Properties of Profit Premium in an Equilibrium Framework*

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January 2019

Abstract

This paper studies the properties of profit premium in a structural equilibrium framework. Due to the motivation that led to its conceptual development, brand value in general and profit premium in particular is supposed to reflect changes in brand equity, and therefore, it is perceived to be positively related to brand equity. This paper finds that this perception does not always hold, as comparative statics in a simple logit model and simulations in a random coefficient logit model show. Specifically, when the experience attributes of a brand affect its marginal cost substantially, then profit premium is negatively related to brand equity. When experience attributes affect marginal cost moderately, profit premium is positively related to brand equity, as intuitively expected. These results are valid irrespective of whether brands are firm-specific or product-specific. We illustrate these findings empirically by estimating profit premiums in the new car market from the Netherlands.

Keywords: brand equity, market equilibrium, Nash-Bertrand, random coefficient logit, empirical IO methods, car market

JEL codes: M31, D43

*We thank Avi Goldfarb, Michel Wedel, and Jie Zhang for useful comments. Financial support from grant PN-II-ID-PCE-2012-4-0066 of the Romanian Ministry of National Education, CNCS-UEFISCDI is gratefully acknowledged.

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1 Introduction

Because brands are capable of incorporating the positive effects of marketing activities, an important problem in marketing is to quantify the efficiency of marketing activities devoted to improve the image of brands. Researchers have focused on measuring, on the one hand, the effect of brands on consumers' preferences, and on the other hand, the value of a brand to its producer. Following Goldfarb et al. (2009), we use the terms *brand value* and *brand equity* distinctively to mean the performance of a brand from the perspective of its producer and the contribution of a brand to consumers' utilities, respectively.

Brand value is in general measured by the difference between a factual quantity (like price or revenue) and a corresponding counterfactual. We follow the recent proposal by Goldfarb et al. (2009) and Ferjani et al. (2009) and define the counterfactual as the unbranded equilibrium quantity, that is, as the quantity computed in a new equilibrium when the product is deprived of its brand equity. We operationalize brand equity as a brand-specific intercept in the utility that is common to all consumers (Kamakura and Russell 1993, Sriram et al. 2007, Goldfarb et al. 2009, Borkovsky et al. 2017). This way the counterfactual depends on the search attributes of the given product, which are just the product attributes available to consumers from the description of the product. Consequently, brand value is measured as the extra value to the producer attributable to brand equity.

The literature conceptualized brand value measurement through different quantities. For example, Aaker (1991) proposed the price premium; Kamakura and Russell (1993) used the sales premium, which is based on market share as a quantity for computing the brand value; Ailawadi et al. (2003) proposed the revenue premium. An important discovery was that, employing the methodology from Berry et al. (1995), one can estimate marginal costs, which makes it possible to compute profit premium as the difference between the profit from the products belonging to a brand and the profit from the unbranded versions of the same products (Kartono and Rao 2006, Goldfarb et al. 2009). This is arguably a potentially outstanding brand value measure since it contains relevant information regarding the financial performance of the brand.[1]

Due to the motivation that led to its conceptual development, brand value is supposed to reflect changes in brand equity, and therefore, it is perceived to be positively related to brand equity. For example, Borkovsky et al. (2017) state that a brand value measure "must properly account for the benefits that a product (including the brand) enjoys relative to a hypothetical unbranded version of the product." In this paper we find that the perception that profit premium is positively related to brand equity does not always hold. We offer a structural framework and a formal explanation for this finding. In addition, we believe it is crucial to better understand the structural relationship between profit premium and brand equity in order to interpret properly numerical outcomes for profit premium. Therefore, in this paper we elaborate on the idea suggested by Goldfarb et al. (2009) who point out that their structural framework allows one "to simulate the thought experiment of a brand's equity changing, and measure its impact on price elasticities, market shares, and margins."

