Cartel Damages in English Procurement Auctions with Endogeneous Entry: Evidence from a Medicine Cartel Case*

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Abstract

This paper studies collusion in an english procurement auction with endogenous entry and estimates cartel damages using data from a medicine cartel case that operated in Brazilian public procurement market. First, we show that in an english auction with endogenous entry, collusive behavior leads to inefficient allocation and also to an increase in price in auctions in which cartel members do not win; in addition to the standard increase in price in auctions that cartel members win. Second, we structurally estimate the cost distribution of cartel and non-cartel members using the data from a medicine cartel case operated in Brazil to simulate the equilibrium in an absence of the medicine cartel. By comparing the observed awarded contracts and their prices with the simulated ones, we find that the medicine cartel generated an overcharge of 10 percent, but it did not created significant inefficient allocation of procurement contracts. Our findings contribute to the recent antitrust debate on how to estimate cartel damages.

Keywords: Collusion; English Procurement Auctions; Endogeneous entry.
JEL classification: C57; D44; D57; L44.

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1 Introduction

This paper studies collusion in an english procurement auction with endogenous entry and estimates cartel damages using data from a medicine cartel case that operated in Brazilian public procurements.

In the first of the paper, we characterize the equilibrium in an english procurement auction with endogenous entry in a presence of a cartel and non-cartel bidding members. Such characterization is non-trivial due to the existence of multiple equilibrium in auction models with endogenous entry.[1] Following Tan and Yilankaya (2006), and assuming that the most efficient cartel bidder will be the representative of this group and that non-cartel bidders know about this collusive behavior, we find the conditions for the existence of an unique equilibrium in ascending second-price auctions with endogenous entry and collusion.[2] We show that to exist a unique equilibrium, where the most efficient cartel bidder has a higher probability to enter than non-cartel players, it is sufficient to satisfy three conditions: (i) cartel private cost cumulative distribution needs to first order stochastically dominates non-cartel's cost cumulative distribution function; (ii) the private cost cumulative distribution of both types of bidders needs to be inelastic and (iii) the reserve price has to be small enough.

After characterizing the equilibrium, we show that, in addition to the standard increase in price in auctions that cartel members win. collusive behavior leads to the inefficient allocation (i.e., the most efficient bidders do not win an auction) and also to an increase in price in auctions in which cartel members do not win.

Second, we structurally estimate the cost distribution of cartel and non-cartel members using the data from a medicine cartel case operated in Brazil. The medicine cartel was composed of 13 companies, among them manufacturers and wholesalers, that colluded between 2007-2012 to reduce competition in public procurement markets for medicines in different Brazilian states. The Brazilian Competition Authority (CADE), produced solid evidence against the cartel. Interesting, the procurement auctions that medicine cartel members bid for are english procurement auctions with relevance entry costs. Therefore, the Brazilian public procurements with the presence of the medicine cartel are perfect environments to structurally estimate the theoretical model developed in the first part of the paper.

[1]The model proposed in this paper assume bidders entry decision is endogenous. Different studies show that a considerable proportion of potential bidders decides to not participate in auctions, revealing that entry is endogenous. Hendricks et al. (2003), for instance, find that the overall participation rate was less than 25% in US Minerals Management Service auctions. Also, the existence of endogenous participation can change predictions found in the traditional auction literature. Other works find similar results on participation rate, like Li and Zheng (2009), Athey et al. (2011) and Krasnokutskaya and Seim (2011).

[2]Ciao and Tan (2010) provides similar conditions for a first-price auction.

Using the model equilibrium conditions allows to create more realistic counterfactual scenarios and to calculate cartel damages other than overcharges. To estimate them, we propose a parametric estimation procedure, adapting Roberts and Sweeting (2013), to recover bidder's private cost distribution.[3] Having recovery the cost distribution, we simulate the equilibrium in an english procurement auction with endogenous entry in absence of the medicine cartel. We then compare the observed awarded contracts and their prices, our real data, with the simulated ones. We find that the medicine cartel generated an overcharge of 10 percent, but it did not created significant inefficient allocation of procurement contracts.[4,5]

Related Literature and Contribution. To the best of our knowledge, there are only few papers that use structural models to create counterfactual scenario in the case of collusion in public procurement. Asker (2010) had access to documents containing detailed information on the internal structure of a cartel, that colluded in some US states collectible stamp markets. The richness of information allowed the author to model the cartel behavior itself and calculate the damage. Another work is Marmer et al. (2016), where they propose a method to identify the set of bidders that potentially colluded and to estimate their damage. The authors used data from the Guaranteed Investment Certificate auctions conducted by US municipalities over the Internet. In the end it was not possible to estimate any cartel damage, because it was not found the presence of collusion in those auctions.[6] Our paper contributes to this growing literature that aims to estimate the cartel damage using counterfactual approach. In the recent years, this literature has called the attention of antitrust authorities from all over the world as they have recently been discussing about methodologies to calculate cartel fines.[7,8,9]

Another branch in the empirical literature on collusion in procurements is concerned with cartel identification. Some authors use reduced form models to explore strategic behaviors that

[3]Differently than the authors, in our model we assume that (i) bidders know *ex-ante* their private cost and (ii) the collusion environment.

[4]In auction literature, efficiency is associated with auction's winner being the bidder that values most the auctioned object, in an ascending case, or being the most efficient bidder, in a lowest price case.

[5]Maier-Rigaud and Schwalbe (2013) is a good reference to study the kinds of cartel damage and the techniques used to estimate them.

[6]Indeed, as the authors pointed out, even after investigations there were no firms convicted for collusion in this market.

[7]Antitrust authority fine a cartel trying to achieve two objectives: (i) to punish the collusion and (ii) to deter future cartelizations. Most of those agents calculate collusion fines using a rule of thumb, charging a proportion of cartel firms turnover, usually of the year before the litigation ended. Most of antitrust agencies still do not adopt cartel's real damage to calculate their fines, claiming the complexity to estimate the damage and a potential increase in the judicialization of their decisions.

[8]In some countries the use of law damages actions is increasing, which also demands calculating collusion damages (Maier-Rigaud and Schwalbe, 2013).

[9]See more in 2017 DLA Piper's study, where they developed a summary of the recent tendencies of the most relevant antitrust agencies.

diverge from a competitive framework. Others propose to use structural model estimation to directly test for the presence of collusion. We follow the first group to find evidence that our data set support the assumptions on bidder's strategies and private cost cumulative distribution from our theoretical model. Note that it is beyond the scope of this work to propose a method to identify collusion, as we know the medicine cartel did operated to reduce competition in procurement auctions in Brazil.[10]

This paper is divided in 7 sections. Section 2 contains all the institutional background, providing detailed information on Brazilian public procurement market for medicines and the medicine cartel case. Section 3 introduces the database used in this work. Section 4 develop the model and its equilibrium outcomes. The empirical evidence that supports the assumptions used in Section 4 model and its equilibrium outcomes are in Section 5. Section 6 presents the structural model estimation proposed in this work. We detail the cartel damages and present a simulation exercise to calculate them in Section 7 and the last section concludes this work. All the mathematical proofs and auxiliary outcomes can be found in the appendix.

2 Institutional Background

In this section we describe the cartel that occurred on public procurements for medicine markets and how these markets are regulated. We divide this section in five parts. The first two explore regulatory laws that structure Brazilian and Sao Paulo public procurement markets. The third part describes the mechanism that public bodies used to acquire medicines. The last two parts discuss the pharmaceutical industry main characteristics and the medicine cartel case.

2.1 Brazilian public procurement market

All Brazilian public bodies (buyers) that wants to buy goods or services (inputs) are subject to the federal Law 8,666/1993, that regulates all the procedures those agents needs to follow in a procurement. According to it, first a buyer needs to develop a notice that contain all the basic information that a potential supplier (seller) needs to know. Usually these information are the quantity demanded, detailed input description, its minimum acceptable quality, contract's term, mechanism used to choose the supplier, the documents a seller has to present etc. After the notice is concluded, it needs to be publicized in an official gazette. The law defines that the mechanism to be used depends on the estimated value of the contract and the input characteristics.

[10] See Porter and Zona (1993,1999), Pesendorfer (2000) and Kawai and Nakabayashi (2015) for the first ones. For the second group see Baldwin et al. (1997), Bajari and Ye (2003), Aryal and Gabrielli (2013).

Federal Law 10,520/2002 regulates the use of Pregao, a mechanism very similar to the usual lowest price open-auction. A Pregao is used to acquire standard inputs, independently of its estimated contract value. Usually Pregao is an English or a two stage auctions and can be physical or electronic.[11][12] [13]

2.2 Sao Paulo's public procurement for standard inputs and medicines

Public acquisition of inputs in the state of Sao Paulo are hold in a decentralized way. Every year buyers decides their expenses accordingly to their own annual budget. After that, if it is necessary to acquire an input, those agents requests authorization to initiate a procurement process.

All Sao Paulo's public procurements for standard inputs uses a electronic platform called Bolsa Eletronica de Compras (BEC). This platform holds the entire procurement process, going through alerting sellers about the incoming procurement, until *ex-post* auction procedures, like appealing against its outcome. All potential sellers that wants to participate in BEC procurements needs to be registered in Sao Paulo's unified registration system, called Caufesp. In BEC's site is possible to download data of procurements held in BEC from 2008 until the current days. BEC's electronic Pregao uses a two stage auction as the mechanism to choose a seller. [14][15][16]

In the auctions we study, buyers acquire medicines to treat patients or to be distributed in public pharmacies with subsidized prices. Different from other standard inputs procurements, medicine notices detail drugs with high precision, using a unique combination of active ingredients, form, concentration, expiring date, number of units and package. A buyer is prohibited to procure medicines of a specific brand. Sellers can be manufacturers or independent wholesalers,

[11] The law 10,520/2002 defines a standard input as an input that performance and quality standards are easily defined using regular market specifications.

[12] In health sector, standard inputs are the ones: (i) essential to medical care offered by agencies registered on SUS (Brazilian public policy for health) and (ii) where performance and quality standards are easily defined using regular market specifications (Law 10,520/2002).

[13] When the input is not standard, Brazilian public agencies can also apply mechanisms that uses rules that are not price specific, to choose a seller. Regularly those rules involves quality and a mix of quality and price indexes. Quality includes precision, safety, endurance and other efficiency measures. Both rules uses a score to rank the bidders, but the difference between them is that the former uses only quality to calculate the score, while the second uses both quality and price. The bidder with the highest score will be the first the buyer will negotiate with. If negotiation fails, the latters starts a new negotiation with the second highest score bidder. This procedure goes on until the negotiation achieve closure.

[14] BEC does not hold only electronic Pregao. There are other two types of mechanisms: (i) Invitation and (ii) other procedures that do not need to pass through a procurement process. For more information, look Law 8,666/1993.

[15] To register in Caufesp, potential sellers needs to upload in the system documents that are regularly asked in standard inputs procurements. If they are valid, potential sellers do not need to present them in each procurement they participate.

[16] BEC's site:https://www.bec.sp.gov.br/BECSP/Home/Home.aspx

making possible that two different sellers offer medicines of the same brand.[17]

2.3 Two stage auction

BEC's Pregao is a two stage lowest price auction. In the first stage, sellers (potential bidders) submit an initial bid before an expiring date. During this stage, bidders identities and their bids are hold on secrecy. Right before the second stage begins, BEC's system reveals the values of the bids qualified to the next stage. Usually all bidders that submit a initial bid are qualified. At the end of this stage bidders identity are still kept on secrecy.[18] [19]

The second stage is similar to a lowest price English auction, where bidders sequentially lower their bids, winning the lowest one. During this stage, bidders can observe the number of participants and each bid they give, but not their true identity. The second stage can last at least 15 minutes, being extended for 3 more minutes each time a valid bid is submitted in the last 3 minutes. Bidders always observe the time left to finish the auction.[20]

2.4 Pharmaceutical industry

Pharmaceutical industry is vertically integrated and according to Palmeira Filho and Pan (2003) this industry productive chain can be divided in four stages. The first one is R&D, where laboratories spends a considerable sum in innovation of new active ingredients. This part is very significant to the industry as the development of new active ingredients can become new monopoly patents. The second is related to pharmacochemical manufacturing, where the active ingredient itself will be manufactured. The next stage is the medicine manufacturing and the last one is the medicine marketing and wholesale stage.

The medicines cartel colluded in the last stage of this industry: selling medicines to public bodies. In Brazil there are two kinds of wholesalers: manufacturers and independent wholesalers. The latter group is the largest group present in the last stage of Brazilian pharmaceutical industry. In the wholesale level, according to some managers from this industry, medicine assumes commodity characteristics. Each wholesaler usually sells drugs of different types and from distinct manufacturers. The same professionals told that one of the most important source of

[17] There are buyers outside the health sector that acquire medicines for internal use.

[18] We call potential bidders players that have a positive probability to participate in an auction. In this work, actual bidders are players that really participated.

[19] What is observed during the whole auction is a combination of letters and numbers, randomly assigned to each actual bidder by BEC's system.

[20] They observe too if the bid was a valid one. A invalid bid is one that it is higher than the last valid bid. Invalid bids are usually submitted by mistake or because different bidders submit same bids in a very small interval of time.

cost is logistic management, like transportation costs and medicine management storage.[21]

2.5 The medicines cartel case

The administrative litigation against the cartel started from a joint investigative operation called Panaceia, that was carried out at 12/04/2011 in the state of Minas Gerais. Panaceia was co-ordinated by Minas Gerais Public Prosecutor's Office (MPMG), the State Secretary of Finance (SEF/MG), the police force from the same state, Anvisa and the Brazilian Department of Economic Law (SDE). This operation had as targets three companies that manufacture and distribute medicines: (i) Hipolabor Farmaceutica Ltda., (ii) Rhamis Distribuidora Farmaceutica Ltda. and (iii) Sanval Comercio e Industria Ltda. Panaceia's objective was to investigate the potential existence of a criminal organization that committed the crimes of tax evasion, drug adulteration and collusion in public procurements for medicines held in Minas Gerais.[22]

A year after the Civil Inquiry was placed, SDE wrote a technical note using collusion's evidence sent by MPMG. The evidence suggested the cartel agreed previously the strategies to be followed by their members. The strategies cited in the technical note are: bid suppression, complementary bids, intentional disqualification, cartel members bidding equal values and/or on regular intervals and independent wholesalers bidding values lower than manufacturers. The technical note concludes pointing to the possibility the cartel was a bigger organization and that it also colluded on procurements of other states.[23][24][25]

Later the General Superintendency (SG) of Cade opened an Administrative Litigation (AP) at 01/04/2015 against a larger group of firms. According to SG, the cartel colluded in public procurements for medicines in the states of Minas Gerais, Sao Paulo, Bahia and Pernambuco, probably between 2007 and 2011, when Panaceia's search and seizure operations first occurred. Table 3 summary all firms prosecuted in this AP and points which ones operated in the state of Sao Paulo.[26][27][28]

[21] There are cases of exclusive contracts, but they are exceptions.

[22] Investigations started from a suspicion that two women died from consuming adulterated medicines.

[23] Civil Inquiry nº 0567.11.000021-1 from 22/06/2011

[24] Among the collusion evidence were telephone interceptions and documents obtained from search and seizure operations.

[25] SDE expected that independent wholesalers had higher costs than manufacturers, as the former is in the last part of the productive chain. We take note that this assumption can be true, but also distribution is wholesaler's main activity and maybe they can be more efficient in this task.

[26] On 2012 May, SDE was extinguished and SG was created, absorbing the former's competence.

