Market Experimentation and New Technology Adoption*

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Abstract

This paper analyzes the adoption and diffusion of a product innovation in a duopoly market. Firms are horizontally and vertically differentiated, and learning about innovation quality is endogenously determined by the volume of consumer purchases (i.e. market experimentation). In equilibrium, both high- and low-quality firms may lead adoption, and the first mover is often uniquely determined by initial beliefs. To maximize efficiency, a social planner prefers sequential adoption over a wider range of beliefs than occurs in equilibrium. A change in market structure, specifically a merger to monopoly, unambiguously improves the efficiency of initial adoption; but the rate of learning via market experimentation may become more or less efficient.

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In many sectors of the economy, firms are presented with frequent opportunities to adopt new technologies to improve their product and service offerings. Economists have long been interested in studying these adoption decisions, as they appear closely connected to economic growth, industrial competitiveness, and consumer welfare.[1] Specifically, the academic literature has directed significant effort toward understanding which factors promote the rapid adoption and diffusion of new innovations. Particular attention has been paid to the role of firm characteristics and market structure.

Despite this longstanding interest, the theoretical literature on new technology adoption has struggled to obtain a general theory to explain the plethora of empirical patterns observed in practice (Baker, 1995). With respect to product innovation, in particular, there is surprisingly little evidence of a consistent relationship between firm characteristics, such as size or market dominance, and the timing of innovation adoption. For example, in a study of post-World War II product innovations, Chandy and Tellis (2000) find that established industry leaders are no less likely than fringe firms to lead in commercializing major product innovations.[2] Nevertheless, when fringe firms do lead in commercializing innovations, the speed at which industry leaders follow is often excruciatingly slow (Henderson and Clark, 1990). Thus, dominant firms are neither incompetent nor unwilling to embrace new technologies; but they are much less likely to do so in a timely fashion as second-movers. A primary challenge in developing a compelling model of innovation adoption comes in explaining such inconsistent patterns of innovative performance by dominant firms.

In this paper, I contribute to this puzzle by developing a continuous-time model of innovation adoption which explains these seemingly inconsistent empirical findings. In the model, consumers and firms are initially uncertain about the quality of a newly discovered product innovation. Early adoption of the innovation confers a valuable

[1] See, for example Comin and Hobijn (2010), Fagerberg et al. (2009), and overall Trajtenberg (1989).

[2] This stands in contrast to theory of the "incumbent's curse" which argues that incumbents are systematically less innovative than new entrants. Many empirical studies in this vein, however, are qualitative in nature (Rosen, 1991). For a recent exception to this trend, see Igami (2017).

first-mover advantage; however, this comes at the price of greater uncertainty, since the innovation's quality can only be learned by observing its market performance. Firms, therefore, face a trade-off between information acquisition and strategic preemption. While such a trade-off is theoretically intuitive, there are, in principle, many reasons why a firm may prefer to delay adoption; for example, expected reductions in cost (e.g. Fudenberg and Tirole, 1985; Reinganum, 1983) or improvements in quality (e.g. Dutta et al., 1995; Riordan and Salant, 1994). To justify economic analysis, it is important to understand whether the trade-off between information acquisition and strategic preemption actually matters for the timing of firms' innovation adoption decisions.

To empirically investigate firms' motivations for specific investment-timing strategies, Klingebiel and Joseph (2016) studied competitors in the German mobile phone industry between 2004 and 2008, when the introduction of new hardware features (product innovations) was at an all-time high. Industry representatives in their study frequently cited the trade-off between information acquisition and strategic preemption as a *primary* motivation for early- or late-adoption of new hardware features. One firm in the industry generally known for early-adoption described the trade-off as follows: *"[Our firm] now tries to be an innovator . . . we have emphasized speed [and risk taking]."* Another, more cautious firm, described the motivation for late-adoption as follows: *"The longer we wait, the easier it is for us to gauge success. . . . [W]e kept monitoring forecasts while Samsung, Sony Ericsson, and a few others tried their luck at development. And the picture became clearer."* Klingebiel and Joseph's (2016) study illustrates that, in many cases, delayed adoption timing was not driven by an inability to access key technological inputs; rather it was a deliberate choice to wait for more information.

In my model, the trade-off between adoption timing and information acquisition is endogenously determined by the volume of consumer purchases, which generate additional information about innovation quality. Firms must, therefore, balance their desire to maximize current profits with their desire to generate additional information. In equilibrium, firms set prices to balance these two considerations and engage a form of *market experimentation*. Under zero discounting, I obtain closed-form expressions for first-mover and second-mover prices as a function of the (expected) quality differential between firms. Since learning is endogenously determined by the volume of consumer purchases, so is the rate of innovation diffusion. In equilibrium, I show that a fringe (i.e. low quality) firm attracts fewer customers than a dominant (i.e. high-quality) firm; hence, learning and innovation diffusion is slowest when adoption is led by fringe firms.

Regarding the question of initial adoption, I show that each firm's initial adoption incentives are succinctly described by two threshold beliefs above and below which immediate and delayed adoption is optimal, respectively. Under minimal assumptions, these thresholds have a unique configuration, and equilibrium adoption timing is characterized by a simple dichotomy: dominant firms prioritize early adoption of highly promising innovations which diffuse rapidly to other firms, while fringe firms prioritize early adoption of less promising innovations which diffuse more slowly. Thus, my model predicts (i) both dominant and fringe firms may be natural first-movers in equilibrium, and (ii) adoption timing will depend on observable innovation characteristics, namely expected quality.

In addition to delivering precise equilibrium predictions, the tractability of my model lends itself well to welfare analysis. By comparing equilibrium adoption timing to a socially efficient benchmark, I find that equilibrium market experimentation may be socially excessive or insufficient. Thus, innovations may diffusion faster or slower than is socially optimal. Regarding initial adoption timing, I find that a social planner prefers to have adoption occur sequentially over a strictly wider range of initial beliefs than occurs in equilibrium. The key intuition for this result lies in the fact that sequential adoption generates valuable information for both consumers and second-movers, a benefit which first-movers do not internalize. As a result, firms are too willing (too reluctant) to adopt innovations with high (low) expected quality.

In addition to welfare analysis, I also consider the impact of a changing market structure; in particular, a merger to monopoly. While such a merger significantly reduces consumer surplus through higher prices, total efficiency of initial adoption timing nevertheless improves. Specifically, a monopolist always chooses the efficient order of adoption, and the range of beliefs over which sequential adoption occurs is a strict improvement over the duopoly case. Monopoly experimentation, however, may become more or less efficient than duopoly experimentation. Market experimentation under monopoly is most efficient for both low and high beliefs, while duopoly experimentation is most efficient for intermediate beliefs. At the heart of the efficiency gains from monopoly is the (partial) internalization of business-stealing effects and information externalities in adoption timing. However, because the monopolist cannot perfectly price discriminate, these effects are not fully internalized. So even a monopoly fails to achieve full efficiency.

The remainder of this paper is organized as follows. Section 1 describes related literature. Section 2 introduces the model. Section 3 characterizes equilibrium adoption timing and innovation diffusion. Section 4 describes the model's main empirical implications. Section 5 investigates the efficiency of equilibrium adoption timing and the effect of changing market structure. Section 6 concludes with an extended discussion. Appendix A contains all proofs of the main results, while auxiliary proofs and lengthy calculations are relegated to Appendix B.

1 Related Literature

This paper belongs primarily to the literatures on new technology adoption and the timing of innovation.[3] Broadly speaking, I contribute to these literatures by providing a novel theoretical explanation for several commonly observed empirical phenomena related the adoption and diffusion of innovations, all within a framework that is flexible enough to accommodate (i) asymmetric market structures, and (ii) normative analysis

[3] A reader can find excellent, comprehensive surveys in Reinganum (1989) and Hoppe (2002).

of market dynamics, competition policy, and welfare.

In line with existing literature, I view technology adoption as an inherently dynamic problem. The early seminal works, Reinganum (1981) and Fudenberg and Tirole (1985), illustrated how strategic timing considerations can explain why innovations tend to diffuse gradually across firms. Both papers model adoption as a pure game of timing in which profits depend only on the date and order of adoption. Each assumes a rich, continuous-time environment; however, to ensure a tractable analysis, they restrict attention to the case of ex-ante identical firms and to an innovation whose profitability is known with certainty. In contrast, my model focuses primarily on the case when firms are *asymmetric* and innovation profitability is *uncertain*.

By no means is this paper the first to study the implications of uncertainty on innovation adoption timing. Early papers which studied the role of uncertainty in adoption timing include Mamer and McCardle (1987) and Jensen (1982) in the context of a symmetric duopoly and Reinganum (1983) in an asymmetric (but static) Cournot duopoly. More recent papers have sought to include various additional elements of uncertainty into adoption timing, such as information spillovers (e.g. Décamps and Mariotti, 2004; Hoppe, 2000), social learning (e.g. Frick and Ishii, 2015; Wagner, 2018), and reputations (e.g. Hendricks, 1992). Each of these papers, however, assume players are ex-ante identical. So it is impossible to study the implications of firm asymmetry, specifically market dominance, on innovation adoption timing.

A central feature of my model is that innovation diffusion and learning is endogenously determined by the volume of consumer purchases. This captures the idea that uncertainty regarding an innovation's true value takes time to be resolved, as is frequently the case when there is two-sided learning in markets (e.g. Aghion et al., 1991; Bergemann and Välimäki, 1997; Judd and Riordan, 1994). In my main specification, consumers and firms observe the frequency at which the new technology generates a good-news signal of innovation quality, and they use this information to update beliefs in real time. This makes my learning environment similar to Keller and Rady (2010) who study strategic experimentation with Poisson bandits.[4]

The technical features of my model are closely related to Bergemann and Välimäki (1997). In their paper, an established incumbent faces off against an entrant of unknown quality. As in my model, learning is endogenously determined by the volume of consumer purchases of the entrant's product, and prices continuously adjust to reflect market beliefs. The focus of our two papers, however, is fundamentally different. The focus of Bergemann and Välimäki (1997) is on characterizing price experimentation and learning dynamics when a new product becomes available to consumers. I, on the other hand, am mostly interested in characterizing the adoption and diffusion of a new innovation when it becomes available to firms. By including innovation adoption decisions, my model extends the scope of Bergemann and Valimaki's analysis by allowing firms to compete along two dimensions: *price* and *product design*. As a result, I am able to investigate a much broader set of questions related to market structure and welfare.

Another related work is a recent paper by Awaya and Krishna (2019) who study the role of asymmetric information on entry-timing decisions into an R&D race. In their paper, firms are identical except for private information about the arrival rate of innovations. They establish the existence of an "upstart equilibrium" in which a less informed start-up is (i) more likely to enter (and win) an R&D race, and (ii) receives a higher payoff than a more informed incumbent. The key difference between our models is the mechanism through which asymmetric investment incentives are generated. In Awaya and Krishna's model, incentives are generated by differences in initial information. In my model, incentives are generated by differences in initial product quality.

[4] In the Appendix B, I consider two alternative learning process: (i) Brownian diffusion, as in Bergemann and Välimäki (1997) and Bolton and Harris (1999), and (ii) Poisson bad news, as in Keller and Rady (2015).

2 Model Description

Two firms, 1 and 2, compete to supply a product to an interval (i.e. unit measure) of consumers. Time is continuous and advances over an infinite horizon. Each firm's product is differentiated along horizontal and vertical dimensions. Vertical differentiation is captured by a firm-specific expected quality q_i, while horizontal differentiation is captured by consumer-specific tastes $\theta \in [0,1]$. A consumer has with taste θ receives utility

$$(1) \quad u_\theta(q_i; i) = \begin{cases} q_i + h(1-\theta) & \text{if } i = 1, \\ q_i + h\theta & \text{if } i = 2, \end{cases}$$

from consuming Firm i's product. Here, the parameter $h > 0$ measures the degree of horizontal differentiation between firms. With this utility specification, consumers with higher values of θ have a naturally stronger preference for Firm 2's product, holding expected quality fixed. For simplicity, I will assume θ is uniformly distributed along $[0,1]$ and that consumers have unit demands at every moment in time. (These assumptions can easily be relaxed.)

At the beginning of the game ($t = 0$), a product innovation is discovered and made available for adoption. To adopt the innovation, Firm i must incur a one-time fixed cost of installation, denoted $C_i \geq 0$. Once adopted, the innovation contributes an uncertain amount $\tilde{q}$ to a firm's expected quality. For simplicity, I assume $\tilde{q}$ can take two possible values; it can either be low (q_L) or high (q_H), where $q_H > 0 > q_L$. Thus, adoption of a high-quality innovation increases demand by raising a product's expected quality, while a low-quality innovation is disliked by consumers and lowers product's expected quality. To highlight the role of uncertainty in motivating adoption-timing decisions, I will make the stylized assumption that adoption is *irreversible*.

All available information about innovation quality at time 0 is summarized by an initial belief, $\alpha_0 = \mathbb{P}(\tilde{q} = q_H)$. Neither firms nor consumers have private means of obtaining additional information about the innovation. Thus, α_0 serves as a common prior held by all agents. Over time, consumer purchases of the innovation (assuming at least one firm has adopted) generate publicly observable signals of innovation quality. I assume these signals arrive according to a Poisson process, the arrival rate of which depends on innovation quality, $\tilde{q}$, and the volume of consumer purchases, n_t. Specifically, a high-quality (low-quality) innovation generates signals at rate $\lambda_H n_t$ ($\lambda_L n_t$). For simplicity, I will assume $\lambda_H = \lambda > 0 = \lambda_L$, so that signal realizations are conclusive good news about innovation quality.

A large body of research across economics, marketing, and psychology indicate that timing advantages exist in a variety of contexts. (see Gilbert and Birnbaum-More, 1996) First-mover advantages frequently arise when pioneering firms are able to, for example, secure exclusive access to high-quality inputs, establish product standards, or cultivate brand loyalty. On the other hand, second-mover advantages may arise due to a firm's ability to improve upon a pioneering technology. To ensure a non-trivial trade-off between adoption timing and information acquisition, I assume that early adoption confers a first-mover advantage[5] as follows: (i) if Firm i adopts the innovation as a first-mover, then Firm i's quality simply becomes $q_i + \tilde{q}$; (ii) if Firm i adopts the innovation as a second-mover, then Firm i's post-adoption quality becomes $q_i + \tilde{q} - x$, where $x \in [0, q_H]$ is a reduced-form measure of second-mover disadvantage. Finally, if simultaneous adoption occurs, then I assume both firms are considered first-movers.

Throughout the paper, I maintain the following assumption:

Assumption 1. $\left|q_i - q_j\right| < h + \max\{q_H, -q_L\}$.

Assumption 1 is a necessary and sufficient condition to ensure that (i) both firms receive positive market share in equilibrium, and (ii) both firms receiving positive market share is efficient. This assumption simplifies my analysis by ensuring prices and market shares are given by standard first-order conditions.

[5] With a second-mover advantage, there is no strategic trade-off between adoption-timing and information acquisition, because a late adopter has access to both more information and a higher-quality innoation.

