Open Source Software Licensing and Entry Incentives

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Abstract

This paper looks at incentives for entry into software markets by open source (OS) platforms which allow software developers to build applications without regard to intellectual property rights. Restrictive OS forces all modifications to be released under the same open terms; while nonrestrictive OS allows applications to be made proprietary by developer. We develop a model of effort provision from software developers under each kind of license and compare the outcome. We identify two motives governing developer effort - user motive to improve surplus as a consumer of the software and revenue motive as a potential proprietor of the new software.

We introduce competition from an incumbent software platform to study how the incentives for platform entry and application development are affected by the incumbent's and entrant's licensing decisions. We also describe the equilibrium license that would result from a competitive entry process under different licensing regimes for the incumbent.

The results of our analysis point to several testable hypotheses and also provide policy implications for studying entry barriers and supply-side effects of mergers in software markets. First, in terms of developer's effort into an OS platform entrant, we show that a non-restrictive license induces effort levels that are at least as high as with a restrictive OS. This validates empirical findings on developers' participation under different open source licenses. Second, we find that entry by an OS platform is less likely when the incumbent has an OS license than with a proprietary incumbent. At the same time, a proprietary incumbent incentivizes too much open source entry relative to what is efficient; in contrast, an OS incumbent does not create high enough incentives for entry by an OS platform.

Finally, looking at equilibrium license choice by the entrants under a competitive entry process, we find that a low valued incumbent attracts restrictive open source entrants irrespective of incumbent license. Whereas high valued incumbents attract entrants with the same license as the incumbent.

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1 Introduction

The principal difference between a traditional proprietary software product and an open source production model is that, in the latter, software developers voluntarily invest their time and effort to develop applications without the encumbrance of copyright and other intellectual property right concerns. In its most widely known form, copyleft licensing forces all modifications on the open source platform to be distributed under the same open licensing terms.

The last few decades have seen a surge in share of open source (OSS) platforms and associated applications in software markets. Linux, which was developed under the GPL open source license serves over 40 percent of the websites on the internet. More recently, Google adopted the Apache open source license for its Android platform. These open source platforms have made successful inroads into their respective markets (web server operating systems in the case of Linux and mobile operating systems in the case of Google) despite competition from well-established proprietary incumbents like Microsoft and Apple.

As open source platforms have grown in size and share, they have also come under antitrust scrutiny. In 2010, the merger between Oracle that had a proprietary database and Sun, with a copyleft open source database, raised several questions about how to assess the competitive restraints provided by open source licenses. Google has also faced its own antitrust battles over predatory pricing on its open source platform. At the same time, the market dynamics that governed the development of the Android OS on the Apache license and subsequent antitrust concerns is substantively different the mechanics surrounding restrictive open source licenses such as Linux's GPL or Sun's open source license. Unlike copyleft licenses which force all modifications on a platformed under its license to be kept open and free of copyright, less restrictive licenses, such as the Apache license, allow developers to make some or all of their application on the platform proprietary.

The role of open source licensing in antitrust analysis has received almost no attention in the literature. The current paper attempts to shed light on entry incentives under different licensing regimes. In particular, we describe how the competitive landscape is affected by the licensing choice of incumbent and entrant software platforms and how to think of entry barriers or entry incentives when considering competition from open source platforms.

As open source software licenses have grown in size as well as variety, there has been considerable empirical research into how developer participation and software innovation is affected by the open source license. However, research into the theoretical underpinnings that drive these empirical findings has been scant. Since open source licensing affects developer incentives for participating in open source, it is likely to have a critical impact both on software quality and the market dynamics of how it competes with software developed under different licenses. Our paper attempts to fill in these theoretical gaps in our understanding of how software license affects the entry and development of new software in the market.

We begin by theoretically examining the implications of license restrictions on effort provision by software developers and software entry. Some recent papers have empirically examined the relationship between open source license and software development and there is a general consensus that restrictive licenses are associated with lower levels of participation and performance compared to non-restrictive licenses. Subramaniam et al. (2009) find that restrictive licenses are less likely to generate successful projects. Similarly, Comino et al. (2007) and Lerner and Tirole (2005) find that software distributed under restrictive licenses are less likely to reach a mature and stable release. Fershtman and Gandal (2007) find that output per contributor is much higher when the open source license is less restrictive.

While there is considerable empirical research on this topic, there has been no formal theoretical model to capture the mechanism that leads to varying outcomes in different licenses. Thus, our first goal in this paper is to provide a theoretical framework for understanding the relationship between developers' effort provision and OSS licensing. This allows us to examine the social welfare associated with different licenses. Thus we show that while restrictive licenses may be associated with lower effort provision than non-restrictive licenses, they can sometimes be more efficient. Specifically, we show that a non-restrictive license leads to over-provision of developer effort relative to what is efficient. Hence the empirical finding that less restrictive licenses lead to more developer participation does not necessarily provide a case against subsidizing restrictive licenses like the GPL. This is an important result because in recent years many countries have adopted policies that favor restrictive open source licensing and there is an ongoing debate about whether such public policy is misguided.[1] For example, Schmidt and Schnitzer (2003) question the need for public policy to encourage open source licenses like the GPL. Our paper argues for caution against dismissing restrictive licensing on the basis of the empirical findings.

Second, we look at how the incentives for entry and subsequent effort provision from developers

[1] For example, the city of Munich in Germany migrated from Microsoft operating system to Linux which is licensed under GPL. In the US, the government provides R&D support for projects released under GPL.

is influenced by the licensing choices of the incumbent and entrant platforms. A number of papers have looked at how proprietary software competes with open source.[2] However, the focus of this literature has been only on restrictive OSS licenses. We find that open source licensing interacts with competition in significant ways to influence the development of open source software.

We begin by developing a basic model of software development where user-developers independently exert costly effort into creating applications over a software platform under a given license. There are two motives that govern developer participation in an open source platform - 1) user motive, as a user, developers would like to enhance their consumer surplus either by generating a high quality software of by lowering price by providing price competition to the incumbent through their new platform; and 2) revenue motive, if the license allows applications developed on the platform to be made proprietary, developers can appropriate revenues if they happen to generate a successful applications. Participation incentivized through the user motive generates positive externalities and hence is subject to the classic public-good under-provision. On the other hand, the revenue motive creates inefficient incentives to over-participate in order to win the proprietary prize. The trade-off between these opposing externalities then determines how each license performs in the face of competition form an incumbent software platform.

Our analysis provides a number of interesting results. First, taking the incumbent and entrant's license as given, we show that, in the presence of a proprietary incumbent, entry incentives for both restrictive and non-restrictive platforms are too high relative to what is efficient. In contrast, an open source incumbent provides for too little entry of other open source entrants- both restrictive and non-restrictive. Second, we look at the equilibrium licensing choice made by competing entrants in the presence of an incumbent. We find that if the incumbent software value is low enough, the entrant with a restrictive license outperforms the entrant with a non-restrictive license when faced with competition from a proprietary incumbent. The licensing of the incumbent software also matters for entrant license choice when the incumbent provides software with relatively high value. Here an open source incumbent attracts more restrictive OSS entrants, while a proprietary incumbent facilitates the entry of non-restrictive open source license.

To our knowledge, there are three other papers that also provide a theoretical model of open source licenses. Lerner and Tirole (2005) provide a preliminary model to explain how developer motivations differ across different licenses. However, they do not provide a rigorous framework

[2] Atal and Shankar (2014); Athey and Ellison (2014); Kumar et. al. (2011); Casadesus-Masanell and Ghemawat (2006); Mustonen (2003).

to explain the efficiency implications of license restrictions. Gaudeul (2005) examines the choice between GPL and BSD licenses when developers can hijack the project and sell it for a positive price. She also shows as we do here that the BSD license allows greater effort provision, but also entails the possibility that the project leader loses some profits to developers. Another paper that explores developer investment in restrictive and non-restrictive licenses is D'Antoni and Rossi (2007) who find that compared to non-restrictive licenses, restrictive licenses encourage greater complementary investments that enhance the value of the open source software. Unlike our paper, however, none of the three papers mentioned here look at the role of software licensing in affecting incentives for entry into software markets with an incumbent platform.

The paper is organized as follows. In Section 2, we describe the model and assumptions. In Section 3, we describe developers' effort provision and entry incentives under restrictive and non-restrictive licenses when the incumbent software has a proprietary license. Section 4, we consider the equilibrium when the incumbent holds a restrictive OSS license. Section 5 consider the choice of license by the entrant. Finally, Section 6 concludes. All proofs are in the appendix.

2 Model

We consider a production and entry model of an open source platform with $N \geq 2$ user-developers. Each developer i exerts effort $e_i \geq 0$ into developing an application that runs on the open source platform; let's call it platform A. The cost of effort is quadratic, $C(e) = \frac{1}{\beta} e^{\beta}, \beta \geq N$. The output from developer i's effort is linear in effort $d_i = \alpha e_i$, $\alpha > 0$.

There is a single developer whose contribution to the platform is critical in the sense that without this innovation, the platform does not create any user value. The identity of this user is determined through a stochastic process that depends on the effort invested by the developer. Specifically, we define the stochastic variable, $y_i = e_i \varepsilon_i$ where $\varepsilon_i \sim U[0, 1]$ is independently and identically distributed for each developer. The critical developer is then the one who generates the highest value of y_i. Let us denote the critical developer by k. Then the value of the platform is the sum of every developer's output as long as it includes developer k's output i.e. $v_A = \sum_{i=1}^{N} d_i$.

Hence, under our set-up, the winning application determines the platform's value, while the losing applications do not generate any value for the platform without winning application.

There are two kinds of open source licenses that the new software platform may have - restrictive and non-restrictive denoted by r and nr respectively. Under a restrictive license, the software application generated by the highest output from developer effort cannot be made proprietary by the developer and hence is available to all users and developers at marginal distribution cost which is zero. Under a non-restrictive license, the winning developer who generates the highest value of effort can make the software application proprietary and sell it to other users at a price.