[27] Administrative Litigation nº 08700.012439/2014-03.

[28] We did some empirical exercises trying to detect if there was differences on bidder's behavior before and after Panaceia. We did not found any significant one, which means that possibly bidder's did not changed their behavior during the term 2008-2012. This assumption is feasible, as Panaceia started investigating only three cartel bidders in Minas Gerais. To add, SDE's technical note was publicized only in 2012 May.

Like SDE's technical note, SG conclusion was that the cartel previously met to agree: (i) bids, (ii) which companies would win, make complementary bids and participate, (iii) sidepayments and (iv) to monitor if members were truly following the agreed strategies.[29]

At 17/07/2016, Novafarma agreed with SG a Cessation Commitment Term (TCC) of actions, homologated by Cades's court two month's later. In that term, Novafarma confirmed the cartel existence and that it was a member of it, exposed the identities of other members and presented more evidence. The TCC suggest the cartel operated in more states and that the number of members was even larger. By the date of this work, this medicine cartel case has not been judged by Cade's court yet.[30]

3 Database

The database uses data from two different sources. Most part of it comes from procurement documents stored in BEC's site. They supply: (i) medicine full description, (ii) date the auction was held, (iii) actual bidders identity, (iv) winner's and buyer's identities, (iv) delivery city, (v) medicine branch, (vi) last bid of each actual bidder, (vii) number of initial and qualified proposals sent in the first stage.

The rest of the data is bidder specific, mainly obtained from the Commercial Council of the State of Sao Paulo (Jucesp) site. There we got bidders cities, which enabled us to calculate the distance between the former and the delivery city.[31][32]

The database is structured as an unbalaced panel, where i is the input being auctioned and t is the date the auction occurred. The panel is unbalaced because we do not observe the each input being procured every period. To define each group of medicine we used a shorter description of the drug. Date is set as month/year.[33]

A discussion that usually appears in empirical auction literature is how to build a set of potential bidders. There is not a consensus about it and different ways were suggested: (i) to use all the planholders, (ii) establish a cutoff point based on distance or time or (iii) use all bidders that participated in auctions for a input. In this work, we decided to use the third

[29] Because the litigation is on secrecy, it is not possible to obtain more details on cartel strategies or its internal structure.

[30] Like the PA, Novafarma TCC contents are on secrecy.

[31] Jucesp's site: https://www.jucesponline.sp.gov.br/

[32] For firms not registered in the state of Sao Paulo, we got the data in the site of their state commercial council or in the federal procurement platform called Comprasnet.

[33] Our database has few information on observable auction characteristics. Structuring it like a panel helps to control for unobserved auction heterogeneity.

option.[34][35]

We used the definition of potential bidder to expand the original database in a way that, for a given i, each observation corresponds to a potential bidder. We call it hereafter the Potential database. This database has 502,449 observations of 29,341 auctions of medicines procured all over the state of Sao Paulo, between 2008's February and 2012's July. During this period, it was bought 2,714 different medicines by 78 different buyers. We observe in total 253 bidders, where 10 of them were cartel members. There are a total of 84,874 observed bids.[36]

Table 5 shows that 52.47% of auctions had cartel participation and that none had more than 5 cartel members in it. From all auctions that had cartel participation, 70.53% had just one cartel member.

Table 6 shows that most part of bids are concentrated in 2010-2011 term, with 2011 containing 33.65% of the entire sample. We see the same pattern for the two types of bidders. Most part of auctions were held in 2011 (38.40%). Cartel bidders had a considerable participation, compared to non-cartel. The number of cartel observations was 30.70% of non-cartel, even the former type containing only 4.11% of the number of the latter group.

Figure 1 focus only on the ten bidders with the highest participation and winning frequencies. Both panels show that even if the cartel was composed of only 10 bidders, half of them were among the top 10 participant and winners of our Potential database. Table 7 contains info on cartel and non-cartel participation and their rate of success. Panel A focus on cartel participation, detailing the number and percentage of auctions it participated. Panel B show cartel's and non-cartel's rate of success (winning) accounting only auction it participated. This Table shows that the cartel participated in 52.47% of the entire sample of auctions and its rate of success was relevant, winning 50.63% of the auctions that participated. Those outcomes shows evidence that the cartel was composed of players that on average were more efficient than non-cartel bidders.

4 Model

It only makes sense to collude if this behavior results in favorable outcomes to cartel members. A necessary condition to it is that collusion reduces competition. In this section we assume the

[34] See Li and Zheng(2009), Roberts and Sweeting (2011) and Athey et al. (2011) for some examples.

[35] Maybe the most used method is the first option, but we can not observe this information in our database. Even if we had it it would still lack precision, as potential bidders do not need to acquire the notice to participate in BEC's procurement.

[36] In reality the original database had 751,577 observations. But we excluded auctions where: (i) cartel bidders were the winner and the second lowest bid (the collusive behavior explained in the next section makes this point clearer), (ii) there was only one type of potential bidder (cartel or non-cartel) and (iii) bids were missing. Later, we calculated the frequency distribution a medicine was procured. We kept until the 98^{th} percentile, so it would be possible to estimate the Logit regressions we present in the appendix.

cartel will achieve its objective as a consequence of two assumptions: (i) the collusive behavior they adopt and (ii) that cartel bidders on average are more efficient than non-cartel ones, i.e., they are on average lower cost players. As a consequence, the collusion has the potential to exclude from the auction competitors that, in a competitive environment, would had changed the auction outcome.[37]

The last section brought evidences the cartel was very participative and had a high rate of success. In this section we model a collusion environment in the context of Sao Paulo's market of public procurement for medicines. We try to answer two different questions. First, how can we set a equilibrium strategy that supports the evidence showed in the database: cartel bidders were on average more efficient and had a higher probability to enter? What are the sufficient conditions for the existence and uniqueness of those equilibriums?

An important assumption we make is that non-cartel bidders knows about the collusive behavior adopted by cartel bidders (symmetric information).[38]

We divide this section into two parts. First we present model's general setup and in the last part we develop the model and it's equilibrium conditions.

4.1 Setup

Auction. Sao Paulo's buyers use a two stage auction to choose the medicine suppliers. From the rule of the game itself, auction's first stage is not binding, so we ignore it and build the model only considering the second stage. The second stage of BEC's auction is similar to a lowest price English auction, where it starts with a initial value that decreases continuously until it remains only one bidder.[39]

Buyer. We assume that a public body wants to buy a quantity q of a input and uses a lowest price English auction to choose the seller that will supply all q. The public body does not know about the collusion. We consider that all agents are risk-neutral.

Potential bidders. We assume two types τ of bidders: cartel and non-cartel. There are N potential bidders, where $cart = \{1, \ldots, C\}$ and $ncart = \{C+1, \ldots, N\}$ are the sets of cartel and non-cartel potential bidders, respectively. Potential bidders entry decision is endogenous, so

[37]Auction outcome is set as who wins the auction and/or the value of the payment the public body has to transfer to the winner.

[38]We also develop a model where non-cartel bidders do not know about the collusion and compare the theoretical outcomes between the two models(asymmetric model). See more in the appendix.

[39]We assume this initial value is the maximum payment the public body is willing to transfer to the winner.

bidders problem can be designed as a two stage game: (i) first a bidder makes its entry decision and (ii) if enter, he plays its bid.

To supply the input, a bidder i of type τ incur in a positive private cost $c_{i\tau}$. Nature independently drawn those costs from a continuous probability distribution $f_\tau(.)$ on $[\underline{c}, \bar{c}]$, in the beginning of game's first stage. We assume that bidders are in a independent private value (IPV) environment, which means that $c_{i\tau}$ is private and is not correlated with rivals costs. Private cost distributions are common knowledge.[40]

We also assume that non-cartel private cost cumulative distribution (first order) stochastically dominates cartel cumulative distribution, i.e, $F_{ncart}(c) \leq F_{cart}(c)$, $\forall\, c \in [\underline{c}, \bar{c}]$, with strict inequality for at least one c. A consequence of it is that cartel bidders will be on average more efficient than non-cartel bidders, one of the necessary conditions so that collusion can change the game outcome to its own favor.

4.2 The game

First stage. A model of auction with endogenous entry can have different assumptions on potential bidders knowledge about its own private cost. We follow Samuelson (1985) and assume that all players knows exactly their own private cost during the whole game. There are a couple of characteristics of the market we study in this work that justify this assumption. Selling medicine is a standard activity, which makes evaluating the costs of this service quite simple for potential bidders. Another point is that we observe in our database bidders trying to sell the same product, to different buyers, multiple times, which makes reasonable to assume that they have sufficient experience to forecast the private cost of an auction.[41][42]

Assume that if a bidder decides to enter, it will incur in a sunk entry cost $d \in (\underline{c}, \bar{c})$ common to all bidders. This entry cost can have different interpretations, like a cost to get more information about the procured input, a bid preparation cost or even an opportunity cost of entering in the auction. In our case, it suits better the latter two examples.

Second stage. In the second stage, $n \leq N$ actual bidders that decided to incur in d play a regular lowest price English auction. Because the mechanism is an open auction, actual bidders observe the number of entrants. We assume that exist an exogenous public reserve price with a

[40]We assume the cumulative distribution is twice continuously differentiable.

[41]There are other two cases in the literature: (i) in the first stage bidders do not receive any information about their private cost and discover it only if it plays the second stage; (ii) bidders receive a imperfect signal of their private cost in the first stage and discover their true private cost in the second stage. See Levin and Smith (1994) and Ye (2007) for more on this literature.

[42]Probably these players sells for both private and public markets.

lower bound of $c_0 > \frac{d}{q} + \underline{c}$, otherwise it would never be optimal for a bidder to enter.[43]

Competitive behavior. In this work when potential bidders behave competitively, it means that, after knowing their private cost, all of them plays the equilibrium strategy defined in the next section.

Collusive behavior. We follow the literature and the evidence from Cade's technical note, and assume that before bidders make their entry decision, but after they know their private cost, the cartel decides which bidder, among them, will enter and bid in the auction. This representative cartel player will be the most efficient cartel bidder and will play competitively against actual non-cartel bidders. The others cartel bidders will not participate in the auction. Note that by this definition we exclude the latter players from the set of potential bidders, which reduces the potential competition that others potential bidders face.[44][45][46]

Timming

We can summarize the game as following:

- Before the first stage, nature will draw from $f_\tau(.)$ the independent private costs of type τ bidders.
- Next, the cartel choose the representative cartel bidder.
- First stage: knowing their own private cost and the auction entry cost, potential bidders make their entry decision.
- Second stage: After entering, actual bidders bid a positive value and the lowest bid wins.

To find model's equilibrium strategy we solve by backward induction, finding first the bidding strategy equilibrium and later the entry strategy.

[43] Public here means that all potential bidders knows the reserve price, but not necessarily the econometrician.

[44] Se Krishna (2009), Asker (2010) and Marmer et al. (2016) for a better discussion on collusion strategies. Usually the literature assume a previous knockout auction as the mechanism a cartel uses to choose its representative cartel bidder.

[45] In our database we observe auctions with more than one actual cartel bidder. We assume those guys are playing complementary bids just to simulate competition. As will be more clear later, in this game, for equilibrium concerns, to not participate will be equal to participate and make complementary bids.

[46] For the complementary bid case, we assume that exist a sidepayment mechanism that compensates cartel bidders that enters only to play it. But to make the discussion more simple we do not enter in details of it. See Krishna (2009) for more on that discussion.

4.3 Equilibrium

4.3.1 Bidding equilibrium strategy.

Lemma 1 *An actual bidder of type τ can not do better than staying in the auction until the current bid reaches the level of it's private cost.*

This equilibrium is the same of English auctions with exogenous entry and it only depends on bidder's private cost.[47]

In this game, the winning price will be equal to the second lowest private cost, or to the reserve price if winner plays alone. This strategy perfectly inform losers private cost, while winner's cost is censured. The unique information that is possible to get directly from bid is that the latter is lower than the winning price.[48]

4.3.2 Entry equilibrium strategy

When a bidder decides to enter in an auction? It is intuitive that bidders enter when they expect to have a positive net profit, i.e., when their expected profit, when all potential bidders plays the bidding and entry equilibrium strategies described in this section is higher than the entry cost.[49]

An equilibrium in this game will be a pair of within type strategies (c^1_τ, β), where c^1_τ is the entry strategy equilibrium of type τ bidder and β is the bidding strategy from Lemma 1. We focus our attention to a within type Bayesian-Nash equilibria, where each potential bidder uses a cutoff strategy. This cutoff strategy can be described as:

Definition 1 *If $c_{i\tau} < c^1_\tau$, bidder i decides to participate in the auction and plays the equilibrium bidding strategy, otherwise, he will not participate.*

We set the discussion on cutoff points because, as pointed out by Tan and Yilankaya (2006), it is sufficient to describe the equilibria on cutoff strategies. As already discussed in this literature, equilibrium cutoff points are characterized by indifference, i.e., in our case they will be points where bidders net expected payoff will be equal to zero (or the minimum cutoff possible if it's not profitable to participate).

Proposition 1 shows that the equilibrium cutoff point will be the private cost of the indifferent bidder, when all potential bidders plays the equilibrium bidding and cutoff strategies. Because

[47] See Milgrom and Weber (1982) for exogenous entry and Gentry and Li (2014) for a discussion on the endogenous case.

[48] Like Li and Zheng (2009) noted, differently than on ascending English auctions, it is necessary the assumption of a positive reserve price. If not, the bidder would bid ∞ if it played alone.

[49] Hereafter we just call them as bidding and entry equilibrium strategies.

we assume that on average cartel bidders are more efficient than non-cartel players, and so it will be for the representative cartel bidder, we consider the case where the latter plays a cutoff point higher than non-cartel. That means the cartel bidder has a higher probability to enter. We call it the intuitive equilibrium.

In the same proposition we characterize also the net expected payoff of indifferent bidders, the sufficient conditions that guarantees the existence and uniqueness of the intuitive equilibrium in this model.

Proposition 1 *Assume that $F_\tau(.)$ is inelastic for all c and all τ. Assume also that c_0 is not so large and that $F_{ncart}(c)$ (first-order) stochastically dominates $F_{cart}(c)$. Then the pair $(c^1_{cart}, c^1_{ncart})$ is a unique cutoff equilibrium, with $c^1_{cart} > c^1_{ncart}$, iff it satisfy both:*

$$q(1 - F_{ncart}(c^1_{ncart}))^{N-C}(c_0 - c^1_{cart}) - d = 0 \tag{1}$$

$$q(1 - F_{ncart}(c^1_{ncart}))^{N-C-1}\left[(1 - F_{cart}(c^1_{cart}))^{C}(c_0 - c^1_{cart}) + \int_{c^1_{ncart}}^{c^1_{cart}} (1 - F_{cart}(c))^{C} dc\right] - d \leq 0, \tag{2}$$

where (2) is satisfied with equality when $c^1_{ncart} > \underline{c}$.

Note that because cartel bidders adopt the collusive behavior, the representative cartel bidder knows that the only rivals it can face are non-cartel players. Because we assume the cutoff point of the former is higher than the latter, an indifferent representative cartel bidder wins only when it plays alone. Non-cartel bidders plays competitively, but knows that if they face a cartel bidder, this player will be the most efficient of that type. So an indifferent non-cartel bidder wins when it plays alone, or when the representative cartel bidder enters and has a higher private cost.

Our model has two important outcomes. First, the intuitive equilibrium means that cartel bidders have a higher probability to enter than non-cartel bidders. The second is that from the first order stochastic dominance assumption, cartel bidders also have a higher probability to win.