The solution concept I consider is Markov-perfect equilibrium (henceforth, equilibrium), where current market beliefs, α_t, serve as a state variable. In such an equilibrium, Firm i's strategy is a triple (p_i, ϕ_i, τ_i^F). The first component, $p_i(\alpha_t)$, is the price charged by Firm i as a function of current market beliefs and, implicitly, the stage of the game. The second component, $\phi_i \in [0,1]$, is a (constant) rate at which Firm i adopts the innovation, assuming Firm j has not yet adopted. The third component, τ_i^F, is a stopping time that describes when Firm i adopts the innovation as a second-mover (i.e. after Firm j has adopted). The definition of an equilibrium in this environment is standard; the strategy (p_i, ϕ_i, τ_i^F) must maximize, at every time t, the expected present-discounted value of Firm i's profits given Firm j's strategy, and vice versa for (p_j, ϕ_j, τ_j^F).

3 Equilibrium Analysis

3.1 Pricing

Begin by taking (ϕ_1, τ_1^F) and (ϕ_2, τ_2^F) as given; I refer to these as *adoption strategies*. For $i = 1, 2$, let T_i denote the actual date at which Firm i adopts the innovation.[6] We can then divide the game into three stages:

- Stage 1 (Neither firm has adopted): $t < \min\{T_1, T_2\}$
- Stage 2 (One firm has adopted): $\min\{T_1, T_2\} \leq t < \max\{T_1, T_2\}$
- Stage 3 (Both firms have adoped): $t \geq \max\{T_1, T_2\}$

In Stages 1 and 3, the quality differential between Firms 1 and 2 does not depend on market beliefs; hence, equilibrium prices are computable via routine flow-profit maximization. (See Appendix A.) In Stage 2, when just one firm has adopted the innovation, the quality differential between firms depends non-trivially on market beliefs. In particular, if Firm j is the first-mover in Stage 2, then the quality differential is $q_i - q_j - q^e(\alpha)$,

[6] For example, if $\phi_1 > 0$ and $\phi_2 = 0$, then $T_1 = 0$ and $T_2 = \tau_2$.

where $q^e(\alpha) \equiv \alpha q_H + (1-\alpha)q_L$ denotes expected innovation quality. Given prices $p_i(\alpha)$ and $p_j(\alpha)$, the respective market shares Firm i and Firm j receive are

$$n_i^{(0,1)}(\alpha) \equiv \frac{(q_i - p_i(\alpha)) - \big(q_j + q^e(\alpha) - p_j(\alpha)\big) + h}{2h} \quad \text{and} \quad n_j^{(1,0)}(\alpha) \equiv 1 - n_i^{(0,1)}(\alpha).$$

Since $n_i^{(0,1)}$ and $n_j^{(1,0)}$ affect both profits and the evolution of beliefs, equilibrium prices in Stage 2 are found via dynamic programming. Standard arguments show the value functions $V_i^F(\alpha)$ and $V_j^L(\alpha)$ satisfy a pair of HJB equations:

$$(2) \quad \rho V_i^F(\alpha) = \max_{p_i(\alpha)} \left\{ \begin{array}{r} p_i(\alpha) n_i^{(0,1)}(\alpha) + \alpha\lambda n_j^{(1,0)}(\alpha)\left[V_i^F(1) - V_i^F(\alpha)\right] \\ -\lambda n_j^{(1,0)}(\alpha)\alpha(1-\alpha)(V_i^F)'(\alpha) \end{array} \right\},$$

$$(3) \quad \rho V_j^L(\alpha) = \max_{p_j(\alpha)} \left\{ \begin{array}{r} p_j(\alpha) n_j^{(1,0)}(\alpha) + \alpha\lambda n_j^{(1,0)}(\alpha)\left[V_j^L(1) - V_j^L(\alpha)\right] \\ -\lambda n_j^{(1,0)}(\alpha)\alpha(1-\alpha)(V_j^L)'(\alpha) \end{array} \right\}.$$

Each firm's discounted value is the sum of three terms: (i) flow profit, (ii) the expected gain/loss resulting from good news arrival, and (iii) the value of information. In an ideal setting, we would compute equilibrium prices by first solving (2) and (3) and then maximizing $V_i^F(\cdot)$ and $V_j^L(\cdot)$ by choice of $p_i(\cdot)$ and $p_j(\cdot)$, respectively. Unfortunately, closed-form solutions to non-linear differential equations of this type do not exist. So $p_i^{(0,1)}(\alpha)$ and $p_j^{(1,0)}(\alpha)$ are computable only through numerical methods. Following Bergemann and Välimäki (1997), I bypass this technical limitation by analyzing equilibrium prices as the discount rate, ρ, approaches zero.[7]

In undiscounted dynamic programming, there is ambiguity about which optimality criterion should be used. This is especially true in an infinite-horizon model such as mine, because *any* pricing policy that generates positive flow profits will result in infinite lifetime value. In light of this, a commonly used alternative optimality criterion in undiscounted problems is the maximization of long-run average profit, defined as

$$v(\alpha_0) = \sup_{p(\cdot)} \mathbb{E}_{\alpha_0} \left\{ \lim_{T\to\infty} \frac{1}{T} \int_0^T \Pi(\alpha_t) dt \right\}.$$

[7]See Bolton and Harris (2000) for an analysis of strategic experimentation under zero discounting.

This optimality condition meaningfully refines the set of optimal pricing policies to a non-trivial set of possibilities. However, there is no guarantee that a long-run average optimal pricing policy will be a fair approximation of an optimal pricing policy under positive discounting.[8] Without such continuity, we cannot guarantee that qualitative conclusions obtained from studying the zero-discounting game will remain valid under small but positive discounting.

To solve this problem of potential discontinuity in the zero-discounting limit, Dutta (1991) proposes a refinement of the long-run average optimality criterion, which he refers to as *strong* long-run average optimality. Using this optimality criterion, firms seek to maximize their expected lifetime deviation from the long-run average:

$$V(\alpha_0; 0) = \sup_{p(\cdot)} \mathbb{E}_{\alpha_0} \left\{ \lim_{T\to\infty} \int_0^T [\Pi(\alpha_t) - \nu(\alpha_t)]\, dt \right\}.$$

By scaling flow payoffs by the long-run average $\nu_i(\alpha)$, the strong long-run average remains finite; hence, we may apply standard dynamic programming techniques. Furthermore, the strong-long run average is continuous in ρ, in the sense that any strong long-run average pricing policy is a limit point of optimal pricing policies under positive discounting as $\rho \to 0$. (See Dutta (1991) and Krylov (1980) for more detail.)

Using the strong long-run average criterion, I recover analytical solutions for each firm's equilibrium pricing policy under zero discounting.

Lemma 3.1 (Dynamic Equilibrium Pricing). *In Stage 2, Firm i's equilibrium price and market share is:*

- *First-mover:*

$$p_i^{(1,0)}(\alpha) = \frac{2}{3}\alpha(q_H - x) + \frac{1}{3}(q_i - q_j + q^e(\alpha)) + h, \qquad n_i^{(1,0)}(\alpha) = \sqrt{\frac{\nu_i^{(1,0)}(\alpha)}{2h}}.$$

[8] In other words, there is no guarantee that a long-run average optimal pricing policy is the limit of a sequence of discounted-value optimal pricing policies as $\rho \to 0$.

- *Second-mover*

$$p_i^{(0,1)}(\alpha) = \frac{2}{3}\alpha(q_H - x) - \frac{2}{3}\left(q_i - q_j + q^e(\alpha)\right) + \sqrt{2h\,v_i^L(\alpha)}, \qquad n_i^{(0,1)}(\alpha) = 1 - \sqrt{\frac{v_j^{(1,0)}(\alpha)}{2h}}.$$

The dynamics of equilibrium pricing are not a central focus of this paper; however, it is worth commenting on the basic incentives at work, as they generate the market shares which I utilize extensively in later sections. When learning is endogenous, a first-mover has a strong incentive to increase its market share, in order to accelerate the arrival of good news. A first-mover effectively subsidizes market experimentation by setting lower prices than is myopically optimal. For a comprehensive analysis of equilibrium price dynamics under market experimentation, the reader is referred to Bergemann and Välimäki (1997).

3.2 Equilibrium Diffusion: The Second-Mover's Problem

Next I turn to the study innovation diffusion by solving the second-mover's problem. Formally, Firm i as a second-mover faces an optimal stopping problem. In the waiting stage, the quality differential remains $q_i - q_j - q^e(\alpha)$, and Firm i receives flow profit $\Pi_i^{(0,1)}(\alpha)$ (caculated using Lemma 3.1). In the stopping stage, the quality differential becomes $q_i - q_j - x$, and Firm i receives a constant flow profit, denoted $v_i^{(1,1),F}$, forever. Then the value from stopping is:

$$V_i^{(1,1),F} = \int_0^\infty e^{-\rho t} v_i^{(1,1),F} dt - C_i = \rho^{-1} v_i^{(1,1),F} - C_i.$$

By comparison, the value from waiting at time t is:

$$V_i^{(0,1)}(\alpha(t)) = \Pi_i^{(0,1)}(\alpha(t))dt + (1 - \rho dt)\mathbb{E}_t[V_i^F(\alpha(t + dt))],$$

where $V_i^F(\alpha)$ denotes Firm i's value from optimal stopping. Clearly, $V_i^F(\alpha)$ satisfies the functional equation

$$V_i^F(\alpha) = \max\left\{V_i^{(1,1),F}, V_i^{(0,1)}(\alpha)\right\}.$$

Because $q_i - q_j - q^e(\alpha)$ is decreasing in α, the solution to Firm i stopping problem is described by a threshold $\alpha_i^* \in [0,1]$ above which immediate adoption is optimal. Expressed as a free-boundary problem, the value function $V_i^F(\cdot)$ and α_i^* satisfy

$$\rho V_i^F(\alpha) = \begin{cases} \Pi_i^{(0,1)}(\alpha) + \alpha\lambda_j^L(\alpha)\left[V_i^F(1) - V_i^F(\alpha)\right] - \lambda_j^L(\alpha)\alpha(1-\alpha)(V_i^F)'(\alpha) & \text{if } \alpha < \alpha^*, \\ V_i^{(1,1),F} & \text{if } \alpha \geq \alpha^*. \end{cases}$$

along with a value-matching condition $\rho V_i^F(\alpha^* -) = V_i^{(1,1),F}$.

Similar to Keller and Rady (2015), smooth pasting does not hold at α_i^*.[9] Without this additional boundary condition, it is impossible to analytically characterize α_i^*. Nevertheless, I am able to fully characterize the rate of innovation diffusion with one simple observation:

Lemma 3.2. *There is no equilibrium in which Firm i is a second-mover whenever $\alpha_0 > \alpha_i^*$.*

Lemma 3.2 follows from there being a first-mover advantage in adoption timing. If $\alpha_0 \geq \alpha_i^*$, then Firm i prefers immediate adoption as a second-mover. With a first-mover advantage, this payoff is strictly less than the payoff attained from simultaneous adoption. Thus, Firm i is willing to be a second-mover in equilibrium only if $\alpha_0 < \alpha_i^*$ (i.e. immediate adoption is not optimal). Since no news is bad news, market beliefs become more pessimistic over time absent a good-news signal realization. Thus, an immediate implication of Lemma 3.2 is that second-movers will always wait for the arrival of good news.

Corollary 3.1 (Waiting for good news)**.** *Conditional on being a second-mover in equilibrium, Firm i adopts at date $\tau_i^F = \inf\{t > 0 : \alpha(t) = 1\}$.*

[9] To see why, consider the case when $\alpha(t) = \alpha_i^*$. By definition, it is optimal for Firm i to adopt the innovation; however, if Firm i delays adoption is delayed by a small amount of time dt, then Firm i continues to receive the same profit (by value matching) and $\alpha(t)$ will almost surely fall to below α_i^*, in which case Firm i will strictly prefer waiting. For such a delay to be strictly sub-optimal, $\frac{d}{d\alpha}V_i^F(\alpha_i^* -)$ must be negative; hence, $V_i^F(\cdot)$ has a convex kink exactly at α_i^*.

For a high-quality innovation, the CDF of τ_i^F is:

$$F_{\tau_i^F}(t) = 1 - \exp\left(-\lambda \int_0^t n_j^{(1,0)}(\alpha(s))ds\right).$$

This CDF is strictly decreasing in $n_j^{(1,0)}(\cdot)$. Hence, to obtain a stochastic comparison of τ_i^F to τ_j^F, it suffices to obtain a point-wise comparison of $n_i^{(1,0)}(\alpha)$ to $n_j^{(1,0)}(\alpha)$.

Lemma 3.3 (First-mover market shares)**.** *Suppose $q_i > q_j$. Then, under zero discounting, $n_i^{(1,0)}(\alpha) > n_j^{(1,0)}(\alpha)$ for all $\alpha \in [0,1]$.*

This comparison leads directly to my first main result.

Theorem 3.1 (Innovation diffusion)**.** *All else equal, a high-quality innovation will diffuse more slowly (on average) when adoption is lead by a fringe firm. In particular, for all $\alpha_0 \in (0,1)$, we have $q_i > q_j \iff \tau_i^F > \tau_j^F$ in the sense of first-order stochastic dominance.*

Theorem 3.1 provides an informational explanation for why the diffusion rate of a given innovation depend on the characteristics of a first-mover. Holding all else equal, an innovation that is adopted by a fringe firm has less mass-market appeal and is consumed by a relatively small subset of consumers. Hence, new product performance reveals less information about innovation quality, and it takes longer (on average) for a second-mover to become confident enough to justify adoption.

It is important to emphasize that expected innovation quality is held fixed in the statement of Theorem 3.1. So the above comparison of dominant-led and fringe-led innovation diffusion implicitly assumes there are no systematic differences between firms with respect to initial adoption. Currently, there is no guarantee that such an assumption will be true. So, to fully characterize equilibrium innovation diffusion and how it compares between firms, I must also characterize Stage 1 initial adoption incentives.

3.3 Initial Adoption: The First-Mover's Problem

Next, I turn to the question of initial adoption. In Stage 1, so long as neither firm has adopted, each firm chooses a rate $\phi_i \in [0,1]$ at which to adopt as a first-mover. Under

Table 1: Notation Summary (Long-Run Average Profits from Adoption Timing)

High Quality ($\tilde{q} = q_H$)

	Lead	Follow
Lead	$v_1^{(1,1)},\ v_2^{(1,1)}$	$v_1^{(1,1),L},\ v_2^{(1,1),F}$
Follow	$v_1^{(1,1),F},\ v_2^{(1,1),L}$	$v_1^{(0,0)},\ v_2^{(0,0)}$

Low Quality ($\tilde{q} = q_L$)

	Lead	Follow
Lead	$v_1^{(1,1)},\ v_2^{(1,1)}$	$v_1^{(1,0)}(0),\ v_2^{(0,1)}(0)$
Follow	$v_1^{(0,1)}(0),\ v_2^{(1,0)}(0)$	$v_1^{(0,0)},\ v_2^{(0,0)}$

zero discounting, Firm i will choose ϕ_i to maximize its expected long-run average profit. There are three possible long-run average values:

1. *First-mover value:*

$$v_i^L(\alpha_0) = \alpha_0 v_i^{(1,1),L} + (1-\alpha) v_i^{(1,0)}(0)$$

2. *Second-mover value:*

$$v_i^F(\alpha_0) = \alpha_0 v_i^{(1,1),F} + (1-\alpha) v_i^{(0,1)}(0)$$

3. *Simultaneous/Non-adoption value:*

$$v_i^{(1,1)} = v_i^{(0,0)} = \frac{\left(\frac{1}{3}(q_i - q_j) + h\right)^2}{2h}$$

See Table 1 for a summary of notation; formulas are located in Appendix A.

