Start-up acquisitions and innovation strategies

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Abstract

This paper provides a theory of strategic innovation project choice by incumbents and start-ups. We apply this theory to identify the effects of prohibiting start-up acquisitions. We differentiate between killer acquisitions (when the incumbent does not commercialize the acquired start-up's technology) and acquisitions with commercialization. A restrictive acquisition policy reduces the variety of research approaches pursued by the firms and thereby the probability of discovering innovations. Furthermore, it leads to strategic duplication of the entrant's innovation by the incumbent. These negative innovation effects of restrictive acquisition policy have to be weighed against the pro-competitive effects of preserving potential competition.

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1 Introduction

In the last few decades, merger policy in Europe and the U.S. has focused mostly on horizontal and, to a lesser extent, on vertical mergers. Even horizontal mergers rarely trigger interventions unless they are expected to involve substantial additions of market shares of incumbents. Recently, many competition policy practitioners and academics have argued that this approach to merger control may be flawed. Reflecting the growing importance of the digital economy, there is an increasing concern that even mergers between firms that are not currently competing in the same market might be problematic because they eliminate potential competition.[1] Such worries even arise when "the target firm has no explicit or immediate plans to challenge the incumbent firm on its home turf, but is one of several firms that is best placed to do so in the next several years" (Shapiro, 2018). However, the issue becomes more pressing when there are strong signs that potential entry could materialize in the short term and that the main rationale for the merger is to avoid such entry. It is well-known that incumbent firms in the digital economy and other sectors, including for instance the pharmaceutical industry, systematically buy up innovative start-ups. For instance, Alphabet, Amazon, Apple, Facebook and Microsoft acquired start-ups worth a total of 31.6 billion USD in 2017. Google has acquired about one firm per month between 2001 and 2018 (Bourreau and de Streel, 2019).[2] There are several conceivable motives for such behavior. For instance, the acquiring firms may be better at commercializing the ideas of the start-ups, so that an acquisition may be efficient. Recent evidence suggests, however, that anti-competitive motives may also be important. The work of Cunningham, Ederer and Ma (2019) for the pharmaceutical industry is a particularly compelling case in point. The authors argue that incumbent firms often engage in so-called *killer acquisitions* by purchasing start-ups with the sole purpose of eliminating potential competition without intending to commercialize the entrant's innovation.[3] Even when incumbents do commercialize the innovation, there are concerns that this can widen the technological lead of a dominant incumbent, making entry ever harder (e.g. Bryan and Hovenkamp, 2019).

Such considerations suggest rethinking what arguably is standard practice in most jurisdictions, namely to wave through acquisitions of small innovative start-ups by incumbent firms. Nevertheless, there are several open issues. First, the current debate is not very clear about whether acquisitions with or without commercialization are more problematic; as suggested above, different authors emphasize different problems. Second, as several observers have pointed out, even pure killer acquisitions may not only have negative welfare effects.[4] Whether the acquirer commercializes the entrant's product or not, the prospect of selling the shop to the incumbent should have positive effects on the entrant's incentive to engage in innovation in the first place. Indeed, going back at least to Rasmussen

[1]This concern is reflected in policy reports such as Crémer, de Montjoye and Schweitzer (2019) for the European Commission, the *Furman Report* in the UK or the *Final Report of the Stigler Committee on the Digital Economy*, available on https://research.chicagobooth.edu/stigler/media/news/committee-on-digital-platforms-final-report; see also Salop (2016), Salop and Shapiro (2017), Hovenkamp and Shapiro (2017), Bryan and Hovenkamp (2020).

[2]Examples include Facebook's takeovers of WhatsApp, Instagram and Oculus CR, Google's acquisition of DoubleClick, Waze and YouTube, and Microsoft's purchases of GitHub and LinkedIn.

[3]The use of the "killer" metaphor in the literature is not uniform. For instance, by contrast with Cunningham et al. (2019), other authors apply the expression "kill zone" to start-up activities that are so close to those of dominant incumbents that they may trigger acquisitions, without implying that the incumbent would not commercialize the start-up's technologies.

[4]See for instance Bourreau and de Streel (2019) and Crémer et al. (2019) and the *Furman Report*.

(1988), several academic papers have made this point in formal models (see the discussion in Section 2). Third, the literature says very little on the effects of merger policy on the incumbent's innovation incentives. All told, it is not clear under which circumstances positive innovation effects will exist and when they will dominate the adverse effects on static competition.

Against this background, our paper engages in an in-depth analysis of the innovation effects of start-up acquisitions that incorporates cases with and without commercialization. To this end, we provide a theory of the strategic choice of innovation projects by incumbents and start-ups. Contrary to most models in the literature, which only analyze how much resources firms invest in innovation, we allow firms to choose which innovation projects to invest in as well as how much to invest in those projects. Ex ante, projects only differ with respect to investment costs; ex post, only one of them will lead to an innovation. In our setting, firms strategically select their research projects, taking into account the competitor's choice. To see why project choice matters, consider an incumbent who wants to stifle competition. Acquiring an innovative entrant is one way to achieve this goal, but alternatively the incumbent can engage in own innovation activities, deliberately investing into the same projects that the entrant also invests in. This can help to preempt successful innovations that would allow the entrant to compete. Contrary to innovations that expand the technological frontier or entrant innovations that enhance competition, innovations by the incumbent into projects that the entrant is also investing in turn out to be socially undesirable: They create additional costs and soften competition, without generating any countervailing benefit. With these considerations in mind, our framework allows firms to choose how much to invest in each conceivable research project. Our analysis concludes that prohibiting start-up acquisitions reduces the variety of innovation projects and, on a closely related note, the probability of innovation. This is true in spite of countervailing effects that prohibiting acquisitions has on entrants and incumbents. Moreover, we show that prohibiting killer acquisitions encourages socially useless investments by the incumbent which duplicate those of the entrant.

In our model, an incumbent monopolist possesses a technology that allows her to operate in a product market without incurring any further innovation cost. By contrast, an entrant has to innovate in order to compete. If the entrant successfully innovates, this can either result in a drastic or a non-drastic innovation. If commercialized, a drastic innovation would turn the entrant into a monopolist (with higher profits than the incumbent previously obtained), whereas the non-drastic innovation would merely allow the entrant to compete with the incumbent. The incumbent can also engage in innovation activities. A drastic innovation allows her to increase her monopoly profits, whereas a non-drastic innovation serves a preemptive purpose by preventing the entrant from becoming a viable competitor. Once the innovation outcomes become common knowledge, the incumbent can offer to acquire the entrant. We assume that an acquisition takes place if and only if it increases joint profits, where the trading surplus is split according to exogenously given shares reflecting bargaining power.[5] The firm possessing the innovation technology will then decide whether to commercialize it at some fixed cost or not. We will allow for both possibilities, killer acquisitions in the narrow sense (without commercialization of the entrant's technology) and acquisitions with commercialization of the new technology.

We first describe the equilibrium structure in a laissez-faire regime where acquisitions

[5] This assumption is in line with several related papers, e.g., Phillips and Zhdanov (2013), Cabral (2018) and Kamepalli, Rajan and Zingales (2019)

are allowed but where the incumbent does not want to commercialize the technology after the acquisition (i.e., the killer acquisitions case). Though multiple equilibria may arise, they display some important common features. In any equilibrium of the game, both firms invest the maximal amount in all projects that are relatively cheap, and neither firm invests at all in the most expensive projects. Moreover, in all equilibria, there exists an interval of projects in which only the entrant invests, and these are the most expensive projects either firm invests in. This will be crucial for our main result because it implies that it is the entrant whose incentives determine the variety of innovation projects. We also show how the variety of innovation projects depends on characteristics of the market environment: It is increasing in the entrant's bargaining power and the probability that a successful innovation is drastic, and it can either increase or decrease in the intensity of potential competition.

Our main result compares the equilibrium structure under laissez-faire with the one that emerges when killer acquisitions are not allowed. There are two main differences between the two cases. First, the variety of research projects which are developed is higher in any equilibrium with acquisitions than in any equilibrium without acquisitions. This observation reflects the entrant's enhanced incentives to invest in costly projects when a sale to the incumbent is a possible option. As a result of the increase in the variety of projects invested in, the probability of innovation is also larger under a laissez-faire policy. Second, when start-up acquisitions are allowed, the incumbent has less incentives for socially wasteful investments in projects that the entrant also invests in. Rather than engaging in such activities, the incumbent can acquire the entrant if this serves to reduce competition. These clear results show the importance of taking into account that firms have different types of research projects to invest in: Prohibiting acquisitions does not have a clear-cut negative effect on overall innovation efforts. While it indeed reduces the entrant's incentives to invest in socially useful projects, it increases the incumbent's incentives to duplicate the entrant's investment. All told, therefore, the prohibition of killer acquisitions tends to affect investments in an undesirable way, leading to a lower probability of innovation, without necessarily reducing overall research expenses. Thus, in spite of the heterogeneous policy adjustments of the two firms, from a welfare perspective, the innovation effect of prohibiting killer acquisitions is unambiguously negative in our setting. Summing up, our analysis thus suggests that the familiar anti-competitive concerns about start-up acquisitions need to be balanced against the positive effects on innovation.

Importantly, our framework can also be applied in the case when the non-drastic innovation is large enough that the incumbent would want to commercialize it after an acquisition. It turns out that the insights are similar to the case of killer acquisitions. In particular, the negative effects of start-up acquisitions on static competition often remain present even if the incumbent commercializes the innovation. For some parameter values, however, the innovation effect disappears.

Going beyond our model, it even seems conceivable that acquisitions with commercialization may be more problematic than those without. For instance, when incumbents buy up small start-ups with the goal of using these firms' knowledge (rather than merely killing the potential entrant), they can combine this knowledge with their own knowledge to expand their technological lead. This is likely to restrict potential competition in the long term by reducing incentives for innovation.[6] This suggests that, paradoxically, acquisitions with commercialization might be a larger problem than killer acquisitions.

[6]This argument is reminiscent of some discussions in Cabral (2018).

Section 2 reviews the literature. Section 3 introduces the assumptions and analyzes the acquisition, commercialisation and product market stages. Section 4 characterizes the innovation behavior when the value of non-drastic innovations is so low that the incumbent will not commercialize them; Section 5 deals with the complementary case with commercialization. Section 6 provides a discussion. Section 7 concludes. All proofs are in the appendix.

2 Relation to the literature

Cunningham et al. (2019) not only provide empirical evidence for the existence of killer acquisitions, but also develop a theoretical model to explain the rationale behind discontinuing development. However, their model does not analyze innovation decisions, which is the focus of our paper. Our analysis links two research areas. One area asks how mergers between incumbents affect their innovation and those of competitors. A second area deals with the effects of anticipated acquisitions of start-ups on innovation and entry. We now discuss how our paper relates to these areas.

Federico, Langus and Valletti (2017, 2018) argue that the incumbents' innovation effort decreases after a merger in a symmetric oligopoly market.[7] In a model where firms simultaneously choose prices and investment levels, Motta and Tarantino (2018) show that mergers lead to higher prices and lower R&D investment. Bourreau, Jullien and Lefouili (2018) disentangle different effects of horizontal mergers on innovation incentives, arguing that the overall effect will depend on whether the innovation involves quality improvements or horizontal differentiation.[8] All these papers consider only the choice of one-dimensional innovation effort. By contrast, Letina (2016) allows (ex-ante symmetric) incumbent firms to invest in a continuum of research projects instead of choosing a uni-dimensional innovation effort. He shows that a merger reduces research duplication, but also lowers R&D diversity, which translates into a lower probability of a successful innovation in the market.[9] While maintaining the emphasis on different research approaches, we address a fundamentally different question, namely how the possibility of acquiring entrants affects the innovations of incumbents and entrants.[10]

This last feature links the paper to the literature that investigates the anticipated effects of the possibility of acquiring small firms or at least buying their technological knowledge. The contributions in this literature can be classified according to whether they identify positive, ambiguous or negative effects of allowing acquisitions on innovation.

[7] In an otherwise similar setting with different assumptions on the innovation technology, Denicolo and Polo (2018) obtain the opposite result.

[8] A related literature investigates the effects of the number of firms on innovation, see. e.g. Yi (1999), Norbäck and Persson (2012) and Marshall and Parra (2019). More broadly related, many papers discuss the relation between other measures of competitive intensity and innovation; see Vives (2008) and Schmutzler (2013) for unifying approaches.

[9] In Gilbert (2019), the firms can also invest in multiple research projects but the projects have i.i.d. probability of success. He finds that mergers often decrease R&D diversity. In another model with multiple research projects, Moraga-González, Motchenkova and Nevrekar (2019) show that mergers can potentially increase welfare by alleviating biases in the direction of innovation, in particular, when perfect price discrimination is possible. More broadly related are Bryan and Lemus (2017) who study the direction of innovation, Letina and Schmutzler (2019) who consider research variety in innovation contests, and Bavly, Heller and Schreiber (2020) who introduce asymmetric beliefs about the success of different projects.

[10] Moreover, in the current paper, we allow firms to partially invest in each project.

The first strand of literature goes back to Rasmussen (1988) who identified an incentive to enter a market just to get bought by the current incumbent, suggesting that a more lenient merger policy can increase overall welfare by incentivizing entry.[11] Phillips and Zhdanov (2013) consider a setting where large firms can sell their own product and the target's product after the acquisition. Contrary to our model, a laissez-faire policy not only fosters the entrant's innovation activity, but also the incumbent's innovation activity.[12] The paper also finds empirical evidence for positive innovation effects. All these papers emphasize desirable effects of a laissez-faire policy on innovation. In this sense, they are similar to our analysis. However, the papers do not shed light on the strategic choices of innovation projects and on the distinction between acquisitions with and without commercialization. We show that even for pure killer acquisitions, the effect of an anticipated acquisitions on innovation is positive.

The second strand of literature contains Bryan and Hovenkamp (2020). Contrary to our paper and those cited above, these authors consider the innovation decisions of start-ups who produce inputs for competing incumbents, without contemplating entry into this competition. While they also find that a laissez-faire policy fosters the level of innovation, they identify distortions in the direction of innovation coming from catering to the needs of the dominant firm in the market rather than the laggard. More broadly related, Cabral (2018) studies a setting where asymmetric competitors can pay to acquire each others' knowledge (a mere technology transfer rather than an acquisition). He concludes that allowing for technological transfer increases the innovation rate for incremental innovation, but decreases it for radical innovation.[13] Though the setting is quite different from ours, Cabral (2018) shares with us the distinction between different types of innovation.

Finally, Kamepalli et al. (2019) strongly argue that a laissez-faire policy has negative effects on the innovation activities of start-ups. Their analysis focuses on platform markets with network externalities. Their central argument is that "techies" (potential early adopters of a new technology) are more willing to bear the switching costs required to understand the start-up's product if they expect this firm to be in the market for long (rather than be acquired). Thus a no-acquisition policy attracts more of these early adopters, which in turn attracts "normal consumers" who benefit from network externalities.

To sum up, our paper is the first to investigate how the policy towards start-up acquisitions affects the innovation portfolios of incumbents and entrants. Contrary to the literature, we focus on differential effects on incumbents and entrants, respectively. This way, we show that prohibiting start-up acquisitions has unambiguously negative effects on the variety of approaches and the probability of innovation. This conclusion does not depend substantially on whether the innovation of the entrant is commercialized or not.

[11] For similar reasoning, see Mason and Weeds (2013).

[12] The difference reflects the additional value from applying an innovation to the target's product as well as the own product.

[13] Motivated by cases from the digital economy (e.g. AMD/Intel), the paper distinguishes between technological leadership and market leadership, where the latter, roughly speaking, corresponds to a firm's ability to monetize a given technology. Through incremental innovations, a laggard becomes a technology leader, but not a market leader; through radical innovations a firm also becomes a market leader.

3 The model

In this section, we first introduce our assumptions and some notation. We then present a simple, but important auxiliary result on the acquisition behavior of the incumbent.

3.1 Assumptions and Notation

We will consider two variants of a multi-stage game, corresponding to a *laissez-faire policy (A)* which tolerates acquisitions and a *no-acquisition policy (N)*, respectively. We capture the laissez-faire policy in a four-stage game between two firms, an entrant ($j = e$) and an incumbent ($j = i$). The entrant does not possess a technology that would enable him to compete in the market, but he can invest in R&D. The incumbent owns a technology with which she can sell products. In addition, she can invest in R&D as well.

In the first stage of the game, the *innovation stage*, the firms choose in which research projects to invest, and for each project, with which intensity to develop it. We assume there is a continuum of projects (approaches to innovation) θ, normalized to $\Theta = [0, 1)$. Only one project, $\hat{\theta} \in \Theta$, will lead to an innovation. All other projects lead to a dead end and produce no valuable output. We assume that each project is equally likely to be successful. Firms can vary the effort intensity on each project. We capture this by assuming that, for any $\theta \in [0, 1)$, each firm chooses a research intensity $r_j(\theta) \in [0, 1]$. If $\hat{\theta}$ is the project leading to innovation, then $r(\hat{\theta})$ is the probability that firm j will develop the innovation. We restrict the firms' choices to the set of measurable functions $r : [0, 1) \rightarrow [0, 1]$, which we denote with R.

The cost of investing with intensity r_j in project θ is given as $r_j(\theta)C(\theta)$, where the cost function $C : [0, 1) \rightarrow \mathbb{R}_+$ is continuous, differentiable, strictly increasing and convex. Moreover, we assume that $\lim_{\theta \rightarrow 1} C(\theta) = \infty$ and that $C(0)$ is low enough that there is some investment in any equilibrium. The total investment cost of firm j is thus $\int_0^1 r_j(\theta)C(\theta)d\theta$.

A successful research project $\hat{\theta}$ can lead to two levels of innovation. With probability p, the successful project results in a high technological state (H), corresponding to a drastic innovation over the technology currently held by the incumbent. With probability $1 - p$, the successful project results in a low technological state (L). L corresponds to a non-drastic innovation, which would allow the entrant to compete with the incumbent and obtain positive profits from the product market. Note that L could be either superior or inferior to the technology initially held by the incumbent. If a single firm develops the innovation, it receives a patent on that technology. If both firms develop the innovation, they each receive the patent with probability 1/2. We assume that only the firm holding the patent can use the new technology. Once the successful project has been realized, both firms learn which technology level the project delivers. Thus, after the patent has been allocated, the firms have *interim technology states* $t_i^I \in \{\ell, L, H\}$ and $t_e^I \in \{0, L, H\}$, where state ℓ corresponds to the situation where the incumbent holds her initial technology.

