Deposit Requirements in Auctions*

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Abstract

We examine the role of a winner-pay deposit requirement in a second-price auction with a reservation price, in which all buyers have post-auction outside options and the winner may default if a better option arrives. We find that buyers bid lower than their values if and only if the deposit is positive, and the optimal deposit and reservation price should be set to fully deter the winner's default for the outside option. The optimal reservation price is lower than that of Myerson (1981), however, counter-intuitively, the optimal deposit is higher when buyer options get worse.

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1 Introduction

In auctions with high-value items, like cars, houses, etc., winners often need additional time to gather payments to complete final transactions after the auctions conclude. In the meantime, the winners may have chances to conduct post-auction search for better outside options before the final transaction, and they will renege on the deal if the outside option turns out to be better. The sellers are faced with the issue of buyer commitment.[1] In such scenarios, it is a common practice that the winner is required to put down a non-refundable fee, also called deposit, before the final transaction is settled. For example, in auto and real estate auctions, deposit requirements have been commonly and broadly adopted by sellers in practice.[2] On the one hand, from the winner's perspective, paying the deposit secures the original transaction opportunity while keeping the future outside options open. On the other hand, from the seller's perspective, the deposit can function as a deterrent against potential buyer deviations to outside options and the deposit becomes the seller's revenue when it is forfeited upon the winner's default. Motivated by these observations, in this paper, we study the effects of such a deposit requirement on buyers' bidding strategies, seller revenue as well as optimal auction design.

We consider a single-object second price auction with future arrival of random outside options for all bidders, in which the winner can renege on the auction deal if a better option arrives. We incorporate a deposit requirement into the auction model, and analyze its role in mitigating buyers' reneging incentives. At the beginning of the auction, the seller announces the reserve price and the amount of deposit that a buyer is required to pay immediately after winning, the latter of which is called *post-bidding winner-pay deposit*.[3] All bidders place their bids simultaneously and the bidder with the highest bid wins the auction given the highest bid is above the reserve price. The prevailing auction price is determined by the second highest bid if it is higher than the reserve price. Otherwise, it equals to the reserve price. Upon winning, the winner chooses whether to pay the deposit. If the winner chooses not to pay, then he or she can only wait for the outside option, which would deliver a random alternative price which can be better or worse than that of the auction. If the winner chooses to pay the deposit, then he or she can later decide whether to complete the original transaction

[1]For example, Resnick and Zeckhauser (2002) observe that the most common complaint by sellers in online auctions is winning bidders do not follow through on the transactions. Dellarocas and Wood (2008) find that 81 percent of the negative feedback given to buyers in eBay auctions result from "bidders who backed out of their commitment to buy the items they won."

[2]Sellers of eBay auto auctions are allowed to set a deposit, and the winners after paying the deposit make the final payments within a week or 10 days. In UK real estate auctions, a buyer after winning is required to pay a deposit immediately and then completes the final payment within 28 days.

[3]In Appendix C, we discuss another case, called *all-pay deposit* where all potential bidders are required to pay the deposit before submitting their bids.

deal or take the outside option. All losing bidders leave the auction to wait for their outside options with random prices.

We first characterize the equilibirum bidding strategy for any given deposit and reservation price. The equilibrium strategy we identify has an important feature that the equilibrium bid should make a bidder indifferent between "waiting for outside option directly" and "winning with the own bid as the auction price and paying the deposit." Such equilibrium strategy in fact is a dominant strategy, and moreover, a higher deposit leads to a lower bid, and bidders bid truthfully when the deposit is zero. The full characterization of the equilibrium bidding strategy consists of two scenarios depending on the comparison between the deposit and the reserve price: deposit is higher than the reserve price in Scenario (I); and the opposite holds for Scenario (II). In each scenario, the bidding equilibrium is increasing and involves a value threshold for entry.

After characterizing the equilibrium bidding strategy, we next examine the optimal auction design with the choices of the deposit and the reserve price. For this purpose, our analysis includes the following two steps: *Step one*. For each given the deposit level, we pin down the optimal reservation price. For this purpose, we separately analyze the seller's revenue maximization problems under the two scenarios mentioned above, and then identify the seller's optimal choice on the reserve price by comparing revenues across the two scenarios. We find that if the deposit is less than a cutoff r^{so}, the optimal reserve price falls in Scenario (II), which means that the optimal reservation price is higher than the deposit; if, however, the deposit is higher than r^{so}, the optimal reserve price is invariant and it falls in Scenario (I), which means that the optimal reservation price is lower than the deposit.

Step two. We examine the optimal deposit the seller should set to maximize revenue with the reserve price being set optimally as a function of the deposit. The tradeoff on choosing the optimal deposit can be illustrated as follows: Setting high deposits gives bidders an incentive to bid low in the auction, but this in turn leads to a lower probability for the winner's default that means no full payment from the winner. We find that the latter positive effect always dominates the former negative one, and thus the seller's revenue is maximized when a sufficiently high deposit is set by the seller that fully deters the winner to deviate and take the outside option. Furthermore, our analysis implies that the existence of outside options calls for a lower optimal reserve price, compared to that of Myerson (1981). There are two reasons for this. First, lowering reserve price from the Myerson level can induce more participation by bidders in the auction, which potentially generates a higher payment to the seller. Second, although a lower reserve price would let low-valuation bidders enter, it does not necessarily reduce the payments to the seller, as a lower reserve price gives a room for requiring a deposit in the auction, which results in a lower default possibility from the bidders. Our finding indicates that the overall effect of a lower reserve price combined with a deposit is beneficial to the seller when outside buyer options exist. The combination better balances between buyers' participation/bidding incentives and the winners' default incentive, which generates a higher seller revenue.

We finally examine the impacts of different distributions of the outside option on equilibrium bidding strategy, seller revenue, the optimal reserve, and the optimal deposit. Our analysis shows that given any reserve price and deposit, a distribution giving better outside options to buyers in the sense of first order stochastic dominance, leads to lower equilibrium bids. We also find that a better outside option distribution in buyers' perspective would definitely entail lower seller revenue at the optimal design. Both results are quite intuitive.

As revealed in our analysis, the optimal deposit is meant to fully deter winners' post-auction default. Therefore, intuitively, worse outside option distributions in buyers' perspective would require lower deposit to be set to deter the winners' deviations, as in Myerson (1981) no deposit is required at optimum when buyers have no outside options. However, counter-intuititvely, a distribution giving worse outside options to buyers in the sense of first order stochastic dominance rather requires a higher optimal deposit to be set by the seller. The puzzle is resolved after we realize that a worse outside option distribution in buyers' perspective would increase buyers' equilibrium bids. Given this, a higher deposit must be in place to deter the winners' post-auction default when the lowest outside option price realizes. On one hand, a higher deposit brings down the equilibrium bids; on the other hand, it makes less profitable for the winners to default once the deposit is paid by them. We also find that a distribution giving worse outside options to buyers in the sense of hazard ratio dominance, which means first order stochastic dominance,[4] requires a higher optimal reservation price to be set by the seller. This finding is pretty consistent with the usual intuition.

To the best of our knowledge, our paper is the first study that examines the impacts of deposit requirement on bidding strategies and optimal auction design. Although no previous study has investigated the same questions imposed here, this study is related to the literatures on auctions without buyer commitment and auctions with outside options.

Our paper is related to the growing literature on auctions without buyer commitment. Resnick and Zeckhauser (2002), which is one of the first studies on sellers' and buyers' behavior in online marketplaces, observe that the most common complaint by sellers is that the winning bidder does not follow through on the transaction. Using

[4]Please refer to Appendix B of Krishna (2002) for details.

a sample with more than 50,000 eBay auctions, Dellarocas and Wood (2008) find that 81 percent of the negative feedback given to buyers result from "bidders who backed out of their commitment to buy the items they won." Engelmann, Frank, Koch, and Valente (2015) study an auction model where the seller can give a second-chance offer to the second-highest bidder if the auction winner fails to pay. Their analysis shows that the availability of the offer reduces bidders' willingness to bid in the auction and thus lowers the seller's revenue, even when no default actually happens. In addition, buyer reputation systems might reward bidders who have a reputation for defaulting, running counter to the idea that they create a deterrent against such behavior.

Asker (2000) studies an auction model where bidders face an uncertainty on their final valuations of the object, and this uncertainty will only be resolved after the simultaneous bidding has taken place. Asker shows that in this case the inclusion of the withdrawal right (allowing the winner to default) raises seller's expected revenue. Zheng (2001, 2009) considers a situation where bidders who face budget constraints can default on their bids. He shows that the default risk induced by financial constraints deeply affects equilibrium bidding strategies and seller revenues in auctions. Focusing on a common value setting, Harstad and Rothkopf (1995) find that the ability to withdraw a winning bid can serve as a valuable insurance against the winner's curse.[5]

Krähmer and Strausz (2015) investigate the effects of withdrawal right on optimal sales contracts that involve only one buyer and one seller. In their contracting environment, the buyer, after having observed his private valuation, has the choice between exercising his option as specified in the contract and withdrawing from it and obtaining his outside option. Their results show that the inclusion of default rights is equivalent to introducing ex-post participation constraints in the sequential screening model, and even though sequential screening is still feasible with ex-post participation constraints, the seller no longer benefits from it. Instead, the optimal selling contract is static and coincides with the optimal posted price contract in the static screening model.[6] Our study is also closely related to Armstrong and Zhou (2016), who study optimal search deterrence in essentially a one-seller-and-one-buyer setting. In their model, the buyer needs to incur a cost to search for an outside option. Their focus is largely on seller's choice between a buy-it-now discount offer and an exploding offer. They also study the optimal selling mechanism and their analysis reveals that at the optimum the seller might charge a non-refundable deposit.

Although our study and the literature on auctions without buyer commitment share the common feature that an auction winner may renege on the original transaction, the focus of our paper is different. We attempt to provide a rationale for the seller's

[5]Also see related studies by Rothkopf (1991), Roelofs (2002).

[6]See related studies by Ben-Shahar and Posner (2011) and Eidenmüller (2011), for example.

adoption of deposits, which is commonly observed in real world auctions. In this sense, our purpose is to characterize the optimal auction design where both the deposit and the reserve price are allowed, and to answer the question of whether the seller should tolerate a post-auction winner default at optimum, when there will be post-auction outside options to buyers.

This paper is part of another growing literature on auctions with outside options. Cherry, Frykblom, Shogren, List, and Sullivan (2004) conduct a lab experiment to examine whether bidders take into account the existence of outside options when formulating their bidding strategies in second-price auctions. Their results show that bidders reduce their bids whenever their resale values exceed the price of the outside option. Kirchkamp, Poen, and Reiss (2009) study equilibrium bidding behavior of bidders in first-price and second-price auctions with outside options. They show that first-price auctions yield more revenues to sellers than second-price auctions, which explains why first-price auctions are more commonly used in practice. Lauermann and Virág (2012) study how the presence of outside options influences whether an auctioneer prefers "opaque" or "transparent" auctions, which differ according to the information that bidders receive. They show that an auctioneer might have incentives to choose opaque auctions in order to reduce the values of the bidders' outside options. Figueroa and Skreta (2007, 2009) examine revenue-maximizing auctions for multiple objects, where bidders' outside options depend on their private information and are endogenously chosen by the seller. Their results show that depending on the shape of the outside options, an optimal mechanism may or may not allocate the objects efficiently.

Unlike these existing models considering either only losing bidders having post-auction outside options or all bidders knowing their outside options before bidding, we instead consider a different scenario where outside options arrive after all bidders have already submitted their bids and they are available to all bidders including the auction winner (i.e., the winner can choose to either complete the original transaction or default and take the outside option). More importantly, our analysis focuses on revealing the role of the deposit requirement in auctions in affecting ex-ante bidding behavior and seller's expected revenue.

The rest of the paper is organized as follows. In Sections 2 and 3, we present the model and characterize bidders' equilibrium strategies. Seller's optimal choices on reserve price and deposit are examined in Section 4. Section 5 provides the comparative statics of outside option distributions on bidding strategy, seller revenue, optimal reserve price and optimal deposit. Section 6 concludes. All technical proofs are relegated to Appendix A. Computation details and discussion on other types of deposit requirements are in online Appendices B and C.

2 Deposit Requirement in Second-Price Auctions

2.1 Model setup

A seller sells an indivisible object to N risk neutral bidders through a sealed bid second price auction, where $1 < N < \infty$. The seller's value of the object is normalized as zero. Bidders' private values, denoted by $v_i,\ i = 1, 2, ..., N$, are independent draws from a common atomless distribution $F(\cdot)$ over the support $[0, \bar{v}]$, with $\overline{v} > 0$. Let $f(\cdot) \equiv F'(\cdot) > 0$. We further assume that an outside option will arrive after the bidding stage, which allows bidder i to obtain the same object by paying price p_i. Prices p_i, $i = 1, 2, ..., N$, are random draws from a continuous distribution function $\Phi(\cdot)$ with density $\varphi(\cdot) > 0$ over $[0, \bar{v}]$. F and Φ are common knowledge among the bidders and the seller, and they are *regular* in the sense that the hazard rates $\frac{f(\cdot)}{1-F(\cdot)}$ and $\frac{\varphi(\cdot)}{1-\Phi(\cdot)}$ are increasing.[7]

Figure 1 depicts the timing of the game, which is comprised of four stages. At stage $t = 0$, the seller announces a reserve price $r \in [0, \bar{v}]$ and a *post-bidding* non-refundable deposit $D \in [0, +\infty)$.[8] All bidders observe r, D and their own private values.

