The welfare effects of category captaincy*

Marie-Laure Allain†and Clémence Christin‡

Preliminary and incomplete

Abstract

We investigate the welfare consequences of the choice of a category captain by a retailer, who can select one of his two suppliers to provide services that may boost demand (*vertical practice*). The welfare effects of the vertical practice depend on the timing of the decisions. When the tariff offers are made after the choice of the category captain and once the effort is exerted, a category captain improves the retailer's profit, consumer surplus and welfare, but reduces the rival's profit. The retailer may however choose the less efficient supplier to enjoy a larger share of a smaller joint profit, thus failing to maximize welfare. When, by contrast, the effort is chosen after the tariff stage, then tariffs are inefficient, consumers are harmed, and welfare may decrease. We extend these results when the effort exerts spillovers on demand for the rival product (horizontal practice).

Jel Codes: L13, L41, L42.

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†CREST, CNRS, Institut Polytechnique de Paris. ENSAE, 5, avenue Henry Le Chatelier - 91764 Palaiseau 91128 Palaiseau, France; allain@ensae.fr.

‡CREM, Université de Caen Normandie, Esplanade de la Paix, 14 000 Caen; clemence.christin@unicaen.fr.

1 Introduction

Category management is a set of methods used by retailers to organize entire categories of products, that is, groups of interrelated products that can be substitutes or complements, as independent business units. Since the 1990s, these practices have become a crucial aspect of the retailers' strategies (see for instance Gajanan et al., 2007). Category management can be either implemented by the retailer itself, or delegated to a "category captain", defined by the United Kingdom's Competition Commission as "a supplier invited by a retailer to provide advice, research and make recommendations to optimize how that retailer stocks and sells all products (including competing products) within a particular grocery category" (see UK Competition Commission, 2005). Typically, the category captain analyzes consumer data to optimize the sales, shares information on market trends and consumer shopping behavior,[1] and gives advice on how to present the products (e.g. which kind of complementary goods to display in adjacent shelves[2]) and at what time of the year to introduce innovations or organize promotional policies. A category captain is thus involved in decisions that affect not only its products, but also those of its competitors.

The information and advice provided by category captains may bring efficiency benefits, but such arrangements are also a growing concern for competition authorities. The French Competition Authority, for instance, points out that the status of category captains is "an informal process marked by a significant lack of transparency" (see Autorité de la Concurrence, 2010). Overall, Competition authorities mainly mention two risks associated to category captaincy: First, the category captain can encourage concerted practices both between retailers and

[1]For example, Mars and Wrigley (a company owned by Mars) have been awarded in 2016 by the professional press for having "conducted extensive global research to learn more about shopper behavior and experience to develop customized recommendations and maximize cross-category sales at checkout. One national grocery retailer following Wrigley's recommendations experienced a doubledigit dollar sales lift, with strong growth in unit sales, household penetration and basket size."(Progressive Grocer, 2016)

[2]For instance, in the ready-to-eat cereals market, Kellogg Co. advised retailers to reorganize the display of the products in the category. As a result, "retailers that implemented Target Age Flow saw base sales performance five points higher than retailers that set the aisle by manufacturer, according to Kellogg "(Progressive Grocer, 2017).

between suppliers.[3] Second, exclusionary effects have also been identified: the Conwood vs. US Tobacco case provides an extreme example of a category captain recommending to suppress a competitor's products from the shelves.[4] More generally, as noticed by Leary (2004), "the idea that a manufacturer would provide advice about the pricing and promotion of competitive brands [. . .] set off every antitrust alarm." However, there is no *per se* regulation on category captaincy. In the US, the FTC only recommends guidelines to minimize the potential for anticompetitive conduct (see FTC 2001).

Competition authorities generally agree to recommend a close monitoring of decisions by a category captain that may directly affect its competitors' products. Leary (2004) rephrases this in terms of vertical and horizontal practices: "advice on the resale of the manufacturer's own product should be viewed as vertical; advice on the resale of a competitor's product should be viewed as horizontal." The standard view is then that horizontal practices should be subject to caution, while vertical practices are deemed innocuous. In this paper, we challenge this view by analyzing the indirect effects of category captaincy, focusing on pure vertical decisions by the category captain. We analyze the effect of category captaincy on buyer power and on profit-sharing in the vertical structure. To do so, we design a framework, in which the category captain makes no decision about prices or assortment of its competitors' products, thereby ruling out antitrust concerns raised by horizontal practices. Furthermore, we assume away sources of vertical inefficiencies (we consider an efficient common agency framework – see *e.g.*, Bernheim and Winston, 1986– and allow for two-part tariffs) and do not consider any informational motivation for the choice of category captain (there is no asymmetric information in the vertical structure). Yet, we show that category captaincy can still lead to inefficiencies.

We consider a framework in which two suppliers produce differentiated products sold through a monopolistic retailer; the products are asymmetric (for the same price, one product is preferred by more than half of the consumers). A supplier chosen by the retailer as category captain can then exert an effort that increases

[3] See, *e.g.*, Desrochers et al. (2003), for instance, who assemble findings and commentaries from a round table organized on this issue by the Journal of Public Policy and Marketing with the American Antitrust Institute.

[4] See Conwood Co. v. United States Tobacco Co., 290 F. 3d 768 (6th Cir. 2002).

consumers' valuation for its own product. In that framework, we show that the welfare effects of category captaincy depend crucially on the timing of decisions. In the baseline setting, the retailer first selects the captain, who chooses its effort before take-it-or-leave-it two-part tariffs are offered by the suppliers. Choosing a category captain then increases welfare, because the tariffs and effort are efficient; however, category captaincy always harms the rival manufacturer and may lead to the exclusion of the rival product. In addition, the retailer may choose the less efficient supplier for category captain, so as to benefit from an increased buyer power, and to secure "a larger share of a smaller pie"(see, *e.g.*, Montez, 2007, or Chambolle and Villas Boa, 2015). This may lead to the exclusion of one product motivated by buyer power considerations. By contrast, if the effort is chosen after the tariffs, category captaincy may then be detrimental not only to consumers, but also to total welfare. Indeed, tariffs must then be inefficient to allow for efficient efforts. Convertly, when the effort decision is made before the choice of the category captain, then the retailer selects the less efficient supplier for category captain, which has negative consequences for welfare. Finally, we discuss the robustness of our results when we assume that the effort exerted by the category captain creates a positive or negative spillover on its rival's demand.

This paper contributes to the growing literature on category management. A stream of literature analyzes the effects of the retailer delegating pricing decisions to the category captain (see for instance Kurtulus and Toktay, 2009, and Nijs et al., 2013). Other papers consider the category captain as the provider of some kind of effort that boosts demand for a product line. We focus on the latter issue. In a closely related paper, Subramanian et al. (2010) analyze a retailer's decision to select category captain(s) among two suppliers. They model the category captain as the provider of a two-dimensional effort, involving horizontal effects: part of the effort is "category-enhancing" (it increases demand for the whole product line, including the competitors' products, and can thus be welfare improving), while the other part is "share-shifting" (it increases the captain's market share at the expense of its competitors). They show that the category captain selects a mix of these two efforts, and that the category-enhancing effort is larger when the retailer selects a single category captain rather than two. By contrast, we assume that the category captain is not able to directly hurt its rivals, and focus on a purely vertical practice.

This paper also contributes to the literature on vertical restraints. Since the seminal papers of Matthewson and Winter (1984) and Rey and Tirole (1986), there has been a large literature on the impact of vertical relations and vertical contracts on the provision of sales services when these services are provided by retailers (see, for instance, Martimort and Pouyet, 2017, or Hunold and Muthers, 2017 for recent contributions). Little is known however about the impact of vertical structure on the provision of sales services by the supplier. This paper fills a gap and shows how such an effort can affect buyer power (see *e.g.*, Chipty and Snyder, 1999, Inderst and Wey, 2003 and 2007, and Inderst and Mazzarotto, 2006, for a survey) and the efficiency of a vertical structure.

The general model is presented in section 2. In section 3, we characterize the equilibrium tariffs and analyze the impact of a category captain on joint profit and on the sharing of profits. In section 4, we first consider a benchmark in which the category captain can fully appropriate its own effort. This enables us to exhibit the main mechanisms at stake in the retailer's choice, in a simple and general framework. In section 5 we check the robustness of our results to changes in the timing of decisions, and develop two important extensions, namely the case in which the retailer has some bargaining power, and the case in which the effort of the category captain benefits its rival. Section 6 concludes.

2 The model

We consider a monopolist retailer R selling one line of products. Two manufacturers, U_A and U_B, produce differentiated goods in this line of products. Without loss of generality, the production costs are normalized to 0. We assume that the manufacturers make take-it-or-leave-it offers to the retailer in the form of two-part tariff contracts $\{w_K, F_K\}$, where w_K is the unit price of product K ($K \in \{A, B\}$) and F_i is a fixed fee paid by the retailer to the supplier.

When the two goods are sold, the inverse demand functions are given by $P_K(q_K, q_L, e, i)$, where $\{K, L\} = \{A, B\}$, i is the identity of the category captain ($i \in \{A, B\}$) and e the effort it exerts (see below). We assume that the two goods are differentiated, which implies

$$\frac{\partial P_K}{\partial q_K} < \frac{\partial P_K}{\partial q_L} < 0,$$

We assume asymmetric horizontal differentiation: Consumers have a higher valuation for the first unit of good A than for good B. In other words, if they are sold at the same price, more consumers will buy good A than good B. More precisely, we assume that, for any nonnegative (q_k, q_l),

$$\begin{aligned} P_A(q_k, q_l, e, A) &> P_B(q_k, q_l, e, B) \\ P_A(q_k, q_l, e, B) &> P_B(q_k, q_l, e, A). \end{aligned}$$

The retailer can choose a category captain who exerts an effort that benefits the whole line through a potential increase in demand. Here, effort is a black box that may represent multi-dimensional activity by the category captain, including consumer data processing, marketing effort, monitoring of the stores, strategic consulting, etc. This effort e entails a quadratic cost $C(e) = e^2$; it increases the inverse demand for the category captain's product ($P_K(q_K, q_L, e, K)$, increases in e), and may increase or decrease the inverse demand for the rival product ($P_K(q_K, q_L, e, L)$, may increase or decrease in e). We assume that the identity of the category captain does not affect demand if no effort is made, that M_K's demand is affected by the effort more if M_K is the category captain rather than if his rival is, and finally that the effort is more efficient for a the product that is preferred by a larger share of consumers: for all $X \in \{A, B\}$, and $\{K, L\} = \{A, B\}$,

$$\begin{aligned} P_X(q_K, q_L, 0, A) &= P_X(q_K, q_L, 0, B) \\ \frac{\partial P_K}{\partial e}(.,., e, K) &\geq \left| \frac{\partial P_K}{\partial e}(.,., e, L) \right| \\ \frac{\partial P_A}{\partial e}(q_K, q_L, e, A) &> \frac{\partial P_B}{\partial e}(q_K, q_L, e, B). \end{aligned}$$

We use the following notations.

$$\pi = (P_A(q_A, q_B, e, i) - w_A)q_A + (P_B(q_B, q_A, e, i) - w_B)q_B$$

is the retailer's profit, gross of the fixed fees, when the two products are sold, and we denote by π_{KL} the second-order derivatives of this profit function with respect to q_K and q_L (that is, $\pi_{KL} = \frac{d^2\pi}{dq_K dq_L}$ for $\{K, L\} \in \{A, B\}$). To ensure concavity of the retailer's profit, we assume that π_{AA} and π_{BB} are negative, and that $\Delta = \pi_{AA}\pi_{BB} - \pi_{AB}\pi_{BA} > 0$. We also assume that $\pi_{AB} < 0$.[5]

To ensure that in equilibrium, the quantity produced by each producer is larger when he is a category captain than when his competitor is, we make the following assumption:

- **Assumption 1** $\frac{\partial^2 P_K}{\partial q_K \partial q_L} \leq 0$ for all $\{K, L\} \subset \{A, B\}$.

We will use the following assumptions when necessary:

- **Assumption 2** $\frac{\partial^2 P_K}{\partial q_K \partial e} = 0$ for $K \in \{A, B\}$.
- **Assumption 3** $\frac{\partial^2 P_K}{\partial q_L \partial e} \leq 0$ for $\{K, L\} = \{A, B\}$.

In particular, these assumptions ensure that the quantity produced by the category captain is larger when U_A is category captain than when U_B is, and that the quantity produced by the competitor is larger when U_B is category captain than when U_A is (Proofs in Appendix TBC).

Assumptions 1, 2 and 3 are satisfied for a wide range of demand functions. In particular, they are satisfied by the following linear specification, which we will use in what follows when needed to focus on some particular features of category captaincy. In this specification, t denotes the substitutability between the two goods, X is the comparative advantage of product A, and ρ captures the spillover

[5]This assumption ensures that the equilibrium quantity of each product will increase in the other product's unit wholesale price (see section 3.1), and that, everything else being equal, the optimal quantity q_K chosen by the retailer decreases in q_L.

effect:[6]

$$
\begin{aligned}
P_A(q_A, q_B, e, A) &= X(1+e) - q_A - tq_B \\
P_B(q_A, q_B, e, A) &= 1 + \rho e - q_B - tq_A \\
P_A(q_A, q_B, e, B) &= X(1+\rho e) - q_A - tq_B \\
P_B(q_A, q_B, e, B) &= 1 + e - q_B - tq_A.
\end{aligned}
$$

The timing of the game is as follows:

1. The retailer chooses its category captain.
2. The two manufacturers U_K, $K \in \{A, B\}$ (whether a CC or not) simultaneously make take-it-or-leave-it offers to the retailer.[7] The category captain U_K offers a contract $\{w_K, F_K, e\}$ that specifies a two-part tariff and a level of effort, while the rival supplier offers a two-part tariff contracts $\{w_L, F_L\}$. The retailer then accepts or rejects the offers.[8] If the retailer rejects the offer of a supplier, he does not sell the products of this supplier. If the retailer rejects the offer of the category captain, the latter does not exert any effort.
3. Finally, the retailer sets its quantities.

In Section 3 we derive our main equilibrium analysis in this general setup. In section 4 we focus on the welfare analysis, assuming away spillovers. In Section 5 we discuss the robustness of our results to changes in the baseline setup: We first discuss the assumption that the category captain is not involved in the pricing decisions; we then discuss the robustness of our results to changes in the timing of the game and in the bargaining assumptions; finally, we further investigate the effects of category captaincy in the presence of spillovers (in particular regarding product eviction and profit sharing).

[6]The utility functions behind this linear specification are the following. When U_A is category captain, we have $U^A = X(1+e)q_A + (1+\rho e)q_B - \frac{q_A^2+q_B^2+2tq_Aq_B}{2}$; when U_B is category captain, we have $U^B = X(1+\rho e)q_A + (1+e)q_B - \frac{q_A^2+q_B^2+2tq_Aq_B}{2}$.

[7]Note that the observability of the offers is irrelevant in the baseline framework, as only the retailer makes decisions after the tariff stage.

[8]This modeling choice implies that we do not consider complement goods, as the outcome of tariff negotiations would be degenerate in our framework (the retailer would have zero profit irrespective of the identity of the category captain).

3 Equilibrium analysis

In this section, we solve the game by backward induction. For each subgame associated to a category captain, we derive the downstream equilibrium quantities, the equilibrium tariffs and effort, and we compare the firms' profits.

3.1 Equilibrium tariffs

In this section, we assume that in Stage 1, U_i ($i \in \{A, B\}$) has been chosen as category captain We solve the last two stages of the game in order to characterize the equilibrium tariffs.

Stage 3: Retailer orders quantities. Assume that in Stage 2, the retailer has accepted both contracts. It then sets quantities to maximize its profit: $\max_{q_A, q_B} \pi$. First order conditions are then:

$$\frac{d\pi}{dq_A} = \frac{\partial P_A}{\partial q_A}(q_A, q_B, e, i)q_A + P_A(q_A, q_B, e, i) - w_A + \frac{\partial P_B}{\partial q_A}(q_A, q_B, e, i)q_B = 0,$$
$$\frac{d\pi}{dq_B} = \frac{\partial P_A}{\partial q_B}(q_B, q_A, e, i)q_A + \frac{\partial P_B}{\partial q_B}(q_B, q_A, e, i)q_B + P_B(q_B, q_A, e, i) - w_B = 0.$$

The system of first order conditions yields the continuation equilibrium quantities, denoted $\tilde{q}_A(w_A, w_B, e, i)$ and $\tilde{q}_B(w_A, w_B, e, i)$.

Totally differentiating the retailer's first order conditions, we point out that

$$\frac{d\tilde{q}_K}{dw_K} = \frac{\pi_{LL}}{\Delta} < 0, \qquad \frac{d\tilde{q}_L}{dw_K} = -\frac{\pi_{KL}}{\Delta} > 0. \tag{1}$$

Note that, in some cases, the retailer may choose to sell only one product, excluding the other product from the market.

If, by contrast, the retailer has accepted only the offer of producer U_K ($K \in \{A, B\}$), or if he wishes to sell product K only, then he chooses the quantities q_A so as to maximize his profit $(P_K^M(q_K, e, i) - w_K)q_K$, where $P_K^M(q_K, e, i)$ is the restriction of the inverse demand function for product K when product L is not on the market.[9] We denote by $q_K^M(w_K, e, i)$ the optimal quantity in that case.

