) < \Pi_{i,C}(\hat{c}_i)$ is to prove

$$(\frac{4a+n-1}{4(n+1)} - \frac{\hat{c}_i}{2})(\frac{(a-\hat{c}_i)(1-F(\hat{c}_i))^n}{2}) > \int_{\hat{c}_i}^{1} (a-p_i^B(t))(1-F(t))^{n-1}dt$$

If $a > \frac{9n+11}{4}$, then $(\frac{4a+n-1}{4(n+1)} - \frac{\hat{c}_i}{2}) > 2 \;\; \forall c_i \in [0,1]$. Under uniform distribution of cost structure, we have $\Pi_{i,C}(\hat{c}_i) > (a-\hat{c}_i)(1-\hat{c}_i)^n$. Since $p_i^B(.)$ is increasing and $p_i^B(\hat{c}_i) > \hat{c}_i$, we have $\Pi_{i,B}(\hat{c}_i) = \int_{\hat{c}_i}^1 (a-p_i^B(t))(1-t)^{n-1}dt < \int_{\hat{c}_i}^1 (a-\hat{c}_i)(1-\hat{c}_i)^{n-1}dt = (a-\hat{c}_i)(1-\hat{c}_i)^n$. Thus $\Pi_{i,B}(\hat{c}_i) < \Pi_{i,C}(\hat{c}_i)$.

Proof of Proposition 5.

Under Bertrand environment, according to the definition of $\gamma^B(\boldsymbol{q}(\boldsymbol{c}))$ and Proposition 3, we have

$$\begin{aligned}
\gamma^B(\boldsymbol{q}(\boldsymbol{c})) &= E_{\boldsymbol{c}}[\sum_{i=1}^n \pi_i(\boldsymbol{c}) - \sum_{i=1}^n q_i(\boldsymbol{c})(p(\boldsymbol{c}) - c_i)] \\
&= \sum_{i=1}^n E_{c_i}[\Pi_i(c_i) + c_i Q_i(c_i)] - E_{\boldsymbol{c}}[\sum_{i=1}^n q_i(\boldsymbol{c})p(\boldsymbol{c})] \\
&= \sum_{i=1}^n E_{c_i}[\Pi_i^B(0) - \int_0^{c_i} Q_i(dt)dt + c_i Q_i(c_i)] - E_{\boldsymbol{c}}[\sum_{i=1}^n q_i(\boldsymbol{c})p(\boldsymbol{c})] \\
&= \sum_{i=1}^n [\int_0^1 (a-p_i^B(t))(1-F(t))^{n-1}dt - \int_0^1\int_0^{c_i} \frac{(a-t)(1-F(t))^{n-1}}{2} f(c_i)dtdc_i \\
&+ \int_0^1 \frac{t(a-t)(1-F(t))^{n-1}}{2} f(t)dt] - \int_0^1 \frac{a^2-t^2}{4} n(1-F(t))^{n-1} f(t)dt \\
&< n\int_0^1 \frac{a-t}{4}[2+2F(t)-(a-t)f(t)](1-F(t))^{n-1}dt
\end{aligned}$$

The last inequality comes from the fact that $p_i^B(t) > t \; \forall t \in [0,1)$. Thus a sufficient condition for $\gamma^B(\boldsymbol{q}(\boldsymbol{c})) \leq 0$ is $2+2F(t)-(a-t)f(t) \leq 0 \; \forall t \in [0,1]$, which is $(a-t)f(t) \geq 2+2F(t) \; \forall t \in [0,1]$.

Proof of Proposition 6.

$$\gamma^B(\boldsymbol{q}(\boldsymbol{c})) = \sum_{i=1}^{n}[\int_0^1 (a - p_i^B(t))(1 - F(t))^{n-1}dt - \int_0^1 \int_0^{c_i} \frac{(a-t)(1-F(t))^{n-1}}{2} f(c_i)dtdc_i$$
$$+ \int_0^1 \frac{t(a-t)(1-F(t))^{n-1}}{2} f(t)dt] - \int_0^1 \frac{a^2 - t^2}{4} n(1-F(t))^{n-1} f(t)dt$$
$$> n \int_0^1 \frac{a-t}{4}[2F(t) - (a-t)f(t)](1-F(t))^{n-1}dt$$

The last inequality comes from the fact that $p_i^B(t) < p^M(t) = \frac{a+t}{2}\ \forall t \in [0, 1)$. Thus a sufficient condition for $\gamma^B(\boldsymbol{q}(\boldsymbol{c})) \geq 0$ is $2F(t) - (a-t)f(t) \geq 0\ \forall t \in [0, 1]$, which is $(a-t)f(t) \leq 2F(t)\ \forall t \in [0, 1]$.