Regulation Rather than Firms' discretion: On the Optimality of Regulated Capacity in Competitive Markets

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January 24, 2020

Abstract

The liberalization policies in the eighties and in the nineties governed the transition to market competition in many industries. However, many of these industries —specifically those that require substantial infrastructure investment— recently experienced a new wave of regulatory reforms. These "re-regulatory" experiences posit the fundamental question of whether both deregulation and market competition —as these liberalization policies had aimed at achieving— are desirable. Using a two-stage framework, in which cost-heterogeneous firms invest in capacity and then compete to serve consumers, we show that the competitive market fails to achieve the efficient outcome. We propose a regulatory framework in which the regulator sets a mandatory level of capacity, and firms that invest in capacity receive a compensation payment. We show that this regulatory framework is able to restore market efficiency —achieves the first-best— and it also reduces market price volatility.

Keywords: Deregulation; Market competition; Regulated capacity; Demand uncertainty.

JEL Classification Numbers: D41, D80, L5, L11.

*I would like to thank Mirko Abbritti, Mark Armstrong, José Azar, Martin Cripps, Peter Hartley, Bard Harstad, Markus Kinateder, Gilat Levy, Shiva Shekhar, Robert Somogyi, Xun Tang, and the participants in many seminars for their helpful comments. I am also grateful to the Ramón Areces Foundation which has provided support for this research (grant number CISP16A4779). All views and any errors in this article are my own.

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"Whenever competition is feasible it is, for all its imperfections, superior to regulation as a means of serving the public interest."

— Alfred E. Kahn ("Father" of the deregulation policies), 1977

"Markets do not have a conscience, they do not provide social policy, and they do not do things they are not paid to do."

— Kellan Fluckiger (Alberta Dept. of Energy) quoted in the NYT article "Experts Assess Deregulation as Factor in '03 Blackout", Sep. 16, 2005

1 Introduction

Before the eighties, many industries in the most advanced economies were dominated by heavily-regulated, government-owned monopolies; among others, this was the case in the telecommunications industry, the electricity industry, the transport industry, and the water treatment industry. Since firms that operate in these sectors typically need to undertake substantial prior investment in infrastructure or capacity, these (usually gigantic) regulated monopolists were often entrusted with the development (or purchase) and the operation of such infrastructure or capacity (e.g. natural gas and coal power plants, airplanes, desalination plants, etc.).

During the eighties and the nineties, many countries initiated a process of deregulation, privatization and liberalization of these sectors.[1] One of the changes introduced in this process was to substitute these regulated, government-owned monopolists by private firms competing in markets, and to allow new (private) competitors to freely enter these industries —see Armstrong and Sappington (2006). Thus, such government-based, centrally-planned investment decisions were replaced by private and competitive firms' investment decisions. In this new *laissez-faire* scenario, firms would invest in a certain facility, additional capacity, or a new plant only if the revenue they expected to obtain (at least) would be able to cover the cost of investing in such a facility, new capacity, or plant.

However, after years of deregulation and market competition, some market authorities and policymakers have prompted a re-think of these market-based rules in some of these sectors, underlined

[1]Due to presence of vast economies of scale, part of the business in some of these industries remained as heavily-regulated (natural) monopolies even after the implementation of these liberalization policies (e.g. electric power transmission, railway infrastructure, etc.).

by a specific growing concern regarding supply reliability. Since market competition drives prices down closer to marginal (operating) costs, expected revenues for some firms may not be enough to cover the costs of the initial investment. Thus, market competition may discourage firms from investing in new or additional facilities, capacity, or plants, which in turn potentially threatens supply reliability. This concern led to the debate and proposal of regulatory interventions, leading thus to a so-called "re-regulation" process. This new regulation usually involved some form of minimum investment requirement or essential mandatory available capacity that firms must guarantee in exchange for some monetary transfer or subsidy typically settled through a public tender.

For instance, in the airlines industry, the US Government implemented the "Alternate Essential Air Service" (AEAS) in 2004 and, in the European Union, policymakers regulated in 2008 the so-called "Public service obligations" (PSO). These programs guarantee a minimum mandatory frequency of service and capacity (seats) in certain routes for a subsidy —see Calzada and Fageda (2014) and Williams (2016). Following the liberalization of the electricity sector, many countries and regions have imposed "capacity requirements" of different forms, by which the regulators guarantee that there is enough generation capacity to meet expected peak demand. This is the case, for instance, in the California Independent System Operator (CAISO) and the Midcontinent Independent System Operator (MISO) in the USA, and Ofgem in the UK, among many others —see Joskow (2013) and Newbery (2015). Similarly, in the telecommunications industry, if a competitive market drives the prices of services such as calling or messaging close to zero, no firm will have an incentive to invest in network expansion and/or spectrum.[2] Therefore, government intervention and regulation of essential services is usually required —see Kreutzmann-Gallash et al. (2013).

These re-regulatory experiences seem to challenge a key idea behind the liberalization and privatization policies implemented in the eighties and in the nineties: suggesting that both full deregulation and market competition may not be as desirable as policymakers thought, as these two market features may not be compatible with security of supply. Hence, this hypothesis posits the question of whether, in those industries that require a substantial initial investment to operate in the market, full deregulation leads to the socially-optimal market outcome. Obviously, one might argue that even though these industries were open to market competition, some of them were (and

[2]The current situation of the telecommunications sector in India documented by The Economist (14 December 2019) is a particularly relevant and eye-opening case in point.

still are) heavily concentrated; if this is the case, some regulation will be desirable to correct market power. However, if one is to assume that these markets are truly competitive (as these liberalization policies intended), does full deregulation yield the socially-optimal (efficient) market outcome? If not, are these re-regulation policies able to restore market efficiency?

To answer these fundamental questions, we analyze a two-stage market environment in which costs-heterogeneous firms invest in capacity in the first stage, when demand is uncertain, and then compete to produce and sell a certain good to consumers. This general framework (which can be readily adapted to any particular application) is analyzed under the usual assumptions of a perfectly competitive market. We study this market environment under two scenarios, whose outcomes are compared through the lens of the usual welfare measures.

First, we study the deregulated market outcome, in which firms endogenously determine their investment in capacity. Then, we compare the equilibrium firms' investment in capacity with the capacity chosen by a planner that seeks to maximize welfare. We show that there exist some market parameters for which the welfare-maximizing investment in capacity is equal to peak demand (we call to this the "full-market-coverage" solution). Since the competitive market always yields an investment in capacity strictly smaller than peak demand,[3] it is straightforward to see that in these cases the competitive market does not achieve the most efficient solution.

Besides formally showing the previous relevant results, another of the main goals of this paper is to characterize some general conditions —some of which are identified in the aforementioned industries— for which "full-market-coverage" is the socially-optimal solution. In particular, this is the case if consumers' reservation price (willingness to pay) for the good in question is relatively high in comparison to firms' costs.[4] In other words, and more generally, this is the case if the marginal benefit (for consumers) of an additional unit of investment in capacity is higher than its marginal cost (for firms). However, as explained, the competitive market is unable to deliver an investment in capacity equal to peak demand. Thus, even though consumers are willing to pay to have enough spare capacity to be served (also at peak demand), market competition fails to do so.

[3] In line with the classic literature on capacity decisions and market uncertainty —see, for instance, Abel (1983), Caballero (1991) and Guiso and Parigi (1999)—, we find that, in equilibrium, firms underinvest in capacity. That is, aggregate firms' investment is strictly smaller than expected peak demand. Since this result is not novel in the literature, we relegate it (and its derivation) to the appendix.

[4] For instance, this is the case of necessities such as water or electricity. For these goods, consumers' reservation price is usually quite high —see Eto et al. (2001), Hensher et al. (2005) and Reichl et al. (2013).

Hence, this Pareto-improving transfer from consumers to firms in exchange for additional capacity does not occur: a market for such a Pareto-improving exchange is missing.

Then, in the light of the aforementioned "re-regulatory" experiences, we propose the following regulatory framework. We study a market with regulated capacity, in which the regulator imposes in the first stage a mandatory level of investment in capacity —in particular, a mandatory level of investment in capacity high enough to satisfy expected peak demand. In this scenario, firms compete in the first-stage to achieve this mandatory level of capacity, and receive an (endogenously determined) capacity compensation payment (subsidy), which is passed-through to consumers.

We show that this regulatory framework, which (by construction) yields the investment in capacity that the social planner would choose (i.e. equal to peak demand), is more efficient than the competitive market. This is because, as we formally show, in a competitive market the capacity compensation payment is just enough to cover firms' investment costs in additional capacity. Since we are dealing with cases in which the marginal benefit (for consumers) of an extra unit of capacity is greater than its marginal cost (for firms), then it follows that the marginal benefit (for consumers) of an extra unit of capacity is also greater than the capacity compensation that they pay to firms in exchange for additional capacity. Therefore, regulated capacity makes it possible for the Pareto-improving transfer from consumers to firms in exchange for additional capacity to happen; i.e. regulation fills the gap of the aforementioned missing market.

Finally, as an additional and ancillary result, we also show that this regulatory framework unambiguously reduces market price volatility and, under some particular assumptions, it is also weakly-preferred by risk-averse consumers.[5]

This paper is related to and fits at the intersection of three strands of the literature. First, it is related to the classical literature on the optimal regulation of the provision of private goods by utilities facing demand uncertainty —inaugurated by Brown and Johnson (1969), and followed by Visscher (1973), Panzar and Sibley (1978), and Kay (1979) (among others). Obviously, according to the reality at the time when these papers were written, all these previous authors assumed that the good was produced and served by a monopolist, for which both capacity investment and market prices were regulated. Instead, following the reality brought by the deregulation and liberalization

[5]The reason to study price volatility is due to the regulators' usual concern about price-spikes risk. The fact that consumers are worse-off when facing market price volatility is a well-established idea in the literature —see Turnovsky et al. (1980), Schmitz et al. (1981), Helms (1985), Cowan (2006), Borenstein (2007) and Bellemare et al. (2013).

policies and the recent “re-regulatory” experiences, we consider that the regulator establishes just the minimum essential (mandatory) capacity and lets firms compete both to achieve this level of mandatory capacity and also serve the good to consumers in the market (second) stage. Moreover, contrary to all these previous papers, we explicitly introduce cost-heterogeneous firms in our model to acknowledge that the good is no longer provided by one (gigantic) utility, but rather there might be different firms (with different investment and operating costs) providing it.

Second, this paper also fits into the literature on capacity pre-commitment and competition in “markets of fixed size”.[6] Up to now, many papers have been written exploring the microeconomics of investment in capacity in the 2-firm (duopoly) case —for instance, Davidson and Deneckere (1986), Hviid (1991), Gabszewicz and Poddar (1997), Reynolds and Wilson (2000), Anupindi and Jiang (2008) and Lepore (2012). The duopoly model was an appropriate one with which to study some of the aforementioned industries several years ago when, as Fabra et al. (2011) state, industries seem trapped in a concentrated structure. However, it does not seem the adequate framework for studying markets in which liberalization and privatization have led to true market competition.[7] To the best of our knowledge, this is the first study that examines investment incentives and welfare assuming instead that firms act as price takers, in an environment of perfect competition.

Finally, this paper is also related to the set of papers that have previously discussed either empirically or using case-studies the potential market effects of the regulation of capacity investment. These papers has been written in the context of specific industries such as, for instance, the healthcare industry —see Ferrier et al. (2010)—, the power industry —see Spees et al. (2013)—, or the telecommunications industry —see Grajek and Röller (2012). However, a paper that studies capacity regulation (and its market implications) using a theoretical model is absent in the literature. Therefore, this paper fills this gap in the literature by providing some (general) analytical results of the consequences of introducing regulated capacity in deregulated competitive industries.

The paper proceeds as follows. Section 2 provides the theoretical framework and characterizes

[6] To show our main result, we restrict our analysis to “markets of fixed size” —see Cripps and Ireland (1988)— which typically present price-inelastic demands. As de Frutos and Fabra (2011) note, this specification is well-suited to analyze industries in which long-run investments are followed by market competition. This is the case of all the industries mentioned above. As an extension, we show that our main results also hold if a downward-slopping demand is assumed instead.

[7] Still, one might argue that even though some markets became more competitive after the reforms, others seem to be heavily concentrated. Even though there are some markets that still seem to be concentrated, as argued above, it is still interesting to study the market outcome under the desideratum of market competition.

the competitive market solution. In Section 3 we solve the social planner's problem and discuss the conditions under which the competitive market does not yield the efficient solution. Section 4 defines the regulatory framework and shows that it is able to restore market efficiency, and provides additional results. In Section 5 we extend our baseline model in several directions. Finally, Section 6 concludes and all proofs are included in the appendix.

2 Theoretical model

We present a two-period model of a market for a homogeneous good, similar to the approach recently used, among others, by Creane and Jeitschko (2016) and Grüner and Siemroth (2016). As these authors do, we assume that firms can be of two types. The timing in this economy is as follows. In the first stage, both types of firms invest in capacity; by doing so, they incur in an investment cost. Capacity decisions are irreversible. In the second stage, investment decisions become publicly observable, demand is realized and Walrasian (competitive) trade takes place. We assume that firms produce the good up to their capacity limits at some positive marginal (operating) cost, while production above capacity is impossible (infinitely costly).

Per-unit capacity investment costs and operating costs are assumed to be inversely related for both types of firms. This assumption, which captures different production technologies, is necessary to observe cost-heterogeneous firms producing and selling the good in the market. Moreover, this assumption is also consistent with the cost structure of the firms in the industries considered in this paper.[8] By contrast, if we were to assume that per-unit capacity costs and marginal (operating) costs are not inversely related, the market would always operate in equilibrium with just one technology. This particular case, in which there are firms of one type only in equilibrium in the market, is also considered in our analysis, since we explicitly deal with corner solutions.

We proceed by backward induction. First, we deal with the equilibrium in the Walrasian market. Then, given the outcome in the second stage, we discuss the capacity investment stage.

[8]For instance, it captures coal generation (high per-unit capacity cost, low production marginal cost) versus natural gas generation (low per-unit capacity cost, high production marginal cost) in the electricity industry; dam storage (high per-unit capacity cost, low production marginal cost) versus desalination (low per-unit capacity cost, high production marginal cost) in the water industry; "low-cost" companies (high per-unit capacity cost, low production marginal cost) versus traditional companies (low per-unit capacity cost, high production marginal cost) –typically former monopolists– in the airline industry and the telecommunications industry; pipelines (high per-unit capacity cost, low production marginal cost) versus LNG-regasification terminals (low per-unit capacity cost, high production marginal cost) in the natural gas sector, etc.

2.1 Walrasian market exchange

In the second stage, demand is realized and Walrasian (competitive) trade takes place. On the supply side, we assume that there is a mass (measure) of atomless, risk-neutral firms which can be of two types, $i \in \{1, 2\}$. Each firm has a fixed, perfectly divisible capacity. Let us denote $k_i \in \mathbb{R}_+$ type-i firms' aggregate capacity, and define $\boldsymbol{k} \equiv (k_1, k_2)$. In addition, let us define K as total (aggregate) capacity in the market, where $K \equiv k_1 + k_2$. Firms can produce the good up to their capacity limits at a marginal cost of $c_i \in \mathbb{R}_{++}$, while production above capacity is impossible; without loss of generality, we assume that $c_1 < c_2$.

All firms act as price-takers and seek to maximize profits. Therefore, as explained by Creane and Jeitschko (2016), at prices above their marginal costs, firms find it optimal to produce and sell their output, whereas at prices below their marginal costs, it is optimal not to produce. Whenever the price is equal to a firms' marginal cost, the firm is indifferent between producing and not. Thus, following these authors, the supply schedule in this market is as follows[9]

$$q^S(p; \boldsymbol{k}) = \begin{cases} 0, & \text{if} \quad p < c_1, \\ \gamma k_1, \ \gamma \in (0,1) & \text{if} \quad p = c_1, \\ k_1 + \gamma k_2, \ \gamma \in [0,1) & \text{if} \quad p = c_2, \\ K, & \text{if} \quad p > c_2 \end{cases} \tag{1}$$

Firms supply the homogeneous good to a mass θ of infinitesimal and identical consumers. Each consumer buys one unit of the homogeneous good if its market price is less than or equal to a (finite) reservation price, denoted $\bar{p}$.[10] We assume that θ is a strictly-positive random variable distributed according to a cumulative distribution function, $F(\theta)$, defined for the support $[0, 1]$.[11] We assume that $F(\cdot)$ is strictly increasing and, as a consequence, $F^{-1}(\cdot)$ (the quantile function), is well-defined. All firms learn the realization of θ at the beginning of the second stage. Consumers observe firms' prices and make consumption decisions. The market is cleared when all consumers are served —if aggregate capacity exceeds demand— or until aggregate capacity is exhausted —if

[9] For the sake of simplicity, we have deliberately omitted to include the "intermediate case" where $q^S(p; \boldsymbol{k}) = k_1$ if $c_1 < p < c_2$ (which is redundant and unnecessary, given the inelastic nature of the demand). This simplification is not innocuous if we consider a downward-slopping demand, as we do in Section 5.2 (where we modify slightly the supply schedule to account for the potential "intermediate cases").

[10] To rule out uninteresting cases, we further assume that $c_i < \bar{p}$ for $i \in \{1, 2\}$.

[11] Our results can be easily generalized for any positive support $[\underline{\theta}, \overline{\theta}]$.

demand exceeds aggregate capacity.

Definition 1. *An equilibrium in the competitive market, given* $\boldsymbol{k}$*, is any price,* p^**, such that*

(i) (market-clearing) there exists $\gamma^* \in [0,1]$ *that reflects firms' output decisions given by equation (1) such that* γ^* *yields* $q^S(p^*;\cdot) = \max\{\theta, K\}$*;*[12] *and*

(ii) (profit-maximizing) there is no other $\tilde{p}^*$ *that yields strictly greater profits for some firm.*[13]

The following Lemma captures some general conditions that every equilibrium satisfies given different realizations of the demand.

Lemma 1. *In every possible equilibrium:*

1. *If* $\theta < k_1$*, then* $p^* = c_1$*, and both types of firms earn zero profit.*
2. *If* $k_1 \leq \theta < K$*, then* $p^* = c_2$*, type-1 firms' profits are strictly positive, and type-2 firms earn zero profit.*
3. *If* $K \leq \theta$*, then* $p^* = \bar{p}$ *and both types of firms earn strictly positive profits.*

If firms' capacities are greater than aggregate demand, then there will be enough capacity to serve all consumers and competition drives prices to the operating cost of the marginal supplier (cases #1 and #2 in Lemma 1). In the third case, capacity is scarce. The part of the demand that is not covered suffers a shortage, and (excess) demand drives the prices up to the reservation price ($\bar{p}$).

2.2 Capacity investment stage

Next, we examine the problem that firms have to solve in the first period. A mass of potential (atomless) firms decide whether to enter the market or not and, upon entry, they decide the amount of investment in capacity, k_i, at a per-unit cost of $c_{k_i} \in \mathbb{R}_{++}0$ for $i \in \{1,2\}$. Without loss of generality, we assume that $c_{k_1} > c_{k_2}$. Investment decisions are taken before knowing the realization of demand. However, capacity levels are chosen anticipating period-2 (expected) profits.

[12]As done by Creane and Jeitschko (2016), in our definition an equilibrium does not specify which firms are specifically active in the market.

[13]Our definition of market prices selects the "most profitable" prices in case of multiple equilibria. We do so in order to foster investment (that is, "less profitable" equilibrium prices would result in lower investment), which makes the underinvestment result more robust.