There is an existing incumbent platform, let's call it B. Platform B has an existing application that generates value v_B for the platform. We consider two different licensing terms for software B as well - proprietary and restrictive OSS, denoted by P and R respectively. This influences the price at which v_B is sold.

The timing of the game is as follows. In stage 1, a competitive process determines the licensing choice and entry decision for the new platform A. Next, N developers exert effort to develop applications on software platform A. After effort is provided, the platform value, v_A, is determined. In stage 2, depending on the licensing terms, the price for A and B are determined through Bertrand price competition. User-developers make their consumption choices based on the prices and values generated by the two platforms. We restrict our attention to symmetric equilibrium in effort provision. The solution concept is Subgame Perfect Nash Equilibrium.

Let us begin by looking at effort provision under each license when there is no incumbent platform in the market. In any kind of license, potentially, the developer receives two kinds of surplus. First, if the developer can make the software proprietary (as is the case under a non-restrictive license), she may receive revenue from selling her application. Second, as a user of the application on the platform, the developer receives user value from the final product. For any developer i, let us denote profit from proprietary revenue by π_i and user surplus from consumption by u_i. The total surplus to eth developer is then $S_i = \pi_i + u_i$.

In the case of a restrictive license (R) for platform A no single developer can make the platform proprietary. In particular, critical developer, k is forced to keep his contributions open and hence cannot appropriate any revenue from the output she generates. Hence, developer surplus from effort only arises from the user value that the platform creates for its developers and the surplus to developer i, given the effort of the remaining $(N-1)$ developers is

$$S_i\left(e_i; e_{j\neq i}\right) = u_i\left(e_i; e_{j\neq i}\right) = \alpha\left(e_i + \sum_{j\neq i} e_j\right) - \frac{1}{\beta} e_i^{\beta}.$$

We restrict $\beta \geq \left(N + \frac{3}{2}\right)$ in order to ensure a symmetric equilibrium with positive effort exists.

The surplus function is concave and hence the effort level that maximizes developer surplus is

$$e_r^* = (\alpha)^{\frac{1}{\beta-1}} .$$

The value of the platform is

$$v_A = N (\alpha)^{\frac{1}{\beta-1}} .$$

The first-best effort provision, where the social planner maximizes the expected value generated to all N user-developers will set the marginal social surplus from a developer's effort to zero. Thus efficient effort e_o solves $N\alpha - e_o^{(\beta-1)} = 0$, or

$$e_o = (N\alpha)^{\frac{1}{\beta-1}} . \tag{1}$$

It is evident from (**??**) and (1) that a restrictive license produces effort provision below the efficient level. This arises from the standard public good under-provision problem as the restrictive license does not allow any developer to appropriate the value of her effort provided to the other developers.

Next let us consider effort provision under a non-restrictive license. Under our set-up, only developer k who realizes the critical innovation to the platform possesses any market power. In other words, the remaining developers with non-critical output do not create anything with stand-alone user-value. Thus, the critical developer, k, makes his output proprietary and appropriates all the value from the platform by setting a monopoly price that equals the value of the platform, so that $S_k = Nv_A - \frac{1}{\beta} e_i^\beta$, whereas the other developers earn negative surplus, i.e. $S_i = 0 - \frac{1}{\beta} e_i^\beta$. for all $i \neq k$. In direct contrast to the restrictive open source license, developer effort under a non-restrictive license is entirely driven by incentives.

Since the identity of developer k is unknown ex ante, the expected surplus to the developer from exerting effort e_i is

$$E\left[S_i\left(e_i; e_{j\neq i}\right)\right] = \Pr\left[i = k\right] N\alpha \left(e_i + \sum_{j\neq i} e_j\right) - \frac{1}{\beta} e_i^\beta .$$

Since we are looking for a symmetric equilibrium, we derive the probability density function for k, $\Pr\left(i = k\right) = \Pr\left(y_i > y_j, \text{ for all } j \neq i\right)$ assuming that developer i chooses e_i and all other developers choose $e_j = e_{nr}$, $j \neq i$. This gives us the following.[3]

[3]See Proof of Proposition 1 in the appendix for derivation of the probability density function.

$$\Pr(i=k) = \begin{cases} \frac{1}{N}\left(\frac{e_i}{e_{nr}}\right)^{N-1} & \text{if } e_i < e_{nr}, \\ \frac{1}{N}\left[N-(N-1)\left(\frac{e_{nr}}{e_i}\right)\right] & \text{if } e_i \geq e_{nr}. \end{cases}.$$

The expected surplus to developer i is:

$$E\left[S_i\left(e_i; e_{nr}\right)\right] = \begin{cases} \alpha\left(\frac{e_i}{e_{nr}}\right)^{N-1}\left[e_i + (N-1)\, e_{nr}\right] - e_i^{\beta} & \text{if } e_i < e_{nr}, \\ \alpha\left[N-(N-1)\left(\frac{e_{nr}}{e_i}\right)\right]\left[e_i + (N-1)\, e_{nr}\right] - e_i^{\beta} & \text{if } e_i \geq e_{nr}. \end{cases}$$

Solving the first order condition and setting $e_i = e_{nr}$, we get positive equilibrium effort under an NR license as:

$$e_{nr}^{*} = \left[\alpha N + \alpha\,(N-1)^2\right]^{\frac{1}{\beta-1}}.$$

The value of the platform is

$$v_A = N\left[\alpha N + \alpha\,(N-1)^2\right]^{\frac{1}{\beta-1}}$$

The following proposition compares effort provision and software value across the two licenses and with the efficient outcome.

Proposition 1 *In the absence of competition ($v_B = 0$), relative to the first-best outcome, a restrictive OSS license under-provides effort and software value while a non-restrictive OSS license over-provides effort and software value, i.e. $e_r^* \leq e_o \leq e_{nr}^*$ and $v_A\left(e_r^*\right) \leq v_A\left(e_o\right) \leq v_A\left(e_{nr}^*\right)$.*

Proposition 1 supports the empirical finding that developer participation is lower under restrictive licenses than under non-restrictive licenses. However, the proposition highlights the fact that neither license provides optimal level of effort relative to cost. Public good characteristics of a restrictive license tend to depress effort below the efficient level. Here, developers, incentivized as users to participate, try to free-ride on each other's effort. On the other hand, a non-restrictive license generates revenue incentives as developers exert effort in order to become the winning proprietor of the software platform. This creates a tournament among developers and leads to over-investment by the losing developers. Thus, it is unclear whether the non-restrictive license is more optimal than the restrictive license. We see some aspects of over-provision in the development of apps for Google's Android phones. Android hosts a larger number of apps and also has a greater number of developers working on its platform. Yet app revenues are lower than for iOS. To the extent that Apple's proprietary model of compensating app developers allows it to suppress app development to maintain higher quality, it would indicate more efficient effort provision.

3 Effort Provision and Entry with an Incumbent Platform

In this section, we look at how competition from an incumbent proprietary software platform interacts with software licensing and affects entry of and effort provision into a new platform. We take the value of the existing software (B) as exogenously given and consider two different licenses for the new open source software platform - a restrictive open source license denoted by r and a non-restrictive open source license denoted by nr.

To set a benchmark, let us first define the first-best efficient effort level when the existing software has value v_B. In stage 2 of the development process when v_A is realized, the efficient outcome is for all developers to use the software with the higher value. Hence efficient software value in stage 2 is $\max\{v_A, v_B\}$.

Given this stage 2 surplus, expected stage 1 surplus to all N user-developers from effort e can be derived as follows.

If $e < \frac{v_B}{N\alpha}$, then $v_A < v_B$. So, total expected welfare across all developers is $W(e) = Nv_B$. On the other hand, if $e \geq \frac{v_B}{N\alpha}$, then total expected welfare across all developers is $W(e) = Nv_A - \frac{N}{\beta}e^{\beta}$. Effort into platform A is efficient if and only if $N^2\alpha e - \frac{N}{\beta}e^{\beta} > Nv_B$, i.e. $v_B \leq \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$.

For α high enough and β low enough, the surplus maximized through effort in this range is higher than the baseline value, Nv_B. When $v_B > \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, so that entry is efficient, the optimal effort, e_{co}, is given by:

$$e_{co} = (N\alpha)^{\frac{1}{\beta-1}}.$$

Thus to summarize, entry by a new open source platform is efficient if and only if $v_B \leq \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$. This range increases with N and α, and decreases with β. If entry is efficient, optimal effort level is $e_{co} = (N\alpha)^{\frac{1}{\beta-1}}$.

The lemma below describes the first-best outcome for entry and effort.

Lemma 1 *Suppose there is an incumbent platform (B) that provides value v_B to each user, then entry by a new platform is efficient if and only if $v_B \leq \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$. When entry is efficient the optimal effort is $e_{co} = e_o = (N\alpha)^{\frac{1}{\beta-1}}$.*

When there is an incumbent platform of value $v_B > 0$, given entry and effort that generates a new open source platform of value v_A, there is price competition between the incumbent platform and the new platform depending on the restrictions imposed by the entering platform's license.

In the following subsections, we consider the outcome the equilibrium effort and entry when there is an existing software. We first take the open source license as given and then derive the equilibrium license choice when the platform owner competes for developers.

3.1 Restrictive Open Source License

If the entering software A adopts a restrictive license, then in the price competition stage by definition its price $p_A = 0$. Since developers can always consume A for free and earn a consumer surplus of v_A, the proprietary seller of B must set a price of $p_B = \max\{v_B - v_A, 0\}$.[4] Thus with a restrictive license, user surplus is always $u_i = v_A - \frac{1}{\beta}e_i^{\beta}$, which is the same as the case when there was no incumbent competition. This means that equilibrium effort provision under a restrictive license is not affected by competition from a proprietary incumbent. This result is stated in the proposition below.

Proposition 2 *Given an incumbent platform (B) of value $v_B > 0$, the following is true about entry by a new platform A that has a restrictive open source license.*

a) Platform A always enters the market and effort provision into A is independent of the value of platform B; specifically $e_r^{P*} = e_r^*$.

b) Entry by the restrictive open source platform is inefficient if $v_B > \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$.

c) If $v_B \leq \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, so that entry is efficient, then developers on platform A under-provide effort relative to the efficient level, i.e. $e_r < e_{co}$.