[9]We assume that the profit function of the retailer is then concave: $2\frac{\partial P_K^M}{\partial q_K} + q_K \frac{\partial^2 P_K^M}{\partial q_K^2} < 0$.

3.1.1 Stage 2: Equilibrium contracts

Consider now the offers of the suppliers in Stage 2. Without loss of generality, we assume that the unit wholesale prices are "reasonable" in the sense that the monopoly quantities must be positive.

We first show that contracts are efficient.

Lemma 1 *In equilibrium,* $w_A = w_B = 0$ *and the category captain chooses the effort that maximizes the total industry profit.*

Proof. See Appendix A ■

In equilibrium, tariffs are efficient and the profits of U_A and U_B consist of the fixed fees. This result is standard in a common agency framework (see for instance Bernheim and Whinston, 1998, Rey and Vergé, 2010, or O'Brien and Shaffer, 1997).

From now on we will denote the subgame equilibrium quantities, for a given effort e and a given category captain U_A or U_B as follows: $\tilde{q}_A(e, A)$ and $\tilde{q}_B(e, B)$. Under Assumptions 1, 2 and 3, comparing the equilibrium quantities produced by the category captain and by its competitor when U_A is the CC and when it is U_B, and we show that (the proof is available upon request):

$$\tilde{q}_A(e, A) \geq \tilde{q}_B(e, B) \tag{2}$$

$$\tilde{q}_A(e, B) \geq \tilde{q}_B(e, A) \tag{3}$$

The quantity sold by the category captain is larger when U_A is the category captain than when it is U_B, and similarly, the quantity produced by the competitor is also larger when U_A is the category captain.

3.2 Profit sharing

Finally, we consider how the existence of a category captain affects the way the joint profit of the industry is shared between the three firms.

Proposition 1 *The category captain captures the additional profit created by the effort it exerts.*

Proof. Assume that K is the category captain, with $\{K, L\} = \{A, B\}$. We have shown that in equilibrium the variable prices w_K, w_L are zero. Hence the retailer receives the joint profit and pays each supplier a fixed fee that is determined by his own participation constraint. We know that, once a category captain is chosen, he exerts the effort that maximizes the joint profit, while in stage 3 the retailer also chooses the quantities that maximize the joint profit. Let us denote π^i the joint profit in the subgame in which U_i ($i \in \{A, B, \emptyset\}$) is the category captain ($i = \emptyset$ meaning that there is no category captain), $\pi^{sqK,i}$ the joint profit when negotiation with the supplier U_K ($K \in \{A, B\}$) fails and U_i is category captain (or there is no category captain if $i = \emptyset$).

When U_K is the category captain, the retailers's profit is $\pi_R^K = \pi^K - F_K - F_L$, where the two fixed fees are determined by the retailer's participation constraints:

$$\pi_K^K = F_K^K = \pi^K - \pi^{sqK,K}, \qquad \pi_L^K = F_L^K = \pi^K - \pi^{sqL,K}.$$

By contrast when there is no category captain, the retailers's profit is $\pi_R^\emptyset = \pi^\emptyset - F_K^\emptyset - F_L^\emptyset$, where the two fixed fees are determined by the retailer's participation constraints:

$$\pi_K^\emptyset = F_K^\emptyset = \pi^\emptyset - \pi^{sqK,\emptyset}, \qquad \pi_L^\emptyset = F_L^\emptyset = \pi^\emptyset - \pi^{sqL,\emptyset}.$$

Hence, since the status-quo profit when the negotiation with U_K fails is the same when U_K is category captain and when there is no category captain ($\pi^{sqK,K} = \pi^{sqK,\emptyset}$), we find that the sum of the retailer's and supplier L's profits is the same when K is category captain and when there is no category captain:

$$\pi_R^K + \pi_L^K = \pi_R^\emptyset + \pi_L^\emptyset = \pi^{sqK,K}.$$

■

Hence the category captain is able to capture all of the additional profit created by his effort. This result comes from the assumption that, when the negotiation with the category captain K fails, he does not exert any effort, and the situation for the two other firms is thus similar to the failure of the negotiation with K when there is no category captain. The joint profit of the retailer and one of his supplier is thus unchanged if the other supplier becomes a category captain, as compared to the situation where there is no category captain.

Corollary 1 *In equilibrium, the profit shared between the retailer and the captain's rival is larger in the subgame in which U_B is the category captain than in the subgame in which it is U_A.*

This impacts the choice of category captain: for the retailer, the profit left by the category captain to share with the rival is larger when he choses U_B than when he choses U_A. However, the final choice will also depend on the share of the profit that is captured by the retailer, and this depends on the bargaining power of the rival too.

Furthermore, proposition 1 implies that, in any situation where supplier L benefits from K being a category captain, the retailer loses from this choice. This means that the retailer could be worse off with K as category captain than with no category captain. However, as the retailer is free to select a category captain or not, he will choose one only if this improves his profit, thereby reducing the other supplier's profit.

Corollary 2 *The choice of a category captain is always detrimental to his competitor.*

Finally, the retailer will be better off (and supplier L worse off) if K is category captain than if there is no category captain if and only if the following condition is satisfied:

$$\pi^{sqL,K} - \pi^{sqL,\emptyset} \geq \pi^{K} - \pi^{\emptyset}$$

Hence whether the retailer benefits from selecting U_K for category captain depends on how, in equilibrium, the effort of the category captain affects the joint profit of the whole industry compared to how it affects the joint profit of the partial structure composed of the retailer and supplier K when the supplier L is *a priori* excluded. Whenever the additional profit created by the effort of the category captain K is larger when product L is excluded, then the retailer is better off with

K as category captain than without a category captain. In that case, R selects a category captain in equilibrium, at the expense of the rival supplier.

Finally, let us consider what drives the choice of category captain in stage 1. The retailer gains the joint profit minus each supplier's profit. However, each supplier wins his own contribution to the profit, that is, the difference between the joint profit when the two products are sold and the joint profit when his own product is excluded. The retailer chooses the category captain that leaves him the larger profit, that is, he will select K instead of L if:

$$\pi^{sqK,K} + \pi^{sqL,K} - \pi^{K} \geq \pi^{sqK,L} + \pi^{sqL,L} - \pi^{L}$$
$$\Leftrightarrow [\pi^{sqL,K} - pi^{sqL,\emptyset}] - [\pi^{K} - \pi^{\emptyset}] \geq [\pi^{sqK,L} - \pi^{sqK,\emptyset}] + [\pi^{L} - \pi^{\emptyset}]$$

The retailer thus chooses the category captain by comparing the additional profit generated by the captain's effort when the rival is excluded to the additional profit generated by the captain's effort when the rival is not excluded. As the category captain captures all the additional profit created by its effort, the sum of the retailer's and the rival's profits is independent of this effort. Yet the sharing of the profits between the retailer and the rival depends on the category captain's effort, as the rival's profit is the difference between the joint profit and the profit of the industry when only the category captain's product is sold. Hence the retailer selects the category captain that provides an effort that contributes more to the status quo profit than to the joint profit.

Hence the retailer is better off when the sum of these two differences is minimum. On the one hand, he wants to select the category captain that maximizes the joint profit; on the other hand, he also wants to minimize the rent left to the two suppliers. As each of the supplier wins his own contribution to the profit, that is, the difference between the joint profit when the two products are sold and the joint profit when his own product is excluded, the retailer is better off when the sum of these two differences is minimum.

In particular, the following lemma highlights what drives the choice of category captain whenever each supplier excludes his rival when selected.

Lemma 2 *When the two suppliers exclude their rival when selected for category captain, the retailer's profit is larger in the subgame in which U_B is the category* *captain than in the subgame in which it is U_A.*

Proof. In the subgame where U_A is category captain, if U_B is excluded, the retailer's profit is $\pi^{sqA,A}$, that is, the profit generated by product B when product A is excluded and no effort is exerted. In the subgame where U_B is category captain, if U_A is excluded, the retailer's profit is $\pi^{sqB,B}$, that is, the profit generated by product A when product B is excluded and no effort is exerted. By assumption, the latter is larger than the former.

■

These first results highlight potential anticompetitive properties of category captaincy. In particular we have shown that, when suppliers make take-it-or-leave-it offers, the choice of a category captain is always detrimental to the competitor. In section 5 we will allow for balanced bargaining power in tariff negotiations. In what follows, we further investigate the welfare effects of having a category captain, and the impact of category captaincy on profit sharing. To do so, we first consider in section 4 the case where the effort exerted by the category captain only affects demand for its own product. In section 5.4 we extend the model to allow for spillovers.

4 Purely vertical effect

In this section, we consider that the effort exerted by a category captain fully targets the demand for its own product : there is no spillover. We assume that the effort exerted by the category captain increases the inverse demand for its own product, but has no effect on the rival product: $P_K(q_K, q_L, e, i)$ is increasing in e iff $K = i$. From now on, we will make Assumptions 2 and 3.

Note that, in the absence of spillover, the presence of a category captain distorts the equilibrium quantities: compared to the situation without category captain, the quantity produced by the captain increases while the quantity produced by the rival supplier decreases (see Appendix B).

4.1 Selection of the category captain

As shown in the previous section, the equilibrium tariffs are efficient ($w_A = w_B = 0$). We therefore focus first on the effort chosen by each supplier when he is selected for the category captain position, before analyzing the stage 1 selection of the category captain by the retailer.

Proposition 2 *In equilibrium, U_A always exerts a larger effort than U_B when chosen for CC: $e_A^* > e_B^*$.*

Proof. By (i), when U_A is the category captain, the optimal effort e_A^* is given by:

$$FOC_A(e_A^*) = \frac{\partial P_A}{\partial e}(\tilde{q}_A(e_A^*, A), \tilde{q}_B(e_A^*, A), e_A^*, A)\tilde{q}_A(e_A^*, A) - 2e_A^* = 0$$

while when U_B is the category captain, the optimal effort e_B^* is given by:

$$FOC_B(e_B^*) = \frac{\partial P_B}{\partial e}(\tilde{q}_B(e_B^*, B), \tilde{q}_A(e_B^*, B), e_B^*, B)\tilde{q}_B(e_B^*, B) - 2e_B^* = 0.$$

In order to compare e_A^* and e_B^*, we compute the first order condition $FOC_A(e_B^*)$ and show that it is positive.

$$\begin{aligned} FOC_A(e_B^*) &= \frac{\partial P_A}{\partial e}(\tilde{q}_A(e_B^*, A), \tilde{q}_B(e_B^*, A), e_B^*, A)\tilde{q}_A(e_B^*, A) - 2e_B^* \\ &> \frac{\partial P_A}{\partial e}(\tilde{q}_A(e_B^*, A), \tilde{q}_B(e_B^*, A), e_B^*, A)\tilde{q}_B(e_B^*, B) - 2e_B^* \end{aligned}$$

because, for a given effort (here, e_B^*), the equilibrium quantity of the category captain is larger when U_A is the category captain than when it is U_B (see equations (2) and (3)).

As we have assumed that $\frac{\partial P_A}{\partial e}(.,.,.,A) > \frac{\partial P_B}{\partial e}(.,.,.,B)$, we have

$$FOC_A(e_B^*) > \frac{\partial P_B}{\partial e}(\tilde{q}_A(e_B^*, A), \tilde{q}_B(e_B^*, A), e_B^*, B)\tilde{q}_B(e_B^*, B) - 2e_B^*.$$

Under Assumptions 2 and 3, we have:

$$\frac{\partial P_B}{\partial e}(\tilde{q}_A(e_B^*, A), \tilde{q}_B(e_B^*, A), e_B^*, B) > \frac{\partial P_B}{\partial e}(\tilde{q}_B(e_B^*, B), \tilde{q}_A(e_B^*, B), e_B^*, B)$$

because $\tilde{q}_B(e_B^*, B) < \tilde{q}_A(e_B^*, A)$, and $\tilde{q}_A(e_B^*, B) > \tilde{q}_B(e_B^*, A)$. Hence

$$\begin{aligned} FOC_A(e_B^*) > FOC_B(e_B^*) = 0 \\ \Rightarrow e_A^* > e_B^* \end{aligned}$$

QED.

■

Proposition 2 states that the effort exerted by the category captain is higher in the subgame where U_A is the category captain than in the other subgame. We now consider the choice of category captain by the retailer in stage 1. First, we show that the retailer is always better off with a category captain than without.

Proposition 3 *In equilibrium, the retailer always chooses a category captain.*

Proof. When U_K is category captain, the profit of the retailer is:

$$\begin{aligned} \pi_R^K &= P_K^M(q_K^M(e,K), e, K)q_K^M(e,K) + P_L^M(q_L^M(0,K), 0, K)q_L^M(0,K) \\ &\quad - [P_K(\tilde{q}_K(e,K), \tilde{q}_L(e,K), e, K)\tilde{q}_K(e,K) + P_L(\tilde{q}_L(e,K), \tilde{q}_K(e,K), e, K)\tilde{q}_L(e,K)] \end{aligned}$$

Using the envelop theorem yields:

$$\frac{d\pi_R^K}{de} = \frac{\partial P_K^M}{\partial e} q_K^M(e,K) - \frac{\partial P_K}{\partial e}\tilde{q}_K(e,K) - \frac{\partial P_L}{\partial e}\tilde{q}_L(e,K)$$

As there are no spillovers, $\frac{\partial P_L}{\partial e} = 0$ and we obtain:

$$\frac{d\pi_R^K}{de} = \frac{\partial P_K^M}{\partial e}(q_K^M, 0, e, K)q_K^M(e,K) - \frac{\partial P_K}{\partial e}(\tilde{q}_K, \tilde{q}_L, e, K)\tilde{q}_K(e,K).$$

Since the quantity q_K chosen by the retailer is decreasing with respect to q_L, for a given effort e the monopoly quantity is larger than the duopoly quantity: $q_K^M(e,K) \geq \tilde{q}_K(e,K)$. Hence using assumptions 2 and 3, we have $\frac{\partial P_K}{\partial e}(q_K^M, 0, e, K) > \frac{\partial P_K}{\partial e}(\tilde{q}_K, \tilde{q}_L, e, K)$.

As a consequence, the profit of the retailer increases with respect to e when U_K is category captain. This implies, in particular, that the profit of retailer is larger when there is a category captain ($e_K^* > 0$) than when there is no category captain. ■

Hence we know that the retailer will chose one of the two suppliers for the category captain position. To analyze the drivers of this choice, let us first compare the total profit of the industry depending on the identity of the category captain.

Lemma 3 *The equilibrium joint profit is larger in the subgame in which U_A is the category captain than in the subgame in which it is U_B.*

Proof. See Appendix C. ■

The total industry profit is thus larger when U_A is category captain. However, the strong supplier U_A also benefits from a better bargaining position than his competitor, which can increase the incentives of the retailer to opt for the weaker supplier U_B as a category captain.

In what follows, we highlight this tradeoff with the linear demand system described above.

Note first that the profit earned by the retailer is always larger when it chooses to have a category captain than when it has no category captain.

Without a category captain, the retailer sells both products if $t \leq \frac{1}{X}$, and only product A otherwise.[10]

The effort exerted by the category captain always increases total profit and tends to increase the status-quo profit of the retailer. As a consequence, it is never profitable for the retailer not to choose a category captain.

Let us now compare the retailer's profit depending on whether he selects U_A or U_B as a category captain, for a given level of effort e. Given the values of F_A and F_B, the profit of the retailer when choosing i as category captain is given by:

$$\pi(e_i^*) - F_A - F_B = \sum_{K=A,B} \left[P_K^M(q_K^M(0, e_i^*, i)) q_K^M(0, e_i^*, i) - P_K(\tilde{q}_K(.), \tilde{q}_L(.), e_i^*, i) q_K(.) \right].$$

Appendix D describes the equilibrium of the subgame composed of Stages 1 to 3 with the linear demand system. Focusing first on the retailer's profit for a given effort level, we obtain the following lemma.

[10] When both products are sold, $q_A = \frac{X-t}{2(1-t^2)}$, $q_B = \frac{1-tXt}{2(1-t^2)}$, $p_A = \frac{X}{2}$, $p_B = \frac{1}{2}$ and profits are $\pi_R = \frac{t(2X-t(1+X^2))}{4(1-t^2)}$, $\pi_A = \frac{(X-t)^2}{4(1-t^2)}$ and $\pi_B = \frac{(1-tX)^2}{4(1-t^2)}$. When only A is sold, $q_A = p_A = \frac{X}{2}$, and profits are $\pi_R = \frac{1}{4}$ and $\pi_A = \frac{X^2-1}{4}$.

Lemma 4 *In the linear specification, for an exogenously given and undifferentiated effort e, it is always more profitable for the retailer to choose U_B as a category captain.*

Proof. Straightforward from profit comparisons - see appendix D. ■

The intuition is as follows, and relies on the idea that granting U_B a category captain position gives the retailer more bargaining power in the negotiations, because he receives a larger share of a smaller pie (see for instance Montez, 2007). Indeed, we know that, for a given level of effort e, the industry as a whole is better off if the retaier chose U_A as a category captain: our assumptions ensure that the joint profit is larger when the effort is dedicated to the preferred poduct (product A) [TBC].