In the second stage, the *acquisition stage*, the incumbent can acquire the entrant by paying the profits that the latter could achieve by competing on the market plus a share of the (bargaining) surplus $\beta \in (0, 1)$. We will assume that the acquisition is successful if and only if the bargaining surplus is strictly positive. If the entrant is acquired, then any patents held by the entrant are transferred to the incumbent.[14]

[14]We could allow for the possibility that the entrant can attempt to acquire the incumbent. That would not affect our results, and is unlikely to be a realistic option in the situations we are modeling.

In the third stage, the *commercialization stage*, the patent holder can bring the new technology to the market at some commercialization cost $\kappa > 0$.[15] We denote the technology states resulting after the acquisition and commercialization stages as *final technology states* $t_i^F \in \{\ell, L, H\}$ and $t_e^F \in \{0, L, H\}$.

Finally, in the *product market stage*, the firms collect product market profits. Denote the profit of firm $j \in \{i, e\}$ as a function of technology states as $\pi_j(t_j, t_k)$ for $j \neq k$.[16] We make the following assumptions regarding the functions $\pi_j(t_j, t_k)$.

Assumption 1.

(i) The profit function is symmetric in its arguments, so that we can write $\pi(t_j, t_k) := \pi_j(t_j, t_k)$ *for* $j \in (i, e)$.

(ii) Without an innovation, the entrant cannot compete. Thus, $\pi(0, t_j) = 0$ *for* $t_j \in (\ell, L, H)$.

(iii) Technology H corresponds to a drastic innovation, so that the respective owner gets the monopoly profit $\pi(H) \equiv \pi(H, \ell) = \pi(H, 0) > max\{\pi(L, 0), \pi(\ell, 0)\}$ *and* $\pi(\ell, H) = 0$.

(iv) Competition decreases total profits, that is, $\max\{\pi(L, 0), \pi(\ell, 0)\} > \pi(\ell, L) + \pi(L, \ell)$.

First, (i) captures the notion that, for each firm, its own state and its competitor's state determine profits in the same way. Next, (ii), reflects the basic asymmetry that (contrary to the incumbent) the entrant needs an innovation to compete. Further, (iii), we suppose that a firm that has access to the drastic innovation earns the monopoly profit. The other firm has zero profits. Finally, (iv) assumes that competition reduces total profits, that is, that the two firms could achieve higher profits by coordinating (and using the superior technology) than by competing. Note that, for the incumbent, technology L is not necessarily an improvement over the status-quo technology ℓ: $\pi(L, 0) > \pi(\ell, 0)$ and $\pi(L, 0) < \pi(\ell, 0)$ are both possible.

Assumption 2. *Commercialization costs satisfy*

$$(i)\ \pi(L, \ell) \geq \kappa;$$
$$(ii)\ \pi(H) - \pi(\ell, 0) \geq \kappa.$$

Given this assumption, the entrant commercializes a successful innovation, no matter whether it is of an L or H-type. The incumbent commercializes the drastic innovation. For the non-drastic innovation, we will allow for the possibility that the incumbent wants to commercialize it, as well as the possibility that she does not want to do so.

We denote the firms' values after the *interim technology states* t_i^I and t_e^I have been realized as $v_i(t_i^I, t_e^I)$ and $v_e(t_e^I, t_i^I)$, respectively. These values not only depend on the policy regime (laissez-faire or regulation), but also on the parameters (p, β, κ), the cost function $C(\theta)$ and the nature of competition and the innovation size as reflected in the profits $\pi_i(t_i^I, t_e^I)$ and $\pi_e(t_e^I, t_i^I)$. When either firm has technology state H, the values are

[15] Implicit is the assumption that the commercialization of drastic and non-drastic innovations is equally costly. None of our main insights depends on this assumption.

[16] It may be helpful to think of technology states as real numbers corresponding to product quality or (inverse) cost level.

independent of the competitor state; thus, we simply write $v_i(H)$ and $v_e(H)$. The expected profit of the incumbent who chooses an investment function $r_i(\theta)$ when facing an entrant who chooses $r_e(\theta)$ is

$$\begin{aligned}\mathbb{E}\Pi_i(r_i,r_e) = &-\int_0^1 r_i(\theta)C(\theta)d\theta + \int_0^1 r_i(\theta)(1-r_e(\theta))\left[pv_i(H)+(1-p)v_i(L,0)\right]d\theta \\ &+\int_0^1 (1-r_i(\theta))r_e(\theta)\left[(1-p)v_i(\ell,L)\right]d\theta + \int_0^1 (1-r_i(\theta))(1-r_e(\theta))v_i(\ell,0)d\theta \\ &+\int_0^1 r_i(\theta)r_e(\theta)\left[p\left(\frac{1}{2}v_i(H)\right)+(1-p)\left(\frac{1}{2}v_i(L,0)+\frac{1}{2}v_i(\ell,L)\right)\right]d\theta.\end{aligned}$$

The first integral captures the innovation costs that the incumbent incurs by using the innovation strategy r_i. The second integral represents the incumbent's expected profits when he successfully innovates and the entrant does not. The third integral captures his expected profits in the opposite case, when he is not successful but the entrant is. The fourth integral represents expected profits when neither firm innovates, and the fifth is for the case when both firms successfully innovate.

The expected profit of the entrant is given analogously as

$$\begin{aligned}\mathbb{E}\Pi_e(r_e,r_i) = &-\int_0^1 r_e(\theta)C(\theta)d\theta + \int_0^1 r_e(\theta)(1-r_i(\theta))\left[pv_e(H)+(1-p)v_e(L,\ell)\right]d\theta \\ &+\int_0^1 r_e(\theta)r_i(\theta)\left[\frac{p}{2}v_e(H)+\frac{1-p}{2}v_e(L,\ell)\right]d\theta.\end{aligned}$$

We are interested in characterizing all subgame-perfect equilibria of the game. This amounts to finding all functions $r_i, r_e \in R$ such that for any $r_i', r_e' \in R$

$$\begin{aligned}\mathbb{E}\Pi_i(r_i,r_e) &\geq \mathbb{E}\Pi_i(r_i',r_e) \\ \mathbb{E}\Pi_e(r_e,r_i) &\geq \mathbb{E}\Pi_e(r_e',r_i).\end{aligned}$$

Obviously, for any (r_i, r_e) which constitute an equilibrium, any pair of functions $(\tilde{r}_i, \tilde{r}_e)$ which only differ from (r_i, r_e) on a set of measure zero, also constitutes an equilibrium. We omit the necessary "almost everywhere" qualifications from the statements of our formal results for ease of exposition.

For the characterization of the equilibrium investment behavior, the *critical projects* $\theta_e^1, \theta_e^2, \theta_i^1, \theta_i^2$, defined as the solutions to the equations below, will play a crucial role. Essentially, they correspond to projects for which, depending on the investment behavior of the competitor, a firm's benefits from investing in the project equal the costs. These critical projects are given by:

$$\begin{aligned}C(\theta_e^1) &= pv_e(H)+(1-p)v_e(L,\ell) \\ C(\theta_e^2) &= \frac{1}{2}\left(pv_e(H)+(1-p)v_e(L,\ell)\right) \\ C(\theta_i^1) &= pv_i(H)+(1-p)v_i(L,0)-v_i(\ell,0) \\ C(\theta_i^2) &= \frac{p}{2}v_i(H)+(1-p)\left(\frac{1}{2}v_i(L,0)+\frac{1}{2}v_i(\ell,L)\right)-(1-p)v_i(\ell,L).\end{aligned}$$

Thus, θ_j^1 $(j = i, e)$ is defined by the requirement that the cost of this project equals the expected value increase to the firm if it invests in a successful project when the other firm does not invest in this project; θ_j^2 is defined by the requirement that the cost of this project equals the expected value increase to the firm if it invests in a successful project when the other firm invests as well.

To sum up, the incumbent and the entrant play a dynamic game of complete information. Under a laissez-faire policy (A), the game has four stages, namely (i) the investment stage, (ii) the acquisition stage, (iii) the commercialization stage and (iv) the market competition stage. The timeline of the model is summarized below.

1. **Investment stage:** The firms simultaneously invest in research projects. If only one firm is successful, it receives the patent on the underlying technology. If both firms are successful, the patent is allocated randomly with equal probability. If neither firm is successful, neither firm receives the patent. Interim technology states t_i^I and t_e^I are realized.

2. **Acquisition stage:** If the entrant holds the patent, the firms negotiate an acquisition. The acquisition is successful if and only if it strictly increases total profits. If there is an acquisition, the incumbent pays the entrant $\pi(t_e^I, t_i^I) - \kappa$, the profits which the entrant could obtain on the market less the commercialization costs, plus a share β of the bargaining surplus.

3. **Commercialization stage:** The firm holding the patent (if any) decides whether to commercialize the technology, thereby incurring costs κ. At the end of the commercialization stage, the firms' final technology states t_i^F and t_e^F are realized.

4. **Market stage:** Firms receive profits, given by $\pi(t_j^F, t_k^F)$. Total payoffs are realized by subtracting potential development and commercialization costs and adding/ subtracting potential acquisition payments.

A *no-acquisition policy (N)* prevents the incumbent from taking over the entrant, corresponding to a three-stage version of the game (without the acquisition stage).

3.2 The Acquisition Subgame

We first summarize the result of the acquisition subgame emerging in the laissez-faire case after the realization of the interim technology states.

Lemma 1 (Acquisitions). *The incumbent acquires the entrant if and only if the entrant holds a patent for the L technology. After the acquisition, the incumbent commercializes the technology if and only if* $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$.

The incumbent only acquires the entrant if the latter has access to technology L, because only then is there competition between the two firms which the acquisition can eliminate. Whether or not the entrant kills the entrant's technology will depend on the value of the non-drastic innovation. If $\pi(L, 0) - \pi(\ell, 0) < \kappa$, commercialization is not worthwhile – the only motive for the acquisition is the elimination of competition. If $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$, the incumbent enjoys the benefits from eliminating competition as well as those from an improved technology.

4 Investments without Commercialization of Start-Up Innovations

We now analyze the investment behavior for the case that $\pi(L,0) - \pi(\ell,0) < \kappa$. We will refer to this as the case without commercialization because, according to Lemma 1, the incumbent will not commercialize non-drastic innovations, so that any acquisition will be a killer acquisition. We first derive the results under the laissez-faire policy. Thereafter, we compare them with the no-acquisition policy.

4.1 Equilibrium Investments under Laissez-Faire

To derive the equilibrium investments in the laissez-faire case, we need some preliminary results. First, we use Lemma 1 to obtain the value functions after the realization of the innovation.

Lemma 2 (Payoffs).
Under the laissez-faire policy, suppose the no-commercialization case applies.
(i) The entrant's values after the realization of the innovation results are

$v_e(H) = \pi(H) - \kappa$

$v_e(L,\ell) = \pi(L,\ell) - \kappa + \beta(\pi(\ell,0) - \pi(L,\ell) - \pi(\ell,L) + \kappa)$

$v_e(0,t_i) = 0$ *for* $t_i \in \{\ell, L, H\}$.

(ii) The incumbent's values after the realization of the innovation results are

$v_i(H) = \pi(H) - \kappa.$

$v_i(L,0) = v_i(\ell,0) = \pi(\ell,0)$

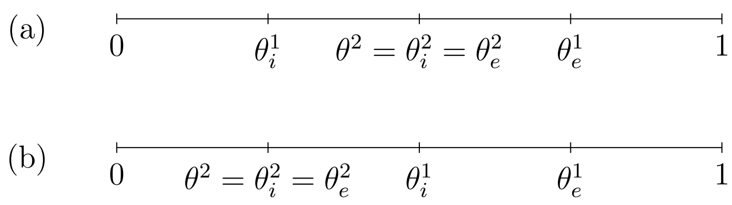
$v_i(\ell,L) = \pi(\ell,0) - v_e(L,\ell)$

$v_i(\ell,H) = 0.$

The results follow from noting that the interim technological states are also the final technological states, except for: (i) $(t_i^I, t_e^I) = (\ell, L)$, in which case an acquisition takes place but the incumbent does not commercialize the L-technology, so that the final state becomes $(t_i^F, t_e^F) = (\ell, 0)$ and (ii) $(t_i^I, t_e^I) = (L, 0)$ when the incumbent discovers the L-technology but does not commercialize it, so that the final state also becomes $(t_i^F, t_e^F) = (\ell, 0)$. Next, we summarize the relations between the critical projects.

Lemma 3. *Under the laissez-faire policy, suppose the no-commercialization case applies. The following relations hold for the critical projects:*
(i) $\theta_i^2 = \theta_e^2 =: \theta^2$*; (ii)* $\theta^2 < \theta_e^1$*; (iii)* $\theta_i^1 < \theta_e^1$
(iv) $\theta_i^1 \leq \theta^2$ *if and only if* $pv_i(H) - 2pv_i(\ell,0) \leq (1-p)v_e(L,\ell)$.

The result shows that the critical projects must satisfy one of the following two orders.



Lemma 3(i) tells us that, irrespective of the entrant's bargaining power, the projects in which the incumbent is willing to invest if the entrant does are exactly those which the entrant is also willing to invest in if the incumbent does. To understand why, remember that those thresholds are formed by comparing payoffs from investing and not investing in a project, conditional on the other firm also investing in this project. If a project in which both firms invest delivers an H technology, both firms receive the same expected net payoff from investing, because not investing means losing the high innovation to the rival and receiving 0 for sure instead of obtaining the high monopoly profit with probability 1/2. If such a project delivers an L technology, the entrant trivially gains the acquisition price times the probability that he ends up with the patent. By contrast, the incumbent can stay the monopolist and save on the acquisition price if she receives the patent, which happens with probability 1/2. Thus, the expected benefits of investing (conditional on the other firm investing) are the same for entrants and incumbents. Lemma 3(*ii*) means that the entrant is always willing to invest in a larger range of projects if he is the sole innovator than if the incumbent also invests in these projects. This is intuitive, as the incumbent's investment reduces the entrant's probability of receiving a patent. From Lemma 3(*iii*) we learn that, conditional on the other firm not investing, the entrant is willing to invest in more expensive projects than the incumbent. This is due to the well-known Arrow replacement effect: The incumbent does not increase her profits when discovering the L innovation and the profit increase from the H innovation is lower than that of the entrant, since without the innovation the entrant receives zero profits. Hence, the entrant's willingness to pay to be the sole innovator is greater than that of the incumbent. This will be important for our main result, namely that prohibiting acquisitions has a negative effect on innovations in equilibrium.

For the equilibrium characterization, it will be decisive whether $\theta_i^1 \leq \theta^2$, that is, whether the condition in (*iv*) holds. Intuitively, when this condition is satisfied, there will be a set of projects in which the incumbent is willing to invest only if the entrant also invests in those projects, so that she can block the entrant.

Our next result provides a full characterization of the equilibrium R&D investments.

Proposition 1 (Equilibrium R&D investment)**.** *Suppose that the no-commercialization case applies under a laissez-faire policy.*

(a) If $\theta_i^1 \leq \theta^2$ *then there exists a unique equilibrium. In this equilibrium*

$$r_e(\theta) = 1 \text{ and } r_i(\theta) = 1, \text{ for } \theta \in [0, \theta^2],$$
$$r_e(\theta) = 1 \text{ and } r_i(\theta) = 0, \text{ for } \theta \in (\theta^2, \theta_e^1],$$
$$r_e(\theta) = 0 \text{ and } r_i(\theta) = 0, \text{ for } \theta \in (\theta_e^1, 1).$$

(b) If $\theta_i^1 > \theta^2$*, the equilibrium is not unique. Functions* r_i *and* r_e *constitute an equilibrium if and only if conditions (i)-(iv) hold:*

(i) $r_e(\theta) = 1$ *and* $r_i(\theta) = 1$*, for* $\theta \in [0, \theta^2]$

(ii) $r_e(\theta) = 1$ *and* $r_i(\theta) = 0$*, for* $\theta \in (\theta_i^1, \theta_e^1]$*,*

(iii) $r_e(\theta) = 0$ *and* $r_i(\theta) = 0$*, for* $\theta \in (\theta_e^1, 1)$*.*

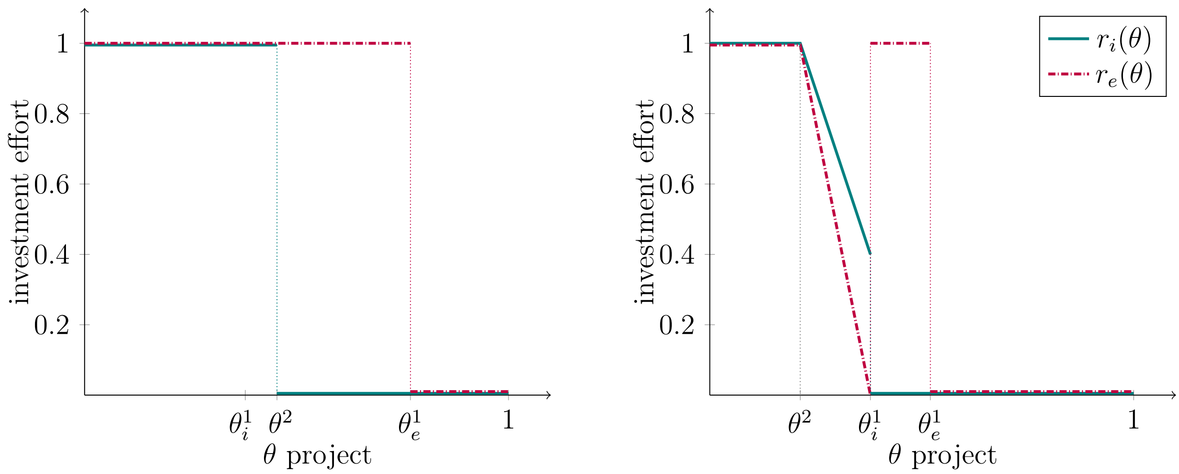
(iv) for any $\theta \in (\theta^2, \theta_i^1]$ *either:*

$r_e(\theta) = 1$ *and* $r_i(\theta) = 0$*, or*

$$r_e(\theta) = 0 \text{ and } r_i(\theta) = 1, \text{ or}$$
$$r_e(\theta) = \frac{C(\theta_i^1) - C(\theta)}{C(\theta_i^1) - C(\theta^2)} \text{ and } r_i(\theta) = \frac{C(\theta_e^1) - C(\theta)}{C(\theta_e^1) - C(\theta^2)}.$$

We will refer to equilibria where $r_e(\theta) \in \{0, 1\}$ and $r_i(\theta) \in \{0, 1\}$ $\forall \theta \in [0, 1)$ as *simple equilibria* for the remainder of this paper. It is immediate from the proposition that for any choice of parameters, a simple equilibrium always exists. The structure of the equilibrium is straightforward (see Figure 1). In case (a), both firms invest fully (that is, with $r = 1$) in all projects in the interval $[0, \theta^2]$, while only the entrant invests in the projects in the interval $(\theta^2, \theta_e^1]$. Neither firm invests in projects in the interval $(\theta_e^1, 1)$. The simple equilibrium also exists in case (b). However, in case (b) there are other equilibria as well. The ambiguity results from the fact that in any project in the interval $[\theta^2, \theta_i^1)$ each firm only wants to invest if the other one does not. As a result, in any equilibrium either only one of the firms invests fully into the project whereas the other one does not invest at all, or both firms invest with probability between 0 and 1.