Firms have a well-defined expectation of market profit in each of the three possible scenarios in Lemma 1. Let us define $E\pi_i(k_i)$ as the type-i firms' period-2 expected revenue at period 1, which is the weighted average of the conditional expectation of the revenue, given by $(p^* - c_i) \times q^S(p^*; \cdot)$, with weights $P(\theta < k_1)$, $P(k_1 \leq \theta < K)$, and $P(K \leq \theta)$ —see Lemma 1.

Again, since we are interested in the market outcome under perfect competition, we solve for the capacity investment of both types of firms assuming that the long-run, free-entry equilibrium condition is satisfied.[14] Thus, following Mankiw and Whinston (1986) and Creane and Jeitschko (2016), we assume that the free-entry equilibrium is given by the largest measure of firms' aggregate capacities such that firms just expect to recover their capacity investment costs.[15]

Definition 2. *k_i^* is the free-entry, type-i equilibrium aggregate capacity if*

$$k_i^* \equiv \max\{k_i \in [0, \infty) \ s.t. \ E\pi_i(k_i) - c_{k_i} k_i \geq 0\}, \quad \text{for } i \in \{1, 2\}$$

Next, it can be shown that, given firms' costs, there always exists a unique $\boldsymbol{k}^* = (k_1^*, k_2^*)$ in equilibrium in the first stage. Moreover, we can also show that, in equilibrium, underinvestment always occurs in the sense that the market does not provide enough capacity to serve all the consumers in the highest possible realization of the demand. That is, a competitive market does not achieve the "full-market-coverage" solution ($K = 1$).[16]

The following results, which will be useful in the following sections, follow immediately from the period-one equilibrium characterization —see Proposition A.1.

Lemma 2. *Firms' expected profits, $E\pi_i(k_i) - c_{k_i} k_i$, are a (strictly) decreasing function of k_i in a neighborhood around $\boldsymbol{k^*}$.*

Lemma 3. *In equilibrium, $k_2^* > 0 \implies \frac{\partial K^*}{\partial \bar{p}} > 0$, $\frac{\partial K^*}{\partial c_2} < 0$, and $\frac{\partial K^*}{\partial c_{k_2}} < 0$.*

[14] The assumption that capacity investment is a long-run decision is widespread in the literature –see Davidson and Deneckere (1986), Reynolds and Wilson (2000), de Frutos and Fabra (2011), Grajek and Röller (2012). The assumption of free-entry is not only a useful theoretical point of departure but also a standard assumption in the context of competitive markets –see Mankiw and Whinston (1986) and Tremblay and Tremblay (2012).

[15] As Mankiw and Whinston (1986) do, and considering that the number of potential entrants is large and their size is infinitesimally small (perfect competitors), and that capacity is perfectly divisible, we assume that the free-entry aggregate capacity equilibrium exactly satisfies the zero-profit condition.

[16] As mentioned above, the fact that firms facing irreversible capacity decisions and market uncertainty underinvest in capacity is a well-established result in the literature –Abel (1983), Caballero (1991) and Guiso and Parigi (1999). Since this result is not novel, we relegate it (and its derivation) to Appendix A (see Proposition A.1).

Throughout the rest of the paper, we discuss our main results assuming that in equilibrium, both types of firms invest in capacity. For the sake of expositional clarity, the discussion of our results for the cases in which $k_i^* = 0$ (feasible corner solutions) are included in the appendix.

3 Welfare analysis

3.1 The Social Planner's problem

In this Section we consider a benevolent central planner that seeks to maximize welfare. That is, we assume now that in the first stage a central planner chooses capacity investment (for both types of firms), k_i for $i \in \{1, 2\}$, such that total welfare in the society is maximized. Our goal is to illustrate whether, by choosing capacity levels, the planner is able to improve welfare relative to the case in which capacity levels are given by the competitive market. Obviously, welfare would be enhanced if the benevolent planner chose capacities once the demand is realized, or if the planner knew in the first stage what the realization of demand would be in the second stage. If so, aggregate capacity would be chosen just to satisfy all consumers in the second stage. However, since demand is (ex-ante) uncertain, the more interesting question is if a planner who does not know the realization of the demand can achieve a Pareto-efficient market outcome (relative to the competitive market).

Let us denote $\mathcal{W}(\boldsymbol{k}|\cdot)$ as (ex-ante) total welfare in this market, as a function of aggregate capacities, $\boldsymbol{k}$. In this market, total welfare is defined as follows

$$\begin{aligned}\mathcal{W}(\boldsymbol{k}|\cdot) = \int_0^{k_1} (\bar{p} - c_1)\theta dF(\theta) + \int_{k_1}^{K} (\bar{p} - c_1)k_1 dF(\theta) + \int_{k_1}^{K} (\bar{p} - c_2)(\theta - k_1) dF(\theta) + \\ + \int_{K}^{1} (\bar{p} - c_1)k_1 dF(\theta) + \int_{K}^{1} (\bar{p} - c_2)k_2 dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2\end{aligned} \tag{2}$$

which is given by the difference between consumers' willingness to pay and firms' operating costs in each of the possible realizations of the demand, minus firms' investment costs.[17]

We are interested in characterizing aggregate capacities, $\boldsymbol{k}$, that maximizes equation (2), which provides us the solution to the social planner's problem. We define such double as the vector of socially-optimal (aggregate) capacities.

[17]In the appendix we show that this exact expression can be obtained by adding consumer and producer surplus.

Definition 3. $\hat{\boldsymbol{k}} \equiv (\hat{k}_1, \hat{k}_2)$ *is the vector of socially-optimal aggregate capacities if*

$$\hat{\boldsymbol{k}} \equiv \arg\max_{k_i \in \mathbb{R}_+} \mathcal{W}(\boldsymbol{k}|\cdot), \quad \textit{for } i \in \{1,2\}$$

We can show that, given firms' costs, there always exists a unique $\hat{\boldsymbol{k}}$ of socially-optimal aggregate capacities.

Proposition 1. *Given capacity investment costs,* c_{k_i} *for* $i \in \{1,2\}$*, there exists a unique vector of socially-optimal aggregate capacities,* $\hat{\boldsymbol{k}}$.

In the appendix we show that, among the (unique) feasible solutions to the planner's problem, there are some in which aggregate investment is equal to 1 ("full-market-coverage"). However, we know that the competitive market is not able to achieve that —see Appendix A, Proposition A.1. It follows that, in all these cases, the competitive market does not achieve the socially-optimal (efficient) solution.

Theorem 1. *There exist some parameters such that the free-entry equilibrium in the competitive market does not achieve the socially-optimal solution in the capacity-investment stage.*

As we formally show in Appendix A —see Proposition A.1—, a solution in which capacity is enough to satisfy all consumers in the highest possible realization of the demand (i.e. "full-market-coverage") cannot be achieved. Because period-2 firms' profits drop to zero if there is spare available capacity, building too much capacity (up to $K = 1$) implies that firms make zero profit in the second stage and, as a consequence, capacity investment costs are not recovered.

Still, we will later show that under the circumstances in which "full-market-coverage" is the socially-optimal solution, for all the aggregate capacity outcomes greater than the competitive market solution, i.e. for all K such that $K \in [K^*, 1]$, the marginal benefit of an additional unit of capacity is greater than its marginal cost. In other words, consumers' additional surplus of an extra unit of capacity (up to $K = 1$) is greater than firms' costs. Therefore, a transfer from consumers to firms such that firms cover (at least) the additional investment costs in exchange for additional investment in capacity is welfare-enhancing. However, in our set-up, such a "market for capacity" is missing. In other words, the competitive market fails because it is subject to a "missing market" problem.

3.2 Comparative Statics

We now focus in detail on the particular circumstances under which the competitive market does not yield the efficient outcome, which (as shown above) is the case if "full-market-coverage" is the socially-optimal solution. In particular, as shown in Appendix A (see the proof of Proposition 1), Theorem 1 holds if

$$\underbrace{(\bar{p} - c_2) \int_{K^*}^{1} \theta dF(\theta)}_{\mathcal{B}(\cdot)} > c_{k_2}; \tag{3}$$

in other words, if (3) holds, then $\hat{K} = 1$ and, as a consequence, the competitive market does not achieve the socially-optimal solution.

The left-hand side (hereafter LHS) of this expression –denoted $\mathcal{B}(\cdot)$– captures the expected (marginal) surplus of additional type-2 capacity (the "peaking" technology) —from aggregate market capacity, K^*, up to 1. Recall that, when the realized demand is such that $\theta \in [K^*, 1]$, capacity is scarce in the competitive market case. As a consequence, there are $\theta - K^*$ consumers that are not served and, therefore, that get zero surplus. Additional investment in type-2 capacity makes it possible for some of these consumers to obtain a (strictly) positive surplus, given by the difference between the reservation price, $\bar{p}$, and type-2 firms' operating costs, c_2. The right-hand side (hereafter RHS) of (3) captures the (marginal) cost of additional investment in type-2 capacity. Therefore, "full-market-coverage" is the socially-optimal solution (and, as a consequence, the competitive market does not yield the efficient outcome) if the expected surplus of additional type-2 capacity (from K^* up to 1) is greater than the cost of investment in this additional type-2 capacity.

Bearing expression (3) in mind, we next do some comparative statics to characterize the conditions under which "full-market-coverage" is the socially-optimal solution, in which case the competitive market fails to achieve the first-best. For that purpose, the following Lemma includes some additional features from the LHS of expression (3).

Lemma 4. $\frac{\partial \mathcal{B}(\cdot)}{\partial \bar{p}} > 0$, $\frac{\partial \mathcal{B}(\cdot)}{\partial c_2} < 0$, *and* $\frac{\partial \mathcal{B}(\cdot)}{\partial c_{k_2}} < 0$.

First, inequality (3) holds (*ceteris paribus*) if the consumers' reservation price, $\bar{p}$, is sufficiently high. Recall that, according to Lemma 4, $\mathcal{B}(\cdot)$ is strictly increasing in $\bar{p}$. That is, the LHS of inequality (3) increases with $\bar{p}$. On the other hand, the RHS remains unchanged as $\bar{p}$ changes.

Therefore, a greater reservation price makes inequality (3) more likely to hold. This result can be generalized for any costs parameters and for any distribution of the demand.

Proposition 2. *For all c_i, c_{k_i} and $F(\cdot)$, there exists $\bar{p}$ (sufficiently high) such that the competitive market does not yield the socially-optimal outcome.*

Goods for which the reservation price is typically high are, for instance, basic goods and necessities (such as electricity and water). For these goods, the expected benefit of having $\hat{K}$ at peak demand (i.e. when $\theta \in [K^*, 1]$) is relatively high —because $\bar{p}$ is relatively high. Therefore, for these goods, a competitive market is likely to fail to attain the socially-optimal solution.

Second, inequality (3) is more likely to hold if c_2 is relatively low. Mathematically, using Lemma 4, it follows that the LHS of inequality (3) is decreasing in c_2 (for any feasible c_2), while the RHS does not change as c_2 changes. Therefore, a decrease in c_2 increases the LHS, making the inequality more likely to hold. However, since c_2 is bounded below by c_1, we cannot derive a similar (general) result as for $\bar{p}$.[18]

The intuition of this result is as follows. The variable c_2 determines net surplus for consumers served by type-2 firms when the realization of the demand is high enough. In particular, if $\theta \in [K^*, 1]$, net surplus is given by $(\bar{p} - c_2)$. If c_2 increases, the gross surplus obtained in the $K = 1$ scenario decreases, making this option less desirable for consumers. Conversely, if c_2 decreases, the gross surplus gained with $K = 1$ goes up, making this option more attractive for consumers.

Next, we can also show that inequality (3) is more likely to hold if c_{k_2} is relatively low. The mathematical reasoning is similar as in the previous cases. We show in Lemma 4 that the LHS of inequality (3) decreases as c_{k_2} increases. Therefore, the RHS of inequality (3) increases as c_{k_2} increases. Combining both results, we see that a decrease in c_{k_2} increases the LHS and decreases the RHS and, as a consequence, a lower c_{k_2} makes this inequality more likely to hold.

Proposition 3. *For all $\bar{p}$, c_2 and $F(\cdot)$, there exists c_{k_2} (sufficiently low) such that the competitive market does not yield the socially-optimal outcome.*

This result is also intuitive. An increase in c_{k_2} increases the investment cost of type-2 firms.

[18] To illustrate this, let us assume the following parameters: $\bar{p} = 8$, $c_1 = 1$, $c_2 = 1.5$, $c_{k_2} = 6$; and let us assume that $\theta \sim \mathcal{U}[0, 1]$. Then, $\mathcal{B}(\cdot) \approx 3.54 < 6 = x$. Since $c_2 > c_1$, in the limit, c_2 will be extremely close to 1 (the value of c_1). Therefore, in the limit, $\mathcal{B}(\cdot) \approx 4 < 6 = x$. Since c_2 cannot decrease further than 1 (when $k_i^* > 0$ for $i \in \{1, 2\}$), we cannot find a c_2 sufficiently low such that inequality (3) holds.

Therefore, if the cost of additional capacity goes up, building additional capacity becomes less attractive from a welfare point of view.

Finally, the distribution of the demand $F(\cdot)$ is also key in determining whether inequality (3) holds or not. In particular, the LHS depends on $\int_{K^*}^1 \theta dF(\theta)$, that is, on the expected value of the demand conditional on being on the right tail of the distribution θ, where the right tail is given by the interval $\theta \in [K^*, 1]$ (peak demand). In actuarial science notation, $\int_{K^*}^1 \theta dF(\theta)$ can be formally defined as a conditional (right) tail expectation —$\text{CRTE}_{K^*}(\theta)$.[19]

In general, it is not possible to show that the greater the (unconditional) expected value of the demand is, the greater $\text{CRTE}_{K^*}(\theta)$ is.[20] However, it is straightforward that, as greater mass probability is concentrated in the right tail of the distribution of the demand (long left-tailed distribution), the expected value of peak demand becomes larger and, as a consequence, the more likely inequality (3) is to hold.

Proposition 4. *For all $\bar{p}$, c_i and c_{k_i}, there exists a (sufficiently long left-tailed) distribution of the random variable θ –with associated c.d.f. $F(\cdot)$– such that the competitive market does not yield the socially-optimal outcome.*

This condition establishes that the greater the expected realization of the demand (at peak demand), the greater the benefit of having $K = 1$ is. Intuitively, we know that at peak demand, if $k_1 + k_2 < 1$, there is zero net surplus. On the other hand, when $k_1 + k_2 = 1$, peak consumers are served at a cost lower than consumers' reservation price. Then, the greater the expected number of consumers at peak demand, the greater the number of consumers served at a cost lower than consumers' reservation price, and the greater the (overall) surplus.

A final point worth mentioning is that, as discussed above, in the welfare expression we do not include the welfare losses due to potential shortages in the case in which $k_1 + k_2 < 1$.[21] Recall

[19] Let X be a random variable, with support $[\underline{x}, \overline{x}]$. Given $a \in (\underline{x}, \overline{x})$, the a-Conditional Right Tail Expectation of X, $\text{CRTE}_a(\text{X})$, is defined as follows $\text{CRTE}_a(X) \equiv \mathbb{E}\left[X \mid X \geq a\right]$, where $\mathbb{E}$ is the expectation operator. Using this definition, it follows that $\int_{K^*}^1 \theta dF(\theta)$ is equivalent to the $\text{CRTE}_{K^*}(\theta)$.

[20] A potential counterexample is as follows. Let us assume the following parameters: $\bar{p} = 8$, $c_2 = 1$ and $c_{k_2} = 6$. We consider two distributions of the demand on the support [0,1], with c.d.f. $F_1(\cdot)$ and $F_2(\cdot)$ respectively. On the one hand, $\theta_1 \sim \text{Beta}(3.001, 3)$. On the other hand, $\theta_1 \sim \text{Beta}(2, 2)$. Then, $\mathbb{E}(\theta_1) = 0.5000833$ and $\mathbb{E}(\theta_2) = 0.5$, i.e. $\mathbb{E}(\theta_1) > \mathbb{E}(\theta_2)$. For the first one, we have that $K^{*,1} = 0.2843318$ and for the second one, $K^{*,2} = 0.2378968$, and the conditional (right) tail expectations are as follows: $\int_{K^{*,1}}^1 \theta_1 dF(\theta_1) = 3.295214$ and $\int_{K^{*,2}}^1 \theta_2 dF(\theta_2) = 3.345139$. That is, $\int_{K^{*,1}}^1 \theta_1 dF(\theta_1) < \int_{K^{*,2}}^1 \theta_2 dF(\theta_2)$.

[21] We do so for two main reasons. First, because in a "textbook" welfare analysis, the consumer surplus expression does not capture this potential negative loss –one exception is, for instance, by Viswanathan and Edison (1989).

that in the perfectly competitive market there is capacity underinvestment —see Proposition A.1. Hence, if the realization of the demand is high enough, some of the consumers will not be served. If we were to assume a consumer loss in the case of a shortage, this would enter equation (2) as a negative parameter. Therefore, the presence of consumer losses if a shortage occurs decreases the welfare in the perfectly competitive market relative to the socially-optimal solution, and thus makes the former case less desirable in terms of welfare.

All the previous information can be summarized in the following Result.

Result 1. *The "full-market-coverage" is the socially-optimal solution if one or more of the following conditions are satisfied:*

a) Consumers' reservation price for the good is relatively high (e.g. necessities)

b) Type-2 firms' costs are low relatively to the reservation price (e.g. cheap peaking technology and/or cheap capacity investment for peaking firms)

c) The distribution of the demand is relatively long-left tailed (e.g. peak demand is relatively highly frequent)

d) The cost of a shortage for consumers is relatively high (e.g. necessities)

If one or more of these conditions are satisfied, then the competitive market does not achieve the socially-optimal solution.

4 Capacity regulation

4.1 Equilibrium in a market with regulated capacity

In the previous Section we have shown that, when "full-market-coverage" is the socially-optimal solution, the competitive market does not achieve the efficient solution: the capacity yielded by the competitive market is always "too low" relative to the socially-optimal solution. This happens even though the price consumers are willing to pay net of type-2 (peaking) operating costs for the extra capacity is greater than the extra cost that firms would incur in providing such capacity.

Second, because the size of the losses may vary greatly depending on the industry and the nature of the good (a shortage in water is not the same as a shortage in leisure flights).

One obvious policy measure is for the policy-maker is to affect the level of investment in capacity through regulation, i.e. to establish a regulated (mandatory) capacity level.[22] Thus, for the cases in which "full-market-coverage" is the socially-optimal solution, the regulator can stimulate investment in capacity up to the social-optimal level by implementing a revenue-neutral transfer from consumers (who presumably are willing to pay for additional capacity) to firms that invest in additional capacity in the first period.

Analytically, we do so by assuming that the regulator sets a capacity target, $K^T \in (0,1]$, such that firms' aggregate capacity investment for both types of firms achieve this target. As explained above, to achieve this capacity level, the firms that invest in capacity receive a subsidy (or transfer) $x \in \mathbb{R}_+$ per unit of capacity built. We assume that this capacity compensation transfer is paid by consumers. The equilibrium compensation transfer, together with firms' capacities, can be (endogenously) determined using again the free-entry equilibrium as our solution concept.[23]

Definition 4. *Assume that the social planner imposes K^T as the regulated capacity level. Then, $k_i^{*,r}$ and $x_i^{*,r}$ are the free-entry, type-i equilibrium aggregate capacity and the equilibrium transfer, respectively, if*

$$\left(k_i^{*,r}, x^{*,r}\right) \equiv \max\{k_i \in [0,\infty) \text{ and } x \in [0,\infty) \text{ s.t. } E\pi_i(k_i) - c_{k_i}k_i + xk_i \geq 0 \text{ and } K^T = k_1 + k_2\},$$

for $i \in \{1,2\}$.