In order to study the structural relationship between profit premium and brand equity we consider a model of demand and supply along the lines of Berry et al. (1995). This model defines demand as a random coefficient logit

[1] It should be recalled, however, that this is a static concept. In a seminal contribution Borkovsky et al. (2017) propose an appealing concept for measuring brand value, which is based on the expected present value of future profits associated to a brand. The methodology for implementing this concept is based on a dynamic model of brand equity management and a static demand model with endogenous prices. We have two remarks related to this. On the one hand, several researchers believe that an appropriate brand value measure should capture brand equity dynamics. On the other hand, application of Borkovsky et al.'s methodology routinely to an arbitrary market is not straightforward. One reason for this is that in order to capture intertemporal variation in brand equity one needs detailed data to get statistically significant estimates of per period brand equities, and data structures that allow for this are rather rare for certain important markets. In order to come up with reliable profit premium estimates in these situations one is confined to use the static Goldfarb et al. (2009) methodology.

model that is derived from utilities that depend on search attributes and brand equity. Products are also characterized by experience attributes, which can signal features beyond search attributes (Nelson 1970). We assume that these experience attributes of a given brand are captured by brand-specific intercepts, which correspond to brand equities in our methodology. The supply side of the model specifies prices as a Nash equilibrium for profit maximizing firms. Similar to the demand side, the marginal cost used for defining profit is expected to depend on brand-specific features, which we capture by specifying marginal cost with brand-specific intercepts (Kartono and Rao 2006).

Goldfarb et al. (2009) in their study of the ready-to-eat cereals market assume that marginal cost does not depend on brand-specific parameters, so they compute unbranded marginal cost by assuming that the production technology is preserved. There are markets, however, where this assumption does not appear to be plausible. In fact in markets where products are more differentiated, marginal costs are likely to show higher variation across brands and, therefore, brand-specific intercepts in the marginal cost are expected to capture an important part of this variation. For example, in the car market, which we study empirically in this paper, it is difficult to accept that the unbranded version of a luxury brand preserves the production technology of the original product. Further, these brand-specific intercepts in the marginal cost are likely to be positively correlated with those in the utility, which can be interpreted as a positive effect of experience attributes on marginal cost. We note that product development expenditure for improving experience attributes do affect marginal cost in certain relevant situations. Often, in addition to product development there are also marketing-related volume-specific costs that affect marginal costs like sales commission and expenditure on advertising efforts related to the turnover generated by these advertising efforts. In the car market it is important to take experience attributes into account also because variable costs in an average car production process exceed 60%. This way the high proportion of marginal cost in the price of a car influences the financial performance of the brand significantly, so one cannot ignore their role in studying the relationship between brand equity and profit premium.

The main contribution of the paper is the finding that, when experience attributes of a brand influence its marginal cost substantially, profit premium is negatively related to brand equity. We have also obtained the result that profit premium is positively related to brand equity when the effect of experience attributes on marginal cost is moderate. The latter result is not surprising, but to date it has not been shown rigorously in the literature. We obtain our results analytically for the simple (non-random coefficient) logit demand model through comparative statics with respect to brand equity and corroborate them by Monte Carlo simulations for the random coefficient logit model. Our results are valid irrespective of whether brands are firm-specific or product-specific.

In order to verify the theoretical findings we conduct an empirical study of the new car market from the Netherlands. In this study, using yearly sales in the period 2003-2008 and car characteristics data, we estimate demand and brand equities as well as marginal cost specifications including brand-specific intercepts. The results confirm theoretical predictions to a large extent. The demand and supply side brand-specific intercepts are highly positively correlated, as expected. The estimates of brand-specific intercepts from marginal cost are quite high, which leads to a large number of negative profit premium values. The fact that there is a large number of negative profit premiums is counterintuitive but in order to explain why they occur we provide two arguments (see Section 5.4). These arguments suggest that this phenomenon can also occur in markets different from the car market.

The remainder of the paper is structured as follows. Section 2 describes the model wherein Section 2.3 provides the definition of profit premium. Section 3 presents our analytical results for the simple logit model on how the brand value measures are related to brand equity. Section 4 presents the corresponding Monte Carlo simulations for the random coefficient logit. Section 5 presents the empirical results for the Dutch car market including a brief description of the data and the estimation method used. Section 6 concludes and provides some recommendations for managers. Partial technical results are included in the Appendix.