Figure 1: Equilibrium portfolio of entrant and incumbent for case (a) of Proposition 1 (left) and case (b) of Proposition 1 (right).



Note: We consider Bertrand competition with homogeneous goods and cost-reducing innovation. A more detailed description of the example can be found in the Online Appendix B.2.

We now develop an intuition for when each case occurs, which is related to the condition in Lemma 3(iv). In the specification we use for Figure 1, it turns out that the bargaining power β of the entrant plays a particularly clear role in determining which of the two cases specified in Proposition 1 arises. On the one hand, if the entrant's bargaining power is high ($\beta = 1$), he can gain a lot from innovating and being acquired by the incumbent, while the incumbent has a relatively large incentive to block the entrant. Case (a) occurs and the equilibrium is of the form depicted in the left plot of Figure 1. On the other hand, if the entrant's bargaining power is low, the incumbent's incentive to block the entrant is relatively low and the entrant has little to gain from innovating, which is depicted in the right plot, representing a non-simple equilibrium in case (b). Another important parameter is the probability of a high innovation. Case (a) occurs when p is low, so that the probability of the non-drastic innovation is high. Thus, it is important for the incumbent to curtail competition by blocking the entrant from obtaining a patent. When p is high, the blocking incentive is relatively less important for the incumbent; instead, the hope of obtaining the drastic innovation looms large.

Proposition 1 is interesting for several reasons. First, note there always exists a set of ideas in which only the entrant invests. Of all developed research ideas, these are the most costly to develop. Furthermore, even if the entrant decided not to invest in these ideas, the incumbent would not replace the entrant's investments. These features will be crucial for our main result – which is that the variety of research ideas developed and the probability of discovering an innovation decrease if acquisitions are prohibited. Decreasing the incentives for the entrant to invest in innovation will cause the entrant to reduce investment *in exactly the projects which cost the most and which only the entrant would develop*. Thus, in contrast to a reduction of the incumbent's incentive to invest, a reduction in the entrant's incentives to invest will cause less variety in developed research ideas and lower probability that the innovation will be developed.

4.2 Comparative statics

Since the equilibrium is characterized by a pair of functions, a change in parameters could, in principle, lead to changes in equilibrium behavior which are difficult to compare. As we will see, a useful way to summarize the equilibrium is to consider the *variety* of research projects that firms are pursuing in equilibrium, and the *probability* that the innovation will be discovered by either firm. Formally, given any two research strategies of the firms r_i and r_e we define the variety of research projects pursued as

$$\mathcal{V}(r_i, r_e) = \int_0^1 \mathbf{1}(r_i(\theta) + r_e(\theta) > 0) d\theta.$$

Thus, the variety of research projects pursued captures the size of the set of research projects that at least one of the firms develops with positive probability. The probability that an innovation will be discovered is given by

$$\mathcal{P}(r_i, r_e) = 1 - \int_0^1 (1 - r_i(\theta))(1 - r_e(\theta)) d\theta.$$

The following results follows immediately from Proposition 1.

Corollary 1. *Consider the no commercialization case in the laissez-faire regime. If (r_i, r_e) is an equilibrium, then $\mathcal{V}(r_i, r_e) = \theta_e^1$. If (r_i, r_e) is a simple equilibrium, then $\mathcal{P}(r_i, r_e) = \theta_e^1$.*

This result again highlights the importance of the observation that it will be the entrant who decides to invest in the most expensive research projects being developed. Thus, to understand how a marginal change in the parameters affects the equilibrium variety of research projects or the probability that the innovation will be developed, it is sufficient to understand how the incentives of the entrant are affected.

Proposition 2 (Comparative statics)**.** *Consider any equilibrium (r_i, r_e) under a laissez-faire policy in the no-commercialization case. Then, the variety $\mathcal{V}(r_i, r_e)$ is increasing (i) in the probability of an H-innovation (p) and (ii) in the bargaining power of the entrant (β); it is (iii) increasing in the entrant's profits $\pi(L, \ell)$, but decreasing in the incumbent's profits $\pi(\ell, L)$ under competition.*

Results (i) and (ii) are intuitive, as both an increase in p and in β make the innovation more valuable to the entrant. However, this intuition and the unambiguous result depend crucially on the fact that it is the marginal effect on the entrant that determines the equilibrium variety of research projects. According to *(iii)*, the profits of the two firms when the entrant successfully enters the market with an L technology affect variety in opposite directions. Again, the intuition can be obtained by considering the entrant's incentives. Even if the entrant is always acquired and thus competition will never realize, his competition profit increases his outside option, but decreases the acquisitions surplus. Since he only receives a share β of the acquisitions surplus, the former effect dominates for his innovation incentives, and thereby for project variety. By a similar logic, the competition profit of the incumbent decreases acquisition surplus. However, the entrant's outside option is not affected. Concluding, project variety decreases when the incumbent gains a larger profit under competition.

Note that, in any simple equilibrium, the variety of research projects and the probability of discovering an innovation coincide. Therefore, if we restrict attention to simple equilibria, then the comparative statics with respect to the probability that an innovation will be discovered are the same as in Proposition 2.

4.3 Investments under the No-Acquisition Policy

The behavior of firms in the commercialization and market stages remain unchanged when acquisitions are prohibited. By Lemma 1, the policy affects the game only when the entrant has a non-drastic innovation, that is, $(t_i^I, t_e^I) = (\ell, L)$, in which case interim technology states t_j^I now correspond to final technology states t_j^F for $j \in \{i, e\}$. Thus, the values of incumbent and entrant under the no-acquisition policy are given by the expressions in Lemma 2, except that $v_i(\ell, L) = \pi(\ell, L)$ and $v_e(L, \ell) = \pi(L, \ell) - \kappa$.

Lemma 4. *Consider the case without commercialization under the no-acquisitions policy. The following relations hold: (i)* $\theta_e^2 < \theta_i^2$*; (ii)* $\theta_e^2 < \theta_e^1$*; (iii)* $\theta_i^1 < \theta_e^1$*.*

In contrast to the laissez-faire case, the set of projects in which the incumbent and entrant are willing to invest simultaneously does not coincide, so that $\theta_e^2 \neq \theta_i^2$. The entrant now has a smaller incentive to invest in projects that the incumbent invests in than vice versa. The reason for this gap is that a successful L-innovation results in an increase in profits for the entrant that is smaller than the corresponding decrease in profits for the incumbent. This follows directly from Assumption 1 (*iv*). Relation (*ii*) and (*iii*) have an equivalent interpretation as relations (*ii*) and (*iii*) in Lemma 3. Note that, as before, the relationship between θ_i^1 and θ_i^2 is not clear. If $\theta_i^2 > \theta_i^1$, then the incumbent would want to invest in projects in the interval (θ_i^1, θ_i^2) only if the entrant is also investing into those projects. Again, the incumbent would not invest were it not to block the entrant.

Based on Lemma 4, we can characterize the equilibrium structure under the no-acquisition policy. Proposition 8 in the Appendix A.6 provides a full characterization. Lemma 4 shows that there are three cases to consider, depending on the position of θ_i^1 relative to the other critical values. Importantly, however, the equilibria share several common features. First, for any project below θ_e^1, at least one firm exerts positive effort. Thus, as in the laissez-faire case, θ_e^1 determines the variety of approaches: The entrant always invests in the most expensive project pursued by any firm and, even if the incumbent also invests in this project, the entrant always exerts more investment effort. Second, no firm exerts any effort above θ_e^1. It might happen that the incumbent's blocking incentive is so strong that $\theta_i^2 > \theta_e^1$. However, since the entrant does not want to invest in projects more expensive than θ_e^1 irrespective of the investment behavior of the incumbent, there is nothing to block for the incumbent. Thus, neither of the two firms invests in projects $\theta \in (\theta_e^1, 1)$.

4.4 Policy Effects

We now analyze the effects of the no-acquisition policy. We first show that this policy negatively affects the variety of pursued research projects as well as the probability that an innovation will be discovered.

Proposition 3 (The effect of preventing acquisitions)**.** *Consider the no commercialization case for fixed parameter values.*

(a) In any equilibrium under the no-acquisition policy, the variety of research projects is strictly smaller than in any equilibrium under the laissez-faire policy.

(b) In any equilibrium under the no-acquisition policy, the probability of a successful innovation is strictly smaller than in any simple equilibrium under the laissez-faire policy.

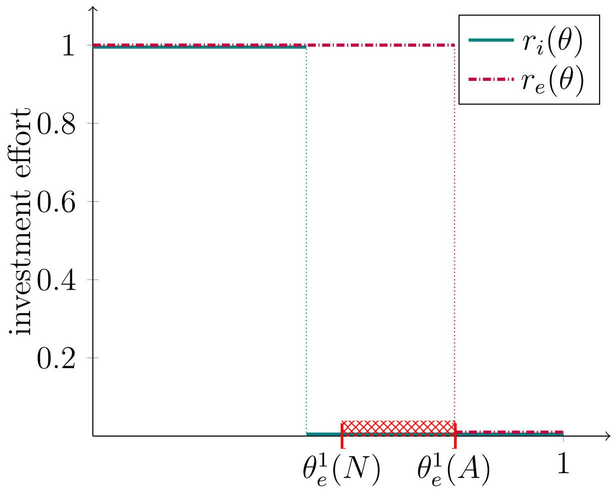
Denote the critical values θ_e^1 under the laissez-faire and no-acquisition policies as $\theta_e^1(A)$ and $\theta_e^1(N)$, respectively. To obtain an intuition for Proposition 3, recall that, under the laissez-faire policy, by Proposition 1 only the entrant invests in approaches close to $\theta_e^1(A)$, and this critical value determines the equilibrium project variety. Preventing acquisitions reduces the entrant's expected payoff from investing in innovation, since he cannot sell the firm when it would be profitable to do so. This causes the entrant to stop investing in projects in the set $(\theta_e^1(N), \theta_e^1(A))$. At the same time, the incumbent's incentives to invest in projects *in which the entrant does not invest* are the same with or without acquisitions. The reason is simple: If the entrant does not invest in the successful project, there will be no reason to acquire him. Hence, when acquisitions are not possible, no firm will invest in the interval $(\theta_e^1(N), \theta_e^1(A))$ which corresponds to a decrease in project variety as stated in part (a) of Proposition 3. In any simple equilibrium under the laissez-faire policy, we know that $\mathcal{P}(r_i(A), r_e(A)) = \theta_e^1(A)$. Since in any non-simple equilibrium under the no-acquisition policy $\mathcal{P}(r_i(N), r_e(N)) \leq \theta_e^1(N)$, part (b) immediately follows.

Finally, as an input into our subsequent welfare analysis, we consider the determinants of the size of the innovation-reducing effect of restricting acquisitions. We use the notation $\Delta_{\mathcal{V}} \equiv \mathcal{V}(r_i(A), r_e(A)) - \mathcal{V}(r_i(N), r_e(N))$ to capture the size of the policy effect on variety.

Proposition 4. *Consider any equilibrium under a laissez-faire policy $(r_i(A), r_e(A))$ and any equilibrium under the no-acquisition policy $(r_i(N), r_e(N))$. Then the size of the policy effect $\Delta_{\mathcal{V}}$ is (i) decreasing in p, (ii) increasing in β and (iii) decreasing in both firms' profits under competition, $\pi(\ell, L)$ and $\pi(L, \ell)$.*

To understand the result, recall that, in the case without commercialization, in both policy regimes, the variety of research projects is determined by the most expensive project the entrant is willing to invest in. Thus, the policy effect on the difference of these critical projects, $\theta_e^1(A) - \theta_e^1(N)$, will pin down the change in variety. If the share p of drastic

Figure 2: Change in project variety moving from a laissez-faire equilibrium to a no-acquisition equilibrium



innovations increases, the variety of projects will increase in both policy regimes because it is more lucrative to innovate. However, variety will increase less under the laissez-faire policy because the payoff from low innovation for the entrant is higher when acquisitions are allowed and a higher p means that the probability of being acquired is lower. Part (i) of the Proposition represents this logic. Part (ii) states that, with higher bargaining power of the entrant, the decrease in variety resulting from a restriction on acquisitions is larger. In fact, bargaining power only affects variety under laissez-faire, and it clearly increases it since a higher bargaining power means that the entrant receives a higher share of the acquisition surplus. Finally, higher profits under competition, which can be interpreted as less intense competition, decrease the effect size of a no-acquisition policy on project variety according to part (iii). This reflects a subtle interplay between profit effects on the bargaining surplus and the entrant's outside option.[17]

Beyond affecting variety and thereby the probability of innovation, policy has more subtle impacts on the allocation of research investments. In particular, policy influences the duplication of research projects in a non-obvious way, where duplication is measured by the probability that both firms discover the innovation. This leads to the following definition of duplication:

$$\mathcal{D}(r_i, r_e) = \int_0^1 r_i(\theta) r_e(\theta) d\theta.$$

To understand the effects of policy on duplication, it is important to understand how it

[17]Under laissez-faire, a higher competitive profit improves the entrant's outside option (his competitive profit), but decreases the acquisition surplus. Under the no-acquisition policy, in contrast, there is no acquisition surplus, so that the entrant's innovation incentives are relatively more affected by his competitive profit. Thus, an increase in such profit weakens the effect on project variety. For the incumbent's competitive profit, remember that it merely reduces the acquisition surplus. When acquisitions are not allowed, it does not affect the entrant's innovation incentives. Consequently, a higher incumbent profit decreases the effect size of a restrictive regime on variety.

affects firms' incentives to invest into projects that the other firm also chooses, as captured by the critical values θ_i^2 and θ_e^2.

Corollary 2. *In the case without commercialization, the following statements hold:* *(i)* $\theta_i^2(A) < \theta_i^2(N)$ *and (ii)* $\theta_e^2(A) > \theta_e^2(N)$.

Intuitively, (i) prohibiting acquisitions makes it more important for the incumbent to invest into projects that the entrant is also investing in: Without the option of an acquisition, own investments are the only means of potentially avoiding competition when the entrant has engaged in a project that leads to an L-innovation. Thus, conditional on the entrant investing in a project, the incumbent gains more from investing in it under a no-acquisition policy than under laissez-faire. As to the entrant, (ii) duplicating the incumbent's investments is less attractive under the no-acquisitions policy than under laissez-faire because of the absence of prospective gains from selling the firm.

The net effects of these reactions to policy changes are complex. We summarize them in Proposition 9 in Online Appendix B.1. If $\theta_i^2(N) < \theta_i^1(N)$, then the negative policy effect on entrant's incentives dominates and there is less duplication under the no-acquisition policy. If $\theta_i^2(N) \geq \theta_i^1(N)$, then the negative effect will only obtain if the entrant's bargaining power β is sufficiently high; otherwise, there will be more duplication under the no-acquisition policy.

5 Acquisitions with commercialization

We now dispense with the assumption that $\pi(L,0) - \pi(\ell,0) < \kappa$ and instead assume that $\pi(L,0) - \pi(\ell,0) \geq \kappa$. According to Lemma 1, the non-drastic innovation is now sufficiently valuable that the incumbent commercializes it, both after a takeover and if she discovers it herself. We show how the equilibrium differs from the case with commercialization, focusing on the laissez-faire policy.

Under laissez-faire, values after the realization of the innovation results are the same as in the case without commercialization except after a non-drastic innovation. Thus, Lemma 2 applies to the case with commercialization if we replace

$$\begin{aligned} &v_e(L,\ell) = \pi(L,\ell) - \kappa + \beta(\pi(L,0) - \pi(\ell,L) - \pi(L,\ell)) \\ &v_i(\ell,L) = \pi(L,0) - \kappa - v_e(L,\ell) = (1-\beta)(\pi(L,0) - \pi(L,\ell)) + \beta\pi(\ell,L) \text{ and} \\ &v_i(L,0) = \pi(L,0) - \kappa. \end{aligned}$$

Intuitively, $v_e(L,\ell)$ differs from the case without commercialization because it affects both the stand-alone value for the firm and the takeover price. Similar reasoning applies to the incumbents values $v_i(\ell,L)$ and $v_i(L,0)$, which also account for the fact that the incumbent now incurs the commercialization cost.

The following result summarizes the order of the critical projects.