Proposition 2 a) highlights an important feature of the restrictive OSS license which is that its entry is not affected by proprietary competition from an incumbent. As we show below, this is not true for software developed under non-restrictive OSS licenses. Moreover, as part b) shows, entry by a new software is not always efficient under a restrictive license. If incumbent software value is high, then from Lemma 1 we know that entry of a new software platform is not efficient. Yet entry always occurs under a restrictive open source license. This is because the new platform allows user-developers to appropriate some surplus from the existing monopolist platform owner of B.

If entry is efficient, a restrictive open source license under-provides effort. This is because, the developer participation is driven only by incentives as a user. Since there is no revenue to be made

[4] We make the following indifference assumptions for users. If a user is indifferent between buying and not buying she always buys the product. If she is indifferent between the lower quality product and the higher quality software, she chooses the latter.

by the winning developer under a restrictive open source license, the public good aspect of OSS development results in under-provision as stated in part c).

3.2 Non-restrictive Open Source License

Now let us move on to the case where A is developed under a non-restrictive license. Here, price competition between two proprietary sellers ensues in stage 2 with new platform value v_A. There are three cases for developer i in stage 2. First, if she is the winning developer who makes the new platform proprietary and $v_A > v_B$, then she can charge a price of $p_A = (v_A - v_B)$ to the remaining $(N-1)$ users by making her innovation proprietary. In that case her total payoff is $v_A + (N-1)(v_A - v_B)$. Here developer surplus is generated through proprietary revenue. Second, if developer i does not win the market but someone else from project A does, then she uses platform A at a price of $(v_A - v_B)$ so that her user surplus is v_B and she does not earn any revenue. Finally, if $v_A \leq v_B$, then the winning developer is irrelevant since no one will buy the lower quality platform. However, as with the restrictive platform the entrant platform value v_B can be used as leverage to compete against the incumbent to get a positive user surplus of v_A. In this case, all developers receive surplus solely as users and no revenue is earned through software sales. Thus, depending on the value of the incumbent software developer effort is driven by a combination of user motive and revenue motive in contrast to the previous case where restrictive open source software development occurs solely through user incentives.

In the price-competition phase then, the price of the incumbent platform is $p_B = v_B - v_A$. The probability that i makes the critical contribution is the same as before. Given that the other $N-1$ developers choose effort e^P_{nr},

$$\Pr(i = k) = \begin{cases} \frac{1}{N}\left(\frac{e_i}{e^P_{nr}}\right)^{N-1} & \text{if } e_i < e^P_{nr}, \\ \frac{1}{N}\left[N - (N-1)\left(\frac{e^p_{nr}}{e_i}\right)\right] & \text{if } e_i \geq e^P_{nr}. \end{cases}$$

Given $v_A = \alpha e_i + (N-1)\alpha e^P_{nr}$, $v_A \geq v_B$ if and only if $e_i \geq \frac{v_B - (N-1)\alpha e^P_{nr}}{\alpha}$. Comparing the cut-offs for e_i, we see that $\frac{v_B - (N-1)\alpha e^P_{nr}}{\alpha} \geq e^P_{nr}$ if and only if $e^P_{nr} \leq \frac{v_B}{N\alpha}$. Thus we have the following cases to consider:

1) If $e^P_{nr} < \frac{v_B}{N\alpha}$, then the effort level required to ensure that v_A crosses the incumbent software value v_B will also be ensure a positive probability of being the critical developer who can appropriate revenue during price competition. Then, expected developer i's surplus is

$$E\left[S_i\left(e_i;e_{nr}^P\right)\right]=\begin{cases} v_A-\frac{1}{\beta}e_i^\beta \text{ if } e_i<\frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ \left[1-\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right]\left[Nv_A-(N-1)v_B\right]+\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)v_B-\frac{1}{\beta}e_i^\beta \text{ if } e_i\geq\frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} \end{cases}$$

If a symmetric equilibrium exists with $e_i^*=e_{nr}^P<\frac{v_B}{N\alpha}$, then we can see from the expression for expected surplus above that $E\left[S_i\left(e_{nr}^P;e_{nr}^P\right)\right]=v_A-\frac{1}{\beta}\left(e_{nr}^P\right)^\beta$ which is surplus that the developer receives as a user of the software. As in the case of the restrictive open source license, this surplus is independent of v_B and identical to the surplus in the absence of competition. So, if a symmetric equilibrium exists in this range, $e_{nr}^{P*}=e_r^{P*}=e_r^*$.

2) If $e_{nr}^P\geq\frac{v_B}{N\alpha}$, then symmetric effort from all developers will ensure that v_A crosses incumbent software value, i.e. $e_i=e_{nr}^P$ generates $v_A\geq v_B$. Expected developer surplus is

$$E\left[S_i\left(e_i;e_{nr}^P\right)\right]=\begin{cases} v_A-\frac{1}{\beta}e_i^\beta \text{ if } e_i<\frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ \frac{1}{N}\left(\frac{e_i}{e_{nr}^P}\right)^{N-1}\left[Nv_A-(N-1)v_B\right]+\left[1-\frac{1}{N}\left(\frac{e_i}{e_{nr}^P}\right)^{N-1}\right]v_B-\frac{1}{\beta}e_i^\beta \text{ if } \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}\leq e_i\leq e_{nr}^P, \\ \left[1-\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right]\left[Nv_A-(N-1)v_B\right]+\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)v_B-\frac{1}{\beta}e_i^\beta \text{ if } e_i\geq e_{nr}^P. \end{cases}$$

If it exists, the symmetric equilibrium maximizes $E\left[S_i\left(e_i;e_{nr}^P\right)\right]=\left[1-\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right]\left[Nv_A-(N-1)v_B\right]+\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)v_B-\frac{1}{\beta}e_i^\beta$. The first part of the surplus expression is revenue earned by the winning developer; while the second part is user-surplus received by the non-critical developers. Any symmetric equilibrium, e_{nr}^{P*} is it exists in this range will solve

$$\left(N^2-N+1\right)\alpha e_{nr}^{P*}-\left(e_{nr}^{P*}\right)^\beta=(N-1)v_B. \tag{2}$$

The following proposition describes the equilibrium entry and effort in a new open source software with a non-restrictive license.

Proposition 3 *Given an incumbent platform (B) of value $v_B>0$, the following is true about entry by a new platform A that has a non-restrictive open source license.*

a) Platform A always enters the market.

b) Equilibrium effort for every i, is $e_i^*=\begin{cases} e_{nr}^{P*} & \text{if } v_B\leq N\alpha^{\frac{\beta}{\beta-1}} \\ e_r^* & \text{if } v_B>N\alpha^{\frac{\beta}{\beta-1}}. \end{cases}$ *, where e_{nr}^{P*} solves* (2) *and* $e_{nr}^{P*}>e_r^*$.

Part a) states that developer effort into A is always positive in the presence of an incumbent. In the absence of an entering platform, the owner of platform B appropriates all of its value through a monopoly price leaving no user surplus. A new platform however small in value generates price competition for the monopolist owner of B and hence forces her to lower its price. This incentive to generate positive user surplus by creating price competition for the existing monopolist always incentivizes entry and hence, irrespective of license, entry will always occur. This also means that as with the restrictive open source license, entry can be inefficient.

However, in contrast to the restrictive open source entrant where development was always only driven by user motive, now with a non-restrictive license, developer effort can also be driven by the potential to earn revenue as a proprietor. When the value of the incumbent software is low, it is relatively easy for A to beat v_B even with small levels of developer effort. Hence the revenue incentive is strong. At the other end, when v_B is high it is extremely costly to provide the high levels of effort required to dominate the market during price competition. Here, the incentive for effort provision is driven more by the desire to gain user surplus through price competition with the proprietor of B. As part b) describes, when v_B is high enough, all effort provision is driven by the user motive to lower price and hence effort is independent of v_B and exactly the same as the effort provided under a restrictive license.

The next proposition compares effort provision and entry to the first-best outcome.

Proposition 4 *Suppose there is a proprietary incumbent platform of value* $v_B > 0$*, and the new platform (A) is developed under a non-restrictive open source license, then whenever entry is efficient, i.e.,* $v_B \leq \frac{(\beta-1)}{\beta} (N\alpha)^{\frac{\beta}{\beta-1}}$*, there is over-provision of effort relative to the efficient level* $\left(e^P_{nr} > e^P_o\right)$ *if* $v_B < \frac{(N-1)}{N} (N\alpha)^{\frac{\beta}{\beta-1}}$ *and there is under-provision of effort relative to the efficient level* $\left(e^P_{nr} \leq e^P_o\right)$ *if* $v_B \geq \frac{(N-1)}{N} (N\alpha)^{\frac{\beta}{\beta-1}}$*.*

As explained before the incentive to develop a new platform under an open source license has two motives - 1) a user motive to drive down prices through competition; and 2) a revenue motive to earn profits as a proprietor. Effort driven by user motive creates a public good externality that leads to under-provision. On the other hand, effort driven by revenue motive creates a negative externality as developer's over-invest in trying to become the critical developer. At low levels of v_B, the revenue incentive dominates. This results in an over-provision of effort. As v_B increases, the revenue incentive becomes weaker and user motive starts to dominate. As the positive externality from effort becomes larger, effort starts to fall below the efficient level. Figure 1 shows the region where entry and effort are inefficient under each type of entrant license.

In the next section we re-evaluate our findings when the incumbent has a restrictive open source license instead of a proprietary one.

4 Incumbent Competition from Restrictive Open Source Software

When platform B is distributed under a restrictive OSS license, it is available to all N user-developers for free, i.e., $p_B = 0$. If A is also developed under a restrictive license, then developers simply use the higher value platform in stage 2. In this case, the payoff to a developer is $\max\{v_A, v_B\}$ which is the same as the welfare function. The proposition below describes the entry and effort equilibrium here.