At $t = 1$, the bidders decide whether to submit bids or simply wait for the outside options. We denote the bid from bidder i by $b_i \in [r, +\infty)$. A nonparticipant's bid is denoted by "$\varnothing$." The highest bidder wins the auction. All other losing bidders can only wait for the outside options at $t = 3$. The full payment of the winner i is denoted by κ. κ equals r if there is only one valid bid, and it equals the second-highest bid otherwise. Slightly abusing notations, we have $\kappa \equiv \max\{r, b_{-i}^{(1)}\}$, where $b_{-i}^{(1)}$ stands for losing bidders' highest bid. If there are no valid bids, the auction game ends and the seller keeps the object.

At $t = 2$, the winning bidder i is required to pay a non-refundable deposit D for the option of purchasing the object from the seller by making the full payment at $t = 3$. A winning bidder i's decision on paying D is denoted by $e_i \in \{0, 1\}$, with $e_i = 1$ for "paying D" and $e_i = 0$ for "not paying D."

At $t = 3$, the outside option with price p_i arrives. If the winning bidder i did not pay D at $t = 2$, i.e., $e_i = 0$, the seller keeps the object and bidder i then only faces the outside option. If $e_i = 1$, the winning bidder i has two options: either completing the purchase from the seller by paying the remaining payment $\kappa - D$, or taking the outside option by paying p_i. For a winning bidder i who paid the deposit, we denote his decision

[7]This assumption on the distributions of $F(\cdot)$ and $\Phi(\cdot)$ in fact facilities the analysis for the seller's choices on reserve price and deposit. See details in Section 4.

[8]There may exist a non-trivial cost for relisting and/or time cost for the seller to negotiate with the second highest bidder, if the object is unsold. This justifies why a seller wants to impose a deposit requirement to reduce winner default in the auction.

on whether to complete the original transaction by or take the outside option o_i, with $o_i = 1$ for "taking the outside option," $o_i = 0$ for "completing the original transaction," and $o_i = NP$ for "no purchase." For all other cases (a losing bidder or a winning bidder who did not pay the deposit), we denote such a bidder j's decision on whether to take the outside option by $\breve{o}_j$, with $\breve{o}_j = 1$ for "taking the outside option p_j" and $\breve{o}_j = NP$ for "no purchase."

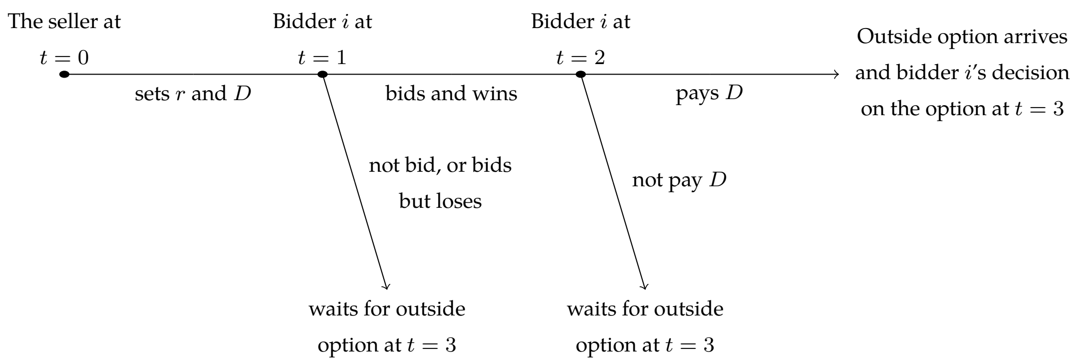

Figure 1: Timing

2.2 Strategies and equilibrium concept

Before proceeding further, we first define the strategies of the seller and bidders, and the equilibrium concept we will use in the following analysis.

Strategies of the seller. Before the bidding stage starts, the seller sets reserve price r and deposit D at $t = 0$.

Strategies of a bidder. Given r and D, a bidder must make four decisions: first, how much to bid in the bidding stage at $t = 1$; second, conditional on winning the auction, whether to pay the deposit at $t = 2$; third, conditional on winning the auction and paying the deposit at $t = 2$, whether to take the outside option, or to complete the original transaction by paying $\kappa - D$, or not to purchase at all at $t = 3$; and fourth, conditional on not winning the auction at $t = 1$ or winning at $t = 1$ but not paying the deposit at $t = 2$, whether to take the outside option at $t = 3$.

The bidding strategy for bidder i at $t = 1$ is a mapping from r, D and his/her private value v_i to his/her bid:

$$b_i(v_i, r, D) : [0, \overline{v}] \times [0, \overline{v}] \times [0, \infty) \to [r, +\infty) \cup \{\varnothing\}.$$

Note that b_i is valid if and only if $b_i \geq r$.

Conditional on winning at the bidding stage, the strategy for winning bidder i on whether to pay the deposit is a mapping from his/her private value v_i, deposit D and full payment amount κ to the deposit payment decision e_i at $t = 2$:

$$e_i(v_i, D, \kappa) : [0, \overline{v}] \times [0, \infty) \times [r, \infty) \rightarrow \{0, 1\}.$$

Contingent on paying the deposit at $t = 2$, the strategy for winning bidder i on whether to complete the original transaction or purchase from the outside option is a mapping from his/her private value v_i, full payment amount κ, the deposit D, and outside option price p_i to decision o_i at $t = 3$:

$$o_i(v_i, \kappa, D, p_i) : [0, \overline{v}] \times [r, \infty) \times [0, \infty) \times [0, \overline{v}] \rightarrow \{0, 1\} \cup \{NP\}.$$

Conditional on not winning the auction or winning but not paying the deposit, the strategy for bidder i on whether to purchase from the outside option is a mapping from his/her private value v_i and outside option price p_i to decision $\breve{o}_i$ at $t = 3$:

$$\breve{o}_i(v_i, p_i) : [0, \overline{v}] \times [0, \overline{v}] \rightarrow \{0, 1\} \cup \{NP\}.$$

Equilibrium concept. We consider a perfect Bayesian equilibrium (PBE), which can be solved using backward induction. We first characterize bidder i's strategies regarding the outside option at $t = 3$, i.e., $o_i(v_i, \kappa, D, p_i)$ and $\breve{o}_i(v_i, p_i)$. We then examine a winning bidder i's decision on paying the deposit at $t = 2$, i.e., $e_i(v_i, D, \kappa)$. Next, we characterize the bidding strategies $b_i(v_i, r, D)$, $i = 1, 2, ..., N$ at $t = 1$. In the last step, we find the seller's optimal r and D at $t = 0$.

Bidders' optimal strategies $e_i(v_i, D, \kappa)$, $o_i(v_i, \kappa, D, p_i)$ and $\breve{o}_i(v_i, p_i)$ are straightforward as will be illustrated in next section. The nontrivial part of the equilibrium analysis is the bidding strategies at $t = 1$ and the seller's strategies at $t = 0$, which are the focus of the following analysis.

3 Bidders' Equilibrium Strategies

We first analyze bidders' behavior given seller's choices of r and D. Before proceeding further, we present the following result that will simplify the equilibrium characterization in the subsequent subgames (see Appendix A for the proof).

Lemma 1. *It is a dominated strategy for a bidder to submit any bid greater than his private value, i.e.,* $b_i > v_i$.

Lemma 1 allows us to focus our analysis on $b_i \leq v_i$ (which also implies that $\kappa \leq v_i$ and thus $\kappa - D \leq v_i$ conditional on bidder i winning the auction) for the equilibrium characterization in the subsequent subgames.

3.1 Bidders' Decisions on taking the outside options at $t = 3$

When bidder i did not submit a valid bid, or submitted a valid bid but did not win the auction, or did not pay D conditional on winning, the only choice bidder i has at $t = 3$ is the outside option with random price p_i. The characterization of bidder i's strategy $\breve{o}_i(\cdot,\cdot)$ is clear: takes the outside option if $v_i \geq p_i$; otherwise, chooses not to purchase.

Next, we consider the winner's strategy $o_i(\cdot,\cdot,\cdot,\cdot)$ upon paying the deposit. Conditional on winning and paying the deposit, bidder i's decision on whether to exercise the outside option at $t = 3$ is given as follows:

$$o_i(v_i, \kappa, D, p_i) = \begin{cases} 0 & \text{if } \big[e_i = 1 \text{ and } v_i \geq \kappa - D \text{ and } p_i \geq \kappa - D\big]; \\ 1 & \text{if } \big[e_i = 1 \text{ and } v_i \geq \kappa - D \text{ and } p_i < \kappa - D\big], \\ & \text{or } \big[e_i = 1 \text{ and } v_i < \kappa - D \text{ and } p_i \leq v_i\big]; \\ NP & \text{if } \big[e_i = 1 \text{ and } v_i < \kappa - D \text{ and } p_i > v_i\big]. \end{cases} \tag{1}$$

(1) provides the optimal decision $o_i(v_i, \kappa, D, p_i)$ for a winning bidder i upon paying the deposit D. Given $e_i = 1$ where bidder i had paid the deposit D at $t = 2$, the bidder has three choices at $t = 3$: takes the outside option with price p_i, or completes the original transaction with payment $\kappa - D$, or not to purchase. Clearly, it is optimal for bidder i to choose either of the first two when $v_i \geq \min\{p_i, \kappa - D\}$. Therefore, when $v_i \geq \kappa - D$, the bidder would definitely purchase the object: completing the original transaction with an additional payment of $\kappa - D$ if $p_i \geq \kappa - D$, or taking the outside option with price p_i if $p_i < \kappa - D$. However, when $v_i < \kappa - D$,[9] the bidder would only take the outside option if $p_i \leq v_i$; otherwise, he chooses not to purchase.

3.2 Winner's decision on paying the deposit at $t = 2$

Next we discuss a winning bidder i's decision on paying deposit D at $t = 2$. Following what we have shown in (1), if bidder i pays D, i.e., $e_i = 1$, then there exist two possibilities: either $\kappa - D \geq 0$ or $\kappa - D < 0$. Recall that we must have $v_i - \kappa \geq 0$ at equilibrium by Lemma 1 and thus $v_i - (\kappa - D) \geq 0$. In the case where $\kappa - D \geq 0$, it is optimal

[9]This is an out of equilibrium event though, as this implies that the bidder bids higher than v_i.

for the bidder to take the outside option if $p_i < \kappa - D$; otherwise, the bidder should complete the original transaction by paying $\kappa - D$. In the case where $\kappa - D < 0$, we have p_i greater than $\kappa - D$ for sure and the bidder always chooses to complete the original transaction. We can therefore construct a winning bidder i's payoff from paying D at $t = 2$, denoted by $\pi_D(v_i, \kappa, D)$, as follows:

$$\pi_D(v_i,\kappa,D) \;=\; \begin{cases} \int_0^{\kappa-D}(v_i - p_i)\varphi(p_i)dp_i + \int_{\kappa-D}^{\overline{v}}[v_i - (\kappa - D)]\varphi(p_i)dp_i - D & \text{if } \kappa - D \geq 0; \\ \int_0^{\overline{v}}[v_i - (\kappa - D)]\varphi(p_i)dp_i - D & \text{if } \kappa - D < 0. \end{cases} \tag{2}$$

If bidder i does not pay D, i.e., $e_i = 0$, then the only option the bidder will have at $t = 3$ is to wait for the outside option, and therefore, bidder i's expected payoff from not paying D at $t = 2$, denoted by $\pi_{ND}(v_i, \kappa, D)$, is given by

$$\pi_{ND}(v_i,\kappa,D) = \int_0^{v_i}(v_i - p_i)\varphi(p_i)dp_i. \tag{3}$$

From (2) and (3), we know that conditional on winning, bidder i is willing to pay D if and only if $\pi_D(v_i, \kappa, D) > \pi_{ND}(v_i, \kappa, D)$. Define $L = \pi_D(v_i, \kappa, D) - \pi_{ND}(v_i, \kappa, D)$, and our analysis includes the following two cases.

Under Case (a) where $\kappa - D \geq 0$, we have

$$\begin{aligned} &L(v_i,\kappa,D) \\ &= \int_0^{\kappa-D}(v_i - p_i)\varphi(p_i)dp_i + \int_{\kappa-D}^{\overline{v}}[v_i - (\kappa - D)]\varphi(p_i)dp_i - D - \int_0^{v_i}(v_i - p_i)\varphi(p_i)dp_i \\ &= v_i - \kappa - \int_{\kappa-D}^{v_i}\Phi(p_i)dp_i. \end{aligned} \tag{4}$$

Re-writing (4) shows $L(v_i, \kappa, D) = \int_{\kappa-D}^{v_i}[1 - \Phi(p_i)]dp_i - D$, which implies that given κ and D, there should exist a unique threshold for bidder i's private value, denoted by $v_i'(\kappa, D) \in [0, \overline{v}]$, such that $L(v_i, \kappa, D) > 0$ if and only if $v_i > v_i'(\kappa, D)$.

Under Case (b) where $\kappa - D < 0$, $L(v_i, \kappa, D)$ can be written as follows

$$\begin{aligned} L(v_i,\kappa,D) &= \int_0^{\overline{v}}[v_i - (\kappa - D)]\varphi(p_i)dp_i - D - \int_0^{v_i}(v_i - p_i)\varphi(p_i)dp_i \\ &= v_i - \kappa - \int_0^{v_i}\Phi(p_i)dp_i, \end{aligned} \tag{5}$$

implying that there should exist a unique threshold $v_i''(\kappa, D) \in [0, \overline{v}]$, such that $L(v_i, \kappa, D) > 0$ if and only if $v_i > v_i''(\kappa, D)$.