More precisely, consider first that, assuming a constant effort e irrespective of the identity of the category captain, the profit of the retailer with a given category captain equals:

$$P_A^m(q_A^A)q_A^A + P_B^m(q_B^B)q_B^B - \sum_{K \in \{A,B\}} p_K q_K$$

This profit can thus be decomposed as the sum, for both products ($K \in \{A, B\}$), of the additional profit created by the exclusion of the competitor in the negotiation stage, that is, $\Delta_K^{icc} = P_K^m(q_K^K)q_K^K - p_K q_K$ when U_i is category captain.

Consider first the difference between Δ_A^{Acc} and Δ_B^{Bcc}. A category captain is never excluded in equilibrium when both contracts have been accepted (at least for effort values close enough to the equilibrium values), while his competitor is excluded for t large enough. We can show that the additional profit created by the exclusion of the competitor in the negotiation stage is always lower for U_A when he is category captain than for U_B when he is category captain: $\Delta_A^{Acc} \leq \Delta_B^{Bcc}$. The reason is that, though both the monopoly profit and the duopoly profit are higher for U_A than for U_B, the difference is larger in the duopoly case because of the competitiveness of U_A. Furthermore, when t is large enough, the rival of the category captain is excluded, but U_B is excluded for lower values of t than U_A (which means $\Delta_A^{Acc} = 0$ for all t above a threshold, and $\Delta_B^{Bcc} = 0$ for all t above a higher threshold).

Consider then the difference between Δ_A^{Bcc} and Δ_B^{Acc}, that is, the additional profit created by the exclusion of the competitor in the negotiation stage, when the competitor is the category captain. Again, when the two goods are independent enough (t low), the difference between monopoly and duopoly profits is larger for U_B than for U_A for the same reason: in other words, for t close to 0, $\Delta_B^{Acc} \geq \Delta_A^{Bcc}$. But U_B is excluded from the downstream competition game quicker than A, and then Δ_B^{Acc} becomes constant (and equal to the joint profit when A is excluded in the negotiation stage). By contrast, Δ_A^{Bcc} becomes flat for much larger t. As the joint profit when U_B is excluded in the negotiation stage is larger than the joint profit when A is excluded in the negotiation stage, Δ_B^{Acc} and Δ_A^{Bcc} intersect and, for larger values of t, $\Delta_A^{Bcc} \geq \Delta_B^{Acc}$.

Finally, when e is fixed, the first effect dominates, and $\Delta_A^{Acc} + \Delta_B^{Acc} \leq \Delta_A^{Bcc} + \Delta_B^{Bcc}$ for all t: if the effort played no role, it would always be more profitable for the rretailer to choose U_B as a category captain. This comes from the negotiation effect higlighted above.

We now compare the profit of the retailer at the equilibrium level of effort depending on its choice of category captain, and obtain the following proposition.

Proposition 4 *In the linear specification, there exists a threshold level of substitutability t^* such that the retailer chooses U_A as category captain when $t < t^*$ and chooses U_B otherwise. The threshold t^* is decreasing with respect to X.*

Proof. Straightforward from the comparison of $\pi(e^*, A) - F_A - F_B$ and $\pi(e^*, B) - F_A - F_B$. ■

The intuition is as follows. We have shown in lemma 4 that, if the two category captains exerted the same effort, the retailer would always prefer to select U_B as a category captain, so as to benefit from a better bargaining power. Yet we also know from Lemma 2 that when U_A is category captain, it exerts a larger effort than U_B when it is category captain. This changes the ordering of Δ_A^{Acc} and Δ_B^{Bcc} when t is very low: for t close to 0, $\Delta_A^{Acc} \geq \Delta_B^{Bcc}$ because the effort exerted by A tends to increase the benefit of excluding the competitor. Yet when t increases starting from 0, the effort decreases and this effect is compensated by the bargaining effect described at e constant. Hence for t large enough, $\Delta_A^{Acc} \leq \Delta_B^{Bcc}$. Endogenizing the effort does not change the general shape of Δ_A^{Bcc} and Δ_B^{Acc}.

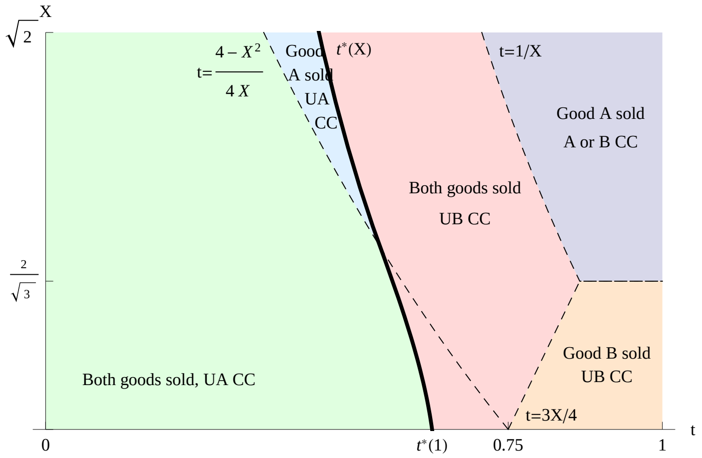

Figure 1: Choice of category captain by the retailer and corresponding market structure depending on X and t. The lower (respetively higher) dashed line corresponds to the threshold above which the retailer sells only good A (resp. B) when U_A (resp. U_B) is category captain.

4.2 Welfare analysis

4.2.1 Welfare effect of having a category captain

Finally, let us focus on the welfare effects of introducing a category captain. As a first step, we analyse the effect of category captaincy on product variety. The European Commission guidelines on vertical restraints indeed notice that "category management agreements [...] may sometimes distort competition between suppliers, and finally result in anticompetitive foreclosure of other suppliers, where the category captain is able, due to its influence over the marketing decisions of the distributor, to limit or disadvantage the distribution of products of competing suppliers."(par. 210).

In the linear specification, the downstream equilibrium has different features according to which supplier is selected for category captain. In particular, product variety may differ. Figure 2 compares the variety of products sold in the subgame equilibrium where U_A is the category captain to that in the subgame where U_B is the category captain. In the red area, both products are sold, while in the blue area, only A is sold and in the orange area, only B is sold. The area where only B is sold exists only when U_B is the category captain. Furthermore, the area where both goods are sold is larger when U_B is the category captain. Finally, note that when U_B is the category captain, the area where only product A is sold (blue zone) corresponds to the area where the category captain exerts no effort.

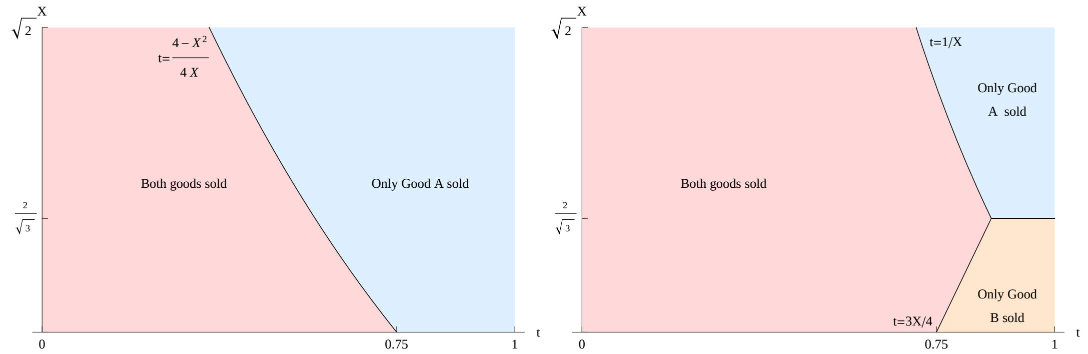

Figure 2: Market structure when U_A is category captain (left) and when U_B is category captain (right).

Proposition 5 *In the linear specification,*

- *introducing a category captain increases the scope for product exclusion. Furthermore, it creates situations in which the preferred product (product A) is excluded.*
- *However, introducing a category captain also increases consumer surplus and welfare.*

Proof. We show in Appendix E that, without a category captain, product B is excluded for all $t \geq \frac{1}{X}$; with a category captain, product B is excluded for $t \in [\frac{4-X^2}{4X}, t^*]$ and for $t \geq \frac{1}{X}$ and $X \geq \frac{2}{\sqrt{3}}$ (with $t^* \leq min\{\frac{3X}{4}, \frac{1}{X}\}$), and product A is excluded for all $t \geq \frac{3X}{4}$ and $X \leq \frac{2}{\sqrt{3}}$. TBC *** ■

Figure 3 compares the areas where at least one product is excluded from the market in two situations: when there is no category captain, and with the category captain selected in equilibrium. First, in the pink and green areas, one of the products is excluded whether the retailer selects a category captain or not, but the identity of the excluded product may vary. Indeed, when X is large (pink area), the retailer always chooses to exclude the less preferred product (product B). However when X is small (green area), while product B is excluded when there

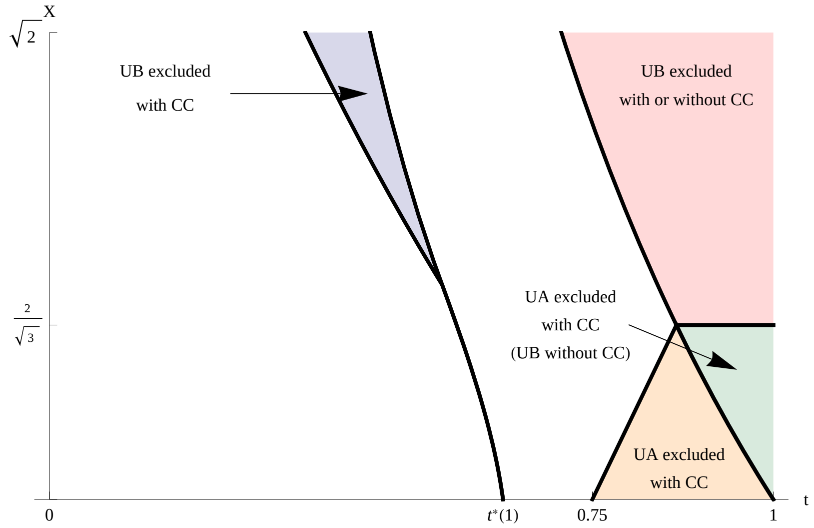

Figure 3: Products excluded in equilibrium without CC or with the chosen CC

is no category captain, with a category captain it is product A that is excluded, because in that area, the retailer chooses U_B as category captain. Second, in the blue and orange areas, no product would be excluded without a category captain, but the choice of a category captain creates product exclusion – of product B in the blue area, and of product A in the orange area.

That the introduction of a category captain increases consumer surplus and welfare is mostly driven by the assumption that only the category captain can exert an effort. A good benchmark would be to compare the equibrium in our game with a situation where the retailer can himself choose the effort level and to which demand it is affected. Assume thus that the retailer exerts the effort himself. In that case, he actually always aims at increasing the demand for product B. Whenever the downstream equilibrium excludes a product, the retailer has no outside option when negotiating with this supplier. In that case, he does not gain any benefit from exerting an effort, and thus chooses the lowest possible effort so as to exit from the monopolization downstream equilibrium. When $t \geq \frac{1}{X}$, if the effort aims at increasing demand for product A, then product B is always exluded: in that case the retailer does not gain any benefit from exerting an effort aiming at increasing demand for product A, while it is profitable for him to increase the demand for product B. By contrast, when $t \leq \frac{1}{X}$, the retailer can always choose an effort so as to guarantee that both products will be sold in equilibrium. However, the retailer's bargaining power is larger when he aims at increasing demand for product B.

4.2.2 Welfare effect of the choice of category captain

In this section we consider the linear demand system specified above. We first compare the welfare according to the identity of the category captain. We obtain the following proposition.

Proposition 6 *In the absence of spillover, the choice of U_B as CC is always detrimental to joint profit. In the linear specification, consumer surplus and welfare in equilibrium are all larger when the retailer chooses U_A rather than U_B as category captain.*

Proof. For the effect of the choice of U_B as CC on joint profit, see lemma 3. For the effect on consumer surplus and welfare in the linear specification, see Appendix E.

■

In other words, whenever the retailer chooses U_B as category captain in equilibrium, this is neither optimal from the point of view of the firms, nor from the point of view of consumers. Interestingly, the larger the asymmetry between the two suppliers (that is, the larger X), the more likely the retailer is to choose the "wrong"category captain. This effect is driven by buyer power considerations: although it is always better to select U_A as a category captain, in terms of industry profit, it may be more profitable for the retailer to select the other supplier as a category captain, in order to benefit from a better bargaining position.

In that case, we may observe the exclusion of one product motivated by buyer power considerations. (TBC)

5 Discussion and Extensions

5.1 Pricing decisions by the category captain

In our setup, allowing the category captain to decide on final prices would not improve the efficiency of the industry, because we allow for two-part wholesale tariffs that are efficient, hence the retailer sets final prices that maximize the joint profit. If the category captain were allowed to make pricing decisions, he would thus not be able to choose a better price from the industry perspective. In particular, if the category captain took final prices decisions after the wholesale tariffs negotiation stage, he would choose the same prices as the retailer. If by contrast he took final prices decisions before the negotiation stage, he could even create distortions by setting prices that reduce the joint profit but increase the share he receives : in this case his interest is to set too high a price (or too low a quantity) for the rival product, so as to minimize the retailer's outside option (that is, the variable profit of the retailer when he sells only the rival product). Locally, increasing the price for the rival product above the joint profit maximizing price will affect more the retailer's outside option than the equilibrium joint profit, and it will thus benefit the category captain. *To study the effect of pricing decisions by a category captain, we would need to consider informational advantage by the CC which is out of the scope of our framework.*

5.2 Balanced bargaining power

In this section we consider the case where the effort is fully appropriable by the category captain. Proposition 1 shows that the whole additional profit created by the effort of the category captain is captured by the CC. However, we have assumed so far that the supplier makes take-it-or-leave-it offers to the retailer, so that the bargaining power of the retailer is limited. Indeed, his only bargaining power comes from his outside option in each of the two separate negotiations.

In this section we explore the robustness of our results to the introduction of a balanced bargaining power in the tariff negotiation.

Assume that the effort level cannot be directly chosen by the retailer, and that the buyer power affects only the tariff stage.[11] We slighty modify the timing of decisions. After the choice of category captain in stage 1, the captain chooses its effort level in stage 2.1 (the effort is observed by the retailer; the cost of effort is actually paid only if the contract is accepted by the category captain in stage 2.2), and tariffs are negotiated in stage 2.2. Finally, the retailer chooses the quantities in stage 3.

During the 2.2 tariff negotiation stage, with probability α ($\alpha \in [0,1]$), the two manufacturers U_K ($K \in \{A, B\}$) simultaneously make take-it-or-leave-it offers to the retailer in the form of two-part tariff contracts $\{w_K, F_K\}$, and with probability $(1-\alpha)$ the offer is made by the retailer (each supplier is then offered a two-part tariff contract). The firms that have been offered contracts then accept or reject the offers. If an offer is rejected, the product is not sold. We derive the equilibrium properties.

Tariffs and quantities. In stage 3, the equilibrium quantities are unchanged, as the retailer sets quantities to maximize its profit: $\max\limits_{q_A, q_B} \pi$.

Consider now stage 2. Assume that U_K is the category captain, and that he exerts the effort e. In stage 2.2, when the suppliers make the offers, the result is the same as in the previous section. If by contrast the retailer makes the offers, his program is:

$$\begin{array}{ll} \max\limits_{\{w_K, F_K, w_L, F_L\}} & (P_K(\tilde{q}_K(.), \tilde{q}_L(.), e, K) - w_K)\tilde{q}_K(.) + (P_L(\tilde{q}_L(.), \tilde{q}_K(.), e, K) - w_L)\tilde{q}_L(.) - F_K - F_L \\ s.t. & w_K \tilde{q}_K(.) + F_K - e^2 \geq 0 \\ s.t. & w_L \tilde{q}_L(.) + F_L \geq 0 \end{array}$$

The participation constraint of the suppliers are binded. Note that the retailer compensates the category captain for the cost of effort: offering the captain a fixed fee below the cost of effort would drive the captain to reject the contract

[11]Note that all the results would hold if the retailer were able to offer a contract specifying the effort required by the CC in stage 2 - that is, with the timing of the baseline model, except that the offers are made by the retailer with probability $(1-\alpha)$. Proofs available upon request.

thereby saving the effort cost. The retailer's program thus boils down to

$$\max_{\{w_A, w_B\}} \quad P_A(\tilde{q}_A(.), \tilde{q}_B(.), e, i)\tilde{q}_A(.) + P_B(\tilde{q}_B(.), \tilde{q}_A(.), e, i)\tilde{q}_B(.) - e^2$$

Again, totally differentiating and reintegrating the retailer's first order conditions determined in Stage 4 yields the following system of first order conditions:

$$w_A \frac{d\tilde{q}_A(.)}{dw_A} + w_B \frac{d\tilde{q}_B(.)}{dw_A} = 0,$$
$$w_A \frac{d\tilde{q}_A(.)}{dw_B} + w_B \frac{d\tilde{q}_B(.)}{dw_B} = 0$$

Hence when the suppliers make the offers, cost-based tariffs $w_A = w_B = 0$ are the unique solution.[12] As a consequence, when the retailer makes the offers, suppliers make zero profit.