Proposition 5 *Suppose the incumbent platform (B) has a restrictive OSS license, then the following is true about entry incentives and effort provision into new platform (A) that is also distributed under a restrictive OSS license.*

a) Entry occurs if and only if $v_B < \left(N - \frac{1}{\beta}\right)\alpha^{\frac{\beta}{\beta-1}}$ *and if entry occurs equilibrium effort is* $e_r^{R*} = e_r^{P*} = e_r^* = \alpha^{\frac{1}{\beta-1}}$.

b) Entry incentives are lower than with a proprietary incumbent and also lower than the efficient level.

c) If entry occurs, equilibrium effort is less than the efficient level, i.e., $e_r^{R*} < e_{co}$.

Proposition 6 brings out a stark distinction between the competitive effects of different open source licenses. While an existing restrictive open source license can deter entry of competing software projects, as shown in propositions 2 and 3, proprietary software always incentivizes new software development. Thus, an open source entrant faces lower entry barriers with a proprietary incumbent platform than with an open source one. Moreover, as long as entry occurs, effort under a restrictive license is independent not only of incumbent value but also of incumbent license type.

The under-provision of effort highlighted in Proposition 6 also implies that a platform with a restrictive open source license has too low an incentive to enter a software market that already has an open source incumbent. Thus both entry and effort is lower than efficient here.

Now let us move on to the case where platform A has a non-restrictive license; then stage 2 profit to the winning developer is limited by the availability of the free alternative platform B in the market. Specifically, if $v_A \leq v_B$, then everyone uses software B so that the winning developer in A does not make any profits and she gets a surplus of v_B as a user. On the other hand, if $v_A > v_B$, then the winning developer in A can charge a price of $p_A = (v_A - v_B)$ and her total payoff is the sum of her surplus as a user of platform A and her profits from selling A to the remaining $(N-1)$ developers, i.e., $v_A + (N-1)(v_A - v_B)$. If developer i does not produce the critical innovation but some other developer within the project does, then i pays a price of $(v_A - v_B)$ to the winning developer so that her surplus as a user in stage 2 is v_B.

So, given e_{nr}^R, if $e_i > \frac{\left[v_B-(N-1)\alpha e_P^R\right]}{\alpha}$, then $\pi_i\left(e_i, e_{nr}^R\right) = \Pr(i=k)\left[v_A + (N-1)(v_A - v_B)\right] + (1-\Pr(i=k))\, v_B - \frac{1}{\beta}e^{\beta}$. If $e_i < \frac{\left[v_B-(N-1)\alpha e_P^R\right]}{\alpha}$, then $\pi_i\left(e_i, e_{nr}^R\right) = v_B - \frac{1}{\beta}e^{\beta}$.

As before, we have two cases:

1) $e_{nr}^R \leq \frac{v_B}{N\alpha}$

$$E\left[S_i\left(e_i; e_{nr}^R\right)\right] = \begin{cases} v_B - \frac{1}{\beta}e_i^{\beta} \text{ if } e_i < \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ \left[1-\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right]\left[Nv_A-(N-1)v_B\right] + \frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)v_B - \frac{1}{\beta}e_i^{\beta} \text{ if } e_i \geq \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} \end{cases}$$

Here the only symmetric equilibrium possible is $e_{nr}^{R*} = 0$.

2) $e_{nr}^R > \frac{v_B}{N\alpha}$,

$$E\left[S_i\left(e_i; e_{nr}^R\right)\right] = \begin{cases} v_B - \frac{1}{\beta}e_i^{\beta} \text{ if } e_i < \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ \frac{1}{N}\left(\frac{e_i}{e_{nr}^P}\right)^{N-1}\left[Nv_A-(N-1)v_B\right] + \left[1-\frac{1}{N}\left(\frac{e_i}{e_{nr}^P}\right)^{N-1}\right]v_B - \frac{1}{\beta}e_i^{\beta} \text{ if } \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} \leq e_i < e_{nr}^P, \\ \left[1-\frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right]\left[Nv_A-(N-1)v_B\right] + \frac{1}{N}(N-1)\left(\frac{e_{nr}^P}{e_i}\right)v_B - \frac{1}{\beta}e_i^{\beta} \text{ if } e_i \geq e_{nr}^P. \end{cases}$$

Developer surplus maximization leads us to the following entry and effort equilibrium with a non-restrictive open source entrant.

Proposition 6 *Suppose the incumbent platform (B) has a restrictive OSS license, then the following is true about entry incentives and effort provision into new platform (A) that is distributed under a non-restrictive OSS license.*

a) Entry occurs if and only if $v_B \leq \frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$ *and if entry occurs equilibrium effort is* $e_{nr}^{R*} = e_{nr}^{P*}$ *where* e_{nr}^{P*} *solves (2).*

b) Entry is less than efficient and also less than restrictive OSS entry.

c) Whenever entry occurs, equilibrium effort exceeds both the efficient level of effort and the effort under a restrictive open source entrant.

The results here are starkly different from the case where we had a proprietary incumbent. In particular as part a) and b) state, the incentives for entry relative to first-best is completely reversed. Now, a restrictive OSS incumbent provides too low an incentive to enter. This is because at high values of the incumbent software the under-provision of effort towards creating user-value reduces the expected platform value and depresses the incentive to enter below what is efficient.

Further, we also find in part c) that when entry does occur, it is always associated with effort over-provision. This is because, in the absence of any incentive to create price competition to generate user-surplus, all effort is motivated only through the revenue motive that requires in this case that effort be high enough to move the platform value above the incumbent value.

5 Software Entrant's License Choice

Given an incumbent software with value v_B, let us determine the license choice in the first stage by the platform owner of A.

First, let us assume that the incumbent software if proprietary. We assume that the platform owner chooses a license that would attract developers and hence the equilibrium license will be the one that maximizes developer surplus. From the previous analyses of entry and effort we have the following result for developer surplus under each type of license adopted by the entrant. Let $S_r^P(v_B)$ and $S_{nr}^P(v_B)$ denote the surplus values under R and NR licenses respectively, given a proprietary incumbent platform, B.

$$S_r^P(v_B) = \left(N - \frac{1}{\beta}\right)(\alpha)^{\frac{\beta}{\beta-1}}.$$

$$S_{nr}^P(v_B) = \begin{cases} N\alpha e_{nr}^{P*} - \frac{1}{\beta}\left(e_{nr}^{P*}\right)^{\beta} \text{ if } v_B \leq N(\alpha)^{\frac{\beta}{\beta-1}}, \\ \left(N - \frac{1}{\beta}\right)(\alpha)^{\frac{\beta}{\beta-1}} \text{ if } v_B > N(\alpha)^{\frac{\beta}{\beta-1}}. \end{cases}$$

where e_{nr}^{P*} solves (2).

With a restrictive OSS incumbent, the surplus for a restrictive OSS entrant remains the same, i.e. $S_r^R = S_r^P$ for $v_B \leq \left[N - \frac{1}{\beta}\right]\alpha^{\frac{\beta}{\beta-1}}$. For an entrant with a non-restrictive license, the surplus is

$$S_{nr}^R(v_B) = \begin{cases} N\alpha e_{nr}^{P*} - \frac{1}{\beta}\left(e_{nr}^{P*}\right)^{\beta} \text{ if } v_B \leq \frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}, \\ 0 \text{ if } v_B > \frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}. \end{cases}$$

The following proposition describes how the entrant platform owner chooses the open source license.

Proposition 7 *Given an incumbent software platform of value v_B, the entrant's license choice is described as follows.*

a) If the incumbent is proprietary then there exists $v'_B < N\alpha^{\frac{\beta}{\beta-1}}$ such that if $v_B \leq v'_B$, the entrant platform, A, adopts a restrictive open source license; if $v'_B < v_B < N\alpha^{\frac{\beta}{\beta-1}}$, a non-restrictive license is adopted; and if $v_B > N\alpha^{\frac{\beta}{\beta-1}}$, the entrant is indifferent between restrictive or non-restrictive open source license.

b) If the incumbent holds a restrictive OSS license, then for $v_B \leq v'_B$, the entrant platform, A, adopts a restrictive open source license; if $v'_B < v_B < \frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, a non-restrictive license is adopted; and if $v_B > \frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, the entrant adopts a restrictive OSS license.

In the case of a proprietary incumbent, the license for the entrant's software trades-off two externalities. On the one hand developer participation that is motivated by gain in user surplus through increased competition with the incumbent software is associated with positive externalities. At the other end, developer effort incentivized through revenue gains as a proprietor of the new software platform leads to over-investment as there is only one proprietor. The restrictive OSS license does not provide any revenue motive and hence always results is effort that is too low. While, the non-restrictive OSS license can lead to over-provision of effort when the incumbent software value is low. When the incumbent software value is high, the non-restrictive OSS license also under-provides effort as the user-motive for effort is strong. However, since the user motive is mitigated by the revenue motive, this under-provision is lower than what we would see with a restrictive OSS license. This makes the non-restrictive license more appealing to developers. At high values for the incumbent product, the over-provision has a greater effect on developer surplus relative to under-provision due to convex effort costs. In that case, the restrictive OSS license wins out. In terms of subsidizing entry, or antitrust policy, we see that with a proprietary incumbent, entry barriers are too low. Hence, there is a case to be made for restricting open source entry into markets where there is a high value proprietary incumbent. Figure 2 shows how the license chosen by the entrant cahnges with incumbent software value.

Looking at part b), with a restrictive OSS incumbent, if entry occurs under both types of licenses for the entrant, we know from Proposition 7 that effort is over-provided under nr and under-provided under r. Here we have to compare the surplus from over-provision with the surplus from under-provision. As in the case of a proprietary incumbent, the incentives to revenue appropriation are too high when v_B is very low. This makes the restrictive OSS license more optimal. However, we also know from Proposition 7 that entry from a restrictive OSS entrant occurs at a higher v_B threshold than a non-restrictive OSS entrant. This means that for high values of v_B as well we will see entry from restrictive OSS license.