From the discussion above, the deposit strategy for a winning bidder i at $t = 2$ can be stated as follows:

$$e_i(v_i, \kappa) \;=\; \begin{cases} 1 & \text{if } \big[\kappa - D \geq 0 \text{ and } v_i > v_i'(\kappa, D)\big], \\ & \text{or } \big[\kappa - D < 0 \text{ and } v_i > v_i''(\kappa, D)\big]; \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

3.3 Bidding strategy at $t = 1$

From the characterizations of bidder i's strategies on the outside option and the deposit payment above, we are now ready to examine his/her equilibrium bidding strategy at the bidding stage ($t = 1$). A first useful observation is that at equilibrium, a bidder never puts a bid which could potentially leads to a high auction price that makes him find it optimal not to pay the deposit upon winning.[10] Winning with such a high price cannot make him better off than not winning and waiting for the outside option. Note that a bidder's expected payoff from waiting for the outside option is fixed, and his expected payoff from paying the deposit D upon winning drops with the prevailing auction price. This means that at the equilibrium, he would not bid higher than a level such that with this level as the prevailing auction price, he would find it in his interest not to pay the deposit upon winning. A second useful observation is that a bidder never loses any chance to make him better off. This means that he would not bid lower than a level such with this level as the prevailing auction price, he would find it in his interest to pay the deposit upon winning. These two observations, together, mean that a bidder should bid at a level such that with this level as the prevailing auction price, he is indifferent between pay the deposit upon winning and simply waiting for the outside option, provided such a bid level exists. We call this a "payoff indifference condition." If such a level of bid does not exist, which means even bidding the reservation price would make him worse off than simply waiting for the outside option if he pays the deposit upon winning, then a bidder should not participate.

A weakly dominant equilibrium bidding strategy $b(v, r, D)$, which is srictly increasing, can then be identified using the above "payoff indifference condition." The bidding strategy would give a minimum type $\check{v}$ who bids at reservation price r if $b(\bar{v}, r, D) \geq r$:

$$b(\check{v}, r, D) = r. \tag{7}$$

Otherwise, we let $\check{v} = \bar{v}$, i.e. no types participate. When $r < D$, if $b(\bar{v}, r, D) \geq D$, there

[10] A small reputation concern (but this not essential for our argument) would strictly prevent this kind of bids from happening.

exists a higher value threshold, denoted by $\hat{v}$, such that

$$b(\hat{v}, r, D) = D. \tag{8}$$

If $b(\overline{v}, r, D) < D$, we let $\hat{v} = \overline{v}$, i.e. no types bid above D.

We next pin down this dominant bidding strategy by considering two cases: Case (I) where $r < D$ and Case (II) where $r \geq D$.

Case (I): $r < D$

If $v_i \in [\check{v}, \hat{v}]$, "payoff indifference condition." means

$$\begin{aligned} \int_0^{\overline{v}} [v_i - (b - D)]\varphi(p_i)dp_i - D &= \int_0^{v_i} (v_i - p_i)\varphi(p_i)dp_i \\ \Leftrightarrow \quad b &= v_i - \int_0^{v_i} \Phi(p_i)dp_i. \end{aligned} \tag{9}$$

Simplification gives $b = \int_0^{v_i}[1 - \Phi(p_i)]dp_i$.

If $v_i \in (\hat{v}, \overline{v}]$, "payoff indifference condition." means

$$\begin{aligned} \int_0^{b-D} (v_i - p_i)\varphi(p_i)dp + \int_{b-D}^{\overline{v}} [v_i - (b - D)]\varphi(p_i)dp_i - D &= \int_0^{v_i} (v_i - p_i)\varphi(p_i)dp_i \\ \Leftrightarrow \quad b &= v_i - \int_{b-D}^{v_i} \Phi(p_i)dp_i. \end{aligned} \tag{10}$$

Simplification gives $D = \int_{b-D}^{v_i}[1 - \Phi(p_i)]dp_i$.

The bidder's bidding strategy at $t = 1$ under Case (I) can thus be summarized as follows:

$$b(v_i, r, D) \;=\; \begin{cases} v_i - \int_{b(v_i,r,D)-D}^{v_i} \Phi(p_i)dp_i & \text{if } v_i > \hat{v}; \\ v_i - \int_0^{v_i} \Phi(p_i)dp_i & \text{if } v_i \in [\check{v}, \hat{v}]; \\ \varnothing & \text{if } v_i < \check{v}. \end{cases} \tag{11}$$

where $\check{v}$ is given by $r = \int_0^{\check{v}}[1 - \Phi(p_i)]dp_i$ if $r < \int_0^{\overline{v}}[1 - \Phi(p_i)]dp_i$, otherwise $\check{v} = \overline{v}$; and $\hat{v}$ is given by $D = \int_0^{\hat{v}}[1 - \Phi(p_i)]dp_i$ if $D < \int_0^{\overline{v}}[1 - \Phi(p_i)]dp_i$, otherwise $\hat{v} = \overline{v}$.

Case (II): $r \geq D$

In this case, a bidder's bidding strategy is only captured by (10), which can be summarized as follows:

$$b(v_i, r, D) \;=\; \begin{cases} v_i - \int_{b(v_i,r,D)-D}^{v_i} \Phi(p_i)dp_i & \text{if } v_i \geq \check{v}; \\ \varnothing & \text{if } v_i < \check{v}. \end{cases} \tag{12}$$

where $\check{v}$ is given by $D = \int_{r-D}^{\check{v}} [1 - \Phi(p_i)]dp_i$ if $D < \int_{r-D}^{\overline{v}} [1 - \Phi(p_i)]dp_i$, otherwise, $\check{v} = \overline{v}$.

It is easy to check that in (9) $\frac{db}{dv_i} = 1 - \Phi(v_i) > 0$, and in (10) $\frac{db}{dv_i} = \frac{1-\Phi(v_i)}{1-\Phi(b-D)} > 0$ and $\frac{db}{dD} = \frac{-\Phi(b-D)}{1-\Phi(b-D)} < 0$. Therefore, in equilibrium $b(v_i, r, D)$ is increasing in v_i and decreasing in D. Moreover, (10) coincides with (9) when $v_i = \hat{v}$.

Furthermore, the construction of the bidding strategy above tells us that if the seller chooses not to charge any deposit from the winner, i.e., $D = 0$, the only solution is then given by $b_i = v_i$, that is, bidders submit their true values in equilibrium.[11] This implies that the existence of the outside option does not affect bidders' bidding strategy in the auction. Nonetheless, if $D > 0$, truthful bidding does not constitute an equilibrium. Since bids decrease with D, we must have that buyers bid lower than their values when deposit is positive.

The bidding equilibrium and its properties are summarized in the following proposition.

Proposition 1. *Given reserve price r and deposit D set by the seller, in a second price sealed bid auction, at $t = 1$, there exists an increasing bidding equilibrium in dominant strategy such that bidder i, $i = 1, ..., N$, uses strategies in* (11) *or* (12)*, depending on $r \lesseqgtr D$. The equilibrium bids decrease with deposit, but are independent of the number of bidders. Buyers bid truthfully with zero deposit; and they bid lower than their values if deposit is positive.*

Proposition 1 shows how bidders respond to the outside option in a second price auction, where the seller is allowed to charge a deposit paid by the winner. Since the bids are in general lower than the buyers' values, at equilbrium the winner must purchase: he either buys from the seller or the outside option. Since bids decrease with deposit, we must have that with a higher deposit, the probability that a winner defaults must get lower: the original transaction becomes more attractive after the deposit is paid.

The next lemma examines how thresholds $\check{v}$ and $\hat{v}$ change with reserve price r and deposit D (see Appendix A for the proof).

Lemma 2. *(i) Under Case (I) where $r < D$, $\check{v}$ is increasing in r but independent of D and $\hat{v}$ is increasing in D but independent of r; (ii) under Case (II) where $r \geq D$, $\check{v}$ is increasing in both r and D.*

Lemma 2 says that when the reserve price is smaller than the deposit, bidder participation decreases with the reserve price but is independent of the deposit amount, while bidder participation decreases with both the reserve price and the deposit when the reserve price is larger than the deposit. These results will be utilized in next section when we study seller revenue.

[11] Given $r \geq 0$, if $D = 0$, then $b(\check{v}, r, D) \geq 0$ and the bidding strategy in (12) can be written as $\int_b^{v_i} [1 - \Phi(p_i)]dp_i = 0$, which implies truthful bidding from bidders.

4 Seller Revenue

4.1 The seller's choices on r and D

In this section, we examine the seller's optimal choices on deposit D and reserve price r at $t = 0$. Note that along the equilibrium path, a bidder always pays the deposit conditional on winning. The seller's revenue equals the auction price if the winner does not default; the deposit if the winner defaults; and zero if there is are no valid bids. Given the bidders' equilibrium strategies characterized in Proposition 1, we have the seller's expected revenue in the auction as follows.

Case (I) where $r < D$. In this case, we have $\hat{v}(r, D) > \check{v}(r, D)$, where $b(\check{v}, r, D) = r$ and $b(\hat{v}, r, D) = D$. Recall by Lemma 2, for Case (I), $\check{v}$ is increasing in r but independent of D, $\hat{v}$ is increasing in D but independent of r. Denote the seller's expected revenue under Case (I) by $\mathbb{E}_S^I[R(r, D)]$. To distinguish bidding strategies in the different value intervals, we write bidding strategies as $b(v_i, r, D)$ and $\tilde{b}(v_i, r, D)$ for $v_i \in [\check{v}, \hat{v}]$ and $v_i \in (\hat{v}, \bar{v}]$, respectively. Then, $\mathbb{E}_S^I[R(r, D)]$ is given by

$$
\begin{aligned}
&\mathbb{E}_S^I[R(r, D)] \\
&= N(1 - F(\check{v}(r)))Q(\check{v}(r))r \\
&\quad + N\int_{\check{v}(r)}^{\hat{v}(D)}\int_{\check{v}(r)}^{v_i} b(x, r, D)dQ(x)dF(v_i) + N\int_{\hat{v}(D)}^{\bar{v}}\int_{\check{v}(r)}^{\hat{v}(D)} b(x, r, D)dQ(x)dF(v_i) \\
&\quad + N\int_{\hat{v}(D)}^{\bar{v}}\int_{\hat{v}(D)}^{v_i}\left[\left(1 - \Phi(\tilde{b}(x, r, D) - D)\right)\tilde{b}(x, r, D) + \Phi(\tilde{b}(x, r, D) - D)D\right]dQ(x)dF(v_i),
\end{aligned}
$$

where $Q(\cdot) \equiv F^{N-1}(\cdot)$ and $q(\cdot) \equiv Q'(\cdot)$. The bidding strategies b and $\tilde{b}$ are given by (11). The first term is the case that only the highest value of all bidders is above $\check{v}$. The second term is the case where the highest value is between $\check{v}$ and $\hat{v}$ and the second highest value is also between $\check{v}$ and $\hat{v}$. The third term is the case where the highest value is higher than $\hat{v}$ while the second highest value is between $\check{v}$ and $\hat{v}$. Finally, the last term is the case where both the highest and second highest values are above $\hat{v}$.

Interchanging the order of integration we obtain that

$$
\begin{aligned}
&\mathbb{E}_S^I[R(r,D)] \\
&= N(1-F(\check{v}(r)))Q(\check{v}(r))r \\
&\quad + N\int_{\check{v}(r)}^{\hat{v}(D)} \Big(F(\hat{v}(D)) - F(x)\Big) b(x,r,D)dQ(x) + N\int_{\check{v}(r)}^{\hat{v}(D)} \Big(1 - F(\hat{v}(D))\Big) b(x,r,D)dQ(x) \\
&\quad + N\int_{\hat{v}(D)}^{\bar{v}} (1-F(x))\Big[\Big(1-\Phi(\tilde{b}(x,r,D)-D)\Big)\tilde{b}(x,r,D) + \Phi(\tilde{b}(x,r,D)-D)D\Big]dQ(x) \\
&= N(1-F(\check{v}(r)))Q(\check{v}(r))r + N\int_{\check{v}(r)}^{\hat{v}(D)} \Big(1-F(x)\Big) b(x,r,D)dQ(x) \\
&\quad + N\int_{\hat{v}(D)}^{\bar{v}} (1-F(x))\Big[\Big(1-\Phi(\tilde{b}(x,r,D)-D)\Big)\tilde{b}(x,r,D) + \Phi(\tilde{b}(x,r,D)-D)D\Big]dQ(x).
\end{aligned}
\tag{13}
$$

Case (II) where $r \geq D$. Recall that by Case (II) in Lemma 2, $\check{v}(r,D)$ is increasing in both r and D. The seller's expected revenue under Case (II), denoted by $\mathbb{E}_S^{II}[R(r,D)]$, is given by

$$
\begin{aligned}
&\mathbb{E}_S^{II}[R(r,D)] \\
&= N(1-F(\check{v}(r,D)))Q(\check{v}(r,D))\Big[\Big(1-\Phi(r-D)\Big)r + \Phi(r-D)D\Big] \\
&\quad + N\int_{\check{v}(r,D)}^{\bar{v}}\int_{\check{v}(r,D)}^{v_i} \Big[\Big(1-\Phi(b(x,r,D)-D)\Big)b(x,r,D) + \Phi(b(x,r,D)-D)D\Big]dQ(x)dF(v_i),
\end{aligned}
$$

where the bidding strategy b is given by (12). Here, the first term is the case where only the highest value of all bidders is above $\check{v}$, while the second term is the case where both the highest and second highest values are above $\hat{v}$. Interchanging the order of integration we obtain that

$$
\begin{aligned}
&\mathbb{E}_S^{II}[R(r,D)] \\
&= N(1-F(\check{v}(r,D)))Q(\check{v}(r,D))\Big[\Big(1-\Phi(r-D)\Big)r + \Phi(r-D)D\Big] \\
&\quad + N\int_{\check{v}(r,D)}^{\bar{v}} (1-F(x))\Big[\Big(1-\Phi(b(x,r,D)-D)\Big)b(x,r,D) + \Phi(b(x,r,D)-D)D\Big]dQ(x).
\end{aligned}
\tag{14}
$$

In the following analysis, we first establish some useful lemmas to characterize the impacts of r and D on the seller's revenue functions (13) and (14), separately, and then show the optimal choices for the seller by comparing these two cases.