Hence for a given effort, there is a unique continuation equilibrium where $w_A = w_B = 0$. This means that, for a given effort, the equilibrium quantities are the same irrespective of who makes the offers. With balanced bargaining power, equilibrium profits are shared as follows:

$$\begin{aligned}
\Pi_R^K &= (1-2\alpha)\pi^K + \alpha(\pi^{sqK,K} + \pi^{sqL,K}) - (1-\alpha)e^2 \\
\Pi_K^K &= \alpha(\pi^K - \pi^{sqK,K}) - \alpha e^2 \\
\pi_L^K &= \alpha(\pi^K - \pi^{sqL,K}).
\end{aligned}$$

Effort and Profit sharing In stage 2.1, the category captain U_K maximizes its profit.

Lemma 5 *When the retailer has some bargaining power ($\alpha < 1$), the category captain chooses the same effort as in the baseline model.*

Proof. The category captain U_K chooses e_K to maximize $\alpha(\pi^K - \pi^{sqK,K} - e_K^2)$, that is, to maximize $\pi^K - e_K^2$. ■

[12] Note first that these conditions imply that w_A and w_B are of the same sign. If $w_A \neq 0$, the conditions also imply that $\frac{d\tilde{q}_A(.)}{dw_A}\frac{d\tilde{q}_B(.)}{dw_B} = \frac{d\tilde{q}_A(.)}{dw_B}\frac{d\tilde{q}_B(.)}{dw_A}$. Yet the assumption that cross-effects are dominated by direct effect prevents this to happen.

Finally, we consider how the existence of a category captain affects the way the joint profit of the industry is shared between the three firms. When the bargaining power is balanced, the result in Proposition 1 no longer holds.

Proposition 7 *When the retailer has some bargaining power ($\alpha < 1$), the category captain no longer captures the whole benefit of its activity.*

Proof. Assume that U_K is the category captain, and consider his effort e as given. With balanced bargaining powers, we have:

$$\Pi_R^K - \Pi_R^\emptyset = (1-2\alpha)\left(\pi^K - \pi^\emptyset\right) + \alpha(\pi^{sqL,K} - \pi^{sqL,\emptyset}) - (1-\alpha)e_K^2$$
$$\Pi_L^K - \Pi_L^\emptyset = \alpha\left[(\pi^K - \pi^\emptyset) - (\pi^{sqL,K} - \pi^{sqL,\emptyset})\right].$$

Hence

$$(\Pi_R^K + \Pi_L^K) - (\Pi_R^\emptyset + \Pi_L^\emptyset) = (1-\alpha)\left(\pi^K - e_K^2 - \pi^\emptyset\right) \geq 0$$

Any solution e_K^* maximizes $\pi^K - e_K^2$ and therefore satisfies $\pi^K(e_K^*) - (e_K^*)^2 \geq \pi^K(0) = \pi^\emptyset$. It is thus clear that the category captain does not capture the whole benefit of its activity, as the sum of the profits of the retailer and of the upstream competitor increase when U_K becomes category captain. ■

Consider now the effect of the category captain on its rival's profit.

Proposition 8 *Regardless of the retailer's bargaining power ($\alpha \leq 1$), the choice of a category captain is never profitable for its rival.*

Proof. The rival is worse off with the category captain if and only if :

$$\Pi_L^K - \Pi_L^\emptyset \leq 0$$
$$\Leftrightarrow (\pi^K - \pi^\emptyset) \leq (\pi^{sqL,K} - \pi^{sqL,\emptyset})$$

The choice of a category captain can hurt the rival U_L if and only if the net gain from the category captain is lower when both products are sold than when only the captain's product is.

Note that, if the effort of the category captain were $e = 0$, we would have $\pi^{sqL,\emptyset} = \pi^{sqL,K}$ and $\pi^K = \pi^\emptyset$.

A sufficient condition for the rival supplier to be worse off with the category captain is thus that the derivative of profits satisfies:

$$\frac{\partial \pi^K}{\partial e}(e) \leq \frac{\partial \pi^{sqL,K}}{\partial e}(e),$$

for all $e \in [0, e^*]$. Whenever there are no spillovers, this condition is satisfied when assumptions 2 and 3 are. Indeed,

$$\begin{aligned} \pi^K &= \underset{q_K, q_L}{Argmax}[P_K(q_K, q_L, e, K)q_K + P_L(q_L, q_K, 0, K)q_L] \\ \pi^{sqL,K} &= \underset{q_K}{Argmax}[P_K(q_K, 0, e, K)q_K] \end{aligned}$$

In the absence of spillovers, the envelop theorem yields:

$$\begin{aligned} \frac{\partial \pi^K}{\partial e}(e) &= \frac{\partial P_K}{\partial e}(q_K^d, q_L^d, e, K)q_K^d \\ \frac{\partial \pi^{sqL,K}}{\partial e}(e) &= \frac{\partial P_K}{\partial e}(q_K^m, 0, e, K)q_K^m, \end{aligned}$$

with q_K^m the equilibrium quantity when product K is the only product available, and q_K^d the equilibrium quantity when the two products can be sold (obviously $q_K^m \geq q_K^d$). Under assumptions 2 and 3, we have $(0 \leq) \frac{\partial P_K}{\partial e}(q_K^d, q_L^d, e, K) \leq \frac{\partial P_K}{\partial e}(q_K^m, 0, e, K)$, hence it is never possible that the rival benefits from the CC. ■

5.3 Timing of effort

In the baseline model, we assume that the level of effort is chosen by the firm that has already been selected as the category captain, and that the choice of effort and tariffs are simultaneous. In this section, we test the robustness of our results to changes in the timing of effort. Note first that the baseline timing is equivalent to a timing in which the category captain can commit to exerting a given effort before the tariff negotiation stage. We now consider two other variants: first when the effort is chosen after the tariff stage, second when effort is chosen before the choice of category captain.

5.3.1 Effort decision after the tariff stage

Assume that the category captain decides on the level of effort once the tariffs have been set, and before the retailer sets its quantities. Accordingly, we modify stage 2 of the game as follows: in stage 2.1 suppliers make secret take-it-or-leave-it, two-part tariff offers, and in stage 2.2 the category captain chooses its effort level. We assume that the tariffs offered in stage 2.1 are not observable by the rivals.

In stage 3, the subgame equilibrium is unchanged. Consider now the stage 2.2 effort decision of the category captain, say U_K. The category captain's profit is as follows:

$$\pi_K = w_K \tilde{q}_K(w_K, w_L^a, e, K) + F_K - e^2,$$

where w_L^a is the anticipated equilibrium wholesale tariff offered by the rival in the previous stage (it is not observable by the category captain). The optimal effort $e_K^{BR}(w_K, w_L^a)$ solves the following first order condition:

$$w_K \frac{\partial \tilde{q}_K(w_K, w_L^a, e, K)}{\partial e} - 2e = 0$$

If the tariffs decided upon in stage 2.1 are efficient, that is, if $w_K = 0$, then the category captain's variable profit is independent of the effort level, and the captain only bears the cost of effort. He is thus better off exerting no effort: $e^{BR}(0, .) = 0$. This implies that the category captain may distort his tariff offer in stage 2, in order to credibly commit to exerting a positive effort.

Proposition 9 *When the effort is chosen after the tariff stage, the wholesale tariffs must be inefficient to yield a positive effort.*

Proof. Assume U_A is the category captain (the analysis is similar when it is U_B), and consider the tariffs offers by U_B. All firms anticipate the continuation equilibrium efforts $e_A^{BR}(w_A, w_B^a, F_A, F_B)$ when the two products are sold, and e_A^M when only A is sold. As in the baseline framework, U_B maximizes its profit provided that the participation constraint of the retailer is satisfied.

$$\max_{w_B} (P_A(\tilde{q}_A(.), \tilde{q}_B(.), e_A^{BR}, A) - w_A)\tilde{q}_A(.) + P_B(\tilde{q}_B(.), \tilde{q}_A(.), e_A^{BR}, A)\tilde{q}_B(.)$$
$$- (P_A^M(q_A^M(w_A, e_A^M, A), e_A^M, A) - w_A) q_A^M(w_A, e_A^M, A)$$

The last term, $(P_A^M(q_A^M(w_A, e_A^M, A), e_A^M, A) - w_A)q_A^M(w_A, e_A^M, A)$, is independent of w_B. U_B's program thus boils down to:

$$\max_{w_B}(P_A(\tilde{q}_A(.), \tilde{q}_B(.), e_A^{BR}, A) - w_A)\tilde{q}_A(.) + P_B(\tilde{q}_B(.), \tilde{q}_A(.), e_A^{BR}, A)\tilde{q}_B(.)$$

This condition is the same as in the baseline model, hence in equilibrium, $w_B = 0$.

Consider now the category captain's profit maximization in stage 2.1, which yields the following first order condition:

$$0 = w_A\left(\frac{\partial \tilde{q}_A}{\partial w_A} + \frac{\partial \tilde{q}_A}{\partial e_A}\frac{\partial e_A^{BR}}{\partial w_A}\right) + \frac{\partial P_A}{\partial e}\frac{\partial e_A^{BR}}{\partial w_A}\tilde{q}_A - 2e_A^{BR}\frac{\partial e_A^{BR}}{\partial w_A}.$$

Injecting the stage 2.2 FOC and rearranging yields the following condition:

$$0 = w_A\frac{\partial \tilde{q}_A}{\partial w_A} + \frac{\partial P_A}{\partial e}\frac{\partial e_A^{BR}}{\partial w_A}\tilde{q}_A$$

for $w_A = 0$, the right hand side simplifies as $\frac{\partial P_A}{\partial e}\frac{\partial e_A^{BR}}{\partial w_A}\tilde{q}_A$, which is positive: hence in equilibrium, $w_A > 0$. ■

In other words, efficient tariffs imply an inefficient effort, and tariffs must be inefficient to reach a positive effort level. This creates a trade-off between the tariff efficiency and the effort efficiency that may be a new source of welfare-reducing effects.

When producers have all the bargaining power, the retailer would like to induce a high effort from the category captain, so as to increase its status-quo profit with respect to the other supplier; but at the same time, in exchange for a higher effort, the retailer must then accept a higher unit tariff, which decreases its profit. we have focused on a few specific pairs (X, t), namely $(11/10, 1/4)$, $(6/5, 4/5)$, $(7/5, 1/4)$ and $(7/5, 11/10)$, and found that in all those cases, the cost is not worth the effort: the profit of the retailer is then always higher without a category captain than with any category captain.

When the retailer has all the bargaining power, the cost of inducing high effort is often offset by its benefit for the retailer, who captures the whole profit of the industry. We thus obtain the following proposition:

Proposition 10 *In the linear specification, when the retailer has all the bargaining power and effort is set after tariffs:*

- *in equilibrium when a category captain is selected, consumer surplus is always lower than without a category captain.*
- *welfare is sometimes lower than without category captain.*

When the retailer has all the bargaining power, it captures the joint profit of the industry, as manufacturers have no status-quo profit. The category captain still makes a positive effort, provided that the unit part of the tariff is positive, because fixed fees are sunk at the effort-setting stage. As a consequence, within a framework in which it cannot set efficient contracts, the retailer chooses the joint-profit maximizing unit prices w_A and w_B. As a consequence, despite this inefficiency, the retailer then still chooses to have a category captain for most values of t and X.[13]

This choice is, however, always detrimental to consumers. Indeed, consumers face higher prices for both goods, due to the increased marginal cost of the retailer. As a result, the quantity purchased from the manufacturer that is not category captain is always lower than in the absence of a category captain. In addition, the quantity purchased from the category captain may also be lower than if there were no category captain: the benefit of a better quality is then offset by the loss from a higher price. The consumer surplus is thus always lower in the presence of a category captain than without a category captain.

The effect of the presence of a category captain on total welfare is therefore ambiguous in this framework, as there is a tradeoff between the increase of the joint profit and the decrease of the consumer surplus.

Note that, in the area in which U_B is chosen as category captain, all agents are better off than if U_A had been chosen. Compared to the baseline timing (when the retailer has all the bargaining power), the choice of category captain may also be distorted: in some cases the retailer may select U_B for category captain, although in the baseline setting U_A would be chosen. This is because U_B distorts less the equilibrium outcome. TBC

[13] There exists a small area in which the retailer is indifferent between having a category captain the makes an effort 0 or not having a cateogry captain. Computations are available upon request.

5.3.2 Effort decision before the category captain's selection

We now consider that the effort is chosen before the tariff stage. Note first that, as long as the effort is set after the choice of category captain by the retailer, the equilibrium outcome is the same as in the baseline model. However, things change when the suppliers can commit to an effort level before the choice of the category captain by the retailer. In that case, the two suppliers are competing *ex ante* for the category captain position.

Consider a variant of the baseline model where, in stage 0, both suppliers commit to exerting a given effort e_A, e_B if selected; in stage 1, the retailer selects a category captain; in stage 2, two-part tariffs contracts are negotiated, and in stage 3, the retailer order quantities. In this setup, we show that the equilibrium outcome is inefficient, as competition for the CC position is likely to dissipate the suppliers' profit, to distort the choice of the category captain, and to yield effort distortion, namely efforts above the joint-profit maximizing level.

For the sake of simplicity, we assume here that the profit functions and the effort costs are such that any supplier is able to exert an effort that excludes his rival when he is selected for category captain (**Assumption 4**).

Proposition 11 *When the suppliers can commit to an effort before the category captain selection, and when the retailer's bargaining power is low, U_B is selected for category captain. Compared to the baseline setup, the effort choice is usually suboptimal for the industry. When the retailer has a high bargaining power, by constrats, U_A is selected and the effort is efficient.*

Sketch of the proof

In the last stage, the subgame equilibrium outcome is the same than in the baseline model, as the retailer maximizes his variable profit given the effort and wholesale prices. We solve here the model for $\alpha = 0$ and $\alpha = 1$; by continutity the results extend in the neighborhood.

The retailer has no bargaining power Assume first that, as in the baseline model, the retailer has no bargaining power: the suppliers make take-it-or-leave-it offers ($\alpha = 1$). In stage 2, given e and the chosen category captain, tariffs are efficient: $w_{Ki} = 0$ and the fixed fees are as in the baseline model.

In stage 2, the retailer selects the category captain that leaves him the best profit: he thus selects U_X so as to maximize $\pi^{SQA,X} + \pi^{SQB,X} - \pi^X$.

In stage 1, the two suppliers thus compete to be selected for category captain. We have shown in Proposition 1 that, for a given category captain U_K, the total profit shared by the retailer and the rival supplier is constant, and amounts to the profit when the category captain is excluded:

$$\pi_R^K + \pi_L^K = \pi^{sqK,K}$$

Furthermore, we have show in proposition 3 that, for a given category captain, the retailer's profit increases in the effort of this category captain. Then the best offer a supplier can make is to exclude his rival, as in that case the retailer receives the whole joint profit $\pi^{sqK,K}$. As $\pi^{sqB,B} \geq \pi^{sqA,A}$, supplier U_B is able to make the better offer to the retailer.

More precisely, the best offer U_A can make is to commit to e_A^* whenever this effort is such that U_B is excluded, and to the minimal effort that excludes U_B, which we denote as $\underline{e_A}$, otherwise. Indeed, this offer maximizes the retailer's profit given the choice of U_A for category captain. Furthermore, when e_A^* is such that the rival product is not excluded, that is, when $e_A^* \leq \underline{e_A}$, U_A's profit decreases in e_A for $e_A \geq e_A^*$, while the retailer's profit is constant once product B is excluded, that is, for $e_A \geq \underline{e_A}$, hence in that case U_A commits to the effort $\underline{e_A}$. This offer grants the retailer a profit of $\pi^{sqA}(0)$, the profit generated when product B is the only product on the market and no effort is exerted.

Assume now that U_A commits to this effort $Max\{e_A^*, \underline{e_A}\}$. Then if U_B is not selected for the CC position, he will receive zero profit. The best strategy for U_B is then to offer the effort that satisfies the following program:

$$\underset{e}{Max}\pi^B(e) - \pi^{sqB,B}$$
$$s.t.\pi_R^B(e) \geq \pi^{sqA}(0)$$

We have shown in the proof of proposition 3 that $\pi_R^B(e)$ increases in e. Hence either e_B^* satisfies the participation constraint of the retailer, or U_B must increase its effort to be selected for category captain. His equilibrium offer is thus the effort that maximizes his profit provided that is satisfies the retailer's participation constraint.

$$max\{e_B^*, \hat{e_B}\}$$

where $\hat{e_B}$ is defined by $\pi_R^B(\hat{e_B}) = \pi^{sqA}(0)$. In equilibrium this offer is accepted by the retailer.[14] Under Assumption 4, it is more profitable for supplier U_B to commit to such an effort than to offer any effort such that the retailer will select U_A for category captain, in which case U_B would receive a zero profit. This is thus the equilibrium offer by U_B and it is accepted by the retailer.

Hence in equilibrium, U_B is always selected for category captain. As we have seen in lemma 3, this choice is not optimal for the joint profit: U_A is a better choice. Furthermore, whenever $\hat{e_B} > e_B^*$, the effort is higher than the optimal effort conditional to U_B being the category captain.

The retailer has all bargaining power Assume now that the retailer has all the bargaining power, and makes take-it-or-leave-it offers to the suppliers in stage 2 ($\alpha = 0$).