5 Empirical Evidence

The collusion model was built under an important assumption: that only the representative cartel and non-cartel bidders compete in an auction. CADE's technical note concludes the cartel adopted the collusive behavior in some auctions, but does not say which ones. So in the first part of this section we show evidence that corroborates the possibility that only the representative cartel and non-cartel bidders competed.[50]

We also check if the observed auction outcomes are consistent with our model's assumptions and equilibrium outcomes. Specifically, we look in the second part of this section if on average: (i) representative cartel bidders have a probability to enter in an auction higher than non-cartel and (ii) the former have a probability to win higher than the latter. Our model tells that those predictions are driven by stochastic dominance and the collusive behavior assumptions. We also explore if some observable characteristics can explain the differences in entry and winning patterns between the two types of bidders.

5.1 Evidence of collusive behavior

Like mentioned in Section 3, we observe in the Potential database, auctions with more than one actual cartel member. Apparently this goes against the collusive behavior assumed in Section 4. But we also discussed that, for equilibrium concerns, there is no difference between cartel bidders others than the representative cartel player not entering or playing complementary bids. So in this section we look for evidence that supports the assumption that the formers were entering just to play complementary bids. Specifically we look for evidence that bid differences between the representative cartel bidder and other cartel bidders, that eventually participated, were much higher than between the former and non-cartel bidders.

For that we estimate both probability and cumulative distributions of the relative difference between the lowest bid of an auction and its "rivals" last bid. Note we used quotation marks, as we also look for the relative differences between a lowest bid made by a cartel member and bids made by others cartel members. We only use auctions that had competition, so we exclude from our sample auctions that had only one bidder or that all actual bidders were cartel members. To estimate the distributions, first we rank the bids of each bidder's type in an increasing order. Later, we calculate the relative difference of the lowest bid of an auction with each "rival's" last bid. We separately estimate the probability distributions for auctions where the winner was

[50] Even if we had access to these documents, probably we would not have the entire universe of auctions the cartel operated. Usually investigations does not reach all the auctions with potential collusion.

cartel and non-cartel bidders. [51]

Figure 2 shows the probability distributions of the relative differences between rank 1 bid and rank 2 cartel bid (black line), and rank 1 bid and rank 1 non-cartel bid (red line), in auctions won by a cartel bidder. Both distributions shows that the red line is much more concentrated in the left side than the black line. This means the bid difference between the two most efficient cartel bidders were higher than between the two most efficient bidders of each type.

Figure 3 includes other bidders. It shows that in auctions won by the cartel, there is a huge gap between the bids of the winner and other cartel bidders. The gap between the winner and the most efficient non-cartel bidder is low, while is greater for less efficient non-cartel bidders.

Doing the same for auctions won by non-cartel bidders, the left tail of the relative difference between the rank 1 bid and rank 1 cartel bid (solid line) is much more heavier than between the rank 1 bid and rank 2 cartel bid (dashed line). If we expand the figure plotting the probability distributions of the differences for higher ranks, this pattern is even more outstanding. From those figures its possible to conclude that, in auctions where a non-cartel bidder was the winner, there is a great gap between the lowest bid made by a cartel bidder and its cartel companions. This is a strong evidence that the latter group were entering only to play complementary bids.

In the end, the patterns presented by the four figures above do not go against the assumption the cartel adopted the collusive behavior and show strong evidence that some cartel bidders were entering only to play complementary bids.

5.2 Differences in entry and winning patterns

Before we do the simulations, it is relevant to demonstrate that the database does not contradict our model assumptions and equilibrium outcomes. We explore in this section two of them: (i) representative cartel bidders have a higher entry probability than non-cartel bidders and (ii) the former are more efficient than the latter. But note that its not possible to directly estimate from our database those statistics for representative cartel bidders. Instead we calculate cartel's entry and winning probabilities. If on average those statistics for the formers are higher than for non-cartel, it will be higher for representative cartel bidders.

First we create two dummies: (a) $d\_entrant_{ia}$ is 1 when a potential bidder i participates in the auction a and 0 otherwise and (b) d_win_{ia} is 1 when bidder i wins the auction a and 0 otherwise. The first dummy is used to calculate bidder's sample probability to enter. With the second dummy, we calculate bidder's sample probability to win. We use the Potential database and a sub sample with only actual bidders, called Actual database, to analyze on entry and

[51] To estimate both probability distributions we used a standard nonparametric approach.

winning probabilities, respectively.

Calculating the sample mean of *d_entrant* for each type of bidder, we find that cartel and non-cartel bidders had, on average, an entry probability of 28.98% and 16.37%, respectively. To calculate the winning probability is more tricky. It is possible to observe directly from database actual bidder's probability to win. We recover the unconditional probability using Bayes theorem and find that both cartel and non-cartel bidders had winning probabilities around 4%. Note that both outcomes supports (i) and (ii), as the entry and winning probabilities for representative cartel bidders are higher than for average cartel bidders, by definition.[52]

Next, we go further and explore how auctions and bidders observable characteristics affects entry and winning strategies between the two types of bidders. We estimate a set of linear models, where the dependent variables are *d_entrant* and *d_win* and the independent ones are auctions and bidders observable characteristics. We also control our models for medicine, bidders, time and city fixed-effects, with interaction between the last two.[53][54]

Table 8 gives a brief description of the variables used in the regressions. Table 9 and 10 describe variables summary statistics for Potential and Actual databases, respectively. All monetary values are in 2008's R$ and distance in kilometers. In both databases, cartel bidders had, on average, previously entered and won more than non-cartel bidders. The former are on average closer to the buyer. We note that cartel bidders on average bid values higher than non-cartel. Possibly that happens because of the complementary bids played by some cartel bidders.

To study how observable characteristics affect entry patterns of each bidder's type, we estimate linear models using *d_entrant* as dependent variable. Tables 11 and 12 reports the outcomes for cartel and non-cartel bidders, respectively. For both bidders, we estimate four different models using alternative distance variables. Table 13 reports that the linear models predicts our data quite well.[55]

Both types of bidders will likely participate more in procurements where they have to sell a higher quantity of medicines. This result are possibly related to scale economies involved in the transaction of larger quantity of medicines. More past participation in auctions of a specific

[52]Note that in this case, bidder's unconditional winning probability is equal to bidder's joint probability of entering and winning the auction.

[53]We also estimated Logit regressions. The outcomes are qualitatively similar to the linear, but the predictions of the former do not fit well. Because of that we decided to put the estimations results in the appendix. We control the Logit models for all the fixed-effects mentioned before, least city's fixed-effect. We had some computational problems when controlling for it.

[54]Because of computational limitation to estimate non-cartel entry Logits, all regressions were made using a sub sample. We calculated the percentiles of the number a medicine is procured and kept until percentile 98^{th}. The qualitative outcomes, for both linear and Logit models, were equal to linear regressions using the entire sample.

[55]All regressions use the logarithm of the variables with monetary and distance measures.

medicine are positively correlated with bidder's entry decision in future auctions of the same input. Having won higher contract value's in the past has the same effect. Instead, winning more in the past is negatively correlated with entry.

A quite surprising outcome is distance positive effect on cartel's entry probability. At first glance this result does not make any sense. We would expect that this correlation was instead negative. But this result is in line with Porter and Zona (1999). The authors find that the entry probability of bidders that colluded in Ohio school milk market had a positive correlation with delivery distance, and that could be a consequence of cartel strategy. In our case, this result could be an outcome of cartel bidders entering just to play complementary bids. Differently, distance effect on non-cartel bidder is negative, as would be expected in a regular case.[56]

Next we study how variables from Actual database affects the probability of winning. Tables 14 and 15 shows linear models outcomes separately for cartel and non-cartel bidders, respectively. Like before we estimate four different models using alternative distance variables.[57]

As expected, we find that for both types of bidders, their winning probability is negatively correlated with the number of competitors and the magnitude of their bid. Quantity of medicines being procured has the same effect. The regression outcomes on entry help to explain this outcome: auctions that procure higher quantities of medicines are correlated with auctions that have more actual bidders, i.e., more competition.

Note that for both types of bidders, the number of previous auctions a bidder participated and won has a negative correlation with the probability to win, where this last effect, in two cases, are not significant for cartel bidders. Bidder's highest contract value is positively correlated with winning probability for both types of bidder.

Distance effect on cartel goes the same as in the entry estimation, but it looses significance in some cases. Differently, for non-cartel bidders, this variable has a negative correlation with probability to win. Like in the entry regressions, linear models predicts well the data.

5.3 Concluding remarks.

This section shows evidence that the database does not contradicts our model assumptions and equilibrium outcomes. There is a huge gap between the bids of the representative cartel bidder and the actual cartel players. Instead, this gap was smaller comparing the former and non-cartel

[56] *Ceteris paribus*, the greater is the distance between bidder and buyer cities, the greater should be the transportation cost.

[57] We estimate using two assumptions on non-cartel bidder's knowledge about cartel's collusive behavior. The estimation outcome for both sets were very similar, so we report only the estimate for the symmetric case. The outcomes using the asymmetric case can be found in the appendix.

bidders. This pattern is aligned with the case where some cartel bidders are entering only to play complementary bids, with the objective to simulate competition.

Representative cartel players had, on average, probabilities of entering and winning higher than non-cartel bidders, which supports the assumption of the former type of bidder being on average more efficient than the latter. Only distance variables looks to affect differently both types of bidders probabilities.

To quantify cartel's damage its necessary to recover the primitives of bidders private cost distribution using structural estimation. The next section details the methodology proposed in this work.

6 Estimation Methodology

To recover bidder's private cost distribution we adapt the parametric estimation procedure used in Roberts and Sweeting (2013). Differently than Roberts and Sweeting, we make two distinct assumptions: (i) bidders know *ex-ante* their private cost and (ii) there is collusion.[58][59]

First, assume the probability distribution function of c, of a type τ bidder, $f_{\tau,a}(c)$, is proportional to a lognormal with location parameters $\mu_{\tau,a}$ and squared scale $\sigma^2_{\tau,a}$, on a $[\underline{c}, \bar{c}]$ interval. We set $\mu_{cart,a} < \mu_{ncart,a}$ and $\sigma_{ncart,a} = \sigma_{cart,a}$ $\forall a$. As Levy (1973) pointed out, this will guarantee that non-cartel private cost cumulative distribution (first order) stochastically dominates cartel cumulative distribution. Roberts and Sweeting (2013) also show that if the difference between the local parameters of the two cumulative distributions is sufficiently large, there will be a unique intuitive equilibrium.[60]

We estimate the primitives using Ackerberg's (2009) method of simulated maximum likelihood (SMLE) with importance sampling. We have chosen this method for two reasons. First, a simulation method is necessary, as the likelihood functions in our problem have higher dimensions. Functions like that are usually very difficult to compute and one of the solutions found by the literature is to use simulation to solve the problem numerically. So, like Roberts and Sweeting (2013), we exclude the possibility of using classical MLE.

Second, Ackerberg's method reduces the computational burden that arises when exist heterogeneity between auctions. In our case, usual simulation procedures would involve on resolving the model each time the value of model's primitives changed, which happens because of auction's

[58]The authors use an ascending English auction.

[59]Roberts and Sweeting (2013) use an AS model, where bidders receive an imperfect signal of their private cost in the first stage and discover their true private cost if it enters.

[60]We assume that the measure of the interval is big enough, so that the truncation in the private cost distribution does not affect the outcomes.

heterogeneity. Imagine that a econometrician has a database with N observations, will use S simulations and classical SMLE numeric optimization requires to evaluate R functions. That mean this model would need to be solved $N*S*R$ times. So if R is quite large, the traditional SMLE can be very time consuming.

Ackerberg's propose to use both importance sampling and a change of variables to lessen the computational burden. The idea of the method is that the parameters used to solve the likelihood function will not be drawn anymore from a distribution that depends on model's primitives. This allows to first solve a large number of games for different value of parameters, drawn from a distribution that does not depend on model's primitives, then calculate the auction's likelihood function for each of those games and lastly estimate model's primitives. Using this method reduces the number of times the model needs to be solved to $N*S$.

Like Roberts and Sweeting (2013), we assume private cost distribution parameters are distributed according to:

$$\begin{aligned}
\mu_{a,cart} &\sim N(X_a\beta_1, \omega^2_{\mu,col}), \\
\mu_{a,diff} = \mu_{a,ncart} - \mu_{a,cart} &\sim TRN(X_a\beta_2, \omega^2_{\mu,diff}, 0, \infty), \\
\sigma_a &\sim TRN(X_a\beta_3, \omega^2_{\sigma}, 0.5, \infty), \\
d_a &\sim TRN(X_a\beta_4, \omega^2_{d}, 0, \infty),
\end{aligned}$$

where X_a is a vector of observed auction characteristics and TRN(μ,σ^2,a,b) is a normal probability distribution, truncated on [a,b], with μ and σ^2 as location and scale parameters. The model primitives we estimate is $\Lambda = \{\beta_1, \beta_2, \beta_3, \beta_4, \omega^2_{\mu,cart}, \omega^2_{\mu,diff}, \omega^2_{\sigma}, \omega^2_{d}\}$. A particular draw of $\{\mu_{a,cart}, \mu_{a,diff}, \sigma_a, d_a\}$ is denoted as γ.

Denote y_a as an outcome for auction a. These outcomes include the number of each type of potential and actual bidders, last bid made by each participant and who made it, which type of bidder won and if the cartel participated. The log-likelihood function for a sample of auctions can be written as:

$$\sum_{a=1}^{A} log(\int L_a(y_a|\gamma)\phi(\gamma|X_a, \Lambda)\mathrm{d}\gamma), \tag{3}$$

where $L_a(y_a|\gamma)$ is the likelihood function of outcome y_a given the set of parameters γ. $\phi(\gamma|X_a, \Lambda)$ is γ's probability distribution given the vector of observable variables, the set of parameters Λ and the equilibrium strategies.

The computational issue is that the integral in (3) is high dimensional and there is no analytical solution. In this case, it is possible to estimate it using a numerical simulation algorithm:

$$\int L_a(y_a|\gamma)\phi(\gamma|X_a,\Lambda)\mathrm{d}\gamma) \approx \frac{1}{S}\sum_{s=1}^{S} L_a(y_a|\gamma_s), \tag{4}$$

where γ_s is one of S draws from $\phi(\gamma|X_a,\Lambda)$. That makes clear the necessity to make new draws of γ_s and re-solve the model each time one of the primitives in Λ changes, as $\phi(.)$ depends on it. Ackerberg's solution is to note that the integral in (3) can be written as:

$$\int L_a(y_a|\gamma)\phi(\gamma|X_a,\Lambda)\mathrm{d}\gamma) = \int L_a(y_a|\gamma)\frac{\phi(\gamma|X_a,\Lambda)}{g(\gamma|X_a)}g(\gamma|X_a)\mathrm{d}\gamma), \tag{5}$$

where $g(\gamma|X_a)$ is the importance sampling density. Note that it's support does not depend on Λ. The expression in (5) can be simulated as:

$$\frac{1}{S}\sum_{s=1}^{S} L_a(y_a|\gamma_s)\frac{\phi(\gamma_s|X_a,\Lambda)}{g(\gamma_s|X_a)} \tag{6}$$

Differently from before, in this procedure, γ_s is draw from $g(\gamma_s|X_a)$, a distribution that does depend on Λ. What changes during the optimization procedure is only the weight $\frac{\phi(\gamma_s|X_a,\Lambda)}{g(\gamma_s|X_a)}$. Which makes necessary only to calculate a new $\phi(\gamma_s|X_a,\Lambda)$ when Λ changes, instead of re-solving the model.