Suppose Firm j chooses $\phi_j = 0$. Then Firm i can receive $v_i^L(\alpha_0)$ by choosing $\phi_i = 1$ or receive $v_i^{(0,0)}$ by choosing $\phi_i = 0$. Choosing $\phi_i = 1$ is optimal if and only if

$$(4) \quad L_i(\alpha_0) := v_i^L(\alpha_0) - v_i^{(0,0)}$$

is positive; I refer to $L_i(\alpha_0)$ as Firm i's "leader curve". Now suppose Firm j chooses $\phi_j = 1$. Then Firm i can either receive $v_i^F(\alpha_0)$ by choosing $\phi_i = 0$ or receive $v_i^{(1,1)}$ by choosing $\phi_i = 1$. In this case, choosing $\phi_i = 0$ is optimal if and only if

$$(5) \quad F_i(\alpha_0) := v_i^F(\alpha_0) - v_i^{(1,1)}$$

is positive; I refer to $F_i(\alpha_0)$ as Firm i's "follower curve".

The leader and follower curves, $L_i(\alpha_0)$ and $F_i(\alpha_0)$, characterize Firm i's relative incentive to be a first-mover or second-mover in adoption. As we should expect, these incentives are monotonic with respect to initial beliefs.

Lemma 3.4. *$L_i(\alpha_0)$ is increasing in α_0; $F_i(\alpha_0)$ is decreasing in α_0.*

It is easy to verify that $L_i(0) < 0 < L_i(1)$ and $F_i(1) < 0 < F_i(0)$. Hence, the threshold values $\alpha_i^L = \inf\{\alpha : L_i(\alpha) \geq 0\}$ and $\alpha_i^F = \sup\{\alpha : F_i(\alpha) \geq 0\}$ are well-defined; I refer to α_i^L as Firm i's "leader threshold" and to α_i^F as Firm i's "follower threshold". Intuitively, α_i^L is the lowest initial belief at which Firm i is willing to be a first-mover (i.e. choose $\phi_i = 1$); while α_i^F is the highest initial belief at which Firm i is willing to be a second-mover (i.e. choose $\phi_i = 0$). Taken together, leader and follower thresholds fully characterize initial adoption incentives.

Lemma 3.5 (Leader and Follower Thresholds)**.**

1. $(1, \tau_i^F) \in BR_i(0, \tau_j^F) \iff \alpha_0 \geq \alpha_i^L$.
2. $(0, \tau_i^F) \in BR_i(1, \tau_j^F) \iff \alpha_0 \leq \alpha_i^F$.

The implication of Lemma 3.5 is that equilibrium adoption timing in Stage 1 is completely determined the comparison of initial beliefs, α_0, to the adoption thresholds α_1^L, α_2^L, α_1^F, and α_2^F. More precisely, we have:

Corollary 3.2. *There is an equilibrium in which Firm i is a first-mover and Firm j is a second-mover if and only if $\alpha_i^L \leq \alpha_0 \leq \alpha_j^F$; there is an equilibrium in which firms adopt simultaneously if and only if $\alpha_0 \geq \max\{\alpha_1^F, \alpha_2^F\}$; and there is an equilibrium in which firms never adopt if and only if $\alpha_0 \leq \min\{\alpha_1^L, \alpha_2^L\}$.*

For Corollary 3.2 to be of practical use, we must find a way to compare leader and follower thresholds between the two firms. To do this, it suffices to perform comparative statics on α_i^L and α_i^F with respect to the quality differential $q_i - q_j$. Direct simplification yields the following expressions for $L_i(\alpha_0)$ and $F_i(\alpha_0)$:

$$L_i(\alpha_0) = \left(\frac{3h + q_i - q_j}{9h}\right)\Xi(\alpha_0) + \frac{1}{18h}\zeta(\alpha_0), \tag{6}$$

$$(7) \quad F_i(\alpha_0) = -\left(\frac{3h + q_i - q_j}{9h}\right)\Xi(\alpha_0) + \frac{1}{18h}\zeta(\alpha_0),$$

where $\Xi(\alpha) = \alpha x + \alpha q_L$ and $\zeta(\alpha) = \alpha x^2 + (1-\alpha)q_L^2$. By differentiating (6) and (7) with respect to q_i, we obtain:

$$(8) \quad \frac{\partial L_i(\alpha_0)}{\partial q_i} = \frac{1}{9h}\Xi(\alpha_0) = -\frac{\partial F_i(\alpha_0)}{\partial q_i}.$$

Hence, an increase in a firm's own quality results in a stronger (weaker) incentive to be a first-mover (second-mover) if and only if $\Xi(\alpha_0) > 0$. By letting $\hat{\alpha} = -q_L/(x - q_L)$ denote the belief which solves $\Xi(\alpha) = 0$, the following comparison is immediate.

Lemma 3.6 (Rotation Property). *Suppose $q_i > q_j$, and let $\hat{\alpha} = x/(x - q_L)$. Then:*

$$L_i(\alpha_0) \begin{cases} < L_j(\alpha_0) & \text{if } \alpha_0 < \hat{\alpha}, \\ = L_j(\alpha_0) & \text{if } \alpha_0 = \hat{\alpha}, \\ < L_j(\alpha_0) & \text{if } \alpha_0 > \hat{\alpha}, \end{cases} \quad \textit{and} \quad F_i(\alpha_0) \begin{cases} > F_j(\alpha_0) & \text{if } \alpha_0 < \hat{\alpha}, \\ = F_j(\alpha_0) & \text{if } \alpha_0 = \hat{\alpha}, \\ < F_j(\alpha_0) & \text{if } \alpha_0 > \hat{\alpha}. \end{cases}$$

The above rotation property illustrates that a firm's leader and follower curves become steeper as own quality increases – see Figure 1. This effect is driven by the convexity of long-run average profit (see Appendix A). As q_i increases, Firm i's profit becomes more sensitive to changes in α_0. Hence, Firm i experiences greater gains from adopting a high-quality innovation, but also suffers worse losses from adopting a low-quality innovation. Depending on initial beliefs, the incentive to be a first- or second-mover in adoption may, therefore, be increasing or decreasing as a firm becomes more dominant. Lemma 3.6 shows that the relative incentive to be a first-mover (second-mover) in adoption is stronger for a more dominant firm if, and only if, $\alpha_0 > \hat{\alpha}$ ($\alpha_0 < \hat{\alpha}$).

To develop intuition for the rotation point $\hat{\alpha}$, notice that $\Xi(\alpha) = \alpha x + (1-\alpha)q_L$ is exactly the (long-run) quality advantage gained by being a first-mover in adoption. When beliefs are $\alpha_0 > \hat{\alpha}$, a first-mover expects its quality advantage will *increase* by leading adoption. Since long-run average profits are convex, the expected gains from being a first-mover are greater for a more dominant firm. On the other hand, if $\alpha_0 < \hat{\alpha}$,

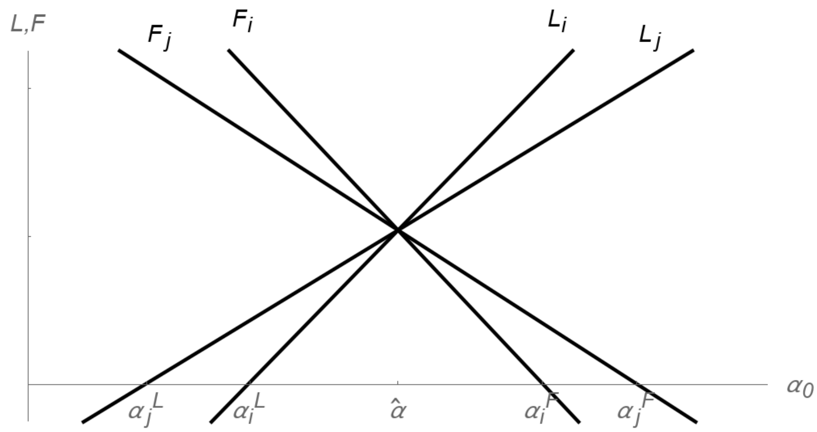

Figure 1: Rotation Property ($q_i > q_j$)

then a first-mover expects its quality will *decrease* with adoption. In this case, adoption entails a relatively large downside risk, which a fringe firm is most willing to tolerate.

In addition to threshold comparative statics, Equations (6) and (7) gives us the sign of $L_i(\alpha)$ and $F_i(\alpha)$ at the rotation point $\hat{\alpha}$. In particular, we have:

$$L_i(\hat{\alpha}) = L_j(\hat{\alpha}) = F_i(\hat{\alpha}) = F_j(\hat{\alpha}) = \frac{1}{9h}\zeta(\hat{\alpha}) > 0.$$

Therefore, leader and follower thresholds have just one possible configuration.

Lemma 3.7. *If $q_i > q_j$, then* $0 < \alpha_j^L < \alpha_i^L < \hat{\alpha} < \alpha_i^F < \alpha_j^F < 1$.

Lemma 3.7, taken together with Corollary 3.2, directly implies the following characterization of initial adoption.

Theorem 3.2 (Initial Adoption)**.** *Suppose $q_i > q_j$. With the exception of boundary cases, there is a unique equilibrium for all $\alpha_0 \in [0,1] \setminus [\alpha_i^L, \alpha_i^F]$:*

1. $\alpha_0 \in [0, \alpha_j^L) \rightarrow$ *Non-adoption*
2. $\alpha_0 \in (\alpha_j^L, \alpha_i^L) \rightarrow$ *Firm j adopts, Firm i waits*
3. $\alpha_0 \in (\alpha_i^F, \alpha_j^F) \rightarrow$ *Firm i adopts, Firm j waits*
4. $\alpha_0 \in (\alpha_j^F, 1] \rightarrow$ *Simultaneous adoption*

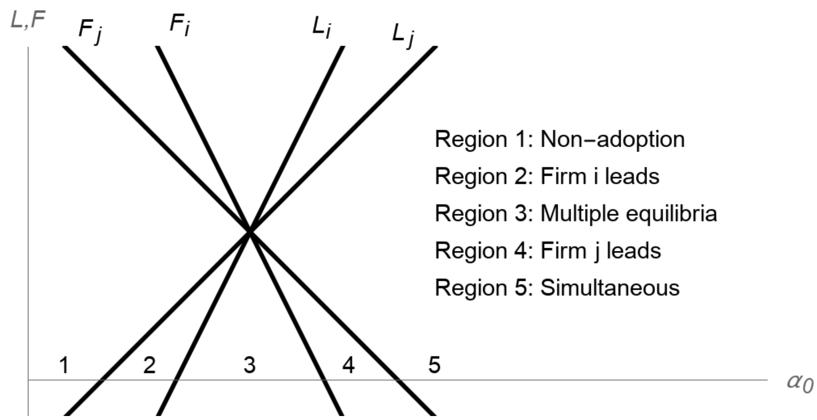

Figure 2: Structure of Equilibrium Adoption Regions ($q_i > q_j$)

For intermediate beliefs $\alpha_0 \in [\alpha_i^L, \alpha_i^F]$*, there are two asymmetric pure-strategy equilibria (i.e. Firm i adopts, Firm j waits) and one mixed-strategy equilibrium in which Firm i adopts at rate* $\phi_i^* = L_j(\alpha_0)/(L_j(\alpha_0) + F_j(\alpha_0))$.

Theorem 3.2 highlights the central role uncertainty plays in firms' adoption-timing decisions. Rather than adoption-timing being always led by a dominant or fringe firm, our equilibrium predicts that self-selection will occur into different adoption-timing strategies, based on current position in the market (dominant or fringe) and on excepted innovation quality – see Figure 2. By Lemma 3.6, dominant firms have (i) the strongest incentive to lead when $\alpha_0 > \hat{\alpha}$, and (ii) the strongest incentive to follow when $\alpha_0 < \hat{\alpha}$. Intuitively, this reversal is due to how firm profits are affected by changes in own-quality. As a firm's quality increases, long-run average profit becomes more variable to changes in market beliefs. A dominant firm is, therefore, exposed to more risk as a first-mover. In equilibrium, dominant firms offset this extra risk by forgoing early adoption of less promising innovations.

For a fringe firm, the logic of adoption incentives is exactly reversed. As a firm's quality advantage over its rival decreases, profits become less variable in expected innovation quality. A fringe firm, therefore, faces less risk as a first-mover, which implies a stronger relative incentives to lead adoption when α_0 is low, because the downside risk is low. At the same time, however, a fringe firm has a weaker relative incentive to lead adoption when α_0 is high, because upside risk is also low. The result is that, in equilibrium, fringe firms tend to forgo early adoption of highly promising innovations.

4 Empirical Implications

Theorems 3.1 and 3.2 illustrate the close connection between market beliefs, firm characteristics, and innovation adoption timing. In this section, I further discuss these results and describe several of their main empirical implications, some of which are supported by existing data and others which may provide directions for future work.

A first implication of my results is that dominant and fringe firms prioritize early adoption of qualitatively different technologies in equilibrium. Dominant firms prioritize early adoption of highly promising innovations which diffuse rapidly. Fringe firms, on the other hand, prioritize early adoption of less promising innovations which diffuse more slowly. As a result, we should expect to find that, on average, dominant firms (i) wait longer before adopting as second-movers, and (ii) are more successful as first-movers.[10] Elements of this first prediction have been widely documented in the management literature on firms' responsiveness to technological change, in particular the failure of dominant firms to adequately respond to so-called *disruptive innovation* (e.g. Christensen, 1993; Cooper and Schendel, 1976; Henderson and Clark, 1990).[11] In contrast to existing theories, however, my model provides an explanation of this phenomenon, commonly referred to as the *Innovator's dilemma*, without making any assumptions about dominant firms' inherent ability or willingness to adopt new innovations. Rather, systematic differences in innovation diffusion between firms is driven

[10] More precisely, these empirical patterns should exist whenever equilibrium adoption timing is unique. Whenever there are multiple equilibria, it is possible that firms will play a mixed-strategy equilibrium in which these comparative statics are reversed.

[11] An common heuristic for identifying disruptive innovation is to look for poor initial market performance followed by rapid improvements in quality (see Christensen, 1997). In my model, this is consistent with a high-quality innovation having low initial expected quality before generating good news.

by differences in initial adoption timing and subsequent market experimentation.

Second, Theorem 3.2 shows that both dominant and fringe firms can be natural leaders of innovation adoption, depending on initial market beliefs. This adds an element of nuance to an empirical literature which has largely focused on a binary paradigm – are dominant industry leaders more or less innovative than the competitive fringe?[12] General conclusions in this area have been historically elusive, and many results appear sensitive to industry characteristics (Gilbert, 2006; Motta, 2004). My results suggest that empirical inconsistencies in adoption timing across industries may be driven by unobserved heterogeneity in initial market beliefs about innovation quality. A promising direction for future empirical work would be in developing measures of (ex-ante) expected innovation quality and correlating them with firms' adoption timing decisions.