Let us define r^{so} and D^{so} as follows:

$$D^{so} = \int_0^{\hat{v}^{so}} [1 - \Phi(p_i)]dp_i \text{ and } r^{so} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)]dp_i, \tag{15}$$

where $\hat{v}^{so} = \overline{v}$ and $\check{v}^{so} \in (0, \overline{v})$ is uniquely determined by $\frac{1-F(\check{v}^{so})}{f(\check{v}^{so})}(1 - \Phi(\check{v}^{so})) = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)]dp_i$. Clearly, D^{so} is strictly greater than r^{so}. Note that $b(\hat{v}^{so}, r^{so}, D^{so}) = D^{so}$ and $b(\check{v}^{so}, r^{so}, D^{so}) = r^{so}$.

Let us first examine $\mathbb{E}_S^I[R(r, D)]$ with $r < D$. If the seller chooses any $D > D^{so}$, $\hat{v}$ takes the corner solution $\bar{v}$, i.e., $\hat{v} = \bar{v}$, and the seller's expected revenue can be rewritten as follows: $\mathbb{E}_S^I[R(r, D)] = N(1-F(\check{v}(r)))Q(\check{v}(r))r + N\int_{\check{v}(r)}^{\overline{v}}(1-F(x))b(x, r, D)dQ(x)$, which does not change with D since $b(x, r, D)$ does not depend on $D \geq D^{so}$.

Lemma 3. *$\mathbb{E}_S^I[R(r, D)]$ does not change with D for any $D \geq D^{so}$.*

Given the restrictions of $r < D$ in Case (I) and $r \geq D$ in Case (II), we then show that the seller obtains zero revenue by setting $r \geq D^{so}$ (see Appendix A for the proof).

Lemma 4. *(i) Under Case (I) where $r < D$, given any $D > D^{so}$, $\mathbb{E}_S^I[R(r, D)] = 0$ for all $r \in [D^{so}, D]$. (ii) Under Case (II) where $r \geq D$, $\mathbb{E}_S^{II}[R(r, D)] = 0$ for all $r \geq D^{so}$.*

From Lemmas 3 and 4(b), we can restrict our following analysis to the area of $(D, r) \in [0, D^{so}]^2$. Fix $D \in [0, D^{so}]$, let us use $r^I(D)(\leq D)$ and $r^{II}(D)(\geq D)$ to denote the optimal reserve prices under Case (I) and Case (II) with the relevant constraint on r, respectively. We then establish the following two important lemmas (see Appendix A for the proofs). The proofs for these lemmas utilize the unrestricted maximizations of the seller revenue functions of Cases (I) and (II) without taking into account the restrictions on (D, r). We use $\tilde{r}^I(D)$ and $\tilde{r}^{II}(D)$ to denote the unrestricted optimal reserve prices maximizing (13) and (14) for given $D \in [0, D^{so}]$.

Lemma 5. *For any $D \in [r^{so}, D^{so}]$, (i) under Case (I) where $r < D$, $r^I(D) = r^{so}$; (ii) under Case (II) where $r \geq D$, $r^{II}(D) = D$.*

The next lemma shows $r^I(D)$ and $r^{II}(D)$ for any $D \in [0, r^{so})$.

Lemma 6. *For any $D \in [0, r^{so})$, (i) under Case (I) where $r < D$, $r^I(D) = D$; (ii) under Case (II) where $r \geq D$, $r^{II}(D)$ is given by $\tilde{r}^{II}(D)$.*

Figure 2 provides a graphic illustration for how $r^I(D)$ varies when $D \in [0, D^{so}]$ changes, using the uniform distribution for both $F(\cdot)$ and $\Phi(\cdot)$ and two bidders (See computation details in Section 4.2). The X-axis is the reserve price and the Y-axis is the

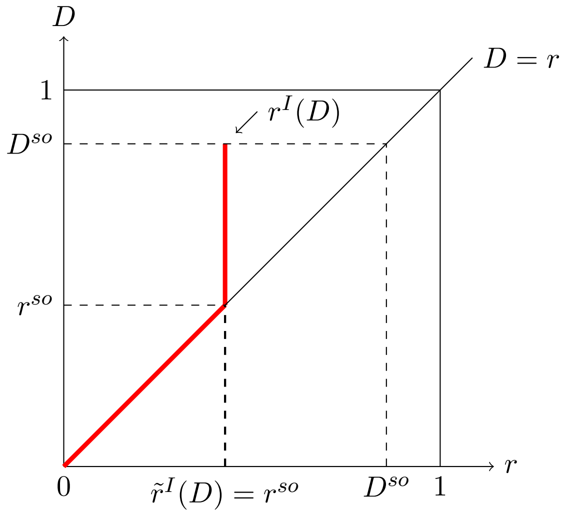

Figure 2: Case (I) where $r < D$

deposit. The unrestricted optimal reserve price $\tilde{r}^I(D)$, which is equal to r^{so}, is a vertical line. The seller's choice is marked by bold red: for $D \in [r^{so}, D^{so}]$, $r^I(D)$ is equal to r^{so}, and for $D \in [0, r^{so})$, $r^I(D)$ is equal to D, which is located on the 45-degree line.

We provide Figure 3 below to graphically illustrate how $r^{II}(D)$ varies when $D \in [0, D^{so}]$ changes, again using the uniform distribution for both $F(\cdot)$ and $\Phi(\cdot)$ and two bidders (See computation details in Section 4.2). The unrestricted optimal reserve price $\tilde{r}^{II}(D)$ for $D \in [r^{so}, D^{so}]$ is in light green. The seller's choice is marked by bold red: for $D \in [r^{so}, D^{so}]$, $r^{II}(D)$ is equal to D (which is located on the 45-degree line), and for $D \in [0, r^{so})$, $r^{II}(D)$ is equal to $\tilde{r}^{II}(D)$.

After establishing the lemmas above, we can then compare the seller's optimal choices of r across Cases (I) and (II) for a fixed $D \in [0, D^{so}]$. Let us use $r^*(D)$ to denote the optimal reserve price to achieve the global maximum of seller revenue (across Cases (I) and (II)), and we can then establish the following result.

Proposition 2. *(i)* $r^*(D) = r^{so}$ *for any* $D \in [r^{so}, D^{so}]$*; (ii)* $r^*(D) = \tilde{r}^{II}(D)$ *for any* $D \in [0, r^{so})$.

The proof for Proposition 2 can be stated as follows. Given the fact that $\mathbb{E}_S^I[R(r, D)] = \mathbb{E}_S^{II}[R(r, D)]$ when $r = D$ (which means that $\check{v} = \hat{v}$), Lemma 5 indicates that for any $D \in [r^{so}, D^{so}]$, $\mathbb{E}_S^I[R(r^I(D) = r^{so}, D)] > \mathbb{E}_S^I[R(r^I(D) = D, D)] = \mathbb{E}_S^{II}[R(r^{II}(D) = D, D)]$. Therefore, $r^*(D) = r^{so}$ for any $D \in [r^{so}, D^{so}]$ in part (i). For part (ii), Lemma 6 gives that for any $D \in [0, r^{so})$, $\mathbb{E}_S^I[R(r^I(D) = D, D)] = \mathbb{E}_S^{II}[R(r^{II}(D) = D, D)] < \mathbb{E}_S^{II}[R(r^{II}(D) = \tilde{r}^{II}(D), D)]$. Hence, $r^*(D) = \tilde{r}^{II}(D)$ for any $D \in [0, r^{so})$. Note that in this case $r^*(D)$ is not necessary to be monotone in D when $D \in [0, r^{so})$.

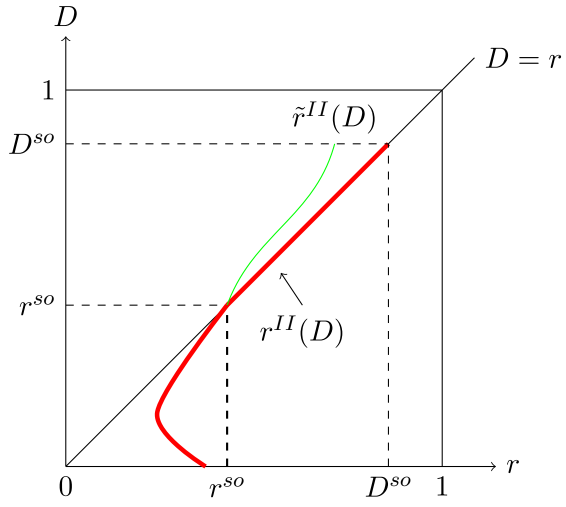

Figure 3: Case (II) where $r \geq D$

Figure 4 below graphically shows the optimal reserve price $r^*(D)$ that achieve the global maximum of seller revenue across Cases (I) and (II). $r^*(D)$ is marked by bold blue: For $D \in [r^{so}, D^{so}]$, $r^*(D)$ is equal to r^{so}, and for $D \in [0, r^{so})$, $r^*(D)$ is equal to $\tilde{r}^{II}(D)$.

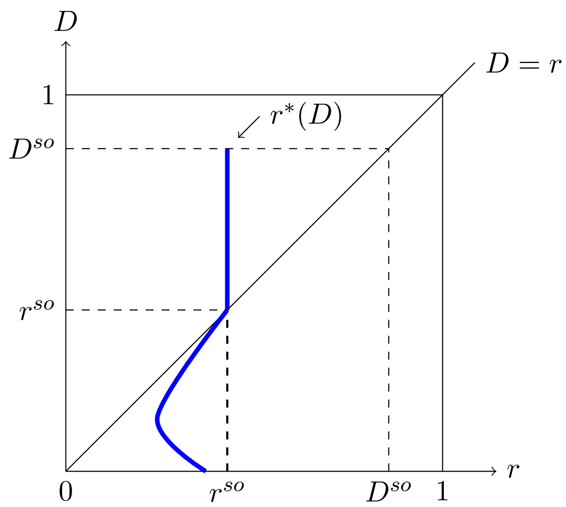

Figure 4: $r^*(D)$ for $D \in [0, D^{so}]$

Define function $R^*(D)$ as follows

$$R^*(D) = \begin{cases} \mathbb{E}_S^I[R(r^*(D), D)] & \text{if } D \in [r^{so}, D^{so}]; \\ \mathbb{E}_S^{II}[R(r^*(D), D)] & \text{if } D \in [0, r^{so}). \end{cases} \tag{16}$$

Using the results of (16), we can then establish the following result (see Appendix A for the proof).

Proposition 3. *$\frac{dR^*(D)}{dD} \geq 0$ for $D \in [0, D^{so}]$, and $\frac{dR^*(D)}{dD} = 0$ when $D = D^{so}$. As a result, (D^{so}, r^{so}) defined in (15) maximizes the seller's revenue. In addition, any (D, r^{so}) wth $D > D^{so}$ also maximizes seller's revenue. The optimal deposit and reservation price are independent of the number of bidders.*

Proposition 3 describes the optimal deposits. Note that charging any deposit $D > D^{so}$ will generate the same expected revenue as $D = D^{so}$ to the seller by Lemma 3, which says that seller revenue does not change with $D > D^{so}$. It works as follows: the winner pays D to the seller and then in the full payment stage the seller pays $D - \kappa$ back to the winner. So the total payment to the seller is still the auction price κ. Given the bidders are usually financially constrainted, it is more practical for the seller to set a lower optimal D.

Furthermore, it is easy to check that when the price of the outside option is equal to 0 with probability 1, all bidders will not enter the auction but choose to wait for outside options directly, regardless of r and D set by the seller; in this case (15) implies that the optimal deposit D^{so} is 0. Another extreme case is that there exists no outside option for bidders after the auction, i.e., $\Phi(.) = 0$. In this case, the bidding strategy and the seller revenue are the same as those of the standard second-price auction (with Myerson's reserve price) and (15) implies that the optimal deposit D^{so} is equal to $\overline{v}$. Note that since the seller's revenue is not affected, charging any deposit D will not play a role in these two extreme cases.

It is clear by (15) that at optimum, the highest type bids D^{so}, which is equal or smaller than the optimal deposit. As a result, the winner would not default even if the outside price is the lowest, i.e. zero. We thus have the following result.

Theorem 1. At optimum, the seller fully deters the winner's default. The winner always completes the original transaction determined by the auction.

Proposition 3 and Theorem 1 indicate that a deposit requirement plays an important role in the auction design. When there exists an ex-post outside option for the winner, the seller can do better by setting a deposit. The tradeoff the seller faces is the following.

Setting a high deposit gives bidders an incentive to bid lower in the auction, but it also makes the winner less likely to default. We find that the latter positive effect always dominates the former negative effect, and thus the seller's revenue is maximized when a sufficiently high deposit is set to fully deter the winner from taking the outside option.

Most of time buyers (bidders) face budget and/or time constraints, especially for items with high values. Therefore, charging a sufficiently large amount for deposit may not be implementable, and this is usually what we observe in practice. In this case, let us denote a upper bound of deposit by $\bar{D}$, and then the seller's optimal choices on deposit and reserve price can be characterized as follows.

Corollary 1. If $\bar{D} \in [0, D^{so}]$ is imposed, then the optimal deposit and reserve price are given by $\bar{D}$ and $r^*(\bar{D})$. In this case, the seller does not fully deter the winner's default.

Note that when $\bar{D}$ is strictly less than D^{so} in the auction, Corollary 1 implies that the probability that the winner defaults from the current transaction and takes the outside option is positive.