Roberts and Sweeting (2013) point out that the estimator proposed will only be accurate and consistent if: (i) γ_s draws are in the range where its probability distribution is relatively high and (ii) the number of simulations grows fast enough relative to the sample size. To reduce the bias in the parameters that are drawn, we use a two stage procedure. In first stage, estimate Λ assuming g(.) as a multivariate uniform distribution, over a large range of parameters. In the second stage, use the estimated parameters $\hat{\Lambda}$ and $g(\gamma_s|X_a) = \phi(\gamma_s|X_a,\hat{\Lambda})$ to construct the new set of γ_s. With the new set of γ_s and using $\hat{\Lambda}$ as a initial guess, it is possible to estimate model's primitives.

6.1 Calculating the likelihood function

.

Like Roberts and Sweeting (2013) we use auction's outcome to calculate the likelihood function. Because in English auctions the private cost of a winning bidder is censured, we do not calculate the likelihood function directly from bidders bidding strategy, as the literature usually does, but instead we use the probability a bidder i wins an auction a. Also because of the censoring problem, we use conservative assumptions to calculate the likelihood function: (i) an auction second lowest bid is equal to the private cost of the second most efficient bidder; (ii) the winning bidder has a private cost lower than the second lowest bid; (iii) all others actual bidders have private cost between the cutoff point and the second lowest bid; (iv) only the lowest bid among actual cartel bidders will be informative, the others will be considered like non entrants; (v) bidders that wins bidding the reserve price have a private cost lower than it.

For example, the likelihood function when a cartel bidder wins the auction with a winning price equal to b_{2a} can be written as:

$$L_a(y_a|\gamma) \propto \left(\int_{\underline{c}}^{b_{2a}} Prob\left(\min_{j \in cart} c_j = c|\gamma \right) \mathrm{d}c \right) \left(f_{ncart}(b_{2a}|\gamma)\right)$$
$$\left(\int_{b_{2a}}^{c^i_{ncart}} f_{ncart}(c|\gamma)\mathrm{d}c \right)^{n_{ncart}} \left(1 - F_{ncart}(c^1_{ncart})\right)^{N-C-n_{ncart}},$$

where c^1_{ncart} is model's cutoff equilibrium.

6.2 Summary of estimation stages

We can summarize the two of stages of our estimation procedure as:

1. First stage
   (a) First draw the set of parameters $\gamma_{s,a}$ from a multivariate uniform distribution $g(\gamma_{s,a}|X_a)$, for a simulation s of auction a.
   (b) Next using the set $\gamma_{s,a}$, find the equilibrium cutoff points for each s and a.
   (c) Having the cutoff points, calculate each simulation likelihood function.
   (d) Construct the multivariate normal distribution ϕ using a initial guess for Λ and auction's observable characteristics.
   (e) Calculate $\hat{\Lambda}$ by MLE using (6).
2. Second stage
   (a) Using $\hat{\Lambda}$, draw $\gamma_{s,a}$ from $g(\gamma_{s,a}|X_a) = \phi(\gamma_{s,a}|X_a, \hat{\Lambda})$, for a simulation s of auction a.
   (b) Next using the set $\gamma_{s,a}$, find the equilibrium cutoff points for each s and a.

(c) Having the cutoff points, calculate each simulation likelihood function.

(d) Construct the multivariate normal distribution ϕ using $\hat{\Lambda}$ and auction's observable characteristics.

(e) Calculate Λ by MLE using (6).

(f) Estimate primitives standard deviation using bootstrap method.

6.3 Identification discussion

We briefly discuss why we chose a parametric identification and not a nonparametric one. The use of nonparametric techniques to estimate structural models of auctions with endogenous entry has been a challenge to auction's literature.

As Gentry and Li (2014) show, a researcher can only retrieve directly from data two statistics: (i) the probability to enter of type τ bidder and (ii) actual bidders private cost cdf. Recovering the latter can be troublesome in English auctions, as the bidding strategy censors the winner's private cost. Marmer et al.(2016) develop a de-censoring estimator to overcome this difficulty. In the appendix we extend their method to auctions with endogenous entry. But still, is not possible to recover the unconditional private cost distribution directly from data.[61]

To estimate the unconditional private cost cumulative distribution, Gentry and Li (2014) prove that it is necessary to use a excludable variation on bidder's entry cost. To estimate pointwise bidder's private cost distribution, its necessary a continuous excludable variation. A discrete variation, like the one caused by varying the number of potential bidders, would only allow to estimate boundaries for those distributions. Because our database does not have a variable that creates this continuous variation, we decided to use a parametric estimation to recover our model primitives.[62]

7 Cartel Damage and Simulation

This section is divided into two parts. The first one discuss the damages that can arise from collusion, while the last part apply a simulation exercise to capture the relevance of cartel damages described in the first part of this section.

[61] There are other identification issues that English auction's bidding strategy can create, like jump bidding. See Haile and Tamer (2003) for a deeper discussion.

[62] Let $z \in Z$ be a set of auction observable characteristics. We say that z creates an excludable variation in d when $F_\tau(c|N,z) = F_\tau(c)$ and $d(N,z) = d(z)$.

7.1 Cartel Damages

Which types of damages a collusive behavior can generate? Collusion in public procurements has the potential to generate two kinds of damages: (i) overcharge and (ii) inefficiency. The first damage occurs when public procurement's price in the collusive environment is higher than it would be in a competitive one. The second effect appears when the most efficient potential bidder does not win the auction. Important to note that this inefficiency can be an outcome from endogenous entry assumption itself, but we show next that the collusive behavior can elevate this inefficiency frequency.[63][64]

To study collusion damages it is necessary to compare the collusion environment with the counterfactual scenario, i.e., the same auction but in a competitive environment. Until now we did not say anything about equilibrium strategies in the latter environment, where we assume there is no collusion. Lemma 1 shows that the bidding strategy does not depend on the environment, being the same in the competitive model. All the others assumptions made in section 4 remains the same. We present in Proposition 2 the competitive equilibrium strategy and the conditions for the existence of a unique equilibrium, where it is the intuitive one.

Proposition 2 *Assume that $F_\tau(.)$ is inelastic for all $c \in [\underline{c}\ \bar{c}]$ and all $\tau \in \{cart, ncart\}$. Also assume that c_0 is not so large and that $F_{ncart}(.)$ (first-order) stochastically dominates $F_{cart}(.)$. Than the pair $(c^2_{cart}, c^2_{ncart})$ is competitive model's unique cutoff equilibrium, with $c^2_{cart} > c^2_{ncart}$, iff it satisfies both:*

[63] The cartel does not have the power to reduce the quantity q because this quantity is set by the public body. It is important to say that in a dynamic environment, the collusive behavior can create, for some agents, barriers to entry or even incentives to exit the market. We also show that in some cases collusion in procurements can lower the price.

[64] For example, assume a competitive environment where entry is exogenous and we have N_{cart} cartel bidders and N_{ncart} non-cartel bidders. Also assume that in an endogenous model, each set of potential bidder is equal to the ones described earlier. If $\bar{c}_{ncart} < \bar{c}_{cart}$, where the first and the second are the private costs of the most efficient non-cartel and cartel bidders, respectively, the former bidder wins the auction. But assume that in the endogenous model, non-cartel bidder's cutoff point c^*_{ncart} is smaller than $\bar{c}_{ncart}$. Now the cartel bidder wins the auction and this auction is inefficient.

$$q(1 - F_{cart}(c^2_{cart}))^{C-1}(1 - F_{ncart}(c^2_{ncart}))^{N-C}(c_0 - c^2_{cart}) - d = 0 \tag{7}$$

$$q(1 - F_{ncart}(c^2_{ncart}))^{N-C-1}\left[(1 - F_{cart}(c^2_{cart}))^C(c_0 - c^2_{cart}) + \int_{c^2_{ncart}}^{c^2_{cart}} (1 - F_{cart}(c))^C dc\right] - d \leq 0, \tag{8}$$

where (8) is satisfied with equality when $c^2_{ncart} > \underline{c}$.

The appendix shows that the sign of the difference between c^2_{ncart} and c^1_{ncart} is ambiguous, but that $c^2_{cart} < c^1_{cart}$. The ambiguity comes from the existence of a positive and negative effects on non-cartel cutoff point, when the cartel cutoff point increase. The first effect occurs because now non-cartel bidders expect to face less efficient cartel bidders. While the negative effect happens because the formers know that the probability of participating alone in an auction decreases. From the ambiguity mentioned above we detail cartel's damage under each possible outcome.

7.1.1 When $c^2_{ncart} > c^1_{ncart}$

Table 1 summarize possible auction's outcomes and collusion damages that can be generated when $c^2_{ncart} > c^1_{ncart}$. We assume the winner in the collusive environment would participate in the competitive framework.

Table 1: Auction outcome and cartel's damage

		Winner: Cartel	Winner: Non-cartel
Wins	Alone	↑Price Alocative	↑Price
	Not alone	↑Price	↑Price

Note. This Table contains auctions outcomes and the cartel damages that can be associated with them when $c^2_{ncart} > c^1_{ncart}$.

The outcome where the representative cartel bidder wins alone results in overcharge if, in a competitive environment instead, this player would face actual competition, as the winning bid in this last scenario would be smaller than the reserve price. But if in the counterfactual scenario a non-cartel bidder turns to be the winner, than there is also an inefficiency. Differently, when the representative cartel bidder wins an auction competing against a rival, where this rival can only be a non-cartel bidder in a collusive environment, there can only exist overcharge. That happens because the representative cartel bidder is already facing the most efficient non-cartel player, but not the second most efficient cartel bidder, and it could be that the latter is more efficient than the most efficient non-cartel bidder.

Quite surprising is to know that both scenarios where a non-cartel bidder wins can still suffer from overcharge. That happens because in this case, when non-cartel bidders know about cartel's collusive behavior, they are more cautious in their entry decision, i.e., they play a cutoff point lower than in the competitive case. A consequence is that non-cartel's best reaction to the collusive behavior can potentially exclude others non-cartel bidders that would have entered in a competitive case. This outcome shows that possibly, antitrust agencies should include, when calculating collusion fines, procurements where the cartel lost.

7.1.2 When $c^2_{ncart} < c^1_{ncart}$

The next Table summarizes the possible outcomes an auction can have and the damages that can be generated when $c^2_{ncart} < c^1_{ncart}$. Like before, we assume that the winner in the collusive environment would participate in the competitive framework.

Table 2: Auction outcome and cartel's "damage"

		Winner	
		Cartel	Non-cartel
Wins	Alone	↑Price	No Damage
	Not alone	Price (Ambigous)	↓Price

Note. This Table contains potential auctions outcomes and the cartel damages that can be associated with them when $c^2_{ncart} < c^1_{ncart}$.

Differently than in the last case, the existence of overcharge when the representative cartel bidder wins alone is a consequence alone of the collusive behavior. Note that from $c^2_{ncart} < c^1_{ncart}$, non-cartel bidders would still not participate in the competitive framework, while it is possible that a cartel bidder would enter in this last framework. In this case the price paid in the competitive environment would be lower than the reserve price paid in the collusive framework.

Surprisingly, when the representative bidder wins an auction facing actual competition, the collusive behavior can increase or reduce the auction's price in the collusive environment compared to the competitive one. The first outcome occurs when a bidder of any type that does not participate in the former environment, would decide to enter in the counterfactual scenario and would end in second place instead. The last effect happens when non-cartel's cutoff point in the competitive environment is sufficiently smaller than in the collusive framework, in a way that all non-cartel bidders who entered in the latter decides to not participate in the former.

The collusion does not generate any damage when a non-cartel bidder wins an auction alone. That happens because this player would still be the only agent that decides to participate in a counterfactual scenario. Differently, an auction won by a non-cartel bidder that faced actual competition in a collusive environment, can have a lower price than in the competitive framework. This will happen if the bidder that ended in second place in the former environment decides to not participate in the latter, because it's private cost is higher than the cutoff of it's type in the competitive framework.

7.2 Simulating cartel's damage.

The last sub section presented three outcomes: (i) the sign of the difference between non-cartel cutoff points in the two frameworks is ambiguous; (ii) a cartel can increase or reduce prices (iii) and have the potential to generate inefficiency. In this subsection we use simulations to understand better the outcomes mentioned before.

For that we follow Roberts and Sweeting (2013) and assume that the probability distribution function $f_\tau(c)$, of a type τ bidder, is proportional to a lognormal with location parameters μ_τ and squared scale σ_τ^2, on a $[\underline{c}, \bar{c}]$ interval. We set $\mu_{cart} < \mu_{ncart}$ and $\sigma_{ncart} = \sigma_{cart}$.

We draw the private cost distribution parameters and the entry cost from a uniform distribution:

$$\begin{aligned} \mu_{cart} &= 5.7 \sim U[5.5, 7.5], \\ \mu_{diff} = \mu_{ncart} - \mu_{cart} &= 0.5 \sim U[0.2, 0.9], \\ \sigma &= 0.4 \sim U[0.05, 1.5], \\ d &= 41.5 \sim U[0, 50], \end{aligned}$$

where $\gamma = [\mu_{cart}, \mu_{diff}, \sigma, d]$.

We simulate a representative auction using the median values of the observable characteristics of Potential database auctions. The representative auction has a quantity of 719 medicines being procured, with a reserve price equal to R\$ 721.19 and $C = 2$ and $N - C = 12$. The simulation steps can be summarized as:

1. Find for a given γ and auction observable characteristics, the pair of cutoff points for each

environment using Proposition 1 and Proposition 2.

2. Draw, for each simulation, a private cost for each potential bidder.

3. Using the cutoff points and the private cost values, build the distribution of bids for each potential bidder.

4. Find auctions outcomes: (i) the expected public body payment, (ii) number of entrants and (iii) the number of efficient auctions.

Table 17 summarizes the simulations mean outcomes. We simulated 1 million of representative auctions for each environment.

The simulation shows, for the parameters values assumed, the equilibrium strategy that prevails is the one described in subsection 7.1.2 and the cartel participates almost in all auctions in the collusive environment. As pointed out in subsection 7.1.2, the collusive behavior does not have the power to generate inefficiency. In this case, all inefficiency comes from the assumption of endogenous entry. But as Table 17 shows, this outcome is not significant. In both environments the average cartel bid are lower than non-cartel, where the difference is larger in the collusive framework. Finally, collusion generated an overcharge of almost 10%.[65]

7.3 Concluding remarks.

This section shows evidence that the database does not contradicts our model assumptions and equilibrium outcomes. There is a huge gap between the bids of the representative cartel bidder and the other actual cartel players. Instead, this gap was smaller comparing the former and non-cartel bidders. This pattern is aligned with the case where some cartel bidders are entering only to play complementary bids, with the objective to simulate competition.

Representative cartel players had, on average, probabilities of entering and winning higher than non-cartel bidders, which supports the assumption that the former type of bidder is on average more efficient than the latter. Only distance variables looks to affect differently both types of bidders probabilities.

8 Conclusion

This work shows the importance of considering endogenous entry in environments with collusion. For that, we develop a lowest price English auction model assuming the collusion existence. When

[65]We also did simulations for representative auction in an asymmetric information model. We found a similar value for the overcharge.

bidders make their own decision to enter in an auction and non-cartel bidders knows about the collusive behavior adopted by the cartel, two outcomes, still underexplored by the literature of collusion in procurement, can occur: (i) increase of inefficiency and (ii) impact the prices of auctions where the cartel lost. Those are important outcomes that antitrust agencies should consider when calculating collusion fines, as an optimal collusion fine uses the value of cartel damages on it's calculation.