Lemma 5. *Consider the case with commercialization under the laissez-faire policy. The following relations hold:*

(i) $\theta_i^2 = \theta_e^2 =: \theta^2$;

(ii) $\theta^2 < \theta_e^1$;

(iiia) $\theta_i^1 \in (0, \theta^2]$ *if* $(1-p)v_i(\ell, L) \leq 2v_i(\ell, 0) - pv_i(H) - (1-p)v_i(L, 0)$*;*

(iiib) $\theta_i^1 \in (\theta^2, \theta_e^1]$ *if* $(1-p)v_i(\ell, L) \in (2v_i(\ell, 0) - pv_i(H) - (1-p)v_i(L, 0), v_i(\ell, 0)]$*;*

(iiic) $\theta_i^1 \in (\theta_e^1, 1)$ *if* $(1-p)v_i(\ell, L) > v_i(\ell, 0)$*.*

The result suggests that, contrary to the case without commercialization, it is possible that the critical project θ_i^1 of the incumbent lies above the critical project θ_e^1 of the entrant. According to Lemma 5, this will be the case if $(1-p)v_i(\ell, L) > v_i(\ell, 0)$ or, equivalently,

$$(1-p)\left[(1-\beta)(\pi(L,0) - \pi(L,\ell)) + \beta\pi(\ell, L)\right] > \pi(\ell, 0).$$

In words, this case occurs when p and β are small (that is, an L-innovation is likely and the incumbent captures most of the bargaining surplus), $\pi(L, 0)$ is large and $\pi(\ell, 0)$ is small (so that L constitutes a significant innovation, even if it is a non-drastic one) and when $\pi(L, \ell)$ is small (that is, if competition is intense). These conditions imply that the entrant has low incentives to innovate: conditional on innovating, he is likely to discover an L-innovation (low p), which generates a low outside option (due to low $\pi(L, \ell)$) and hence low acquisition price (due to low β). By contrast, the incumbent has higher incentives to invest, because discovering the L-innovation is a desirable outcome for her (due to high $\pi(L, 0)$ and small $\pi(\ell, 0)$).

The possibility that the critical project θ_i^1 of the incumbent lies above the critical project θ_e^1 of the entrant has repercussions for the equilibrium structure, which is characterized in our next result.

Proposition 5 (Equilibrium R&D investment)**.** *Consider the commercialization case under a laissez-faire policy.*

(a) If $\theta_i^1 \in (0, \theta^2]$*, then there exists a unique equilibrium which is identical to the case without commercialization.*

(b) If $\theta_i^1 \in (\theta^2, \theta_e^1]$*, the equilibrium is not unique. The equilibrium structure corresponds exactly to the case without commercialization.*

(c) If $\theta_i^1 \in (\theta_e^1, 1)$*, the equilibrium is not unique. Functions* r_i *and* r_e *constitute an equilibrium if and only if conditions (i)-(iv) hold:*

$$\begin{aligned}
&\textit{(i)}\ \ r_e(\theta) = 1 \textit{ and } r_i(\theta) = 1, \textit{ for } \theta \in [0, \theta^2] \\
&\textit{(ii)}\ \ r_e(\theta) = 0 \textit{ and } r_i(\theta) = 1, \textit{ for } \theta \in (\theta_e^1, \theta_i^1], \\
&\textit{(iii)}\ \ r_e(\theta) = 0 \textit{ and } r_i(\theta) = 0, \textit{ for } \theta \in (\theta_i^1, 1). \\
&\textit{(iv)}\ \ \textit{for any } \theta \in (\theta^2, \theta_e^1] \textit{ either:} \\
&\quad r_e(\theta) = 1 \textit{ and } r_i(\theta) = 0, \textit{ or} \\
&\quad r_e(\theta) = 0 \textit{ and } r_i(\theta) = 1, \textit{ or} \\
&\quad r_e(\theta) = \tfrac{C(\theta_i^1) - C(\theta)}{C(\theta_i^1) - C(\theta^2)} \textit{ and } r_i(\theta) = \tfrac{C(\theta_e^1) - C(\theta)}{C(\theta_e^1) - C(\theta^2)}.
\end{aligned}$$

Thus, as long as the order of the critical projects is the same as in the case without commercialization (that is, Parts (a) and (b) of Proposition 5 apply), so is the equilibrium structure. In particular, the variety of the projects is determined by the critical project θ_e^1 of the entrant. However, in the case with commercialization, whenever

$(1-p)v_i(\ell, L) > v_i(\ell, 0)$ then $\theta_i^1 > \theta_e^1$, so that the variety of the developed innovation projects is determined by the incumbent's critical project according to Proposition 5(c). Intuitively, with commercialization, the incumbent's incentive to innovate partly comes from increasing her monopoly profit even when she only obtains a non-drastic innovation. Compared to the entrant, she may therefore have higher incentives to invest when the competitor does not.

Next, we analyze the no-acquisition policy. First, we consider the values after the realization of the innovation results. Compared to the laissez-faire policy, only the values for $(t_e, t_i) = (L, \ell)$ are different. In this case, $v_e(L, \ell) = \pi(L, \ell) - \kappa$ and $v_i(\ell, L) = \pi(\ell, L)$. We obtain the following result on the ordering of critical projects.

Lemma 6. *Consider the case with commercialization under the no-acquisition policy. The following relations hold: (i) $\theta_e^2 < \theta_i^2$; (ii) $\theta_e^2 < \theta_e^1$.*

This leads to the following result on the effects of prohibiting acquisitions.

Proposition 6 (The effect of preventing acquisitions). *Consider the case with commercialization for fixed parameter values.*

(a) In any equilibrium under the no-acquisition policy, the variety of research projects is weakly smaller than in any equilibrium under the laissez-faire policy.

(b) In any equilibrium under the no-acquisition policy, the probability of a successful innovation is weakly smaller than in any simple equilibrium under laissez-faire.

(c) If $\theta_i^1(A) < \theta_e^1(A)$, the inequalities in (a) and (b) are strict; otherwise the policy has no effect on variety.

As in the case without commercialization, a restrictive merger policy can never have a beneficial effect on the variety of innovation projects. However, according to case (c), policy has no effect on variety if $\theta_i^1(A) > \theta_e^1(A)$. Then, variety under laissez-faire is driven by the incumbent's critical project $\theta_i^1(A)$ rather than by $\theta_e^1(A)$. However, the acquisition policy does not affect $\theta_i^1(A)$, so that there is no effect on variety.

In summary, from the perspective of innovation effects, the main difference between the cases with and without commercialization is that, in the former case, there are situations where policy has no effects on variety. With this caveat, the innovation effects are qualitatively similar in the two cases.

6 Discussion and extensions

6.1 Consumer surplus effects

So far, our analysis suggests a positive short-term effect of a restrictive acquisition policy on competition, but a negative effect on innovation. It would thus be desirable to know under which circumstances the former effect is likely to dominate the latter. The above analysis provides some understanding of the determinants of the size of the adverse effect of prohibiting acquisitions on the probability of innovation. However, this is clearly insufficient for understanding the circumstances in which intervention is called for. On the one hand, to understand how large the adverse innovation effect of a restrictive policy on welfare is, we need to know what determines the size of the effects of any reduction in innovation probability on consumer surplus. On the other hand, to evaluate the pro-competitive effects of a restrictive policy, we need to understand what drives the size of the pro-competitive effect of being in a duopoly rather than in a monopoly.

To address these issues, we will focus on the special case that $L = \ell$, so that competition in the case of entry would be symmetric. We denote consumer surplus as $S(t_i^F, t_e^F)$ and assume that $S(H,0) > S(L,L) > S(L,0)$.

Thus consumers prefer both the high-state monopoly and the (low-state) duopoly to the low-state monopoly, and they also enjoy higher surplus in the high-state monopoly than in the low-state duopoly. While the former statement applies much more generally, the justification of the latter statement relies on H corresponding to a drastic innovation.[18] Denote the probability that in policy regime R both firms have final state L as $prob^R(L,L)$ and the probability of a monopoly with technology H (L) as $prob^R(H,0)$ $(prob^R(L,0))$.[19] Then, the expected consumer surplus under the laissez-faire policy is:

$$prob^A\left(H,0\right)S\left(H,0\right)+prob^A\left(L,0\right)S\left(L,0\right).$$

Under the no-acquisition policy, the expected consumer surplus is:

$$prob^N\left(H,0\right)S\left(H,0\right)+prob^N\left(L,L\right)S\left(L,L\right)+prob^N\left(L,0\right)S\left(L,0\right).$$

These expressions illustrate the countervailing effects of prohibiting acquisitions. The following result provides a simple description of the policy effect on welfare.

Proposition 7. *Suppose $L = \ell$, so that the no-commercialization case applies. The expected consumer surplus effect of prohibiting start-up acquisitions is:*

$$prob^N\left(L,L\right)\left[S\left(L,L\right)-S\left(L,0\right)\right]+\left[prob^N\left(H,0\right)-prob^A\left(H,0\right)\right]\left[S\left(H,0\right)-S\left(L,0\right)\right].$$

The decomposition summarizes the differences between the two cases in a succinct fashion. On the one hand, the policy measure will introduce desirable competition with probability $prob^N(L,L)$. The first term captures the effect of having the competitive surplus $S\left(L,L\right)$ rather than the non-competitive surplus $S\left(L,0\right)$. On the other hand, the policy measure introduces a change in the probability of the drastic innovation $(prob^N\left(H,0\right) - prob^A\left(H,0\right))$, which, in many cases, will be negative.[20]

To understand the result, it is crucial to understand what determines the components of the terms in the decomposition. This is simple for the expressions $S\left(L,L\right)-S\left(L,0\right)$ and $S\left(H,0\right)-S\left(L,0\right)$, which are fully determined by the innovation technology and properties of the market environment. $S\left(H,0\right) - S\left(L,0\right)$ depends on the size of the innovation and, closely related, on its effect on demand. $S\left(L,L\right)-S\left(L,0\right)$ captures the value of competition to consumers. The remaining terms are endogenous. $prob^N(L,L)$ is the product of the entrant's innovation probability under no-acquisition and the conditional probability $1-p$ that this innovation will be non-drastic. $prob^N\left(H,0\right) - prob^A\left(H,0\right)$ is the product of the

[18]For instance, the last condition holds when a homogeneous Bertrand duopoly is compared to a monopoly and a higher state corresponds to lower costs.

[19]Note that these probabilities are determined by the equilibrium innovation strategies (r_i, r_e) which we have characterized previously.

[20]In cases where this effect is not negative, the consumer surplus effect of restricting acquisitions is trivially positive because of the increase in competition.

innovation effect of the acquisition policy (as discussed in Section 4) and the conditional probability p that an innovation is drastic.

These general considerations lead to some insights for the comparative statics of the consumer surplus effect. This is straightforward for the entrant's bargaining power: Clearly, β only enters $prob^N(H,0) - prob^A(H,0)$; it has no effects on the remaining terms in the decomposition. Moreover, from Proposition 4, we know that β increases $prob^N(H,0) - prob^A(H,0)$ and thus the adverse innovation effect of a restrictive merger policy (and there is no such effect when $\beta = 0$). Thus, a restrictive merger policy will always be justified for sufficiently low bargaining power of the entrant, but not necessarily when this bargaining power increases.

The effect of the (conditional) probability of drastic innovations p is more subtle. As discussed in Section 4, this parameter affects the probability of innovation and the size of the policy effects. Moreover, by definition, a higher p corresponds to an increase in the conditional probability of a drastic innovation, given that an innovation emerges. As these effects are potentially countervailing, the net effect is unclear. For instance, under the conditions of Proposition 4, an increase in p reduces the policy effect on innovation probabilities, which counteracts the direct positive effect of p. Thus, in general, we cannot expect a clear positive effect of p on $prob^N(H,0) - prob^A(H,0)$. Similarly, an increase in the conditional probability p of a drastic innovation potentially has ambiguous effects on the probability of competition ($prob^N(L,L)$).[21]

Such ambiguities also arise for the effects of increasing competition intensity on the product market. To illustrate, consider a reduction in duopoly profits $\pi(L,L)$. Under the no-acquisitions policy, this means that the entrant gains less from successfully innovating and competing with the incumbent, which, in turn, should reduce his innovation investment. Consequently, he is less likely to innovate and enter the market. However, in many reasonable parameterizations of the model, lower product market profits go hand in hand with higher consumer surplus $S(L,L)$. Despite the clear effect on the probability of drastic innovation (as described in Proposition 4), the total expected effect of more intense product market competition on consumer surplus can therefore be positive or negative.

6.2 One-dimensional innovation efforts

We now briefly discuss what would happen in an alternative setting where firms can only choose total innovation efforts rather than which projects to invest in. We summarize the main ideas here; the details are in the appendix. In this simple benchmark model, we assume that both firms exert innovation effort that determines the probability x_j of innovation success independently across firms. The necessary costs are $K(x_j)$. The cost function $K : [0,1] \to \mathbb{R}_+$ is continuous, differentiable, strictly increasing and convex, with $K(0) = 0$, $\lim_{x \to 1} K(x) = \infty$.

This simple model differs from our model only in the way in which innovation probabilities are determined. Thus, the analysis of the acquisition and commercialization stages applies as before. As acquisitions only arise when the entrant has invested successfully into a non-drastic innovation, the effect of prohibiting acquisitions is driven by the differences in the values of the firms in this situation.

[21] While the direct effect on the probability that an L-innovation rather than an H-innovation emerges is negative, we conjecture that the effect on the overall innovation probability is positive under the no-acquisition policy.

For the entrant, the incentive to invest is higher when acquisitions are allowed than when they are not. The intuition is straightforward: When acquisitions are allowed, entrants will sell their innovation to the incumbent, thereby sharing the gain from eliminating competition with the incumbent. Conversely, the incumbent's innovation incentives are lower under laissez-faire than under a no-acquisitions policy. Again, the intuition follows by investigating the firm's profit in the case that the entrant has generated a non-drastic innovation. Under a laissez-faire policy, the incumbent can avoid the duopoly case by means of acquisitions. Thus, she has a lower incentive to block the entrant via innovation investment.

The differences in innovation incentives with and without acquisitions depend in particular on the bargaining power of the entrant, but also on the intensity of competition in the final market stage. To provide an intuition, consider the extreme case where $\beta = 0$ and the entrant does not receive any surplus above his outside option when the start-up is acquired. Thus the entrant's innovation incentives are not directly affected by the prospect of an acquisition, but only indirectly through the reaction of the incumbent. Under laissez-faire, the incumbent reduces her innovation effort, since she can appropriate the entire surplus. The decrease in innovation effort by the incumbent induces an increase by the entrant, since for him, innovation efforts are strategic substitutes. Which effect dominates then depends on the intensity of competition in the market stage. If competition is very intense, in other words, duopoly profits are very low, the incumbent's reaction to the possibility of acquiring the entrant is much stronger and overcompensates the positive indirect effect on the entrant's innovation effort.

Conversely, in the other polar case of $\beta = 1$, the prospect of acquisitions has no direct effect on the incumbent's profits and hence on her innovation incentives, so that the potential negative effect of allowing acquisitions disappears entirely. In this case, only the positive effect on the entrant's incentives is present and even reinforced by the incumbent, who considers innovation efforts as strategic complements because her main incentive is to fight the entry threat. Consequently, the overall effect of allowing acquisitions is to increase innovation incentives.

In sum, in this simple model, a ban on acquisitions has ambiguous effects on innovation.

7 Conclusion

Recently, there has been an intense debate on the interactions between mergers and innovation, with particular emphasis on the role of start-up acquisitions. Motivated by this debate, our paper provides a theory of the strategic choice of innovation projects by incumbents and start-ups which takes into account the endogeneity of acquisition and commercialization decisions by the incumbent.

Very generally, both firms invest as much as possible into low-cost projects, whereas neither invests at all in high-cost projects. For projects with intermediate costs, at least one of the firms invest. This structure is independent of whether acquisitions are allowed or not and whether the incumbent commercializes the entrant's innovation after an acquisition or not.

We obtain several robust findings on the effects of acquisition policy. First, prohibiting start-up acquisitions weakly reduces the diversity of research approaches pursued by the firms and thereby the probability of developing breakthrough innovations, where the effect is strictly negative for killer acquisitions (without commercialization of the start-up's innovation). Second, a restrictive merger policy may induce the incumbent to strategically duplicate projects of the entrant to prevent competition.

These findings suggest that, even for pure killer acquisitions, the case for prohibition is not straightforward. On the one hand, any acquisition of a potential entrant leads to the well-known anti-competitive effect resulting from the elimination of a potential competitor. On the other hand, the chance to be acquired provides incentives to start-ups to invest in research projects that the incumbent would not develop herself (and it constrains the incumbent's desire to engage in socially wasteful projects). While the number of start-ups that are being acquired by incumbents in the digital economy sector is certainly a cause for concern, our analysis also shows that start-up acquisitions can help to keep the variety of innovative projects high, suggesting that an appropriate design of merger policy towards start-ups requires a lot of care. It also provides some understanding of the circumstances in which interventions may be warranted.

While our approach focuses on the short-term effects of acquisition policy, we believe that our analysis can potentially be useful to inform multi-period models of start-up acquisitions. It would be interesting to analyze how incumbents and potential entrants target their innovation activities when entry can take place repeatedly and when the incumbent's technology improves as a result of acquisitions. Is increasing dominance of the incumbent an inevitable outcome? Will the innovation process eventually slow down because it becomes too hard for entrant to compete? While these questions are beyond the scope of the current paper, our analysis suggests that it is expedient to take the policy effects on project choice into account, rather than only the effects on the overall innovation level.

A Appendix

A.1 Proof of Lemma 1

Proof. There are three possible cases. Either the entrant holds no patents, or he holds the H patent or the L patent. We will examine the three cases in turn. First, suppose that the entrant holds no patents. Then, since the entrant cannot compete without an innovation, the incumbent's profits are the same with or without the acquisition. Thus, the incumbent has no incentive to acquire the entrant. Next, suppose the entrant holds a patent on the H technology. Without an acquisition, the entrant commercializes the technology and obtains the payoff $\pi(H) - \kappa$ while the incumbent obtains $\pi(\ell, H) = 0$. With the acquisition, the incumbent commercializes the technology and obtains the payoff $\pi(H) - \kappa$. Thus the total payoffs are equal with or without the acquisition. Since the acquisition (by assumption) only goes through if the total payoffs strictly increase, the incumbent does not acquire the entrant.