In that case, for a given category captain and a given level of effort, in stage 2, the retailer makes efficient offers ($w_A = w_B = 0$) as we have shown in section 5.2. The fixed fees capture the net profit of the two suppliers. The resulting net profits of the three firms are then:

$$\Pi_R^K = \pi^K - e^2, \Pi_K^K \qquad = \pi_L^K = 0$$

In stage 0, each supplier then commits to the effort that maximizes the joint profit when he is the category captain, and the retailer thus selects U_A for category captain. In that case, the effort is efficient.

Requiring the suppliers to commit to a level of effort before they are selected for category captain by the retailer would thus create distortions as the inefficient supplier would always be selected. Furthermore, the effort exerted by this category captain is also likely to be distorted, as he will offer an effort above e_B^* as soon as $\pi_R^B(e_B^*) \leq \pi^{sqA}(0)$. Hence allowing for competition for the category cap-

[14]Note that we do the usual assumptions to break out tie, as this game has the same structure than an asymmetric Bertrand competition game.

tain position when the suppliers can commit to a given effort level would create distortions in the provision of effort by the CC and leads to situations that are worse for wefare than the baseline equilibrium.

Note that we have assumed here that suppliers cannot pay the retailer to be selected as category captain. Allowing for such transferts would restore some efficiency, as the efficient supplier would be able to be selected more often. Indeed, he would be able to offer the retailer a better transfert, because the total profit received by the category captain is larger when U_A is captain than when it is U_B, as $\pi^A - \pi^{sqA,A} > \pi^B - \pi^{sqB,B}$.

TBC.

5.4 Spillovers

Industry reports state that the work of a category captain can increase the sales of its competitors' products and not only its own. Progressive Grocer (2017) relates for instance the case of the fresh mushrooms market, where the producer Monterey Mushrooms has been appointed as category captain by a Southwest US retailer: "after Monterey fine-tuned the retailer's assortment, everyday retails and promotional strategy, it saw units increase 4.4 percent, while those in its market grew 2.6 percent [...] and sales grew 6.9 percent, compared with 4.6 percent for the market". We therefore assume in this section that the effort exerted by a category captain can be partially appropriated by the rival manufacturer. We consider the linear specification.

We assume in this Section that in case of failure of the negotiation with the category captain, the latter does not exert any effort. If, however, the negotiation with the category captain succeeds, it exerts an effort regardless of its own output. It may then happen that the category captain obtains a positive fixed fee while its product is not on the market, simply because the effort exerted benefits the rival product. This seems to us the most reasonable assumption regarding effort. Results would however be qualitatively similar if we assumed that the effort remains even if the negotiation with the category captain fails.[15]

[15]Proof available upon request.

5.4.1 Effort choice of the category captain

In Appendix F, we determine the equilibrium of the subgame composed of Stages 2 to 4 in the presence of a spillover. We determine the optimal effort depending on the choice of category captain, and the market structure in equilibrium, and we obtain the following proposition:

Proposition 12 *In the linear specification with spillovers, $e_A^* > e_B^*$: in equilibrium, U_A always exerts a larger effort than U_B when chosen for CC.*

Proof. See Appendix F. ■

Though the equilibrium efforts depend on the intensity of the spillovers, the supplier of the preferred product, U_A always provides a larger effort than his competitor when he is chosen for category captain, as in the baseline model without spillovers. The intuition is as follows. As $\rho = 0$, there is no direct effect of the effort exerted by the category captain on the competitor's demand. As U_A enjoys a larger demand, but faces the same cost of effort, the marginal benefit of effort is greater for U_A, and the optimal effort is larger. Note that, when $X = 1$, the effort exerted by U_A and U_B are the same.

5.4.2 Choice of category captain

Category captain vs. no category captain Comparing the industry profit in equilibrium when U_A is chosen for category captain and when U_B is chosen, we obtain the following proposition:

Proposition 13 *In the linear specification with spillovers, the joint profit is always larger when the retailer chooses U_A rather than U_B as category captain.*

Proof. Immediate from the comparison of profits obtained in Appendix F. ■

We show that choosing A as category captain is always a better choice than having no category captain. The effort exerted by the category captain A always increases total profit and tends to increase the status-quo profit of the retailer.

Proposition 14 *In the linear specification with spillovers, when the spillover ρ is large enough (larger than tX), the retailer is better off choosing no category captain.*

Proof. Immediate from the comparison of profits obtained in Appendix F. ∎

When the spillover is sufficiently large, the effect of the category captain on the joint profit may be larger than its effect on the statu-quo profit of the retailer in its negotiation with the category captain. Indeed, when both products are sold, the rival's product also benefits from the category captain's effort. As a consequence, in that case, choosing a category captain increases the profit of both manufacturers but decreases the profit of the retailer.

Choice of category captain Comparing the retailer's profits in the subgames, we obtain the following proposition.

Proposition 15 *If the spillover ρ is lower than tX, then:*

- *When the spillover ρ is low enough, the retailer chooses U_A as a category captain when products are differentiated enough and U_B when products are close enough substitutes.*
- *When the spillover ρ is strong enough, the retailer always chooses U_A as a category captain.*

In figure 4, we depict the choice of category captain by the retailer in the linear specification with $X = 1.09$. The differentiation parameter t is in abscissa while the spillover parameter ρ in ordinate. Below the blue line, B is preferred as category captain while above the retailer selects U_A as category captain. Note that the white area at the top left corresponds to the area where $\rho > tX$, that is the area in which the retailer chooses no category captain. When U_A is the category captain, the turquoise and green areas are where both goods are sold, and in all other areas only U_A is sold. When U_B is the category captain, both goods are sold in the turquoise, green and blue areas; in the yellow area only U_A is sold, and in the pink area only U_B is sold.

Spillovers tend to induce the retailer to choose the efficient supplier as category captain. However, some inefficiencies remain even with strong spillovers.

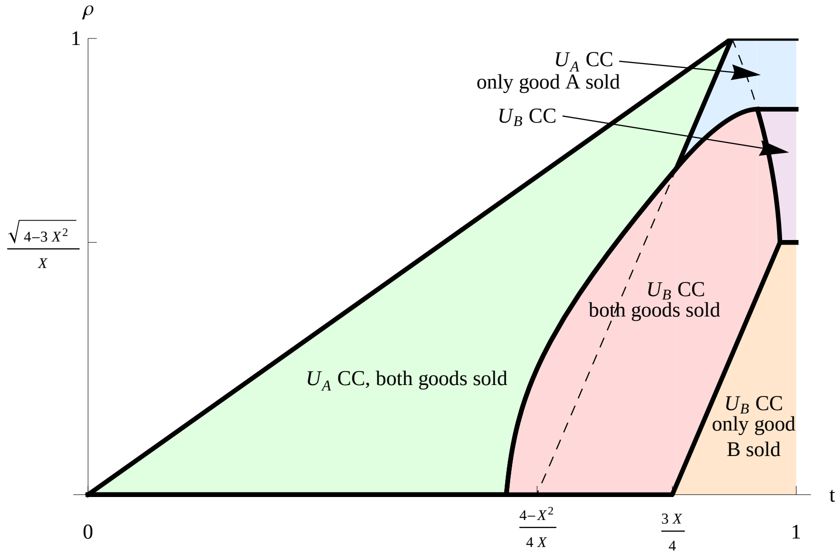

Figure 4: Market structure for X=1.09. t is in abscissa and ρ in ordinate.

6 Conclusion

In this paper, we investigate the choice of category captain by a monopolist retailer. We view the category captain as the provider of a service that boosts demand for all products, including the products of its rival. We consider that the retailer faces two suppliers: a leader, who produces a well-established brand, and a challenger that sells a product facing a smaller demand. We show that, when chosen for category captain, the leader provides a larger effort than the challenger, and the joint profit is then larger. However, choosing the challenger for category captain grants the retailer a better bargaining power, because the retailers' outside option in the negotiation with the leader is then higher. We show that the optimal choice of the retailer depends on the intensity of product competition: if competition is fierce, the retailer is better off choosing the leader, whereas if competition is softer, and spillovers are low enough, the retailer may prefer to select the challenger for category captain.

This model sheds some light on the competitive consequences of category captain arrangements. Empirical evidence is scarce, but recent results by Alan et al (2017) or Kim et al (2016) highlight that, while both the retailer's private label and the captain benefit from CC because of pricing and assortment changes, some competing manufacturers benefit from CC while others suffer. Kim et al moreover show that, despite this anticompetitive effect, consumer surplus is likely to have benefitted from the implementation of category captaincy. Specifically, the manufacturers that closely compete with the captain benefit, whereas the manufacturers that are in close competition with the private label suffer because the retailer protects its private label.

We plan to further develop this analysis along several dimensions. First, we will allow for negative spillovers, that are often mentioned by Competition Authorities as a possible anti-competitive effect of category captaincy agreements. Second, introducing several lines of products will allow us to investigate whether the retailer will choose the same or different category captains on each line.

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Appendix

A Efficient tariffs: proof of lemma 1

Assume U_A is the category captain (the analysis is similar when it is U_B). Consider the negotiation with the rival of the category captain, U_B. The optimal tariff (F_B, w_B) offered by U_B, is the best response to the contract (w_A, F_A, e) offered by U_A. It is such that U_B maximizes its profit provided that the participation constraint of the retailer is satisfied. The participation constraint of the retailer ensures that he is better off accepting the contract offered by U_B and selling product B, rather than either selling product A only, or selling nothing. Then, U_B sets the fixed fee F_B so that the participation constraint of the retailer is binding, that is, omitting the argument when obvious:

$$\begin{aligned}
&\max_{w_B, F_B} && F_B + w_B \tilde{q}_B(.) \\
&s.t. && \sum_{K \in \{A,B\}} (P_K(\tilde{q}_K(.), \tilde{q}_L(.), e, A) - w_K)\tilde{q}_K(.) - F_A - F_B \\
&\geq && \max\{0, (P_A^M(q_A^M(w_A, e, A), e, A) - w_A)q_A^M(w_A, e, A) - F_A\}.
\end{aligned}$$

In equilibrium, the retailer's participation constraint is binding, hence the fixed fee is:

$$F_B = \sum_{K \in \{A,B\}} (P_K(\tilde{q}_K(.), \tilde{q}_L(.), e, A) - w_K)\tilde{q}_K(.) - (P_A^M(q_A^M(w_A, e, A), e, A) - w_A)q_A^M(w_A, e, A).$$

The last term, $P_A^M(q_A^M(w_A, e, A), e, A) - w_A)q_A^M(w_A, e, A))$, is independent of w_B. U_B's program thus boils down to:

$$\max_{w_B} (P_A(\tilde{q}_A(.), \tilde{q}_B(.), e, A) - w_A)\tilde{q}_A(.) + (P_B(\tilde{q}_B(.), \tilde{q}_A(.), e, A))\tilde{q}_B(.).$$

Totally differentiating and reintegrating the retailer's first order conditions determined in Stage 3, we obtain the following simplified first order condition:

$$w_B \frac{d\tilde{q}_B(.)}{dw_B} = 0,$$

The unique solution is $w_B = 0$. Note that second-order conditions are satisfied by assumption.

Consider now the contract offer by U_A. Given (w_B, F_B), the optimal contract (F_A, w_A, e) offered by U_A, is such that U_A maximizes its profit provided that the participation constraint of the retailer is satisfied, that is, omitting the arguments when obvious:

$$\begin{aligned}
\max_{w_A, F_A, e} \quad & F_A + w_A \tilde{q}_A(.) - e^2. \\
s.t. \quad & \sum_{K \in \{A,B\}} (P_K(\tilde{q}_K(.), \tilde{q}_L(.), e, A) - w_K)\tilde{q}_K(.) - F_A - F_B \\
\geq \quad & \max\{0, (P_B^M(q_B^M(w_B, 0, \emptyset), 0, \emptyset) - w_B)q_B^M(w_B, 0, \emptyset) - F_B\}.
\end{aligned}$$

The participation constraint of the retailer determines the fixed fee:

$$F_A = \sum_{K \in \{A,B\}} (P_K(\tilde{q}_K(A), \tilde{q}_L(.), e, A) - w_K)\tilde{q}_K(.) - (P_B^M(q_B^M(w_B, 0, \emptyset), 0, \emptyset) - w_B)q_B^M(w_B, 0, \emptyset).$$

The last term, $(P_B^M(q_B^M(w_B, 0, \emptyset), 0, \emptyset) - w_B)q_B^M(w_B, 0, \emptyset)$, is independent of w_A.

U_A's program thus boils down to:

$$\max_{w_A, e} P_A(\tilde{q}_A(.), \tilde{q}_B(.), e, A)\tilde{q}_A(.) + (P_B(\tilde{q}_B(.), \tilde{q}_A(.), e, A) - w_B)\tilde{q}_B(.) - e^2.$$

Totally differentiating and reintegrating the retailer's first order conditions determined in Stage 3, we obtain the following first order condition with respect to w_A:

$$w_A \frac{d\tilde{q}_A(.)}{dw_A} = 0.$$

The unique solution is such that $w_A = 0$.

Hence the category captain's program simplifies as:

$$\max_e P_A(\tilde{q}_A(.), \tilde{q}_B(.), e, A))\tilde{q}_A(.) + P_B(\tilde{q}_B(.), \tilde{q}_A(.), e, A))\tilde{q}_B(.) - P_B^M(q_B^M(0, A))q_B^M(0, A) - e^2.$$

The category captain chooses the effort that maximizes the total industry profit.

B Impact of a category captain on quantities (purely vertical case)

In this section, we show that, compared to a situation with no category captain, the category captain produces more and the rival less.

Consider first that there is no category captain. The optimal quantities, denoted by $(\tilde{q}_A^{\emptyset}, \tilde{q}_B^{\emptyset})$, are determined by the first-order conditions of the retailer's program:

$$FOC_A^{\emptyset}(q_A, q_B) = \frac{d\pi}{dq_A} = \frac{\partial P_A}{\partial q_A}(q_A, q_B, 0, \emptyset)q_A + P_A(q_A, q_B, 0, \emptyset) + \frac{\partial P_B}{\partial q_A}(q_A, q_B, 0, \emptyset)q_B = 0,$$

$$FOC_B^{\emptyset}(q_A, q_B) = \frac{d\pi}{dq_B} = \frac{\partial P_A}{\partial q_B}(q_B, q_A, 0, \emptyset)q_A + \frac{\partial P_B}{\partial q_B}(q_B, q_A, 0, \emptyset)q_B + P_B(q_B, q_A, 0, \emptyset) = 0.$$

Assume now that K is the category captain, and that it exerts the effort e. The optimal quantities, denoted by $(\tilde{q}_A^K, \tilde{q}_B^K)$, are now determined by the followig first-order conditions:

$$FOC_A^K(q_A, q_B) = \frac{\partial P_A}{\partial q_A}(q_A, q_B, e, K)q_A + P_A(q_A, q_B, e, K) + \frac{\partial P_B}{\partial q_A}(q_A, q_B, e, K)q_B = 0,$$

$$FOC_B^K(q_A, q_B) = \frac{\partial P_A}{\partial q_B}(q_B, q_A, e, K)q_A + \frac{\partial P_B}{\partial q_B}(q_B, q_A, e, K)q_B + P_B(q_B, q_A, e, K) = 0.$$

If $q_L = \tilde{q}_L^{\emptyset}$ and $q_K = \tilde{q}_K^{\emptyset}$, the first order condition that defines q_K is equal to

$$\begin{aligned} FOC_K^K(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}) &= \frac{\partial P_K}{\partial q_K}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K)\tilde{q}_K^{\emptyset} + P_K(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K) + \frac{\partial P_L}{\partial q_K}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K)\tilde{q}_L^{\emptyset} \\ &= \frac{\partial P_K}{\partial q_K}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K)\tilde{q}_K^{\emptyset} + P_K(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K) + \frac{\partial P_L}{\partial q_K}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, 0, \emptyset)\tilde{q}_L^{\emptyset} \end{aligned}$$

By assumption, the second term is such that: $P_K(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K) \geq P_K(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, 0, \emptyset)$

According to Assumption 2, the first term is such that: $\frac{\partial P_K}{\partial q_K}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, e, K)\tilde{q}_K^{\emptyset} = \frac{\partial P_K}{\partial q_K}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}, 0, \emptyset)\tilde{q}_K^{\emptyset}$.

Hence

$$FOC_K^K(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset}) \geq FOC_K^{\emptyset}(\tilde{q}_K^{\emptyset}, \tilde{q}_L^{\emptyset})$$

Therefore, if q_L is unchanged, in equilibrium $\tilde{q}_K^K \geq \tilde{q}_K^{\emptyset}$: the quantity of the good produced by the category captain is larger than when there is no category captain.