We also argue that using equilibrium conditions to construct counterfactual scenarios is one of the most promising options to calculate collusion damages. This method has the power to calculate cartel damages other than overcharging. We do simulations using data from procurements where the cartel participated and find that cartel overcharged almost 10%. In those simulations the collusion was not capable of generating significant inefficiency.

This work also contributes with the literature of auctions with endogenous entry, finding sufficient conditions for the existence and uniqueness of an intuitive equilibrium both in competitive and collusive models.

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Appendix

This appendix starts developing the proofs for the equilibria conditions in sections 4 and 7. We also develop a model with collusion, where non-cartel bidders do not know about the collusion existence (asymmetric information). In the last we compare the outcomes of the tree models.

Collusion: Symmetric information (M1)

Here we present the equilibrium proof of the model presented in section 4. We first characterize in Lemma 2 indifferent bidder's net expected profit. Later we divide Proposition 1 proof in three others: (i) Intuitive equilibrium existence, (ii) Non existence of the non intuitive equilibrium and (iii) Intuitive equilibrium uniqueness.

Lemma 2 *Let $c_\tau^1 \in [\underline{c}, \bar{c}]$ be the cutoff point of type τ bidder. Without loss of generality, assume $c_0 > c_{cart}^1 \geq c_{nccart}^1 \geq \underline{c}$. The net expected payoff of the indifferent representative cartel and non-cartel bidders are, respectively:*

$$q(1 - F_{ncart}(c_{ncart}^1))^{N-C}(c_0 - c_{cart}^1) - d \tag{9}$$

$$q(1 - F_{ncart}(c_{ncart}^1))^{N-C-1} \left[(1 - F_{cart}(c_{cart}^1))^C (c_0 - c_{cart}^1) + \int_{c_{ncart}^1}^{c_{cart}^1} (1 - F_{cart}(c))^C dc \right] - d \tag{10}$$

Proof. The proof for the indifferent representative cartel bidder is straightforward. This player knows that the other cartel bidders will not participate and knows they will not deviate from this behavior. Because by assumption all actual non-cartel bidders have a lower cutoff point, the indifferent representative cartel bidder will win only if he participates alone, in which case it bids the reserve price.

We can write the net expected payoff of the indifferent non-cartel bidder as:

$$(1-F_{ncart}(c^1_{ncart}))^{N-C-1}\left\{q(c_0-c^1_{ncart})(1-F_{cart}(c^1_{cart}))^C+\right.$$

$$\left.\int\limits_{c^1_{ncart}}^{c^1_{cart}} q(c-c^1_{cart})\mathrm{d}[1-(1-F_{cart}(c))^C]\right\}-d$$

Note that we use the implicit minimum's pdf as a measure in the integrals. Using integration by parts we have:

$$q(1-F_{ncart}(c^1_{ncart}))^{N-C-1}\left\{(c_0-c^1_{ncart})(1-F_{cart}(c^1_{cart}))^C+\left((c^1_{cart}-c^1_{ncart})-\right.\right.$$

$$-(c^1_{cart}-c^1_{ncart})(1-F_{cart}(c^1_{cart}))^C+(c^1_{ncart}-c^1_{ncart})(1-F_{cart}(c^1_{ncart}))^C$$

$$\left.\left.-(c^1_{cart}-c^1_{ncart})+\int\limits_{c^1_{ncart}}^{c^1_{cart}}(1-F_{cart}(c))^C\mathrm{d}c\right)\right\}-d$$

Doing some arrangements we find (10). ■

Proposition 3 *Let c^1_{cart} and c^1_{ncart} be cartel and non-cartel cutoff points, respectively. If $F_{ncart}(.)$ first order stochastically dominates $F_{cart}(.)$, than there will always exist a equilibrium where $c^1_{ncart}<c^1_{cart}$ that satisfies:*

$$q(1-F_{ncart}(c^1_{ncart}))^{N-C}(c_0-c^1_{cart})-d=0 \tag{11}$$

$$q(1-F_{ncart}(c^1_{ncart}))^{N-C-1}\left[(1-F_{cart}(c^1_{cart}))^C(c_0-c^1_{cart})\right.$$

$$\left.+\int\limits_{c^1_{ncart}}^{c^1_{cart}}(1-F_{cart}(c))^C\mathrm{d}c\right]-d\leq 0, \tag{12}$$

where (12) is satisfied with equality when $c^1_{ncart}>\underline{c}$.

Proof. First, let's prove that bidders will play the cutoff strategy. Assume that all bidders, less i, uses the cutoff strategy. The expected payoff of it is decreasing and continuous in its private cost. So it will exist a c_τ^1 where the net expected payoff of this bidder will be 0. If $c_{i\tau} > c_\tau^1$, his net expected payoff will be negative and will not be optimal for it to participate. But if his private cost is smaller than the cutoff point, he will enter. His cutoff point will be $c_\tau^1 = \underline{c}$ iff his payoff is nonpositive when his private cost is $\underline{c}$. Otherwise, $c_\tau^1 > \underline{c}$ if he enters having a private cost of c_τ^1 and its net expected payoff is zero. Now set a pair (x,y) that satisfies:

$$q(1 - F_{ncart}(y))^{N-C}(c_0 - x) - d = 0 \tag{13}$$

$$q(1 - F_{ncart}(y))^{N-C-1}\left[(1 - F_{cart}(x))^C(c_0 - x) + \int_y^x (1 - F_{cart}(c))^C \mathrm{d}c\right] - d \leq 0 \tag{14}$$

The pair $(c_{cart}^1, c_{ncart}^1)$ will be a intuitive equilibrium when $(c_{cart}^1, c_{ncart}^1) = (x, y)$. From (13) we can set $x = \phi(y)$, where $\phi(y)$ is decreasing in y. As (13) is continuous it exists a $\bar{b}$ so that:

$$q(1 - F_{ncart}(\bar{b}))^{N-C}(c_0 - \bar{b}) - d = 0 \tag{15}$$

Note that $\bar{b} > \underline{c}$, because if not, $d = q(c_0 - \underline{c})$, which can not be true. Let h(y) be:

$$h(y) = q(1 - F_{ncart}(y))^{N-C-1}\left[(1 - F_{cart}(\phi(y)))^C(c_0 - \phi(y)) \right.$$
$$\left. + \int_y^{\phi(y)} (1 - F_{cart}(c))^C \mathrm{d}c\right] - d$$

Because $\phi(y)$ and the cumulative distributions are continuous in y, so it will be h(.). Taking h(y) in $\bar{b}$ and $\underline{c}$, respectively we have:

$$h(\bar{b}) = q(1 - F_{ncart}(\bar{b}))^{N-C-1}(1 - F_{cart}(\bar{b}))^C(c_0 - \bar{b}) - d = \frac{(1 - F_{cart}(\bar{b}))^C}{(1 - F_{ncart}(\bar{b}))}d - d < 0,$$

where the last inequality comes from the stochastic dominance assumption.

$$h(\underline{c}) = q\left[(1-F_{cart}(\phi(\underline{c})))^C(c_0-\phi(\underline{c})) + \int_{\underline{c}}^{\phi(\underline{c})}(1-F_{cart}(c))^C\mathrm{d}c\right] - d$$

If $h(\underline{c}) > 0$, than it exists a $c^1_{ncart} \in (\underline{c}, \bar{b})$ so that $h(c^1_{cart}) = 0$. That means $(c^1_{cart}, c^1_{ncart})$, where $c^1_{ncart} = y < \phi(y) = c^1_{cart}$, is a intuitive equilibrium. If $h(\underline{c}) \leq 0$, than it will exist a equilibrium where $c^1_{ncart} = \underline{c} < \phi(\underline{c}) = c^1_{cart}$. ■

Proposition 4 *If $F_{ncart}(.)$ is inelastic in all c, there is no cutoff equilibrium where $c^1_{cart} < c^1_{ncart}$.*

Proof. By contradiction assume instead this equilibrium exist. Write the cutoff equilibrium conditions for both types of bidders as:

$$q(1-F_{cart}(c^1_{cart}))^C(1-F_{ncart}(c^1_{ncart}))^{N-C-1}(c_0-c^1_{ncart}) - d = 0$$

$$q\left[(1-F_{ncart}(c^1_{ncart}))^{N-C}(c_0-c^1_{ncart}) + \int_{c^1_{cart}}^{c^1_{ncart}}(1-F_{ncart}(c))^{N-C}\mathrm{d}c\right] - d \leq 0$$

Using the inequality condition we can write:

$$(1-F_{cart}(c^1_{cart}))^C(1-F_{ncart}(c^1_{ncart}))^{N-C-1}(c_0-c^1_{ncart}) \geq$$
$$(1-F_{ncart}(c^1_{ncart}))^{N-C}(c_0-c^1_{ncart}) + \int_{c^1_{cart}}^{c^1_{ncart}}(1-F_{ncart}(c))^{N-C}\mathrm{d}c$$

$$(1-F_{cart}(c^1_{cart}))^C(1-F_{ncart}(c^1_{ncart}))^{N-C-1}(c_0-c^1_{ncart}) > (1-F_{cart}(c^1_{cart}))^{C-1}$$
$$\left[(1-F_{ncart}(c^1_{ncart}))^{N-C}(c_0-c^1_{ncart}) + \int_{c^1_{cart}}^{c^1_{ncart}}(1-F_{ncart}(c))^{N-C}\mathrm{d}c\right]$$
$$(1-F_{ncart}(c^1_{ncart}))^{N-C-1}(c_0-c^1_{ncart})[(1-F_{cart}(c^1_{cart}))-(1-F_{ncart}(c^1_{ncart}))] >$$
$$(c^1_{ncart}-c^1_{cart})(1-F_{ncart}(c^1_{ncart}))^{N-C}$$
$$\frac{(1-F_{cart}(c^1_{cart}))}{c_0-c^1_{cart}} > \frac{(1-F_{ncart}(c^1_{ncart}))}{c_0-c^1_{ncart}},$$

where the third inequality comes from the fact that $F_{ncart}(.)$ is strictly increasing in c.

But the last line is a contradiction. From inelasticity assumption $\frac{1-F_{ncart}(c^1_{ncart})}{c_0-c^1_{ncart}} > \frac{1-F_{ncart}(c^1_{cart})}{c_0-c^1_{cart}} > \frac{1-F_{cart}(c^1_{cart})}{c_0-c^1_{cart}}$. ■

What rests to show now are the sufficient conditions that guarantees the intuitive equilibrium is the unique equilibrium.

Proposition 5 *If both $F_{ncart}(.)$ and $F_{cart}(.)$ are inelastic and the reserve price not so large, than it exist a unique equilibrium and it is a intuitive one.*

Proof. Assume there is a $y \in (\underline{c}, \bar{b})$, so that (14) is satisfied with equality. Substituting (13) in (14) and dividing both sides by $q(1 - F_{ncart}(y))^{N-C-1}$ we have:

$$(1 - F_{cart}(x))^C (c_0 - x) + \int_y^x (1 - F_{cart}(c))^C \mathrm{d}c - (1 - F_{ncart}(y))(c_0 - x) = 0 \tag{16}$$

Taking the derivative of x:

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{(1 - F_{cart}(y))^C - f_{ncart}(y)(c_0 - x)}{-C(1 - F_{cart}(x))^{C-1} f_{cart}(x)(c_0 - x) + (1 - F_{ncart}(y))}$$

We need to find when $\frac{\mathrm{d}x}{\mathrm{d}y} > 0$, so that $x = \phi(y)$ only one time, or it will never be equal, which happens when $c^1_{ncart} = \underline{c}$ and $c^1_{cart} = \phi(\underline{c})$. Analyzing first the numerator we can write:

$$(1 - F_{cart}(y))^C - f_{ncart}(y)(c_0 - x) > (1 - F_{cart}(y))^C - \frac{(1 - F_{ncart}(y))}{y}(c_0 - x)$$

The inequality comes from the fact that $F_{ncart}(.)$ is inelastic. The sign of the righthand side from the expression above will be the same as the sign of:

$$\begin{aligned}
& y(1 - F_{cart}(y))^C - (1 - F_{ncart}(y))(c_0 - x) \\
&= y(1 - F_{cart}(y))^C - (1 - F_{cart}(x))^C (c_0 - x) - \int_y^x (1 - F_{cart}(c))^C \mathrm{d}c \\
&> y(1 - F_{cart}(y))^C - (1 - F_{cart}(y))^C (c_0 - x) - (x - y)(1 - F_{cart}(y))^C \\
&= (1 - F_{cart}(y))^C [2y - c_0],
\end{aligned}$$

where the first equality comes from (16) and the inequality from the fact that $F_{cart}(.)$ is strictly increasing in c.

It is clear that if $c_0 < 2y$ the numerator will be positive. Now it rests to find the condition that guarantees the denominator is positive too. First note that:

$$\begin{aligned}
&(1 - F_{ncart}(y)) - C(1 - F_{cart}(x))^{C-1} f_{cart}(x)(c_0 - x) > \\
&(1 - F_{ncart}(y)) - C\frac{(1 - F_{cart}(x))^C}{x}(c_0 - x) > \\
&(1 - F_{cart}(x))^C - C\frac{(1 - F_{cart}(x))^C}{x}(c_0 - x)
\end{aligned}$$

From $(1 - F_{ncart}(y)) > (1 - F_{cart}(y)) > (1 - F_{cart}(x))$ and $F_{cart}(.)$ inelasticity. The expression above is positive when:

$$x - C(c_0 - x) > 0$$

This will happen when $c_0 < \frac{C+1}{C}x$. The upper bound condition for the reserve price depends on the relative difference between the two cutoff equilibrium points:

$$\frac{C+1}{C}x < 2y \implies \frac{x}{y} < \frac{2C}{C+1}$$

■

Competitive model(M2)

Here we present the equilibrium proof for the competitive model. We first characterize in Lemma 3, indifferent bidder's net expected profit. Later we divide the proof of Proposition 2 in three others propositions: (i) Intuitive equilibrium existence, (ii) Non existence of the non intuitive equilibrium and (iii) Intuitive equilibrium uniqueness.

Lemma 3 *Let $c^2_\tau \in [\underline{c}, \bar{c}]$ be the cutoff point of type τ bidder. Without loss of generality, assume $c_0 > c^2_{cart} \geq c^2_{ncart} \geq \underline{c}$. The net expected payoff of the indifferent cartel and non-cartel bidders are, respectively:*

$$q(1 - F_{cart}(c^2_{cart}))^{C-1}(1 - F_{ncart}(c^2_{ncart}))^{N-C}(c_0 - c^2_{cart}) - d \tag{17}$$

$$q(1 - F_{ncart}(c^2_{ncart}))^{N-C-1}\left[(1 - F_{cart}(c^2_{cart}))^{C}(c_0 - c^2_{cart}) + \int_{c^2_{ncart}}^{c^2_{cart}} (1 - F_{cart}(c))^{C} dc\right] - d \tag{18}$$

Proof. *Lemma 3*

The proof for the indifferent cartel bidder is straightforward. This player only wins if no other bidder enters. Any other cartel bidder that decides to enter would be more efficient than the indifferent cartel player. By assumption non-cartel cutoff point is lower than cartel. If any of the former bidder participates, it will necessarily has a lower private cost than the indifferent cartel bidder. If the indifferent cartel bidder plays alone, he bids the reserve price. That means the net expected profit of the indifferent cartel bidder is equal to the probability of this bidder playing alone, times the payoff when this event happens.