2 The model

We use a model that allows for measuring brand effects both on the demand and supply sides. It is based on the well-known Berry et al. (1995) model, which is a random coefficient logit on the demand side and a Nash-Bertrand competition model on the supply side.

2.1 Demand

The utility of consumer i from buying product $j \in \mathcal{G}_f$, where $\mathcal{G}_f$ denotes the set of products produced by firm $f \in \{1, ..., F\}$ and F denotes the number of firms, is

$$u_{ij} = \beta_f - \alpha_i p_j + \mathbf{x}_j \beta_i + \delta_i M_j + \xi_j + \varepsilon_{ij},$$

where β_f is a parameter common to all products of firm f, $\mathbf{x}_j$ is a K-dimensional row vector of search attributes of product j whose first component is 1 for intercept, p_j is the unit price of product j, M_j is a measure of marketing expenditures, ξ_j is a product characteristic unknown to the econometrician but observed by consumers, ε_{ij} is an iid type I extreme value distributed error term. Further, the random coefficients have the distributions $\alpha_i \sim N(\alpha, \sigma_\alpha^2),\ \beta_i \sim N(\beta, \Sigma)\ \delta_i \sim N(\delta, \sigma_\delta^2)$, where Σ is a diagonal matrix with diagonal elements $\left(\sigma_1^2, ..., \sigma_K^2\right)$. In the market consumers can choose from J products and the outside alternative, which represents the option of not purchasing any of the J products and its utility is normalized to $u_{i0} = \varepsilon_{i0}$.

The utility specification yields that the probability that product j is purchased is

$$s_j = \int \frac{\exp\left(\beta_f - \alpha_i p_j + \mathbf{x}_j \beta_i + \delta_i M_j + \xi_j\right)}{1 + \sum_{g=1}^{F} \sum_{r \in \mathcal{G}_g} \exp\left(\beta_g - \alpha_i p_r + \mathbf{x}_r \beta_i + \delta_i M_r + \xi_r\right)} f\left(\alpha_i, \beta_i, \delta_i\right) d\alpha_i d\beta_i d\delta_i, \tag{1}$$

where $f\left(\alpha_i, \beta_i, \delta_i\right)$ is the joint density function of the random coefficients $\alpha_i,\ \beta_i,\ \delta_i$. This choice probability, if the number of purchases is large, is equal to the market share of product j. Therefore, in what follows we use the term 'market share' to refer to both quantities.

We define brand equity as the demand side effect of the brand, and since we assume that all products of firm f have the same brand name, the model assigns the firm-specific parameter β_f to measure brand equity.[2] This approach is rather common in the literature (e.g., Jedidi et al. 1999; Chintagunta 1994; Chintagunta et al. 2005, Sriram et al. 2007; Aribarg and Arora 2008; Goldfarb et al. 2009). Since search attributes are included in the utility, we expect β_f to measure the brand-specific effect of experience attributes on the utility.

2.2 Supply

We assume that prices are determined as a Nash equilibrium, where each firm maximizes its own profit with respect to own prices. The profit of firm f is

$$\pi_f = \sum_{h \in \mathcal{G}_f} \left(p_h - c_h\right) s_h,$$

[2] Below in Section 3.2 we also analyze the case when the products of a firm have different brand names. Bronnenberg and Dubé (2017, footnote 3) raise the concern that the brand equity measured by a brand-specific parameter captures all unobserved product-level features, including some features that should not be part of brand equity. The unobserved product characteristic ξ_j in the utility attempts to alleviate this concern.