Consider now the effect of the introduction of the category captain on the equilibirum quantity sold by its rival. With the category captain, we have

$$\begin{aligned} FOC_L^{\emptyset}(\tilde{q}_K^K, \tilde{q}_L^K) &= \frac{\partial P_K}{\partial q_L}(\tilde{q}_L^K, \tilde{q}_K^K, 0, \emptyset)\tilde{q}_K^K + \frac{\partial P_L}{\partial q_L}(\tilde{q}_L^K, \tilde{q}_K^K, 0, \emptyset)\tilde{q}_L^K + P_L(\tilde{q}_L^K, \tilde{q}_K^K, 0, \emptyset) \\ &= \frac{\partial P_K}{\partial q_L}(\tilde{q}_L^K, \tilde{q}_K^K, 0, \emptyset)\tilde{q}_K^K + \frac{\partial P_L}{\partial q_L}(\tilde{q}_L^K, \tilde{q}_K^K, e, K)\tilde{q}_L^K + P_L(\tilde{q}_L^K, \tilde{q}_K^K, e, K) \end{aligned}$$

by assumption 2 and by the absence of spillovers. Yet $\frac{\partial P_K}{\partial q_L}(\tilde{q}_L^K, \tilde{q}_K^K, 0, \emptyset) \geq \frac{\partial P_K}{\partial q_L}(\tilde{q}_L^K, \tilde{q}_K^K, e, K)$ by Assumption 3. Hence $FOC_L^{\emptyset}(\tilde{q}_K^K, \tilde{q}_L^K) \geq FOC_L^K(\tilde{q}_K^K, \tilde{q}_L^K)$ and thus, if the quantity produced by the category captain was unchanged, the rival would produce less with a category captain than without.

By strategic substitutability of the two products, we have the result.

C Proof of Lemma 3

We want to show that the joint profit is larger when U_A is the CC than when U_B is the CC. In a first step, we compare the industry profit in the two cases, for a given effort e. In a second step, we endogenize the effort choice.

C.1 When $\tilde{q}_A^B \geq \tilde{q}_B^B$

In that case, when U_B is CC, the retailer prefers to sell more product A than B. Consider first the difference of the joint profits, for a given pair of quantities $(\tilde{q}_A^B(e), \tilde{q}_B^B(e))$, when U_B is CC and when U_A is CC:

$$\begin{aligned} \Delta\Pi = \tilde{\Pi}_R(B) - \pi_R(A)(\tilde{q}_A^B(e), \tilde{q}_B^B(e)) \\ = P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B)\tilde{q}_A^B(e) + P_B(\tilde{q}_B(B), \tilde{q}_A(B), e, B)\tilde{q}_B(B) \\ - [P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, A)\tilde{q}_A^B(e) + P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A)\tilde{q}_B^B(e)] \\ = \tilde{q}_A^B(e)[P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) - P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, A)] \\ + \tilde{q}_B^B(e)[P_B\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A)] \end{aligned}$$

where

$$P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) - P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, A) = -\int_0^e \frac{dP_A}{de}(\tilde{q}_A^B(e), \tilde{q}_B^B(e), E, A)dE \leq 0$$

and

$$P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A) = \int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE.$$

Hence

$$\begin{aligned}\Delta\Pi &= -\tilde{q}_A^B(e)\int_0^e \frac{dP_A}{de}(\tilde{q}_A^B(e), \tilde{q}_B^B(e), E, A)dE + \tilde{q}_B^B(e)\int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE \\ &\leq -\tilde{q}_A^B(e)\int_0^e \frac{dP_A}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, A)dE + \tilde{q}_B^B(e)\int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE\end{aligned}$$

under Assumptions 2 and 3 and because $\tilde{q}_A^B(e) \geq \tilde{q}_B^B(e)$.

Yet by assumption,

$$\frac{dP_A}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, A) \geq \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B) \geq 0$$

Hence

$$\begin{aligned}\Delta\Pi &\leq -\tilde{q}_A^B(e)\int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE + \tilde{q}_B^B(e)\int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE \\ &\leq [\tilde{q}_B^B(e) - \tilde{q}_A^B(e)]\int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE\end{aligned}$$

where $[\tilde{q}_B^B(e) - \tilde{q}_A^B(e)] \leq 0$ and $\int_0^e \frac{dP_B}{de}(\tilde{q}_B^B(e), \tilde{q}_A^B(e), E, B)dE \geq 0$.

Hence $\Delta\Pi \leq 0$. We have shown that the optimal joint profit when U_B is CC is lower than the joint profit for the same quantities, when U_A is CC. Yet whenU_A is CC the retailer chooses quantities so as to optimize this joint profit, and the optimal joint profit when U_A is CC is thus larger than $\pi_R(A)(\tilde{q}_A^B(e), \tilde{q}_B^B(e))$. QED.

C.2 When $\tilde{q}_A^B(e) \leq \tilde{q}_B^B(e)$

In that case, when U_B is CC, the retailer prefers to sell more product B than A. Consider first the difference of the joint profits, for a given pair of quantities

$(\tilde{q}_A^B(e), \tilde{q}_B^B(e))$, when U_B is CC and when U_A is CC:

$$\begin{aligned}
\Delta\Pi &= \tilde{\Pi}_R(B) - \pi_R(A)(\tilde{q}_B^B(e), \tilde{q}_A^B(e)) \\
&= P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B)\tilde{q}_A^B(e) + P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B)\tilde{q}_B^B(e) \\
&\quad -[P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, A)\tilde{q}_B^B(e) + P_B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, A)\tilde{q}_A^B(e)] \\
&= \tilde{q}_A^B(e)[P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) - P_B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, A)] \\
&\quad +\tilde{q}_B^B(e)[P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) - P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, A)]
\end{aligned}$$

First, we have

$$P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, A) - P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, B) \geq P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A)$$

because $\frac{\partial P_A}{\partial e}(x, y, e, A) \geq \frac{\partial P_B}{\partial e}(x, y, e, B)$. Hence

$$P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, A) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) \geq P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, B) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A)$$

Second,

$$P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, B) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A) = P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) - P_B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, A)$$

because

$$\begin{aligned}
& P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, B) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A) \\
& -[P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) - P_B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, A)] \\
=\ & P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, B) - P_A(\tilde{q}_A^B(e), \tilde{q}_A^B(e), 0, B) \\
& +P_A(\tilde{q}_A^B(e), \tilde{q}_A^B(e), 0, B) - P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) \\
& -[P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), 0, A) - P_B(\tilde{q}_A^B(e), \tilde{q}_A^B(e), 0, A) \\
& +P_B(\tilde{q}_A^B(e), \tilde{q}_A^B(e), 0, A) - P_B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, A)] \\
=\ & \int_{\tilde{q}_A^B(e)}^{\tilde{q}_B^B(e)} \frac{\partial P_A}{\partial q_A}(t, \tilde{q}_A^B(e), 0, B)dt - \int_{\tilde{q}_A^B(e)}^{\tilde{q}_B^B(e)} \frac{\partial P_B}{\partial q_B}(t, \tilde{q}_A^B(e), 0, A)dt \\
& + \int_{\tilde{q}_B^B(e)}^{\tilde{q}_A^B(e)} \frac{\partial P_A}{\partial q_B}(\tilde{q}_A^B(e), t, 0, B)dt - \int_{\tilde{q}_B^B(e)}^{\tilde{q}_A^B(e)} \frac{\partial P_B}{\partial q_A}(\tilde{q}_A^B(e), t, 0, A)dt] \\
=\ & 0
\end{aligned}$$

as we assumed that $\frac{\partial P_K}{\partial q_K}$ and $\frac{\partial P_K}{\partial q_L}$ do not depend on K and L.

Hence

$$P_A(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, A) - P_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) \geq P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, B) - P_B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), 0, A)$$

As $\tilde{q}_A^B(e) \leq \tilde{q}_B^B(e)$, this yields $\Delta\Pi \leq 0$.

We have shown that the optimal joint profit when U_B is CC is lower than the joint profit when U_A is CC and the quantities are $q_A = \tilde{q}_B^B(e)$ and $q_B = \tilde{q}_A^B(e)$. Yet when U_A is CC the retailer chooses quantities so as to optimize this joint profit, and the optimal joint profit when U_A is CC is thus larger than $\pi_R(A)(\tilde{q}_B^B(e), \tilde{q}_A^B(e))$. QED.

C.3 Comparison of the subgame equilibrium joint profit

We have proven that, for a given effort e, the joint profit is larger when U_A is category captain than when U_B is the category captain. This is true when the effort is e_B^*. Besides we have shown in lemma 5 that the category captain chooses the effort that maximizes the total industry profit. Hence when U_A is the category captain, he chooses e_A^* so as to maximize the joint profit: hence the equilibrium joint profit when U_A is category captain is larger than the joint profit when U_A is category captain and exerts the effort e_B^*. QED.

D Effort choices with a linear demand system and no spillover

Consider the following demand system:

$$\begin{aligned} P_A(q_A, q_B, e, A) &= X(1+e) - q_A - tq_B, & P_B(q_A, q_B, e, A) &= 1 - q_B - tq_A \\ P_A(q_A, q_B, e, B) &= X - q_A - tq_B, & P_B(q_A, q_B, e, B) &= 1 + e - q_B - tq_A. \end{aligned}$$

In this appendix we determine the equilibrium of the subgame composed of Stages 1 to 4. We determine the effort levels depending on the choice of category captain, and the market structure in equilibrium, depending on the values of X and t. To simplify notations, we consider from the start that $w_A = w_B = 0$. We first determine the general form of profits that result from the negotiation, before computing these profit when there is no category captain, when U_A is category captain, and finally when U_B is category captain.

D.1 Stage 3: Negotiation.

Supplier U_K, $K \in \{A, B\}$ wants to maximize the fixed fee F_K provided that the participation constraint of the retailer is satisfied, which implies that the following inequations must be satisfied (for $K, L = A, B$):

$$\sum_{i,j=K,L,i\neq j} P_i(\tilde{q}_i^T(e), \tilde{q}_j^K(e), e, T)\tilde{q}_i^K(e) - F_K - F_L \geq P_K^M(q_K^{M,T}(t), t, T)q_K^{M,T}(t) - F_K$$

where $T \in \{A, B, \emptyset\}$ is the category captain, and $t = e$ if $T = K$ and $t = 0$ otherwise. Note that if there is no category captain, that is $T = \emptyset$, then the effort is by definition $e = 0$.

Each supplier binds the retailer's participation constraint, and therefore the profit of the retailer is given by:

$$\pi_R^T = P_K^M(q_K^{M,T}(t), t, T)q_K^{M,T}(e) + P_L^M(q_L^{M,T}(|e-t|), 0, T)q_L^{M,T}(|e-t|) - \sum_{i,j=K,L,i\neq j} P_i(\tilde{q}_i^T(e), \tilde{q}_j^T(e), e, T)\tilde{q}_i^T$$

and fixed fees are given by:

$$F_K^T = \sum_{i,j=K,L,i\neq j} P_i(\tilde{q}_i^T(e), \tilde{q}_j^T(e), e, T)\tilde{q}_i^T(e) - P_L^M(q_L^{M,T}(|e-t|), |e-t|, T)q_L^{M,T}(0)$$

D.2 No category captain

Assume first that there is no category captain.

If the two producers' offers have been accepted, the retailer sells both products and offers the optimal quantities $\tilde{q}_A^{\emptyset} = \frac{X-t}{2(1-t^2)}$ and $\tilde{q}_B^{\emptyset} = \frac{1-tX}{2(1-t^2)}$ whenever $t \leq \frac{1}{X}$. When $t \geq \frac{1}{X}$, then the retailer is better off selling only product A; he then offers a quantity $\tilde{q}_A^{\emptyset} = \frac{X}{2}$.

If by contrast the retailer has accepted only the offer of U_A, or if it wishes to sell product A only, then it chooses the quantity q_A so as to maximize its profit $P_A^M(q_A)q_A$. The optimal quantity is then $q_A^{M,\emptyset} = \frac{X}{2}$.

Finally if the retailer has accepted only the offer of U_B, or if it wishes to sell product B only, then it chooses the quantity q_B so as to maximize its profit $P_B^M(q_B)q_B$. The optimal quantity is then $q_B^{M,\emptyset} = \frac{1}{2}$.

We obtain the fixed fees and retailer's profit in equilibrium:

- If $t < 1/X$, then both goods are sold in equilibrium, and we have:

$$F_A = \frac{(X-t)^2}{4(1-t^2)}, \qquad F_B = \frac{(1-tX)^2}{4(1-t^2)}, \qquad \pi_R^{\emptyset} = \frac{t(2X-t-tX^2)}{4(1-t^2)}.$$

- If $t \geq 1/X$, only product A is sold, $F_A = \frac{X^2-1}{4}$.

D.3 U_A is category captain

Stages 3 and 4: quantity setting and negotiation. If the two producers' offers have been accepted, the retailer chooses quantities q_A and q_B so as to maximize its profit $p_A(q_A, q_B)q_A + p_B(q_A, q_B)q_B$. The optimal quantities are as follows:

- If $t < 1/X$ and $e < \frac{1-tX}{tX}$, the retailer sells a positive quantity of both products:

$$\tilde{q}_A^A(e) = \frac{X(1+e)-t}{2(1-t^2)}, \qquad \tilde{q}_B^A(e) = \frac{1-tX(1+e)}{2(1-t^2)}.$$

  Final prices in equilibrium are given by:

$$P_A(\tilde{q}_A^A(e), \tilde{q}_B^A(e), e, A) = \frac{X(1+e)}{2}, \qquad P_B(\tilde{q}_B^A(e), \tilde{q}_A^A(e), e, A) = \frac{1}{2}.$$

- Otherwise, the retailer sells only product A:

$$\tilde{q}_A(e, A) = \frac{X(1+e)}{2} = P_A(\tilde{q}_A^A(e), \tilde{q}_B^A(e), e, A), \qquad \tilde{q}_B(e, A) = 0.$$

If by contrast the retailer has accepted only the offer of U_A, or if it wishes to sell product A only, then it chooses the quantity q_A so as to maximize its profit $P_A^M(q_A)q_A$. The optimal quantity and price are then $q_A^{M,A}(e) = \frac{X(1+e)}{2} = p_A^M(q_A^{M,A}(e), e, A)$.

Finally if the retailer has accepted only the offer of U_B, or if it wishes to sell product B only, then it chooses the quantity q_B so as to maximize its profit $P_B^M(q_B)q_B$. The optimal quantity and price are then $q_B^{M,A}(0) = \frac{1}{2} = p_A^M(q_B^{M,A}(0), 0, A)$.

We thus obtain the fixed fees and retailer's profit in equilibrium:

- If $t < 1/X$ and $e < \frac{1-tX}{tX}$, then both goods are sold in equilibrium, and we have:

$$F_A^A(e) = \frac{(X(1+e)-t)^2}{4(1-t^2)}, \quad F_B^A(e) = \frac{(tX(1+e)-1)^2}{4(1-t^2)}, \quad \pi_R^A = \frac{t(2X(1+e)-t(1+X^2(1+e)^2))}{4(1-t^2)}.$$

- Otherwise, only good A is sold in equilibrium, and we have:

$$F_A^A(e) = \frac{X^2(1+e)^2-1}{4}, \qquad F_B^A(e) = 0, \qquad \pi_R^A = \frac{1}{4}.$$

Stage 2: Effort choice of the category captain. The category captain U_A sets its effort e so as to maximize its profit, given by $F_A^A(e) - e^2$. The optimal effort depends on the market structure it yields, since $F_A^A(e)$ depends on the market structure. We thus consider two cases in turn.

First, if $t < 1/X$, then the profit of U_A is $\frac{(X(1+e)-t)^2}{4(1-t^2)}$ if $e < \frac{1-tX}{tX}$ and $\frac{X^2(1+e)^2-1}{4}$ otherwise. The optimal effort is then as follows:

- If $t < \frac{4-X^2}{4}$, $e_A^* = \frac{X(X-t)}{4-4t^2-X^2}$, in which case we have:

$$\begin{aligned}
\pi_A^A(e_A^*) &= \frac{(X(1+e_A^*)-t)^2}{4(1-t^2)} - (e_A^*)^2 = \frac{(X-t)^2}{4-4t^2-X^2}, \\
\pi_R^A(e_A^*) &= \frac{t(2X(1+e_A^*)-t(1+X^2(1+e_A^*)^2))}{4(1-t^2)}, \\
&= \frac{t\left[8X(4-X^2)(1-t^2)+16t^3(1+X^2)-t(16+16X^2-X^4)\right]}{4(4-4t^2-X^2)^2}.
\end{aligned}$$

- If $t \in \left[\frac{4-X^2}{4}, \frac{1}{X}\right]$, $e_A^* = \frac{X^2}{4-X^2}$, in which case we have:

$$\pi_A^A(e_A^*) = \frac{X^2(1+e_A^*)^2 - 1}{4} - (e^*)^2 = \frac{4-5X^2}{4(X^2-4)}, \qquad \pi_R^A(e_A^*) = \frac{1}{4}.$$

Second, if $t \geq 1/X$, then the profit of U_A is $\frac{X^2(1+e)^2-1}{4} - e^2$ for all values of e. The optimal effort of U_A is thus $e_A^* = \frac{X^2}{4-X^2}$, and profits are given above.