Notice that in this case, the representation of the net expected payoff of the indifferent non-cartel bidder will be the same as in the model M1. This player continues to win the auction only if: (i) he plays alone, (ii) or the most efficient actual cartel bidder has a higher private cost. The rest of the proof follows Lemma2.[66] ■

Proposition 6 *Let c^2_{cart} and c^2_{cart} be cartel and non-cartel cutoff strategies, respectively. Because $F_{ncart}(.)$ (first order) stochastic dominates $F_{cart}(.)$, it will always exist a intuitive equilibrium $c^2_{ncart} < c^2_{cart}$ that satisfies:*

[66]That does not mean the equilibrium cutoff points of both models will be the same. Probably they will not, as the indifferent cartel bidder now is not the most efficient of that type.

$$q(1-F_{cart}(c^2_{cart}))^{C-1}(1-F_{ncart}(c^2_{ncart}))^{N-C}(c_0-c^2_{cart})-d=0 \tag{19}$$

$$q(1-F_{ncart}(c^2_{ncart}))^{N-C-1}\left[(1-F_{cart}(c^2_{cart}))^{C}(c_0-c^2_{cart}) + \int_{c^2_{ncart}}^{c^2_{cart}}(1-F_{cart}(c))^{C}\mathrm{d}c\right]-d\leq 0, \tag{20}$$

where (20) is satisfied with equality when $c^2_{ncart} > \underline{c}$.

Proof. Proving that the cutoff strategy is an equilibrium follows the same intuition from Proposition 1. Now set the pair (x,y) as an equilibrium that satisfies (19) and (20). It is possible to write:

$$q(1-F_{cart}(x))^{C-1}(1-F_{ncart}(y))^{N-C}(c_0-x)-d=0 \tag{21}$$

$$q(1-F_{ncart}(y))^{N-C-1}\left[(1-F_{cart}(x))^{C}(c_0-x)+\int_{y}^{x}(1-F_{cart}(c))^{C}\mathrm{d}c\right]-d\leq 0 \tag{22}$$

Where (22) is satisfied with equality when $y > \underline{c}$. That means the pair $(c^2_{cart}, c^2_{ncart})$ will be an equilibrium, with $c^2_{ncart} < c^2_{cart}$, when $(c^2_{cart}, c^2_{ncart}) = (x, y)$. From (7.13), set $x = \phi(y)$. Note that $\phi(y)$ is continuously decreasing in y. Because (21) is continuous, it exists a $\bar{b}$ that:

$$q(1-F_{cart}(\bar{b}))^{C-1}(1-F_{ncart}(\bar{b}))^{N-C}(c_0-\bar{b})-d=0 \tag{23}$$

Note that $\bar{b} > \underline{c}$, because if not, $d = q(c_0 - \underline{c})$ which can not be true by definition. Let h(y) as:

$$
\begin{aligned}
h(y) = & q(1-F_{ncart}(y))^{N-C-1}\Bigg[(1-F_{cart}(\phi(y)))^{C}(c_0-\phi(y)) \\
& + \int_{y}^{\phi(y)}(1-F_{cart}(c))^{C}\mathrm{d}c\Bigg]-d
\end{aligned} \tag{24}
$$

Because $\phi(.)$ and the cumulative distributions are continuous in y, so it will be h(.). Analyzing h(.) in $\bar{b}$ and $\underline{c}$, respectively, we have:

$$
h(\bar{b}) = q(1-F_{ncart}(\bar{b}))^{N-C-1}(1-F_{cart}(\bar{b}))^{C}(c_0-\bar{b})-d = \frac{(1-F_{cart}(\bar{b}))}{(1-F_{ncart}(\bar{b}))}d-d<0,
$$

where the inequality comes from the first order stochastic dominance assumption.

$$
h(\underline{c}) = q\left[(1-F_{cart}(\phi(\underline{c})))^{C}(c_0-\phi(\underline{c}))+\int_{\underline{c}}^{\phi(\underline{c})}(1-F_{cart}(c))^{C}\mathrm{d}c\right]-d
$$

Now we just need to analyze $h(\underline{c})$ signal. If $h(\underline{c})>0$, than it exist a $c^2_{ncart}\in(\underline{c},\bar{b})$ so that $h(c^2_{ncart})=0$. That means there is a cutoff equilibrium (c^2_{cart},c^2_{ncart}) where $c^2_{ncart}=y<\phi(y)=c^2_{cart}$. If $h(\underline{c})\leq 0$, than it will exist a equilibrium where $c^2_{ncart}=\underline{c}<\phi(\underline{c})=c^2_{cart}$. ■

Proposition 7 *If $F_{ncart}(.)$ is inelastic in all c, then there is no equilibrium $c^2_{cart}<c^2_{ncart}$.*

Proof. Assume instead that this equilibrium exist. We can write this equilibrium conditions as:

$$
q(1-F_{cart}(c^2_{cart}))^{C}(1-F_{ncart}(c^2_{ncart}))^{N-C-1}(c_0-c^2_{ncart})-d=0 \tag{25}
$$

$$
\begin{aligned}
& q(1-F_{cart}(c^2_{cart}))^{C-1}\Bigg[(1-F_{ncart}(c^2_{ncart}))^{N-C}(c_0-c^2_{ncart})+ \\
& + \int_{c^2_{cart}}^{c^2_{ncart}}(1-F_{ncart}(c))^{N-C}\mathrm{d}c\Bigg]-d\leq 0
\end{aligned} \tag{26}
$$

Using the inequalities we can write:

$$(1-F_{cart}(c^2_{cart}))(1-F_{ncart}(c^2_{ncart}))^{N-C-1}(c_0-c^2_{ncart})-(1-F_{ncart}(c^2_{ncart}))^{N-C}$$

$$(c_0-c^2_{ncart}) \geq \int\limits_{c^2_{cart}}^{c^2_{ncart}} (1-F_{ncart}(c))^{N-C}\mathrm{d}c$$

$$(c_0-c^2_{ncart})(1-F_{ncart}(c^2_{ncart}))^{N-C-1}[(1-F_{cart}(c^2_{cart}))-(1-F_{ncart}(c^2_{ncart}))] >$$

$$(c^2_{ncart}-c^2_{cart})(1-F_{ncart}(c^2_{ncart}))^{N-C}$$

$$\frac{(1-F_{cart}(c^2_{cart}))}{c_0-c^2_{cart}} > \frac{(1-F_{ncart}(b))}{c_0-c^2_{ncart}},$$

where the second inequality comes from the fact that $F_{ncart}(.)$ is strictly increasing in $c \in [\underline{c}, \bar{c}]$.

But the last line is a contradiction, because from the inelasticity assumption $\frac{1-F_{ncart}(c^2_{ncart})}{c_0-c^2_{ncart}} > \frac{1-F_{ncart}(c^2_{cart})}{c_0-c^2_{cart}} > \frac{1-F_{cart}(c^2_{cart})}{c_0-c^2_{cart}}$. ■

Now it rests to show that if both $F_{cart}(.)$ and $F_{ncart}(.)$ are inelastic and the reserve price not so large, than exist only a unique equilibrium and it will be the intuitive one.

Proposition 8 *If both $F_{cart}(.)$ and $F_{ncart}(.)$ are inelastic and $c_0 < 2c^2_{ncart}$, than it exist only the intuitive equilibrium and it is unique.*

Proof. Assume there is a $y \in (\underline{c}, \bar{b})$. So (22) will be satisfied with equality. Substituting (22) in (23) and dividing both sides by $q(1-F_{ncart}(y))^{N-C-1}$ we have:

$$(1-F_{cart}(x))^C(c_0-x)+\int\limits_y^x (1-F_{cart}(c))^C\mathrm{d}c$$
$$-(1-F_{cart}(x))^{C-1}(1-F_{ncart}(y))(c_0-x)=0 \tag{27}$$

From equation above, let x be a implicit function of y. Taking the derivative of x:

We need to show that $\frac{\mathrm{d}x}{\mathrm{d}y} > 0$, so that $x=\phi(y)$ only once, or it will never be equal, which is the case when $c^2_{ncart}=\underline{c}$ and $c^2_{cart}=\phi(\underline{c})$.

Focus first in the numerator. Note that:

$$(1 - F_{cart}(y))^C - (1 - F_{cart}(x))^{C-1} f_{ncart}(y)(c_0 - x) >$$
$$(1 - F_{cart}(y))^C - (1 - F_{cart}(x))^{C-1} \frac{(1 - F_{ncart}(y))}{y}(c_0 - x)$$

The inequality comes from the inelasticity of $F_{ncart}(.)$. Looking only to the righthand side of the former inequality, its sign has to be the same as the sign of:

$$\begin{aligned}
& y(1 - F_{cart}(y))^C - (1 - F_{cart}(x))^{C-1}(1 - F_{ncart}(y))(c_0 - x) \\
& = y(1 - F_{cart}(y))^C - (1 - F_{cart}(x))^C(c_0 - x) - \int_y^x (1 - F_{cart}(c))^C \mathrm{d}c \\
& > y(1 - F_{cart}(y))^C - (1 - F_{cart}(y))^C(c_0 - x) - (x - y)(1 - F_{cart}(y))^C \\
& = (1 - F_{cart}(y))^C[2y - c_0],
\end{aligned}$$

where the first equality comes from (27) and the inequality from the fact that $F_{cart}(.)$ is strictly increasing in c.

It is clear that if $c_0 < 2y$, the numerator will be positive. It rests to find the condition that guarantees the denominator will be positive too. First note that:

$$\begin{aligned}
& C(1 - F_{ncart}(y))(1 - F_{cart}(x))^{C-2} f_{cart}(x)(c_0 - x) \\
& - C(1 - F_{cart}(x))^{C-1} f_{cart}(x)(c_0 - x) > 0,
\end{aligned}$$

where the inequality comes from $(1 - F_{ncart}(y)) > (1 - F_{ncart}(x)) > (1 - F_{cart}(x))$.

So what remains to show is when:

$$(1 - F_{cart}(x)) - f_{cart}(x)(c_0 - x) > 0$$

That happens when $c_0 < x + \frac{(1-F_{cart}(x))}{f_{cart}(x)}$. Note that $2y < x + \frac{(1-F_{cart}(x))}{f_{cart}(x)}$, because $x > y$ and $F_{cart}(.)$ is inelastic. Than $\frac{(1-F_{cart}(x))}{f_{cart}(x)} > x > y$. ■

Collusion: Asymmetric information (M3)

This last model assumes that non-cartel bidders do not know about the collusion, i.e., they have the belief they are in a competitive environment. That means their cutoff points will be the same as in the competitive model. The representative cartel bidder reacts assuming non-cartel's will play like that. Next proposition set the conditions for the equilibrium.

Proposition 9 *Assume c^2_{cart} is cartel's equilibrium cutoff point in the competitive model. A pair $(c^3_{cart}, c^3_{ncart})$ is a cutoff equilibrium, with $c^3_{cart} > c^3_{ncart} = c^2_{ncart}$, iff it satisfy both:*

$$q(1 - F_{ncart}(c^2_{ncart}))^{N-C}(c_0 - c^3_{cart}) - d = 0 \tag{28}$$

$$q(1 - F_{ncart}(c^2_{ncart}))^{N-C-1}\left[(1 - F_{cart}(c^2_{cart}))^C(c_0 - c^2_{cart}) + \int_{c^2_{ncart}}^{c^2_{cart}} (1 - F_{cart}(c))^C dc\right] - d \leq 0, \tag{29}$$

where (29) is satisfied with equality when $c^2_{ncart} > \underline{c}$.

Proof. In this environment the proof that all potential bidders will play the cutoff strategy goes the same as in the other models. Now note that in this model we have two distinct situations.

First, non-cartel bidders believe they are in a competitive environment and that cartel bidders reacts accordingly to it. To calculate its equilibrium cutoff point, non-cartel bidders solve the competitive model. Second, the representative cartel bidder knows non-cartel bidders beliefs and reacts optimally to it. Because the former wins only when none other bidder enter, his net expected profit can be written like in (1). The only difference is that now he will react to c^2_{ncart} and not to c^1_{ncart}. ■

Because the equilibrium cutoff of non-cartel bidders comes from the competitive model and (28) is continuous $\forall c \in [\underline{c}, \bar{c}]$, it is clear there will exist a unique equilibrium in this model, where it is the intuitive equilibrium, if there is only one equilibrium, a intuitive one, in the competitive model.

Proposition 10 *The intuitive equilibrium $(c^3_{cart}, c^3_{ncart})$ exists and is unique if the intuitive equilibrium $(c^2_{cart}, c^2_{ncart})$ exist and is unique.*

Proof. Assume the unique intuitive equilibrium $(c^2_{cart}, c^2_{ncart})$ exists. The net expected profit of the indifferent representative bidder can be written as:

$$q(1 - F_{ncart}(c^2_{ncart}))^{N-C}(c_0 - x)$$

The indifferent representative cartel bidder will find the x that makes the expression above equal to zero. As already discussed, this cutoff point will be equal to its private cost. Note that:

$$q(1 - F_{ncart}(c^2_{ncart}))^{N-C}(c_0 - c^2_{cart}) >$$
$$q(1 - F_{cart}(c^2_{cart}))^{C-1}(1 - F_{ncart}(c^2_{ncart}))^{N-C}(c_0 - c^2_{cart}) = 0$$

Because the expected net profit of the indifferent representative cartel bidder is continuously decreasing in its private cost, $x = c^3_{cart} > c^2_{cart} > c^2_{ncart}$. ■

Collusion outcomes

After equilibria conditions are set, it is possible to study how, jointly, the collusive behavior, the first order stochastic and the informational set of non-cartel bidders assumptions can generate inefficiencies in this market. We do that by comparing the three models cutoff equilibriums. Note that this comparison is possible analytically only when all models have a unique intuitive equilibrium. Otherwise this question can only be answered empirically. As a consequence, in this subsection we assume that all models have a unique intuitive equilibrium. The next theorem presents the outcomes.

Theorem 1 *Assume all models have a unique intuitive equilibrium. Let* $\{(c^i_{cart}, c^i_{ncart})\}^3_{i=1}$ *be the set of intuitive equilibrium of each model.*

1. *Equilibrium cutoff points:*
   (a) $c^3_{ncart} = c^2_{ncart}$ *and the relative difference sign between* c^3_{ncart} *and* c^1_{ncart} *is ambigous*
   (b) $c^2_{cart} < c^1_{cart}$ *and* $c^2_{cart} < c^3_{cart}$
   (c) *The relative difference sign between* c^1_{cart} *and* c^3_{cart} *is ambigous.*
2. *Collusion inefficiencies:*
   (a) *The collusive behavior in model M3 can only generate higher expected prices.*
   (b) *The collusive behavior in model M1 has the potential to generate higher or lower expected prices and alocative inefficiency.*

Proof. We already showed in Proposition 10 the proof for the comparison between model's M2 and M3 cutoff points. So we omit this part of the proof here.

Proving that $\boldsymbol{c^2_{cart} < c^1_{cart}}$ and that the sign of the relative difference between c^1_{ncart} and c^2_{ncart} is ambiguous

We know that if indifferent representative cartel and non-cartel bidders play the cutoff pair $(c^2_{cart}, c^2_{ncart})$, respectively, the net expected profit of those players are positive and equal to zero, respectively. Because a bidder profit is decreasing in it's private cost, the indifferent representative cartel bidder plays a higher cutoff point.