6 Conclusion

We compared effort provision by software developers across two types of open source licenses - restrictive licenses that force all modifications originating from an open source software to be released under the same open terms and non-restrictive licenses that allow such modifications to be made proprietary by the developer. We provided a model of developer effort provision under each kind of license. We found that effort under a non-restrictive license is greater than under a restrictive license thus supporting the overwhelming empirical finding that restrictive licenses adversely affect developer participation. Further, we studied the impact of competition under different licenses by looking at effort provision in the presence of proprietary and open source competition. We found that the existence of competition and the nature of competition affects effort provision under the two licenses and moreover these effects are sometimes asymmetric. We also derived policy implications for open source licensing. We showed that a restrictive license always leads to under-provision of effort relative to the efficient level of effort. On the other hand, a non-restrictive license may lead to higher or lower effort than the efficient level. Entry incentives are also affected by the license of the incumbent as well as the license of the entrant. Finally, we derived the equilibrium license chosen by the entrant taking into account the incumbent software value and license. Our results suggest a more nuanced look at entry barriers and antitrust implications of open source software.

Appendix

Proof of Proposition 1. Let us derive the equilibrium effort under restrictive and non-restrictive licenses. Under restrictive license, the price of the software is zero since no developer can make it proprietary. Hence every developer maximizes her own user surplus, or

$$\max_{e_i \geq 0} \alpha \left(e_i + \sum_{j \neq i} e_j \right) - \frac{1}{\beta} e_i^{\beta}.$$

The objective function is concave and hence the first order condition is necessary and sufficient. Setting $\left.\frac{\partial S_i}{\partial e_i}\right]_{e_r^*} = 0$, we get $e_r^* = (\alpha)^{\frac{1}{\beta-1}}$ for every i. The value of the platform is $v_A(e_r) = N\alpha e_r^* = N(\alpha)^{\frac{\beta}{\beta-1}}$.

Under a non-restrictive license, given that $e_j = e_{nr}$ for all $j \neq i$, the surplus to developer i depends on whether she is a critical developer. Hence, the surplus is $E[S_i(e_i; e_{j \neq i})] =$

$\Pr[i=k]\, N\alpha\left(e_i+\sum_{j\neq i} e_j\right)-\frac{1}{\beta}e_i^{\beta}$.

Let us derive $\Pr[i=k]$.

$$\Pr(i=k)=\Pr(y_i>y_j,\text{ for all } j\neq i).$$

Let us look for a symmetric equilibrium where all users exert identical effort. Suppose all developers, $j\neq i$, choose $e_j=e_{nr}$. Then developer i makes the critical contribution if and only if $e_i\varepsilon_i>e_{nr}\varepsilon_j$ for every $j\neq i$, i.e. $\varepsilon_j<\frac{e_i}{e_{nr}}\varepsilon_i$. Given ε_i, $\Pr\left(\varepsilon_j<\frac{e_i}{e_{nr}}\varepsilon_i \forall j\neq i\right)=\left[\Pr\left(\varepsilon_j<\frac{e_i}{e_{nr}}\varepsilon_i\right)\right]^{N-1}$. Consider the following cases:

a) If $e_i<e_{nr}$, then $\frac{e_i}{e_{nr}}\varepsilon_i\leq 1$, so that $\Pr\left(\varepsilon_j<\frac{e_i}{e_{nr}}\varepsilon_i\right)=\frac{e_i}{e_{nr}}\varepsilon_i$, and

$$\Pr(i=k)=\int_0^1\left(\frac{e_i}{e_{nr}}\varepsilon_i\right)^{N-1}d\varepsilon_i=\frac{1}{N}\left(\frac{e_i}{e_{nr}}\right)^{N-1}.$$

b) If $e_i\geq e_{nr}$, then $\frac{e_i}{e_{nr}}\varepsilon_i\leq 1$ if and only if $\varepsilon_i\leq\frac{e_{nr}}{e_i}$. Thus, for $\varepsilon_i\leq\frac{e_{nr}}{e_i}$, $\Pr\left(\varepsilon_j<\frac{e_i}{e_{nr}}\varepsilon_i\right)=\frac{e_i}{e_{nr}}\varepsilon_i$ and for $\varepsilon_i>\frac{e_{nr}}{e_i}$, $\Pr\left(\varepsilon_j<\frac{e_i}{e_{nr}}\varepsilon_i\right)=1$.

$$\Pr(i=k)=\int_0^{\frac{e_{nr}}{e_i}}\left(\frac{e_i}{e_{nr}}\varepsilon_i\right)^{N-1}d\varepsilon_i+\int_{\frac{e_{nr}}{e_i}}^1 d\varepsilon_i=\frac{1}{N}\left[N-(N-1)\left(\frac{e_{nr}}{e_i}\right)\right].$$

Thus the expected surplus to developer i is:

$$E[S_i(e_i,e_{nr})]=\begin{cases}\alpha e_i^N e_{nr}^{-(N-1)}+\alpha(N-1)e_i^{N-1}e_{nr}^{-(N-2)}-\frac{1}{\beta}e_i^{\beta} & \text{if } e_i<e_{nr},\\ \alpha N e_i+\alpha(N-1)^2 e_{nr}-\alpha(N-1)^2 e_{nr}^2 e_i^{-1}-\frac{1}{\beta}e_i^{\beta} & \text{if } e_i\geq e_{nr}.\end{cases}$$

Hence,

$$\frac{d}{de_i}E[S_i(e_i,e_{nr})]=\begin{cases}\alpha N e_i^{N-1}e_{nr}^{-(N-1)}+\alpha(N-1)^2 e_i^{N-2}e_{nr}^{-(N-2)}-e_i^{\beta-1} & \text{if } e_i<e_{nr},\\ \alpha N+\alpha(N-1)^2 e_{nr}^2 e_i^{-2}-e_i^{\beta-1} & \text{if } e_i\geq e_{nr}.\end{cases}$$

$$\frac{d^2}{de_i^2}E[S_i(e_i,e_{nr})]=\begin{cases}\alpha N(N-1)e_i^{N-2}e_{nr}^{-(N-1)}+\alpha(N-1)^2(N-2)e_i^{N-3}e_{nr}^{-(N-2)}-(\beta-1)e_i^{\beta-2} & \text{if } e_i<e_{nr},\\ -2\alpha(N-1)^2 e_{nr}^2 e_i^{-3}-(\beta-1)e_i^{\beta-2} & \text{if } e_i\geq e_{nr}.\end{cases}$$

Note that if $e_{nr}=0$, then $\frac{d}{de_i}E[S_i(e_i,e_{nr})]\to\infty$ so that $e_i^*>0$. Hence, we know that any symmetric equilibrium must have positive effort level. For $e_i\geq e_{nr}$, the surplus function is concave. At $e_i=0$, the surplus is 0 and $\frac{d}{de_i}E[S_i(e_i,e_{nr})]=0$. At $e_i=e_{nr}$, $E[S_i(e_i,e_{nr})]=\alpha N e_{nr}-\frac{1}{\beta}e_{nr}^{\beta}$ and $\frac{d}{de_i}E[S_i(e_i,e_{nr})]=\alpha\left[N+(N-1)^2\right]-e_{nr}^{\beta-1}$.

So if $\alpha\left[N+(N-1)^2\right]-\beta e_{nr}^{\beta-1}<0$, i.e. if $e_{nr}>\left[\alpha\left(N+(N-1)^2\right)\right]^{\frac{1}{\beta-1}}$, then $e_i^*(e_{nr})=0$. But this cannot be a symmetric equilibrium.

Thus a necessary condition is that $\frac{d}{de_i}E\left[S_i(e_i,e_{nr})\right]\geq 0$ at $e_i=e_{nr}$, or $e_{nr}\leq\left[\alpha\left(N+(N-1)^2\right)\right]^{\frac{1}{\beta-1}}$.

The first order condition yields, $\alpha N+\alpha(N-1)^2 e_{nr}^2(e_i^*)^{-2}-(e_i^*)^{\beta-1}=0$. Setting $e_i^*=e_{nr}$, we get $e_{nr}^*=\left[\alpha\left(N^2-N+1\right)\right]^{\frac{1}{\beta-1}}$. Let us check if surplus is positive here. $\alpha Ne_{nr}-\frac{1}{\beta}e_{nr}^{\beta}>0$ if and only if $e_{nr}<(\alpha\beta N)^{\frac{1}{\beta-1}}$.

$$\begin{aligned} e_{nr} &\leq \left[\alpha\left(N^2-N+1\right)\right]^{\frac{1}{\beta-1}}<(\alpha\beta N)^{\frac{1}{\beta-1}} \\ &\Leftrightarrow \frac{N^2-N+1}{N}<\beta \end{aligned}$$

Since $N\geq 2$ and $\beta\geq N$, this condition holds. The value of the platform is $v_A(e_{nr}^*)=N\alpha^{\frac{\beta}{\beta-1}}\left[\left(N^2-N+1\right)\right]^{\frac{1}{\beta-1}}$

Finally to derive the first-best efficient effort, note that the social planner maximizes aggregate welfare. Hence $W(e)=N^2\alpha e-\frac{1}{\beta}Ne^{\beta}$. This is concave and hence the necessary and sufficient condition for welfare maximization is $\left.\frac{\partial W(e)}{\partial e}\right]_{e_o}=0$ which gives us $e_o=(N\alpha)^{\frac{1}{\beta-1}}$ and value $v_A(e_o)=(N\alpha)^{\frac{\beta}{\beta-1}}$.

Comparing the effort levels across the licenses and with efficient effort we can see that $e_r<e_o<e_{nr}$. Since value of the platform is increasing in effort, it automatically follows that $v_A(e_r)<v_A(e_o)<v_A(e_{nr})$. ■

Proof of Lemma 1. If $v_B\leq\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, then entry by the new platform is efficient and the efficient level of effort provision by each developer is $e_{co}=(N\alpha)^{\frac{1}{\beta-1}}$.