D.4 U_B is the Category Captain

Stages 3 and 4: Quantity setting and negotiation. If the two producers' offers have been accepted, the retailer chooses quantities q_A and q_B so as to profit $p_A q_A + p_B q_B$. The optimal quantities are as follows:

- If $t < 1/X$ and $e < \frac{X-t}{t}$ or $t \geq 1/X$ and $e \in \left[tX-1, \frac{X-t}{t}\right]$, the retailer sells a positive quantity of both products:

$$\tilde{q}_A^B(e) = \frac{X - t(1+e)}{2(1-t^2)}, \qquad \tilde{q}_B^B(e) = \frac{1+e-tX}{2(1-t^2)}.$$

  Final prices in equilibrium are given by:

$$\tilde{P}_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, B) = \frac{X}{2}, \qquad \tilde{P}_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) = \frac{1+e}{2}.$$

- If $e > \frac{X-t}{t}$, the retailer sells only product B:

$$\tilde{q}_A^B(e) = 0, \qquad \tilde{q}_B^B(e) = \frac{1+e}{2} = \tilde{P}_B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B).$$

- Finally, if $t > 1/X$ and $e < tX - 1$, the retailer sells only product A:

$$\tilde{q}_A(e, B) = \frac{X}{2} = \tilde{P}_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, B), \qquad \tilde{q}_B(e, B) = 0.$$

If by constrast the retailer has accepted only the offer of producer A, or if it wishes to sell product A only, then it chooses the quantity q_A so as to maximize its profit $p_A^M(q_A)q_A$. Then the optimal quantity is $q_A^{M,B}(0) = \frac{X}{2} = p_A^M(q_A^{M,B}(0), 0, B)$.

If the retailer has accepted only the offer of producer B, then it chooses the quantities q_B so as to maximize his profit $p_B^M(q_B)q_B$. Then the optimal quantity is $q_B^{M,B}(e) = \frac{1+e}{2} = p_B^{M,B}(q_B^{M,B}(e), e, B)$.

We obtain the fixed fees and the retailer's profit in equilibrium:

- If $t < 1/X$ and $e < \frac{X-t}{X}$ or $t \geq 1/X$ and $e \in \left[tX - 1, \frac{X-t}{t}\right]$, then both goods are sold in equilibrium and we have:

$$F_A^B(e) = \frac{(X - t(1+e))^2}{4(1-t^2)}, \qquad F_B^B(e) = \frac{(1+e-tX)^2}{4(1-t^2)},$$
$$\pi_R^B(e) = \frac{t(2X(1+e) - t((1+e)^2 + X^2))}{4(1-t^2)}.$$

- If $e \geq \frac{X-t}{X}$, then only good B is sold in equilibrium, and we have:

$$F_A^B(e) = 0, \qquad F_B^B(e) = \frac{(1+e)^2 - X^2}{4}, \qquad \pi_R^B(e) = \frac{X^2}{4}.$$

- If $t > 1/X$ and $e < tX - 1$, then only good A is sold in equilibrium, and we have:

$$F_A^B(e) = \frac{X^2 - (1+e)^2}{4}, \qquad F_B^B(e) = 0, \qquad \pi_R^B(e) = \frac{(1+e)^2}{4}.$$

Stage 2: Effort choice of the category captain. The category captain U_B sets its effort e to maximize its profit, which is given by $F_B^B(e) - e^2$. Again, the optimal effort depends on the market structure it will yield. We consider two cases in turn.

First, if $t < 1/X$, then $F_B^B(e) = \frac{(1+e-tX)^2}{4(1-t^2)}$ if $e < \frac{X-t}{t}$ and $F_B^B(e) = \frac{(1+e)^2 - X^2}{4}$ otherwise. The optimal effort is then as follows:

- If $X \geq \frac{2}{\sqrt{3}}$ or $X < \frac{2}{\sqrt{3}}$ and $t \leq \frac{3X}{4}$, then $e_B^* = \frac{1-tX}{3-4t^2}$, in which case we have:

$$\pi_B^B(e_B^*) = \frac{(1+e_B^* - tX)^2}{4(1-t^2)} - e_B^{*2} = \frac{(1-tX)^2}{3-4t^2},$$
$$\pi_R^B(e_B^*) = \frac{t(24X(1-t^2) + 16t^3(1+X^2) - t(16+15X^2))}{4(3-4t^2)^2}$$

- otherwise, $e_B^* = \frac{1}{3}$ and

$$\pi_B^B(e_B^*) = \frac{1}{3} - \frac{X^2}{4}, \qquad \pi_R^B(e_B^*) = \frac{X^2}{4}$$

Second, if $t \geq 1/X$, then $F_B^B(e) = 0$ if $e < tX - 1$, $F_B^B(e) = \frac{(1+e-tX)^2}{4(1-t^2)}$ if $e \in [tX-1, \frac{X-t}{X}]$ and $F_B(e, B) = \frac{(1+e)^2 - X^2}{4}$ otherwise. In that case, the optimal effort is $e_B^* = \frac{1}{3}$ if $X \leq \frac{2}{\sqrt{3}}$ (only good B is sold on the market) and 0 otherwise (only good A is sold on the market) and profits are given above.

D.5 Choice of category captain by the retailer

Depending on the identity of the category captain, in equilibrium, effort choices are as follows:

- If $X \leq \frac{2}{\sqrt{3}}$, then:
  - If $t \leq \frac{4-X^2}{4X}$, $e_B^* = \frac{1-tX}{3-4t^2} \leq e_A^* = \frac{X(X-t)}{4-4t^2-X^2}$,
  - If $t \in \left(\frac{4-X^2}{4X}, \frac{3X}{4}\right]$, $e_B^* = \frac{1-tX}{3-4t^2} < \frac{X^2}{4-X^2}$,
  - If $t \in \left(\frac{3X}{4}, 1\right]$, $e_B^* = \frac{1}{3} < \frac{X^2}{4-X^2}$.
- If $X > \frac{2}{\sqrt{3}}$, then:
  - If $t \leq \frac{4-X^2}{4X}$, $e_B^* = \frac{1-tX}{3-4t^2} \leq e_A^* = \frac{X(X-t)}{4-4t^2-X^2}$,
  - If $t \in \left(\frac{4-X^2}{4X}, \frac{1}{X}\right]$, $e_B^* = \frac{1-tX}{3-4t^2} < \frac{X^2}{4-X^2}$,
  - If $t \in \left(\frac{1}{X}, 1\right]$, $e_B^* = 0 < \frac{X^2}{4-X^2}$.

TO BE COMPLETED: CHOICE OF CC

E Joint profit, consumer surplus and welfare (no spillover)

In this appendix we show that the joint profit of the three firms, consumer surplus and welfare are all larger when the retailer chooses U_A as category captain rather than U_B, for all values of t and X.

When U_A is category captain, the joint profit is given by $\tilde{P}_A(e_A^*, A)\tilde{q}_A(e_A^*, A) + \tilde{P}_B(e_A^*, A)\tilde{q}_B(e_A^*, A) - (e_A^*)^2$, that is:

$$\begin{aligned}\pi^A - C(e_A^*) &= \frac{4 - 8tX + 3X^2}{4(4 - 4t^2 - X^2)} \text{ if } t \in [0, \frac{4 - X^2}{4X}), \\ &= \frac{X^2}{4 - X^2} \text{ otherwise.}\end{aligned}$$

When U_B is category captain, the joint profit is given by $\tilde{P}_A(e_B^*, B)\tilde{q}_A(e_B^*, B) + \tilde{P}_B(e_B^*, B)\tilde{q}_B(e_B^*, B) - (e_B^*)^2$, that is:

$$\begin{aligned}\pi^B - C(e_B^*) &= \frac{4(1 - 2tX) + 3X^2}{4(3 - 4t^2)} \text{ if } X \geq \frac{2}{\sqrt{3}} \text{ or } \left\{X < \frac{2}{\sqrt{3}} \text{ and } t \in [0, \frac{3X}{4})\right\}, \\ &= \frac{1}{3} \text{ if } X < \frac{2}{\sqrt{3}} \text{ and } t > \frac{3X}{4}, \\ &= \frac{X^2}{4} \text{ otherwise.}\end{aligned}$$

Comparing joint profits in the two cases, we obtain that the joint profit is always higher when U_A is category captain than when U_B is category captain.

When U_A is category captain, the consumer surplus is given by:

$$\begin{aligned}SC^A &= X(1 + e_A^*)\tilde{q}_A(e_A^*, A) + \tilde{q}_B(e_A^*, A) - \frac{\tilde{q}_A(e_A^*, A)^2 + \tilde{q}_B(e_A^*, A)^2 + 2t\tilde{q}_A(e_A^*, A)\tilde{q}_B(e_A^*, A)}{2} \\ &\quad - \left(\tilde{P}_A(e_A^*, A)\tilde{q}_A(e_A^*, A) + \tilde{P}_B(e_A^*, A))\tilde{q}_B(e_A^*, A)\right),\end{aligned}$$

that is:

$$\begin{aligned}SC^A &= \frac{(4 + X^2)^2 - 8t(t(2 + X^2) + 4X(1 - t^2))}{8(X^2 - 4(1 - t^2))^2} \text{ if } t \in [0, \frac{4 - X^2}{4X}), \\ &= \frac{X^2}{(4 - X^2)^2} \text{ otherwise.}\end{aligned}$$

When U_B is category captain, the consumer surplus is given by:

$$\begin{aligned}SC^B &= X\tilde{q}_A(e_B^*, B) + (1 + e_B^*)\tilde{q}_B(e_B^*, B) - \frac{\tilde{q}_A(e_B^*, B)^2 + \tilde{q}_B(e_B^*, B)^2 + 2t\tilde{q}_A(e_B^*, B)\tilde{q}_B(e_B^*, B)}{2} \\ &\quad - \left(\tilde{P}_A(e_B^*, B)\tilde{q}_A(e_B^*, B) + \tilde{P}_B(e_B^*, B))\tilde{q}_B(e_B^*, B)\right),\end{aligned}$$

that is:

$$\begin{aligned} SC^B &= \frac{8(1-t^2)(2-4tX+X^2)+X^2}{8(3-4t^2)^2} \text{ if } X \geq \frac{2}{\sqrt{3}} \text{ or } \left\{X < \frac{2}{\sqrt{3}} \text{ and } t \in \left[0, \frac{3X}{4}\right]\right\}, \\ &= \frac{2}{9} \text{ if } X < \frac{2}{\sqrt{3}} \text{ and } t > \frac{3X}{4}, \\ &= \frac{X^2}{8} \text{ otherwise.} \end{aligned}$$

Comparing the consumer surplus in the two cases, we obtain that the consumer surplus is always higher when U_A is category captain than when U_B is category captain.

The welfare when U_i is category captain is given by $SC^i + \pi^i - C(e_i^*)$. As a consequence, the welfare is always higher when U_A is category captain than when U_B is category captain.

Finally, without category captain, consumer surplus is given by:

$$\begin{aligned} SC^{\emptyset} &= \frac{1-2tX+X^2}{8(1-t^2)} \text{ if } t \leq \frac{1}{X} \\ &= \frac{X^2}{8} \text{ otherwise} \end{aligned}$$

while the welfare is thrice the consumer surplus.

F Effort choices with spillovers

Consider the following demand system:

$$\begin{aligned} P_A(q_A, q_B, e, A) &= X(1+e) - q_A - tq_B, \quad P_B(q_A, q_B, e, A) = 1 + \rho - q_B - tq_A \\ P_A(q_A, q_B, e, B) &= X(1+\rho e) - q_A - tq_B, \quad P_B(q_A, q_B, e, B) = 1 + e - q_B - tq_A. \end{aligned}$$

In this appendix we determine the equilibrium of the subgame composed of Stages 2 and 3 in the presence of a spillover, specified by $\rho \in [-1, \min\{1, tX\}]$. We have shown in lemma 1 that $w_A = w_B = 0$. We determine the effort levels depending on the choice of category captain, and the market structure in equilibrium, depending on the values of X, t and ρ. We first consider the case when U_A is category captain, and second the case when U_B is category captain.

We assume in this section that, in case of failure in the negotiation between the category captain and the retailer, the category captain does not exert an effort, and therefore the effort cannot benefit the rival manufacturer. Therefore, is U_A is category captain, then in case of negotiation failure between U_A and the retailer, the inverse demand for product B is $P_B^M(q_B, 0, A) = 1 - q_B$; if, however, U_B is categroy captain, then in case of negotiation failure between U_B and the retailer, the inverse demand for product A is $P_A^M(q_A, 0, B) = X - q_A$.

The general expressions of profits and fixed fees are unchanged as compared to the case without spillovers (see Appendix D).

We now determine the equilibrium of the game.

F.1 U_A is category captain

Stage 3: Quantity setting. If the two producers' offers have been accepted, the retailer chooses quantities q_A and q_B so as to maximize its profit $P_A(q_A, q_B, e, A)q_A + P_B(q_B, q_A, e, B)q_B$. The optimal quantities are as follows:

- If $t < 1/X$ and $[e < \frac{1-tX}{tX-\rho}$ or $\rho > tX]$, the retailer sells a positive quantity of both products:

$$\tilde{q}_A^A(e) = \frac{X(1+e) - t(1+\rho e)}{2(1-t^2)}, \qquad \tilde{q}_B^A(e) = \frac{(1+\rho e) - tX(1+e)}{2(1-t^2)}.$$

  Final prices in equilibrium are given by:

$$P_A^A(e) = \frac{X(1+e)}{2}, \qquad P_B(\tilde{q}_B^A(e), \tilde{q}_A^A(e), e, A) = \frac{(1+\rho e)}{2}.$$

- Otherwise, the retailer sells only product A:

$$\tilde{q}_A^A(e) = \frac{X(1+e)}{2} = P_A(\tilde{q}_A^A(e), \tilde{q}_B^A(e), e, A), \qquad \tilde{q}_B^A(e) = 0.$$

If, by contrast, the retailer has accepted only the offer of U_A, or if it wishes to sell product A only, then it chooses the quantity q_A so as to maximize its profit $p_A^M(q_A, e, A)q_A$. The optimal quantity is $q_A^{M,A}(e) = \frac{X(1+e)}{2} = p_A^M(q_A^{M,A}(e), e, A)$.

Finally if the retailer has accepted only the offer of U_B, or if it wishes to sell product B only, then it chooses the quantity q_B so as to maximize its profit $P_B^M(q_B, 0, A)q_B$. The optimal quantity is then $q_B^{M,A}(0) = \frac{1}{2} = P_B^M(q_B^{M,A}(0), 0, A)$.

Stage 2: Negotiation and effort choice. The retailer's participation constraints yields the fixed fees and retailer's profit in equilibrium:

- If $t < 1/X$ and $[e < \frac{1-tX}{tX-\rho}$ or $\rho > tX]$, then both goods are sold in equilibrium, and we have:

$$F_A^A(e) = \frac{(X(1+e)-t)^2 - \rho e(2(tX(1+e)-1)-\rho e)}{4(1-t^2)},$$
$$F_B^A(e) = \frac{(tX(1+e)-1-\rho e)^2}{4(1-t^2)},$$
$$\pi_R^A(e) = \frac{-e^2(\rho - tX)^2 - 2e(\rho - tX)(1-tX) - t(t-2X+tX^2)}{4(1-t^2)}.$$

- Otherwise, only good A is sold in equilibrium, and we have:

$$F_A^A(e) = \frac{X^2(1+e)^2-1}{4}, \qquad F_B^A(e) = 0, \qquad \pi_R^A(e) = \frac{1}{4}.$$

The category captain U_A sets its effort e so as to maximize its profit, given by $F_A^A(e) - e^2$. The optimal effort depends on the market structure it yields, since $F_A^A(e)$ depends on the market structure. We thus consider two cases in turn.

First, if $t < 1/X$, then $F_A^A(e) = \frac{(X(1+e)-t)^2-\rho e(2(tX(1+e)-1)-\rho e)}{4(1-t^2)}$ if $e < \frac{1-tX}{tX}$ and $F_A^A(e) = \frac{X^2(1+e)^2-1}{4}$ otherwise. The optimal effort is then as follows:

- If $t < \frac{2-X^2}{2X}$, $e_A^* = \frac{X(X-t)+\rho(1-tX)}{4-4t^2-X^2-\rho(\rho-2tX)}$, in which case we have:

$$\begin{aligned}
\pi_A^A(e_A^*) &= \frac{(X(1+e_A^*)-t)^2 - \rho e_A^*(2(tX(1+e_A^*)-1)-\rho e_A^*)}{4(1-t^2)} - (e_A^*)^2 \\
&= \frac{4(X-t)^2 - \rho(2X(t-X)-\rho(1-X^2))}{4(4-4t^2-X^2-\rho(\rho-2tX))}, \\
\pi_R^A(e_A^*) &= \frac{e_A^*(tX-\rho)(2+\rho e_A^* - tX(2+e_A^*)) - t(t-2X+tX^2)}{4(1-t^2)}, \\
&= \frac{2(\rho-tX)(1-tX)(X(t-X)-\rho(1-tX))}{4(1-t^2)(4-4t^2-X^2-\rho(\rho-2tX))} - \frac{t(t-2X+tX^2)}{4(1-t^2)} - \frac{(\rho-tX)^2(X(t-X)-\rho(1-tX))^2}{4(1-t^2)(4-4t^2-X^2-\rho(\rho-2tX))^2}
\end{aligned}$$

- If $t \in \left[\frac{2-X^2}{2X}, \frac{1}{X}\right]$, then:
  - If $\rho > \frac{X^2+4tX-4}{X^2}$, the effort is $e_A^* = \frac{X(X-t)+\rho(1-tX)}{4-4t^2-X^2-\rho(\rho-2tX)}$, both goods are sold and the profits of U_A and of the retailer are given above.
  - If $\rho \leq \frac{X^2+4tX-4}{X^2}$, $e_A^* = \frac{X^2}{4-X^2}$, in which case we have:

$$\pi_A^A(e_A^*) = \frac{X^2(1+e_A^*)^2 - 1}{4} - (e_A^*)^2 = \frac{5X^2-4}{4(4-X^2)}, \qquad \pi_R^A(e_A^*) = \frac{1}{4}.$$

Second, if $t \geq 1/X$, then $F_A^A(e) = \frac{X^2(1+e)^2-1}{4}$ for all values of e. The optimal effort of U_A is thus $e_A^* = \frac{X^2}{4-X^2}$, and profits are given above.