We show next that the indifferent non-cartel bidder can have both the incentive to decrease or increase his cutoff point if the representative cartel bidder increase it's own. Set $(x, \phi(x))$ a pair of cutoff points for the representative cartel and non-cartel bidders, respectively, and that satisfy (8) with equality. Using the theorem of implicit function in (8)

$$\frac{\mathrm{d}\phi(x)}{\mathrm{d}x} = \frac{-C(1-F_{ncart}(\phi(x)))(1-F_{cart}(x))^{C-1}(c_0-x)}{(N-C-1)f_{ncart}(\phi(x))A + (1-F_{ncart}(\phi(x)))(1-F_{cart}(\phi(x)))^C} < 0, \tag{30}$$

where $A = (1-F_{cart}(x))^C(c_0-x) + \int\limits_{\phi(x)}^{x}(1-F_{cart}(c))^C\mathrm{d}c$.

Now assume that (8) is satisfied with equality. Replacing it in (7), we have:

$$\begin{aligned} F =& q(1-F_{cart}(x)))^{C-1}(1-F_{ncart}(y))^{N-C}(c_0-x) \\ & - q(1-F_{ncart}(y))^{N-C-1}\Bigg[(1-F_{cart}(x))^C(c_0-x) \\ & + \int_y^x (1-F_{cart}(c))^C dc\Bigg] = 0 \end{aligned}$$

Using the implicit theorem to find the derivative of y in x:

It is not clear analytically which effect will be higher.

Proving the relation between $\boldsymbol{c^1_{cart}}$ and $\boldsymbol{c^3_{cart}}$ is ambigous.

We need to analyze the sign of the second derivative of $\phi(y)$, where $\phi(y)$=x from (8.6). From the same equation we know that $\phi(y)$ is decreasing in y. We also know that c^3_{ncart} will not react to

c^3_{cart}, differently than c^1_{ncart}, that reacts to c^1_{cart}. This means that if $\frac{\mathrm{d}^2\phi(y)}{\mathrm{d}y^2} > 0$, than $c^3_{cart} < c^1_{cart}$. If the sign of the former is negative, than $c^3_{cart} > c^1_{cart}$. Writing first the derivative of $\phi(y)$:

$$\frac{\mathrm{d}\phi(y)}{\mathrm{d}y} = \frac{-(N-C)f_{ncart}(y)(c_0 - \phi(y))}{(1 - F_{ncart}(y))} < 0$$

Writing the second derivative, the sign will be settled by the numerator:

$$\begin{aligned} T = &- (N-C)\left[(1 - F_{ncart}(y))\left[f'_{ncart}(y)(c_0 - \phi(y)) - \phi'(y)f_{ncart}(y)\right]\right. \\ &\left.- f_{ncart}(y)^2(c_0 - \phi(y))\right] \end{aligned}$$

Note that the sign will depend not only on the magnitude, but also on the sign of $f'_{ncart}(.)$.

Proof of collusion inefficiencies.

The proof of this part is in section 6. ■

Figures

Figure 1: 10 most frequent bidders and winner bidders

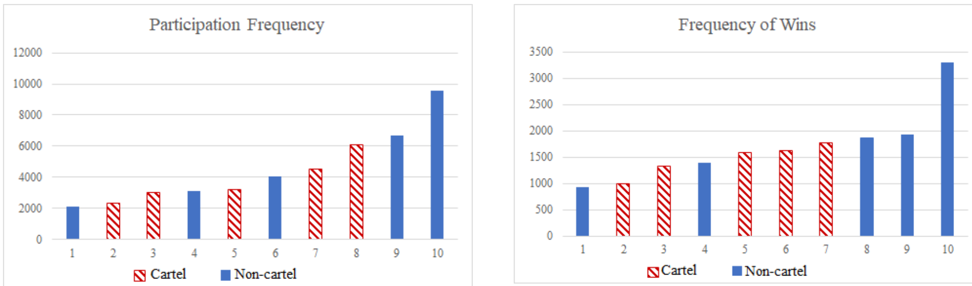


Figure 2: Relative difference between lowest bid and most efficient "rivals"- Pdf and Cdf (Cartel win)

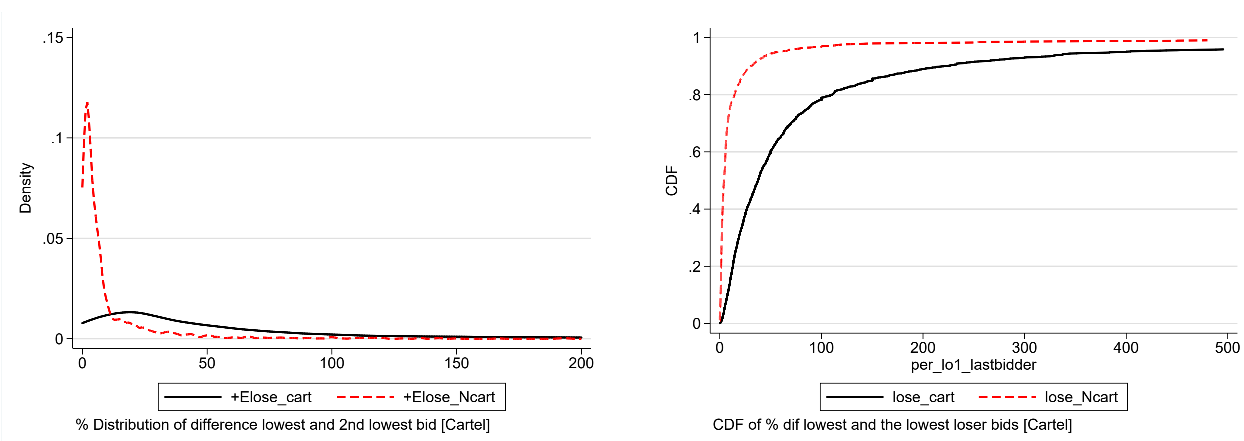


Figure 3: Relative difference between lowest bid and higher bids - Pdf and Cdf (Cartel win)

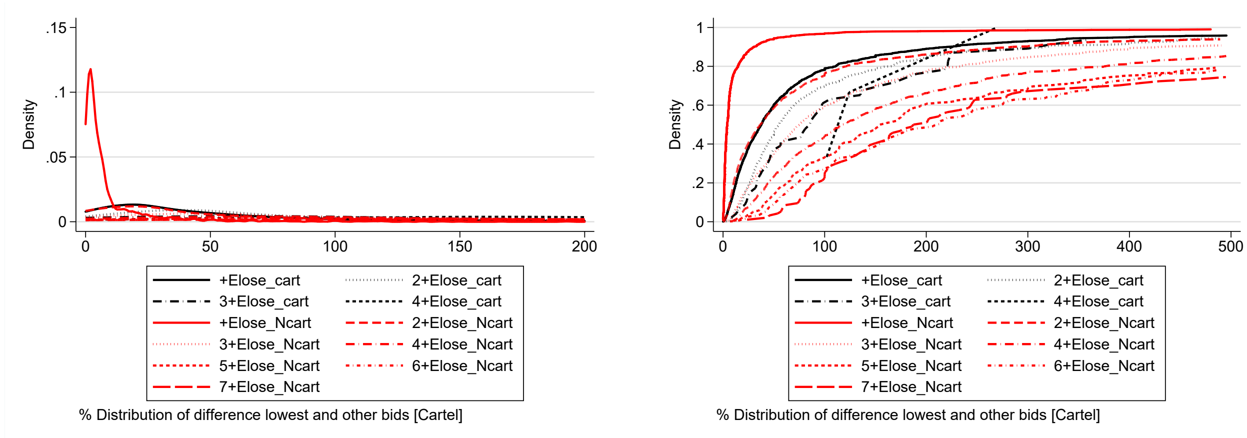


Figure 4: Relative difference between lowest bid and the two lowest cartel bid- Pdf and Cdf (Non-cartel win)

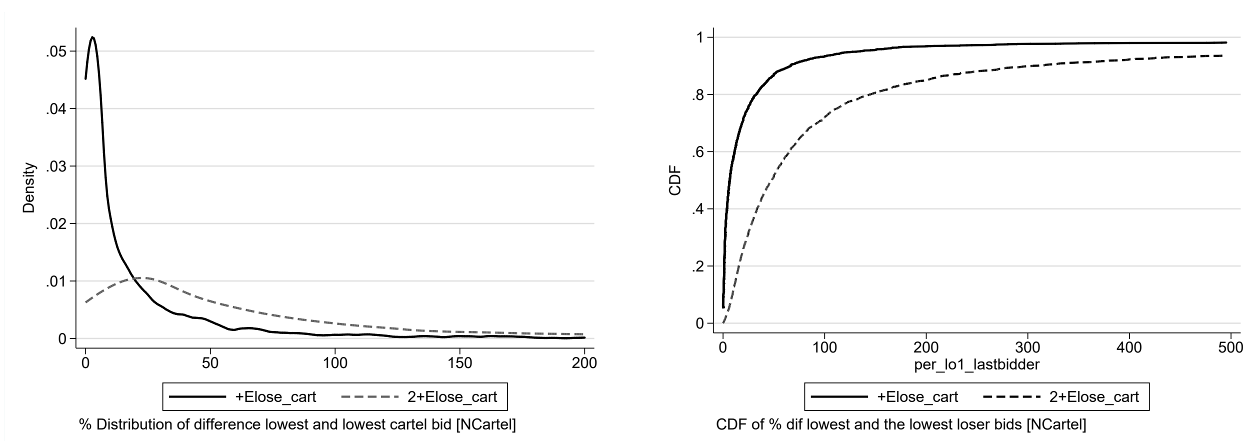
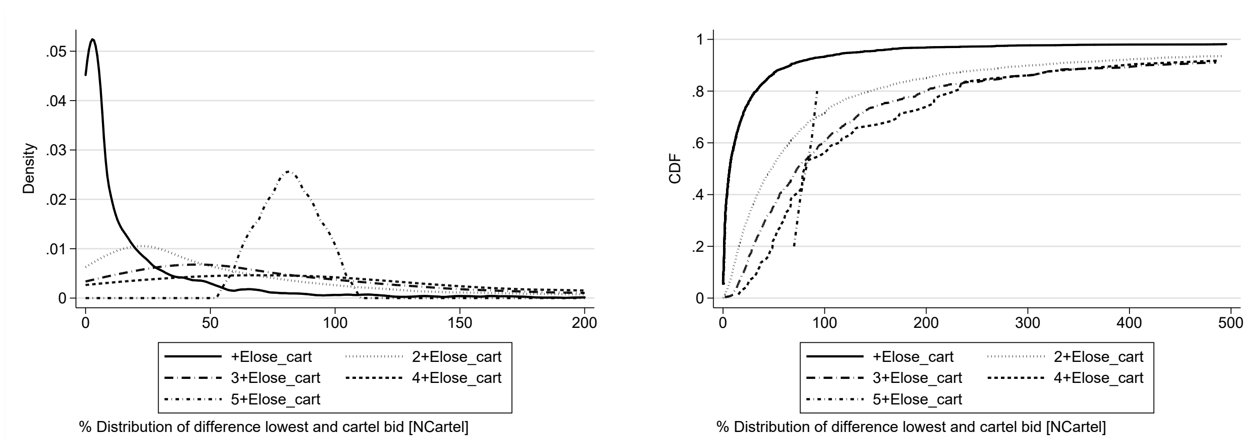


Figure 5: Relative difference between lowest bid and cartel bids - Pdf and Cdf (Non-cartel win)



Tables

Table 3: Cartel bidders and their participation in Sao Paulo's procurements

Bidder	Enter
Comercial Cirurgica Rioclarense Ltda.	X
Cristalia Produtos Químicos Farmaceuticos Ltda.	X
Dimaci Material Cirurgico Ltda.	X
Drogafonte Medicamentos e Material Hospitalar	X
Laboratorio Teuto Brasileiro S.A.	X
Mafra Hospitalar Ltda.	X
NovaFarma Industria Farmaceutica	X
Prodiet Farmaceutica Ltda.	X
Torrent do Brasil Ltda.	X
Macromed Comercio de Material Medico e Hospitalar Ltda.	
Merriam Farma Comércio de Produtos Farmacêuticos Ltda.	
Netfarma Comercial Ltda.	

Note. This table contains all firms prosecuted by Cade. The ones marked with an X participated in procurements for medicines in the state of Sao Paulo.

Table 4: Database general description

Variables	**Obs**
Auction	29,341
Bids	84,874
Public Bodies	78
Medicines	2,714
Cartel bidders	10
Non-cartel bidders	243

Note. This table contains the number of observation for each main info contained in the Potential database.

Table 5: Frequency of cartel members in the same auction

# Cartel members in the same auction	Freq.	%	% if cartel participated
0	13,946	47.53	-
1	10,859	37.00	70.53
2	3,340	11.38	21.69
3	1,043	3.55	6.77
4	144	0.49	0.93
5	9	0.00*	0.00*

Note. The first column of this Table contains the number of cartel members in the same auction. The second and third contains the frequency and the unconditional percentage of it, respectively. The last contains the percentage conditioned on cartel participation.

* Their values were inferior than 0.0001.

Table 6: Number of auctions and actual bidders (type), by year

	Cartel	Non-cartel	Total	Auctions
2008	1,142	5,323	6,465	2,039
2009	2,995	10,951	13,946	4,422
2010	4,107	14,285	18,392	6,470
2011	8,157	22,545	30,702	11,267
2012	3,537	11,832	15,369	5,143
Total	19,938	64,936	84,874	29,341

Note. The first three columns of this Table contains the number of times we observe a cartel member, a non-cartel bidder and the sum of both in the Potential database. The last column contains the distribution of auctions between 2008-2012.

Table 7: Cartel participation and rate of success

Variables	Obs	%
Panel A - Cartel Participation		
# auctions	29,341	100
# auction cartel participated	15,395	52.47
# auction cartel didn't participated	13,946	47.53
Panel B - Rate of Success		
# cartel winning	7,796	26.57
Rate of cartel success		50.63
# non-cartel winning	21,545	73.43
Rate of non-cartel success (if cartel participated)		49.37

Note. This Table contains info on cartel and non-cartel participation and rate of success in Potential database. Panel A focus on cartel participation, detailing the number and percentage of auctions it participated. Panel B presents cartel's and non-cartel's rate of success (winning) accounting only auctions where the former participated.

Table 8: Variables description

Variables	Description
Dependent variables	
$d\_entrant_{ia}$	Dummy equal to 1 when potential bidder i enters in auction a and 0 otherwise.
d_win_{ia}	Dummy equal to 1 when bidder i wins auction a and 0 otherwise.
Independent variables	
Continous variables	
$Quant_a$	Quantity of medicines procured in a.
# Previous $enter_i$	Number of previous auction bidder i entered
# Previous win_i	Number of previous auction bidder i won.
$HContract^*_{it}$	Highest contract value bidder i won in the 12 months before t.
$Dist\_bidder\_buyer^*_{ia}$	Distance between bidder i city and auction's a buyer city.
$MindistC^*_{ia}$	Distance of the closest potential rival (cartel) to auction's a buyer.
$MindistNC^*_{ia}$	Distance of the closest potential rival (non-cartel) to auction's a buyer.
$Mindist\_EntraC^*_{ia}$	Distance of the closest actual rival (cartel) to auction's a buyer.
$Mindist\_EntraNC^*_{ia}$	Distance of the closest actual rival (non-cartel) to auction's a buyer.
$MeandistC\_comp^*_{ia}$	Average distance between auction's a buyer and potential rivals (cartel).
$MeandistNC\_comp^*_{ia}$	Average distance between auction's a buyer and potential rivals (non-cartel).
$MeandistC\_Entracomp^*_{ia}$	Average distance between auction's a buyer and actual rivals (cartel).
$MeandistNC\_Entracomp^*_{ia}$	Average distance between auction's a buyer and actual rivals (non-cartel).
# $Ccompetitors_a$	Auction's a number of actual cartel rivals.
# $NCcompetitors_a$	Auction's a number of actual non-cartel rivals.
Bid^*_{ia}	Bidder's i last bid in auction a.