Third, my results suggest that increasing dominance may be an inevitable feature of innovative industries. In equilibrium, dominant firms lead adoption of the most promising innovations which, on average, increase its advantage over fringe rivals, while fringe firms lead adoption of innovations which, on average, decrease own-product quality. We have known for some time that increasing dominance may emerge when firms can engage in product or process innovation.[13] In most existing studies, however, increasing dominance is generated by dominant firms investing more aggressively into innovation. My results complement this existing work by showing that increasing dominance may also arise from already dominant firms making systematically more productive decisions regarding which innovations to adopt.

[12] Cohen (2010) provides a comprehensive survey of this literature.

[13] See, for example, Athey and Schmutzler (2001); Bagwell and Ramey (1994); Cabral (2018); Klepper (1996); Riordan and Salant (1994).

5 Market Structure and Efficiency

In this section, I investigate the potential for government regulation and changes in market structure to increase or decrease the efficiency of equilibrium adoption timing. Specifically, I compare equilibrium adoption timing to that which would occur in two cases: (i) if a benevolent social planner were to control firms' adoption-timing decisions, and (ii) if the two firms were to merge and become a single joint-profit-maximizing monopolist.

In my analysis of efficient and monopoly experimentation, I will routinely impose the following additional assumption.

Assumption 2 (Contestability)**.** $\left|q_i - q_j\right| < \max\{q_H, -q_L\}$.

Assumption 2 strengthens Assumption 1 to require firm qualities to be sufficiently close in the sense that (i) a fringe first-mover becomes dominant if $\tilde{q} = q_H$, and (ii) a dominant first-mover becomes fringe if $\tilde{q} = q_L$. Thus, adoption timing always has the potential to reverse market dominance. This assumption is mainly for expositional simplicity and is relaxed in Appendix B.

5.1 Social Planner

Consider first the problem of a social planner who seeks to maximize expected total surplus by regulating market shares and the timing of adoption.

5.1.1 Efficient Experimentation

Given expected qualities $q_1^e(\alpha)$, $q_2^e(\alpha)$ and market shares $n_1(\alpha)$, $n_2(\alpha)$, flow total surplus equals

$$TS(\alpha) = \left[q_1^e(\alpha) + h\left(\frac{2 - n_1(\alpha)}{2}\right)\right] n_1(\alpha) + \left[q_2^e(\alpha) + h\left(\frac{1 + n_1(\alpha)}{2}\right)\right] n_2(\alpha).$$

I consider only the case of zero-discounting, where the social planner uses the strong long-run average criterion. The notation used for long-run average total surplus is analogous to the notation from Section 3.1, and so is the derivation of socially efficient market shares. The following proposition summarizes the relevant information needed to conduct welfare analysis. Full details are relegated to Appendix B.

Proposition 5.1 (Efficient Experimentation)**.** *The efficient market share for Firm i as a first-mover in Stage 2 is*

$$n_{i,S}^{(1,0)}(\alpha) = \sqrt{\frac{v_{i,S}^{L}(\alpha) - q_j - \frac{h}{2}}{h}},$$

where $v_{i,S}^{L}(\alpha) = \alpha v_{i,S}^{(1,1),L}(1) + (1-\alpha) v_{i,S}^{(1,0)}(0)$ is long-run average total surplus.

As a second-mover, the planner's optimal adoption date for Firm j is $\tau_{j,S}^{F} = \inf\{t \geq 0 : \alpha(t) = 1\}$. Conditional on $\tilde{q} = q_H$, this stopping time has CDF

$$F_{\tau_{j,S}}(t) = 1 - \exp\left(-\lambda \int_0^t n_{i,S}^{(1,0)}(\alpha_s) ds\right).$$

Thus, a key efficiency question is whether $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is positive or negative.

Proposition 5.2. *Under Assumption 2, the difference $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is strictly increasing and crosses zero once. Thus, there is a (unique) market belief $\tilde{\alpha} \in (0,1)$ such that equilibrium experimentation is (i) excessive if $\alpha < \tilde{\alpha}$ and (ii) insufficient if $\alpha > \tilde{\alpha}$.*

Intuitively, as α increases, the first-mover's market share in both equilibrium and efficient regimes increases. However, because the planner internalizes more of the innovation's expected social benefit, we find that $n_{i,S}^{(1,0)}(\alpha)$ increases more than $n_i^{(1,0)}(\alpha)$. To provide intuition for the crossing itself, consider a first-mover's incentive in each case. When α is low, it is socially optimal for a first-mover to receive a relatively low market share, because both its expected quality and the value of information are low. The firm, however, only values expected profit. So it is privately optimal to continue generating a relatively high volume of sales, in hopes that good news will arrive. When α is high, the intuition is reversed. In this case, it is socially efficient for a first-mover to cultivate a high market share in order to accelerate learning and innovation diffusion. A private firm, however, strictly prefers reducing its volume of sales in order to charge a higher price and increase profits.

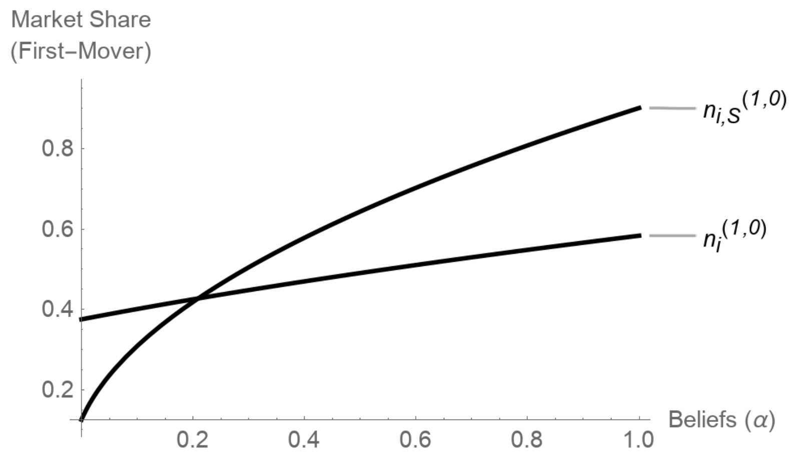

Figure 3: Equilibrium and Efficient Market Shares

5.1.2 Efficient Initial Adoption

Next, I turn to the question of efficient initial adoption. To do this, I define social leader and follower curves using long-run average total surplus. The social leader curve for Firm i is $L_{i,S}(\alpha_0) = v^L_{i,S}(\alpha_0) - v^{(0,0)}_{i,S}$; the social follower curve for Firm i is $F_{i,S}(\alpha_0) = v^F_{i,S}(\alpha_0) - v^{(1,1)}_{i,S}(\alpha_0)$. (See Appendix A for exact formulas.) Upon simplification, these curves can be expressed as:

$$(9)\quad L_{i,S}(\alpha_0) = \left(\frac{h + q_i - q_j}{2h}\right)\Xi(\alpha_0) + \frac{1}{4h}\zeta(\alpha_0) + \alpha(q_H - x)$$

and

$$(10)\quad F_{i,S}(\alpha_0) = -\left(\frac{h + q_i - q_j}{2h}\right)\Xi(\alpha_0) + \frac{1}{4h}\zeta(\alpha_0).$$

The functions $L_{i,S}(\cdot)$ and $F_{i,S}(\cdot)$ fully characterize efficient adoption timing in equilibrium. Specifically, Firm i is an efficient first-mover if and only if:

1. Firm i leading is preferred to non-adoption: $L_{i,S}(\alpha_0) \geq 0$.

2. Firm j following is preferred to simultaneous adoption: $F_{j,S}(\alpha_0) \geq 0$.

3. Firm i leading is preferred to Firm j leading: $L_{i,S}(\alpha_0) \geq F_{i,S}(\alpha_0)$.

Regarding initial adoption, there are two possible sources of inefficiency: order and timing. The *order* of adoption is inefficient whenever Firm i leads adoption but Firm j is a more efficient first-mover (i.e. $L_{i,S}(\alpha_0) < L_{j,S}(\alpha_0)$). The *timing* of adoption is inefficient whenever Firm i adopts as a first-mover and either non-adoption or simultaneous adoption is optimal (i.e. $L_{i,S}(\alpha_0) < 0$ or $F_{j,S}(\alpha_0) < 0$, respectively). To study the former type of inefficiency, note that $L_{i,S}(\alpha_0)$ and $F_{i,S}(\alpha_0)$ both satisfy a rotation property around $\hat{\alpha} = -q_L/(x - q_L)$. From this, we easily determine the efficient order of sequential adoption:

Lemma 5.1. *Suppose $q_i > q_j$. Then:*

$$L_{i,S}(\alpha_0)\begin{cases} < L_{j,S}(\alpha_0) & \text{if } \alpha_0 < \hat{\alpha}, \\ = L_{j,S}(\alpha_0) & \text{if } \alpha_0 = \hat{\alpha}, \\ > L_{j,S}(\alpha_0) & \text{if } \alpha_0 > \hat{\alpha}, \end{cases} \quad \text{and} \quad F_{i,S}(\alpha_0)\begin{cases} > F_{j,S}(\alpha_0) & \text{if } \alpha_0 < \hat{\alpha}, \\ = F_{j,S}(\alpha_0) & \text{if } \alpha_0 = \hat{\alpha}, \\ < F_{j,S}(\alpha_0) & \text{if } \alpha_0 > \hat{\alpha}, \end{cases} .$$

Thus, Firm i is an efficient first-mover if and only if $\alpha_0 \geq \hat{\alpha}$.

An interesting feature of efficient sequential adoption is that the preferred first-mover is (i) always unique, and (ii) changes depending on whether α_0 is above or below the pivotal belief $\hat{\alpha}$. For beliefs $\alpha_0 < \hat{\alpha}$, the socially preferred first-mover is the low-quality firm; the high-quality firm is the preferred for beliefs $\alpha_0 > \hat{\alpha}$. Intuitively, when deciding the order of initial adoption, the social planner faces a dynamic trade-off. On one hand, initial adoption by the high-quality firm ensures the most consumers get early access to the innovation and begin receiving its expected benefit. On the other hand, such early adoption is risky, because the innovation may be low quality. To resolve this trade-off, the social planner prefers the high-quality firm lead adoption if and only if initial beliefs about innovation quality are sufficiently high – namely, above $\hat{\alpha}$.

To determine efficient initial adoption timing, define $\alpha^L_{i,S} = \inf\{\alpha : L_{i,S}(\alpha_0) \geq 0\}$ and $\alpha^F_{i,S} = \sup\{\alpha : F_{i,S}(\alpha) \geq 0\}$, respectively. By comparing these social leader and follower thresholds, we obtain the following characterization of efficient initial adoption.

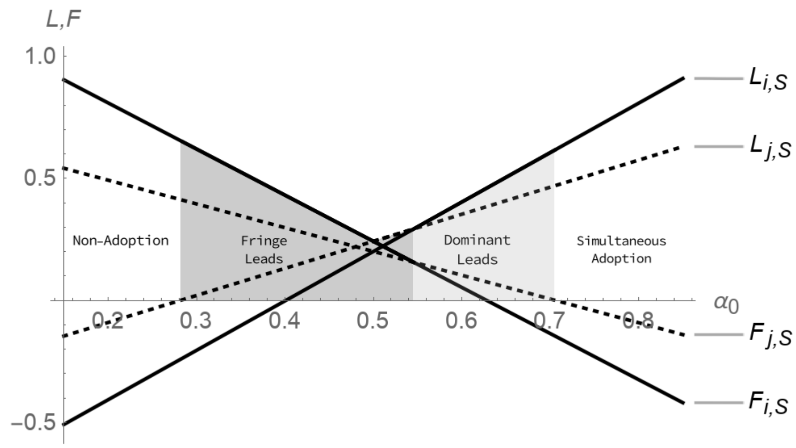

Figure 4: Efficient Initial Adoption ($q_i > q_j$)

Proposition 5.3 (Efficient Initial Adoption). *Suppose $q_i > q_j$. With the exception of boundary cases, there is a unique socially efficient adoption regime for all $\alpha_0 \in [0,1]$:*

1. $\alpha_0 < \alpha^L_{j,S} \rightarrow$ *Neither firm adopts*

2) $\alpha_0 \in (\alpha^L_{j,S}, \hat{\alpha}) \rightarrow$ *Firm j adopts, Firm i waits*

3) $\alpha_0 \in (\hat{\alpha}, \alpha^F_{j,S}) \rightarrow$ *Firm i adopts, Firm j waits*

4) $\alpha_0 > \alpha^F_{j,S} \rightarrow$ *Both firms adopt*

As in equilibrium, sequential adoption occurs in the planner's optimum for a range of intermediate beliefs. An important efficiency question is whether sequential adoption is more or less common under in the efficient regime. Through direct calculation, we obtain:

Lemma 5.2. *Suppose $q_i > q_j$. Then $\alpha^L_{j,S} < \alpha^L_j < \alpha^F_j < \alpha^F_{j,S}$.*

Thus, a social planner prefers sequential adoption over a strictly wider range of initial beliefs than occurs in equilibrium. Intuitively, this is because a social planner fully internalizes the information externality generated by sequential adoption. For low beliefs, firms have *insufficient* incentives to lead adoption, because they do not internalize the benefit of initial adoption for the second-mover. For high beliefs, firms have *excessive* incentives to lead adoption, because they do not internalize the potential risk to consumers from adopting a low-quality innovation.

5.2 Monopoly

To analyze the effect of changing market structure on equilibrium adoption, suppose the two firms are allowed to merge and behave as a monopolist. Once again, we need to consider the efficiency of market experimentation in Stage 2 and as well as the efficiency of initial adoption in Stage 1. Begin with monopoly experimentation.

Proposition 5.4 (Monopoly Experimentation)**.** *Under zero discounting, the monopolist's optimal market shares for a first-mover in Stage 2 is*

$$n_{i,M}^{(1,0)}(\alpha) = \sqrt{\frac{v_{i,M}^{L}(\alpha) - q_j}{2h}},$$

where $v_{i,M}^{L}(\alpha) = \alpha v_{i,M}^{(1,1),L}(1) + (1-\alpha) v_{i,M}^{(1,0)}(0)$ *is long-run average monopoly profit.*

Relative efficiency of monopoly versus duopoly experimentation is then determined by comparing market-share differences.

Lemma 5.3. *Consider the following three market-share differences:*

$$n_{i,S}^{(1,0)}(\alpha) - n_{i}^{(1,0)}(\alpha), \qquad n_{i,S}^{(1,0)}(\alpha) - n_{i,M}^{(1,0)}(\alpha), \qquad n_{i,M}^{(1,0)}(\alpha) - n_{i}^{(1,0)}(\alpha).$$

Under Assumption 2, each of these differences are strictly increasing and cross zero once.

An immediate implication of Lemma 5.3 is:

Theorem 5.1 (Monopoly vs. Duopoly Experimentation)**.** *Under Assumption 2, there is an interval* $(\underline{\alpha}, \overline{\alpha}) \subset (0,1)$ *such that duopoly experimentation is more efficient than monopoly experimentation if and only if* $\alpha \in (\underline{\alpha}, \overline{\alpha})$.