Finally, consider the case when the entrant has a patent for the L technology. If there is no acquisition, the entrant commercializes the technology and obtains payoffs $\pi(L, \ell) - \kappa$, while the incumbent's payoffs are $\pi(\ell, L)$. If the incumbent acquires the entrant and commercializes the technology, he obtains $\pi(L, 0) - \kappa$, while without commercialization he obtains $\pi(\ell, 0)$, thus he will choose to commercialize only if $\pi(L, 0) - \kappa \geq \pi(\ell, 0)$. The entrant obtains the payoff of zero. There are three possible cases. (i) Suppose that $\pi(L, 0) - \kappa \geq \pi(\ell, 0)$, which implies $\pi(L, 0) > \pi(\ell, 0)$. By Assumption 1(iv), in this case $\pi(L, 0) > \pi(\ell, L) + \pi(L, \ell)$, so that $\pi(L, 0) - \kappa > \pi(\ell, L) + \pi(L, \ell) - \kappa$ and the acquisition is profitable. (ii) If $\pi(L, 0) - \kappa < \pi(\ell, 0)$ but $\pi(L, 0) \geq \pi(\ell, 0)$, Assumption 1(iv) implies $\pi(L, 0) - \kappa > \pi(\ell, L) + \pi(L, \ell) - \kappa$, so that $\pi(\ell, 0) > \pi(\ell, L) + \pi(L, \ell) - \kappa$ and the acquisition is profitable. (iii) Finally, if $\pi(L, 0) < \pi(\ell, 0)$ then Assumption 1(iv) gives $\pi(\ell, 0) > \pi(\ell, L) + \pi(L, \ell) - \kappa$, so that the acquisition is again profitable.

The price of the acquisition is either $\pi(L, \ell) - \kappa + \beta(\pi(L, 0) - \kappa - (\pi(\ell, L) + \pi(L, \ell) - \kappa)) = \beta(\pi(L, 0) - \pi(\ell, L) - \kappa) + (1 - \beta)(\pi(L, \ell) - \kappa)$ if the innovation is such that the incumbent intends to commercialize it, or $\pi(L, \ell) - \kappa + \beta(\pi(\ell, 0) - (\pi(\ell, L) + \pi(L, \ell) - \kappa)) = \beta(\pi(\ell, 0) - \pi(\ell, L)) + (1 - \beta)(\pi(L, \ell) - \kappa)$ if the incumbent does not plan to commercialize it. □

A.2 Proof of Lemma 3

Proof. (i) To see this, note that $v_i(H) = v_e(H)$. Thus

$$
\begin{aligned}
C(\theta_i^2) &= \frac{1}{2}\left[pv_e(H + (1-p)\left(v_i(L, 0) - v_i(\ell, L)\right)\right] \\
&= \frac{1}{2}\left[pv_e(H) + (1-p)\left(\pi_i(\ell, 0) - (\pi_i(\ell, 0) - v_e(L, \ell))\right)\right] \\
&= \frac{1}{2}\left[pv_e(H) + (1-p)v_e(L, \ell)\right] = C(\theta_e^2).
\end{aligned}
$$

(ii) Since $C(\theta_e^2) < C(\theta_e^1)$, it follows immediately that $\theta^2 < \theta_e^1$.

(iii) This will hold if and only if

$$\begin{aligned} C(\theta_i^1) &< C(\theta_e^1) \\ pv_i(H) + (1-p)v_i(L,0) - v_i(\ell,0) &< pv_e(H) + (1-p)v_e(L,\ell) \\ (1-p)v_i(L,0) - v_i(\ell,0) &< (1-p)v_e(L,\ell) \\ -pv_i(\ell,0) &< (1-p)v_e(L,\ell) \end{aligned}$$

which is satisfied since $v_i(L,0) = v_i(\ell,0)$ in the case without commercialization acquisitions.

(iv) This follows from

$$\begin{aligned} C(\theta_i^1) &\leq C(\theta^2) \\ pv_i(H) + (1-p)v_i(L,0) - v_i(\ell,0) &\leq \frac{1}{2}\left[pv_e(H) + (1-p)v_e(L,\ell)\right] \\ pv_i(H) - pv_i(\ell,0) &\leq \frac{1}{2}\left[pv_e(H) + (1-p)v_e(L,\ell)\right] \\ pv_i(H) - 2pv_i(\ell,0) &\leq (1-p)v_e(L,\ell). \end{aligned}$$

□

A.3 Proof of Proposition 1

Proof. (*a*) We first show that the proposed investment functions indeed constitute an equilibrium. Then we show uniqueness.

Existence We need to show that, conditional on the opponent choosing the equilibrium, there is no profitable deviation for either firm to some other effort function r_j. Starting from the proposed equilibrium strategy, it is sufficient to show that for none of the firms one of the three following types of deviations can be profitable.

1. Reduction of efforts on a set of projects that has positive measure.
2. Increase of effort on a set of projects that has positive measure.
3. Simultaneous increase of efforts on a set of projects with positive measure and reduction of effort on a set of projects with positive measure.

Accordingly, we show existence in six steps.

Step 1: There exists no profitable deviation in which the incumbent reduces effort on a set of projects with positive measure.
The set of projects on which the incumbent invests is given as $[0, \theta^2]$, and the investment intensity is $r_i(\theta) = 1$ on this entire set. The contribution of project θ to expected profits is

$$\frac{p}{2}v_i(H) + (1-p)\left(\frac{1}{2}v_i(L,0) + \frac{1}{2}v_i(\ell,L)\right) - (1-p)v_i(\ell,L) - C(\theta).$$

For $\theta \in [0, \theta^2]$, this expression is positive and it is therefore not profitable to reduce efforts on the project.

Step 2: There exists no profitable deviation in which the incumbent additionally invests in a set of projects with positive measure.
The set of projects on which the incumbent invests is given as $[0, \theta^2]$. As $\theta_i^1 \leq \theta^2$, investing in projects outside of this set is not profitable, no matter whether the entrant invests in these projects or not.

Step 3: There exists no profitable deviation in which the incumbent simultaneously exerts efforts on a set of projects with positive measure and reduces efforts on a set of projects with positive measure.
Arguing as in Step 1, the effort reduction reduces expected payoffs. Arguing as in Step 2, exerting efforts on projects from $[0,1) \setminus [0, \theta^2]$ further reduces the expected payoffs.

Step 4: There exists no profitable deviation in which the entrant reduces effort on a set of projects with positive measure.
First consider deviations where only efforts on projects from $[0, \theta^2]$ are reduced. As the incumbent invests in those projects in equilibrium, reducing efforts on them would reduce profits by definition of $\theta^2 = \theta_e^2$. Similarly, reducing efforts only on approaches from $[0, \theta_e^1] \setminus [0, \theta^2]$ would reduce profits by definition of θ_e^1, as the incumbent is not investing in these projects. Finally, combining the arguments, there is no profitable deviation where the entrant reduces efforts on projects in $[0, \theta^2]$ as well as on projects in $[0, \theta_e^1] \setminus [0, \theta^2]$, as projects in both subsets contribute positively to expected payoffs.

Step 5: There exists no profitable deviation in which the entrant increases efforts in a set of projects with positive measure.
Any project that the entrant does not already invest in satisfies $\theta > \theta_e^1 > \theta^2$. Thus, investing in such projects is never profitable.

Step 6: There exists no profitable deviation in which the entrant simultaneously increases efforts on a set of projects with positive measure and reduces efforts on a set of projects with positive measure.
This follows by combing the arguments in Step 4 and Step 5.

Uniqueness The proof consists of several steps.

Step 1: In any equilibrium, both firms exert maximal effort in almost every project in $[0, \theta_i^1]$.
Any such project satisfies $\theta \leq \min\{\theta_j^1, \theta_j^2\}$ for $j \in \{i, e\}$. Thus, if either firm does not exert maximal effort on a set of such projects with positive measure, it can increase profits by exerting (e.g.) maximal effort on these projects.

Step 2: In any equilibrium, effort in almost any project in $(\theta_e^1, 1)$ is zero.
As $\theta_e^1 \geq max\{\theta_j^1, \theta_j^2\}$ for $j \in \{i, e\}$, investing in such projects can never be profitable.

Step 3: In any equilibrium, the incumbent does not invest in a positive measure of projects in $(\theta^2, \theta_e^1]$.
Any such project θ satisfies $\theta > \max\{\theta_i^1, \theta_i^2\}$. Hence, investing in such projects is not profitable.

Step 4: In any equilibrium, the entrant exerts maximal effort in almost every project in $(\theta^2, \theta_e^1]$.
Any such project θ satisfies $\theta \leq \theta_e^1$. Moreover, by Step 3, in any such equilibrium, the incumbent does not invest in these projects. Thus, if the entrant does not exert maximal effort on a set of such projects with positive measure, he can profitably deviate by investing in this set.

Step 5: In any equilibrium, both firms invest in almost every project in

$(\theta_i^1, \theta^2]$.
For the entrant, this statement holds because projects in this interval are profitable independent of the incumbent's behavior, as all these projects satisfy $\theta \leq \theta^2 < \theta_e^1$. As the entrant invests in any project in the interval, investments of the incumbent are also profitable given $\theta \leq \theta^2$.

(b) We show that, for any choice of investment functions satisfying the conditions, there is no profitable deviation.

Step 1: There is no profitable deviation of either firm which consists of effort reductions on projects in $[0, \theta^2]$ for any of the equilibria under consideration.
Projects on this interval are profitable for both firms independent of the behavior of the other by definition of θ^2 and $\theta^2 < \theta_i^1 < \theta_e^1$.

Step 2: There is no profitable deviation which consists of effort increases on projects in $(\theta_e^1, 1)$ for any of the equilibria under consideration
Projects on this interval are not profitable for entrants by definition of θ_e^1; they are not profitable for incumbents by definition of θ^2 and the fact that $\theta_e^1 > \theta_i^1$.

Step 3: There is no profitable deviation of the incumbent which consists of effort increases on projects in $(\theta_i^1, \theta_e^1]$ for any of the equilibria under consideration.
These changes are not profitable for the incumbent because the projects satisfy $\theta \geq \theta_i^1 > \theta^2$.

Step 4: There is no profitable deviation of the entrant which consists of effort reductions on projects in $(\theta_i^1, \theta_e^1]$ for any of the equilibria under consideration.
The changes are not profitable for the entrant because the projects satisfy $\theta \geq \theta_i^1$, so that the incumbent does not invest, and they satisfy $\theta < \theta_e^1$, so that the entrant earns positive profits on them if the incumbent does not invest.

Step 5: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta^2, \theta_i^1]$ for those equilibria where firms $j = i, e$ choose either $r_j = 0$ or $r_j = 1$ on this interval.
In almost all the projects θ in this interval, exactly one firm is exerting full effort, whereas the other firm is exerting zero effort. As $\theta > \theta^2$, exerting additional effort would not be profitable for either firm. Conversely, as $\theta < \theta_i^1 < \theta_e^1$, reducing efforts is not profitable either.

Step 6: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta^2, \theta_i^1]$ from the equilibrium where $r_e(\theta) = \frac{C(\theta_i^1) - C(\theta)}{C(\theta_i^1) - C(\theta^2)}$ and $r_i(\theta) = \frac{C(\theta_e^1) - C(\theta)}{C(\theta_e^1) - C(\theta^2)}$.
It suffices to show that, for each player, at every $\theta \in (\theta^2, \theta_i^1]$, the proposed effort $r_j(\theta)$ maximizes expected profits, given the opponent's choice $r_k(\theta)$. The proposed efforts make the firms exactly indifferent between investing and not investing, hence the above investment efforts are weakly optimal for either firm conditional on the effort of the rival. After using the definitions of $C(\theta_i^1)$, $C(\theta_e^1)$ and $C(\theta^2)$ for the investment efforts which make the rival indifferent, we arrive at the above expressions. □

A.4 Proof of Proposition 2

Proof. By Corollary 1, $\mathcal{V}(r_i, r_e) = \theta_e^1$ and by definition $C(\theta_e^1) = pv_e(H) + (1-p)v_e(L, \ell)$. By Lemma 2, we can substitute $v_e(H)$ and $v_e(L, \ell)$ so that

$$C(\theta_e^1) = p(\pi(H) - \kappa) + (1-p)(\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)).$$

Since C is an increasing function, $\partial\mathcal{V}(r_i, r_e)/\partial x > 0$ if and only if

$$\frac{\partial(p(\pi(H) - \kappa) + (1-p)(\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)))}{\partial x} > 0,$$

for any parameter x. We will examine this inequality for the cases *(i)-(iii)* in three steps below.

Step 1: $\partial\mathcal{V}(r_i, r_e)/\partial p > 0$.

$$\begin{aligned} &\frac{\partial(p(\pi(H) - \kappa) + (1-p)(\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)))}{\partial p} = \\ &\pi(H) - \kappa - (\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)) = \\ &\pi(H) - \beta(\pi(\ell, 0) - \pi(\ell, L) - \pi(L, \ell) + \kappa) - \pi(L, \ell) \geq \\ &\pi(H) - \kappa - (\pi(\ell, 0) - \pi(\ell, L) - \pi(L, \ell)) - \pi(L, \ell) = \\ &\pi(H) - \kappa - \pi(\ell, 0) + \pi(\ell, L) > 0. \end{aligned}$$

The first inequality follows from Assumption 1(iv): Either $\max\{\pi(L, 0), \pi(\ell, 0)\} = \pi(\ell, 0)$, then the inequality is trivially satisfied, or $\max\{\pi(L, 0), \pi(\ell, 0)\} = \pi(L, 0)$. Since we consider cases where the innovation is not commercialized, if follows that $\pi(\ell, 0) > \pi(L, 0) - \kappa > \pi(\ell, L) + \pi(L, \ell) - \kappa$. The second inequality follows from Assumption 2(ii).

Step 2: $\partial\mathcal{V}(r_i, r_e)/\partial\beta > 0$.

$$\begin{aligned} &\frac{\partial(p(\pi(H) - \kappa) + (1-p)(\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)))}{\partial\beta} = \\ &(1-p)(\pi(\ell, 0) - \pi(\ell, L) - \pi(L, \ell) + \kappa) > 0, \end{aligned}$$

where the inequality follows from Assumption 1(iv) combined with the restriction to the case where $\pi(\ell, 0) > \pi(L, 0) - \kappa$.

Step 3: $\partial\mathcal{V}(r_i, r_e)/\partial\pi(L, \ell) > 0$ and $\partial\mathcal{V}(r_i, r_e)/\partial\pi(\ell, L) < 0$.

$$\begin{aligned} &\frac{\partial(p(\pi(H) - \kappa) + (1-p)(\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)))}{\partial\pi(L, \ell)} = \\ &(1-p)(1-\beta) > 0, \\ &\frac{\partial(p(\pi(H) - \kappa) + (1-p)(\beta(\pi(\ell, 0) - \pi(\ell, L)) + (1-\beta)(\pi(L, \ell) - \kappa)))}{\partial\pi(\ell, L)} = \\ &-(1-p)\beta < 0. \end{aligned}$$

□

A.5 Proof of Lemma 4

Proof. (i) Note that $v_e(H) = v_i(H)$. $\theta_e^2 < \theta_e^1$ will hold if and only if:

$$C(\theta_e^2) < C(\theta_i^2)$$
$$\frac{1}{2}\left(pv_e(H) + (1-p)v_e(L,\ell)\right) < \frac{1}{2}\left(pv_i(H) + (1-p)(v_i(L,0) - v_i(\ell,L))\right)$$
$$v_e(L,\ell) < v_i(L,0) - v_i(\ell,L)$$
$$\pi(L,\ell) - \kappa < \pi(\ell,0) - \pi(\ell,L)$$

which is satisfied by Assumption 1(iv) combined with $\pi(\ell,0) > \pi(L,0) - \kappa$.
(ii) Since $C(\theta_e^2) < C(\theta_e^1)$, it follows immediately that $\theta_e^2 < \theta_e^1$.
(iii) This will hold if and only if

$$C(\theta_i^1) < C(\theta_e^1)$$
$$pv_i(H) + (1-p)v_i(L,0) - v_i(\ell,0)) < (pv_e(H) + (1-p)v_e(L,\ell))$$
$$-pv_i(\ell,0) < (1-p)v_e(L,\ell)$$

which follows from $v_i(L,0) = v_i(\ell,0)$. □

A.6 Characterization of the equilibrium under the no-acquisition policy

Proposition 8. *Consider the case without commercialization under a no-acquisition policy.*

(a) If $\theta_i^1 < \theta_i^2$ and $\theta_i^1 > \theta_e^2$ then $\theta_e^2 < \theta_i^1 \leq \min\{\theta_e^1, \theta_i^2\}$. The equilibrium is unique. Functions r_i and r_e constitute an equilibrium if and only if conditions (i)-(v) hold:

(i) $r_e(\theta) = 1$ *and* $r_i(\theta) = 1$, *for* $\theta \in [0, \theta_e^2]$

(ii) $r_e(\theta) = 0$ *and* $r_i(\theta) = 1$, *for* $\theta \in (\theta_e^2, \theta_i^1]$

(iii) $r_e(\theta) = \frac{C(\theta)-C(\theta_i^1)}{C(\theta_i^2-C(\theta_i^1))}$ *and* $r_i(\theta) = \frac{C(\theta_e^1)-C(\theta)}{C(\theta_e^1)-C(\theta_e^2)}$, *for* $\theta \in (\theta_i^1, \min\{\theta_i^2, \theta_e^1\}]$

(iv) $r_e(\theta) = 1$ *and* $r_i(\theta) = 0$, *for* $\theta \in (\min\{\theta_i^2, \theta_e^1\}, \theta_e^1]$,

(v) $r_e(\theta) = 0$ *and* $r_i(\theta) = 0$, *for* $\theta \in (\theta_e^1, 1)$.