In the presence of regulated capacity, firms balance the fact that they incur in c_{k_i} when investing in additional capacity, but they also get x which compensates part of the investment costs. Again, in a free-entry, long-run equilibrium, investment continues up to the point where firms' stage-2 (expected) profits plus the capacity payment no longer covers the investment costs. It can be shown that, if the capacity target is greater than some threshold $\tilde{K}$,[24] there always exists a unique double $\boldsymbol{k}^{*,r} = (k_1^{*,r}, k_2^{*,r})$ together with a unique capacity payment $x^{*,r}$ in a free-entry equilibrium.

[22] As mentioned in the introduction, this kind of regulation, in which the regulator imposes a "regulated capacity" level, has been observed in some of the industries that we consider in this study, such as the transportation industry, the natural gas sector, the health industry, and the electricity sector.

[23] Some previous papers, specifically in the financial economics literature, have referred to this kind of equilibrium in which there is some kind of (lower/upper bound) regulation as a "regulated competitive equilibrium". See, for instance, Di Tella (2017), Gersbach and Rochet (2017) and Bianchi (2016).

[24] This requirement, the characterization of which is provided in detail in Appendix A (see the proof of Proposition 5), is necessary to avoid the possibility that the capacity target is smaller than type-1 aggregate capacity, which would imply that type-2 capacity is negative (which is impossible).

Proposition 5. *Given capacity investment costs, c_{k_i} for $i \in \{1,2\}$, there exists a $\tilde{K}$ such that for every $K^T \in [\tilde{K}, 1]$, there exists a unique equilibrium in which type-i firms' aggregate capacities are such that $k_i^{*,r} \geq 0$, for $i = 1,2$, and the unique capacity payment is $x^{*,r} \geq 0$.*

4.2 Restoring welfare through capacity regulation

So far we know that, under some particular circumstances, the competitive market does not yield the socially-optimal outcome —see Theorem 1. In particular, we show that this is the case if "full-market-coverage" is the socially-optimal solution, since the competitive equilibrium always yields a solution in which $K^* < 1$. In Section 3.2 we discussed in detail the particular circumstances, summarized in Result 1, under which "full-market-coverage" is the efficient solution. In the previous section we showed that a market with regulated capacity achieves a unique equilibrium. In this case, aggregate capacity is set by the regulator, and firms that invest in capacity receive a compensation which is paid for by consumers.

With all this information, we now focus on the following question: under the circumstances in which "full-market-coverage" is the socially-optimal (efficient) solution, is this able to be achieved by the competitive market with regulated capacity?

Theorem 2. *There exist some parameters such that the competitive market with regulated capacity, with $K^T = \hat{K}$, is more efficient that the competitive market without regulated capacity.*

Then, according to Theorem 2, the answer to the previous question is positive. In particular, for the parameters for which "full-market-coverage" is the socially-optimal solution —given by inequality 3— regulation is able to achieve it. This happens if the regulator establishes the capacity target precisely at $K^T = 1$. This capacity requirement triggers additional investment in capacity in the market, relative to the unregulated competitive market case —in which $K^* < 1$. The cost of this additional investment is compensated by consumers, who pay a positive capacity payment to the firms, and the extra (expected) surplus obtained by this additional investment outweighs the additional costs, as required by inequality 3. Thus, the solution is Pareto efficient.

Intuitively, by establishing a capacity target K^T, the regulator is creating a "new market for capacity" —for which the market-clearing condition is $K^T = k_1 + k_2$. By doing so, the regulator enables consumers to send this welfare-enhancing cash transfer that we discussed in Section 3, but which was not feasible in the unregulated market case. Thus, the creation of this "market for capacity" solves the missing market problem.

The parameters under which the competitive market with regulated capacity is more efficient that the competitive market without regulated capacity are the same as those discussed in Section 3.2. Thus, the following result follows immediately.

Result 2. *The competitive market with regulated capacity is more efficient that the competitive market without regulated capacity if one or more of the following conditions are satisfied:*

a) Consumers' reservation price for the good is relatively high (e.g. necessities)

b) Type-2 firms' costs are low relatively to the reservation price (e.g. cheap peaking technology and/or cheap capacity investment for peaking firms)

c) The distribution of the demand is relatively long-left tailed (e.g. peak demand is relatively highly frequent)

d) The cost of a shortage for consumers is relatively high (e.g. necessities)

4.3 Discussion

So far, we show that in industries whose firms require substantial investment in capacity before serving consumers, a competitive market may not achieve the socially-optimal level of capacity investment. Moreover, we show that, if this is the case, the efficient solution can be obtained with regulation. One potential drawback of the proposed regulatory framework is that it requires the regulator to know the socially-optimal capacity investment. That is, regulation is welfare-enhancing if, and only if, the regulator knows $\hat{K}$, which may seem a strong assumption.

However, in our particular framework, this should not be that problematic. Notice that, as shown in the previous section, regulation is welfare-enhancing if "full-market-coverage" is the socially-optimal solution (otherwise, the competitive outcome is Pareto efficient). That is, regulation is a good idea if the social-optimal capacity is equal to expected peak demand (the highest possible realization of the demand). Thus, in industries characterized by one or more of the conditions included in Result 2, for the regulator it is enough to calculate expected peak demand to obtain the optimal level of investment. For example, if the regulator knows for a particular industry that the maximum expected demand is equal to 1,000 units, K^T must be such that firms are able to satisfy 1,000 units of the good to consumers.

Another potential concern in our framework arises from the following question: if firms know that consumers' willingness to pay is higher than their costs of building additional capacity, why do these firms not offer additional capacity at a greater price? This question has two potential answers. First, if the market is perfectly competitive, in the second stage prices drop to operating costs. Thus, offering the product at a greater price drives a firm out of the competitive market.

Second, one possible way to escape from this "trap" is for firms to offer an insurance against shortages: the "missing market" for additional capacity could be replaced by an "insurance market" against lack of capacity. That is, firms may offer an insurance against shortages at a price greater than their investment costs and lower than consumers' reservation price. Thus, with this extra revenue, firms could invest in additional "precautionary" capacity that could be used to serve insured consumers in case of a shortage. However, this brings back two problems. First, this may create a free-rider problem among consumers: if some of them pay for the insurance and some others do not do so, the shortage is less likely to affect all consumers. In other words, consumers that do not pay for the insurance benefit from the additional capacity (in the sense that shortages will be less likely to occur) because of those that paid for the insurance.[25]

Moreover, this may also create a "negative signaling" problem for those firms that offer an insurance. Suppose that, a power supply company offers an insurance against blackouts, or an airline offers an insurance against overbooking. Clearly, consumers may think that blackouts or overbookings are more likely in those firms (otherwise, they would not offer such an insurance). If so, consumers may be discouraged to purchase from those firms and, as a consequence, they will be driven out of the market (even though they offer a Pareto efficient deal). Although this interesting point may be a path for future research, our point here is that an insurance would be an effective solution only if it were offered by all the firms in the market, which would require the regulator to compel firms to offer a mandatory insurance.

[25] Obviously, the way to avoid this problem is to make this insurance mandatory for all consumers which, evidently, requires some form of regulation.

4.4 Additional results: price volatility and price-spike risk

In addition to welfare, we also study price-spike risk and price volatility under both scenarios, namely, in the unregulated case and in the presence of capacity regulation.[26] Recall that the market outcome both in the absence and in the presence of regulation are equivalent if $\hat{K} < 1$; as a consequence, price volatility and price-spike risk will be the same in both scenarios. However, we know that the solution to the unregulated market is not the same as the solution to the regulated market case whenever "full-market-coverage" is the socially-optimal solution. Thus, we focus on the study of price volatility and price-spike risk in this particular case.

In order to study price volatility and price-spike risk, we use two well-known concepts, namely, the Second Order Stochastic Dominance (SOSD) and the price volatility, measured as the variance of consumer prices. First, let us define the set of equilibrium market prices in the Walrasian market stage as $\mathcal{C} \subseteq \mathbb{R}_+$. Denote m as the (ex-ante) possible number of equilibria in the price competition stage, and consider the set of m-equilibrium market prices, $\mathcal{C}^m \equiv \{p_1^c, \cdots, p_m^c\} \subseteq \mathbb{R}_{++}^m$. Given the space of equilibrium market prices, we denote the set of lotteries over $\mathcal{C}^m$ as $\mathcal{P} = \Delta(\mathcal{C}^m)$ which is an m-dimensional unit simplex, i.e. $\Delta(\mathcal{C}^m) = \{\alpha_1, \cdots, \alpha_m \mid \alpha_i \geq 0 \ \forall i \in m \text{ and } \sum_i \alpha_i = 1\}$. We capture both a set of equilibrium market prices together with a lottery associated to it in a price-contingent contract for consumers.

Definition 5. *Given a set of m equilibrium market prices, $\mathcal{C}^m \equiv \{p_1^c, \cdots, p_m^c\} \subseteq \mathbb{R}_{++}^m$, where m is the (ex-ante) possible number of equilibria in the Walrasian market stage, and given the set of lotteries over $\mathcal{C}^m$, $\mathcal{P} = \Delta(\mathcal{C}^m)$, such that $\Delta(\mathcal{C}^m) = \{\alpha_1, \cdots, \alpha_m \mid \alpha_i \geq 0 \ \forall i \in m \text{ and } \sum_i \alpha_i = 1\}$, we refer to $\mathcal{L}$ as the price-contingent contract for consumers associated to $\mathcal{C}^m$, denoted as:*

$$\mathcal{L} = \begin{pmatrix} p_1^c & \cdots & p_m^c \\ pr(1) \equiv \alpha_1 & \cdots & pr(m) \equiv \alpha_m \end{pmatrix} \tag{4}$$

and we denote $G_{\mathcal{L}}(p^c)$ as the cdf associated to the price-contingent contract $\mathcal{L}$ such that $G_{\mathcal{L}}(p^c) \equiv pr(p_r^c \leq p^c), \ \forall p^c \in \mathcal{C}, \ r \in \{1, \cdots, m\}$.

[26] The reason for studying price volatility is due to the regulators' common concern about price-spike risk. In fact, some previous authors have also recognized that consumers are worse-off when facing market price volatility –see Turnovsky et al. (1980), Schmitz et al. (1981), Helms (1985), Cowan (2006), Borenstein (2007) and Bellemare et al. (2013).

In order to compare price riskiness between two price-contingent contracts, we employ the commonly used notion of Second Order Stochastic Dominance (SOSD)[27]:

Definition 6. *Given two price-contingent contracts with the same mean, $\mathcal{L}$ and $\mathcal{L}'$, $\mathcal{L}$ Second Order Stochastic Dominates (is less risky than) $\mathcal{L}'$ if and only if $\int_{-\infty}^{p^c} G_{\mathcal{L}}(z)dz \leq \int_{-\infty}^{p^c} G_{\mathcal{L}'}(z)dz, \ \forall p^c \in \mathcal{C}$.*

As shown in Mas-Colell et al. (1995), SOSD implies that one price-contingent contract that is less risky than another –i.e. that Second Order Stochastic Dominates– is (weakly) preferred by all risk-averse consumers.

Even though the concept of SOSD is stronger than the concept of variance in the presence of risk-averse consumers, it requires both price-contingent contracts to have the same mean, which is a restrictive assumption. Therefore, we also study the variance of the price as a measure of price volatility, which does not require such a restrictive assumption.

Definition 7. *Given two price-contingent contracts, $\mathcal{L}$ and $\mathcal{L}'$, $\mathcal{L}$ is less price-volatile than $\mathcal{L}'$ if and only if $var(\mathcal{L}) \leq var(\mathcal{L}')$.*

Bearing these two definitions in mind, we provide two results regarding price riskiness and the volatility of prices. Our first result is stronger than the second one, although it requires a stronger assumption.

Proposition 6. *Assuming that the demand follows a uniform distribution, the price-contingent contract in a market with regulated capacity is less risky than (SOSD) the price-contingent contract in a market with unregulated capacity.*

In other words, if the expected price of both contracts is the same (which holds if $\theta \sim \mathcal{U}[0,1]$), a regulated capacity regime not only reduces price-spike risk, but it also reduces price volatility (measured as the variance). Thus, the price-contingent contract generated by a market with regulated capacity is weakly preferred by all risk-averse consumers to the price-contingent contract generated by a market with unregulated capacity.

Proposition 6 presents a strong result in terms of price volatility. However, it requires a strong assumption. Thus, in the next proposition we present a result in terms of price volatility that does not require restrictive assumptions.

[27]Notice that the concept of SOSD is stronger than the concept of variance, as a measure of price volatility. In fact, we can show that if $\mathcal{L}$ Second Order Stochastic Dominates (is less risky than) $\mathcal{L}' \implies \text{var}(\mathcal{L}) \leq \text{var}(\mathcal{L}')$, but $\text{var}(\mathcal{L}) < \text{var}(\mathcal{L}') \;\not\Longrightarrow$ that $\mathcal{L}$ Second Order Stochastic Dominates (is less risky than) $\mathcal{L}'$.

Proposition 7. *The price-contingent contract in a market with regulated capacity is less volatile (has a lower variance) than the price-contingent contract in a market with unregulated capacity.*

That is, in comparison to an unregulated capacity scenario, the presence of regulated capacity leads to consumers facing less price volatility.

5 Model extensions and variations

In the preceding Sections we developed an intentionally simple model to highlight the mechanisms behind our main results. In particular, we have assumed that there are two types of firms that invest in perfectly divisible capacity and supply the product to a perfectly-inelastic demand. In the following Sections we relax each of these assumptions to show the robustness of our main results.

5.1 More than two types of firms

In the baseline model, we focus our analysis on the case in which there are two types of firms. However, this baseline model can easily be extended to the case in which there are N types of firms, where $N > 2$ and $N \in \mathbb{N}$. In this case, firms' operating (marginal) costs are c_i, and firms' per-unit capacity investment costs are c_{k_i}, for $i = 1, \cdots, N$. As in the baseline case, an inverse relationship between firms' operating and investment costs is assumed.[28]

As in the baseline model, in the second stage, Walrasian (competitive) trade takes place. The assumptions on the supply side remain the same, but we slightly modify the supply schedule to accommodate the additional types of firms. Thus, given firms' capacities $\boldsymbol{k} \equiv (k_1, k_2, \cdots, k_N)$, the new supply schedule is, then, as follows

$$q^S(p; \boldsymbol{k}) = \begin{cases} 0, & \text{if} \quad p < c_1, \\ \gamma k_1, \ \gamma \in (0,1) & \text{if} \quad p = c_1, \\ \sum_{j=1}^{J-1} k_j + \gamma k_J, \ \gamma \in [0,1) & \text{if} \quad p = c_J, \\ K, & \text{if} \quad p > c_N \end{cases} \tag{5}$$

for $J = \{2, \cdots, N\}$, where $K = k_1 + \cdots + k_N$. The assumptions on the demand side remain as in

[28]Otherwise, if there is some type of firm i' for which both investment and operating costs are greater than for some other type of firms i'', it is never optimal for type-i' firms to invest and produce in equilibrium.

the baseline model.

Given the new supply schedule, we can show that the general conditions that every second-stage equilibrium satisfies included in Lemma 1 can be easily extended to the case in which there are N types of firms.

Lemma 5. *In every possible equilibrium:*

1. *If $\theta < k_1$, then $p^* = c_1$, and both types of firms earn zero profit.*
2. *If $\sum_{j=1}^{J-1} k_j \leq \theta < \sum_{j=1}^{J} k_j$, then $p^* = c_J$, type-j firms' profits are strictly positive (for $j = 2, \cdots, J-1$), and type-J firms earn zero profit, for $J = \{2, \cdots, N\}$.*
3. *If $K \leq \theta$, then $p^* = \bar{p}$ and all types of firms earn strictly positive profits.*

For the baseline model, we showed that if there is too much capacity, competition drives the price down to firms' marginal costs. In particular, if $K = 1$, type-N firms' profits in the second stage are equal to zero, which implies that these firms incur losses, which is impossible in equilibrium. Therefore, in this new scenario, every interior solution also requires aggregate capacity to be strictly smaller than peak demand.

Lemma 6. *Every equilibrium in the second stage satisfies $\sum_{i=1}^{N} k_i^* < 1$.*

Again, as in the baseline model, regulated capacity up to 1 will be efficient if the marginal surplus of an additional unit of capacity exceeds its marginal cost, c_{k_2}. This requires that

$$\underbrace{(\bar{p} - c_N) \int_{K^*}^{1} \theta dF(\theta)}_{\mathcal{B}(\cdot)} > c_{k_N}; \tag{6}$$

which is the analog of the condition in 3, and for which a similar interpretation and conclusions as those discussed above apply.

5.2 Downward-slopping demand

So far, our analysis has focused on the case in which market demand is price-unresponsive. This assumption is not only a useful simplification, but is also a common one imposed when studying the so-called "markets of fixed size" (an appropriate category for many of the industries considered in this study) which typically present price-inelastic demands —see Cripps and Ireland (1988) and de Frutos and Fabra (2011). However, our main result can also be easily extended to markets for which the demand is price-responsive (downward-slopping). Thus, our next extension considers this case.

As in the baseline model, in the second stage, Walrasian trade takes place. The assumptions on the supply side remain the same, but we modify slightly the supply schedule to account for the potential "intermediate cases" in which demand is equal to k_1. Thus, given firms' capacities, the supply schedule in this market is as follows

$$q^S(p;\boldsymbol{k}) = \begin{cases} 0, & \text{if} \quad p < c_1, \\ \gamma k_1,\ \gamma \in (0,1) & \text{if} \quad p = c_1, \\ k_1, & \text{if} \quad p \in [c_1, c_2], \\ k_1 + \gamma k_2,\ \gamma \in [0,1) & \text{if} \quad p = c_2, \\ K, & \text{if} \quad p > c_2 \end{cases} \tag{7}$$

On the demand side, we now assume that the market demand function is represented by $q^D(p;\theta)$, which is a bounded, continuous and decreasing function of $p \in [0, \bar{p}]$, and for which there exists $\bar{p}$ (the reservation price) such that $q^D(p;\theta) = 0$ for all $p > \bar{p}$, for all θ. We also assume that $q^D(p;\theta)$ is continuous and increasing in the stochastic parameter θ. Thus, θ affects the position of the demand function, by shifting it to the left or to the right. As in the baselined model, θ is distributed according to a cumulative distribution function, $F(\theta)$, defined for the support $[0,1]$. We further assume that $q^D(0;0) = 0$ and, therefore, $q^D(p;0) = 0$ for all $p \in [0, \bar{p})$, and that $q^D(0;1) = 1$ (peak demand).[29]

As explained in Definition 1, an equilibrium in the second stage, is any price, p^*, such that *(i)* the market is cleared and *(ii)* firms maximize profits. With the new market demand function, the definition of an equilibrium in the second stage remains the same, but the market-clearing condition requires that $q^S(p^*;\boldsymbol{k}) = \max\{q^D(p^*;\theta), K\}$. The following Lemma —which is the analog of Lemma 1— captures some general conditions that every equilibrium satisfies given different realizations of the demand.

[29] As in the baseline model, our results can be easily generalized for any positive support.