Theorem 5.1 shows that despite appropriating a larger share of the returns to innovation adoption, a monopolist may nevertheless engage in less efficient experimentation compared to a duopoly firm. To provide intuition for this somewhat surprising

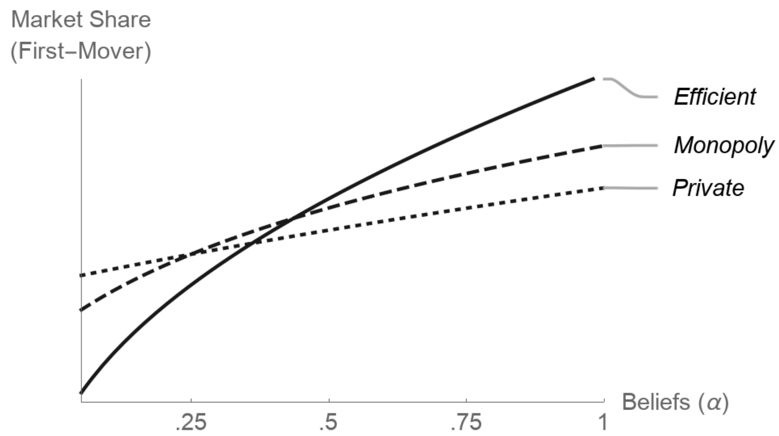

Figure 5: Monopoly vs. Private Experimentation

result, it is helpful to note that both monopoly and social planner take into account the value of first-mover and second-mover adoption. So, for extreme beliefs, it is no surprise that agree about whether equilibrium experimentation is excessive or insufficient. For intermediate beliefs, however, it is possible for monopoly and social planner to disagree about whether equilibrium experimentation is excessive or insufficient. This is because a monopolist's objective is to maximize joint-profits, while a social planner seeks to maximize social surplus. This disagreement generates a range of interior beliefs in which monopoly and social planner prefer opposite changes to equilibrium experimentation. Within this range, monopoly experimentation is *less* efficient than duopoly experimentation – see Figure 5.

Turning to initial adoption, we can construct the monopoly leader and follower curves just as before; define $L_{i,M}(\alpha_0) = v^L_{i,M}(\alpha_0) - v^{(0,0)}_{i,M}$ and $F_{i,M}(\alpha_0) = v^L_{j,M}(\alpha_0) - v^{(1,1)}_{i,M}(\alpha_0)$. (See Appendix A for exact formulas.) Monopoly leader and follower curves simplify as follows:

$$(11)\quad L_{i,M}(\alpha_0) = \left(\frac{2h + q_i - q_j}{4h}\right)\Xi(\alpha_0) + \frac{1}{8h}\zeta(\alpha_0) + \alpha(q_H - x)$$

and

$$(12)\quad F_{i,M}(\alpha_0) = -\left(\frac{2h + q_i - q_j}{4h}\right)\Xi(\alpha_0) + \frac{1}{8h}\zeta(\alpha_0).$$

The monopolist's ideal order of adoption is determined by comparing $L_{i,M}(\alpha_0)$ to $L_{j,M}(\alpha_0)$. From (11) and (12), it is easy to see that $L_{i,M}$ and $F_{i,M}$ satisfy the now-familiar rotation property around $\hat{\alpha}$. Thus, a monopolist will always choose the efficient first-mover.

Proposition 5.5. *Suppose $q_i > q_j$. Then $L_{i,S}(\alpha_0) \geq L_{j,S}(\alpha_0) \iff \alpha_0 \geq \hat{\alpha}$. Thus, the order of sequential adoption under monopoly is always socially efficient.*

Because the order of sequential adoption is always efficient under monopoly, the only possible source of inefficiency is the extent to which sequential adoption occurs under monopoly. Thus, a key efficiency question is how monopoly initial adoption thresholds, $\alpha^L_{i,M} = \inf\{\alpha : L_{i,M}(\alpha) \geq 0\}$ and $\alpha^F_{i,M} = \sup\{\alpha : F_{i,M}(\alpha) \geq 0\}$, compare to their efficient and private counterparts.

Proposition 5.6 (Monopoly Threshold Comparisons)**.** *Suppose $q_i > q_j$. Then $\alpha^L_{j,S} < \alpha^L_{j,M} < \alpha^L_j$ and $\alpha^F_j < \alpha^F_{j,M} < \alpha^F_{j,S}$.*

Thus, the efficiency of initial adoption unambiguously increases under monopoly. The intuition for this result lies in increased appropriability of the surplus generated by adoption. Duopoly firms, due to intense price competition, internalize very little of this surplus. By relaxing this price competition, a monopolist is able to capture a larger share of total surplus which leads to a preference for sequential adoption over a strictly wider range of beliefs. Full efficiency is not achieved, however, because the monopolist does not capture the full surplus from adoption – the social planner prefers sequential adoption over an even wider range of initial beliefs.

6 Discussion and Concluding Remarks

In this paper, I consider the adoption and diffusion of a product innovation within a duopoly market. Uncertainty exists regarding the quality of this innovation, and this generates a unique trade-off between adoption timing and information acquisition. I find that dominant and fringe firms choose qualitatively different strategies in resolving this trade-off, thus providing an explanation for several empirical phenomena. Specifically, dominant firms prioritize early adoption of highly promising innovations which diffuse rapidly to other firms, while fringe firms prioritize early adoption of less promising innovations which diffuse more slowly.

Equilibrium adoption timing may be inefficient along several dimensions. First, sequential adoption is socially desirable over a strictly wider range of initial beliefs than occurs in equilibrium. Thus, an efficient technology policy would encourage the adoption of relatively low expected quality innovations by fringe firms, but also discourage simultaneous adoption of more highly promising innovations.[14] Second, for intermediate beliefs, equilibrium multiplicity admits the possibility that firms may coordinate on an inefficient order of adoption. To correct this inefficiency, a government may wish to design subsidies or taxes to facilitate greater coordination on the socially preferred equilibrium. Third, the rate of market experimentation may be higher or lower than is socially optimal. Hence, a government may wish to engage in a policy of selective information provision to facilitate more efficient learning; for example, by more rigorously testing certain classes of innovations.

By explicitly comparing the monopoly and duopoly solutions, I have identified a novel consideration for antitrust analysis in industries characterized by frequent innovation. Specifically, my results suggest that certain horizontal mergers have the potential to generate significant welfare gains by increasing the efficiency of innovation adoption timing. The mechanism through which these efficiency gains are realized is greater internalization of (i) the social value of innovation adoption, and (ii) the information externality generated by sequential adoption. A key question from the standpoint of an-

[14]A policy to encourage adoption by fringe firms would likely resemble recent government programs aimed at promoting firm modernization. An example of such a program is India's Credit Linked Capital Subsidy Scheme (CLCSS) for Technology Upgradation which is designed expand small businesses' access to capital for workplace modernization. For detail, see `http://www.dcmsme.gov.in/schemes/Credit_link_Scheme.htm`.

titrust is whether consumer welfare increases or decreases under a monopoly regime. While I do not consider this question directly, my results suggest that it is possible for consumer welfare to increase under a monopoly regime, provided that prices increases are not significant enough to offset the welfare gains realized through more efficient adoption timing.

In my model, consumers and firms make their decisions based upon the Poisson arrival of good news about innovation quality. It is natural to ask how the analysis of this paper would be adapted to other learning environments. In Appendix B, I consider alternative specifications in which news arrives via Brownian diffusion or Poisson bad news, as in Bolton and Harris (1999) and Keller and Rady (2015), respectively. In each case, Assumption 1 guarantees that $\alpha(t)$ converges to 0 or 1 almost surely as $t \to \infty$ in Stage 2. Learning is thus complete, and all long-run average values (duopoly profit, monopoly profit, and total surplus) remain identical to my original specification. Hence, all results pertaining to initial adoption timing under zero discounting go through without change.

The only complication that arises when considering alternative learning processes is with respect to the second-mover's problem in Stage 2. Specifically, it now becomes possible for market beliefs to hit the second-mover's adoption threshold in finite time. Innovation diffusion then depends on both the rate of market experimentation, $n_i^{(1,0)}(\alpha)$, and the adoption threshold, α_i^*. In such a situation, it no longer appropriate to consider the zero discounting limit, since α_i^* converges to 1 under zero discounting. In other words, sufficiently patient second-movers are willing to wait arbitrarily long lengths of time before adopting an innovation.

It would be useful to extend my analysis in several directions. By focusing on the case of just a single innovation, I implicitly assume the expected rate of technological change is zero. In reality, we may be interested in how the possibility of follow-up innovation affect firms' incentives to adopt innovations within a given technological generation. Another way to add further richness to my model would be to endogenize the source of first-mover advantage. A promising direction for future work may be found by connecting first-mover advantage to learning-by-doing, whereby an adopting firm becomes better at producing a high-quality innovation over time; for example, by having more experience or information about consumers' preferences. This would certainly complicate analysis of innovation diffusion and the second-mover's problem; but such an extension seems feasible given the simplicity and tractability of the base model.

References

AGHION, P., P. BOLTON, C. HARRIS, AND B. JULLIEN (1991): "Optimal Learning by Experimentation," *Review of Economic Studies*, 58, 621–654.

ATHEY, S. AND A. SCHMUTZLER (2001): "Investment and Market Dominance," *RAND Journal of Economics*, 32, 1–26.

AWAYA, Y. AND V. KRISHNA (2019): "Startups and Upstarts," SSRN Scholarly Paper ID 3380351, Social Science Research Network, Rochester, NY.

BAGWELL, K. AND G. RAMEY (1994): "Coordination Economies, Advertising, and Search Behavior in Retail Markets," *American Economic Review*, 84, 498–517.

BAKER, J. B. (1995): "Fringe Firms and the Incentive to Innovate," *Antitrust Law Journal*, 63, 621–641.

BERGEMANN, D. AND J. VÄLIMÄKI (1997): "Market Diffusion with Two-Sided Learning," *RAND Journal of Economics*, 28, 773–795.

BOLTON, P. AND C. HARRIS (1999): "Strategic Experimentation," *Econometrica*, 67, 349–374.

——— (2000): "Strategic experimentation: The undiscounted case," *Incentives, Organizations and Public Economics–Papers in Honour of Sir James Mirrlees*, 53–68.

CABRAL, L. M. B. (2018): "Standing on the Shoulders of Dwarfs: Dominant Firms and Innovation Incentives," Tech. Rep. 13115, C.E.P.R. Discussion Papers.

CHANDY, R. K. AND G. J. TELLIS (2000): "The Incumbent's Curse? Incumbency, Size, and Radical Product Innovation," *Journal of Marketing*, 64, 1–17.

CHRISTENSEN, C. (1997): *The Innovator's Dilemma: When New Technologies Cause Great Firms to Fail*, Boston, MA: Harvard Business School Press.

CHRISTENSEN, C. M. (1993): "The Rigid Disk Drive Industry: A History of Commercial and Technological Turbulence," *Business History Review*, 67, 531–588.

COHEN, W. M. (2010): "Chapter 4 - Fifty Years of Empirical Studies of Innovative Activity and Performance," in *Handbook of the Economics of Innovation*, ed. by B. H. Hall and N. Rosenberg, North-Holland, vol. 1 of *Handbook of The Economics of Innovation, Vol. 1*, 129–213.

COMIN, D. AND B. HOBIJN (2010): "An Exploration of Technology Diffusion," *American Economic Review*, 100, 2031–2059.

COOPER, A. C. AND D. SCHENDEL (1976): "Strategic responses to technological threats," *Business Horizons*, 19, 61–69.

DÉCAMPS, J.-P. AND T. MARIOTTI (2004): "Investment timing and learning externalities," *Journal of Economic Theory*, 118, 80–102.

DUTTA, P. K. (1991): "What do discounted optima converge to?: A theory of discount rate asymptotics in economic models," *Journal of Economic Theory*, 55, 64–94.

DUTTA, P. K., S. LACH, AND A. RUSTICHINI (1995): "Better Late than Early: Vertical Differentiation in the Adoption of a New Technology," *Journal of Economics & Management Strategy*, 4, 563–589.

FAGERBERG, J., D. C. MOWERY, AND J. CANTWELL (2009): *Innovation and Competitiveness*, Oxford University Press.

FRICK, M. AND Y. ISHII (2015): "Innovation Adoption by Forward-Looking Social Learners," SSRN Scholarly Paper ID 2565769, Social Science Research Network, Rochester, NY.

FUDENBERG, D. AND J. TIROLE (1985): "Preemption and Rent Equalization in the Adoption of New Technology," *Review of Economic Studies*, 52, 383–401.

GILBERT, J. T. AND P. H. BIRNBAUM-MORE (1996): "Innovation timing advantages: From economic theory to strategic application," *Journal of Engineering and Technology Management*, 12, 245–266.

GILBERT, R. (2006): "Looking for Mr. Schumpeter: Where Are We in the Competition-Innovation Debate?" in *Innovation Policy and the Economy, Volume 6*, National Bureau of Economic Research, Inc, 159–215.

HENDERSON, R. AND K. CLARK (1990): "Architectural Innovation: The Reconfiguration of Existing Product Technologies and the Failure of Established Firms," *Administrative science quarterly*, 35.

HENDRICKS, K. (1992): "Reputations in the Adoption of a New Technology," *International Journal of Industrial Organization*, 663–677.

HOPPE, H. C. (2000): "Second-Mover Advantages in the Strategic Adoption of New Technology under Uncertainty," *International Journal of Industrial Organization*, 18, 315–338.

——— (2002): "The Timing of New Technology Adoption: Theoretical Models and Empirical Evidence," *The Manchester School*, 70, 56–76.

IGAMI, M. (2017): "Estimating the Innovator's Dilemma: Structural Analysis of Creative Destruction in the Hard Disk Drive Industry, 1981-1998," *Journal of Political Economy*, 125, 798–847.

JENSEN, R. (1982): "Adoption and diffusion of an innovation of uncertain profitability," *Journal of Economic Theory*, 27, 182–193.

JUDD, K. L. AND M. H. RIORDAN (1994): "Price and Quality in a New Product Monopoly," *Review of Economic Studies*, 61, 773–789.

KELLER, G. AND S. RADY (2010): "Strategic experimentation with Poisson bandits," *Theoretical Economics*, 5, 275–311.

——— (2015): "Breakdowns," *Theoretical Economics*, 10, 175–202.

KLEPPER, S. (1996): "Entry, Exit, Growth, and Innovation over the Product Life Cycle," *American Economic Review*, 86, 562–583.

KLINGEBIEL, R. AND J. JOSEPH (2016): “Entry timing and innovation strategy in feature phones,” *Strategic Management Journal*, 37, 1002–1020.

KRYLOV, N. V. (1980): *Controlled Diffusion Processes*, Stochastic Modelling and Applied Probability, Berlin Heidelberg: Springer-Verlag.

MAMER, J. W. AND K. F. MCCARDLE (1987): “Uncertainty, Competition, and the Adoption of New Technology,” *Management Science*, 33, 161–177.

MOTTA, M. (2004): *Competition Policy: Theory and Practice*, Cambridge University Press.