(b) If $\theta_i^1 < \theta_i^2$ and $\theta_i^1 \leq \theta_e^2$ then $\theta_i^1 \leq \theta_e^2 \leq \min\{\theta_e^1, \theta_i^2\}$. The equilibrium is unique. Functions r_i and r_e constitute an equilibrium if and only if conditions (i)-(iv) hold:

(i) $r_e(\theta) = 1$ *and* $r_i(\theta) = 1$, *for* $\theta \in [0, \theta_e^2]$

(ii) $r_e(\theta) = \frac{C(\theta)-C(\theta_i^1)}{C(\theta_i^2-C(\theta_i^1))}$ *and* $r_i(\theta) = \frac{C(\theta_e^1)-C(\theta)}{C(\theta_e^1)-C(\theta_e^2)}$, *for* $\theta \in (\theta_e^2, \min\{\theta_i^2, \theta_e^1\}]$

(iii) $r_e(\theta) = 1$ *and* $r_i(\theta) = 0$, *for* $\theta \in (\min\{\theta_i^2, \theta_e^1\}, \theta_e^1]$,

(iv) $r_e(\theta) = 0$ *and* $r_i(\theta) = 0$, *for* $\theta \in (\theta_e^1, 1)$.

(c) If $\theta_i^2 \leq \theta_i^1$ then $\theta_e^2 < \theta_i^2 \leq \theta_i^1 < \theta_e^1$. The equilibrium is not unique. Functions r_i and r_e constitute an equilibrium if and only if conditions (i)-(v) hold:

(i) $r_e(\theta) = 1$ *and* $r_i(\theta) = 1$, *for* $\theta \in [0, \theta_e^2]$

(ii) $r_e(\theta) = 0$ *and* $r_i(\theta) = 1$, *for* $\theta \in (\theta_e^2, \theta_i^2]$

(iii) $r_e(\theta) = 1$ *and* $r_i(\theta) = 0$, *for* $\theta \in (\theta_i^1, \theta_e^1]$,

(iv) $r_e(\theta) = 0$ *and* $r_i(\theta) = 0$, *for* $\theta \in (\theta_e^1, 1)$.

(v) *for any* $\theta \in (\theta_i^2, \theta_i^1]$ *either:*

$r_e(\theta) = 1$ *and* $r_i(\theta) = 0$, *or*

$r_e(\theta) = 0$ *and* $r_i(\theta) = 1$, *or*

$r_e(\theta) = \frac{C(\theta_i^1)-C(\theta)}{C(\theta_i^1)-C(\theta_i^2)}$ *and* $r_i(\theta) = \frac{C(\theta_e^1)-C(\theta)}{C(\theta_e^1)-C(\theta_e^2)}$.

Proof. (a) Since the claim here is that the equilibrium is unique, we first show that the equilibrium proposed is indeed an equilibrium to then show that it is unique.

Existence There are no profitable deviation for neither of the two firms. Steps 1 and 2 are identical to Steps 1 and 2 in the proof for (a), so we will start with Step 3:

Step 3: There is no profitable deviation that consists of the incumbent decreasing effort or the entrant increasing effort on projects in $(\theta_e^2, \theta_i^1]$.
Since on this interval the projects satisfy $\theta \leq \theta_i^1 \leq \theta_i^2$, the incumbent will invest fully irrespective of the investment of the entrant. Because $\theta > \theta_e^2$, the entrant earns negative profits if he invests on any such project conditional on the incumbent investing, thus he should exert zero effort in any such project.

Step 4: There is no profitable deviation that consists of the incumbent increasing effort or the entrant decreasing effort on projects in $(\min\{\theta_i^2, \theta_e^1\}, \theta_e^1]$.
Note that this interval only has positive measure if $\min\{\theta_i^2, \theta_e^1\} = \theta_i^2 < \theta_e^1$. Hence we can ignore the case when $\min\{\theta_i^2, \theta_e^1\} = \theta_e^1$. When $\theta_i^2 < \theta_e^1$, then projects on this interval satisfy $\theta \geq \theta_i^2 > \theta_i^1$ and the incumbent will not invest irrespective of the investment of the entrant. Because $\theta < \theta_e^1$, the entrant earns positive profits if he invests on any such project conditional on the incumbent investing, thus he should exert maximal effort.

Step 5: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta_i^1, \min\{\theta_i^2, \theta_e^1\}]$.
It suffices to show that, for each player, at every $(\theta_i^1, \min\{\theta_i^2, \theta_e^1\}]$, the proposed effort maximizes expected profits, given the opponent's choice. With the specified investment strategies, the opponent's choice makes the firm exactly indifferent between investing and not investing, hence the above investment efforts are weakly optimal for either firm. After using the definitions of $C(\theta_i^1)$, $C(\theta_i^2)$, $C(\theta_e^1)$ and $C(\theta_e^2)$, we arrive at the above expressions.

Uniqueness We build the proof in several steps considering each interval separately.

Step 1: In any equilibrium, both firms exert maximal effort in almost every project in $[0, \theta_e^2]$.
Any such project satisfies $\theta \leq \min\{\theta_j^1, \theta_j^2\}$ for $j \in \{i, e\}$. Thus if either firm does not exert maximal effort on a set of such projects with positive measure, it can (weakly) increase profits by exerting maximal effort irrespective of the investment decision of the rival.

Step 2: In any equilibrium, effort in almost any project in $(\theta_e^1, 1)$ is zero.
As any such project satisfies $\theta > \max\{\theta_j^1, \theta_j^2\}$ for $j \in \{i, e\}$, investing in such projects is not profitable.

Step 3: In any equilibrium, the incumbent exerts maximal effort in almost every project in $(\theta_e^2, \theta_i^1]$.

In this interval any project satisies $\theta \leq \min\{\theta_i^1, \theta_i^2\}$, thus it is always profitable to exert as much effort as possible irrespective of the investment of the entrant.

Step 4: In any equilibrium, the entrant exerts no effort in almost every project in $(\theta_e^2, \theta_i^1]$.
Since $\theta_e^1 > \theta > \theta_e^2$, the entrant only want to invest if he is the sole investor. From Step 3, we know that the incumbent would always want to invest, so the entrant is never the sole investor. However, sharing the innovation is not profitable for him in this interval.

Step 5: In any equilibrium, the incumbent does not invest in a positive measure of projects in $(\min\{\theta_i^2, \theta_e^1\}, \theta_e^1]$.
If $\theta_i^2 \geq \theta_e^1$ the interval does not have a positive measure. If $\theta_i^2 < \theta_e^1$, then any project in the interval satisfies $\theta > \theta_i^2 > \theta_i^1$ and it is not profitable for the incumbent to invest in any set of projects irrespective of the investment behavior of the entrant.

Step 6: In any equilibrium, the entrant exerts maximal effort in a positive measure of projects in $(\min\{\theta_i^2, \theta_e^1\}, \theta_e^1]$.
If $\theta_i^2 \geq \theta_e^1$ the interval does not have a positive measure. If $\theta_i^2 < \theta_e^1$, then the entrant only want to invest if he is the sole innovator. From Step 5 we know that the incumbent never invests and, as any project satisfies $\theta \leq \theta_e^1$, the entrant wants to exert maximal effort.

Step 7: In any equilibrium, the optimal investment efforts for projects in $(\theta_i^1, \min\{\theta_i^2, \theta_e^1\}]$ are $r_e(\theta) = \frac{C(\theta)-C(\theta_i^1)}{C(\theta_i^2)-C(\theta_i^1)}$ and $r_i(\theta) = \frac{C(\theta_e^1)-C(\theta)}{C(\theta_e^1)-C(\theta_e^2)}$.
Suppose the incumbent exerts $r_i' > r_i$. Then profits of investing for the entrant are negative and he should not invest. Since $\theta > \theta_i^1$, reducing her effort is increasing the incumbent's profits. Thus, clearly, $r_i' > r_i$ and $r_e' = 0$ cannot be an equilibrium. Conversely, suppose $r_i' < r_i$. Then, profits of investing for the entrant are positive and he should exert maximal effort. In this case, since $\theta \leq \theta_i^2$, increasing her effort is profitable for the incumbent. Therefore, $r_i' < r_i$ and $r_e = 1$ cannot be an equilibrium as well. Now suppose the entrant exerts $r_e' > r_e$. Then the incumbent should exert maximal effort on any such project. Since $\theta > \theta_e^2$, the entrant should then not invest. Thus $r_e' > r_e$ and $r_i = 1$ cannot be an equilibrium. If the entrant exerts $r_e' < r_e$, the incumbent should not exert any effort since $\theta < \theta_i^2$. But then, as $\theta < \theta_e^1$, the entrant should exert maximal effort. Again, $r_e' < r_e$ and $r_i = 0$ cannot be an equilibrium. Consequently, r_e and r_i as specified are the only remaining equilibrium strategies.

(b) **Existence** We begin by showing that there is no profitable deviation for neither of the two firms:

Step 1: There is no profitable deviation of either firm which consists of effort reductions on projects in $[0, \theta_e^2]$.
Projects in this interval are profitable for the entrant irrespective of the behavior of the incumbent since $\theta \leq \theta_e^2 < \theta_e^1$. Given the entrant invests, investment for the incumbent is profitable, since $\theta \leq \theta_i^2$. Consequently, both firms should exert maximal effort.

Step 2: There is no profitable deviation that consists of effort increases on projects in $(\theta_e^1, 1)$ for any equilibria under consideration.
Projects in this interval are not profitable for the entrant by definition of θ_e^1, they are not profitable for the incumbent due to the fact that $\theta_i^1 < \theta_e^1$. By definition of θ_i^2, it does not matter whether it is larger or smaller than θ_e^1 conditional on the entrant not investing.

Step 3: There is no profitable deviation that consists of the incumbent increasing effort or the entrant decreasing effort on projects in $(\min\{\theta_i^2, \theta_e^1\}, \theta_e^1]$ for any equilibrium under consideration.

Note that this interval only has positive measure if $\min\{\theta_i^2, \theta_e^1\} = \theta_i^2 < \theta_e^1$. Hence we can ignore the case when $\min\{\theta_i^2, \theta_e^1\} = \theta_e^1$. When $\theta_i^2 < \theta_e^1$, then projects on this interval satisfy $\theta \geq \theta_i^2 > \theta_i^1$ and the incumbent will not invest irrespective of the investment of the entrant. Because $\theta < \theta_e^1$, the entrant maximizes profits if he fully invests on any such project and the incumbent does not invest.

Step 4: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta_e^2, \min\{\theta_i^2, \theta_e^1\}]$.
It suffices to show that, for each player, at every $(\theta_i^1, \min\{\theta_i^2, \theta_e^1\}]$, the proposed effort maximizes expected profits, given the opponent's choice. In this case, the opponent's choice makes the firm exactly indifferent between exerting any positive effort and not investing, hence the above investment efforts are weakly optimal for either firm. After using the definitions of $C(\theta_i^1)$, $C(\theta_i^2)$, $C(\theta_e^1)$ and $C(\theta_e^2)$, we arrive at the above expressions.

Uniqueness Again we build the proof in several steps considering each interval separately.

Step 1: In any equilibrium, both firms exert maximal effort in almost every project in $[0, \theta_i^1]$.
Any such project satisfies $\theta \leq \min\{\theta_j^1, \theta_j^2\}$ for $j \in \{i, e\}$. Thus if either firm does not exert maximal effort on a set of such projects with positive measure, it can (weakly) increase profits by exerting maximal effort.

Step 2: In any equilibrium, both firms exert maximal effort in almost every project in $[\theta_i^1, \theta_e^2]$.
Any such project satisfies $\theta \leq \min\{\theta_e^1, \theta_e^2\}$, hence, if the entrant does not exert maximal effort on a set of such projects with positive measure, it can (weakly) increase profits by exerting maximal effort. For the incumbent, there is a threshold $\bar{r}_e$, such that, if $r_e > \bar{r}_e$, then he should exert maximal effort, if $r_e > \bar{r}_e$ he should exert no effort, and if $r_e = \bar{r}_e$ he is indifferent for any $\theta \in [\theta_i^1, \theta_e^2]$. Since the entrant will always exert $r_e > \bar{r}_e$, there is no equilibrium where the incumbent would profit from lower than maximal effort.

Step 3: In any equilibrium, effort in almost any project in $(\theta_e^1, 1)$ is zero.
As any such project satisfies $\theta > \max\{\theta_j^1, \theta_j^2\}$ for $j \in \{i, e\}$, investing in such projects is not profitable.

Steps 4, 5 and 6 are identical to steps 5, 6 and 7 in (b), with the exception that the interval in consideration in Step 6 is $(\theta_e^2, \min\{\theta_i^2, \theta_e^1\}]$, while everything else goes through as in Step 7.

(c) As in the proof for Proposition 1, we need to show that there are no profitable deviations, which we will do with several steps:

Step 1: There is no profitable deviation of either firm which consists of effort reductions on projects in $[0, \theta_e^2]$ for any of the equilibria under consideration.
Projects θ in this interval are profitable for both firms irrespective of the behavior of the other firms since $\theta \leq \min\{\theta_j^2, \theta_j^1\}$ for $j \in \{i, e\}$. Thus both should exert maximal effort.

Step 2: There is no profitable deviation that consists of effort increases on projects in $(\theta_e^1, 1)$ for any equilibria under consideration.
Projects in this interval are not profitable for the entrant by definition of θ_e^1, they are not profitable for the incumbent by definition of θ_i^2 and the fact that $\theta_i^1 < \theta_e^1$.

Step 3: There is no profitable deviation that consists of the incumbent decreasing effort or the entrant increasing effort on projects in $(\theta_e^2, \theta_i^2]$ for any

equilibrium under consideration.
Since on this interval the projects satisfy $\theta \leq \theta_i^2 \leq \theta_i^1$, the incumbent will invest fully irrespective of the investment of the entrant. Because $\theta > \theta_e^2$, the entrant earns negative profits if he invests on any such project and the incumbent invests at her maximum.

Step 4: There is no profitable deviation that consists of the incumbent increasing effort or the entrant decreasing effort or on project in $(\theta_i^1, \theta_e^1]$ for any equilibrium under consideration.
Since on this interval the projects satisfy $\theta > \theta_i^1 \geq \theta_i^2$, the incumbent will not invest irrespective of the investment of the entrant. Because $\theta < \theta_e^1$, the entrant earns positive profits if he invests on any such project and the incumbent does not invest, thus he should exert maximal effort on any such project.

Step 5: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta_i^2, \theta_i^1]$ for those equilibria where firms $j = i, e$ choose either $r_j = 0$ or $r_j = 1$ on this interval.
In almost all the projects θ in this interval, exactly one firm is exerting full effort, whereas the other firm is exerting zero effort. As $\theta > \theta_i^2 > \theta_e^2$, exerting additional effort would not be profitable for either firm. Conversely, as $\theta \leq \theta_i^1 < \theta_e^1$, reducing effort is not profitable either.

Step 6: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta_i^2, \theta_i^1]$ for those equilibria where $r_e(\theta) = \frac{C(\theta_i^1)-C(\theta)}{C(\theta_i^1)-C(\theta_i^2)}$ and $r_i(\theta) = \frac{C(\theta_e^1)-C(\theta)}{C(\theta_e^1)-C(\theta_e^2)}$.
It suffices to show that, for each player, at every $\theta \in (\theta_i^2, \theta_i^1]$, the proposed effort maximizes expected profits, given the opponent's choice. With the specified investment efforts, the opponent's choice makes the firm exactly indifferent between investing and not investing, hence the above investment efforts are weakly optimal for either firm. After using the definitions of $C(\theta_i^1)$, $C(\theta_i^2)$, $C(\theta_e^1)$ and $C(\theta_e^2)$, we arrive at the above expressions.

□

A.7 Proof of Proposition 3

Proof. In the equilibrium with acquisition, denote the equilibrium strategies with $r_i^*(A)$ and $r_e^*(A)$. Similarly, in the equilibrium without acquisitions, denote the equilibrium strategies with $(r_i^*(N), r_e^*(N))$.

Part (a) By Proposition 1, in any equilibrium with acquisitions $r_e^*(\theta; A) = 1$ for all $\theta \in [0, \theta^2(A)] \cup [\theta_i^1(A), \theta_e^1(A)]$. Furthermore, for all $\theta \in (\theta^2(A), \theta_i^1(A))$ either $r_i^*(\theta; A) + r_e^*(\theta; A) = 1$ or $r_i^*(\theta; A) = \frac{C(\theta_e^1(A))-C(\theta)}{C(\theta_e^1(A))-C(\theta^2(A))} > 0$, where the last inequality follows from $\theta_e^1(A) > \theta_i^1(A)$. Thus, for all $\theta \in [0, \theta_e^1(A)]$ we have $r_i^*(\theta; A) + r_e^*(\theta; A) > 0$. Then, by Proposition 1, $\mathcal{V}(r_i^*(A), r_e^*(A)) = \theta_e^1(A)$.

By Proposition 8, in equilibria with acquisitions where $\theta_i^2 \leq \theta_i^1$, $r_i^*(\theta, N) + r_e^*(\theta, N) = 1$ for all $\theta \in [0, \theta_i^2(N)] \cup (\theta_i^1, \theta_e^1]$. For $\theta \in (\theta_i^2, \theta_i^1]$ either $r_e(\theta, N) + r_i(\theta, N) = 1$ or $r_e(\theta) = \frac{C(\theta_i^1)-C(\theta)}{C(\theta_i^1)-C(\theta_i^2)} > 0$. Hence $\mathcal{V}(r_i^*(N), r_e^*(N)) = \theta_e^1(N)$. In equilibria with acquisitions where $\theta_i^2 > \theta_i^1$, $r_i^*(\theta, N) + r_e^*(\theta, N) = 1$ for all $\theta \in [0, \max\{\theta_i^1(N), \theta_e^2(N)\}] \cup (\min\{\theta_i^2(N), \theta_e^1(N)\}, \theta_e^1(N)]$. For $\theta \in (\max\{\theta_i^1(N), \theta_e^2(N)\}, \min\{\theta_i^2(N), \theta_e^1(N)\}]$ $r_e(\theta) = \frac{C(\theta)-C(\theta_i^1)}{C(\theta_i^2)-C(\theta_i^1)} > 0$. Hence, again, $\mathcal{V}(r_i^*(N), r_e^*(N)) = \theta_e^1(N)$.