Variables	Description
Indicator variable	
Closestpot_{ia}	1 if potential bidder i is the closest to auction's a buyer and 0 otherswise.
Closest2pot_{it}	1 if potential bidder i is the second closest to auction's a buyer and 0 otherwise.
Closest_{ia}	1 if actual bidder i is the closest to auction's a buyer and 0 otherswise.
Closest2_{it}	1 if actual bidder i is the second closest to auction's a buyer and 0 otherwise.

Note. This Table describes regressions variables. We present first, dependent variables description and later we do the same for independent variables. This latter group we divide in two types: continuous and indicator variables.

*Bid and contract values are in 2008 R$ and all distances are in kilometers.

Table 9: Variables summary statistics-Potential database

Variables	Obs	Mean	Std.Dev	Min	Max
Auction characteristic					
Quant	502,449	86,866.55	1,841,124	0	1.14e+08
Bidder characteristic					
d_entrants					
Cartel	73,442	0.289	0.453	0	1
Non-cartel	429,007	0.163	0.370	0	1
# Previous enter					
Cartel	73,442	4.328	6.184	0	66
Non-cartel	429,007	2.207	3.810	0	66
# Previous win					
Cartel	73,442	1.514	3.014	0	49
Non-cartel	429,007	0.602	1.647	0	40
HContract					
Cartel	73,442	1.34e+07	2.58e+08	0	1.50e+10
Non-cartel	429,007	2.38e+07	2.02e+09	0	3.76e+11
Dist_bidder_buyer					
Cartel	73,442	239.110	181.125	0	889.421
Non-cartel	429,007	350.038	382.227	0	2,515.189
Closestpot					
Cartel	73,442	0.064	0.245	0	1
Non-cartel	429,007	0.057	0.232	0	1
Closestpot2					
Cartel	73,442	0.086	0.281	0	1
Non-cartel	429,007	0.053	0.225	0	1

Variables	Obs	Mean	Std.Dev	Min	Max
Mindist_C					
Cartel	64,589	134.919	142.838	0	889.421
Non-cartel	429,007	117.658	137.337	0	847.172
Mindist_NC					
Cartel	73,442	52.421	99.598	0	2,468.978
Non-cartel	428,518	45.322	84.643	0	2,468.978
MeandistC_comp					
Cartel	64,589	235.883	127.547	0	889.421
Non-cartel	429,007	238.673	117.234	0	847.172
MeandistNC_comp					
Cartel	73,442	353.073	149.349	0	2,468.978
Non-cartel	428,518	349.953	128.279	0	2,468.978

Note. This Table presents the summary statistics of regressions variables observed in the Potential database.

Table 10: Variable's summary statistic-Actual base

Variables	Obs	Mean	Std.Dev	Min	Max
Auction characteristic					
Quant	84,874	101,082.6	1,747,962	0	1.14e+08
# Ccompetitors	84,874	3.680	2.364	0	16
# NCcompetitors	84,874	3.806	2.873	0	18
Bidder characteristic					
d_win					
Cartel	19,938	0.391	0.487	0	1
Non-cartel	64,936	0.331	0.470	0	1
Bid					
Cartel	19,938	1,260.779	36,390.52	0.002	3,537,080
Non-cartel	64,936	1,168.59	28,426.59	1.68e-4	5,778,829
# Previous enter					
Cartel	19,938	6.109	7.757	0	66
Non-cartel	64,936	4.212	6.033	0	66
# Previous win					
Cartel	19,938	2.267	3.867	0	49
Non-cartel	64,936	1.423	2.837	0	40
HContract					
Cartel	19,938	2.8e+07	3.1e+08	0	1.50e+10
Non-cartel	64,936	3.3e+07	2.0e+09	0	3.76e+11
Dist_bidder_buyer					
Cartel	19,938	240.279	156.861	0	889.421
Non-cartel	64,936	332.727	405.835	0	2,515.189

Variables	Obs	Mean	Std.Dev	Min	Max
Closest					
Cartel	19,938	0.293	0.455	0	1
Non-cartel	64,936	0.361	0.480	0	1
Closest2					
Cartel	19,938	0.257	0.437	0	1
Non-cartel	64,936	0.220	0.414	0	1
Mindist_EntraC					
Cartel	9,629	184.836	146.214	0	889.421
Non-cartel	39,041	191.243	150.942	0	847.172
Mindist_EntraNC					
Cartel	18,043	132.597	230.143	0	2,468.978
Non-cartel	54,839	145.723	246.503	0	2,468.978
MeandistC_Entracomp					
Cartel	9,529	225.334	135.089	8.881	888.421
Non-cartel	38,713	241.831	131	8.881	846.172
MeandistNC_Entracomp					
Cartel	17,823	319.422	255.804	6.027	2,467.978
Non-cartel	53,968	330.815	271.146	4.480	2,491.084

Note. This Table presents summary statistics of regressions variables observed in the Actual database.

Table 11: Cartel probability to enter - Linear model

	Model 1	Model 2	Model 3	Model 4
Auction characteristic				
Lnquant	0.047***	0.047***	0.048***	0.048***
	(0.001)	(0.001)	(0.002)	(0.002)
Bidder characteristic				
# Previous enter	0.007***	0.007***	0.008***	0.007***
	(9.6e-4)	(9.7e-4)	(9.7e-4)	(9.8e-4)
# Previous win	-0.009***	-0.009***	-0.006***	-0.006***
	(0.001)	(0.001)	(0.001)	(0.001)
LnHContract	0.021***	0.021***	0.019***	0.019***
	(8.5e-4)	(8.5e-4)	(8.2e-4)	(8.2e-4)
Lndist_bidder_buyer	0.058***	0.060***	0.056***	0.057***
	(0.001)	(0.002)	(0.002)	(0.002)
Closestpot		0.011		
		(0.008)		
Closestpot2		0.022***		
		(0.007)		
LnmindistC			0.004**	
			(0.002)	
LnmindistNC			0.003**	
			(0.001)	
LnmeandistC_comp				0.020***
				(0.004)
LnmeandistNC_comp				0.007
				(0.016)
Constant	-0.390***	-0.404***	-0.505***	-0.642***
	(0.083)	(0.084)	(0.089)	(0.136)
Observations	73,442	73,442	64,589	64,589
R-squared	0.192	0.192	0.198	0.199
Number of inputs	2,714	2,714	1,431	1,431
Bidder, Product Month/Year and City FE	Yes	Yes	Yes	Yes

Note. This Table contains linear regressions outcomes for cartel probability to enter. We estimate four different models, using different distance variables. We include bidder, product, time and city fixed-effects.

Table 12: Non-cartel probability to enter - Linear Model

	Model 1	Model 2	Model 3	Model 4
Auction characteristic				
Lnquant	0.023***	0.023***	0.023***	0.023***
	(6.4e-4)	(6.4e-4)	(6.4e-4)	(6.3e-4)
Bidder characteristic				
# Previous enter	0.011***	0.011***	0.011***	0.011***
	(6.2e-4)	(6.2e-4)	(6.2e-4)	(6.2e-4)
# Previous win	-0.010***	-0.010***	-0.010***	-0.010***
	(0.001)	(0.001)	(0.001)	(0.001)
LnHContract	0.024***	0.024***	0.024***	0.024***
	(5.2e-4)	(5.2e-4)	(5.2e-4)	(5.2e-4)
Lndist_bidder_buyer	-0.024***	-0.020***	-0.024***	-0.024***
	(7.8e-4)	(9.2e-4)	(7.8e-4)	(7.9e-4)
Closestpot		0.030***		
		(0.003)		
Closestpot2		0.014***		
		(0.003)		
LnmindistC			-7.0e-4	
			(8.3e-4)	
LnmindistNC			0.002***	
			(6.2e-4)	
LnmeandistC_comp				0.002
				(0.001)
LnmeandistNC_comp				-0.001
				(0.005)
Constant	-0.009	-0.037	-0.020	-0.011
	(0.060)	(0.060)	(0.060)	(0.069)
Observations	429,007	429,007	428,518	428,518
R-squared	0.172	0.173	0.173	0.173
Number of inputs	2,714	2,714	2,495	2,495
Bidder, Product Month/Year and City FE	Yes	Yes	Yes	Yes

Note. This Table contains linear regressions outcomes for non-cartel probability to enter. We estimate four different models, using different distance variables. We include bidder, product, time and city fixed-effects.

Table 13: Linear models predictions - Summary statistics

	Mean	S. Dev	Min	Max
Sample cartel 1	0.289	-	-	-
Sample cartel 2	0.278	-	-	-
Model 1	0.289	0.230	-0.778	1.372
Model 2	0.289	0.230	-0.758	1.371
Model 3	0.278	0.230	-0.830	1.320
Model 4	0.278	0.232	-0.829	1.340
Obs Cartel	73,442	73,442	64,589	64,589
Sample non-cartel 1	0.163	-	-	-
Sample non-cartel 2	0.163	-	-	-
Model 1	0.163	0.157	-0.397	1.218
Model 2	0.163	0.157	-0.398	1.227
Model 3	0.163	0.157	-0.396	1.215
Model 4	0.163	0.157	-0.398	1.217
Obs non-cartel	429,007	429,007	428,518	428,518

Note. This Table contains the sample entry probabilities and regressions predictions of them, for both types of bidders.

Table 14: Cartel probability to win - Linear model

	Model 1	Model 2	Model 3	Model 4
Auction characteristic				
# Ccompetitors	-0.045***	-0.030***	-0.010***	-0.013***
	(0.002)	(0.002)	(0.002)	(0.002)
Lnquant	-0.023***	-0.021***	-0.018***	-0.019***
	(0.002)	(0.002)	(0.003)	(0.003)
Bidder characteristic				
Lnbid	-0.007***	-0.007***	-0.006***	-0.006***
	(0.001)	(0.001)	(0.001)	(0.001)
# Previous enter	-0.007***	-0.006***	-0.008***	-0.007***
	(0.001)	(0.001)	(0.001)	(0.001)
# Previous win	-0.002	-0.002	0.006**	0.006**
	(0.003)	(0.003)	(0.002)	(0.002)
LnHContract	0.037***	0.036***	0.027***	0.027***
	(0.001)	(0.001)	(0.001)	(0.001)
Lndist_bidder_buyer	0.009**	0.038***	0.009	0.008
	(0.004)	(0.004)	(0.005)	(0.005)
Closest		0.210***		
		(0.012)		
Closest2		0.079***		
		(0.009)		
Lnmindist_EntraC			0.011***	
			(0.003)	
Lnmindist_EntraNC			0.007***	
			(0.002)	
LnmeandistC_Entracomp				0.021***
				(0.006)
LnmeandistNC_Entracomp				0.002
				(0.006)
Constant	1.051***	0.655***	1.162***	1.143***
	(0.204)	(0.195)	(0.102)	(0.114)
Observations	19,938	19,938	9,604	9,458
R-squared	0.335	0.353	0.317	0.318
Number of inputs	2,621	2,621	1,080	1,068
Bidder, Product Month/Year and City FE	Yes	Yes	Yes	Yes

Note. This Table contains linear regression outcomes for cartel probability to win. We estimate four different models, using different distance variables. We include bidder, product, time and city fixed-effects.

Table 15: Non-cartel probability to win - Linear model

	Model 1	Model 2	Model 3	Model 4
Auction characteristics				
# NCcompetitors	-0.043***	-0.029***	-0.015***	-0.017***
	(0.001)	(0.001)	(0.001)	(0.001)
Lnquant	-0.033***	-0.029***	-0.008***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)
Bidder characteristics				
Lnbid	-0.016***	-0.016***	-0.013***	-0.013***
	(8e-4)	(8e-4)	(9e-4)	(9e-4)
# Previous enter	-0.010***	-0.010***	-0.007***	-0.007***
	(8e-4)	(8e-4)	(9e-4)	(9e-4)
# Previous win	-0.006**	-0.005**	-0.007**	-0.007***
	(0.002)	(0.002)	(0.003)	(0.003)
LnHContract	0.052***	0.051***	0.043***	0.043***
	(9e-4)	(9e-4)	(0.001)	(0.001)
Lndist_bidder_buyer	-0.009***	0.018***	-0.005**	-0.007***
	(0.001)	(0.001)	(0.002)	(0.002)
Closest		0.213***		
		(0.006)		
Closest2		0.036***		
		(0.004)		
Lnmindist_EntraC			0.006***	
			(0.001)	
Lnmindist_EntraNC			0.004***	
			(0.001)	
LnmeandistC_Entracomp				0.012***
				(0.003)
LnmeandistNC_Entracomp				0.007***
				(0.002)
Constant	0.618***	0.269**	0.259***	0.242***
	(0.156)	(0.136)	(0.069)	(0.072)
Observations	64,936	64,936	35,849	35,237
R-squared	0.403	0.426	0.363	0.363
Number of inputs	2,689	2,689	2,032	2,018
Bidder, Product Month/Year and City FE	Yes	Yes	Yes	Yes

Note. This Table contains linear regression outcomes for non-cartel probability to win. We estimate four different models, using different distance variables. We include bidder, product, time and city fixed-effects.

Table 16: Fitt probability to win - Linear models

	Mean	S. Dev	Min	Max
Sample cartel 1	0.391	-	-	-
Sample cartel 2	0.193	-	-	-
Sample cartel 3	0.191	-	-	-
Model 1	0.391	0.312	-0.552	1.459
Model 2	0.391	0.318	-0.428	1.481
Model 3	0.193	0.236	-0.630	1.096
Model 4	0.191	0.235	-0.634	1.109
Obs cartel	19,938	19,938	9,604	9,458
Sample non-cartel 1	0.331	-	-	-
Sample non-cartel 2	0.180	-	-	-
Sample non-cartel 3	0.179	-	-	-
Model 1	0.331	0.339	-0.816	1.671
Model 2	0.331	0.345	-0.792	1.667
Model 3	0.180	0.242	-0.543	1.492
Model 4	0.179	0.241	-0.513	1.451
Obs non-cartel	64,936	64,936	35,849	35,237

Note. This Table contains sample conditional winning probabilities and regression predictions of them, for both types of bidders.

Table 17: Representative auction mean outcomes

Outcomes	Collusion	Competition
c^*_{cart}	706.61	602.15
c^*_{ncart}	585.41	567.02
# No Sale	0	0
# Cartel Bidders	0.99	1.84
# Non-Cartel Bidders	4.43	4.13
Alocative Inefficiency Ratio	1.2e-5	1.2e-5
Cartel Bids	247.78	304.07
Non-Cartel Bids	432.81	422.58
Public Payment	339.29	309.17
Overcharge	9.75	

Note. This Table contains the simulations mean outcomes for the representative auction in each environment. We did 1 million simulations. Column 1 and 2 represents the collusive and competitive environments, respectively. c^*_τ is the cutoff point of type τ bidder in a given environment. Bid and public payment values are in 2008 R$.