REINGANUM, J. F. (1981): “On the Diffusion of New Technology: A Game Theoretic Approach,” *Review of Economic Studies*, 48, 395–405.

——— (1983): “Technology Adoption under Imperfect Information,” *Bell Journal of Economics*, 14, 57–69.

——— (1989): “Chapter 14 – The Timing of Innovation: Research, Development, and Diffusion,” in *Handbook of Industrial Organization*, Elsevier, vol. 1, 849–908.

RIORDAN, M. H. AND D. J. SALANT (1994): “Preemptive Adoptions of an Emerging Technology,” *Journal of Industrial Economics*, 42, 247–261.

ROSEN, R. J. (1991): “Research and Development with Asymmetric Firm Sizes,” *RAND Journal of Economics*, 22, 411–429.

TRAJTENBERG, M. (1989): “The Welfare Analysis of Product Innovations, with an Application to Computed Tomography Scanners,” *Journal of Political Economy*, 97, 444–479.

WAGNER, P. A. (2018): “Who goes first? Strategic delay under information asymmetry,” *Theoretical Economics*, 13, 341–375.

A Appendix

A.1 Glossary of Long-run Values

Long-run duopoly profit

$$v_i^{(0,0)} = \frac{\left(\frac{1}{3}(q_i-q_j)+h\right)^2}{2h} \qquad v_i^{(1,1)} = \frac{\left(\frac{1}{3}(q_i-q_j)+h\right)^2}{2h} \qquad v_i^{(1,1),L} = \frac{\left(\frac{1}{3}(q_i-q_j+x)+h\right)^2}{2h}$$

$$v_i^{(1,0)}(0) = \frac{\left(\frac{1}{3}(q_i-q_j+q_L)+h\right)^2}{2h} \qquad v_i^{(1,1),F} = \frac{\left(\frac{1}{3}(q_i-q_j-x)+h\right)^2}{2h} \qquad v_i^{(0,1)}(0) = \frac{\left(\frac{1}{3}(q_i-q_j-q_L)+h\right)^2}{2h}$$

Long-run total surplus

$$v_{i,S}^{(0,0)} = \frac{2h(q_i+q_j)+(q_i-q_j)^2+3h^2}{4h} \qquad v_{i,S}^{(1,1)}(\alpha) = \frac{2h(q_i+q_j+q^e(\alpha))+(q_i-q_j)^2+3h^2}{4h}$$

$$v_{i,S}^{(1,1),L} = \frac{2h(q_i+q_j+2q_H-x)+(q_i-q_j+x)^2+3h^2}{4h} \qquad v_{i,S}^{(1,0)}(0) = \frac{2h(q_i+q_j+q_L)+(q_i-q_j+q_L)^2+3h^2}{4h}$$

$$v_{i,S}^{(1,1),F} = \frac{2h(q_i+q_j+2q_H-x)+(q_i-q_j-x)^2+3h^2}{4h} \qquad v_{i,S}^{(0,1)}(0) = \frac{2h(q_i+q_j+q_L)+(q_i-q_j-q_L)^2+3h^2}{4h}$$

Long-run monopoly profit

$$v_{i,M}^{(0,0)} = \frac{4h(q_i+q_j)+(q_i-q_j)^2+4h^2}{8h} \qquad v_{i,M}^{(1,1)}(\alpha) = \frac{4h(q_i+q_j+q^e(\alpha))+(q_i-q_j)^2+4h^2}{8h}$$

$$v_{i,M}^{(1,1),L} = \frac{4h(q_i+q_j+2q_H-x)+(q_i-q_j+x)^2+4h^2}{8h} \qquad v_{i,M}^{(1,0)}(0) = \frac{4h(q_i+q_j+q_L)+(q_i-q_j+q_L)^2+4h^2}{8h}$$

$$v_{i,M}^{(1,1),F} = \frac{4h(q_i+q_j+2q_H-x)+(q_i-q_j-x)^2+4h^2}{8h} \qquad v_{i,M}^{(0,1)}(0) = \frac{4h(q_i+q_j+q_L)+(q_i-q_j-q_L)^2+4h^2}{8h}$$

A.2 Proofs

Proof of Proposition 3.1. Assuming Firm i is the first-mover, the rate of learning is $\lambda_i^L(\alpha) = \lambda n_i(\alpha)$. Under zero discounting, $p_i(\alpha)$ maximizes $\frac{p_i(\alpha)n_i(\alpha)-v_i^L(\alpha)}{\lambda n_i(\alpha)} = \frac{1}{\lambda}\left(p_i(\alpha) - \frac{v_i^L(\alpha)}{n_i(\alpha)}\right)$. Upon simplification, the first-order condition yields $n_i(\alpha) = \sqrt{v_i^L(\alpha)/2h}$. Likewise, $p_j(\alpha)$ maximizes $\frac{p_j(\alpha)n_j(\alpha)-v_j^F(\alpha)}{\lambda n_i(\alpha)} = \frac{1}{\lambda}\left(\frac{p_j(\alpha)n_j(\alpha)-v_i^L(\alpha)}{n_i(\alpha)}\right)$. Upon simplification, the first-order condition becomes $p_j(\alpha) = 2hn_i(\alpha)n_j(\alpha)+v_j^F(\alpha)$. Use this along with $n_i(\alpha) = \sqrt{v_i^L(\alpha)/2h}$ to obtain $p_j(\alpha) = \sqrt{2hv_i^L(\alpha)} + v_j^F(\alpha) - v_i^F(\alpha)$.

To complete the proof, observe that $\sqrt{2hv_i^L(\alpha)} = 2hn_i(\alpha)$ implies $\sqrt{2hv_i^L(\alpha)} = p_j(\alpha)-p_i(\alpha)+q_i^e(\alpha)-q_j+h$. Using Firm j's first-order condition, it follows that $v_j^F(\alpha)-v_i^L(\alpha) = p_i(\alpha)+q_j-q_i^e(\alpha)-h$. Hence, $p_i(\alpha) = q_i^e(\alpha)-q_j + v_j^F(\alpha) - v_i^L(\alpha)+h$. ■

Proof of Lemma 3.2. If $\alpha_0 > \alpha_i^*$, then beliefs immediately enter stopping region if Firm i chooses to wait, and Firm i therefore receives payoff $\rho^{-1} v_i^{(1,1),F} - C_i$ from waiting at time 0. This is less than $\rho^{-1} v_i^{Sim} - C_i$, which Firm i can get from adopting at time 0. ■

Proof of Lemma 3.3. Under Assumption 1, $\frac{\partial n_i^{(1,0)}(\alpha)}{\partial (q_i - q_j)} = \frac{3h + q_i - q_j + \alpha x + (1-\alpha) q_L}{\left(6hn_i^{(1,0)}(\alpha)\right)^2} > 0.$ ■

Proof of Theorem 3.1. High-quality innovations generate good news with intensity $\Lambda_i(t) = \lambda \int_0^t n_i^{(1,0)}(\alpha(s))ds$ when Firm i leads adoption. Lemma 3.3 implies $n_i^{(1,0)}(\cdot) > n_j^{(1,0)}(\cdot)$. Thus, $\Lambda_i(t) > \Lambda_j(t)$ for all $t > 0$; hence, $\tau_i^F >_{FOSD} \tau_j^F$ for fixed α_0. ■

Proof of Lemma 3.4. Follows immediately from monotonicity of $v_i^L(\alpha)$ and $v_i^F(\alpha)$. ■

Proof of Lemma 3.5. (1) $\alpha_0 \geq \alpha_i^L \iff v_i^L(\alpha) \geq v_i^{Non} \iff (1, \tau_i^*) \in BR_i(0, \tau_j^*)$.

(2) $\alpha_0 \leq \alpha_i^F \iff v_i^F(\alpha) \geq v_i^{Sim} \iff (0, \tau_i^*) \in BR_i(1, \tau_j^*)$. ■

Proof of Corollary 3.2. Follows directly from Lemma 3.5. ■

Proof of Lemma 3.6. Derived in text. ■

Proof of Proposition 3.7. L_i, L_j, F_i, and F_j are positive at $\hat{\alpha}$. Monotonicity of these curves implies $\alpha_i^L, \alpha_j^L < \hat{\alpha} < \alpha_i^F, \alpha_j^F$. Using the rotation property, we have $L_i(\alpha_j^L) < L_j(\alpha_j^L) = 0$ and $F_j(\alpha_i^F) > F_i(\alpha_i^F) = 0$. Therefore, $\alpha_j^L < \alpha_i^L$ and $\alpha_i^F < \alpha_j^F$. ■

Proof of Theorem 3.2. The characterization of pure strategy equilibria follows directly from Corollary 3.2 and Proposition 3.7. The mixed-strategy equilibrium is determined by the indifference condition $\phi_j v_i^{Sim} + (1-\phi_j) v_i^L(\alpha_0) = \phi_j v_i^F(\alpha_0) + (1-\phi_j) v_i^{Non}$, which simplifies to give $\phi_j = L_i(\alpha_0)/(L_i(\alpha_0) + F_i(\alpha_0))$. Observe that $\phi_j \in [0,1]$ if and only if $\alpha_0 \in [\alpha_i^L, \alpha_i^F]$. Thus, for $q_i > q_j$, the mixed-strategy equilibrium exists if and only if $\alpha_0 \in [\alpha_i^L, \alpha_i^F]$. ■

Proof of Proposition 5.1. Similar to Proposition 3.1. For full details, see Appendix B. ■

Proof of Lemma 5.2. First establish that the difference $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is single-crossing. It suffices to prove $\frac{d}{d\alpha}\left[n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)\right] = \frac{\left(v_{i,S}^L\right)'(\alpha)}{2hn_{i,S}^{(1,0)}(\alpha)} - \frac{\left(v_i^L\right)'(\alpha)}{4hn_i^{(1,0)}(\alpha)} > 0$ holds whenever

$n_{i,S}^{(1,0)}(\alpha) = n_i^{(1,0)}(\alpha)$. Thus, it suffices to prove $2(\nu_{i,S}^L)'(\alpha) - \left(\nu_i^L\right)'(\alpha) > 0$. Using the expressions for $\nu_{i,S}^L(\alpha)$ and $\nu_i^L(\alpha)$, it is straightforward to show this condition holds under Assumption 2.

For $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ to be increasing, it suffices to prove $n_{i,S}^{(1,0)}(0) - n_i^{(1,0)}(0) < 0$. Simplification yields $n_{i,S}^{(1,0)}(0) < n_i^{(1,0)}(0) \iff (q_i - q_j + q_L)[3h + 2(q_i - q_j + q_L)] < 0$. Thus, $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is strictly increasing under Assumption 2.

Direct verification shows that Assumption 1 guarantees $n_{i,S}^{(1,0)}(0) < n_i^{(1,0)}(0)$ and $n_{i,S}^{(1,0)}(1) > n_i^{(1,0)}(1)$. Thus, Assumption 2 is sufficient to imply $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is increasing and crosses zero. ■

Proof of Lemma 5.1. Similar to Lemma 3.6. ■

Proof of Proposition 5.3. Similar to Theorem 3.2. ■

Proof of Lemma 5.2. To compare the private and social thresholds, note that

$$L_{i,S}(\alpha) = \frac{9}{2}L_i(\alpha) - \frac{1}{2}\Xi(\alpha) + \alpha(q_H - x) \quad \text{and} \quad F_{i,S}(\alpha) = \frac{9}{2}L_i(\alpha) + \frac{1}{2}\Xi(\alpha).$$

Observe that $L_{i,S}(\alpha_i^L) = -\frac{1}{2}\Xi(\alpha_i^L) + \alpha_i^L(q_H - x) > 0$, since $\alpha_i^L < \hat{\alpha}$. Therefore, $\alpha_{i,S}^L < \alpha_i^L$. Likewise, observe that $F_{i,S}(\alpha_i^F) = \frac{1}{2}\Xi(\alpha_i^F) > 0$, since $\alpha_i^F > \hat{\alpha}$. Therefore, $\alpha_{i,S}^F > \alpha_i^F$. ■

Proof of Proposition 5.4. Similar to Proposition 3.1. For full details, see Appendix B. ■

Proof of Lemma 5.3. Similar to Proposition 5.2. For full details, see Appendix B. ■

Proof of Theorem 5.1. For α sufficiently low, we have $n_{i,S}^{(1,0)}(\alpha) < n_{i,M}^{(1,0)}(\alpha) < n_i^{(1,0)}(\alpha)$. For α is sufficiently high, then we have $n_{i,S}^{(1,0)}(\alpha) > n_{i,M}^{(1,0)}(\alpha) > n_i^{(1,0)}(\alpha)$. Thus, monopoly experimentation is more efficient than private experimentation for extreme beliefs.

For intermediate beliefs, it is possible that monopoly experimentation is less efficient. To see this, note that $\left[n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)\right] - \left[n_{i,S}^{(1,0)}(\alpha) - n_{i,M}^{(1,0)}(\alpha)\right] = n_{i,M}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is strictly increasing. Let $\bar{\alpha}_S$ equal the solution to $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha) = 0$; and let $\bar{\alpha}_M$ equal the solution to $n_{i,M}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha) = 0$. Examine two cases:

Case 1: If $\bar{\alpha}_S < \bar{\alpha}_M$, then $n_i^{(1,0)}(\alpha) < n_{i,S}^{(1,0)}(\alpha) < n_{i,M}^{(1,0)}(\alpha)$ holds for all $\alpha \in (\bar{\alpha}_S, \bar{\alpha}_M)$. Because $n_{i,M}^{(1,0)}(\alpha) - n_{i,S}^{(1,0)}(\alpha)$ decreasing and $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is increasing, there is a range of beliefs in $(\bar{\alpha}_S, \bar{\alpha}_M)$ over which $0 < n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha) < n_{i,M}^{(1,0)}(\alpha) - n_{i,S}^{(1,0)}(\alpha)$. Over this range, private experimentation is more efficient.

Case 2: If $\bar{\alpha}_M < \bar{\alpha}_S$, then $n_{i,M}^{(1,0)}(\alpha) < n_{i,S}^{(1,0)}(\alpha) < n_i^{(1,0)}(\alpha)$ holds for all $\alpha \in (\bar{\alpha}_M, \bar{\alpha}_S)$. Because $n_{i,S}^{(1,0)}(\alpha) - n_{i,M}^{(1,0)}(\alpha)$ is increasing and $n_i^{(1,0)}(\alpha) - n_{i,S}^{(1,0)}(\alpha)$ is decreasing, there is a range of beliefs in $(\bar{\alpha}_M, \bar{\alpha}_S)$ over which $n_{i,M}^{(1,0)}(\alpha) - n_{i,S}^{(1,0)}(\alpha) < n_i^{(1,0)}(\alpha) - n_{i,S}^{(1,0)}(\alpha) < 0$. Over this range, private experimentation is more efficient.

In both cases, private experimentation is most efficient for an interior range of α. ■

Proof of Proposition 5.5. Follows directly from the rotation property. ■

Proof of Proposition 5.6. Similar to Lemma 5.2. For full details, see Appendix B. ■

B Supplementary Appendix (for Online Publication)

B.1 Monopoly Pricing and Long-Run Values

Suppose firms 1 and 2 merge to become a joint-profit maximizing monopolist. We need to know (i) whether the monopoly continues to sell the two products, and (ii) whether the monopoly continues to serve the entire market.