Finally: $\theta_e^1(N) < \theta_e^1(A)$.
To show this, it is sufficient that

$$\begin{aligned}
C(\theta_e^1(N)) &< C(\theta_e^1(A)) \\
p\pi(H) + (1-p)\pi(L,\ell) - \kappa &< pv_e(H) + (1-p)v_e(L,\ell;A) \\
p\pi(H) + (1-p)\pi(L,\ell) - \kappa &< p(\pi(H)-\kappa) + (1-p)v_e(L,\ell;A) \\
\pi(L,\ell) - \kappa &< v_e(L,\ell;A) \\
\pi(L,\ell) - \kappa &< \beta(\pi(\ell,0) - \pi(\ell,L)) + (1-\beta)(\pi(L,\ell)-\kappa) \\
\pi(L,\ell) - \kappa &< \pi(\ell,0) - \pi(\ell,L) \\
\pi(\ell,L) + \pi(L,\ell) - \kappa &< \pi(\ell,0)
\end{aligned}$$

where simple algebra leads to the last inequality, which holds by Assumption 1(iv) and $\pi(\ell,0) > \pi(L,0) - \kappa$.

Part (b) If $(r_i^*(A), r_e^*(A))$ constitutes a simple equilibrium, then $\mathcal{P}(r_i^*(A), r_e^*(A)) > \mathcal{P}(r_i^*(N), r_e^*(N))$.
By Proposition 1, if $(r_i^*(A), r_e^*(A))$ is a simple equilibrium, then $\mathcal{P}(r_i^*(A), r_e^*(A)) = \theta_e^1(A)$. By Part (a), $\theta_e^1(N) \geq \mathcal{P}(r_i^*(N), r_e^*(N))$, and $\theta_e^1(A) > \theta_e^1(N)$.

□

A.8 Proof of Propositions 4

Proof. By Proposition 3(a), we get $\mathcal{V}(r_i^*(A), r_e^*(A)) - \mathcal{V}(r_i^*(N), r_e^*(N)) = \theta_e^1(A) - \theta_e^1(N) > 0$.

Applying the implicit function theorem on the equations determining $\theta_e^1(A)$ and $\theta_e^1(N)$, we get that

$$\begin{aligned}
\frac{\partial(\theta_e^1(A) - \theta_e^1(N))}{\partial x} &= \\
&\frac{\frac{\partial}{\partial x}(p(\pi(H)-\kappa) + (1-p)(\beta(\pi(\ell,0) - \pi(\ell,L)) + (1-\beta)(\pi(L,\ell)-\kappa)))}{C'(\theta_e^1(A))} \\
&- \frac{\frac{\partial}{\partial x}(p(\pi(H)-\kappa) + (1-p)(\pi(L,\ell)-\kappa))}{C'(\theta_e^1(N))}
\end{aligned}$$

for any parameter x.

(i) $\partial(\mathcal{V}(r_i(A), r_e(A)) - \mathcal{V}(r_i(N), r_e(N)))/\partial p < 0$**:**

$$\begin{aligned}
\frac{\partial(\theta_e^1(A) - \theta_e^1(N))}{\partial p} &= \frac{\pi(H) - \kappa - \beta(\pi(\ell,0) - \pi(\ell,L)) - (1-\beta)(\pi(L,\ell)-\kappa)}{C'(\theta_e^1(A))} \\
&- \frac{\pi(H) - \kappa - \pi(L,\ell) + \kappa}{C'(\theta_e^1(N))} < 0 \\
&\Leftrightarrow \frac{\pi(H) - \pi(L,\ell) - \beta(\pi(\ell,0) - \pi(\ell,L) - \pi(L,\ell) + \kappa)}{\pi(H) - \pi(L,\ell)} < \frac{C'(\theta_e^1(A))}{C'(\theta_e^1(N))}
\end{aligned}$$

The LHS of the above inequality is strictly smaller than one by Assumption 1 combined with $\pi(\ell,0) > \pi(L,0) - \kappa$. The RHS is the relation of the slopes of the cost function at the threshold resarch projects with and without acquisitions, respectively. Remember that $\theta_e^1(A) > \theta_e^1(N)$. Then, since costs are convex, the inequality holds.

(ii) $\partial(\mathcal{V}(r_i(A), r_e(A)) - \mathcal{V}(r_i(N), r_e(N)))/\partial\beta > 0$

$$\frac{\partial(\theta_e^1(A) - \theta_e^1(N))}{\partial\beta} = \frac{(1-p)(\pi(\ell,0) - \pi(\ell,L) - \pi(L,\ell) + \kappa)}{C'(\theta_e^1(A))} > 0$$

which follows from Assumption 1 and $\pi(\ell, 0) > \pi(L, 0) - \kappa$.

(iii) $\partial(\mathcal{V}(r_i(A), r_e(A)) - \mathcal{V}(r_i(N), r_e(N)))/\partial\pi(L,\ell) < 0$ **and**
$\mathcal{V}(r_i(A), r_e(A)) - \mathcal{V}(r_i(N), r_e(N)))/\partial\pi(\ell, L) < 0$

$$\begin{aligned}\frac{\partial(\theta_e^1(A) - \theta_e^1(N))}{\partial\pi(L,\ell)} &= \frac{(1-p)(1-\beta)}{C'(\theta_e^1(A))} - \frac{(1-p)}{C'(\theta_e^1(N))} < 0\\ &\Leftrightarrow (1-\beta) < \frac{C'(\theta_e^1(A))}{C'(\theta_e^1(N))}\end{aligned}$$

$$\frac{\partial(\theta_e^1(A) - \theta_e^1(N))}{\partial\pi(\ell,L)} = \frac{-(1-p)\beta}{C'(\theta_e^1(A))} - 0 < 0$$

where the first part of *(iii)* requires convexity of research costs, while the second part does not.

□

A.9 Proof of Lemma 5

Proof.
(i)

$$\begin{aligned}C(\theta_e^2) &= \frac{1}{2}\left(pv_e(H) + (1-p)v_e(L,\ell)\right)\\ &= \frac{p}{2}v_i(H) + \frac{1}{2}\left((1-p)(v_i(L,0) - v_i(\ell,L))\right)\\ &= \frac{p}{2}v_i(H) + (1-p)\left(\frac{1}{2}(v_i(L,0) + \frac{1}{2}v_i(\ell,L))\right) - (1-p)v_i(\ell,L)\\ &= C(\theta_i^2).\end{aligned}$$

(ii) Since $C(\theta_e^2) < C(\theta_e^1)$, it follows immediately that $\theta^2 < \theta_e^1$.
(iiia) $\theta_i^1 \in (0, \theta^2]$ if:

$$\begin{aligned}C(\theta_i^1) &\le C(\theta^2)\\ pv_i(H) + (1-p)v_i(L,0) - v_i(\ell,0) &\le \frac{p}{2}v_i(H) + \frac{(1-p)}{2}\left(v_i(L,0) - v_i(\ell,L)\right)\\ 2(1-p)v_i(L,0) - 2v_i(\ell,0) &\le -pv_i(H) + (1-p)\left(v_i(L,0) - v_i(\ell,L)\right)\\ (1-p)v_i(\ell,L) &\le 2v_i(\ell,0) - pv_i(H) - (1-p)v_i(L,0).\end{aligned}$$

(iiib) $\theta_i^1 \in (\theta^2, \theta_e^1]$ if:

$$\begin{aligned}&C(\theta_i^1) > C(\theta^2) \text{ and}\\ &C(\theta_i^1) \le C(\theta_e^1).\end{aligned}$$

By (iiia), the first inequality holds if

$$(1-p)v_i(\ell, L) > 2v_i(\ell, 0) - pv_i(H) - (1-p)v_i(L, 0).$$

The second inequality holds if

$$\begin{aligned} pv_i(H) + (1-p)v_i(L,0) - v_i(\ell,0) &\leq pv_e(H) + (1-p)v_e(L,\ell) \\ (1-p)v_i(L,0) - (1-p)v_e(L,\ell) &\leq v_i(\ell,0) \\ (1-p)v_i(\ell,L) &\leq v_i(\ell,0) \end{aligned}$$

(iiic) $\theta_i^1 \in (\theta_e^1, 1)$ if $C(\theta_i^1) > C(\theta_e^1)$ which by (iiib) holds if

$$(1-p)v_i(\ell, L) > v_i(\ell, 0).$$

□

A.10 Proof of Proposition 5

Proof. (a) and (b) The proof is analogous to the proof of Proposition 1(a) and 1(b) and is therefore omitted.

(c) We show that, for any choice of investment functions satisfying the conditions, there is no profitable deviation.

Step 1: There is no profitable deviation of either firm which consists of effort reductions on projects in $[0, \theta^2]$ for any of the equilibria under consideration.
Since $\theta^2 < \theta_e^1 < \theta_i^1$, projects on this interval are profitable for both firms independent of the behavior of the other firm.

Step 2: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta^2, \theta_e^1]$ for those equilibria where firms $j = i, e$ choose either $r_j = 0$ or $r_j = 1$ on this interval.
As $\theta > \theta^2$, when $r_j = 1$ the best reply of the other firm is $r_k = 0$. As $\theta < \theta_e^1 < \theta_i^1$, when $r_j = 0$ the best reply of the other firm is $r_k = 1$.

Step 3: There is no profitable deviation of either firm that consists of increasing or decreasing effort in $(\theta^2, \theta_e^1]$ from the equilibrium where $r_e(\theta) = \frac{C(\theta_i^1)-C(\theta)}{C(\theta_i^1)-C(\theta^2)}$ and $r_i(\theta) = \frac{C(\theta_e^1)-C(\theta)}{C(\theta_e^1)-C(\theta^2)}$.
It suffices to show that, for each player, at every $\theta \in (\theta^2, \theta_e^1]$, the proposed effort $r_j(\theta)$ maximizes expected profits, given the opponent's choice $r_k(\theta)$. The proposed efforts make the firms exactly indifferent between investing and not investing, hence the above investment efforts are weakly optimal for either firm conditional on the effort of the rival. After using the definitions of $C(\theta_i^1)$, $C(\theta_e^1)$ and $C(\theta^2)$ for the investment efforts which make the rival indifferent, we arrive at the above expressions.

Step 4: There is no profitable deviation of the incumbent which consists of effort decreases on projects in $(\theta_e^1, \theta_i^1]$ for any of the equilibria under consideration.
These changes are not profitable for the incumbent because the projects satisfy $\theta \leq \theta_i^1$.

Step 5: There is no profitable deviation of the entrant which consists of effort increases on projects in $(\theta_e^1, \theta_i^1]$ for any of the equilibria under consideration.
The changes are not profitable for the entrant because the projects satisfy $\theta > \theta_e^1 > \theta^2$.

Step 6: There is no profitable deviation which consists of effort increases on projects in $(\theta_i^1, 1)$ for any of the equilibria under consideration
Projects on this interval are not profitable for entrants by definition of θ_e^1 and the fact that $\theta_i^1 > \theta_e^1$; they are not profitable for incumbents by definition of θ_i^1. □

A.11 Proof of Lemma 6

Proof.
(i) The inequality holds if $C(\theta_e^2) < C(\theta_i^2)$ which is equivalent to

$$\frac{1}{2}\left(pv_e(H) + (1-p)v_e(L,\ell)\right) < \frac{1}{2}\left(pv_i(H) + (1-p)\left(v_i(L,0) - v_i(\ell, L)\right)\right)$$
$$v_e(L,\ell) + v_i(\ell, L) < v_i(L,0)$$

which always holds by Assumption 1(iv).
(ii) Since $C(\theta_e^2) < C(\theta_e^1)$, it follows immediately that $\theta_e^2 < \theta_i^2$. □

A.12 Proof of Proposition 6

Proof. In the equilibrium with acquisition, denote the equilibrium strategies with (r_i, r_e). Similarly, in the equilibrium without acquisitions, denote the equilibrium strategies with $(\bar{r}_i, \bar{r}_e)$.

We first prove an intermediate results and then the claim of the proposition follows from Steps 1-4.

Lemma 7. *If (r_i, r_e) constitutes an equilibrium with laissez-faire policy, then $\mathcal{V}(r_i, r_e) = \max\{\theta_e^1(A), \theta_i^1(A)\}$. If $(\bar{r}_i, \bar{r}_e)$ constitutes an equilibrium with no-acquisitions policy, then $\mathcal{V}(\bar{r}_i, \bar{r}_e) = \max\{\theta_e^1(N), \theta_i^1(N)\}$.*

Proof. We prove the two claims in turn. First, consider the equilibrium with acquisitions. Note that $\int_0^{\max\{\theta_e^1(A),\theta_i^1(A)\}} \mathbf{1}(r_i(\theta) + r_e(\theta) > 0)d\theta < \max\{\theta_e^1(A), \theta_i^1(A)\}$ implies that there is a set of positive measure of projects $\theta < \max\{\theta_e^1(A), \theta_i^1(A)\}$ in which no agent is investing. This cannot hold in any equilibrium, since either the entrant or the incumbent would find it profitable to invest in those projects. Next, consider the projects $\theta > \max\{\theta_e^1(A), \theta_i^1(A)\}$. By Lemma 5(ii), $\theta^2(A) < \theta_e^1(A)$ so that neither the entrant nor the incumbent can profitably invest in projects $\theta > \max\{\theta_e^1(A), \theta_i^1(A)\}$, regardless of the investment of the other firm. Hence, in any equilibrium $\int_{\max\{\theta_e^1(A),\theta_i^1(A)\}}^1 \mathbf{1}(r_i(\theta) + r_e(\theta) > 0)d\theta = 0$, which proves that $\mathcal{V}(r_i, r_e) = \max\{\theta_e^1(A), \theta_i^1(A)\}$.

Now consider the equilibrium without acquisitions. For the same reason as above, in any equilibrium it must be $\int_0^{\max\{\theta_e^1(N),\theta_i^1(N)\}} \mathbf{1}(\bar{r}_i + \bar{r}_e > 0)d\theta = \max\{\theta_e^1(N), \theta_i^1(N)\}$. By Lemma 6(ii), $\theta_e^2(N) < \theta_e^1(N)$, so that the entrant cannot profitably invest in any project $\theta > \max\{\theta_e^1(N), \theta_i^1(N)\}$, regardless of the investment of the incumbent. Conditional on entrant not investing in projects $\theta > \max\{\theta_e^1(N), \theta_i^1(N)\}$, then it also is not profitable for the incumbent to invest in those projects. Hence in any equilibrium $\int_{\max\{\theta_e^1(N),\theta_i^1(N)\}}^1 \mathbf{1}(\bar{r}_i + \bar{r}_e > 0)d\theta = 0$, which proves that $\mathcal{V}(\bar{r}_i, \bar{r}_e) = \max\{\theta_e^1(N), \theta_i^1(N)\}$. □

Step 1: $\theta_i^1(A) = \theta_i^1(N)$.
To show this, it is sufficient that

$$C(\theta_i^1(A)) = C(\theta_i^1(N))$$
$$pv_i(H;A) + (1-p)v_i(L,0;A) - v_i(\ell,0;A) = pv_i(H;N) + (1-p)v_i(L,0;N) - v_i(\ell,0;N)$$

which holds since $v_i(t,0;A) = v_i(t,0;N)$ for all $t \in \{\ell, L, H\}$.
Step 2: $\theta_e^1(N) < \theta_e^1(A)$.
To show this, it is sufficient that

$$C(\theta_e^1(N)) < C(\theta_e^1(A))$$
$$pv_e(H;N) + (1-p)v_e(L,\ell;N) < pv_e(H;A) + (1-p)v_e(L,\ell;A)$$
$$v_e(L,\ell;N) < v_e(L,\ell;A)$$

which always holds.
Step 3: If $\theta_e^1(A) > \theta_i^1(A)$, then $\mathcal{V}(r_i, r_e) > \mathcal{V}(\bar{r}_i, \bar{r}_e)$ and if (r_i, r_e) is a simple equilibrium then $\mathcal{P}(r_i, r_e) > \mathcal{P}(\bar{r}_i, \bar{r}_e)$.
Since $\theta_e^1(A) > \theta_e^1(N)$ by Step 2 and $\theta_i^1(A) = \theta_i^1(N)$ by Step 1, then it must be $\theta_e^1(A) > \max\{\theta_e^1(N), \theta_i^1(N)\}$. Then by Lemma 7, $\mathcal{V}(r_i, r_e) > \mathcal{V}(\bar{r}_i, \bar{r}_e)$. Since $\mathcal{P}(r_i, r_e) \leq \mathcal{V}(r_i, r_e)$ for any (r_i, r_e) and $\mathcal{P}(r_i, r_e) = \mathcal{V}(r_i, r_e)$ for simple equilibria, then also $\mathcal{P}(r_i, r_e) > \mathcal{P}(\bar{r}_i, \bar{r}_e)$
Step 4: If $\theta_e^1(A) \leq \theta_i^1(A)$, then $\mathcal{V}(r_i, r_e) = \mathcal{V}(\bar{r}_i, \bar{r}_e)$ and if (r_i, r_e) is a simple equilibrium then $\mathcal{P}(r_i, r_e) \geq \mathcal{P}(\bar{r}_i, \bar{r}_e)$.
If $\theta_e^1(A) \leq \theta_i^1(A)$, then by Steps 1 and 2, $\theta_e^1(N) < \theta_i^1(N)$. Then, by Lemma 7, $\mathcal{V}(r_i, r_e) = \theta_i^1(A) = \theta_i^1(N) = \mathcal{V}(\bar{r}_i, \bar{r}_e)$. Since $\mathcal{P}(r_i, r_e) \leq \mathcal{V}(r_i, r_e)$ for any (r_i, r_e) and $\mathcal{P}(r_i, r_e) = \mathcal{V}(r_i, r_e)$ for simple equilibria, then also $\mathcal{P}(r_i, r_e) \geq \mathcal{P}(\bar{r}_i, \bar{r}_e)$. □

A.13 Proof of Proposition 7

Proof. Subtracting the two expressions for expected consumer surplus gives the welfare difference

$$\begin{aligned} &prob^N(L,L)\,S(L,L) + \left[prob^N(H,0) - prob^A(H,0)\right] S(H,0) + \\ &\qquad \left[prob^N(L,0) - prob^A(L,0)\right] S(L,0) = \\ &prob^N(L,L)\left[S(L,L) - S(L,0)\right] + \left[prob^N(H,0) - prob^A(H,0)\right] S(H,0) + \\ &\qquad \left[prob^N(L,0) + prob^N(L,L) - prob^A(L,0)\right] S(L,0) \end{aligned}$$

The result then follows because

$$prob^N(L,0) + prob^N(L,L) - prob^A(L,0) = prob^A(H,0) - prob^N(H,0)\,.$$

□

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B Online Appendix

B.1 The effect on duplication

As mentioned in the main text, duplication is measured by the probability that both firms discover the innovation:

$$\mathcal{D}(r_i, r_e) = \int_0^1 r_i(\theta) r_e(\theta) d\theta.$$

To assess the effect of banning start-up acquisitions on duplication, we will focus on the case without commercialization as in Section 4. We distinguish between equilibria where $\theta_i^2(N) \leq \theta_i^1(N)$ and equilibria where $\theta_i^2(N) > \theta_i^1(N)$.