Next, we compare the optimal reserve price in our model r^{so} to that of an auction model without the outside option, characterized by Myerson (1981). Let us denote Myerson's optimal reserve price by r^m, which is given by $\frac{1-F(r^m)}{f(r^m)} = r^m$. From the characterization of r^{so} in (15) we have the following

$$\frac{1 - F(\check{v}^{so})}{f(\check{v}^{so})} = \frac{\int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i}{1 - \Phi(\check{v}^{so})} \geq \frac{\check{v}^{so}(1 - \Phi(\check{v}^{so}))}{1 - \Phi(\check{v}^{so})} = \check{v}^{so}.$$

This implies that $r^m \geq \check{v}^{so}$, as $\frac{1-F(v)}{f(v)}$ is decreasing in v. Furthermore, given that $r^{so} = \int_0^{\check{v}^{so}} [1 - \Phi(p_i)] dp_i \leq \check{v}^{so}$, we have $r^m \geq \check{v}^{so} \geq r^{so}$. Clearly, $r^m = \check{v}^{so} = r^{so}$ holds if and only if there exists no outside option, i.e., $\Phi(\cdot) = 0$. Summarizing the comparison gives the following result.

Proposition 4. *$r^m \geq \check{v}^{so} \geq r^{so}$. In particular, $r^m = \check{v}^{so} = r^{so}$ holds only when no outside option exists, i.e., $\Phi(\cdot) = 0$.*

4.2 Numerical example 1

To exemplify the seller's choices on r and D, we have the following numerical example. Assume for simplicity that there are only two bidders ($N = 2$), and both bidder valuations and the price for the outside option are uniformly distributed on the unit interval ($v_i \sim U[0, 1]$ and $p_i \sim U[0, 1]$), computations show $r^{so} = 0.33333$ and $D^{so} = 0.5$.[12]

[12] Note that if no ex-post outside option exists after the auction, i.e., $\Phi(.) = 0$, the optimal reserve price is $r^m = 0.5$, which is greater than $\check{v}^{so} = 0.42265$ and $r^{so} = 0.33333$.

Moreover, when the seller sets $r \geq 0.5$, no valid bids can be submitted and the seller's revenue decreases to zero. All details regarding the computations are provided in Appendix B.

Figures 5 and 6 provide two different angels of the same 3-D graph of the seller revenue as a function of reserve price and deposit. The X-axis is reserve price, the Y-axis is deposit, and the Z-axis is seller revenue. Consistent with our theoretical predictions, the figures show that when the deposit D is fixed, the seller's expected revenue is single peaked with respect to the reserve price r (Figure 5). Furthermore, the seller revenue function with the optimal reserve price, defined as $R^*(D)$ in (16), is increasing in D for $D \in [0, D^{so}]$ and becomes flat for any $D > D^{so}$ (Figure 6). The global maximum of the seller revenue ($= 0.30157$) is achieved at $r^{so} = 0.33333$ and $D^{so} = 0.5$.

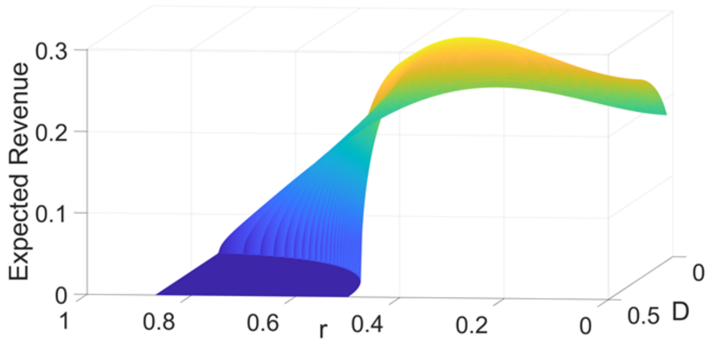

Figure 5: First angel of the 3-D graph

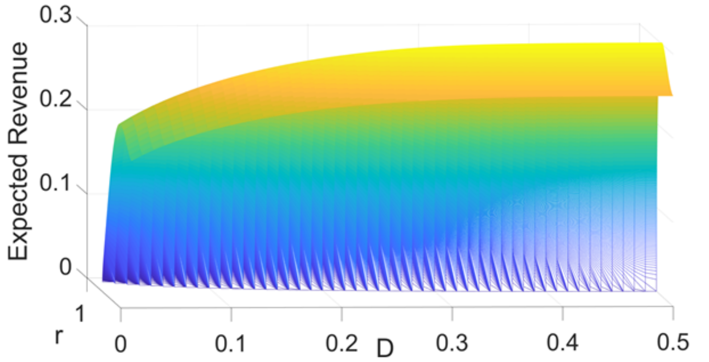

Figure 6: Second angel of the 3-D graph

5 Comparative Statics Analysis

5.1 The Impact of Outside Option

We first study for a given reserve price and deposit how equilibrium bid changes across different outside option distributions. Fix r and D, let us use $b(v_i, r, D, \Phi_1)$ and $b(v_i, r, D, \Phi_2)$ to denote the bidding functions corresponding to distributions of outside options $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, respectively. The following result can then be established.

Proposition 5. *Given reserve price r and deposit D, equilibrium bid submitted by a bidder is higher when buyer outside option distribution gets worse in the sense of first-order stochastic dominance. In other words, if $\Phi_1(\cdot) < \Phi_2(\cdot)$, we have $\check{v}_1 < \check{v}_2$, $\hat{v}_1 < \hat{v}_2$, and $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.*

Second, we attempt to rank seller revenues with the optimal choices on reserve price and deposit across different distributions of outside option price. Let us denote the optimal reserve price and deposit and seller revenue by r_i^{so}, D_i^{so}, and $R^*(r_i^{so}, D_i^{so}, \Phi_i)$ corresponding to $\Phi_i(\cdot), i = 1, 2$. We then have the following result.

Proposition 6. $R^*(r_1^{so}, D_1^{so}, \Phi_1) > R^*(r_2^{so}, D_2^{so}, \Phi_2)$ *if* $\Phi_1(\cdot) < \Phi_2(\cdot)$. *In other words, optimal seller revenue is higher if outside option distribution gets worse (in the sense of first-order stochastic dominance) in buyers' perspective.*

We next examine how the optimal deposit D^{so} change across different outside option distributions. Recall $\hat{v}_1 = \hat{v}_2 = \overline{v}$, which correspond to bids D_1^{so} and D_2^{so}, respectively. It is obvious that if $\Phi_1(\cdot) < \Phi_2(\cdot)$, $D_1^{so} = \int_0^{\bar{v}} [1 - \Phi_1(p_i)]dp_i > \int_0^{\bar{v}} [1 - \Phi_2(p_i)]dp_i = D_2^{so}$. This comparison is formally presented in the following proposition.

Proposition 7. $D_1^{so} > D_2^{so}$ *if* $\Phi_1(\cdot) < \Phi_2(\cdot)$.[13] *In other words, the sellers sets a higher optimal deposit when outside option distribution gets worse (in the sense of first-order stochastic dominance) in buyers' perspective.*

Proportion 7 is quite counter-intuitive as one might well expect that a worse outside option distribution should allow the seller less replying on a deposit to deter the post-auction default. Recall in Myerson (1981), no deposit is required when there are no outside options for buyers. The logic of the result in Proportion 7 can be explained as follows. A worse outside option distribution to buyers (in the sense of first order stochastic dominance) induces bidders to submit higher bids, which makes the ex-post

[13] We assume these distributions cover the full support. The discussion on the degenerate cases are discussed following Proposition 3.

default become more likely to happen. As revealed previously, the optimal deposit is meant to fully deter default of winners. For this purpose, a higher deposit must be in place for the following reasons. First, setting a higher optimal deposit lowers the equilibrium bid, which makes the default easier to deter. Second, a higher deposit paid by the winners makes the default less profitable even the bidding strategy remains the same.

We now turn to the comparisons of threshold $\check{v}^{so}$ and the optimal reserve price r^{so}. Let us define function $\tau(v_i)$ as $\tau(v_i) \equiv \frac{\int_0^{v_i}[1-\Phi(p_i)]dp_i}{1-\Phi(v_i)}$. Differentiating $\tau(v_i)$ with respect to v_i yields

$$\begin{aligned}\tau'(v_i) &= 1 + \frac{\int_0^{v_i}(1-\Phi(p_i))dp_i}{1-\Phi(v_i)} \cdot \frac{\varphi(v_i)}{1-\Phi(v_i)} \\ &= 1 + \tau(v_i) \cdot \frac{\varphi(v_i)}{1-\Phi(v_i)} \\ &> 0.\end{aligned}$$

$\tau(v_i)$ is increasing in v_i and moreover $\tau(0) = 0$. If $\Phi_1(\cdot)$ dominates $\Phi_2(\cdot)$ in terms of the hazard rate, i.e., $\frac{\varphi_1(\cdot)}{1-\Phi_1(\cdot)} < \frac{\varphi_2(\cdot)}{1-\Phi_2(\cdot)}$, we clearly have $\tau_1(\cdot) < \tau_2(\cdot)$.[14] This immediately implies that, in order for $\frac{1-F(\check{v}_1^{so})}{f(\check{v}_1^{so})} = \tau_1(\check{v}_1^{so})$ and $\frac{1-F(\check{v}_2^{so})}{f(\check{v}_2^{so})} = \tau_2(\check{v}_2^{so})$ to be satisfied, we must have $\check{v}_1^{so} > \check{v}_2^{so}$ and $r_1^{so} = \int_0^{\check{v}_1^{so}}[1-\Phi_1(p_i)]dp_i > \int_0^{\check{v}_2^{so}}[1-\Phi_2(p_i)]dp_i = r_2^{so}$. We thus have the following results.

Proposition 8. *$\check{v}_1^{so} > \check{v}_2^{so}$ and $r_1^{so} > r_2^{so}$, if $\frac{\varphi_1(\cdot)}{1-\Phi_1(\cdot)} < \frac{\varphi_2(\cdot)}{1-\Phi_2(\cdot)}$. In other words, the sellers sets a higher optimal reservation price when outside option distribution gets worse (in the sense of hazard rate dominance) in buyers' perspective.*

Combining Propositions 4 and 8 provides a new explanation to the common observation in most online auction platforms that many sellers set very low reserve prices in practice compared with offline auctions. From the analysis above, we know that in such online auction marketplaces where auctions for the similar items are organized sequentially and regularly and therefore outside options do arrive with relatively high probabilities, the sellers have to lower the reserve price in order to attract bidders to participate in the current auctions.

5.2 Numerical example 2

To further illustrate the impacts of an outside option, we conduct additional numerical analyses. Here, we assume that bidder i's valuation still follows a uniform dis-

[14] This also implies that $\Phi_1(\cdot)$ first order stochastically dominates $\Phi_2(\cdot)$, that is, $\Phi_1(\cdot) \leq \Phi_2(\cdot)$, see more details in Appendix B of Krishna (2002).

tribution but the distribution of the outside option price takes the form of $\Phi(p_i) = p_i^\alpha$ over $[0, 1]$, where $\alpha \geq 0$. We then compute how a change in α would affect the sellers' choices on the optimal reserve price r^{so}, the optimal deposit D^{so}, and the optimal expected revenue R^*. All computational details are relegated to Appendix B.

Figures 7 and 8 illustrate how r^{so} and D^{so} change with α. Consistent with our theoretical predictions, the figures show that when α increases, which corresponds to a worse chance for the outside option, both r^{so} and D^{so} increase.

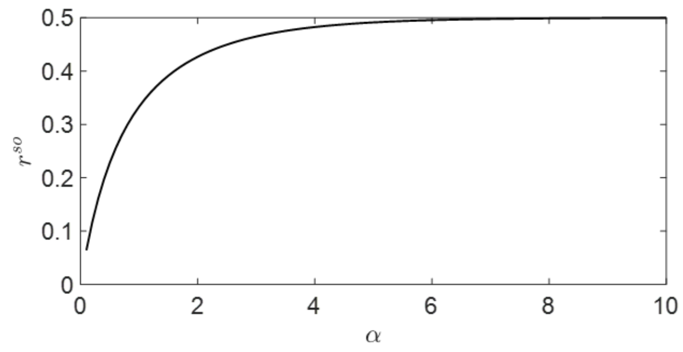

Figure 7: The relationship between r^{so} and α

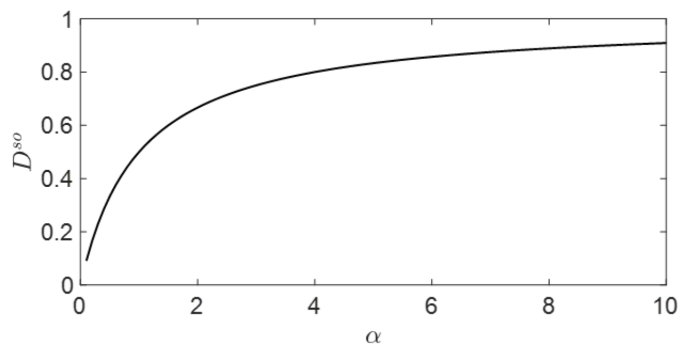

Figure 8: The relationship between D^{so} and α

Figure 9 shows how R^* changes with α. Again, consistent with the theoretical prediction, an increase in α generates a higher R^*, which is captured by the black curve. To further illustrate the role of a deposit in the auction with an ex-post outside option, we compute and plot how the seller revenue with $D = 0$ and the associated optimal reserve price, denoted by $r^*(D = 0)$, changes with α. As shown by the red dotted curve, the seller revenue increases in α as well. More interestingly, the difference between the two revenues is not monotonic. The improvement on seller revenue from charging a deposit is small when the likelihood of having an outside option, captured by α, is either too small or too large, but it gives a relatively large improvement when the likelihood is in the medium range.