F.2 U_B is category captain

Stage 3: Quantity choices by the retailer. If the two producers' offers have been accepted, the retailer chooses quantities q_A and q_B so as to maximize its profit profit $p_A q_A + p_B q_B$. The optimal quantities are as follows.

- If $(t < 1/X \cap ((\rho < t/X \cap e < \frac{X-t}{t-\rho X}) \cup \rho \geq t/X))$ or $(t \geq 1/X \cap ((\rho < \frac{t}{X} \cap \frac{1-tX}{\rho tX-1} < e < \frac{X-t}{t-\rho X}) \cup (\frac{t}{X} \leq \rho \leq \frac{1}{tX} \cap e > \frac{1-tX}{\rho tX-1})))$, the retailer sells a positive quantity of both products:

$$\tilde{q}_A^B(e) = \frac{X(1+\rho e) - t(1+e)}{2(1-t^2)}, \qquad \tilde{q}_B^B(e) = \frac{1+e-tX(1+\rho e)}{2(1-t^2)}.$$

  Final prices in equilibrium are given by:

$$P_A^B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, B) = \frac{X(1+\rho e)}{2}, \quad P_B^B(\tilde{q}_B^B(e), \tilde{q}_A^B(e), e, B) = \frac{1+e}{2}.$$

- If $\rho < t/X$ and $e > \frac{X-t}{t-\rho X}$, the retailer sells only product B:

$$\tilde{q}_A^B(e) = 0, \qquad \tilde{q}_B^B(e) = \frac{1+e}{2} = P_B^B(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, B).$$

- Finally, if $t > 1/X$ and $((\rho < \frac{1}{tX} \cap e < \frac{tX-1}{1-\rho tX}) \cup \rho \geq \frac{1}{tX})$, the retailer sells only product A:

$$\tilde{q}_A^B(e) = \frac{X(1+\rho e)}{2} = P_A(\tilde{q}_A^B(e), \tilde{q}_B^B(e), e, B), \qquad \tilde{q}_B^B(e) = 0.$$

If by constrast the retailer has accepted only the offer of producer A, or if it wishes to sell product A only, then it chooses the quantities q_A so as to maximize its profit $p_A^M(q_A)q_A$. Then the optimal quantity is $q_A^{M,B}(e) = \frac{X}{2}$, and the resulting outside-option for the retailer is $\frac{X^2}{2} - F_A$. If the retailer has accepted only the offer of producer B, then it chooses the quantities q_B so as to maximize his profit $p_B^M(q_B)q_B$. Then the optimal quantity is $q_B^{M,B}(e) = \frac{1+e}{2} = p_B^M(q_B^{M,B}(e), e, B)$.

Stage 2: Negotiation and effort choice. The retailer's participation constraints yields the fixed fees and retailer's profit in equilibrium:

- If $(t < 1/X \cap ((\rho < t/X \cap e < \frac{X-t}{t-\rho X}) \cup \rho \geq t/X))$ or $(t \geq 1/X \cap ((\rho < t/X \cap \frac{1-tX}{\rho tX-1} < e < \frac{X-t}{t-\rho X}) \cup (t/X \leq \rho \leq \frac{1}{tX} \cap e > \frac{1-tX}{\rho tX-1})))$, then both goods are sold in equilibrium and we have:

$$\begin{aligned} F_A^B(e) &= \frac{(X(1+\rho e) - t(1+e))^2}{4(1-t^2)}, \\ F_B^B(e) &= \frac{(1+e)^2 - 2(1+e)(1+\rho e)tX + (e\rho(2+e\rho)+t^2)X^2}{4(1-t^2)}, \\ \pi_R^B(e) &= \frac{2tX(1+e)(1+\rho e) - \rho eX^2(2+\rho e) - t^2((1+e)^2+X^2)}{4(1-t^2)}. \end{aligned}$$

- If $\rho < t/X$ and $e > \frac{X-t}{t-\rho X}$, then only good B is sold in equilibrium, and we have:

$$F_A^B(e) = 0, \qquad F_B^B(e) = \frac{(1+e)^2 - X^2}{4}, \qquad \pi_R^B(e) = \frac{X^2}{4}.$$

- If $t > 1/X$ and $((\rho < \frac{1}{tX} \cap e < \frac{tX-1}{1-\rho tX}) \cup \rho \geq \frac{1}{tX})$, then only good A is sold in equilibrium. However, because U_B makes an effort as category captain, the retailer still pays U_B to ensure that the effort benefits its relationship with

U_A. We thus have:

$$F_A^B(e) = \frac{X^2(1+\rho e)^2 - (1+e)^2}{4}, \qquad F_B^B(e) = \frac{\rho e X^2(2+\rho e)}{4},$$
$$\pi_R^B(e) = \frac{(1+e)^2 - X^2\rho e(2+\rho e)}{4}.$$

The category captain U_B sets its effort e to maximize its profit, which is given by $F_B^B(e) - e^2$. Again, the optimal effort depends on the market structure it will yield. We consider two cases in turn.

1. Assume first that $t < 1/X$, then:

   - if $\rho < t/X$, the fixed fee paid by the retailer to U_B is given by: $F_B^B(e) = \frac{(1+e)^2-2(1+e)(1+\rho e)tX+(e\rho(2+e\rho)+t^2)X^2}{4(1-t^2)}$ if $e < \frac{X-t}{t-\rho X}$ and $F_B^B(e) = \frac{(1+e)^2-X^2}{4}$ otherwise.
   - if $\rho \geq t/X$, then both goods are sold on the final market, and $F_B^B(e) = \frac{(1+e)^2-2(1+e)(1+\rho e)tX+(e\rho(2+e\rho)+t^2)X^2}{4(1-t^2)}$ for all e.

   The optimal effort is then as follows.

   - First, if $X < \frac{2}{\sqrt{3}}$, $t \geq \frac{3X}{4} - \frac{\sqrt{4-3X^2}}{4}$ and $\rho < \frac{4t-3X}{X}$ then $e_B^* = 1/3$, in which case B is the only product sold and we have:

   $$\pi_B^B(e_B^*) = \tfrac{(1+e_B^*)^2-X^2}{4} - (e_B^*)^2 = \frac{1}{3} - \frac{X^2}{4}, \qquad \pi_R^B(e_B^*) = \frac{X^2}{4}.$$

   - Second, if
     - $X < \frac{2}{\sqrt{3}}$ and $[\frac{X}{2} \leq t \leq \frac{3X}{4} - \frac{\sqrt{4-3X^2}}{4}$ and $\rho < \frac{4t-3X}{X}$, or $t \leq \frac{-X}{4} + \frac{\sqrt{3(4-X^2)}}{4}$ and $\rho \leq \frac{1-tX}{tX-X^2}$, or $\frac{-X}{4} + \frac{\sqrt{3(4-X^2)}}{4} \leq t \leq \frac{3X}{4} - \frac{\sqrt{4-3X^2}}{4}$ and $-\sqrt{\frac{3(1-t^2)}{X^2}} + \frac{t}{X} \leq \rho \leq \frac{1-tX}{tX-X^2}]$; or
     - $\frac{2}{\sqrt{3}} \leq X \leq \sqrt{2}$ and $[\frac{X}{2} \leq t \leq \frac{1}{X}$ and $\rho \leq \frac{4t-3X}{X}$, or $t \leq \frac{-X}{4} + \frac{\sqrt{3(4-X^2)}}{4}$ and $\rho \leq \frac{1-tX}{tX-X^2}$, or $\frac{-X}{4} + \frac{\sqrt{3(4-X^2)}}{4} \leq t \leq \frac{1}{X}$ and $-\sqrt{\frac{3(1-t^2)}{X^2}} + \frac{t}{X} \leq \rho \leq \frac{1-tX}{tX-X^2}]$; or
     - $\frac{-X}{4} + \frac{\sqrt{3(4-X^2)}}{4} \leq t \leq \frac{1}{X}$ and $\rho \leq -\sqrt{\frac{3(1-t^2)}{X^2}} + \frac{t}{X}$ and $[t \leq \frac{X}{2}$ and $\rho \leq \frac{t}{X}$, or $\frac{X}{2} \leq t \leq \frac{1}{X}$ and $\frac{4t-3X}{X} \leq \rho \leq \frac{t}{X}]$.

then the optimal effort of U_B is zero: $e_B^* = 0$, in which case both goods are sold and we have

$$F_A^B(0) = \frac{(X-t)^2}{4(1-t^2)},$$
$$F_B^B(0) = \frac{1-2tX+t^2X^2}{4(1-t^2)},$$
$$\pi_R^B(0) = \frac{2tX - t^2(1+X^2)}{4(1-t^2)}.$$

- Otherwise, $e_B^* = \frac{1-(1+\rho)tX+\rho X^2}{3-4t^2+\rho X(2t-\rho X)}$, in which case both goods are sold and we have:

$$\pi_R^B(e_B^*) = \frac{-t^2(1+e_B^*)^2 + 2(1+e_B^*)(1+\rho e_B^*)tX - (e_B^*\rho(2+e_B^*\rho)+t^2)X^2}{4(1-t^2)}$$
$$= \frac{4tX(6-\rho^3X^4+\rho(2+3X^2))+\rho X^2(\rho^3X^4-\rho(1+6X^2)-6)-8t^3X(2\rho X^2+\rho+3)}{4(-3+4t^2-2\rho tX+\rho^2X^2)^2}$$
$$+ \frac{t^2(X^2(\rho(12\rho X^2+\rho+6)-15)-16)+16t^4(X^2+1)}{4(-3+4t^2-2\rho tX+\rho^2X^2)^2}, \qquad (4)$$

and

$$\pi_B^B(e_B^*) = \frac{(1-tX)^2 + 2e_B^*(\rho X^2 - tX(1+\rho)+1) + (e_B^*)^2(1-2\rho tX+\rho^2X^2)}{4(1-t^2)} - (e_B^*)^2$$
$$= \frac{(1-t^2)((4t-\rho X)(6X+\rho X-4t)-16tX^2(t-\rho X))-6\rho^2X^4(1-2t^2)-4\rho tX^3(1+\rho^2X^2)+\rho^4X^6+t^2X^2}{4(-3+4t^2-2\rho tX+\rho^2X^2)^2} \qquad (5)$$

2. Second, if $t \geq 1/X$, then three cases may occur depending on the value of ρ.

   - For $\rho > 1/(tX)$, then only A is sold and $F_B^B(e) = \frac{X^2\rho e(2+\rho e)}{4}$ regardless of e,
   - For $\rho \in \left[\frac{t}{X}, \frac{1}{tX}\right]$, then $F_B^B(e) = \frac{X^2\rho e(2+\rho e)}{4}$ if $e < \frac{1-tX}{\rho tX-1}$ and $F_B^B(e) = \frac{(1+e)^2-2(1+e)(1+\rho e)tX+(e\rho(2+e\rho)+t^2)X^2}{4(1-t^2)}$ otherwise (both goods are then sold).
   - For $\rho < t/X$, $F_B^B(e) = \frac{X^2\rho e(2+\rho e)}{4}$ if $e < \frac{1-tX}{\rho tX-1}$, $F_B^B(e) = \frac{(1+e)^2-2e(1+e)(1+\rho e)tX+(e\rho(2+e\rho)+t^2)X^2}{4(1-t^2)}$ if $e \in \left[\frac{1-tX}{\rho tX-1}, \frac{X-t}{r-\rho X}\right]$ and $F_B^B(e) = \frac{(1+e-X)(1+e+X)}{4}$ otherwise (that is, when only B is sold).

The optimal effort is then as follows:

- For $X \leq \frac{4}{\sqrt{13}}$, then:
  - If $t \in \left[\frac{1}{X}, \frac{3X+\sqrt{4-3X^2}}{4}\right]$, then $e_B^* = 1/3$ for $\rho < max\left\{0, \frac{4t-3X}{X}\right\}$; $e_B^* = \frac{1-(1+\rho)tX+\rho X^2}{3-4t^2+\rho X(2t-\rho X)}$ for $\rho \in \left[\frac{4t-3X}{X}, \frac{1}{2} + \frac{\sqrt{16-16tX+X^2}}{2X}\right]$; otherwise $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$, in which case profits are as follows:

$$\pi_B^B(e_B^*) = \frac{(2+\rho e_B^*)\rho e_B^* X^2}{4} - (e_B^*)^2 = \frac{\rho^2 X^2}{4(4-\rho^2 X^2)}$$

$$\pi_R(e_B^*) = \frac{(1+e_B^*)^2 - (2+\rho e_B^*)\rho e_B^* X^2}{4}$$

$$= \frac{4+4(1-\rho)\rho X^2 - (3-\rho)(1+\rho)\rho^2 X^4 + \rho^4 X^6}{4(2-\rho^2 X^2)^2}.$$

  - If $t \in \left[\frac{3X+\sqrt{4-3X^2}}{4}, 1\right]$, then $e_B^* = 1/3$ for $\rho < max\left\{0, \frac{\sqrt{4-3X^2}}{X}\right\}$; $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$ otherwise. Profits are given above.
- For $X \in \left[\frac{4}{\sqrt{13}}, \frac{2}{\sqrt{3}}\right]$, then:
  - If $t \in \left[\frac{1}{X}, \frac{3X+\sqrt{4-3X^2}}{4}\right]$, then $e_B^* = 1/3$ for $\rho < max\left\{0, \frac{4t-3X}{X}\right\}$; $e_B^* = \frac{1-(1+\rho)tX+\rho X^2}{3-4t^2+\rho X(2t-\rho X)}$ for $\rho \in \left[\frac{4t-3X}{X}, \frac{1}{2} + \frac{\sqrt{16-16tX+X^2}}{2X}\right]$; $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$ otherwise. Profits are given above.
  - If $t \in \left[\frac{3X+\sqrt{4-3X^2}}{4}, \frac{16+X^2}{16X}\right]$, then $e_B^* = 1/3$ for $\rho < max\left\{0, \frac{\sqrt{4-3X^2}}{X}\right\}$; $e_B^* = \frac{1-(1+\rho)tX+\rho X^2}{3-4t^2+\rho X(2t-\rho X)}$ for $\rho \in \left[\frac{1}{2} - \frac{\sqrt{16-16tX+X^2}}{2X}, \frac{1}{2} + \frac{\sqrt{16-16tX+X^2}}{2X}\right]$; $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$ otherwise. Profits are given above.
  - Finally, if $t \in \left[\frac{16+X^2}{16X}, 1\right]$, then $e_B^* = 1/3$ for $\rho < max\left\{0, \frac{\sqrt{4-3X^2}}{X}\right\}$ and $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$ otherwise. Profits are given above.
- Finally, for $X \in \left[\frac{2}{\sqrt{3}}, \sqrt{2}\right]$:
  - If $\rho \leq 0$,
    * For $\frac{2}{\sqrt{3}} \leq X \leq \sqrt{2}$ and $\frac{1}{X} \leq t \leq \frac{\sqrt{3}}{2}$ and $\frac{t}{X} - \frac{\sqrt{3(1-t^2)}}{X} \leq \rho \leq 0$, then $e_B^* = \frac{1-(1+\rho)tX+\rho X^2}{3-4t^2+\rho X(2t-\rho X)}$; both goods are sold and profits are given above;
    * For $\frac{2}{\sqrt{3}} \leq X \leq \frac{4}{3}$ and [$t \geq \frac{3X}{4}$, or $\frac{1}{X} \leq t \leq \frac{3X}{4}$ and $-1 \leq \rho \leq \frac{4t-3X}{X}$], or for $\frac{4}{3} \leq X \leq \sqrt{2}$ and [$\frac{1}{X} \leq t$ and $\rho \leq \frac{4t-3X}{X}$] then

$e_B^* = 1/3$ and B is the only product sold. Profits are given above.

  * Otherwise $e_B^* = 0$ and only A is sold. Profits are given above.

- If $\rho \geq 0$,
  * If $t \in \left[\frac{1}{X}, \frac{16+X^2}{16X}\right]$, then $e_B^* = \frac{1-(1+\rho)tX+\rho X^2}{3-4t^2+\rho X(2t-\rho X)}$ for $\rho \in \left[\frac{1}{2} - \frac{\sqrt{16-16tX+X^2}}{2X}, \frac{1}{2} + \frac{\sqrt{16-16tX+X^2}}{2X}\right]$, and $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$ otherwise. Profits are given above.
  * If $t \in \left[\frac{16+X^2}{16X}, 1\right]$ then $e_B^* = \frac{\rho X^2}{4-\rho^2 X^2}$ for all values of $\rho \in [0, 1]$.

Finally, we use Mathematica to compute the optimal choice of category captain as displayed in Figure 5.4.2.