Proposition 9 (The effect of start-up acquisitions on duplication)**.** *Suppose that the no-commercialization case applies.*

(a) When $\theta_i^2(N) \leq \theta_i^1(N)$, compare any simple equilibrium with acquisitions and any simple equilibrium without acquisitions. Duplication is strictly smaller in the equilibrium without acquisitions.

(b) When $\theta_i^2(N) > \theta_i^1(N)$, compare any simple equilibrium with acquisitions and any equilibrium without acquisitions. There exists a threshold bargaining power $\tilde{\beta} \in [0, 1)$ such that:

(i) if $\beta < \tilde{\beta}$ duplication is larger in the equilibrium without acquisitions, and

(ii) if $\beta \geq \tilde{\beta}$ duplication is smaller in the equilibrium without acquisitions.

Proof. First note that, without acquisitions, simple equilibria only exist when $\theta_i^2(N) < \theta_i^1(N)$. In any simple equilibrium with acquisitions, $\mathcal{D}(r_i(A), r_e(A)) = \theta^2(A)$ by Proposition 1 and in any simple equilibrium without acquisitions $\mathcal{D}(r_i(N), r_e(N)) = \theta_e^2(N)$ by Proposition 8. For $\theta_e^2(N) < \theta^2(A)$, we need

$$\begin{aligned}
C(\theta_e^2(N)) &< C(\theta^2(A)) \\
v_e(L, \ell; N) &< v_e(L, \ell; A) \\
\pi(L, \ell) - \kappa &< \beta(\pi(\ell, 0) - \pi(\ell, L)) + (1 - \beta)(\pi(L, \ell) - \kappa) \\
\pi(L, \ell) - \kappa &< \pi(\ell, 0) - \pi(\ell, L)
\end{aligned}$$

which holds by Assumption 1 and the no-commercialization case condition $\pi(L, 0) - \pi(\ell, 0) \geq \kappa$. Hence $\mathcal{D}(r_i(A), r_e(A)) - \mathcal{D}(r_i(N), r_e(N)) > 0$, which establishes part (a) of the proposition.

For part (b), we need to consider duplication in any equilibrium without acquisitions. When $\theta_i^2(N) > \theta_i^1(N)$, only non-simple equilibria exist without acquisitions. In these cases, by Proposition 8, duplication is given by:

$$\mathcal{D}(r_i(N), r_e(N)) = \theta_e^2(N) + \int_{\max\{\theta_e^2(N), \theta_i^1(N)\}}^{\min\{\theta_i^2(N), \theta_e^1(N)\}} r_i(\theta; N) r_e(\theta; N) d\theta.$$

Now we show that there exists a threshold $\tilde{\beta}$ which determines the sign of the effect of acquisitions on duplication. When $\beta = 0$, then $\theta^2(A) = \theta_e^2(N)$, thus

$$\mathcal{D}(r_i(A), r_e(A); \beta = 0) - \mathcal{D}(r_i(N), r_e(N); \beta = 0) = -\int_{\max\{\theta_e^2(N), \theta_i^1(N)\}}^{\min\{\theta_i^2(N), \theta_e^1(N)\}} r_i(\theta; N) r_e(\theta; N) d\theta \leq 0.$$

When $\beta = 1$, then $\theta^2(A) = \theta_i^2(N)$, thus

$$\mathcal{D}(r_i(A), r_e(A); \beta = 1) - \mathcal{D}(r_i(N), r_e(N); \beta = 1) =$$
$$\theta_i^2(N) - \theta_e^2(N) - \int_{\max\{\theta_e^2(N), \theta_i^1(N)\}}^{\min\{\theta_i^2(N), \theta_e^1(N)\}} r_i(\theta; N) r_e(\theta; N) d\theta > 0.$$

The last inequality follows from the following two observations, which are implied by Proposition 8: (i) $0 \leq \min\{\theta_i^2(N), \theta_e^1(N)\} - \max\{\theta_e^2(N), \theta_i^1(N)\} \leq \theta_i^2(N) - \theta_e^2(N)$, and (ii) $r_i(\theta; N) r_e(\theta; N) \leq 1$ for all θ and $r_i(\theta; N) r_e(\theta; N) < 1$ for some θ. Finally, the effect of β on the change in duplication is monotone:

$$\frac{\partial(\mathcal{D}(r_i(A), r_e(A)) - \mathcal{D}(r_i(N), r_e(N)))}{\partial \beta} = \frac{\partial \theta^2(A)}{\partial \beta}$$
$$= \frac{(1-p)(\pi(\ell, 0) - \pi(\ell, L) - \pi(L, \ell) + \kappa)}{C'(\theta^2(A))} > 0.$$

□

The intuition for the result is the following: If $\theta_i^2(N) \leq \theta_i^1(N)$, then it is only the change in the entrant's incentive to invest in duplicate projects $\theta_e^2(N)$ which will determine the result. Similar to the effect on variety, banning acquisitions also decreases profitability of duplicate innovations for the entrant, which leads to lower $\theta_e^2(N)$ and thus less duplication.

In the complementary case, $\theta_i^2(N) > \theta_i^1(N)$, the incumbent invests in a positive measure of projects for the mere purpose of reducing the entrant's probability of a successful L innovation. This creates additional duplication for more costly projects. Allowing for acquisitions will reduce this duplication, while still increasing the duplication for rather cheap projects as discussed in the previous paragraph. The overall effect of allowing acquisitions then depends on the relative size of those countervailing effects, which is determined by the bargaining power.

The above analysis suggests that there are two distinct reasons for duplication: Duplication just due to high relative payoff of innovation, such that both investing in the same project still pays off, and duplication due to the blocking incentives of the incumbent. The latter is stronger when acquisitions are not allowed and, while duplication by the incumbent has the negative side-effect of preventing a successful L innovation and thereby competition, duplication by the entrant has the mirrored positive side-effect of increasing competition and can thus not only be considered as wasteful. However, even if duplication might be decreased by allowing acquisitions, the positive side-effect of duplication by the entrant is also shut down because his successful L innovation will just be acquired and will not lead to more product market competition.

B.2 Illustrative example for Figures 1 and 2

We consider an example of Bertrand competition with homogeneous goods, linear costs c and linear demand $Q = 1 - P$. Hence, if only one firm is operating on the market, it will receive monopoly profits. We assume that $\ell = L$, hence if the entrant enters with an L technology, both firms have the same marginal costs and receive zero profits. The L technology corresponds to high marginal costs $\bar{c}$, while the H technology corresponds to low marginal costs $\underline{c}$, where $\underline{c} < \bar{c} \leq 1$. This leads to the following profits: $\pi(L,0) = (\frac{1-\bar{c}}{2})^2$, $\pi(H,0) = \pi(H,L) = (\frac{1-\underline{c}}{2})^2$, $\pi(L,L) = 0$ and $\pi(0,t_j) = 0\ \forall t_j \in \{L,H\}$. Moreover, commercialization costs κ are set to zero. Note that with this specification Assumptions 1 and 2 are satisfied. The costs of research are given by

$$C(\theta) = s\sqrt{\frac{\theta}{1-\theta}}$$

where $s > 0$ is a slope parameter. With acquisitions the critical θs are then given by:

$$\theta_e^1(A) = \frac{(p\pi(H) + (1-p)(\beta(\pi(L,0) - \pi(L,L)) + (1-\beta)\pi(L,L))^2}{s^2 + (p\pi(H) + (1-p)(\beta(\pi(L,0) - \pi(L,L)) + (1-\beta)\pi(L,L))^2}$$

$$\theta_e^2(A) = \frac{(\frac{1}{2}(p\pi(H) + (1-p)(\beta(\pi(L,0) - \pi(L,L)) + (1-\beta)\pi(L,L)))^2}{s^2 + (\frac{1}{2}(p\pi(H) + (1-p)(\beta(\pi(L,0) - \pi(L,L)) + (1-\beta)\pi(L,L)))^2}$$

$$\theta_i^1(A) = \frac{(p(\pi(H) - \pi(L,0)))^2}{s^2 + (p(\pi(H) - \pi(L,0)))^2}$$

$$\theta_i^2(A) = \theta_e^2$$

Without acquisitions, $\theta_e^1(N)$ is given by:

$$\theta_e^1(N) = \frac{(p\pi(H) + (1-p)\pi(L,L))^2}{s^2 + (p\pi(H) + (1-p)\pi(L,L))^2}$$

The parameter values chosen for the Figures are $s = 0.015$, $\bar{c} = 0.75$, $\underline{c} = 0.50$ and $p = 0.25$, where we depicted the extreme cases of $\beta = 0$ and $\beta = 1$.

B.3 Simple Innovation Model and effects on Innovation Probability

In this section we show that, in a model where firms only choose the amount of resources they invest in research, banning acquisitions will have an ambiguous effect on innovations.

Let x_j be the probability that the firm $j \in \{i,e\}$ discovers the innovation, with the associated cost given by $K(\cdot)$, where K is strictly increasing and convex. Apart from the innovation stage, the model is unchanged.

Profits and best responses The expected profit of the incumbent and the entrant, given x_i and x_j, can be written as

$$\begin{aligned}\mathbb{E}\Pi_i(x_i,x_e) &= x_i(1-\frac{1}{2}x_e)\left[pv_i(H) + (1-p)v_i(L,0)\right] \\ &\quad + x_e(1-\frac{1}{2}x_i)(1-p)v_i(\ell,L) + (1-x_i)(1-x_i)v_i(\ell,0) - K(x_i) \\ \mathbb{E}\Pi_e(x_e,x_i) &= x_e(1-\frac{1}{2}x_i)\left[pv_e(H) + (1-p)v_e(L,\ell)\right] - K(x_e).\end{aligned}$$

Consequently, the first-order conditions and, implicitly, the best responses of the firms are

$$\begin{aligned} K'(x_i(x_e)) = &(1-x_e)\left[pv_i(H) + (1-p)v_i(L,0) - v_i(\ell,0)\right] \\ &+ \frac{1}{2}x_e\left[pv_i(H) + (1-p)(v_i(L,0) - v_i(\ell,L))\right] \\ K'(x_e(x_i)) = &(1-\frac{1}{2}x_i)\left[pv_e(H) + (1-p)v_e(L,\ell)\right] \end{aligned}$$

The Nash equilibrium solves the above system of equations and is denoted by (x_i^*, x_e^*).[22]

Note that, when acquisitions are allowed, $v_i(L,0) - v_i(\ell,L)$ reduces to $v_e(L,\ell)$. In case of being the sole successful innovator, expected profits are larger for the entrant than for the incumbent. Part of it is explained by the Arrow replacement effect: The gains of achieving the H-innovation are larger to the entrant, as he does not have to sacrifice current monopoly profits, in contrast to the incumbent. In addition, when the L-innovation is realized, the entrant can pocket the acquisition price. In case both are successful, they expect to have the same innovation payoff. The reason for this is that, with H-innovation, both win a monopoly position and, with non-drastic innovation, the entrant receives the acquisition price, while the incumbent can "save" on the amount she would have otherwise to paid to the entrant.

If acquisitions are not allowed, the only terms changing in the above first-order conditions are $v_i(\ell,L)$ and $v_e(L,\ell)$. In this scenario, the entrant stays in the market and gains duopoly profits less commercialization costs $\bar{v}_e(L,\ell) = \pi(L,\ell) - \kappa$ and the incumbent receives $\bar{v}_i(\ell,L) = \pi(\ell,L)$. Because acquisitions are not feasible, the incumbent's payoff is lower when the entrant successfully innovates(compared to the case when acquisitions are possible), *increasing* her incentives to innovate in order to drive out the entrant. However, the entrant also receives lower profits when he obtains a non-drastic innovation, which *reduces* his innovation incentives overall. Due to these counteracting effects, the net effect of a ban on acquisitions on the sum of investment levels is not clear ex-ante.

Effect of Acquisitions on Innovation Probability We consider cases where $\pi(\ell,0) > \pi(L,0)-\kappa$, hence $v_i(L,0) = v_i(\ell,0)$. To simplify the comparison between a ban and no ban on acquisitions, we introduce a new parameter μ, where μ represents the probability that the acquisition will be allowed. The first order conditions of the entrant and incumbent for a given regime μ are given by:

$$\begin{aligned} K'(x_i(x_e);\mu) =&(1-x_e)p(v_i(H) - v_i(\ell,0)) \\ &+ \frac{1}{2}x_e(pv_i(H) + (1-p)\left[\mu v_e(L,\ell) + (1-\mu)(v_i(\ell,0) - \bar{v}_i(\ell,L))\right]) \\ K'(x_e(x_i);\mu) =&(1-\frac{1}{2}x_i)(pv_e(H) + (1-p)\left[\mu v_e(L,\ell) + (1-\mu)\bar{v}_e(L,\ell)\right]). \end{aligned}$$

The probability of an innovation, and its change when μ increases are given by:

$$\begin{aligned} &Pr(\text{Innovation}) = x_i^*(\mu) + x_e^*(\mu) - x_i^*(\mu)x_e^*(\mu) \\ \Rightarrow\ &\frac{dPr(\text{Innovation})}{d\mu} = (1-x_e^*(\mu))\frac{dx_i^*(\mu)}{d\mu} + (1-x_i^*(\mu))\frac{dx_e^*(\mu)}{d\mu}. \end{aligned}$$

[22]Second order conditions are satisfied due to convexity of $K(x)$

We use the implicit function theorem on the first order conditions of the incumbent and entrant to evaluate the effect on the innovation efforts, $\frac{dx_i^*(\mu)}{d\mu}$ and $\frac{dx_e^*(\mu)}{d\mu}$. Plugging those expressions into the above derivative of the innovation probability, we get:

$$\frac{dPr(\text{Innovation})}{d\mu} = \frac{\frac{1}{2}x_e^*(\mu)(1-p)(v_i(\ell,L) - \bar{v}_i(\ell,L)) * I}{|J|} + \frac{(1-\frac{1}{2}x_i^*(\mu))(1-p)(v_e(L,\ell) - \bar{v}_e(L,\ell)) * E}{|J|}$$

where

$$I = \frac{1}{2}(1-x_i^*(\mu))(pv_e(H) + (1-p)(\mu v_e(L,\ell) + (1-\mu)\bar{v}_e(L,\ell))) - (1-x_e^*(\mu))K''(x_e^*(\mu))$$

and

$$E = \frac{1}{2}(1-x_e^*(\mu))\left[pv_i(H) + (1-p)(v_i(\ell,0) - \mu v_i(\ell,L) - (1-\mu)\bar{v}_i(\ell,L)) - 2p(v_i(H) - v_i(\ell,0))\right] + (1-x_i^*(\mu))K''(x_i^*(\mu)).$$

Note that the Jacobian matrix J is the collection of second-order partial derivatives and is negative definite assuming sufficiently strict convexity of the cost function $K(x)$. Hence the determinant of the Jacobian matrix $|J|$ is positive and the sign of the effect of acquisitions on innovation probability is the same as the sign of weighted sum of I and E.

This sign is not clear ex-ante. If $\beta = 0$, so that $v_e(L,\ell) = \bar{v}_e(L,\ell)$, then the sign of the effect on innovation probability is determined by

$$\frac{dPr(\text{Innovation})}{d\mu}|_{\beta=0} \gtrless 0 \Leftrightarrow (pv_e(H) + (1-p)\bar{v}_e(L,\ell)) \gtrless 2\frac{(1-x_e^*(\mu))K''(x_e^*(\mu))}{1-x_i^*(\mu)}.$$

This condition is more likely to be satisfied for large competition intensity in a duopoly, i.e. relatively small $\pi(L,\ell) = \bar{v}_e(L,\ell) + \kappa = \bar{v}_i(\ell,L)$. With relatively small profits in a duopoly with non-drastic innovation, the first order reaction of the incumbent towards the possibility of acquisition leads to a relatively big decrease of innovation effort, while the entrant's reaction is only of second order. He only increases his efforts as a best response to the effort decrease of the incumbent. The overall effect is thus negative.

If the entrant has all bargaining power, i.e. $\beta = 1$ and $v_i(\ell,L) = \bar{v}_i(\ell,L)$, we get a similar expression for the sign of the effect:

$$\frac{dPr(\text{Innovation})}{d\mu}|_{\beta=1} \gtrless 0$$
$$\Leftrightarrow (1-p)(v_i(L,0) - \bar{v}_i(\ell,L)) + p(2v_i(\ell,0) - v_i(H)) \gtrless -2\frac{(1-x_i^*(\mu))K''(x_i^*(\mu))}{1-x_e^*(\mu)}$$

If drastic innovation is not too profitable, i.e. $v_i(H) < 2v_i(\ell,0)$, a more lenient regime towards acquisitions will increase innovation probability, irrespective of product market competition intensity when both firms are active.

The above analysis shows that, in a model where firms cannot target their innovation efforts towards specific projects with respect to innovation competition, at minimum, one needs to make specific assumptions on the bargaining power of the potential entrant β in addition to competition intensity in the product market as measured by $\pi(\ell,L) + \pi(L,\ell)$.