Lemma 7. *In every possible equilibrium:*

1. *If $q^D(p^*;\theta) < k_1$, then $p^* = c_1$, and both types of firms earn zero profit.*

2. *If $q^D(p^*;\theta) = k_1$, then $p^* \in [c_1, c_2]$, type-1 firms' profits are positive, and type-2 firms earn zero profit.*

3. *If $k_1 < q^D(p^*;\theta) < K$, then $p^* = c_2$, type-1 firms' profits are strictly positive, and type-2 firms earn zero profit.*

4. *If $K = q^D(p^*;\theta)$, then $p^* \in [c_2, \bar{p}]$, type-1 firms' profits are strictly positive, and type-2 firms' profits are positive.*

5. *If $K < q^D(p^*;\theta)$, then $p^* = \bar{p}$ and both types of firms earn strictly positive profits.*

As in the baseline model, Lemma 7 implies that type-2 firms make some positive profit only if aggregate capacity is scarce relative to the realization of the demand. Therefore, as is the case with price-inelastic demand, if there is too much capacity, competition drives the price down to firms' marginal cost, which precludes firms from obtaining revenue in the second stage to compensate the investment cost incurred in the first stage. Therefore, as we did in the baseline model, we can show that aggregate capacity is always strictly smaller than peak demand evaluated at a price equal to type-2 firms' operating cost.

Lemma 8. *Every equilibrium in the second stage satisfies $k_1^* + k_2^* < q^D(c_2, 1)$.*

The previous result directly implies that, if the stochastic parameter θ exceeds a threshold value, capacity is scarce (which is the scenario under which all firms earn a positive profit).

Lemma 9. *There exist $\tilde{\theta}$ such that $K^* < q^D(c_2, \theta)$ for all $\theta > \tilde{\theta}$.*

As in the baseline model, regulated capacity up to $q^D(c_2, \theta)$ will be efficient if the marginal surplus of an additional unit of capacity exceeds its marginal cost, c_{k_2}. With a downward-slopping demand, this requires that

$$\underbrace{(\bar{p} - c_2) \int_{\tilde{\theta}}^{1} q^D(p, \theta) dF(\theta)}_{\mathcal{B}(\cdot)} > c_{k_2}; \tag{8}$$

which is the analog of the condition in 3, and for which a similar interpretation and conclusions as those discussed above apply.

5.3 Lumpy investment

In the baseline model, we assumed that capacity is perfectly divisible. However, this may be not be the case in some of the aforementioned industries in which firms need to invest in capacity additions in a "lumpy" fashion –see Gilbert and Harris (1984). This is the final extension of our baseline model that we consider.

"Lumpy" capacity investment may affect our baseline results in two different ways. First, it may affect the equilibrium investment in capacity in the first stage. In particular, we may face a situation in which the equilibrium aggregate investment, K^*, is such that the zero-profit condition is not binding. If so, and bearing in mind that firms' profits decreases as k_i increases in a neighborhood around the equilibrium –see Lemma 2—, aggregate investment must be smaller than in the baseline model (otherwise, some firms will incur losses). Therefore, in this case, the competitive market outcome is less efficient than in the baseline scenario.

Second, it may also affect the capacity investment target imposed by the regulator. In particular, the regulator may face a situation in which, due to "lumpiness", $k_1 + k_2 > K^T = 1$ (as opposed to the baseline model, in which this condition could be satisfied with equality). In this case, the regulated market outcome is less efficient that in the baseline model, since firms need to invest in a level of capacity greater than that needed by consumers (even at peak demand).

Therefore, lumpy investment may affect the efficiency of both the unregulated market outcome and the regulated market outcome. Therefore, we need to evaluate on a case-by-case basis if the loss of efficiency due to lumpy investment is greater in the regulated or in the unregulated market to reach a conclusion: if it is greater in the unregulated market than in the regulated market, then regulation is welfare-improving in more cases than in the baseline model. Otherwise, regulation is welfare-improving in fewer cases relative to the baseline model.

6 Conclusions

We began this paper by questioning the compatibility of two of the main goals of the liberalization and privatization reforms introduced in the late eighties –namely, market deregulation and competition– with security of supply. We show that, indeed, these two goals are incompatible in some industries; specifically, in those that require a substantial initial capacity investment to operate in the market. Then, following the reality in many of these industries, we study the potential welfare effects of implementing capacity investment regulation. This kind of regulation had led to a new phase in the market restructuring process, usually known as the "re-regulation" phase. Is this post-liberalization regulation desirable in these kinds of industries in the presence of market competition?

To answer this question, we have extended earlier work on capacity investment under conditions of demand uncertainty to study the welfare implications of the implementation of regulated capacity in liberalized and competitive markets. We set up a theoretical model with cost heterogeneous firms that in the first stage invest in capacity (when demand is uncertain) to produce and sell a homogeneous good in the second stage (when demand is realized and known).

Our main findings are as follows. First, we support that in the absence of capacity regulation, a competitive market leads to capacity underinvestment. This capacity underinvestment can be solved in the presence of regulated capacity. Second, we show that regulated capacity leads to a more efficient market outcome relative to the unregulated capacity case under some specific circumstances. In particular, this result holds if i) consumers' reservation price is relatively high, and/or ii) "peaking firms" costs are relatively low, and/or iii) the distribution of the demand is relatively long left-tailed, and/or iv) the cost of a shortage is relatively high for consumers. In these cases, this result holds even though regulated capacity entails a capacity compensation paid by consumers. Finally, we show that a market with regulated capacity unambiguously reduces market price volatility relative to an unregulated capacity regime.

These results are in line with reality. In fact, regulators have introduced regulated capacity in industries such as the electricity industry, the health industry, the water industry or the renewable energy industry. A common point of these industries is precisely the relatively high cost of shortages and/or the existence of a relatively high reservation price.[30] However, regulated capacity is typically absent in industries that do not serve basic goods and, consequently, for which consumers' reservation price is not that high. This is the case, for instance, in the industry of leisure flights.[31]

[30]In the renewables industry, underinvestment in renewable capacity will lead to fines not only for companies –due to national pollution laws– but also for countries –due to environmental agreements and protocols. These recent environmental laws and agreements have increased the "reservation price" of not having enough renewable capacity.

[31]As argued above, this is not true in general in the airlines industry. For instance, some routes to remote areas within a country or routes to islands and overseas territories are regulated with the purpose of guaranteeing minimum flight services.

On the other hand, the absence of regulated capacity in this industry has led to huge oscillations in prices as a response to seasonal demand patterns –see Borenstein and Rose (1994) and Gaggero and Piga (2011).

This model provides some general results that apply to liberalized and competitive industries with regulated capacity. Future work should focus on the optimal design of regulated capacity target level taking into account the idiosyncratic features of different industries, such as imperfect competition or externalities.

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Appendix A: Proofs

Proof of Lemma 1: given firms' capacities, $\boldsymbol{k}$,

1. If $\theta < k_1$, the demand is fully served by type-1 firms only. By definition 1, an equilibrium requires the "market-clearing" condition, which implies that $q^s(p^*;\cdot) = \max\{\theta, K\}$. In this case, since $\theta < k_1 \le K$, then $q^s(p^*;\cdot) = \theta$, that is, $\gamma^* k_1 = \theta$, for some $\gamma^* \in (0,1)$ which, by equation 1, implies that $p = c_1$. Therefore, all consumers are served at a price c_1. Type-1 firms that serve the demand make zero profit (because the price is equal to the marginal cost). Firms that do not serve the demand (the rest of the firms) make zero profit.

2. If $k_1 \le \theta < K$, the demand is fully served by all type-1 firms and some type-2 firms. By the "market-clearing" condition, and considering that $k_1 \le \theta < K$, then $q^s(p^*;\cdot) = \theta$, that is, $k_1 + \gamma^* k_2 = \theta$, for some $\gamma^* \in [0,1)$. By equation 1, this implies that $p = c_2$. Therefore, all consumers are served at a price c_2. All type-1 firms serve the demand, so these firms make a positive profit (the price at which they serve is greater than the marginal cost). Type-2 firms that serve the demand make zero profit (because the price is equal to the marginal cost). Type-2 firms that do not serve the demand (the rest of the firms) make zero profit.

3. If $K \le \theta$, the demand is fully served by all type-1 firms and all type-2 firms (up to available capacity). By the "market-clearing" condition, and considering that $\theta \ge K$, then $q^s(p^*;\cdot) = K$. By equation 1, this implies that $p > c_2$. By definition 1, an equilibrium requires the "profit-maximizing" condition. Therefore, the equilibrium price will be a price that clear the markets and that maximizes firms' profits. That is, $p^* = \bar{p} > c_2$ (recall that $\bar{p}$ is the reservation price, so firms cannot sell at any price above $\bar{p}$). Therefore, consumers are served at a price $\bar{p}$, and all firms make a positive profit (the price at which they serve is greater than the marignal cost). □

Proposition A.1. *Given capacity investment costs, c_{k_i} for $i \in \{1,2\}$, there exists a unique equilibrium in which aggregate capacities of the firms are such that $k_i^* \ge 0$, for $i = 1,2$. Moreover, every equilibrium satisfies $k_1^* + k_2^* < 1$.*

Proof: let us begin with type-1 firms. By definition 2, we know that in equilibrium, $E\pi_1(k_1) - c_{k_1}k_1 = 0$. That is, $E\pi_1(k_1|\theta < k_1) * P(\theta < k_1) + E\pi_1(k_1|k_1 \le \theta < K) * P(k_1 \le \theta < K) + E\pi_1(k_1|K \le \theta) * P(K \le \theta) - c_{k_1}k_1 = 0$. By Lemma 1, we know that $E\pi_1(k_1|\theta < k_1) = 0$. Moreover, by Lemma 1 we also know that if $k_1 \le \theta < K$, then $p^* = c_2$, and if $K \le \theta$, then $p^* = \bar{p}$; in both cases, all type-1 firms serve the demand up to their capacities. Therefore, the free-entry equilibrium condition can be rewritten as follows

$$\int_{k_1}^{K} (c_2 - c_1)k_1 f(\theta)d\theta + \int_{K}^{1} (\bar{p} - c_1)k_1 f(\theta)d\theta - c_{k_1}k_1 = 0$$

Solving the integrals and rearranging, we arrive at:

$$(c_2 - c_1)F(k_1) = (\bar{p} - c_1) - (\bar{p} - c_2)F(K) - c_{k_1} \tag{A.1}$$

Next, we deal with type-2 firms. By definition 2, we know that in equilibrium, $E\pi_2(k_2) - c_{k_2}k_2 = 0$. That is, $E\pi_2(k_2|\theta < k_2) * P(\theta < k_2) + E\pi_1(k_2|k_2 \le \theta < K) * P(k_2 \le \theta < K) + E\pi_2(k_2|K \le$

$\theta) * P(K \leq \theta) - c_{k_2}k_2 = 0$. By Lemma 1, we know that $E\pi_2(k_2|\theta < k_2) = 0$ and that $E\pi_2(k_2|k_1 \leq \theta < K) = 0$. Moreover, by Lemma 1 we also know that if $K \leq \theta$, then $p^* = \bar{p}$; in this case, all type-2 firms serve the demand up to their capacities. Therefore, the free-entry equilibrium condition can be rewritten as follows

$$\int_K^1 (\bar{p} - c_2)k_2 f(\theta)d\theta - c_{k_2}k_2 = 0$$

Solving the integral, we arrive at:

$$(\bar{p} - c_2)k_2[1 - F(K)] - c_{k_2}k_2 = 0$$

Rearranging, we arrive at:

$$F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right) = K \tag{A.2}$$

Replacing A.2 into A.1, we arrive at:

$$(c_2 - c_1)F(k_1) = (\bar{p} - c_1) - (\bar{p} - c_2)F\left[F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)\right] - c_{k_1}$$

Solving for k_1 we arrive at:

$$k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) \tag{A.3}$$

Replacing A.3 into A.2

$$k_2 = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right) - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$$

or, equivalently,

$$k_2 = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right) - k_1 \tag{A.4}$$

Therefore, aggregate capacity is

$$K = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right) \tag{A.5}$$

We focus on the conditions under which there is an interior solution (we deal with corner solutions in Appendix B), i.e. $k_i \in (0, 1)$ for $i \in \{1, 2\}$. First, $k_1 > 0$ implies that $F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) > 0$, which requires that $c_2 - c_1 > c_{k_1} - c_{k_2}$. In addition, $k_2 > 0$ implies that $F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right) - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) > 0$; rearranging, this condition implies that $c_{k_2}(c_2 - c_1) < (\bar{p} - c_2)(c_{k_1} - c_{k_2})$. Combining both conditions, i.e. $c_2 - c_1 > c_{k_1} - c_{k_2}$ and $c_{k_2}(c_2 - c_1) < (\bar{p} - c_2)(c_{k_1} - c_{k_2})$ —this implies that $c_{k_2} < \bar{p} - c_2$, which is also a required condition for $K > 0$. Thus, if these two conditons are satisfied, equilibrium capacities are given by equations A.3 and A.4.

In the interior solution case, equilibrium uniqueness is guaranteed by the fact that $F(\cdot)$ is strictly increasing, which implies that $F^{-1}(\cdot)$ is also strictly increasing.

Finally, we know that at an interior solution, $K = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$. As we have just discussed, an interior solution requires two conditions, namely, $c_2 - c_1 > c_{k_1} - c_{k_2}$ and $c_{k_2}(c_2 - c_1) < (\bar{p} - c_2)(c_{k_1} - c_{k_2})$. As discussed, both conditions are compatible if, and only if, $(\bar{p} - c_2) > c_{k_2}$. Therefore, an interior solution requires that $(\bar{p} - c_2) > c_{k_2}$. If so, then $1 > \frac{c_{k_2}}{\bar{p} - c_2} > 0$, which implies that $1 - \frac{c_{k_2}}{\bar{p} - c_2} \in$

$(0,1)$. That is, $K = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) \in (0,1)$. That is, at an interior solution, $K = k_1 + k_2 < 1$ is satisfied. □

Proof of Lemma 2: first, we know that by definition, type-1 firms' profits are

$$E\pi_1(k_1) - c_{k_1}k_1 := \int_{k_1}^{K} (c_2 - c_1)k_1 f(\theta)d\theta + \int_{K}^{1} (\bar{p} - c_1)k_1 f(\theta)d\theta - c_{k_1}k_1$$

Solving the integrals and rearranging, we arrive at

$$E\pi_1(k_1) - c_{k_1}k_1 := \left[(c_2 - \bar{p})F(K) - (c_2 - c_1)F(k_1) + (\bar{p} - c_1) - c_{k_1}\right] k_1$$

Taking F.O.C. w.r.t. k_1

$$\frac{\partial E\pi_1(k_1) - c_{k_1}k_1}{\partial k_1} = \left[(c_2 - \bar{p})f(K) - (c_2 - c_1)f(k_1)\right] k_1 + (c_2 - \bar{p})F(K) - (c_2 - c_1)F(k_1) + \bar{p} - c_1 - c_{k_1}$$

Evaluated at the equilibrium, reduces to

$$\left.\frac{\partial E\pi_1(k_1) - c_{k_1}k_1}{\partial k_1}\right|_{\boldsymbol{k^*}} = \left[(c_2 - \bar{p})f(K^*) - (c_2 - c_1)f(k_1^*)\right] k_1^* +$$

$$+(c_2 - \bar{p})\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right) - (c_2 - c_1)\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) + \bar{p} - c_1 - c_{k_1}$$

Rearranging, we arrive at

$$\left.\frac{\partial E\pi_1(k_1) - c_{k_1}k_1}{\partial k_1}\right|_{\boldsymbol{k^*}} = \left[(c_2 - \bar{p})f(K^*) - (c_2 - c_1)f(k_1^*)\right] k_1^*$$

Since $\bar{p} > c_2$ and $c_2 > c_1$, then $\left.\frac{\partial E\pi_1(k_1) - c_{k_1}k_1}{\partial k_1}\right|_{\boldsymbol{k^*}} < 0$. Provided that the profit function is twice continuously differentiable, this inequality also holds in a neighborhood around $\boldsymbol{k^*}$.

Next, we know that by definition, type-2 firms' profits are

$$E\pi_2(k_2) - c_{k_2}k_2 := \int_{K}^{1} (\bar{p} - c_2)k_2 f(\theta)d\theta - c_{k_2}k_2$$

Solving the integrals and rearranging, we arrive at

$$E\pi_2(k_2) - c_{k_2}k_2 := (\bar{p} - c_2)k_2\left[1 - F(K)\right] - c_{k_2}k_2$$

Taking F.O.C. w.r.t. k_2

$$\frac{\partial E\pi_2(k_2) - c_{k_2}k_2}{\partial k_2} = (\bar{p} - c_2)\left[1 - F(K)\right] - (\bar{p} - c_2)k_2 f(K) - c_{k_2}$$

Evaluated at the equilibrium, reduces to

$$\left.\frac{\partial E\pi_2(k_2) - c_{k_2}k_2}{\partial k_2}\right|_{\boldsymbol{k^*}} = (\bar{p} - c_2)\left[1 - \left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)\right] - (\bar{p} - c_2)k_2^* f(K^*) - c_{k_2}$$

Rearranging, we arrive at

$$\left.\frac{\partial E\pi_2(k_2) - c_{k_2}k_2}{\partial k_2}\right|_{\boldsymbol{k^*}} = -(\bar{p} - c_2)k_2^* f(K^*)$$

Since $\bar{p} > c_2$, then $\left.\frac{\partial E\pi_2(k_2) - c_{k_2}k_2}{\partial k_2}\right|_{\boldsymbol{k^*}} < 0$. Provided that the profit function is twice continuously differentiable, this inequality also holds in a neighborhood around $\boldsymbol{k^*}$. □

Proof of Lemma 3: at an interior solution, as shown in the proof of Proposition A.1, $K = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$ (see equation A.5).

First, we find $\frac{\partial K}{\partial \bar{p}}$. Applying the chain rule, $\frac{\partial K}{\partial \bar{p}} = F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)\frac{c_{k_2}}{(\bar{p}-c_2)^2}$.

By the inverse function theorem, we know that $F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) = \frac{1}{f\left(F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)\right)}$, where $f(\cdot)$ is the pdf associated to the random variable θ.

That is, $F'(\cdot) = f(\cdot)$. That is, $F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) = \frac{1}{f(K)}$. Therefore, $\frac{\partial K}{\partial \bar{p}} = \frac{\frac{c_{k_2}}{(\bar{p}-c_2)^2}}{f(K)} > 0$.

Second, we find $\frac{\partial K}{\partial c_2}$. Applying the chain rule, $\frac{\partial K}{\partial c_2} = F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)\frac{-c_{k_2}}{(\bar{p}-c_2)^2}$.

By the inverse function theorem, we know that $F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) = \frac{1}{f\left(F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)\right)}$. That is, $F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) = \frac{1}{f(K)}$. Therefore, $\frac{\partial K}{\partial c_2} = \frac{\frac{-c_{k_2}}{(\bar{p}-c_2)^2}}{f(K)} < 0$.

Finally, we find $\frac{\partial K}{\partial c_{k_2}}$. Applying the chain rule, $\frac{\partial K}{\partial c_{k_2}} = F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)\frac{-1}{\bar{p}-c_2}$.