Given that a new platform is developed, welfare from effort remains the same as before, i.e. $W(e)=N^2\alpha e-\frac{1}{\beta}Ne^{\beta}$. Hence, conditional on entry being efficient, welfare is maximzed at $e_{co}=e_o=(N\alpha)^{\frac{1}{\beta-1}}$ and the maximized welfare is $W(e_{co})=N\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$. Entry by the new platform is efficient if and only if this maximized welfare with entry exceeds the welfare with the incumbent platform, i.e. $N\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}\geq Nv_B$ or $\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}\geq v_B$. ■

Proof of Proposition 2. a) Since the stage 2 payoff to the developer is the same with or without competition, the stage 1 effort remains the same so that $e_r^{P*}=e_r^*$.

b) From Lemma 1, we know that entry by a new platform is inefficient if $v_B>\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$. On the other hand, entry always occurs in equilibrium. This is because, in the absence of entrant, the monopolis incumbent will set price of the platform equal to its value leaving no surplus to the user. Hence, entry by a restrictive OSS platform is inefficient.

c) If entry is efficient, from Lemma 1, we know that $e_{co}=(N\alpha)^{\frac{1}{\beta-1}}$. However, $e_r^{P*}=e_r^*=(\alpha)^{\frac{1}{\beta-1}}<e_{co}$. ■

Proof of Proposition 3. a) As in the case of a restrictive license, a new platform provides a positive surplus to developers while a monopolist incumbent leaves zero user surplus. Hence entry is aways profitable.

b) Next let us derive the equilibrium effort.

Given that the other $N-1$ developers choose effort e_{nr}^P, the probability of i being the critical developer is as before

$$\Pr(i=k) = \begin{cases} \frac{1}{N}\left(\frac{e_i}{e_{nr}^P}\right)^{N-1} \text{ if } e_i < e_{nr}^P, \\ \frac{1}{N}\left[N-(N-1)\left(\frac{e_{nr}^p}{e_i}\right)\right] \text{ if } e_i \geq e_{nr}^P. \end{cases}$$

$v_A \geq v_B$ if and only if $e_i \geq \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}$. Comparing the cut-offs for e_i, we see that $\frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} \geq e_{nr}^P$ if and only if $e_{nr}^P \leq \frac{v_B}{N\alpha}$. Thus we have the following cases to consider:

1) $e_{nr}^P < \frac{v_B}{N\alpha}$

$$E\left[S_i\left(e_i;e_{nr}^P\right)\right] = \begin{cases} v_A - \frac{1}{\beta}e_i^\beta \text{ if } e_i < \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ v_A + \left[1-\left(\frac{e_{nr}^P}{e_i}\right)\right](N-1)(v_A-v_B) - \frac{1}{\beta}e_i^\beta \text{ if } e_i \geq \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} > e_{nr}^P. \end{cases}$$

$$\frac{d}{de_i}E\left[S_i\left(e_i;e_{nr}^P\right)\right] = \begin{cases} \alpha - e_i^{\beta-1} \text{ if } e_i < \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ \alpha\left[N-(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right] + (N-1)\left(\frac{e_{nr}^P}{e_i^2}\right)(v_A-v_B) - e_i^{\beta-1} \text{ if } e_i \geq \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} > e_{nr}^P. \end{cases}$$

It can be checked that the $E\left[S_i\left(e_i;e_{nr}^P\right)\right]$ is concave in both segments. A symmetric equilibrium, if it exists in this region will maximize $v_A - \frac{1}{\beta}e_i^\beta$, which is the same surplus as with an R license. Hence in this region the symmetric equilibrium if it exists will be $e_{nr}^{P*} = (\alpha)^{\frac{1}{\beta-1}}$. This equilibrium will exist if and only if the slope at $e_i = \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}$ is $\frac{d}{de_i}E\left[S_i\left(e_i,e_{nr}\right)\right] < 0$ or $v_B > N\alpha^{\frac{\beta}{\beta-1}}$.

2) If $e_{nr}^P \geq \frac{v_B}{N\alpha}$,

$$E\left[S_i\left(e_i,e_{nr}\right)\right] = \begin{cases} v_A - \frac{1}{\beta}e_i^\beta \text{ if } e_i < \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ v_B + \left(\frac{e_i}{e_{nr}^P}\right)^{N-1}(v_A-v_B) - \frac{1}{\beta}e_i^\beta \text{ if } \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} \leq e_i \leq e_{nr}^P, \\ v_A + \left[1-\left(\frac{e_{nr}^P}{e_i}\right)\right](N-1)(v_A-v_B) - \frac{1}{\beta}e_i^\beta \text{ if } e_i \geq e_{nr}^P. \end{cases}$$

$$\frac{d}{de_i}E\left[S_i\left(e_i,e_{nr}\right)\right] = \begin{cases} \alpha - e_i^{\beta-1} \text{ if } e_i < \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha}, \\ \alpha\left(\frac{e_i}{e_{nr}^P}\right)^{N-1} + \frac{1}{e_i}(N-1)\left(\frac{e_i}{e_{nr}^P}\right)^{N-1}(v_A-v_B) - e_i^{\beta-1} \text{ if } \frac{v_B-(N-1)\alpha e_{nr}^P}{\alpha} \leq e_i \leq e_{nr}^P, \\ \alpha\left[N-(N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right] + (N-1)\left(\frac{e_{nr}^P}{e_i^2}\right)(v_A-v_B) - e_i^{\beta-1} \text{ if } e_i \geq e_{nr}^P. \end{cases}$$

If it exists, at a symmetric equilibrium, $\left(N^2-N+1\right)\alpha e_{nr}^{P*} - \left(e_{nr}^{P*}\right)^\beta = (N-1)v_B$. It can be checked that the second order condition is satosfied at this symmetric effort equilibrium. Moreover, this equilibrium will exist and is unique if and only if $v_B \leq N\alpha^{\frac{\beta}{\beta-1}}$.

Finally, $\left(e_{nr}^{P*}\right)$ here is decreasing in v_B. At $e_{nr}^{P} = (\alpha)^{\frac{1}{\beta-1}}$, $(N-1)\left\{N\alpha^{\frac{\beta}{\beta-1}} - v_B\right\} > 0$. Hence $e_{nr}^{P*} > (\alpha)^{\frac{1}{\beta-1}}$. ■

Proof of Proposition 4. Clearly if $v_B > N\alpha^{\frac{\beta}{\beta-1}}$ and $e_{nr}^{P*} = \alpha^{\frac{1}{\beta-1}}$, we have under-provision of effort. If $v_B \leq N\alpha^{\frac{\beta}{\beta-1}}$, then at $e_{co} = (N\alpha)^{\frac{1}{\beta-1}}$, $\left(N^2 - N + 1\right)\alpha e_{nr}^{P*} - \left(e_{nr}^{P*}\right)^{\beta} - (N-1)\, v_B > 0$ if and only if $v_B < \frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}$. Since $N\alpha^{\frac{\beta}{\beta-1}} > \frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}$ there is a range of effort where $e_{co} > e_{nr}^{P*} > e_r^*$. ■

Proof of Proposition 5. The surplus function $N\alpha e^* - \frac{1}{\beta}(e^*)^{\beta}$ is concave in e^* with a maximum at $(N\alpha)^{\frac{1}{\beta-1}}$. Hence the license that generates an equilibrium effort closest to $(N\alpha)^{\frac{1}{\beta-1}}$ will win the entry competition. If $v_B > N(\alpha)^{\frac{\beta}{\beta-1}}$, then license type is irrelevant since both licenses generate the same equiibrium effort and hence the same value v_A.

If $v_B \leq N(\alpha)^{\frac{\beta}{\beta-1}}$, then e_{nr}^{P*} which solves (2) is higher than $e_r^P = \alpha^{\frac{1}{\beta-1}}$. e_{nr}^{P*} If $\alpha^{\frac{1}{\beta-1}} < e_{nr}^{P*} < (N\alpha)^{\frac{1}{\beta-1}}$, then the non-restrictive licence will win. This is because both licenses under-provide relative to maximum, but the non-restrictive license has less under-provision of effort.

If $e_{nr}^{P*} > (N\alpha)^{\frac{1}{\beta-1}}$, then we have to compare the under-provision under restrictive license with the overprovision in the non-restrictive license.

Note that $e_{nr}^{P*}(v_B)$ is continuously decreasing in v_B. At $v_B = N(\alpha)^{\frac{\beta}{\beta-1}}$, e_{nr}^{P*} which solves (2) is exactly equal to $\alpha^{\frac{1}{\beta-1}} < (N\alpha)^{\frac{1}{\beta-1}}$. So we know that when v_B decreases a little below $N(\alpha)^{\frac{\beta}{\beta-1}}$, we will get $\alpha^{\frac{1}{\beta-1}} < e_{nr}^{P*} < (N\alpha)^{\frac{1}{\beta-1}}$ and the non-restrictive license wins.