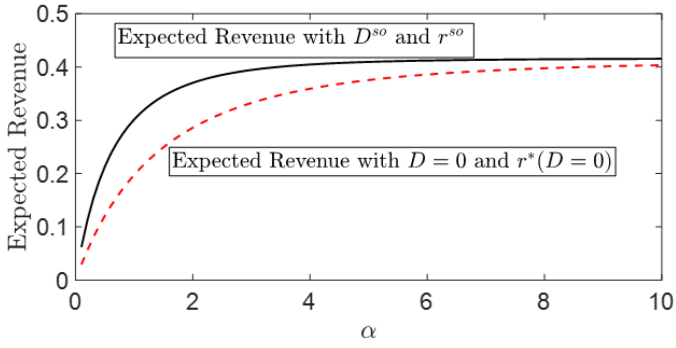

Figure 9: The relationship between R^* and α

6 Concluding Remarks

In this paper, we study the role of a deposit requirement in an auction with post-auction outside options to all buyers. We first characterize the equilibrium bidding strategies for bidders, that is, it is a dominant strategy to submit a bid that makes the bidder indifferent between "simply waiting for the outside option" and "winning with the own bid as the auction price and paying the deposit." We then examine the seller's optimal choices of reserve price and deposit amount and further show that the seller achieves the maximum revenue when a sufficiently high deposit, if it is possible to implement, is set to fully deter any default possibility from the winner. Our study thus provides a jusitification for the widely adopted deposit requirement in settings wth post-auction buyer options where potential winner default is a well recognized concern.

We also examine how different distributions of the outside option price would affect seller revenue, the optimal reserve price, and the optimal deposit. Our results show that a distribution which gives a better chance for the outside option leads to a lower optimal deposit and a lower optimal reserve price, and generates a lower expected seller revenue.[15]

Quitely surprisingly, we find worse option distribution to buyers unambigously requires a higher optimal deposit to be set. This result means that observing a low deposit being set in practice is not an indicator for the insignificance of the concerned post-auction buyer default; on the contrary, it is rather an indicator for the significance of

[15]These findings are potentially testable using empirical data.

this issue. Our study reveals that a lower reservation price than that of Myerson (1981) should be set at optimum, and it gets lower when the option distribution gets better in buyers' perspective. This finding provides alternative explanation for the adoption of low reservation price in practice.

We find that the equilibrium bidding strategy, the optimal deposit and reservation price are independent of the number of bidders. An immediate implication is that our results apply to a setting wth stochastic number of bidders. It is worth mentioning that we have also examined several other extensions on other types of deposit requirements in Appendix C. The first extension we consider is called the all-pay deposit, where the seller requires all potential bidders to pay a certain amount as the deposit before submitting their bids. After the auction ends, the seller refunds the deposits to all other losing bidders, except the winner. For example, Sotheby's requires such a deposit for items with high values. We show that although a bidder needs to pay the deposit before bidding under the all-pay deposit requirement, it does not affect the expected surplus of the bidder and the construction of the equilibrium bid, as the deposit will be paid back conditional on losing in the auction. As a result, the all-pay deposit is strategically equivalent to the winner-pay deposit we consider in the auction game.

The second extension we consider is related to the proportional deduction of the deposit from the full payment. Instead of fully deducted from the final payment, the seller may require that only a certain percentage of the deposit can be deducted the final payment. In this case, the deduction proportion becomes a choice variable to the seller, and we are interested in comparing seller revenues across different proportion rules, answering the question of whether there exists an optimal proportion to the seller. Our analysis shows that if submitting negative-valued bids is allowed, the seller's expected revenue with optimal reserve price and deposit is independent of the deduction proportion rule. We call this property the "deduction proportion independence." However, in practice, it is unlikely (if not impossible) to allow negative-valued bids. In this case, we show that charging a deposit which is proportional to the final payment generates a lower revenue to the seller.

Another extension that we have not considered should also be mentioned before we close. It is commonly observed that sellers implement deposit requirements in first-price auctions, such as UK real estate auctions where most of time the format is first-price, the winners are required to pay a deposit and then complete the final payment within 28 days. It would thus be interesting to examine the impacts of the deposit requirement in first-price auctions, and further compare with second-price auctions in terms of bidding strategies and seller revenues.

Appendix A: Proofs

Proof of Lemma 1

Bidding higher than own value v_i only makes the bidder to win additionally when the prevailing auction price is in between his value and his bid. Clearly, winning with such a price is weakly dominated by not winning and thus simply waiting for the outside options.

□

Proof of Lemma 2

Case (I) where $r - D < 0$. Given $r = \check{v} - \int_0^{\check{v}} \Phi(p_i)dp_i$, differentiating $\check{v}$ with respect to r yields

$$1 = \frac{\partial \check{v}}{\partial r} - \Phi(\check{v})\frac{\partial \check{v}}{\partial r} \quad \Leftrightarrow \quad \frac{\partial \check{v}}{\partial r} = \frac{1}{1 - \Phi(\check{v})} > 0. \tag{17}$$

Thus, $\check{v}$ is increasing in r. Clearly, it is independent of D.

$\hat{v}$ is defined by $D = \hat{v} - \int_0^{\hat{v}} \Phi(p_i)dp_i = \int_0^{\hat{v}} [1 - \Phi(p_i)]dp_i$. Clearly, $\hat{v}$ increases with D but is independent of $r(< D)$.

Case (II) where $r - D \geq 0$. Given $r = \check{v} - \int_{r-D}^{\check{v}} \Phi(p_i)dp_i$, differentiating $\check{v}$ with respect to r yields

$$1 = \frac{\partial \check{v}}{\partial r} - \left(\Phi(\check{v})\frac{\partial \check{v}}{\partial r} - \Phi(r - D)\right) \quad \Leftrightarrow \quad \frac{\partial \check{v}}{\partial r} = \frac{1 - \Phi(r - D)}{1 - \Phi(\check{v})} > 0. \tag{18}$$

Further, differentiating $\check{v}$ with respect to D shows

$$0 = \frac{\partial \check{v}}{\partial D} - \left(\Phi(\check{v})\frac{\partial \check{v}}{\partial d} + \Phi(r - D)\right) \quad \Leftrightarrow \quad \frac{\partial \check{v}}{\partial D} = \frac{\Phi(r - D)}{1 - \Phi(\check{v})} > 0. \tag{19}$$

We then have that $\check{v}$ is increasing in both r and D. □

Proof of Lemma 4

Case (I) where $r - D < 0$. In this case, the seller revenue with any $D > D^{so}$ is given by $\mathbb{E}_S^I[R(r, D)] = N(1 - F(\check{v}(r)))Q(\check{v}(r))r + N \int_{\check{v}(r)}^{\bar{v}} (1 - F(x))b(x, r, D)dQ(x)$. Further, if the seller sets $r \geq D^{so}$ (but still less than D), then $\check{v}$ takes the corner solution $\bar{v}$, i.e., $\check{v} = \bar{v}$. As a result, no one can submit a valid bid and the seller's revenue decreases to zero.

Case (II) where $r - D \geq 0$. With $r \geq D^{so}$, $\check{v}$ takes the corner solution $\bar{v}$, i.e., $\check{v} = \bar{v}$. As a result, no one can submit a valid bid and the seller's revenue decreases to zero. □

Proof of Lemma 5

Let us separately consider the following two cases:

Part (i). From (7), we have that $b(\check{v}(r), r, D) = r$ which is given by $\int_0^{\check{v}}[1-\Phi(p_i))dp_i = r$, we can then re-write $\mathbb{E}_S^I[R(r,D)]$ as follows

$$\begin{aligned}
&\mathbb{E}_S^I[R(r,D)] \\
&= N(1-F(\check{v}(r)))Q(\check{v}(r))b(\check{v}(r),r,D) + N\int_{\check{v}(r)}^{\hat{v}(D)}\Big(1-F(x)\Big)b(x,r,D)dQ(x) \\
&\quad + N\int_{\hat{v}(D)}^{\bar{v}}(1-F(x))\Bigg[\Big(1-\Phi(\tilde{b}(x,r,D)-D)\Big)\tilde{b}(x,r,D) + \Phi(\tilde{b}(x,r,D)-D)D\Bigg]dQ(x).
\end{aligned} \tag{20}$$

Fix D, let us differentiate $\mathbb{E}_S^I[R(r,D)]$ with respect to r and plug $\frac{\partial b}{\partial v_i} = 1-\Phi(v_i)$ into the equation yields

$$\begin{aligned}
&\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(r,D)]}{N} \\
&= \frac{\partial}{\partial \check{v}(r)}\frac{\mathbb{E}_S^I[R(r,D)]}{N}\cdot\frac{\partial \check{v}(r)}{\partial r} \\
&= \Bigg[-f(\check{v}(r))Q(\check{v}(r))b(\check{v}(r),r,D) + (1-F(\check{v}(r))q(\check{v}(r))b(\check{v}(r),r,D) \\
&\quad + (1-F(\check{v}(r)))Q(\check{v}(r))\frac{\partial b(\check{v}(r),r,D)}{\partial \check{v}((r))} - (1-F(\check{v}(r)))q(\check{v}(r))b(\check{v}(r),r,D)\Bigg]\frac{\partial \check{v}(r)}{\partial r} \\
&= Q(\check{v}(r))\Bigg[-f(\check{v}(r))b(\check{v}(r),r,D) + (1-F(\check{v}(r)))\frac{\partial b(\check{v}(r),r,D)}{\partial \check{v}(r)}\Bigg]\frac{\partial \check{v}(r)}{\partial r} \\
&= Q(\check{v}(r))\Bigg[1-\Phi(\check{v}(r)) - \frac{f(\check{v}(r))}{(1-F(\check{v}(r)))}\Big(\check{v}(r) - \int_0^{\check{v}(r)}\Phi(p_i)dp_i\Big)\Bigg]\frac{\partial \check{v}(r)}{\partial r}.
\end{aligned} \tag{21}$$

Recall that $\frac{\partial \check{v}(r)}{\partial r} = \frac{1}{1-\Phi(\check{v})} > 0$ from (17). When $\check{v} = 0$, the derivative is 0. But as long as F and Φ are regular (increasing hazard rate), then $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(\check{v},\hat{v},D)]}{N} = 0$ should occur when the term in the square brackets is zero. We thus have

$$\begin{aligned}
&\Bigg[1-\Phi(\check{v}(r)) - \frac{f(\check{v}(r))}{(1-F(\check{v}(r)))}\Big(\check{v}(r) - \int_0^{\check{v}(r)}\Phi(p_i)dp_i\Big)\Bigg] = 0 \\
\Leftrightarrow\quad &\frac{(1-F(\check{v}(r)))(1-\Phi(\check{v}(r)))}{f(\check{v}(r))} = \int_0^{\check{v}(r)}[1-\Phi(p_i)]dp_i.
\end{aligned} \tag{22}$$

This gives $\check{v}(r) = \check{v}^{so}$ and further, since $b(\check{v}(r), r, D) = r$, $r = r^{so} = \check{v}^{so} - \int_0^{\check{v}^{so}}\Phi(p_i)dp_i = \int_0^{\check{v}^{so}}[1-\Phi(p_i)]dp_i$. The analysis above indicates that the unrestricted optimal reserve price $\tilde{r}^I(D)$ is exactly equal to r^{so} for any $D \geq 0$. Moreover, for any $D \in [r^{so}, D^{so}]$, we have $r < D$ and the unrestricted optimal reserve price is implementable; therefore, $r^I(D) = \tilde{r}^I(D) = r^{so}$.

Part (ii). From (7), we have that $b(\check{v}(r,D),r,D) = r$ which is given by $\check{v} - \int_{r-D}^{\check{v}}[1-\Phi(p_i))dp_i = r$, we can then re-write $\mathbb{E}_S^{II}[R(r,D)]$ as follows

$$
\begin{aligned}
&\mathbb{E}_S^{II}[R(r,D)] \\
&= N(1-F(\check{v}(r,D)))Q(\check{v}(r,D))\Bigg[\bigg(1-\Phi(b(\check{v}(r,D),r,D)-D)\bigg)r + \Phi(b(\check{v}(r,D),r,D)-D)D\Bigg] \\
&\quad + N\int_{\check{v}(r,D)}^{\bar{v}}(1-F(x))\Bigg[\bigg(1-\Phi(b(x,r,D)-D)\bigg)b(x,r,D) + \Phi(b(x,r,D)-D)D\Bigg]dQ(x).
\end{aligned}
\tag{23}
$$

Fix D, differentiating $\mathbb{E}_S^{II}[R(r,D)]$ with respect to r yields

$$
\begin{aligned}
&\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} \\
&= \frac{\partial}{\partial \check{v}(r,D)}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}\cdot\frac{\partial \check{v}(r,D)}{\partial r} \\
&= \Bigg[-f(\check{v}(r,D))Q(\check{v}(r,D))\Bigg[\bigg(1-\Phi(b(\check{v}(r,D),r,D)-D)\bigg)b(\check{v}(r,D),r,D) + \Phi(b(\check{v}(r,D),r,D)-D)D\Bigg] \\
&\quad + (1-F(\check{v}(r,D)))Q(\check{v}(r,D))\frac{\partial b(\check{v}(r,D),r,D)}{\partial \check{v}(r,D)}\Bigg[\bigg(1-\Phi(b(\check{v}(r,D),r,D)-D)\bigg) \\
&\quad - \varphi(b(\check{v}(r,D),r,D)-D)(b(\check{v}(r,D),r,D)-D)\Bigg]\Bigg]\frac{\partial \check{v}(r,D)}{\partial r} \\
&= Q(\check{v}(r,D))\Bigg[-f(\check{v}(r,D))\Bigg[\bigg(1-\Phi(b(\check{v}(r,D),r,D)-D)\bigg)(b(\check{v}(r,D),r,D)-D) + D\Bigg] \\
&\quad + (1-F(\check{v}(r,D)))\frac{\partial b(\check{v}(r,D),r,D)}{\partial \check{v}(r,D)}\Bigg[\bigg(1-\Phi(b(\check{v}(r,D),r,D)-D)\bigg) \\
&\quad - \varphi(b(\check{v}(r,D),r,D)-D)(b(\check{v}(r,D),r,D)-D)\Bigg]\Bigg]\frac{\partial \check{v}(r,D)}{\partial r}.
\end{aligned}
\tag{24}
$$

Recall that $\frac{\partial \check{v}}{\partial r} = \frac{1-\Phi(r-D)}{1-\Phi(\check{v})} > 0$ from (18). When $\check{v} = 0$, the derivative is 0. But as long as F and Φ are regular (increasing hazard rate), then $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} = 0$ should occur when the term in the big square brackets is zero. This gives the unrestricted optimal reserve price $\tilde{r}^{II}(D)$.