By the inverse function theorem, we know that $F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) = \frac{1}{f\left(F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)\right)}$. That is, $F^{-1'}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) = \frac{1}{f(K)}$. Therefore, $\frac{\partial K}{\partial c_{k_2}} = \frac{\frac{-1}{\bar{p}-c_2}}{f(K)} < 0$. □

Proof of Proposition 1: consider again equation 2. Rearranging, we arrive at:

$$\begin{aligned}\mathcal{W}(\boldsymbol{k}|\cdot) = &\int_0^{k_1}(\bar{p}-c_1)\theta dF(\theta) + \int_{k_1}^{K}(c_2-c_1)k_1 dF(\theta) + \int_{k_1}^{K}(\bar{p}-c_2)\theta dF(\theta)+\\ &+\int_K^1(\bar{p}-c_1)k_1 dF(\theta) + \int_K^1(\bar{p}-c_2)k_2 dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2\end{aligned}$$

$$\begin{aligned}\mathcal{W}(\boldsymbol{k}|\cdot) = &\int_0^{k_1}(\bar{p}-c_1)\theta dF(\theta) + (c_2-c_1)k_1[F(K)-F(k_1)] + \int_{k_1}^{K}(\bar{p}-c_2)\theta dF(\theta)+\\ &+(\bar{p}-c_1)k_1[1-F(K)] + (\bar{p}-c_2)k_2[1-F(K)] - c_{k_1}k_1 - c_{k_2}k_2\end{aligned} \tag{A.6}$$

Our goal is to find the vector $\boldsymbol{k}$ that maximizes equation A.6. We look for both interior solutions (which are the same as in the competitive market case) and corner solutions (which are not).

<u>Interior solution</u>: The Fist Order Conditions (F.O.C.) at an interior solution are $\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_i} = 0$ for $i \in \{1, 2\}$. First,

$$\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2} = 0$$

which implies that,

$(\bar{p}-c_2)Kf(K)+(c_2-c_1)k_1f(K)-(\bar{p}-c_1)k_1f(K)+(\bar{p}-c_2)[1-F(K)]-(\bar{p}-c_2)k_2f(K)-c_{k_2}=0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_2)[1-F(K)]-(\bar{p}-c_2)k_2f(K)-c_{k_2}=0 \Leftrightarrow (\bar{p}-c_2)(K-k_1-k_2)f(K)+(\bar{p}-c_2)[1-F(K)]-c_{k_2}=0 \Leftrightarrow (\bar{p}-c_2)(K-k_1-k_2)f(K)+(\bar{p}-c_2)[1-F(K)]-c_{k_2}=0 \Leftrightarrow (\bar{p}-c_2)[1-F(K)]-c_{k_2}=0.$

Solving for K, we arrive at:

$$K = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$$

Next, the second F.O.C. is

$$\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1}=0$$

which implies that,
$(\bar{p}-c_1)k_1f(k_1)+(\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)k_1f(k_1)+(c_2-c_1)[F(K)-F(k_1)]+(c_2-c_1)k_1[f(K)-f(k_1)]+(\bar{p}-c_1)[1-F(K)]-(\bar{p}-c_1)k_1f(K)-(\bar{p}-c_2)k_2f(K)-c_{k_1}=0 \Leftrightarrow (c_2-c_1)k_1f(k_1)+(\bar{p}-c_2)Kf(K)+(c_2-c_1)[F(K)-F(k_1)]+(c_2-c_1)k_1[f(K)-f(k_1)]+(\bar{p}-c_1)[1-F(K)]-(\bar{p}-c_1)k_1f(K)-(\bar{p}-c_2)k_2f(K)-c_{k_1}=0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)F(K)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1}=0$. Recall that $K=F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$. Then, $(\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)F(K)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1}=0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1}=0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)+c_{k_2}-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1}=0 \Leftrightarrow (\bar{p}-c_2)f(K)(K-k_1-k_2)-(\bar{p}-c_2)+c_{k_2}-(c_2-c_1)F(k_1)+(\bar{p}-c_1)-c_{k_1}=0 \Leftrightarrow (c_2-c_1)F(k_1)=c_2-c_1+c_{k_2}-c_{k_1}$.

Solving for k_1, we arrive at:

$$k_1 = F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \tag{A.7}$$

Therefore, k_2 is just given by the difference between K and k_1. That is,

$$k_2 = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) - F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \tag{A.8}$$

Notice that A.7 and A.8 are exactly the same as A.3 and A.4 respectively. Therefore, we know that the solution given by these two equations is unique (due to strict monotonicity of $F(\cdot)$). Again, we focus on the interior solution case (we deal with corner solutions in Appendix B). Thus, for the interior solution case, the same conditions are required as those required in the competitive market interior solution case. We need to show that the previous equations are actually a solution to the maximization problem. For that purpose, we use the Second partial derivative test, which allow us to determine that the solution obtained is actually a (global) maximum.

The Hessian matrix of $\mathcal{W}(\boldsymbol{k}|\cdot)$, denoted $H(\boldsymbol{k})$, is as follows:

$$H(\boldsymbol{k}) = \begin{pmatrix} \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1^2} & \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1 \partial k_2} \\ \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2 \partial k_1} & \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2^2} \end{pmatrix} = \begin{pmatrix} -(\bar{p}-c_2)f(K)-(c_2-c_1)f(k_1) & -(\bar{p}-c_2)f(K) \\ -(\bar{p}-c_2)f(K) & -(\bar{p}-c_2)f(K) \end{pmatrix}$$

Let us denote $D(\boldsymbol{k})$ the determinant of the Hessian matrix. That is, $D(\boldsymbol{k}) \equiv \det(D(\boldsymbol{k})) = [-(\bar{p}-c_2)f(K)-(c_2-c_1)f(k_1)][-(\bar{p}-c_2)f(K)]-[-(\bar{p}-c_2)f(K)][-(\bar{p}-c_2)f(K)]$. Rearranging, we arrive at $D(\boldsymbol{k})=(c_2-c_1)(\bar{p}-c_2)f(k_1)f(K)$.

Notice that for an interior solution, we have that $D(\boldsymbol{k})>0$; moreover, for an interior solution, we have that $\frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1^2}<0$. Therefore, by the second partial derivative test, we know that these two types of solutions are maxima of the welfare function.

Corner solution: So far, we checked that an interior solution (which is the same as in the competitive market case) do also maximizes welfare. Moreover, by the proof of Proposition A.1, we know that $k_1 + k_2 < 1$.

However, in the Proposition we also include that there exist some parameters for which the vector of socially-optimal aggregate capacities satisfies $k_1 + k_2 = 1$. In particular, we consider the following candidate solution $k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ and $k_2 = 1 - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ (notice that k_2 is obtained as the residual capacity, as shown in the maximization problem). Thus, we need to show that that there exist some parameters for which the welfare function at $k_1 + k_2 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) + 1 - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) = 1$ is greater than the welfare function at any other parameters for which $k_1 + k_2 < 1$ (the previous solution).

Thus, we compare the case $k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ and $k_2 = 1 - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ with an interior solution to the social-planner's problem, denoted as $k_1' \in (0,1)$ and $k_2' \in (0,1)$. In particular, $\mathcal{W}(\boldsymbol{k}|\cdot)\,|_{k_1+k_2=1} = \int_0^{k_1}(\bar{p} - c_1)\theta dF(\theta) + (c_2 - c_1)k_1[1 - F(k_1)] + \int_{k_1}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2$

Thus, we need show that $\int_0^{k_1}(\bar{p} - c_1)\theta dF(\theta) + (c_2 - c_1)k_1[1 - F(k_1)] + \int_{k_1}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2 > \int_0^{k_1'}(\bar{p} - c_1)\theta dF(\theta) + (c_2 - c_1)k_1'[F(K') - F(k_1')] + \int_{k_1'}^{K'}(\bar{p} - c_2)\theta dF(\theta) + (\bar{p} - c_1)k_1'[1 - F(K')] + (\bar{p} - c_2)k_2'[1 - F(K')] - c_{k_1}k_1' - c_{k_2}k_2'$. Since $k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ and $k_1' = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$, then $k_1 = k_1'$. Then $(c_2 - c_1)k_1[1 - F(k_1)] + \int_{k_1}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_2}k_2 > (c_2 - c_1)k_1'[F(K') - F(k_1')] + \int_{k_1'}^{K'}(\bar{p} - c_2)\theta dF(\theta) + (\bar{p} - c_1)k_1'[1 - F(K')] + (\bar{p} - c_2)k_2'[1 - F(K')] - c_{k_2}k_2' \iff \int_{K'}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_2}k_2 > (\bar{p} - c_2)k_1'[1 - F(K')] + (\bar{p} - c_2)k_2'[1 - F(K')] - c_{k_2}k_2' \iff \int_{K'}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_2}k_2 > (\bar{p} - c_2)K'[1 - F(K')] - c_{k_2}k_2'$. Recall that $F(K') = \left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$. Then, $\int_{K'}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_2}k_2 > (\bar{p} - c_2)K'[1 - F(K')] - c_{k_2}k_2' \iff \int_{K'}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_2}k_2 > c_{k_2}K' - c_{k_2}k_2' \iff \int_{K'}^1(\bar{p} - c_2)\theta dF(\theta) - c_{k_2}k_2 > c_{k_2}k_1' \iff \int_{K'}^1(\bar{p} - c_2)\theta dF(\theta) > c_{k_2}k_1' + c_{k_2}k_2 \iff (\bar{p} - c_2)\int_{K'}^1 \theta dF(\theta) > c_{k_2}$.

Therefore, if $(\bar{p} - c_2)\int_{K'}^1 \theta dF(\theta) > c_{k_2}$, then $k_1 + k_2 = 1$, with $k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ is the (unique) socially-optimal solution. □

Proof of Theorem 1: in the proof of Proposition 1 we provide the conditons under which $k_1 + k_2 = 1$ is the socially-optimal solution. Therefore, if these conditions are fulfilled, and since the competitive market always yield a solution such that $k_1 + k_2 < 1$, then, the competitive market does not achieve the socially-optimal solution. □

Proof of Lemma 4: let $\varphi(\cdot) \equiv \int_{K^*}^1 \theta dF(\theta)$. Then, $\mathcal{B}(\cdot) = (\bar{p} - c_2)\varphi(\cdot)$.

First, we show that $\mathcal{B}(\cdot)$ is strictly increasing in $\bar{p}$. Taking the derivative of $\mathcal{B}(\cdot)$ with respect to (hereafter w.r.t.) $\bar{p}$, we arrive at $\frac{\partial \mathcal{B}(\cdot)}{\partial \bar{p}} = \varphi(\cdot) + (\bar{p} - c_2)\frac{\partial \varphi(\cdot)}{\partial \bar{p}}$. We know that, when $k_i^* > 0$ for $i \in \{1, 2\}$, then $K^* = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$ —see the proof of Propostion A.1. To get $\frac{\partial \varphi(\cdot)}{\partial \bar{p}}$ that is, in order to get $\frac{\partial}{\partial \bar{p}}\left[\int_{K^*}^1 \theta dF(\theta)\right]$, we apply the Leibniz integral rule*. Then, $\frac{\partial}{\partial \bar{p}}\left[\int_{K^*}^1 \theta dF(\theta)\right] = -K^* f(K^*)\frac{\partial K^*}{\partial \bar{p}}$, where $f(\cdot)$ is the p.d.f. of the demand (θ). By Lemma 3, $k_2^* > 0$ implies that $\frac{\partial K^*}{\partial \bar{p}} > 0$. Therefore, $\frac{\partial \mathcal{B}(\cdot)}{\partial \bar{p}} = \int_{K^*}^1 \theta dF(\theta) - (\bar{p} - c_2)K^* f(K^*)\frac{\frac{c_{k_2}}{(\bar{p} - c_2)^2}}{f(K^*)}$. Cancelling out terms, we arrive at $\frac{\partial \mathcal{B}(\cdot)}{\partial \bar{p}} = \int_{K^*}^1 \theta dF(\theta) - K^*\frac{c_{k_2}}{\bar{p} - c_2}$. Notice that $\int_{K^*}^1 \theta dF(\theta) > K^*$, while $K^*\frac{c_{k_2}}{\bar{p} - c_2} < K^*$, since

* $\frac{\partial}{\partial x}\int_{h(x)}^{g(x)} r(u)du = r(h(x))h'(x) - r(g(x))g'(x)$

$\bar{p} - c_2 > c_{k_2}$ is required when $k_i^* > 0$ for $i \in \{1, 2\}$ —as shown in the proof of Propostion A.1. Therefore, $\frac{\partial \mathcal{B}(\cdot)}{\partial \bar{p}} > 0$.

Second, we show that $\mathcal{B}(\cdot)$ is strictly decreasing in c_2. Taking the derivative of $\mathcal{B}(\cdot)$ w.r.t. c_2, we arrive at $\frac{\partial \mathcal{B}(\cdot)}{\partial c_2} = -\varphi(\cdot) + (\bar{p} - c_2)\frac{\partial \varphi(\cdot)}{\partial c_2}$. We know that, when $k_i^* > 0$ for $i \in \{1, 2\}$, then $K^* = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$ —see the proof of Propostion A.1. Again, to get $\frac{\partial \varphi(\cdot)}{\partial c_2}$ that is, in order to get $\frac{\partial}{\partial c_2}\left[\int_{K^*}^1 \theta dF(\theta)\right]$, we apply the Leibniz integral rule. Then, $\frac{\partial}{\partial c_2}\left[\int_{K^*}^1 \theta dF(\theta)\right] = -K^* f(K^*)\frac{\partial K^*}{\partial c_2}$, where $f(\cdot)$ is the p.d.f. of the demand (θ). By Lemma 3, we know that $\frac{\partial K^*}{\partial c_2} =< 0$. Then, $\frac{\partial \varphi(\cdot)}{\partial c_2} = \frac{\partial}{\partial c_2}\left[\int_{K^*}^1 \theta dF(\theta)\right] = -K^* f(K^*)\frac{\partial K^*}{\partial c_2} > 0$. Therefore, $\frac{\partial \mathcal{B}(\cdot)}{\partial c_2} = -\varphi(\cdot) + (\bar{p} - c_2)\frac{\partial \varphi(\cdot)}{\partial c_2} = -\varphi(\cdot) + (\bar{p} - c_2)K^* f(K^*)\frac{\frac{c_{k_2}}{(\bar{p}-c_2)^2}}{f(K^*)}$. Cancelling out, we arrive at $\frac{\partial \mathcal{B}(\cdot)}{\partial c_2} = -\varphi(\cdot) + K^* \frac{c_{k_2}}{\bar{p}-c_2}$. As shown in the proof of Propostion A.1, if $k_i^* > 0$ for $i \in \{1, 2\}$, then $\bar{p} - c_2 > c_{k_2}$ is required. Therefore, while $\varphi(\cdot) \in (K^*, 1)$, $K^* \frac{c_{k_2}}{\bar{p}-c_2} < K^*$. Thus, $\frac{\partial \mathcal{B}(\cdot)}{\partial c_2} = -\varphi(\cdot) + K^* \frac{c_{k_2}}{\bar{p}-c_2} < 0$.

Third, we show that $\mathcal{B}(\cdot)$ is strictly decreasing in c_{k_2}. Taking the derivative of $\mathcal{B}(\cdot)$ w.r.t. c_{k_2}, we arrive at $\frac{\partial \mathcal{B}(\cdot)}{\partial c_{k_2}} = -\varphi(\cdot) + (\bar{p} - c_2)\frac{\partial \varphi(\cdot)}{\partial c_{k_2}}$.

We know that, when $k_i^* > 0$ for $i \in \{1, 2\}$, then $K^* = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$ —see the proof of Propostion A.1. Again, to get $\frac{\partial \varphi(\cdot)}{\partial c_{k_2}}$ that is, in order to get $\frac{\partial}{\partial c_{k_2}}\left[\int_{K^*}^1 \theta dF(\theta)\right]$, we apply the Leibniz integral rule. Then, $\frac{\partial}{\partial c_{k_2}}\left[\int_{K^*}^1 \theta dF(\theta)\right] = -K^* f(K^*)\frac{\partial K^*}{\partial c_{k_2}}$, where $f(\cdot)$ is the p.d.f. of the demand (θ). By Lemma 3, we know that $\frac{\partial K^*}{\partial c_{k_2}} < 0$. The previous implies that $\frac{\partial \varphi(\cdot)}{\partial c_{k_2}} = \frac{\partial}{\partial c_{k_2}}\left[\int_{K^*}^1 \theta dF(\theta)\right] = -K^* f(K^*)\frac{\partial K^*}{\partial c_{k_2}} > 0$. Therefore, $\frac{\partial \mathcal{B}(\cdot)}{\partial c_{k_2}} = -\varphi(\cdot) + (\bar{p} - c_2)\frac{\partial \varphi(\cdot)}{\partial c_{k_2}} = -\varphi(\cdot) + (\bar{p} - c_2)K^* f(K^*)\frac{\frac{1}{\bar{p}-c_2}}{f(K^*)}$. Cancelling out, we arrive at $\frac{\partial \mathcal{B}(\cdot)}{\partial c_{k_2}} = -\varphi(\cdot) + K^*$. Moreover, because $F(\cdot)$ is strictly increasing, $\varphi(\cdot) \in (K^*, 1)$. Thus, $\frac{\partial \mathcal{B}(\cdot)}{\partial c_{k_2}} = -\varphi(\cdot) + K^* < 0$. □

Proof of Proposition 2: assume for the sake of contradiction (a.f.s.o.c.) that $\exists\ c_i$, c_{k_i} and $F(\cdot)$ such that $\forall\ \bar{p}$ we have that $(\bar{p} - c_2)\int_K^1 \theta dF(\theta) \leq c_{k_2}$. By Lemma 4, we know that $\frac{\partial \mathcal{B}(\cdot)}{\partial \bar{p}} > 0$. However, c_{k_2} does not change with $\bar{p}$. Therefore, as $\bar{p}$ increases, $\mathcal{B}(\cdot)$ also increases. Eventually, for $\bar{p}$ sufficiently high, $\mathcal{B}(\cdot) > c_{k_2}$, which implies that $(\bar{p} - c_2)\int_K^1 \theta dF(\theta) > c_{k_2}$, which is a contradiction. □

Proof of Proposition 3: a.f.s.o.c. that $\exists\ c_i$, c_{k_1}, $\bar{p}$ and $F(\cdot)$ such that $\forall\ c_{k_2}$ we have that $(\bar{p} - c_2)\int_K^1 \theta dF(\theta) \leq c_{k_2}$. By Lemma 4, we know that $\frac{\partial \mathcal{B}(\cdot)}{\partial c_{k_2}} < 0$. Eventually, for c_{k_2} sufficiently close to (but strictly greater than) 0, $\mathcal{B}(\cdot) > c_{k_2}$, which implies that $(\bar{p} - c_2)\int_K^1 \theta dF(\theta) > c_{k_2}$, which is a contradiction. □

Proof of Proposition 4: a.f.s.o.c. that $\exists\ c_i$, c_{k_i} and $\bar{p}$ such that for all distributions of the random variable θ we have that $(\bar{p} - c_2)\int_K^1 \theta dF(\theta) \leq c_{k_2}$. This implies that $\mathcal{B}(\cdot) \leq c_{k_2}$. That is, $(\bar{p} - c_2)\int_K^1 \theta dF(\theta) < c_{k_2}$. As shown in the proof of Propostion A.1, if $k_i^* > 0$ for $i \in \{1, 2\}$, then $\bar{p} - c_2 > c_{k_2}$ is required. Moreover, we know that $\int_K^1 \theta dF(\theta) \in (K^*, 1)$.