It can be checked that $e_{nr}^{P*}(0) > (N\alpha)^{\frac{1}{\beta-1}}$, then we NR wins for all v_B. Developer surplus is, therefore lowest when $v_B = 0$ since equilibrium effort under NR is furthest away from the maximum. So let us compare the surplus at $v_B = 0$ with $e^* = \alpha^{\frac{1}{\beta-1}}$ with $e_{nr}^{P*}(0)$, where $e_{nr}^{P*}(0) = \left[\alpha\left(N^2 - N + 1\right)\right]^{\frac{1}{\beta-1}}$. Surplus with NR is lower than with R if and only

$$\left[\frac{1}{\beta}\left(N^2 - N + 1\right) - N\right]\left(N^2 - N + 1\right)^{\frac{1}{\beta-1}} + N - \frac{1}{\beta} > 0 \tag{3}$$

If (3) does not hold then, the non-restrictive open source license is always adopted. If (3) then a restrictive open source license is adopted for low v_B. In particular if v_B' solves

$$N\alpha\left(\alpha^{\frac{1}{\beta-1}}\right) - \frac{1}{\beta}\left(\alpha^{\frac{1}{\beta-1}}\right)^{\beta} = N\alpha e_{nr}^{P*}\left(v_B'\right) - \frac{1}{\beta}\left(e_{nr}^{P*}\left(v_B'\right)\right)^{\beta}.$$

where $e_{nr}^{P*}\left(v_B'\right)$ solves

$$\left(N^2 - N + 1\right)\alpha e_{nr}^{P*} - \left(e_{nr}^{P*}\right)^{\beta} = (N-1)\, v_B'.$$

■

Proof of Proposition 6. Given e_r^R effort from the other developers, developer i's payoff from e_i is $\alpha e_i + (N-1)\alpha e_r^R - \frac{1}{\beta}e_i^\beta$ if $\alpha e_i + (N-1)\alpha e_r^R - \frac{1}{\beta}e_i^\beta > v_B$, and v_B otherwise. Since developer surplus is concave in e_i, positive at $e_i = 0$ and has a slope of $-\infty$ as $e_i \to \infty$, there is a maximum at $e_i^* = \alpha^{\frac{1}{\beta-1}} = e_r^{R*}$. As long as the value produced by this effort exceeds v_B, we will see entry by an OSS platform that has a restrictive license. The condition for entry is then given as $\left(N - \frac{1}{\beta}\right)\alpha^{\frac{\beta}{\beta-1}} > v_B$. Thus the equilibrium effort is

$$e_r^{R*} = \begin{cases} \alpha^{\frac{1}{\beta-1}} \text{ if } v_B < \left(N - \frac{1}{\beta}\right)\alpha^{\frac{\beta}{\beta-1}}, \\ 0 \text{ otherwise.} \end{cases}$$

Comparing it to the efficient entry and effort outcome, we clearly see that if entry occurs and is efficient, there is underprovision since $(N\alpha)^{\frac{1}{\beta-1}} > \alpha^{\frac{1}{\beta-1}}$. Comparing the entry cut-offs, $\left(1 - \frac{1}{\beta}\right)(N\alpha)^{\frac{\beta}{\beta-1}} > \left(N - \frac{1}{\beta}\right)\alpha^{\frac{\beta}{\beta-1}}$ if and only if $(\beta - 1)N^{\frac{\beta}{\beta-1}} - N\beta + 1 > 0$. The difference is increasing in N and at $N = 1$, it is zero and hence it is positive for all N. This means that cut-off where entry stops for an OSS entrant with a restrictive license is smaller than the efficient cut-off of v_B where entry should stop. Thus we have less than efficient entry.

We can also compare the entry and effort incentives for an OSS entrant to the case where the incumbent was proprietary (which is the same as the case where there was no incumbent, or $v_B = 0$). Here $e_r = e_r^P = \alpha^{\frac{1}{\beta-1}} = e_r^R$ for $v_B < \left(N - \frac{1}{\beta}\right)\alpha^{\frac{\beta}{\beta-1}}$. If $v_B > \left(N - \frac{1}{\beta}\right)\alpha^{\frac{\beta}{\beta-1}}$, then $e_r^{R*} = 0 < e_r^P$. ■

Proof of Proposition 7. If developer i is a critical developer and $v_A > v_B$, i.e. $e_i > \frac{v_B - (N-1)\alpha e_{nr}^P}{\alpha}$. When this happens, given that the price of the incumbent software is zero, the highest price that the proprietor of software A can charge is $v_A - v_B$ leaving a surplus of v_B to the other user-developers. If $v_A < v_B$, i.e. $e_i < \frac{v_B - (N-1)\alpha e_{nr}^P}{\alpha}$, then no one benefits from the new software as it cannot be sold. Hence developer surplus is v_B. So the payoff to developers is

1) If $e_{nr}^P < \frac{v_B}{N\alpha}$

$$E\left[S_i\left(e_i; e_{nr}^P\right)\right] = \begin{cases} v_B - \frac{1}{\beta}e_i^\beta \text{ if } e_i < \frac{v_B - (N-1)\alpha e_{nr}^P}{\alpha}, \\ v_A + \left[1 - \left(\frac{e_{nr}^P}{e_i}\right)\right](N-1)(v_A - v_B) - \frac{1}{\beta}e_i^\beta \text{ if } e_i \geq \frac{v_B - (N-1)\alpha e_{nr}^P}{\alpha} > e_{nr}^P. \end{cases}$$

$$\frac{d}{de_i}E\left[S_i\left(e_i; e_{nr}^P\right)\right] = \begin{cases} -e_i^{\beta-1} \text{ if } e_i < \frac{v_B - (N-1)\alpha e_{nr}^P}{\alpha}, \\ \alpha\left[N - (N-1)\left(\frac{e_{nr}^P}{e_i}\right)\right] + (N-1)\left(\frac{e_{nr}^P}{e_i^2}\right)(v_A - v_B) - e_i^{\beta-1} \text{ if } e_i \geq \frac{v_B - (N-1)\alpha e_{nr}^P}{\alpha} > e_{nr}^P. \end{cases}$$

As before payoff is concave in both ranges. But now profit is decreasing in the first range. Hence, we cannot have a symmetric equilibrium with positive effort in this range. The only equilibrium that is possible here is $e_{nr}^{R*}=0$. This will be an equilibrium if, given $e_{nr}^{R}=0$, payoff is decreasing everywhere, or the highest payoff is lower than v_B.

In the second segment, given $e_{nr}^{R}=0$, $\frac{d}{de_i}E\left[S_i\left(e_i;e_{nr}^{P}\right)\right]=\alpha N-e_i^{\beta-1}$. At $e_i=\frac{v_B}{\alpha}$, $\alpha N-\left(\frac{v_B}{\alpha}\right)^{\beta-1}<0$ if and only if $v_B>N^{\frac{1}{\beta-1}}\alpha^{\frac{\beta}{\beta-1}}$.

Suppose $v_B<N^{\frac{1}{\beta-1}}\alpha^{\frac{\beta}{\beta-1}}$, then there is a local maximum for e_i at $e_i(0)=(N\alpha)^{\frac{1}{\beta-1}}$. The maximized payoff is $\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}-(N-1)v_B<v_B$ if and only if $v_B>\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$.

$N^{\frac{1}{\beta-1}}\alpha^{\frac{\beta}{\beta-1}}>\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$ if and only if $\beta>(\beta-1)$ which is true. This means that $e_{nr}^{R*}=0$ is a symmetric equilibrium if and only if $v_B>\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$.

2) $e_{nr}^{P}\geq\frac{v_B}{N\alpha}$,

$$E\left[S_i\left(e_i;e_{nr}^{P}\right)\right]=\begin{cases} v_B-\frac{1}{\beta}e_i^{\beta} \text{ if } e_i<\frac{v_B-(N-1)\alpha e_{nr}^{P}}{\alpha}, \\ v_B+\left(\frac{e_i}{e_{nr}^{P}}\right)^{N-1}(v_A-v_B)-\frac{1}{\beta}e_i^{\beta} \text{ if } \frac{v_B-(N-1)\alpha e_{nr}^{P}}{\alpha}\leq e_i\leq e_{nr}^{P}, \\ v_A+\left[1-\left(\frac{e_{nr}^{P}}{e_i}\right)\right](N-1)(v_A-v_B)-\frac{1}{\beta}e_i^{\beta} \text{ if } e_i\geq e_{nr}^{P}. \end{cases}$$

$$\frac{d}{de_i}E\left[S_i\left(e_i;e_{nr}^{P}\right)\right]=\begin{cases} -e_i^{\beta-1} \text{ if } e_i<\frac{v_B-(N-1)\alpha e_{nr}^{P}}{\alpha}, \\ \alpha\left(\frac{e_i}{e_{nr}^{P}}\right)^{N-1}+\frac{1}{e_i}(N-1)\left(\frac{e_i}{e_{nr}^{P}}\right)^{N-1}(v_A-v_B)-e_i^{\beta-1} \text{ if } \frac{v_B-(N-1)\alpha e_{nr}^{P}}{\alpha}\leq e_i\leq e_{nr}^{P}, \\ \alpha\left[N-(N-1)\left(\frac{e_{nr}^{P}}{e_i}\right)\right]+(N-1)\left(\frac{e_{nr}^{P}}{e_i^2}\right)(v_A-v_B)-e_i^{\beta-1} \text{ if } e_i\geq e_{nr}^{P}. \end{cases}$$

$$\frac{d^2}{de_i^2}E\left[S_i\left(e_i;e_{nr}^{P}\right)\right]=\begin{cases} -(\beta-1)e_i^{\beta-2} \text{ if } e_i<\frac{v_B-(N-1)\alpha e_{nr}^{P}}{\alpha}, \\ 2(N-1)\frac{1}{e_i}\alpha\left(\frac{e_i}{e_{nr}^{P}}\right)^{N-1}+\frac{1}{e_i^2}(N-1)(N-2)\left(\frac{e_i}{e_{nr}^{P}}\right)^{N-1}(v_A-v_B)-(\beta-1)e_i^{\beta-2} \text{ if } \frac{v_B-(N}{} \\ -2(N-1)\left(\frac{e_{nr}^{P}}{e_i^3}\right)\left[(N-1)\alpha e_{nr}^{P}-v_B\right]-(\beta-1)e_i^{\beta-2} \text{ if } e_i\geq e_{nr}^{P}. \end{cases}$$

As before, we know that if a symmetric equilibrium exists, it will solve, $\left(N^2-N+1\right)\alpha e_{nr}^{P*}-\left(e_{nr}^{P*}\right)^{\beta}=(N-1)v_B$. Hence, here effort remains unchanged, $e_{nr}^{R*}=e_{nr}^{P*}$.

Comparing the entry cut-offs, we can see that $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}<\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, so that entry is less than efficient.

Further, we can also see that $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}<\left[N-\frac{1}{\beta}\right](\alpha)^{\frac{\beta}{\beta-1}}$, so that entry is lower than under a restrictive OSS license as well.

We know from before that $e_{nr}^{R*}>\alpha^{\frac{1}{\beta-1}}$ and that $e_{nr}^{R*}>(N\alpha)^{\frac{1}{\beta-1}}$ if and only if $v_B<\frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}$.

$\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}<\frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}$ if and only if $\beta(N-2)+1>0$ is true. Since $e_{nr}^{R*}>0$ only if $v_B<\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, we always get effort over-provision.