Now let us establish the following important property: For any $D \in [r^{so}, D^{so}]$ and $r \geq D$, we have $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} \leq 0$, specially $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} = 0$ if and only if $D = r^{so}$ and $r = D$. To do so, given $b(\check{v}(r,D),r,D) = r$, we re-write the terms in the large square brackets of last line in (24) and further define it as Λ:

$$
\begin{aligned}
\Lambda \equiv &-\Bigg[\bigg(1-\Phi(r-D)\bigg)(r-D) + D\Bigg] \\
&+ \frac{(1-F(\check{v}(r,D)))}{f(\check{v}(r,D))}\frac{\partial r}{\partial \check{v}(r,D)}\Bigg[1-\Phi(r-D) - \varphi(r-D)(r-D)\Bigg].
\end{aligned}
\tag{25}
$$

Note that Λ shares the same sign with $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N}$. Plugging $\frac{\partial r}{\partial \check{v}} = \frac{1-\Phi(\check{v})}{1-\Phi(r-D)}$ into Λ, and simplifying it shows

$$\begin{aligned}\Lambda = &-\left[\left(1-\Phi(r-D)\right)(r-D)+D\right] \\ &+\frac{(1-F(\check{v}(r,D)))(1-\Phi(\check{v}(r,D)))}{f(\check{v}(r,D))}\left[1-\frac{\varphi(r-D)}{1-\Phi(r-D)}(r-D)\right].\end{aligned} \tag{26}$$

Step (1). When $D = r^{so}$ and $r = D$, i.e., $\check{v} = \check{v}^{so}$, we have

$$\Lambda = -r^{so} + \frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0. \tag{27}$$

Step (2). When $D = r^{so}$ and $r - r^{so} > 0$, we can then write Λ as follows:

$$\begin{aligned}\Lambda = &-\left[\left(1-\Phi(r-r^{so})\right)(r-r^{so})+r^{so}\right] \\ &+\frac{(1-F(\check{v}(r,D)))(1-\Phi(\check{v}(r,D)))}{f(\check{v}(r,D))}\left[1-\frac{\varphi(r-r^{so})}{1-\Phi(r-r^{so})}(r-r^{so})\right].\end{aligned} \tag{28}$$

Note that $-\left[\left(1-\Phi(r-r^{so})\right)(r-r^{so})+r^{so}\right] < -r^{so}$, and further, since $\check{v}(r,D) > \check{v}^{so}$ and the hazard rates of F and Φ are increasing, we have $\frac{(1-F(\check{v}(r,D)))(1-\Phi(\check{v}(r,D)))}{f(\check{v}(r,D))}$ decreases with $\check{v}(r,D)$ and $\frac{1-\Phi(r-r^{so})}{\varphi(r-r^{so})}$ decreases with r, implying that the following inequality should hold

$$\begin{aligned}\frac{(1-F(\check{v}(r,D)))(1-\Phi(\check{v}(r,D)))}{f(\check{v}(r,D))}\left[1-\frac{\varphi(r-r^{so})}{1-\Phi(r-r^{so})}(r-r^{so})\right] &< \frac{(1-F(\check{v}(r,D)))(1-\Phi(\check{v}(r,D)))}{f(\check{v}(r,D))} \\ &< \frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})}.\end{aligned}$$

Therefore, we have $\Lambda < -r^{so} + \frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0$.

Step (3). Let us consider any $D \in (r^{so}, D^{so}]$. Define $\omega = r - D$ and denote the inverse bidding function by $b^{-1}(v, r, D)$. (26) can be re-written as follows:

$$\begin{aligned}\Lambda = &-\left[(1-\Phi(\omega))\omega + r^{so} + (D-r^{so})\right] \\ &+\left[\frac{1-F(b^{-1}(\omega+r^{so}+(D-r^{so}),r,D))}{f(b^{-1}(\omega+r^{so}+(D-r^{so}),r,D))}\left(1-\Phi(b^{-1}(\omega+r^{so}+(D-r^{so}),r,D))\right)\right]\left[1-\frac{\varphi(\omega)}{1-\Phi(\omega)}\omega\right].\end{aligned} \tag{29}$$

We now use D' to denote the deposit D when $D > r^{so}$ and D'' to denote the deposit D when $D = r^{so}$. Given the same v and $D' > D''$, we have $b(v, r, D') < b(v, r, D'')$, as $b(v, r, D)$ is decreasing in D. Further, $b(v, r, D)$ is increasing in v; given the same b, we should have $v' > v''$ for $b(v', r, D') = b(v'', r, D'')$. This property implies that with the same bid $\omega + r^{so}$, we should have $b^{-1}(\omega + r^{so}, r, D') > b^{-1}(\omega + r^{so}, r, D'')$. Thus, from equation (29), we have

$$\begin{aligned}
&\left[\frac{1 - F(b^{-1}(\omega + r^{so} + (D' - r^{so}), r, D'))}{f(b^{-1}(\omega + r^{so} + (D' - r^{so}), r, D'))}\left(1 - \Phi(b^{-1}(\omega + r^{so} + (D' - r^{so}), r, D'))\right)\right] \\
&< \left[\frac{1 - F(b^{-1}(\omega + r^{so}, r, D'))}{f(b^{-1}(\omega + r^{so}, r, D'))}\left(1 - \Phi(b^{-1}(\omega + r^{so}, r, D'))\right)\right] \\
&< \left[\frac{1 - F(b^{-1}(\omega + r^{so}, r, D''))}{f(b^{-1}(\omega + r^{so}, r, D''))}\left(1 - \Phi(b^{-1}(\omega + r^{so}, r, D''))\right)\right] \\
&< \left[\frac{1 - F(b^{-1}(r^{so}, r, D''))}{f(b^{-1}(r^{so}, r, D''))}\left(1 - \Phi(b^{-1}(r^{so}, r, D''))\right)\right] \\
&= \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})}.
\end{aligned} \tag{30}$$

Since $\frac{1-\Phi(\omega)}{\varphi(\omega)}$ decrease with ω, we have $\Lambda < -r^{so} + \frac{(1-F(\check{v}^{so}))(1-\Phi(\check{v}^{so}))}{f(\check{v}^{so})} = 0$. The property shown above indicates that for any $D \in [r^{so}, D^{so}]$ and $r \geq D$, $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} \leq 0$, in particular, $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} = 0$ when $D = r^{so}$ and $r = D$. This established property indicates that the unrestricted optimal reserve price $\tilde{r}^{II}(D)$ must be smaller than or equal to D, i.e., $\tilde{r}^{II}(D) \leq D$ for any $D \in [r^{so}, D^{so}]$ (the equality holds if and only if $D = r^{so}$ and $r = D$), and therefore, under Case (II) where $r \geq D$, $\tilde{r}^{II}(D)$ is not implementable and $\mathbb{E}_S^{II}[R(r, D)]$ is maximized at the boundary condition of $r^{II}(D) = D$. □

Proof of Lemma 6

Part (i). From part (i) of Lemma 5, we have shown that the unrestricted optimal reserve price $\tilde{r}^I(D)$ is equal to r^{so} for any $D \geq 0$. From (22), since $\frac{(1-F(\check{v}(r)))(1-\Phi(\check{v}(r)))}{f(\check{v}(r))}$ decreases with $\check{v}(r)$ and $\int_0^{\check{v}(r)}[1 - \Phi(p_i)]dp_i$ increases with $\check{v}(r)$, we have the following inequality for $D \in [0, r^{so})$ and $r < D$:

$$\begin{aligned}
\frac{(1 - F(\check{v}(r)))(1 - \Phi(\check{v}(r)))}{f(\check{v}(r))} - \int_0^{\check{v}(r)}[1 - \Phi(p_i)]dp_i &> \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} - r^{so} \\
&= 0.
\end{aligned} \tag{31}$$

The inequality above implies that $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^I[R(r,D)]}{N} > 0$ for any $D \in [0, r^{so})$ and $r < D$. Therefore, under case (I) where $r < D$, $\tilde{r}^I(D)$ is not implementable for any $D \in [0, r^{so})$ and $\mathbb{E}_S^I[R(r, D)]$ is maximized at the boundary condition of $r^I(D) = D$.

Part (ii). Given the fact that $\frac{(1-F(\check{v}(r,D)))(1-\Phi(\check{v}(r,D)))}{f(\check{v}(r,D))}$ decreases with $\check{v}(r, D)$, (26) gives the following inequality for any $D \in [0, r^{so})$ and $D = r$, that is,

$$\begin{aligned}\Lambda &= \frac{(1 - F(\check{v}(r, D)))(1 - \Phi(\check{v}(r, D)))}{f(\check{v}(r, D))} - D \\ &> \frac{(1 - F(\check{v}^{so}))(1 - \Phi(\check{v}^{so}))}{f(\check{v}^{so})} - r^{so} \\ &= 0.\end{aligned} \tag{32}$$

The inequality above implies that $\frac{\partial}{\partial r}\frac{\mathbb{E}_S^{II}[R(r,D)]}{N} > 0$ for any $D \in [0, r^{so})$ and $r = D$. In this case, we should have that for any $D \in [0, r^{so})$ the unrestricted optimal reserve price $\tilde{r}^{II}(D)$ is greater than D, and therefore, $\tilde{r}^{II}(D)$ is implementable under case (II) where $r \geq D$, i.e., $r^{II}(D) = \tilde{r}^{II}(D)$. □

Proof of Proposition 3

For $D \in [r^{so}, D^{so}]$, $R^*(D) = \mathbb{E}_S^I[R(r^*(D), D)]$, and we have the following three steps: *Step* (i). Let us differentiate $\mathbb{E}_S^I[R(r^*(D), D)]$ with respect to $\hat{v}(D)$, which gives the following equation:

$$\begin{aligned}&\frac{\partial}{\partial \hat{v}(D)}\frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N} \\ &= \Bigg[(1 - F(\hat{v}(D)))b(\hat{v}(D), r, D) - (1 - F(\hat{v}(D)))\Bigg[\Bigg(1 - \Phi(\tilde{b}(\hat{v}(D), r, D) - D)\Bigg)\tilde{b}(\hat{v}(D), r, D) \\ &\quad + \Phi(\tilde{b}(\hat{v}(D), r, D) - D)D\Bigg]\Bigg]q(\hat{v}(D)).\end{aligned} \tag{33}$$

Since $\tilde{b}(\hat{v}(D), r, D) - D = b(\hat{v}(D), r, D) - D = 0$, (33) can be re-written as follows:

$$\frac{\partial}{\partial \hat{v}(D)}\frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N} = \Bigg[(1 - F(\hat{v}(D)))D - (1 - F(\hat{v}(D)))D\Bigg]q(\hat{v}(D)) = 0. \tag{34}$$

This indicates that the choice of $\hat{v}(D)$ has no impact on $\mathbb{E}_S^I[R(r^*(D), D)]$. For convenience, we denote the corresponding $\hat{v}(D)$ by $\hat{v}^*(D)$.

Step (ii). Total differentiation of $\mathbb{E}_S^I[R(r^*(D), D)]$ with respect to D yields

$$\begin{aligned}&\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N} \\ &= \underbrace{\frac{\partial}{\partial \check{v}(r)}\frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N}}_{=0} \cdot \frac{\partial \check{v}(r)}{\partial r} \cdot \frac{dr}{dD} + \underbrace{\frac{\partial}{\partial \hat{v}(D)}\frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N}}_{=0} \cdot \frac{d\hat{v}(D)}{dD} + \frac{\partial}{\partial D}\frac{\mathbb{E}_S^I[R(r^*(D), D)]}{N}.\end{aligned} \tag{35}$$

Step (i) indicates that the second term is equal to zero, and from envelope theorem, we have $\frac{\partial}{\partial \check{v}} \frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} = 0$. We thus have the following equation

$$\begin{aligned}
&\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} \\
&= \int_{\hat{v}^*(D)}^{\bar{v}} (1-F(x))\Bigg[-\varphi(\tilde{b}(x,r,D)-D)\Bigg(\Phi(\tilde{b}(x,r,D)-D)(\frac{\partial \tilde{b}(x,r,D)}{\partial D}-1)-1\Bigg)\tilde{b}(x,r,D) \\
&\quad + \Bigg(1-\Phi(\tilde{b}(x,r,D)-D)\Bigg)\Phi(\tilde{b}(x,r,D)-D)(\frac{\partial \tilde{b}(x,r,D)}{\partial D}-1) \\
&\quad + \varphi(\tilde{b}(x,r,D)-D)\Bigg(\Phi(\tilde{b}(x,r,D)-D)(\frac{\partial \tilde{b}(x,r,D)}{\partial D}-1)-1\Bigg)D + \Phi(\tilde{b}(x,r,D)-D)\Bigg]dQ(x).
\end{aligned} \tag{36}$$

Since $\frac{\partial \tilde{b}(x,r,D)}{\partial D} = \frac{-\Phi(\tilde{b}(x,r,D)-D)}{1-\Phi(\tilde{b}(x,r,D)-D)}$, we have

$$\frac{\partial \tilde{b}(x,r,D)}{\partial D} - 1 = \frac{-\Phi(\tilde{b}(x,r,D)-D)}{1-\Phi(\tilde{b}(x,r,D)-D)} - \frac{1-\Phi(\tilde{b}(x,r,D)-D)}{1-\Phi(\tilde{b}(x,r,D)-D)} = \frac{-1}{1-\Phi(\tilde{b}(x,r,D)-D)}. \tag{37}$$