However, if the distribution of the random variable θ is long left-tailed, $f_1(\theta)$ (the p.d.f. of the random variable) is almost flat in the left region, which implies that the c.d.f. is also almost flat in the left region. If so, the inverse of the c.d.f., denoted as $F_1^{-1}(\theta)$ will be extremely close to 1 for $\theta > 0$. If so, $\int_K^1 \theta dF(\theta)$ will also be extremely close to 1. That is, we can find some distribution for which $F_1^{-1}(\theta)$ is arbitarily closer to 1 for all θ in the support of the distribution. That is, we can find a distribution for which $\int_K^1 \theta dF_1(\theta)$ is arbitrarily closer to 1. Say, $\int_K^1 \theta dF_1(\theta) = 1 - \epsilon$, where ϵ is a (arbitrarily) small and positive number.

Because $(\bar{p} - c_2) > c_{k_2}$ when $k_i^* > 0$ for $i \in \{1, 2\}$ (see the proof of Propostion A.1), if ϵ is infinitesimally small, $(\bar{p} - c_2) * (1 - \epsilon) > c_{k_2}$. That is, $(\bar{p} - c_2) \int_K^1 \theta dF_1(\theta) > c_{k_2}$, which implies that, for the distribution of the random variable θ whose c.d.f. is $F_1(\cdot)$, $(\bar{p} - c_2) \int_K^1 \theta dF(\theta) > c_{k_2}$, which is a contradiction. □

Proof of Proposition A.5: let us begin with type-1 firms. By definition 4, we know that in equilibrium, $E\pi_1(k_1) - c_{k_1}k_1 + xk_1 = 0$. That is, $E\pi_1(k_1|\theta < k_1) + E\pi_1(k_1|k_1 \leq \theta < K^T) + E\pi_1(k_1|K^T \leq \theta) - c_{k_1}k_1 + xk_1 = 0$. By Lemma 1, we know that $E\pi_1(k_1|\theta < k_1) = 0$. Moreover, by Lemma 1 we also know that if $k_1 \leq \theta < K^T$, then $p^* = c_2$, and if $K^T \leq \theta$, then $p^* = \bar{p}$; in both cases, all type-1 firms serve the demand. Therefore, the free-entry equilibrium condition can be rewritten as follows

$$\int_{k_1}^{K^T} (c_2 - c_1)k_1 dF(\theta) + \int_{K^T}^{1} (\bar{p} - c_1)k_1 dF(\theta) - c_{k_1}k_1 + xk_1 = 0$$

Solving the integrals and rearranging, we arrive at:

$$(c_2 - c_1)F(k_1) = (\bar{p} - c_1) - (\bar{p} - c_2)F(K^T) - c_{k_1} + x \tag{A.9}$$

Next, we deal with type-2 firms. By definition 4, we know that in equilibrium, $E\pi_2(k_2) - c_{k_2}k_2 + xk_2 = 0$. That is, $E\pi_2(k_2|\theta < k_2) + E\pi_1(k_2|k_2 \leq \theta < K^T) + E\pi_2(k_2|K^T \leq \theta) - c_{k_2}k_2 + xk_2 = 0$. By Lemma 1, we know that $E\pi_2(k_2|\theta < k_2) = 0$ and that $E\pi_2(k_2|k_1 \leq \theta < K^T) = 0$. Moreover, by Lemma 1 we also know that if $K^T \leq \theta$, then $p^* = \bar{p}$; in this case, all type-2 firms serve the demand. Therefore, the free-entry equilibrium condition can be rewritten as follows

$$\int_{K^T}^{1} (\bar{p} - c_2)k_2 dF(\theta) - c_{k_2}k_2 + xk_2 = 0$$

Solving the integrals and rearranging, we arrive at:

$$F^{-1}\left(1 - \frac{c_{k_2} - x}{\bar{p} - c_2}\right) = K^T \tag{A.10}$$

Replacing A.10 into A.9, we arrive at:

$$(c_2 - c_1)F(k_1) = (\bar{p} - c_1) - (\bar{p} - c_2)F\left[F^{-1}\left(1 - \frac{c_{k_2} - x}{\bar{p} - c_2}\right)\right] - c_{k_1} + x$$

Solving for k_1 we arrive at:

$$k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) \tag{A.11}$$

We know that, by construction, $K^T = k_1 + k_2$. Thus, replacing A.11 into this condition, we arrive at

$$k_2 = K^T - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) \tag{A.12}$$

Notice that k_1 is similar as in the case in which capacity is not regulated, while k_2 is again defined as the residual ($k_2 = K^T - k_1$). Thus, both k_i for $i \in \{1,2\}$ are unique.

Finally, to get the capacity payment, we need to solve for x in equation A.10:

$$x = c_{k_2} - (\bar{p} - c_2)\left[1 - F(K^T)\right] \tag{A.13}$$

Notice that $x \geq 0 \iff K^T \geq F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$. Thus, we use $F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$ as our candidate threshold $\tilde{K}$.

We focus on the conditions under which there is an interior solution (we deal with corner solutions in Appendix B), i.e. $k_i \in (0,1)$ for $i \in \{1,2\}$. First, $k_1 > 0$ implies that $F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) > 0$, which requires that $c_2 - c_1 > c_{k_1} - c_{k_2}$. Then, $k_1 > 0$ and $k_2 = K^T - k_1$. Therefore, an equilibrium requires that $k_2 \geq 0$, that is, $K^T \geq k_1 = F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$.

Moreover, an interior solution also requires $c_{k_2}(c_2 - c_1) < (\bar{p} - c_2)(c_{k_1} - c_{k_2})$. If so, then $F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) > F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) = k_1$ which guarantees that $k_2 > 0$. Therefore, the threshold $\tilde{K} = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$ guarantees that, in equilibrium, $k_2 > 0$ and $x \geq 0$. That is, every interior equilibrium requires that $K^T \geq F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$. □

Proof of Theorem 2: in a competitive market with regulated capacity with $K^T = \hat{K} = 1$, we know that, in equilibrium, $k_1^{*,r} = F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$, $k_2^{*,r} = 1 - F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$, and $x^{*,r} = c_{k_2}$ —see equations (A.11), (B.7) and (A.13), which are evaluated at $K^T = 1$. From the proof of Proposition 1, we know that if $\hat{K} = 1$, then $\hat{k_1} = F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$ and $\hat{k_2} = 1 - F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$. Thus, we have that $k_i^{*,r} = \hat{k}_i$ for $i \in \{1,2\}$. Moreover, in the presence of $x^{*,r} = c_{k_2} > 0$, this implies that there is a transfer from consumers to firms, so no surplus is lost. Therefore, the competitive market with regulated capacity yields the same welfare as the planner's problem, so the solution is socially-optimal. □

Proof of Proposition 6: the price-contingent contract in a market with unregulated capacity, $\mathcal{L}$, is

$$\mathcal{L} = \begin{pmatrix} p_{(\theta<k_1^*)} & p^c_{(k_1^* \leq \theta < K^*)} & p^c_{(K^* \leq \theta)} \\ \text{pr}(\theta < k_1^*) & \text{pr}(k_1^* \leq \theta < K^*) & \text{pr}(K^* \leq \theta) \end{pmatrix} = \begin{pmatrix} -c_1 & -c_2 & -\bar{p} \\ k_1 & k_2 & 1-K \end{pmatrix} \tag{A.14}$$

and the price-contingent contract in a market with regulated capacity, $\mathcal{L}^{\text{m}}$, is

$$\mathcal{L}^{\text{r}} = \begin{pmatrix} p^c_{(\theta<k_1^{*,\text{r}})} & p^c_{(k_1^{*,\text{r}} \leq \theta \leq 1)} \\ \text{pr}(\theta < k_1^{*,\text{r}}) & \text{pr}(k_1^{*,\text{r}} \leq \theta \leq 1) \end{pmatrix} = \begin{pmatrix} -c_1 - x & -c_2 - x \\ k_1 & 1 - k_1 \end{pmatrix} \tag{A.15}$$

where p^c_s is the equilibrium market price in state s.

(Clarification: throughtout this proof, we write the prices in negative to capture the fact that, the greater the price is, the worst is the outcome for the consumer).

By equations A.3 and A.11, $k_1^* = k_1^{*,\text{r}}$. Bearing that in mind, first, we show that both price-contingent contracts have the same mean. Let us begin with the mean of $\mathcal{L}$:

$$E(\mathcal{L}) = c_1 k_1 + c_2 k_2 + \bar{p}(1 - K)$$

$$E(\mathcal{L}) = c_1 k_1 + c_2 k_2 + \bar{p} - \bar{p}k_2 - \bar{p}k_1$$

$$E(\mathcal{L}) = \bar{p} - a_2 k_2 - a_1 k_1$$

where $a_1 \equiv \bar{p} - c_1$ and $a_2 \equiv \bar{p} - c_2$.

Next, we calculate the mean for $\mathcal{L}^{\mathrm{r}}$:

$$E(\mathcal{L}^{\mathrm{r}}) = (c_1 + x)k_1 + (c_2 + x)(1 - k_1)$$

By equation A.13, we know that $x = c_{k_2}$ when $K^T = 1$. Then:

$$E(\mathcal{L}^{\mathrm{r}}) = (c_1 + c_{k_2})k_1 + (c_2 + c_{k_2})(1 - k_1)$$

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_{k_2} k_1 + c_2 + c_{k_2} - c_2 k_1 - c_{k_2} k_1$$

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_2 + c_{k_2} - c_2 k_1$$

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_2 + a_2 \frac{c_{k_2}}{a_2} - c_2 k_1$$

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_2 + a_2 \left(\frac{c_{k_2}}{a_2} + 1 - 1 \right) - c_2 k_1$$

By equation A.5, we know that $K = F^{-1}\left(1 - \frac{c_{k_2}}{a_2}\right)$. Since θ follows a uniform distribution (by assumption), then $K = 1 - \frac{c_{k_2}}{a_2}$. Therefore:

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_2 + a_2 (1 - K) - c_2 k_1$$

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_2 + a_2 - a_2 K - c_2 k_1$$

$$E(\mathcal{L}^{\mathrm{r}}) = c_1 k_1 + c_2 + \bar{p} - c_2 - \bar{p} k_1 - \bar{p} k_2 + c_2 k_1 + c_2 k_2 - c_2 k_1$$

$$E(\mathcal{L}^{\mathrm{r}}) = \bar{p} - a_2 k_2 - a_1 k_1$$

Thus $E(\mathcal{L}) = \bar{p} - a_2 k_2 - a_1 k_1$ and $E(\mathcal{L}^{\mathrm{r}}) = \bar{p} - a_2 k_2 - a_1 k_1$. That is, $E(\mathcal{L}) = E(\mathcal{L}^{\mathrm{r}})$.

To continue with the proof, Figure A.1 is helpful. Notice that this Figure captures the two possible scenarios that we may have: $c_2 < c_1 + c_{k_2}$ in subfigure A.1a; and $c_2 > c_1 + c_{k_2}$ in subfigure A.1b. However, as we will see below, both scenarios are equivalent, since area of A (ΔA) and area of B (ΔB) are equal.

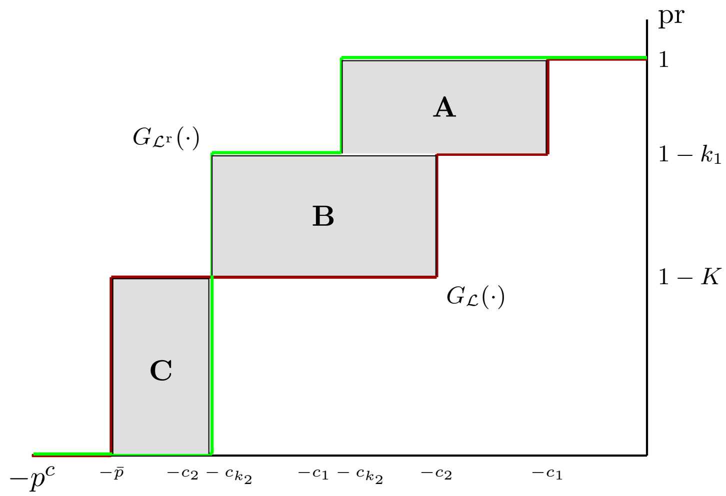
Let us begin with the case included in subfigure A.1a (i.e. $c_2 \leq c_1 + c_{k_2}$). In this subfigure, the cdf associated with the price-contingent contract in the unregulated capacity case, $G_{\mathcal{L}}(z)$, is given by the red (darker) line, and cdf associated with the price-contingent contract in the regulated capacity case, $G_{\mathcal{L}^{\mathrm{r}}}(z)$, is given by the green (lighter) line.

First, it follows that if $p^c \geq c_2 + c_{k_2}$, (or, in subfigure A.1a terms, if $-p^c \leq -c_2 - c_{k_2}$) then $\int_0^{p^c} G_{\mathcal{L}}(z)dz > \int_0^{p^c} G_{\mathcal{L}^{\mathrm{r}}}(z)dz$.

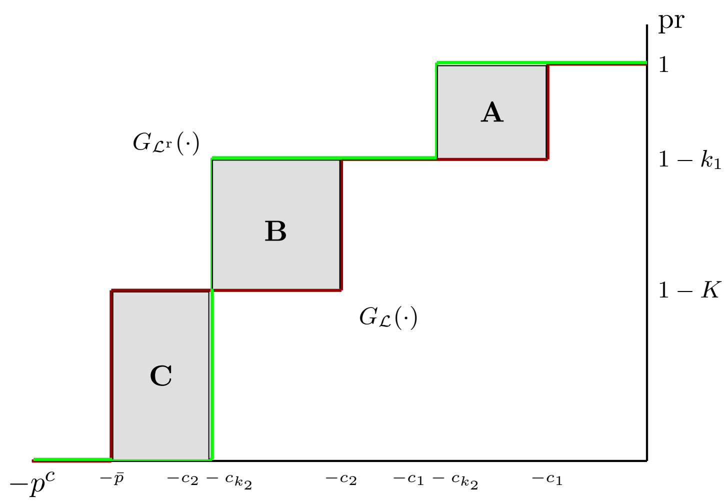
Next, we claim that, the area of A plus area of B, $\Delta A + \Delta B$, is equal to the area of C, ΔC. Proof: solving for these areas: $\Delta A + \Delta B = c_{k_2}(1 - 1 + k_1) + c_{k_2}(1 - k_1 - 1 + K) \iff \Delta A + \Delta B = c_{k_2} K$. Next, $\Delta C = (\bar{p} - c_2 - c_{k_2})(1 - K) \iff \Delta C = a_2 - c_{k_2} - (a_2 - c_{k_2})K \iff \Delta C = a_2(1 - K) - c_{k_2}(1 - K) \iff \Delta C = a_2\left(1 - 1 + \frac{c_{k_2}}{a_2}\right) - c_{k_2}(1 - K) \iff \Delta C = c_{k_2} - c_{k_2}(1 - K) \iff \Delta C = c_{k_2} K$. That is, $\Delta A + \Delta B = c_{k_2} K$ and $\Delta C = c_{k_2} K$, which implies that $\Delta A + \Delta B = \Delta C$. $\square$

Using subfigure A.1a and the fact that $\Delta A + \Delta B = \Delta C$, it follows that if $p^c \in (c_1, c_2 + c_{k_2}, c_1]$ (or, in subfigure A.1a terms, if $-p^c \in [-c_2 - c_{k_2}, -c_1)$), then $\int_0^{p^c} G_{\mathcal{L}}(z)dz > \int_0^{p^c} G_{\mathcal{L}^{\mathrm{r}}}(z)dz$. Finally, if $p^c \leq c_1$ (or, in subfigure A.1a terms, if $-p^c \geq -c_1$), then $\int_0^{p^c} G_{\mathcal{L}}(z)dz \geq \int_0^{p^c} G_{\mathcal{L}^{\mathrm{r}}}(z)dz$. Thus, $\forall p^c \in \mathcal{C}$, we have that $\int_0^{p^c} G_{\mathcal{L}}(z)dz \geq \int_0^{p^c} G_{\mathcal{L}^{\mathrm{r}}}(z)dz$.

Figure A.1: Cdf associated to $\mathcal{L}$ (green) and cdf associated to $\mathcal{L}^r$ (red)



(a) $c_2 \leq c_1 + c_{k_2}$



(b) $c_2 > c_1 + c_{k_2}$

Next, we deal with the case included in subfigure A.1b (i.e. $c_2 > c_1 + c_{k_2}$). Again, the cdf associated with the price-contingent contract in the unregulated capacity case, $G_{\mathcal{L}}(z)$, is given by the red (darker) line, and cdf associated with the price-contingent contract in the regulated capacity case, $G_{\mathcal{L}^r}(z)$, is given by the green (lighter) line.

First, it follows that if $p^c \geq c_2 + c_{k_2}$, (or, in subfigure A.1b terms, if $-p^c \leq -c_2 - c_{k_2}$) then $\int_0^{p^c} G_{\mathcal{L}}(z)dz > \int_0^{p^c} G_{\mathcal{L}^r}(z)dz$.

Then, notice that $\Delta A + \Delta B \leq \Delta C$ (the proof is exactly the same as in the previous case).

Therefore, as in the previous case, it follows that if $p^c < c_2 + c_{k_2}$ (or, in subfigure A.1b terms, if $-p^c \geq -c_2 - c_{k_2}$), then $\int_0^{p^c} G_{\mathcal{L}}(z)dz \geq \int_0^{p^c} G_{\mathcal{L}^r}(z)dz$. Thus, $\forall p^c \in \mathcal{C}$, we have that $\int_0^{p^c} G_{\mathcal{L}}(z)dz \geq \int_0^{p^c} G_{\mathcal{L}^r}(z)dz$. $\square$

Proof of Proposition 7: from the previous proof, we know that $E(\mathcal{L}) = \bar{p} - a_1k_1 - a_2k_2$ and

$E(\mathcal{L}^{\mathrm{r}}) = (c_1 - c_2)k_1 + c_2 + c_{k_2}$, where $a_i \equiv \bar{p} - c_i$. Thus, the variance of $\mathcal{L}$ is

$$Var(\mathcal{L}) = k_1[c_1 - \bar{p} + a_1k_1 + a_2k_2]^2 + k_2[c_2 - \bar{p} + a_1k_1 + a_2k_2]^2 + (1 - k_1 - k_2)[\bar{p} - \bar{p} + a_1k_1 + a_2k_2]^2$$

Rearranging,

$$Var(\mathcal{L}) = k_1\bar{p}^2 + k_2\bar{p}^2 + k_2c_2^2 + 2k_2k_1\bar{p}c_1 + 2k_2k_1\bar{p}c_2 + k_2^2 2\bar{p}c_2 + k_1^2 2\bar{p}c_1 -$$

$$-2k_1\bar{p}c_1 - 2k_2\bar{p}c_2 - k_1^2\bar{p}^2 - k_2^2\bar{p}^2 - k_2^2c_2^2 - 2k_2k_1\bar{p}^2 - 2k_2k_1c_1c_2;$$

$$Var(\mathcal{L}) = k_1a_1^2 - k_1^2a_1^2 + k_2a_2^2 - k_2^2a_2^2 - 2k_2a_2a_1k_1$$

Next, the variance of $\mathcal{L}^{\mathrm{r}}$ is

$$Var(\mathcal{L}^{\mathrm{r}}) = k_1[c_1 + c_{k_2} - (c_1 - c_2)k_1 - c_2 - c_{k_2}]^2 + (1 - K)[c_2 + c_{k_2} - (c_1 - c_2)k_1 - c_2 - c_{k_2}]^2$$

Rearranging,

$$Var(\mathcal{L}^{\mathrm{r}}) = (k_1 - k_1^2 - k_1^2k_2)(c_1^2 + c_2^2 - 2c_1c_2)$$

By assumption, we know that $\bar{p} > c_2$ (see footnote 12). If we were to assume that $\bar{p} = c_2$, then, $Var(\mathcal{L}) = k_1a_1^2 - k_1^2a_1^2 \iff Var(\mathcal{L}) = (k_1 - k_1^2)(c_2^2 + c_1^2 - 2c_2c_1)$. In addition, we know that $Var(\mathcal{L}^{\mathrm{r}}) = (k_1 - k_1^2 - k_1^2k_2)(c_1^2 + c_2^2 - 2c_1c_2)$. Then, if $\bar{p} = c_2$, then $Var(\mathcal{L}) = (k_1 - k_1^2)(c_2^2 + c_1^2 - 2c_2c_1) > (k_1 - k_1^2 - k_1^2k_2)(c_1^2 + c_2^2 - 2c_1c_2) = Var(\mathcal{L}^{\mathrm{r}})$.