For $\beta>N$, $\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}>\frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}$. So $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}<\frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}<\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$. So for $v_B>\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, $e_{nr}^{R*}=e_{co}=0$. For $\frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}\leq v_B\leq\frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, we have $e_{nr}^{R*}=0<e_{co}$. Again for $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}\leq v_B\leq\frac{(N-1)}{N}(N\alpha)^{\frac{\beta}{\beta-1}}$, $e_{nr}^{R*}=0<e_{co}$. Finally for $v_B\leq\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, $e_{nr}^{R*}>(N\alpha)^{\frac{1}{\beta-1}}$. ■

Proof of Proposition 8. a) The surplus function $N\alpha e^* - \frac{1}{\beta}(e^*)^\beta$ is concave in e^* with a maximum at $(N\alpha)^{\frac{1}{\beta-1}}$. Hence the license that generates an equilibrium effort closest to $(N\alpha)^{\frac{1}{\beta-1}}$ will win the entry competition. If $v_B > N(\alpha)^{\frac{\beta}{\beta-1}}$, then license type is irrelevant since both licenses generate the same equiibrium effort and hence the same value v_A.

If $v_B \leq N(\alpha)^{\frac{\beta}{\beta-1}}$, then e_{nr}^{P*} which solves (2) is higher than $e_r^P = \alpha^{\frac{1}{\beta-1}}$. e_{nr}^{P*} If $\alpha^{\frac{1}{\beta-1}} < e_{nr}^{P*} < (N\alpha)^{\frac{1}{\beta-1}}$, then the non-restrictive licence will win. This is because both licenses under-provide relative to maximum, but the non-restrictive license has less under-provision of effort.

If $e_{nr}^{P*} > (N\alpha)^{\frac{1}{\beta-1}}$, then we have to compare the under-provision under restrictive license with the overprovision in the non-restrictive license.

Note that $e_{nr}^{P*}(v_B)$ is continuously decreasing in v_B. At $v_B = N(\alpha)^{\frac{\beta}{\beta-1}}$, e_{nr}^{P*} which solves (2) is exactly equal to $\alpha^{\frac{1}{\beta-1}} < (N\alpha)^{\frac{1}{\beta-1}}$. So we know that when v_B decreases a little below $N(\alpha)^{\frac{\beta}{\beta-1}}$, we will get $\alpha^{\frac{1}{\beta-1}} < e_{nr}^{P*} < (N\alpha)^{\frac{1}{\beta-1}}$ and the non-restrictive license wins.

The highest effort under the non-restrictive license is when $v_B = 0$ (i.e. when there is no competitor). It can be checked that Suppose $e_{nr}^{P*}(0) < (N\alpha)^{\frac{1}{\beta-1}}$, then we NR wins for all v_B. Since This is true if and only if

$$\left(N^2 - N + 1\right)\alpha (N\alpha)^{\frac{1}{\beta-1}} - \left((N\alpha)^{\frac{1}{\beta-1}}\right)^\beta < 0$$

$$N^2 - 2N + 1 < 0$$

which is not true. Hence we know that at $v_B = 0$, we necessarily have over-provision of effort. Developer surplus is, therefore lowest when $v_B = 0$ since equilibrium effort under NR is furthest away from the maximum. So let us compare the surplus at $v_B = 0$ with $e^* = \alpha^{\frac{1}{\beta-1}}$ with $e_{nr}^{P*}(0)$, where $e_{nr}^{P*}(0) = \left[\alpha\left(N^2 - N + 1\right)\right]^{\frac{1}{\beta-1}}$. Surplus with NR is lower than with R if and only if $\left[\frac{1}{\beta}\left(N^2 - N + 1\right) - N\right]\left(N^2 - N + 1\right)^{\frac{1}{\beta-1}} + N - \frac{1}{\beta} > 0$. If this does not hold then a non-restrictive open source license is always adopted. If it holds then a restrictive open source license is adopted for low v_B. In particular if v_B' solves

$$N\alpha\left(\alpha^{\frac{1}{\beta-1}}\right) - \frac{1}{\beta}\left(\alpha^{\frac{1}{\beta-1}}\right)^\beta = N\alpha e_{nr}^{P*}\left(v_B'\right) - \frac{1}{\beta}\left(e_{nr}^{P*}\left(v_B'\right)\right)^\beta.$$

where $e_{nr}^{P*}\left(v_B'\right)$ solves

$$\left(N^2 - N + 1\right)\alpha e_{nr}^{P*} - \left(e_{nr}^{P*}\right)^\beta = (N-1)\,v_B'$$

b) As before, the surplus function $N\alpha e^* - \frac{1}{\beta}(e^*)^\beta$ is concave in e^* with a maximum at $(N\alpha)^{\frac{1}{\beta-1}}$. Hence the license that generates an equilibrium effort closest to $(N\alpha)^{\frac{1}{\beta-1}}$ will win the entry competition. Comparing the entry cut-offs, we have $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}} < \left[N - \frac{1}{\beta}\right](\alpha)^{\frac{\beta}{\beta-1}} < \frac{(\beta-1)}{\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$.

Also, $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}} < N(\alpha)^{\frac{\beta}{\beta-1}}$. At $v_B = N(\alpha)^{\frac{\beta}{\beta-1}}$, $e_{nr}^{P*} = (\alpha)^{\frac{1}{\beta-1}}$.

If $\frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}} < v_B < \left[N - \frac{1}{\beta}\right](\alpha)^{\frac{\beta}{\beta-1}}$, then entry will occur under restrictive license and is efficient. Hence the restrictive license dominates the non-restrictive license.

If $v_B < \frac{(\beta-1)}{N\beta}(N\alpha)^{\frac{\beta}{\beta-1}}$, then entry will occur under both licenses and is efficient. We have to compare the under-provision under restrictive license with the overprovision in the non-restrictive license. Following the same steps as in part a) above, we get v'_B as the threshold belwo which restrictive OSS license generates higher surplus. ■

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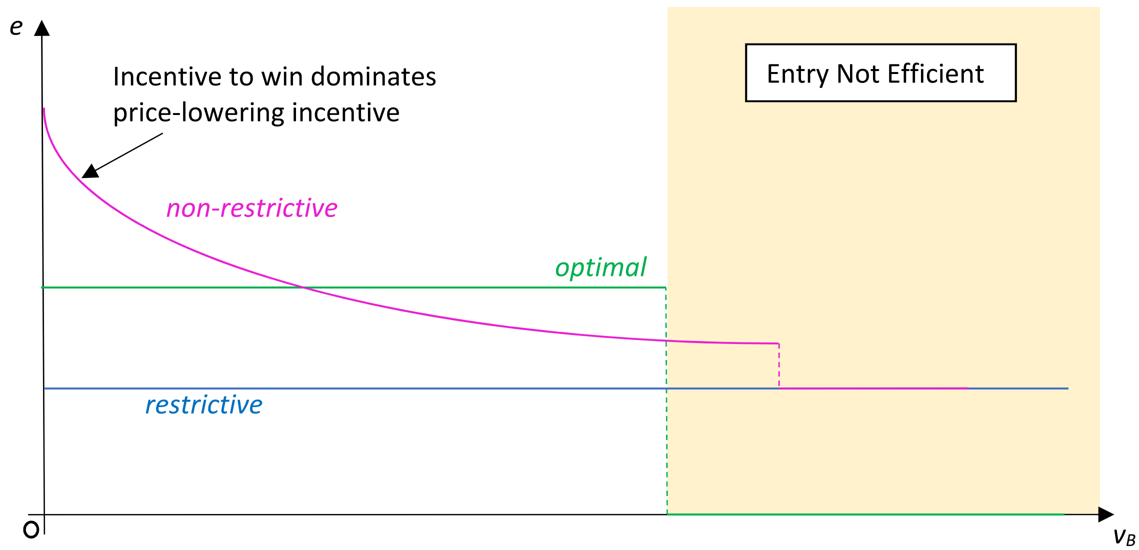

Figure 1: Effort and Entry with a Proprietary Incumbent

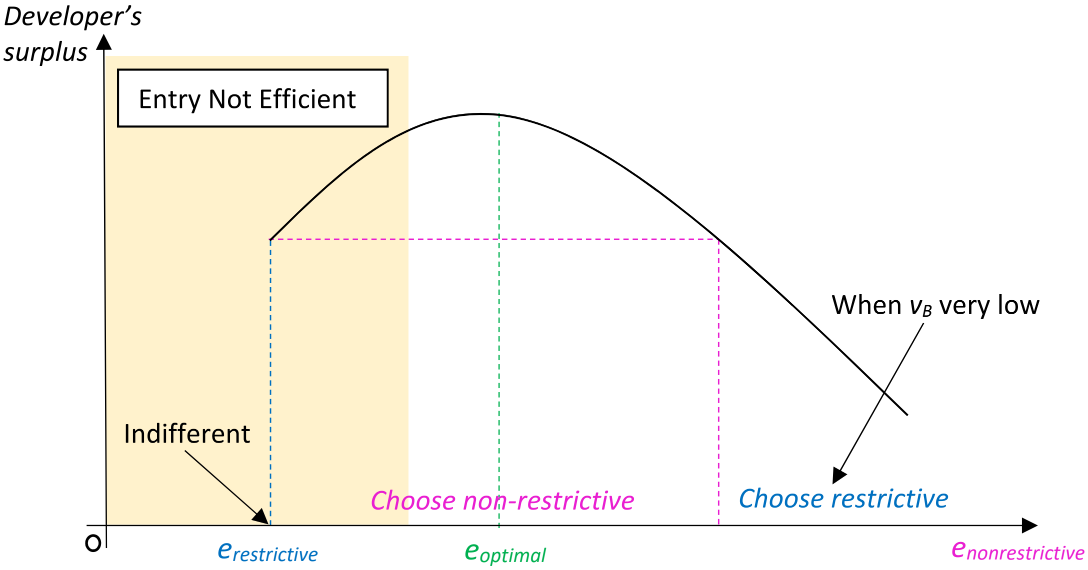

Figure 2: Entrant's License Choice with a Proprietary Incumbent