Given qualities q_1, q_2 and prices p_1, p_2, A consumer purchases good 1 iff

$$q_1 + h(1-\theta) - p_1 \geq \max\{q_2 + h\theta - p_2, 0\} \iff \theta \leq \min\left\{\frac{(q_1 - p_1) - (q_2 - p_2) + h}{2h}, \frac{q_1 - p_1 + h}{h}\right\}.$$

Let this threshold be denoted by θ_1. Likewise, a consumer purchases good 2 iff

$$q_2 + h\theta - p_2 \geq \max\{q_1 + h(1-\theta) - p_1, 0\} \iff \theta \geq \max\left\{\frac{(q_1 - p_1) - (q_2 - p_2) + h}{2h}, \frac{p_2 - q_2}{h}\right\}.$$

Let this threshold be denoted by θ_2. Straightforward calculation establishes that

$$\theta_1 = \frac{1}{2h}\left[(q_1 - p_1) - (q_2 - p_2) + h\right] \iff \theta_2 = \frac{1}{2h}\left[(q_1 - p_1) - (q_2 - p_2) + h\right].$$

Thus, there are only two cases to consider:

Case 1: (Serve the entire market)

$$\theta_1 = \frac{(q_1 - p_1) + (q_2 - p_2) + h}{2h} = \theta_2.$$

Case 2: (Exclude the middle)

$$\theta_1 = \frac{q_1 - p_1 + h}{h} < \frac{p_2 - q_2}{h} = \theta_2.$$

To prove that Case 2: (Exclude the middle) never arises, assume by way of contradiction that $\theta_1^* < \theta_2^*$ at the monopoly solution. Then p_1^* and p_2^* solve the following maximization problem:

$$\max_{p_1, p_2} \; p_1 \cdot \underbrace{\left(\frac{q_1 - p_1 + h}{h}\right)}_{\theta_1} + p_2 \cdot \underbrace{\left(1 - \frac{p_2 - q_2}{h}\right)}_{1-\theta_2} \qquad \text{subject to} \qquad \theta_1 \leq \theta_2.$$

Our hypothesis is that the inequality constraint will be slack at the optimum. So we can disregard it in the first-order conditions, and the optimal prices are easily computed as

$$p_1^* = \frac{q_1 + h}{2} \qquad \text{and} \qquad p_2^* = \frac{q_2 + h}{2}.$$

We must now verify that the constraint $\theta_1 \leq \theta_2$ is satisfied at these prices. Plug p_1^* and p_2^* into the formulas for θ_1 and θ_2 to obtain

$$\theta_1^* = \frac{q_1 - p_1^* + h}{h} = \frac{q_1 + h}{2h} \qquad \text{and} \qquad \theta_2^* = \frac{p_2^* - q_2}{h} = \frac{h - q_2}{2h}.$$

With this solution, however, the condition $\theta_1^* \geq \theta_2^*$ holds only if $q_1 + q_2 \leq 0$, which I rule out by assumption.

Having established the monopolist never excludes the middle, I next wish to show that the monopolist continues to supply both products. Note the monopolist serves the entire market (so $\theta_1 = \theta_2 = \bar{\theta}$) only if the marginal type $\bar{\theta}$ receives zero utility; otherwise, the monopolist could raise both p_1 and p_2 by some small amount $\varepsilon > 0$ to increase profits. Therefore, the marginal type $\bar{\theta}$ satisfies

$$q_1 + h(1 - \bar{\theta}) - p_1 = q_2 + h\bar{\theta} - p_2 = 0,$$

and the prices p_1 and p_2 can be written as functions of $\bar{\theta}$:

$$p_1 = q_1 + h(1-\bar{\theta}) \quad \text{and} \quad p_2 = q_2 + h\bar{\theta}.$$

The monopolist's profit maximization problem then becomes

$$\max_{\bar{\theta}} \; \big(q_1 + h(1-\bar{\theta})\big) \cdot \bar{\theta} + \big(q_2 + h\bar{\theta}\big) \cdot (1-\bar{\theta}) \quad \text{subject to} \quad 0 \leq \bar{\theta} \leq 1.$$

The first-order condition when ignoring the marginal type constraint gives

$$\bar{\theta}^* = \frac{1}{2} + \frac{1}{4h}(q_1 - q_2),$$

which is interior if and only if $|q_1 - q_2| < 2h$. Hence, Assumption 1 is sufficient to guarantee the monopolist supplies both products in equilibrium. Upon simplification, the monopolist's flow profit with product qualities q_1 and q_2 becomes:

$$\Pi^M(q_1, q_2) = \frac{4h(q_1 + q_2) + (q_1 - q_2)^2 + 4h^2}{8h}.$$

Using this, it is straightforward to calculate $v^L_{i,M}(\alpha)$, $v^F_{i,M}(\alpha)$, $v^{(1,1)}_{i,M}(\alpha)$, and $v^{(0,0)}_{i,M}$.

B.2 Relaxing Assumption 2

There are two ways Assumption 2 can fail:

1. $q_i - q_j > q_L$, i.e. Firm i remains dominant after adopting a low-quality innovation.
2. $q_i - q_j < q_H$, i.e. Firm i remains fringe after adopting a high-quality innovation.

To investigate the implications of how an unbalanced market structures affects the efficiency of equilibrium experimentation, it suffices to note that Assumption 2 is used in the proofs of Lemma 5.3. The first place Assumption 2 is when establishing the market share differences

$$n^{(1,0)}_{i,S}(\alpha) - n^{(1,0)}_{i}(\alpha), \qquad n^{(1,0)}_{i,S}(\alpha) - n^{(1,0)}_{i,M}(\alpha), \qquad n^{(1,0)}_{i,M}(\alpha) - n^{(1,0)}_{i}(\alpha)$$

are strictly increasing. Without Assumption 2, we can only guarantee they are (strictly) single crossing.

Lemma B.1. *Under Assumption 1, the market share differences*

$$n_{i,S}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha), \qquad n_{i,S}^{(1,0)}(\alpha)-n_{i,M}^{(1,0)}(\alpha), \qquad n_{i,M}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha)$$

are strictly single crossing.

The second place Assumption 2 gets used is in the verification that market shares cross zero. Direct simplification shows that $q_i-q_j+q_L<0$ is a necessary and sufficient condition for

$$n_{i,S}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha), \qquad n_{i,S}^{(1,0)}(\alpha)-n_{i,M}^{(1,0)}(\alpha), \qquad n_{i,M}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha)$$

to each be negative at $\alpha=0$. Thus, if Assumption 2 fails due to $q_i-q_j+q_L>0$, it follows from Lemma B.1 that $n_{i,S}^{(1,0)}(\alpha)>n_{i,M}^{(1,0)}(\alpha)>n_{i}^{(1,0)}(\alpha)$ for all $\alpha\in[0,1]$. Hence, we have:

Proposition B.1. *Suppose $q_i-q_j+q_L>0$. Then private and monopoly experimentation is always insufficient, but the latter is always more efficient.*

If Assumption 2 fails due to $q_i-q_j+q_H<0$, then it follows that

$$n_{i,S}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha), \qquad n_{i,S}^{(1,0)}(\alpha)-n_{i,M}^{(1,0)}(\alpha), \qquad n_{i,M}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha)$$

are strictly increasing, but they do not necessarily cross zero. In general, there are two possibilities for each difference:

Lemma B.2. *Suppose $q_i-q_j+q_H<0$. For each difference*

$$D(\alpha)\in\left\{n_{i,S}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha), n_{i,M}^{(1,0)}(\alpha)-n_{i}^{(1,0)}(\alpha), n_{i,S}^{(1,0)}(\alpha)-n_{i,M}^{(1,0)}(\alpha)\right\},$$

there are two possibilities: (i) $D(\alpha)$ is negative for all $\alpha\in[0,1]$, or (ii) $D(\alpha)$ is strictly increasing and crosses zero once.

Corollary B.1. *Suppose $q_i-q_j+q_H<0$. Then market experimentation in Stage 2 with Firm i as a first-mover is either (i) always excessive, or (ii) excessive for low values of α and insufficient for high values of α.*

Thus, it remains possible to compare the efficiency of monopoly and duopoly experimentation; but results will depend on the choice of parameter values.

B.3 Alternative Learning Processes

B.3.1 Learning from negative customer experiences.

Suppose a low-quality innovation produces a *negative* customer experience at rate $\lambda n(t)$, where $n(t)$ is the number of consumers purchasing the innovation. Suppose further that a high-quality innovation never generates a negative customer experience. Then beliefs absent bad news gradually evolve upward over time according to the law of motion $\alpha'(t) = \lambda n(t)\alpha(t)(1-\alpha(t))$.

Stage 2 pricing In a bad-news environment, if Firm j is a first-mover and Firm i is a second-mover in Stage 2, then the HJB equations are:

$$\rho V_i^F(\alpha) = \Pi_i^{(0,1)}(\alpha) + (1-\alpha)\lambda_j^L(\alpha)\left[V_i^F(0) - V_i^F(\alpha)\right] + \lambda_j^L(\alpha)\alpha(1-\alpha)\left(V_i^F\right)'(\alpha)$$

and

$$\rho V_j^L(\alpha) = \Pi_j^{(1,0)}(\alpha) + (1-\alpha)\lambda_j^L(\alpha)\left[V_j^L(0) - V_j^L(\alpha)\right] + \lambda_j^L\alpha(1-\alpha)\left(V_j^L\right)'(\alpha).$$

Under zero discounting using the strong long-run average criterion, equilibrium prices and market shares are identical to the good-news case.

Proposition B.2. *In a bad-news learning environment, equilibrium prices and market shares under zero discounting have the same formulas as in Proposition 3.1.*

The reason why prices and market shares are invariant to whether learning occurs via good news or bad news is due to the fact that learning, whether it be from good news or bad news, is complete. Thus, the exact manner in which learning occurs is irrelevant when firms become infinitely patient. All that matters is the current belief $\alpha(t)$ and the firm's long-run value given that belief, neither of which depends on the learning process directly.

The second-mover's problem Once again, Firm i's optimal stopping problem is characterized by a threshold belief α_i^* above which adoption is optimal. In the bad news case, α_i^* is pinned down by value-matching and smooth-pasting conditions. The date of Firm i's adoption as a second-mover is denoted $\tau_i^F = \inf\{t \geq 0 : \alpha(t) = \alpha_i^*\}$. The analysis of innovation is more complicated in the case of bad news, because τ_i^F now directly depends on α_i^* and, hence, the discount rate ρ. Since $\tau_i^F \to \infty$ as $\rho \to 0$, it is no longer appropriate to consider the zero discounting limit, and we must use numerical solution techniques to obtain plausible comparative statics.

Initial adoption Despite the technical complications that arise in analyzing the second-mover's problem, the analysis of initial adoption in equilibrium remains identical to the good-news case. Because learning is complete, the long-run average value to Firm i as a leader and as a follower remain identical to the good-news case. So we obtain an identical characterization of initial adoption.

Proposition B.3. *In a bad-news learning environment, initial adoption has the same structure as described in Proposition 3.2.*

B.3.2 Learning from Brownian news

In good-news and bad-news learning environments, certain customer experiences conclusively reveal an innovation to be high- or low-quality. We may think that such rapid learning is impossible in certain situations. To capture the possibility that learning may instead be gradual, consider the specification of Bergemann and Välimäki (1997), where news about innovation quality is arrives according to a Brownian diffusion process

$$dX(t) = \tilde{q}n(t)dt + \sigma\sqrt{n(t)}dW_t,$$

where $W(t)$ is standard Brownian motion.[15] Thus, consumers and firms gradually infer innovation quality from the evolution of $X(t)$ over time. On average, $X(t)$ will be increasing (decreasing) over time whenever the innovation is high-quality (low-quality).

[15] See Bergemann and Välimäki (1997) for a detailed discussion of the micro-foundation of $X(t)$.

Ito's lemma implies that posterior beliefs $\alpha(t)$ in this setting satisfy the law of motion

$$d\alpha(t) = \sqrt{n(t)}\left[\frac{q_H - q_L}{\sigma}\alpha(t)(1-\alpha(t))\right]d\bar{W}(t),$$

where $\bar{W}(t)$ is standard Brownian motion.

B.3.3 Stage 2 pricing.

Firm i's HJB equation as a first-mover and as a second-mover is

$$\rho V_i^F(\alpha) = \Pi_i^{(0,1)}(\alpha) + \frac{1}{2}\left[\frac{q_H - q_L}{\sigma}\alpha(1-\alpha)\right]n_j^{(1,0)}(\alpha)]\left(V_i^F\right)''(\alpha)$$

and

$$\rho V_i^L(\alpha) = \Pi_j^{(1,0)}(\alpha) + \frac{1}{2}\left[\frac{q_H - q_L}{\sigma}\alpha(1-\alpha)\right]n_j^{(1,0)}(\alpha)\left(V_j^L\right)''(\alpha),$$

respectively. Since news is never conclusive, these HJB equations consists of just two terms: (i) flow payoffs, and (ii) the value of information. Learning is nevertheless complete, because both firms receive positive market share. Since market share is linear in the value of information term, each firm's first-order condition under zero discounting remains identical to the good-news case.

Proposition B.4. *In a Brownian-news learning environment, equilibrium prices and market shares under zero discounting have the same formulas as in Proposition 3.1.*

B.3.4 Second-mover's problem and initial adoption.

Similar to Poisson bad-news case, the second-mover's problem with Brownian news is characterized by a threshold belief α_i^* above which adoption is optimal. Both value-matching and smooth-pasting boundary conditions apply, but numerical solution techniques must be applied for positive discounting to obtain realistic comparative statics. Despite this, long-run average profits and, hence, initial adoption is identical to the good-news case.

Proposition B.5. *In a Brownian-news learning environment, initial adoption has the same structure described in Proposition 3.2.*

In general, initial adoption timing under zero discounting is robust to any choice of complete learning process.