Next, we show that $\frac{\partial Var(\mathcal{L})}{\partial \bar{p}} > 0$.

$$Var(\mathcal{L}) = k_1\bar{p}^2 + k_2\bar{p}^2 + k_2c_2^2 + 2k_2k_1\bar{p}c_1 + 2k_2k_1\bar{p}c_2 + k_2^2 2\bar{p}c_2 + k_1^2 2\bar{p}c_1 -$$

$$-2k_1\bar{p}c_1 - 2k_2\bar{p}c_2 - k_1^2\bar{p}^2 - k_2^2\bar{p}^2 - k_2^2c_2^2 - 2k_2k_1\bar{p}^2 - 2k_2k_1c_1c_2;$$

$$\frac{\partial Var(\mathcal{L})}{\partial \bar{p}} = 2k_1\bar{p} + 2k_2\bar{p} + 2k_2k_1c_1 + 2k_2k_1c_2 + 2k_2^2c_2 + 2k_1^2c_1 - 2k_1c_1 - 2k_2c_2 - 2k_1^2\bar{p} - 2k_2^2\bar{p} - 4k_2k_1\bar{p} =$$

$$= 2k_1\bar{p} + 2k_2\bar{p} + 2k_2k_1c_1 + 2k_2k_1c_2 + 2k_2^2c_2 + 2k_1^2c_1 - 2k_1c_1 - 2k_2c_2 - 2k_1^2\bar{p} - 2k_2^2\bar{p} - 4k_2k_1\bar{p} =$$

$$= k_1a_1 + k_2a_2 - a_1k_1^2 - a_2k_2^2 - 2k_2k_1\bar{p} + k_2k_1c_2 + k_2k_1c_1 =$$

$$= a_1(k_1 - k_1^2 - k_2k_1) + a_2(k_2 - k_2^2 - k_2k_1) = a_1[k_1(1 - k_1 - k_2)] + a_2[k_2(1 - k_2 - k_1)] > 0$$

since $(1 - k_1 - k_2) > 0$ by Proposition A.1.

Moreover, it is straightforward that $\frac{\partial Var(\mathcal{L}^{\mathrm{r}})}{\partial \bar{p}} = 0$. Therefore, a fortiori, for $\bar{p} > c_2$, then $Var(\mathcal{L}) > Var(\mathcal{L}^{\mathrm{r}})$. □

Proof of Lemma 5:

1. If $\theta < k_1$, the proof is similar to case #1 in the proof of Lemma 1.
2. If $k_1 \leq \theta < k_1 + k_2$, the proof is similar to case #2 in the proof of Lemma 1. The demand is fully served by all type-1 firms and some type-2 firms. By the "market-clearing" condition, then $q^s(p^*; \cdot) = \theta$, that is, $k_1 + \gamma^* k_2 = \theta$, for some $\gamma^* \in [0, 1)$. By equation 5, this implies that $p = c_2$. Therefore, all consumers are served at a price c_2. All type-1 firms serve the demand, so these firms make a positive profit (the price at which they serve is greater than the marginal cost). Type-2 firms that serve the demand make zero profit (because the price is equal to the marginal cost). Type-2 firms that do not serve the demand (the rest of the firms) make zero profit.

3. If $k_1 + k_2 \leq \theta < k_1 + k_2 + k_3$, the proof is similar to the previous case. The demand is fully served by all type-1 and type-2 firms and some type-3 firms. By the "market-clearing" condition, then $q^s(p^*; \cdot) = \theta$, that is, $k_1 + k_2 + \gamma^* k_3 = \theta$, for some $\gamma^* \in [0, 1)$. By equation 5, this implies that $p = c_3$. Therefore, all consumers are served at a price c_3. All type-1 and type-2 firms serve the demand, so these firms make a positive profit (the price at which they serve is greater than the marginal cost). Type-3 firms that serve the demand make zero profit (because the price is equal to the marginal cost). Type-3 firms that do not serve the demand (the rest of the firms) make zero profit.

4. If $\sum_{j=1}^{J-1} k_j \leq \theta < \sum_{j=1}^{J} k_j$, the proof is just an extension of the previous case for some $J \in \{2, 3, \cdots, N\}$.

5. If $K \leq \theta$, the proof is similar to case #3 in the proof of Lemma 1. □

Proof of Lemma 6: a.f.s.o.c. that $\sum_{i=1}^{N} k_i^* < 1$. By Lemma 5, this implies that all firms make zero-profit in the second stage. Thus, if firms expect no profit in the second stage, then $k_i^* = 0$ for all $i \in \{1, 2, \cdots, N\}$, which is a contradiction. □

Proof of Lemma 7:

1. If $q^D(p^*; \theta) < k_1$, the demand is fully served by type-1 firms only. By definition, an equilibrium requires the "market-clearing" condition, which implies that $q^S(p^*; \boldsymbol{k}) = q^D(p^*; \theta)$. In this case, $\gamma^* k_1 = q^D(p^*; \theta)$, for some $\gamma^* \in (0, 1)$ which, by equation 7, implies that $p = c_1$. Therefore, all consumers are served at a price c_1. Type-1 firms that serve the demand make zero profit (because the price is equal to the marginal cost). Firms that do not serve the demand (the rest of the firms) make zero profit.

2. If $q^D(p^*; \theta) = k_1$, the demand is fully served by type-1 firms only. By definition, an equilibrium requires the "market-clearing" condition, which implies that $q^S(p^*; \boldsymbol{k}) = q^D(p^*; \theta)$. In this case, $k_1 = q^D(p^*; \theta)$, which, by equation 7, implies that $p = [c_1, c_2]$. Therefore, all consumers are served at a price $p = [c_1, c_2]$. All type-1 firms serve the demand and make a positive profit (because the price is equal to or greater than the marginal cost). Type-2 firms do not serve the demand and make zero profit.

3. If $k_1 < q^D(p^*; \theta) < K$, the demand is fully served by all type-1 and some type-2 firms. By the "market-clearing" condition, then $q^s(p^*; \cdot) = q^D(p^*; \theta)$, that is, $k_1 + \gamma^* k_2 = q^D(p^*; \theta)$, for some $\gamma^* \in [0, 1)$. By equation 5, this implies that $p = c_2$. Therefore, all consumers are served at a price c_2. All type-1 firms serve the demand, so these firms make a strictly positive profit (the price at which they serve is greater than the marginal cost). Type-2 firms that serve the demand make zero profit (because the price is equal to the marginal cost). Type-2 firms that do not serve the demand (the rest of the firms) make zero profit.

4. If $K = q^D(p^*; \theta)$, the demand is fully served by all type-1 and type-2 firms. By the "market-clearing" condition, then $q^s(p^*; \cdot) = q^D(p^*; \theta)$, that is, $k_1 + k_2 = q^D(p^*; \theta)$, which, by equation 5, implies that $p \in [c_2, \bar{p}]$. Therefore, all consumers are served at a price $p \in [c_2, \bar{p}]$. All type-1 firms serve the demand and make a strictly positive profit (because the price is equal to or greater than the marginal cost). All type-2 firms serve the demand and make a positive profit (because the price is equal to or greater than the marginal cost).

5. If $K < q^D(p^*;\theta)$, the demand is fully served by all type-1 firms and all type-2 firms (up to available capacity). By the "market-clearing" condition, and considering that $\theta \geq K$, then $q^s(p^*;\cdot) = K$. By equation 7, this implies that $p > c_2$. By definition 1, an equilibrium requires the "profit-maximizing" condition. Therefore, the equilibrium price will be a price that clear the markets and that maximizes firms' profits. That is, $p^* = \bar{p} > c_2$. Therefore, consumers are served at a price $\bar{p}$, and all firms make a positive profit (the price at which they serve is greater than the marignal cost). □

Proof of Lemma 8: a.f.s.o.c. that $k_1^* + k_2^* \geq q^D(c_2, 1)$. By Lemma 7, this implies that all firms make zero-profit in the second stage. Thus, if firms expect no profit in the second stage, then $k_i^* = 0$ for all $i \in \{1, 2\}$, which is a contradiction. □

Appendix B: Market equilibrium (corner solutions)

In this appendix we extend the proofs for the previous Propositions regarding market equilibrium to accommodate potential feasible corner solution for which $k_i = 0$ for some $i \in \{1,2\}$.

Proof of Proposition A.1 (corner solutions): *(i)* $k_1 \leq 0$ and $k_2 \geq 1$. If $k_2 \geq 1$, then $F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) \geq 1$ which implies that $-\frac{c_{k_2}}{\bar{p}-c_2} \geq 0$, which is impossible, since $c_{k_2} > 0$ and $\bar{p}-c_2 > 0$. *(ii)* $k_1 \geq 1$ and $k_2 \leq 0$. If $k_2 \leq 0$, then the free-entry equilibrium for type-1 firms is $\int_{k_1}^{1}(\bar{p}-c_1)k_1 dF(\theta) - c_{k_1}k_1 = 0$. Solving the integral, we arrive at $(\bar{p}-c_1)k_1[1-F(k_1)] - c_{k_1}k_1 = 0$. Rearranging, we arrive at $k_1 = F^{-1}\left(1-\frac{c_{k_1}}{\bar{p}-c_1}\right)$. Next, $k_1 \geq 1$ implies that $F^{-1}\left(1-\frac{c_{k_1}}{\bar{p}-c_1}\right) \geq 1$, which implies that $-\frac{c_{k_1}}{\bar{p}-c_1} \geq 0$, which is impossible, since $c_{k_1} > 0$ and $\bar{p}-c_1 > 0$. *(iii)* $k_1 \geq 1$ and $k_2 \geq 1$. If $k_1 \geq 1$, then $F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \geq 1$, which implies that $-\frac{c_{k_1}-c_{k_2}}{c_2-c_1} \geq 0$, which is impossible, since $c_{k_1} > c_{k_2}$ and $c_2 > c_1$. *(iv)* $k_1 \leq 0$ and $k_2 \in (0,1)$ (interior). That is, $k_2 = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$ and notice that $k_2 \in (0,1)$ requires that $\bar{p}-c_2 > c_{k_2}$. Next, $k_1 \leq 0$ implies that $F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \leq 0$ which implies that $c_2 - c_1 \leq c_{k_1} - c_{k_2}$. *(v)* $k_1 \in (0,1)$ (interior) and $k_2 \leq 0$. If $k_2 \leq 0$, then $F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) - F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \leq 0$, which implies that $F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) \leq F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$; rearranging, this requires that $c_{k_2}(c_2-c_1) \geq (\bar{p}-c_2)(c_{k_1}-c_{k_2})$. Next, by case *(ii)*, if $k_2 \leq 0$ then $k_1 = F^{-1}\left(1-\frac{c_{k_1}}{\bar{p}-c_1}\right)$, and notice that $k_1 \in (0,1)$ requires that $\bar{p}-c_1 > c_{k_1}$. *(vi)* $k_1 \geq 1$ and $k_2 \in (0,1)$ (interior). As shown in case *(iii)*, this is impossible, since $k_1 \geq 1$ requires that $-\frac{c_{k_1}-c_{k_2}}{c_2-c_1} \geq 0$. *(vii)* $k_1 \in (0,1)$ (interior) and $k_2 \geq 1$. If $k_2 \geq 1$, then $F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) - k_1 \geq 1$, which implies that $F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) \geq 1+k_1$, which implies that $1-\frac{c_{k_2}}{\bar{p}-c_2} \geq 1$, which implies that $-\frac{c_{k_2}}{\bar{p}-c_2} \geq 0$, which is impossible, since $c_{k_2} > 0$ and $\bar{p}-c_2 > 0$.

Therefore, leaving aside the uninteresting case in which $k_1 = 0$ and $k_2 = 0$ (for the remaining conditions), there are two feasible solutions in the long-run equilibrium in the first stage. First, $k_1 = 0$ and $k_2 = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right) \in (0,1)$, which requires that $\bar{p}-c_2 > c_{k_2}$ and $c_2 - c_1 \leq c_{k_1} - c_{k_2}$. Second, $k_1 = F^{-1}\left(1-\frac{c_{k_1}}{\bar{p}-c_1}\right) \in (0,1)$ and $k_2 = 0$, which requires that $c_{k_2}(c_2-c_1) \geq (\bar{p}-c_2)(c_{k_1}-c_{k_2})$ and $\bar{p}-c_1 > c_{k_1}$. In these two cases, equilibrium uniqueness is again guaranteed by the fact that $F(\cdot)$ is strictly increasing, which implies that $F^{-1}(\cdot)$ is also strictly increasing. Finally, it is straightforward to see that in these two cases $K = k_1 + k_2 < 1$. □

Proof of Proposition 1 (corner solutions): consider again equation 2. Rearranging, we arrive at:

$$\mathcal{W}(\boldsymbol{k}|\cdot) = \int_0^{k_1}(\bar{p}-c_1)\theta dF(\theta) + \int_{k_1}^{K}(c_2-c_1)k_1 dF(\theta) + \int_{k_1}^{K}(\bar{p}-c_2)\theta dF(\theta) + \\ + \int_K^1(\bar{p}-c_1)k_1 dF(\theta) + \int_K^1(\bar{p}-c_2)k_2 dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2$$

$$\mathcal{W}(\boldsymbol{k}|\cdot) = \int_0^{k_1}(\bar{p}-c_1)\theta dF(\theta) + (c_2-c_1)k_1[F(K)-F(k_1)] + \int_{k_1}^{K}(\bar{p}-c_2)\theta dF(\theta) + \\ +(\bar{p}-c_1)k_1[1-F(K)] + (\bar{p}-c_2)k_2[1-F(K)] - c_{k_1}k_1 - c_{k_2}k_2 \tag{B.1}$$

Our goal is to find the vector $\boldsymbol{k}$ that maximizes equation B.1. The Fist Order Conditions (F.O.C.) are, thus, $\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_i} = 0$ for $i \in \{1, 2\}$. First,

$$\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2} = 0$$

which implies that,
$(\bar{p}-c_2)Kf(K)+(c_2-c_1)k_1f(K)-(\bar{p}-c_1)k_1f(K)+(\bar{p}-c_2)[1-F(K)]-(\bar{p}-c_2)k_2f(K)-c_{k_2} = 0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_2)[1-F(K)]-(\bar{p}-c_2)k_2f(K)-c_{k_2} = 0 \Leftrightarrow (\bar{p}-c_2)(K-k_1-k_2)f(K)+(\bar{p}-c_2)[1-F(K)]-c_{k_2} = 0 \Leftrightarrow (\bar{p}-c_2)(K-k_1-k_2)f(K)+(\bar{p}-c_2)[1-F(K)]-c_{k_2} = 0 \Leftrightarrow (\bar{p}-c_2)[1-F(K)]-c_{k_2} = 0$.

Solving for K, we arrive at:

$$K = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right)$$

Next, the second F.O.C. is

$$\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1} = 0$$

which implies that,
$(\bar{p}-c_1)k_1f(k_1)+(\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)k_1f(k_1)+(c_2-c_1)[F(K)-F(k_1)]+(c_2-c_1)k_1[f(K)-f(k_1)]+(\bar{p}-c_1)[1-F(K)]-(\bar{p}-c_1)k_1f(K)-(\bar{p}-c_2)k_2f(K)-c_{k_1} = 0 \Leftrightarrow (c_2-c_1)k_1f(k_1)+(\bar{p}-c_2)Kf(K)+(c_2-c_1)[F(K)-F(k_1)]+(c_2-c_1)k_1[f(K)-f(k_1)]+(\bar{p}-c_1)[1-F(K)]-(\bar{p}-c_1)k_1f(K)-(\bar{p}-c_2)k_2f(K)-c_{k_1} = 0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)F(K)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1} = 0$. Recall that $K = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$. Then, $(\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)F(K)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1} = 0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1} = 0 \Leftrightarrow (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)+c_{k_2}-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1} = 0 \Leftrightarrow (\bar{p}-c_2)f(K)(K-k_1-k_2)-(\bar{p}-c_2)+c_{k_2}-(c_2-c_1)F(k_1)+(\bar{p}-c_1)-c_{k_1} = 0 \Leftrightarrow (c_2-c_1)F(k_1) = c_2-c_1+c_{k_2}-c_{k_1}$.

Solving for k_1, we arrive at:

$$k_1 = F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \tag{B.2}$$

Therefore, k_2 is just given by the difference between K and k_1. That is,

$$k_2 = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p}-c_2}\right) - F^{-1}\left(1 - \frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \tag{B.3}$$

Notice that B.2 and B.3 are exactly the same as A.3 and A.4 respectively. Therefore, we know that the solution given by these two equations is unique (due to strict monotonicity of $F(\cdot)$). Moreover, we also know that there are two types of corner solutions, namely, *(a)* $k_1 = 0$ and $k_2 \in (0, 1)$ and, *(b)* $k_1 \in (0, 1)$ and $k_2 = 0$, for which same conditions are required as those required in the competitive market solution case. We need to show that, in all these three cases, the previous equations are actually a solution to the maximization problem. For that purpose, we use the Second partial derivative test, which allow us to determine that the solution obtained is actually a (global) maximum.

The Hessian matrix of $\mathcal{W}(\boldsymbol{k}|\cdot)$, denoted $H(\boldsymbol{k})$, is as follows:

$$H(\boldsymbol{k}) = \begin{pmatrix} \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1^2} & \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1 \partial k_2} \\ \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2 \partial k_1} & \frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2^2} \end{pmatrix} = \begin{pmatrix} -(\bar{p}-c_2)f(K)-(c_2-c_1)f(k_1) & -(\bar{p}-c_2)f(K) \\ -(\bar{p}-c_2)f(K) & -(\bar{p}-c_2)f(K) \end{pmatrix}$$

Let us denote $D(\boldsymbol{k})$ the determinant of the Hessian matrix. That is, $D(\boldsymbol{k}) \equiv \det(D(\boldsymbol{k})) = [-(\bar{p}-c_2)f(K)-(c_2-c_1)f(k_1)][-(\bar{p}-c_2)f(K)]-[-(\bar{p}-c_2)f(K)][-(\bar{p}-c_2)f(K)]$. Rearranging, we arrive at $D(\boldsymbol{k}) = (c_2-c_1)(\bar{p}-c_2)f(k_1)f(K)$.

Notice that for solutions *(b)* $k_1 \in (0,1)$ and $k_2 = 0$, we have that $D(\boldsymbol{k}) > 0$; moreover, for these two types of solutions, we have that $\frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1^2} < 0$. Therefore, by the second partial derivative test, we know that these two types of solutions are maxima of the welfare function.