Plugging (37) into (36) shows

$$\begin{aligned}
&\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} \\
&= \int_{\hat{v}^*(D)}^{\bar{v}} (1-F(x))\Bigg[-\varphi(\tilde{b}(x,r,D)-D)\Bigg(\Phi(\tilde{b}(x,r,D)-D)(\frac{\partial \tilde{b}(x,r,D)}{\partial D}-1)-1\Bigg)\tilde{b}(x,r,D) \\
&\quad + \varphi(\tilde{b}(x,r,D)-D)\Bigg(\Phi(\tilde{b}(x,r,D)-D)(\frac{\partial \tilde{b}(x,r,D)}{\partial D}-1)-1\Bigg)D\Bigg]dQ(x) \\
&= \int_{\hat{v}^*(D)}^{\bar{v}} (1-F(x))\Bigg[-\varphi(\tilde{b}(x,r,D)-D)\Bigg(\frac{-1}{1-\Phi(\tilde{b}(x,r,D)-D)}\Bigg)\tilde{b}(x,r,D) \\
&\quad + \varphi(\tilde{b}(x,r,D)-D)\Bigg(\frac{-1}{1-\Phi(\tilde{b}(x,r,D)-D)}\Bigg)D\Bigg]dQ(x) \\
&= \int_{\hat{v}^*(D)}^{\bar{v}} (1-F(x))\Bigg(\frac{\varphi(\tilde{b}(x,r,D)-D)(\tilde{b}(x,r,D)-D)}{1-\Phi(\tilde{b}(x,r,D)-D)}\Bigg)dQ(x).
\end{aligned} \tag{38}$$

Clearly, $\frac{d}{dD}\frac{\mathbb{E}_S^I[R(r^*(D),D)]}{N} \geq 0$ when D is in the interval of $[r^{so}, D^{so}]$. The equality holds if and only if $\hat{v}^*(D) = \bar{v}$, which implies that the seller charges $D = D^{so}$.

For any $D \in [0, r^{so})$, $R^*(D) = \mathbb{E}_S^{II}[R(r^*(D),D)]$. Let us do the total differentiation of

$\mathbb{E}_S^{II}[R(r^*(D), D)]$ with respect to D, which gives the following equation:

$$\frac{d}{dD}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N} = \underbrace{\frac{\partial}{\partial \check{v}(r,D)}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N}}_{=0} \cdot \frac{\partial \check{v}(r,D)}{\partial r}\cdot\frac{dr}{dD} + \frac{\partial}{\partial D}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N}. \tag{39}$$

Envelope theorem indicates that $\frac{\partial}{\partial \check{v}(r,D)}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N} = 0$. For convenience, we denote the corresponding $\hat{v}(r, D)$ by $\hat{v}^*(r, D)$. Plugging $\frac{\partial b(x,r,D)}{\partial D} = \frac{-\Phi(b(x,r,D)-D)}{1-\Phi(b(x,r,D)-D)}$ into the equation above yields

$$\begin{aligned}
&\frac{d}{dD}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N} \\
&= (1-F(\check{v}^*(r,D)))Q(\check{v}^*(r,D))\Bigg[-\varphi(b(\check{v}^*(r,D),r,D)-D)(\frac{\partial b(\check{v}^*(r,D),r,D)}{\partial D}-1)(b(\check{v}^*(r,D),r,D)-D) \\
&\quad -\Bigg(1-\Phi(b(\check{v}^*(r,D),r,D)-D)\Bigg)(\frac{\partial b(\check{v}^*(r,D),r,D)}{\partial D}-1)+1\Bigg] \\
&\quad + \int_{\check{v}^*(r,D)}^{\bar{v}}(1-F(x))\Bigg[-\varphi(b(x,r,D)-D)(\frac{\partial b(x,r,D)}{\partial D}-1)(b(x,r,D)-D) \\
&\quad + \Bigg(1-\Phi(b(x,r,D)-D)\Bigg)(\frac{\partial b(x,r,D)}{\partial D}-1)+1\Bigg]dQ(x) \\
&= (1-F(\check{v}^*(r,D)))Q(\check{v}^*(r,D))\Bigg(\frac{\varphi(b(\check{v}^*(r,D),r,D)-D)(b(\check{v}^*(r,D),r,D)-D)}{1-\Phi(b(\check{v}^*(r,D),r,D)-D)}\Bigg) \\
&\quad + \int_{\check{v}^*(r,D)}^{\bar{v}}(1-F(x))\Bigg(\frac{\varphi(b(x,r,D)-D)(b(x,r,D)-D)}{1-\Phi(b(x,r,D)-D)}\Bigg)dQ(x).
\end{aligned} \tag{40}$$

When the seller charges any D in the interval of $[0, r^{so})$, we have $\check{v}^*(r, D) < \bar{v}$ which indicates that $\frac{d}{dD}\frac{\mathbb{E}_S^{II}[R(r^*(D),D)]}{N} > 0$, which means $D = D^{so}$ is optimal.

Any (D, r^{so}) wth $D > D^{so}$ also maximizes seller's revenue, since seller revenue does not change with such a change in D by Lemma 3. □

Proof of Proposition 5

Case (I) where $r - D < 0$. First, given that $\Phi_1(\cdot)$ first order stochastically dominates $\Phi_2(\cdot)$, i.e. $\Phi_1(\cdot) < \Phi_2(\cdot)$, which means that $\Phi_1(\cdot)$ gives a worse outside option to the buyers in the sense of first order stochastic dominance, we can easily establish the following facts: (a) $\int_0^{\check{v}_2}[1-\Phi_2(p_i)]dp_i = r = \int_0^{\check{v}_1}[1-\Phi_1(p_i)]dp_i$ implies $\check{v}_1 < \check{v}_2$; (b) $\int_0^{\hat{v}_2}[1-\Phi_2(p_i)]dp_i = D = \int_0^{\hat{v}_1}[1-\Phi_1(p_i)]dp_i$ gives $\hat{v}_1 < \hat{v}_2$.

Second, we compare equilibrium bidding strategies across $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, given $\Phi_1(\cdot) < \Phi_2(\cdot)$. For convenience, we write $b(v_i, r, D, \Phi_k)$ and $\tilde{b}(v_i, r, D, \Phi_k), k = 1, 2$ for $v_i \in [\check{v}_k, \hat{v}_k]$ and $v_i > \hat{v}_k$, respectively. We then have the following:

For $v_i \in [\check{v}_1, \check{v}_2)$, $b(v_i, r, D, \Phi_1) = \int_0^{v_i}[1 - \Phi_1(p_i)]dp_i \geq r$ under $\Phi_1(.)$, but the bidder under $\Phi_2(.)$ does not submit a valid bid, equivalently, $b(v_i, r, D, \Phi_2) = 0$. Thus, $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i = \check{v}_2$, given that the equilibrium bidding strategy is monotone and increasing, the following inequality must be true: $b(v_i, r, D, \Phi_1) > b(\check{v}_1, r, D, \Phi_1) = b(v_i, r, D, \Phi_2) = r$. Thus, $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i \in (\check{v}_2, \hat{v}_1]$, we have $b(v_i, r, D, \Phi_1) = \int_0^{v_i}[1 - \Phi_1(p_i)]dp_i > \int_0^{v_i}[1 - \Phi_2(p_i)]dp_i = b(v_i, r, D, \Phi_2)$. Thus, $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i \in (\hat{v}_1, \hat{v}_2)$, given that the equilibrium bidding strategy is monotone and increasing, we have $\tilde{b}(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_1) = \int_0^{v_i}[1 - \Phi_1(p_i)]dp_i > \int_0^{v_i}[1 - \Phi_2(p_i)]dp_i = b(v_i, r, D, \Phi_2)$. Thus, $\tilde{b}(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i = \hat{v}_2$, given that the equilibrium bidding strategy is monotone and increasing, the following inequality must be true: $\tilde{b}(v_i, r, D, \Phi_1) > b(\hat{v}_1, r, D, \Phi_1) = b(v_i, r, D, \Phi_2) = D$. Thus, $\tilde{b}(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i \in (\hat{v}_2, \bar{v}]$, recall that we can re-write $\tilde{b}(v_i, r, D, \Phi)$ as $\int_{\tilde{b}(v_i,r,D,\Phi)-D}^{v_i}[1 - \Phi(p_i)]dp_i = D$, therefore, we have $\int_{\tilde{b}(v_i,r,D,\Phi_1)-D}^{v_i}[1 - \Phi_1(p_i)]dp_i = D = \int_{\tilde{b}(v_i,r,D,\Phi_2)-D}^{v_i}[1 - \Phi_2(p_i)]dp_i$, which immediately indicates $\tilde{b}(v_i, r, D, \Phi_1) > \tilde{b}(v_i, r, D, \Phi_2)$.

Case (II) where $r - D \geq 0$. Recall that $\int_{r-D}^{\check{v}}[1 - \Phi(p_i)]dp_i = D$. First, given that $\Phi_1(\cdot)$ first order stochastically dominates $\Phi_2(\cdot)$, i.e. $\Phi_1(\cdot) < \Phi_2(\cdot)$, we can easily establish the following fact: $\int_{r-D}^{\check{v}_1}[1 - \Phi_1(p_i)]dp_i = D = \int_{r-D}^{\check{v}_2}[1 - \Phi_2(p_i)]dp_i$ implies $\check{v}_1 < \check{v}_2$.

Second, we compare equilibrium bidding strategies across $\Phi_1(\cdot)$ and $\Phi_2(\cdot)$, given $\Phi_1(\cdot) < \Phi_2(\cdot)$.

For $v_i \in [\check{v}_1, \check{v}_2)$, $b(v_i, r, D, \Phi_1) \geq r$ under $\Phi_1(.)$, but the bidder under $\Phi_2(.)$ does not submit a valid bid, equivalently, $b(v_i, r, D, \Phi_2) = 0$. Thus, $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i = \check{v}_2$, given that the equilibrium bidding strategy is monotone and increasing, the following inequality must be true: $b(v_i, r, D, \Phi_1) > b(\check{v}_1, r, D, \Phi_1) = b(v_i, r, D, \Phi_2) = r$. Thus, $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

For $v_i \in (\check{v}_2, \bar{v}]$, recall that we can re-write $b(v_i, r, D, \Phi)$ as $\int_{b(v_i,r,D,\Phi)-D}^{v_i}[1 - \Phi(p_i)]dp_i = D$, therefore, we have $\int_{b(v_i,r,D,\Phi_1)-D}^{v_i}[1 - \Phi_1(p_i)]dp_i = D = \int_{b(v_i,r,D,\Phi_2)-D}^{v_i}[1 - \Phi_2(p_i)]dp_i$, which immediately indicates $b(v_i, r, D, \Phi_1) > b(v_i, r, D, \Phi_2)$.

Therefore, we can conclude that given reserve price r and deposit D, equilibrium bid submitted by a bidder is higher when buyer outside option distribution gets worse in the sense of first-order stochastic dominance. $\square$

Proof of Proposition 6

Let $\Phi_1(\cdot)$ first-order stochastically dominates $\Phi_2(\cdot)$, i.e., $\Phi_1(\cdot) < \Phi_2(\cdot)$. Denote the optimal reservation price under $\Phi_i(\cdot)$ by $r_i^{so}, i = 1, 2$. Note any sufficiently high D is optimal and fully deters winner defult, and the optimal revenues under $\Phi_i(\cdot), i = 1, 2$ do not depend on D when it is optimaly set. Take such a D, so under (r_2^{so}, D), the winner default is fully detered even with $\Phi_1(\cdot)$. Since $\Phi_1(\cdot) < \Phi_2(\cdot)$, under (r_2^{so}, D), we have $\check{v}_1 < \check{v}_2$ and $b(v_i, r_2^{so}, D, \Phi_1) > b(v_i, r_2^{so}, D, \Phi_2)$ by Proposition 5 and its proof.

Under (r_2^{so}, D), the seller revenue with $\Phi_1(\cdot)$ is given by

$$R(r_2^{so}, D, \Phi_1) = N(1 - F(\check{v}_1))Q(\check{v}_1)r_2^{so} + N \int_{\check{v}_1}^{\bar{v}} \Big(1 - F(x)\Big) b(x, r_2^{so}, D, \Phi_1) dQ(x),$$

and the optimal seller revenue under $\Phi_2(\cdot)$ is given by

$$R^*(r_2^{so}, D, \Phi_2) = N(1 - F(\check{v}_2))Q(\check{v}_2)r_2^{so} + N \int_{\check{v}_2}^{\bar{v}} \Big(1 - F(x)\Big) b(x, r_2^{so}, D, \Phi_2) dQ(x).$$

Define $R(v_1) = N(1 - F(v_1))Q(v_1)r_2^{so} + N \int_{v_1}^{\bar{v}} \Big(1 - F(x)\Big) b(x, r_2^{so}, D, \Phi_1) dQ(x)$. Differentiating $R(v_1)$ with respect to v_1 yields

$$\frac{dR(v_1)}{dv_1} = -Nf(v_1)Q(v_1)r_2^{so} + N(1 - F(v_1))Q'(v_1)(r_2^{so} - b(v_1, r_2^{so}, D, \Phi_1)) < 0, \forall v_1 > \check{v}_1.$$

Let $R^*(r_1^{so}, D, \Phi_1)$ denote the optimal seller revenue under Φ_1. We thus have

$$\begin{aligned} R^*(r_1^{so}, D, \Phi_1) &\geq R(r_2^{so}, D, \Phi_1) \\ &= R(\check{v}_1) \\ &> R(\check{v}_2) \\ &\geq R^*(r_2^{so}, D, \Phi_2). \end{aligned}$$

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