B.4 Appendix B Proofs

Proof of Lemma B.1. It suffices to prove $\frac{d}{d\alpha}\left[n_{i,S}^{(1,0)}(\alpha)-n_i^{(1,0)}(\alpha)\right]>0$ whenever $n_{i,S}^{(1,0)}(\alpha)=n_i^{(1,0)}(\alpha)$. Differentiating the difference yields:

$$\frac{d}{d\alpha}\left[n_{i,S}^{(1,0)}(\alpha)-n_i^{(1,0)}(\alpha)\right]=\frac{\left(v_{i,S}^L\right)'(\alpha)}{2hn_{i,S}^{(1,0)}(\alpha)}-\frac{\left(v_i^L\right)'(\alpha)}{4hn_i^{(1,0)}(\alpha)}.$$

If $n_{i,S}^{(1,0)}(\alpha)=n_i^{(1,0)}(\alpha)$, we require $2\left(v_{i,S}^L\right)'(\alpha)-\left(v_i^L\right)'(\alpha)>0$. Upon simplification, the long-run average total surplus under efficient experimentation is given by

$$v_{i,S}^L(\alpha)=\frac{\left(q_i+q_j+\alpha(2q_H-x)+(1-\alpha)q_L+\frac{3}{2}h\right)}{2}+(1-\alpha)\frac{(q_i-q_j+q_L)^2}{4h}+\alpha\frac{(q_i-q_j+x)^2}{4h}.$$

The long-run average for equilibrium experimentation is given by

$$v_i^L(\alpha)=\alpha\frac{\left(\frac{1}{3}(q_i-q_j+x)+h\right)^2}{2h}+(1-\alpha)\frac{\left(\frac{1}{3}(q_i-q_j+q_L)+h\right)^2}{2h}.$$

Therefore, $n_{i,S}^{(1,0)}(\alpha)-n_i^{(1,0)}(\alpha)$ is single-crossing if and only if

$$2(q_H-x)+(x-q_L)+\frac{(q_i-q_j+x)^2}{2h}-\frac{(q_i-q_j+q_L)^2}{2h}>\frac{\left(\frac{1}{3}(q_i-q_j+x)+h\right)^2}{2h}-\frac{\left(\frac{1}{3}(q_i-q_j+q_L)+h\right)^2}{2h}$$

which in turn is equivalent to

$$2(q_H-x)+\frac{2}{9}(x-q_L)\frac{(3h+4(q_i-q_j)+2q_L+2x)}{h}>0.$$

Under Assumption 1, the left-hand side of this inequality is strictly decreasing in x. Hence, a sufficient condition for the above is

$$2(q_H-q_L)+\frac{2}{9}(q_L-q_L)\frac{(3h+4(q_i-q_j)+2q_L+2q_L)}{h}>0 \iff 2(q_H-q_L)>0,$$

which is true. Thus, $n_{i,S}^{(1,0)}(\alpha)-n_i^{(1,0)}(\alpha)$ is single-crossing under Assumption 1. ■

Proof of Lemma B.2. Suppose $q_i - q_j < -q_H$. Since $q_L < 0 < q_H$, we have $q_i - q_j < -q_L$. Thus, each difference

$$n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha), \quad n_{i,M}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha), \quad n_{i,S}^{(1,0)}(\alpha) - n_{i,M}^{(1,0)}(\alpha)$$

is increasing. Moreover, each difference negative for all $\alpha \in [0,1]$ if negative at $\alpha = 1$; otherwise, the it crosses zero. ■

Proof of Proposition B.1. Through simplification, we have:

$$n_{i,M}^{(1,0)}(0) < n_i^{(1,0)}(0) \iff \frac{(q_i - q_j + q_L)(12h + 5(q_i - q_j + q_L))}{72h} < 0 \iff q_i - q_j + q_L < 0$$

and

$$n_{i,S}^{(1,0)}(0) - n_{i,M}^{(1,0)}(0) < 0 \iff \frac{q_i - q_j + q_L}{2} + \frac{3(q_i - q_j + q_L)^2}{8h} < 0 \iff q_i - q_j + q_L < 0.$$

Therefore, $n_{i,S}^{(1,0)} - n_{i,M}^{(1,0)}(0) > 0$ and $n_{i,M}^{(1,0)}(0) - n_i^{(1,0)}(0) > 0$ whenever $q_i - q_j + q_L > 0$. By Lemma B.1, these market share differences are single-crossing, so they must be positive for all $\alpha \in [0,1]$. ■

Proof of Proposition B.3. The profit maximization problem for Firm i (as a follower) under zero discounting using the strong long-run average is

$$0 = \max_{p_i^{(1,0)}(\alpha)} \left\{ \frac{\Pi_i^{(0,1)}(\alpha) - v_i^F(\alpha)}{n_j^{(1,0)}(\alpha)} \right\} + (1-\alpha)\lambda \left[V_i^F(0) - V_i^F(\alpha) \right] + \lambda\alpha(1-\alpha) \left(V_i^F \right)'(\alpha),$$

where $V_i^F(\cdot)$ denotes Firm i's strong long-run average value as a follower. The profit maximization problem for Firm i (as a leader) under zero discounting using the strong long-run average is

$$0 = \max_{p_i^{(1,0)}(\alpha)} \left\{ \frac{\Pi_i^{(1,0)}(\alpha) - v_i^L(\alpha)}{n_i^{(1,0)}(\alpha)} \right\} + (1-\alpha)\lambda \left[V_i^L(0) - V_i^L(\alpha) \right] + \lambda\alpha(1-\alpha) \left(V_i^L \right)'(\alpha),$$

where $V_i^L(\cdot)$ denotes Firm i's strong long-run average value as a leader. Thus, equilibrium prices and market shares are identical to the original model. ■

Proof of Proposition B.4. The profit maximization problem for Firm i (as a follower) under zero discounting using the strong long-run average is

$$0 = \max_{p_i^{(0,1)}(\alpha)} \left\{ \frac{\Pi_i^{(0,1)}(\alpha) - v_i^F(\alpha)}{n_j^{(1,0)}(\alpha)} \right\} + \frac{1}{2}\left[\frac{q_H - q_L}{\sigma}\alpha(1-\alpha)\right]\left(V_i^F\right)''(\alpha).$$

Likewise, the profit maximization problem for Firm i (as a leader) under zero discounting using the strong long-run average is

$$0 = \max_{p_i^{(1,0)}(\alpha)} \left\{ \frac{\Pi_i^{(1,0)}(\alpha) - v_i^L(\alpha)}{n_i^{(1,0)}(\alpha)} \right\} + \frac{1}{2}\left[\frac{q_H - q_L}{\sigma}\alpha(1-\alpha)\right]\left(V_i^L\right)''(\alpha;0).$$

Thus, equilibrium prices and market shares are identical to the original model. ■

B.5 Proofs Omitted from Appendix A

Proof of Proposition 5.1. Suppose Firm i is a first-mover and Firm j is a second-mover. Under zero discounting, the planner's objective in Stage 2 is to choose $n_i(\alpha)$ to solve

$$\max_{n_i(\alpha)} \left\{ \frac{TS_i^L(\alpha) - v_{i,S}^L(\alpha)}{n_i(\alpha)} \right\},$$

where $v_{i,S}^L(\alpha) = \alpha v_{i,S}^{(1,1),L}(1) + (1-\alpha) v_{i,S}^{(1,0)}(0)$ is long-run average total surplus. Upon simplification, the objective function becomes

$$q_i - q_j + q^e(\alpha) - \left(v_{i,S}^L(\alpha) - q_j - \frac{h}{2} \right) \frac{1}{n_i(\alpha)} + h(1 - n_i(\alpha)).$$

The first-order condition simplifies to become $n_i(\alpha) = \sqrt{(v_{i,S}^L(\alpha) - q_j - \frac{h}{2})/h}$, as desired. ■

Proof of Proposition 5.4. Suppose the monopoly chooses Firm i to be the first-mover in Stage 2. The HJB equation for the monopolist is

$$\rho V_{1,M}^L(\alpha) = \Pi_{1,M}^{(1,0)}(\alpha) + \alpha \lambda_1^L(\alpha)\left[V_{1,M}^{(1,1),L}(1) - V_{1,M}^L(\alpha) \right] - \lambda_1^L(\alpha)\alpha(1-\alpha)(V_{1,M}^L)'(\alpha).$$

Under Assumption 2, the entire market is served. Therefore, the marginal consumer type, $\bar{\theta}$, satisfies $q_1 + h(1-\bar{\theta}) - p_1 = q_2^e(\alpha) + h\bar{\theta} - p_2 = 0$. Hence, $p_1 = q_1 + h(1 - n_1(\alpha))$

and $p_2 = q_2^e(\alpha) + hn_1(\alpha)$, and the monopolist's flow profit is

$$\Pi_{i,M}^{(1,0)}(\alpha) = \left(q_i^e(\alpha) + h(1-n_i(\alpha))\right) n_i(\alpha) + \left(q_j + hn_i(\alpha)\right)(1-n_i(\alpha)).$$

Using the strong long-run average criterion, the HJB equation under zero discounting becomes

$$0 = \max_{n_i(\alpha)} \left\{ \frac{\Pi_{i,M}^{(1,0)}(\alpha) - v_{i,M}^L(\alpha)}{n_i(\alpha)} \right\} + \lambda\alpha[V_{i,M}^{(1,1),L}(1) - V_{i,M}^L(\alpha;0)] - \lambda\alpha(1-\alpha)\left(V_{i,M}^L\right)'(\alpha).$$

Upon simplification, the first-order condition yields $n_i^{(1,0)}(\alpha) = \sqrt{(v_{i,M}^L(\alpha) - q_j)/(2h)}$, as desired. ■

Proof of Lemma 5.3. From Proposition 5.2, the market share difference $n_{i,S}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is strictly increasing and crosses zero once under Assumption 2. Consider the other two differences:

1. Monopoly vs. Private experimentation

   The monopoly and equilibrium experimentation policies are

$$n_{i,M}^{(1,0)}(\alpha) = \sqrt{\frac{v_{i,M}^L(\alpha) - q_j}{2h}} \quad \text{and} \quad n_i^{(1,0)}(\alpha) = \sqrt{\frac{v_i^L(\alpha)}{2h}},$$

   respectively. To establish that $n_{i,M}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is strictly increasing, it suffices to prove that $(v_{i,M}^L)'(\alpha) > (v_i^L)'(\alpha)$ and $n_{i,M}^{(1,0)}(0) < n_i^{(1,0)}(0)$. The first condition is equivalent to

$$\frac{2q_H - x - q_L}{2} + \frac{(q_i - q_j + x)^2}{8h} - \frac{(q_i - q_j + q_L)^2}{8h} > \frac{\left(\frac{1}{3}(q_i - q_j + x) + h\right)^2}{2h} - \frac{\left(\frac{1}{3}(q_i - q_j + q_L) + h\right)^2}{2h}$$

   which, in turn, is equivalent to:

$$(q_H - x) + \frac{1}{72}(x - q_L)\frac{12h + 10(q_i - q_j) + q_L + x}{h} > 0.$$

   This holds under Assumption 1. The second condition holds if and only if

$$\frac{(q_i - q_j + q_L)[12h + 5(q_i - q_j + q_L)]}{72h} < 0 \iff q_i - q_j + q_L < 0.$$

   This holds under Assumption 2. Thus, $n_{i,M}^{(1,0)}(\alpha) - n_i^{(1,0)}(\alpha)$ is strictly increasing.

2. Efficient vs. Monopoly experimentation

   The efficient and monopoly experimentation policies are

$$n_{i,S}^{(1,0)}(\alpha) = \sqrt{\frac{2v_{i,S}^L(\alpha) - 2q_j - h}{2h}} \quad \text{and} \quad n_{i,M}^{(1,0)}(\alpha) = \sqrt{\frac{v_{i,M}^L(\alpha) - q_j}{2h}},$$

   respectively. To establish that $n_{i,S}^{(1,0)}(\alpha) - n_{i,M}^{(1,0)}(\alpha)$ is strictly increasing, it suffices to prove that $2(v_{i,S}^L)'(\alpha) > (v_{i,M}^L)'(\alpha)$ and $n_{i,S}^{(1,0)}(0) < n_{i,M}^{(1,0)}(0)$. The first condition is equivalent to

$$2(q_H - x) + (x - q_L) + \frac{(q_i - q_j + x)^2}{2h} - \frac{(q_i - q_j + q_L)^2}{2h} > \frac{2(q_H - x) + (x - q_L)}{2} + \frac{(q_i - q_j + x)^2}{8h} - \frac{(q_i - q_j + q_L)^2}{8h}$$

   which in turn is equivalent to

$$q_H - x + \frac{1}{72}(x - q_L)\frac{12h + 10(q_i - q_j) + 5q_L + 5x}{h} > 0.$$

   This holds under Assumption 1. The second condition is equivalent to

$$\frac{q_i - q_j + q_L}{2} + \frac{3(q_i - q_j + q_L)^2}{8h} < 0 \iff q_i - q_j + q_L < 0.$$

   This holds under Assumption 2. Thus, $n_{i,S}^{(1,0)}(\alpha) - n_{i,M}^{(1,0)}(\alpha)$ is strictly increasing.

Finally, for these differences to cross zero, we must have:

$$n_{i,M}^{(1,0)}(1) - n_i^{(1,0)}(1) > 0 \iff \frac{5}{6}(q_H - x) + \frac{1}{6}(q_i - q_j + q_H) + \frac{5}{72}\frac{(q_i - q_j + x)^2}{h} > 0$$
$$n_{i,S}^{(1,0)}(1) - n_{i,M}^{(1,0)}(1) > 0 \iff \frac{q_i - q_j + q_H + (q_H - x)}{2} + \frac{3}{8}\frac{(q_i - q_j + x)^2}{h} > 0.$$

Assumption 2 is sufficient for each of these inequalities to hold. ■

Proof of Proposition 5.6. We already know the comparison between private and social thresholds. To compare the private and monopoly leader thresholds, note that

$$L_{i,M}(\alpha) = \frac{9}{4}L_i(\alpha) - \frac{1}{4}\Xi(\alpha) + \alpha(q_H - x).$$

Observe that $L_{i,M}(\alpha_i^L) = -\frac{1}{4}\Xi(\alpha_i^L) + \alpha_i^L(q_H - x) > 0$, since $\alpha_i^L < \hat{\alpha}$. Therefore, $\alpha_{i,M}^L < \alpha_i^L$.

To compare the private and monopoly follower thresholds, note that

$$F_{i,M}(\alpha) = \frac{9}{4}F_i(\alpha) + \frac{1}{4}\Xi(\alpha).$$

Observe that $F_{i,M}(\alpha_i^F) = \frac{1}{4}\Xi(\alpha_i^F) > 0$, since $\alpha_i^F > \hat{\alpha}$. Therefore, $\alpha_{i,M}^F > \alpha_i^F$.

To compare the monopoly and social follower thresholds, note that

$$F_{i,M}(\alpha) = \frac{1}{2}F_{i,S}(\alpha) - \frac{1}{4}\Xi(\alpha).$$

Observe that $F_{i,M}(\alpha_i^F) = -\frac{1}{4}\Xi(\alpha_i^F) < 0$, since $\alpha_i^F > \hat{\alpha}$. Therefore, $\alpha_{i,M}^F < \alpha_{i,M}^F$.

The final comparison between monopoly and social leader thresholds is more involved. First observe that $L_{i,M}(\hat{\alpha}) < L_{i,S}(\hat{\alpha})$. Upon simplification, we have

$$L'_{i,S}(\alpha) - L'_{i,M}(\alpha) = \frac{q_i - q_j}{4h}(x - q_L) + \frac{1}{8h}(x^2 - q_L^2) > 0 \iff 2(q_i - q_j) + x + q_L > 0.$$

Therefore, $\alpha_{i,S}^L < \alpha_{i,M}^L$ whenever $q_i > q_j$. If $q_i - q_j < 0$, then

$$L_{i,S}(\alpha_{i,M}^L) = \frac{q_i - q_j}{2h}\Xi(\alpha_{i,M}^L) + \frac{1}{8h}\zeta(\alpha_{i,M}^L) > 0.$$

Therefore, $\alpha_{i,S}^L < \alpha_{i,M}^L$ whenever $q_i < q_j$, as well. ■