However, for the solution *(a)* $k_1 = 0$ and $k_2 \in (0,1)$,[*] we have that $D(\boldsymbol{k}) = 0$; therefore, the second partial derivative test is inconclusive. Still, because $\frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1^2} < 0$ and $\frac{\partial^2 \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_2^2} < 0$ at this solution, we know that the Hessian matrix is negative semi-definite (but not negative definite), so we can rule out the possibility of that this solution is minimum. However, to determine if a solution of the type $k_1 = 0$ and $k_2 \in (0,1)$ is a maximum or a saddle point, we need to perform a "visual" inspection. In particular, we need to check the behaviour of the welfare function in a neighbourhood of this solution.

Let us first consider $k_1 = 0$ and that $k_2 \in (0,1)$. The first derivative of the welfare function evaluated at this solution $\frac{\partial \mathcal{W}(0,k_2|\cdot)}{\partial k_2} = (\bar{p}-c_2)[1-F(k_2)]-c_{k_2}$. For $k_2^* \equiv k_2 = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$, we know that the function $\mathcal{W}(0,k_2|\cdot)$ has a critical point. To check whether this critical point is a maximum or a minimum, we check the $\mathcal{W}(0,k_2|\cdot)$ in a neighbourhood of k_2^*.

First, let us consider $k_2^*+\varepsilon$, where $\varepsilon > 0$ (arbitrarily small). Then $k_2^*+\varepsilon = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)+\varepsilon$, and because $F(\cdot)$ is strictly increasing, we know that $F^{-1}(\cdot)$ is also strictly increasing; therefore, $F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)+\varepsilon = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}+\zeta\right)$, that is, $k_2^*+\varepsilon = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}+\zeta\right)$ where $\zeta > 0$ (arbitrarily small). Then, $\frac{\partial^2 \mathcal{W}\left(0,F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}+\zeta\right)|\cdot\right)}{\partial k_1^2} = (\bar{p}-c_2)\left[1-\left(1-\frac{c_{k_2}}{\bar{p}-c_2}+\zeta\right)\right]-c_{k_2} = (\bar{p}-c_2)\left(\frac{c_{k_2}}{\bar{p}-c_2}-\zeta\right)-c_{k_2} = -(\bar{p}-c_2)\zeta < 0$. Moreover, similarly, it can be shown that for $k_2^*-\varepsilon$, $\frac{\partial^2 \mathcal{W}\left(0,F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}-\zeta\right)|\cdot\right)}{\partial k_1^2} = (\bar{p}-c_2)\zeta > 0$. Therefore, at values below k_2^*,the function $\mathcal{W}(0,k_2|\cdot)$ increases, and at values above k_2^*,the function $\mathcal{W}(0,k_2|\cdot)$ decreases. Therefore, at $k_1 = 0$ and that k_2^*, the function attains a maximum.

Next, we need to check the behavior of $\mathcal{W}(k_1,k_2|\cdot)$ if a neighborhood of $k_1 = 0$. Obviously, since $k_1 \geq 0$ (by construction), then we need to check the behavior of the function when $k_1 = 0$ an when $k_1 = \varepsilon > 0$, where $\varepsilon > 0$ (arbitrarily small). The first derivative of the welfare function with respect to k_1 is $\frac{\partial \mathcal{W}(\boldsymbol{k}|\cdot)}{\partial k_1} = (\bar{p}-c_2)Kf(K)-(\bar{p}-c_2)F(K)-(c_2-c_1)F(k_1)-(\bar{p}-c_2)k_1f(K)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(K)-c_{k_1}$. At $k_1^* = 0$ and $k_2^* \equiv k_2 = F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$, the derivative is $\frac{\partial \mathcal{W}(0,k_2|\cdot)}{\partial k_1} = (\bar{p}-c_2)k_2f(k_2)-(\bar{p}-c_2)F(k_2)+(\bar{p}-c_1)-(\bar{p}-c_2)k_2f(k_2)-c_{k_1}$, that is, $\frac{\partial \mathcal{W}(0,k_2^*|\cdot)}{\partial k_1} = -(\bar{p}-c_2)\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)+\bar{p}-c_1-c_{k_1} = c_2-c_1+c_{k_2}-c_{k_1}$. Recall that this solution requires that $c_2-c_1 \leq c_{k_1}-c_{k_2}$ (see the footnote). Therefore, $\frac{\partial \mathcal{W}(0,k_2^*|\cdot)}{\partial k_1} \leq 0$.

[*]Recall that this solution requires two conditions, namely, that $\bar{p}-c_2 > c_{k_2}$ and that $c_2-c_1 \leq c_{k_1}-c_{k_2}$, as we show in the proof of Proposition A.1.

Next, we check the derivative for the case in which $k_1 = \varepsilon$, that is, $\frac{\partial \mathcal{W}(\varepsilon,k_2|\cdot)}{\partial k_1}$, and we compare it with the case in which $k_1 = 0$, that is, with $\frac{\partial \mathcal{W}(0,k_2|\cdot)}{\partial k_1}$. In particular, we can show that $\frac{\partial \mathcal{W}(\varepsilon,k_2|\cdot)}{\partial k_1} < \frac{\partial \mathcal{W}(0,k_2|\cdot)}{\partial k_1}$. To see this, consider that $\frac{\partial \mathcal{W}(\varepsilon,k_2|\cdot)}{\partial k_1} = (\bar{p} - c_2)(\varepsilon + k_2)f(\varepsilon + k_2) - (\bar{p} - c_2)F(\varepsilon + k_2) - (c_2 - c_1)F(\varepsilon) - (\bar{p} - c_2)\varepsilon f(\varepsilon + k_2) + (\bar{p} - c_1) - (\bar{p} - c_2)k_2 f(\varepsilon + k_2) - c_{k_1}$; moreover, we also know that $\frac{\partial \mathcal{W}(0,k_2|\cdot)}{\partial k_1} = -(\bar{p} - c_2)F(k_2) + (\bar{p} - c_1) - c_{k_1}$. Thus, we just need to compare $(\bar{p} - c_2)(\varepsilon + k_2)f(\varepsilon + k_2) - (\bar{p} - c_2)F(\varepsilon + k_2) - (c_2 - c_1)F(\varepsilon) - (\bar{p} - c_2)\varepsilon f(\varepsilon + k_2) + (\bar{p} - c_1) - (\bar{p} - c_2)k_2 f(\varepsilon + k_2) - c_{k_1}$ with $-(\bar{p} - c_2)F(k_2) + (\bar{p} - c_1) - c_{k_1}$. In particular, we can show that $-(\bar{p} - c_2)F(k_2) + (\bar{p} - c_1) - c_{k_1} > (\bar{p} - c_2)(\varepsilon + k_2)f(\varepsilon + k_2) - (\bar{p} - c_2)F(\varepsilon + k_2) - (c_2 - c_1)F(\varepsilon) - (\bar{p} - c_2)\varepsilon f(\varepsilon + k_2) + (\bar{p} - c_1) - (\bar{p} - c_2)k_2 f(\varepsilon + k_2) - c_{k_1} \iff -(\bar{p} - c_2)F(k_2) > (\bar{p} - c_2)(\varepsilon + k_2)f(\varepsilon + k_2) - (\bar{p} - c_2)F(\varepsilon + k_2) - (c_2 - c_1)F(\varepsilon) - (\bar{p} - c_2)\varepsilon f(\varepsilon + k_2) - (\bar{p} - c_2)k_2 f(\varepsilon + k_2) \iff -(\bar{p} - c_2)F(k_2) > -(\bar{p} - c_2)F(\varepsilon + k_2) - (c_2 - c_1)F(\varepsilon) \iff (\bar{p} - c_2)F(\varepsilon + k_2) - (\bar{p} - c_2)F(k_2) > -(c_2 - c_1)F(\varepsilon) \iff (\bar{p} - c_2)[F(\varepsilon + k_2) - F(k_2)] > -(c_2 - c_1)F(\varepsilon)$, which is true, since $F(\cdot)$ is strictly increasing. Therefore, it is true that $\frac{\partial \mathcal{W}(\varepsilon,k_2|\cdot)}{\partial k_1} < \frac{\partial \mathcal{W}(0,k_2|\cdot)}{\partial k_1}$. That is, in a neighborhood of $k_1 = 0$, the first derivative is negative, that is, the welfare function attains a maximum at $k_1 = 0$. Therefore, we can conclude that the solution $k_1 = 0$ and that $k_2 \in (0, 1)$ is also a maximum.

So far, we checked that the corner solutions *(a)* $k_1 = 0$ and $k_2 \in (0, 1)$ and *(b)* $k_1 \in (0, 1)$ and $k_2 = 0$ (which are the same as in the competitive market case) do also maximize welfare. Moreover, by the proof of Proposition A.1, we know that in all these cases, $k_1 + k_2 < 1$.

However, in the Proposition we also include that there exist some parameters for which the vector of socially-optimal aggregate capacities satisfies $k_1 + k_2 = 1$. In particular, we consider the following candidate solution $k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ and $k_2 = 1 - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right)$ (notice that k_2 is obtained as the residual capacity, as shown in the maximization problem). Thus, we need to show that that there exist some parameters for which the welfare function at $k_1 + k_2 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) + 1 - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) = 1$ is greater than the welfare function at any other parameters for which $k_1 + k_2 < 1$ (the previous solutions).

Let us first compare the case $k_1 = 1$ and $k_2 = 0$ with case *(b)* $k'_1 \in (0, 1)$ and $k'_2 = 0$. Recall that this case requires that $c_{k_2}(c_2 - c_1) \geq (\bar{p} - c_2)(c_{k_1} - c_{k_2})$ (see the proof of Proposition A.1). Thus, this implies that we compare the case $k_1 = 1$ and $k_2 = 0$ with case *(b)* $k'_1 \in (0, 1)$ and $k'_2 = 0$. Thus, we need show that $\int_0^1 (\bar{p} - c_1)\theta dF(\theta) - c_{k_1} > \int_0^{k'_1} (\bar{p} - c_1)\theta dF(\theta) + (\bar{p} - c_1)k'_1[1 - F(k'_1)] - c_{k_1}k'_1$ $\iff \int_{k'_1}^1 (\bar{p} - c_1)\theta dF(\theta) - c_{k_1} > (\bar{p} - c_1)k'_1[1 - F(k'_1)] - c_{k_1}k'_1$. Recall that $F(k'_1) = \left(1 - \frac{c_{k_1}}{\bar{p} - c_1}\right)$. Thus, $\int_{k'_1}^1 (\bar{p} - c_1)\theta dF(\theta) - c_{k_1} > (\bar{p} - c_1)k'_1[1 - F(k'_1)] - c_{k_1}k'_1 \iff \int_{k'_1}^1 (\bar{p} - c_1)\theta dF(\theta) - c_{k_1} > (\bar{p} - c_1)k'_1[1 - 1 + \frac{c_{k_1}}{\bar{p} - c_1}] - c_{k_1}k'_1 \iff \int_{k'_1}^1 (\bar{p} - c_1)\theta dF(\theta) - c_{k_1} > 0$.

Therefore, if $\int_{k'_1}^1 (\bar{p} - c_1)\theta dF(\theta) - c_{k_1} > 0$, then $k_1 = 1$ and $k_2 = 0$ is the socially-optimal solution.

Finally, we compare the case $k_1 = 0$ and $k_2 = 1$ with case *(b)* $k'_1 = 0$ and $k'_2 \in (0, 1)$. Recall that this case requires that $c_{k_1} - c_{k_2} \geq c_2 - c_1$ (see the proof of Proposition A.1). Thus, this implies that we compare the case $k_1 = 0$ and $k_2 = 1$ with case *(b)* $k'_1 = 0$ and $k'_2 \in (0, 1)$. Thus, we need show that $\int_0^1 (\bar{p} - c_2)\theta dF(\theta) - c_{k_2} > \int_0^{k'_2} (\bar{p} - c_2)\theta dF(\theta) + (\bar{p} - c_2)k'_2[1 - F(k'_2)] - c_{k_2}k'_2$ $\iff \int_{k'_2}^1 (\bar{p} - c_2)\theta dF(\theta) - c_{k_2} > (\bar{p} - c_2)k'_2[1 - F(k'_2)] - c_{k_2}k'_2$. Recall that $F(k'_2) = \left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$. Thus, $\int_{k'_2}^1 (\bar{p} - c_2)\theta dF(\theta) - c_{k_2} > (\bar{p} - c_2)k'_2[1 - F(k'_2)] - c_{k_2}k'_2 \iff \int_{k'_2}^1 (\bar{p} - c_2)\theta dF(\theta) - c_{k_2} > (\bar{p} - c_2)k'_2[1 - 1 + \frac{c_{k_2}}{\bar{p} - c_2}] - c_{k_2}k'_2 \iff \int_{k'_2}^1 (\bar{p} - c_2)\theta dF(\theta) - c_{k_2} > 0$, which is exactly the same condition as before, since $k'_2 = F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$. □

Proof of Proposition 5 (corner solutions): let us begin with type-1 firms. By definition 4, we know that in equilibrium, $E\pi_1(k_1) - c_{k_1}k_1 + xk_1 = 0$. That is, $E\pi_1(k_1|\theta < k_1) + E\pi_1(k_1|k_1 \leq \theta < K^T) + E\pi_1(k_1|K^T \leq \theta) - c_{k_1}k_1 + xk_1 = 0$. By Lemma 1, we know that $E\pi_1(k_1|\theta < k_1) = 0$. Moreover, by Lemma 1 we also know that if $k_1 \leq \theta < K^T$, then $p^* = c_2$, and if $K^T \leq \theta$, then $p^* = \bar{p}$; in both cases, all type-1 firms serve the demand. Therefore, the free-entry equilibrium condition can be rewritten as follows

$$\int_{k_1}^{K^T} (c_2 - c_1)k_1 dF(\theta) + \int_{K^T}^{1} (\bar{p} - c_1)k_1 dF(\theta) - c_{k_1}k_1 + xk_1 = 0$$

Solving the integrals and rearranging, we arrive at:

$$(c_2 - c_1)F(k_1) = (\bar{p} - c_1) - (\bar{p} - c_2)F(K^T) - c_{k_1} + x \tag{B.4}$$

Next, we deal with type-2 firms. By definition 4, we know that in equilibrium, $E\pi_2(k_2) - c_{k_2}k_2 + xk_2 = 0$. That is, $E\pi_2(k_2|\theta < k_2) + E\pi_1(k_2|k_2 \leq \theta < K^T) + E\pi_2(k_2|K^T \leq \theta) - c_{k_2}k_2 + xk_2 = 0$. By Lemma 1, we know that $E\pi_2(k_2|\theta < k_2) = 0$ and that $E\pi_2(k_2|k_1 \leq \theta < K^T) = 0$. Moreover, by Lemma 1 we also know that if $K^T \leq \theta$, then $p^* = \bar{p}$; in this case, all type-2 firms serve the demand. Therefore, the free-entry equilibrium condition can be rewritten as follows

$$\int_{K^T}^{1} (\bar{p} - c_2)k_2 dF(\theta) - c_{k_2}k_2 + xk_2 = 0$$

Solving the integrals and rearranging, we arrive at:

$$F^{-1}\left(1 - \frac{c_{k_2} - x}{\bar{p} - c_2}\right) = K^T \tag{B.5}$$

Replacing B.5 into B.4, we arrive at:

$$(c_2 - c_1)F(k_1) = (\bar{p} - c_1) - (\bar{p} - c_2)F\left[F^{-1}\left(1 - \frac{c_{k_2} - x}{\bar{p} - c_2}\right)\right] - c_{k_1} + x$$

Solving for k_1 we arrive at:

$$k_1 = F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) \tag{B.6}$$

We know that, by construction, $K^T = k_1 + k_2$. Thus, replacing B.6 into this condition, we arrive at

$$k_2 = K^T - F^{-1}\left(1 - \frac{c_{k_1} - c_{k_2}}{c_2 - c_1}\right) \tag{B.7}$$

Notice that k_1 is similar as in the case in which capacity is not regulated, while k_2 is again defined as the residual ($k_2 = K^T - k_1$). Thus, both k_i for $i \in \{1, 2\}$ are unique.

Finally, to get the capacity payment, we need to solve for x in equation B.5:

$$x = c_{k_2} - (\bar{p} - c_2)\left[1 - F(K^T)\right] \tag{B.8}$$

Notice that $x \geq 0 \iff K^T \geq F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$. Thus, we use $F^{-1}\left(1 - \frac{c_{k_2}}{\bar{p} - c_2}\right)$ as our candidate threshold $\tilde{K}$.

A potential corner solution implies that $k_1 = 0$ and $k_2 = K^T$, which occurs if $c_2 - c_1 \leq c_{k_1} - c_{k_2}$. Then, $k_1 = 0$ and $k_2 = K^T$. We consider two scenarios; namely, if $\bar{p} - c_2 > c_{k_2}$, then $\tilde{K} =$

$F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)>0$ and, given $K^T \geq \tilde{K}$, we have that $x \geq 0$; and if $\bar{p}-c_2 \leq c_{k_2}$, then $\tilde{K}=0$ and, given $K^T \geq 0$, we have that $x=c_{k_2}-(\bar{p}-c_2)\left[1-F(K^T)\right] \geq 0$, since $\left[1-F(K^T)\right] \in [0,1]$ and (in this case) $\bar{p}-c_2 \leq c_{k_2}$.

On the other hand, there can be a corner solution in which $k_2=0$ and $k_1=K^T$. This one occurs if $c_2-c_1>c_{k_1}-c_{k_2}$ and $c_{k_2}(c_2-c_1) \geq (\bar{p}-c_2)(c_{k_1}-c_{k_2})$. If so, then $k_1=F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \geq F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$ and this requires that the threshold is $\tilde{K}=F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$, that is, $K^T \geq F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right)$ in equilibrium. In this case, since $K^T \geq \tilde{K}=F^{-1}\left(1-\frac{c_{k_1}-c_{k_2}}{c_2-c_1}\right) \geq F^{-1}\left(1-\frac{c_{k_2}}{\bar{p}-c_2}\right)$, then we know that $x \geq 0$ is fulfilled. □

Appendix C: Additional (miscellaneous) derivations and results

In this Appendix, we show that equation 2 can be also obtained from the sum of Consumer Surplus (CS) and Producer Surplus (PS).

$$
\begin{aligned}
\mathcal{W}(\boldsymbol{k}|\cdot) = & \underbrace{\int_0^{k_1}(\bar{p}-c_1)\theta dF(\theta) + \int_{k_1}^{K}(\bar{p}-c_2)k_1 dF(\theta) + \int_{k_1}^{K}(\bar{p}-c_2)(\theta-k_1)dF(\theta) +}_{\text{CS}} \\
& \underbrace{+\int_{k_1}^{K}(c_2-c_1)k_1 dF(\theta) + \int_{K}^{1}(\bar{p}-c_1)k_1 dF(\theta) + \int_{K}^{1}(\bar{p}-c_2)k_2 dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2}_{\text{PS}} = \\
= & \int_0^{k_1}(\bar{p}-c_1)\theta dF(\theta) + \int_{k_1}^{K}(\bar{p}-c_1)k_1 dF(\theta) + \int_{k_1}^{K}(\bar{p}-c_2)(\theta-k_1)dF(\theta) + \\
& + \int_{K}^{1}(\bar{p}-c_1)k_1 dF(\theta) + \int_{K}^{1}(\bar{p}-c_2)k_2 dF(\theta) - c_{k_1}k_1 - c_{k_2}k_2
\end{aligned}
$$