where c_h denotes the marginal cost of producing product $h \in \mathcal{G}_f$. The fixed costs of production and the number of consumers in the market are omitted because they do not depend on prices.[3] We specify the marginal cost of product $j \in \mathcal{G}_f$ as

$$c_j = \gamma_f + \mathbf{w}_j \gamma + \omega_j, \tag{2}$$

where γ_f is a parameter that measures the brand-specific effect on the marginal cost, $\mathbf{w}_j$ is a vector of attributes that affect marginal cost and ω_j is a marginal cost characteristic unobserved by the econometrician. Intuitively, γ_f is expected to be positively correlated with β_f across firms $f = 1, \ldots, F$ because higher experience attributes for a product are likely to increase the marginal cost of the product. Therefore, in the paper we refer to γ_f as the experience attribute effect on marginal cost. In order to model the dependence of γ_f on β_f, we assume that $\gamma_f = \phi_f \beta_f$. By specifying the coefficient ϕ_f of β_f as firm f-dependent, we allow marginal costs to be heterogenous with respect to experience attributes. This allows for imperfect correlation between the demand and supply side brand-specific parameters. For example, if the ϕ_f coefficients are close to each other for different firms f then the demand and supply brand-specific parameters will be highly correlated. If, however, the ϕ_f's differ from each other significantly, then the correlation will be low. Throughout the paper we maintain that $\phi_f \geq 0$, which reflects our expectation that β_f and γ_f are positively correlated. As mentioned in the Introduction, this positive correlation is realistic because several types of brand equity building costs affect marginal cost. Examples include product development expenditure devoted for improving experience attributes or communication-related volume-specific costs (e.g., merchandising).

As common in the literature we assume that prices can be determined from the first order conditions for profit maximization. These are equivalent to the equations (Berry et al. 1995)

$$\mathbf{p}_f - \mathbf{c}_f = \Delta_f(\mathbf{p})^{-1} \mathbf{s}_f, \;\; f = 1, \ldots, F, \tag{3}$$

where $\mathbf{p}_f$, $\mathbf{c}_f$ and $\mathbf{s}_f$ are the vectors of prices, marginal costs and market shares for the products of firm f, respectively, and $\Delta_f(\mathbf{p})$ is a conformable square matrix having the element in row j and column r equal to $-\partial s_r / \partial p_j$.

2.3 Profit premium

According to the widely accepted definition (Keller 1993), brand value measurement involves a comparison between a certain quantity and a corresponding counterfactual. The literature offers various solutions for the brand used for the counterfactual, namely, private label (Ailawadi et al. 2003), hypothetical unbranded product (Ferjani et al. 2009), or the brand with the lowest market share. Following the proposal of Goldfarb et al. (2009), we define the counterfactual for a brand to be an unbranded quantity, that is, the quantity computed by setting the brand-specific parameters equal to zero. Along these lines, we define brand value of a specific brand as the incremental gain realized over the unbranded state of the same brand. In the unbranded state the brand enters the computations without brand equity but it retains its search attributes. Specifically, in order to compute the counterfactual prices and market shares for the products of firm f we take the unbranded version of these products by putting $\beta_f = (\gamma_f =)\, 0$, while keeping the parameters and variables corresponding to the other firms unchanged.

Within this framework we define *profit premium* as the difference between the profit from the products belonging to a brand and the profit from the same unbranded products. Specifically, the profit premium for firm f is $prp_f =$

[3] In Section 3.1.1 we discuss special features when fixed costs appear in the profits.

$\sum_{j\in\mathcal{G}_f}\left[(p_j-c_j)\,s_j-\left(p_j^c-c_j^c\right)s_j^c\right]$, where c_j^c is the counterfactual marginal cost of product j computed from (2) by taking $\gamma_f=0$.

3 Properties of profit premium in the simple logit

According to the above definitions, brand value can be regarded as an explicit function of brand equity. Here we present results on how the price, sales, revenue, and profit premium behave as a function of brand equity in a version of the model without consumer heterogeneity. Specifically, we provide conditions under which these quantities are either increasing or decreasing in brand equity. We observe that if, for example, price is increasing in brand equity, then price premium is also increasing in brand equity. More generally, the brand value measures defined as the premiums specified in Section 2.3 behave in a way similar to the corresponding quantities. Therefore, we study whether price, market share, revenue, and profit are increasing or decreasing in brand equity by computing their derivatives with respect to their own brand equity.

In the model an exogenous increase in brand equity of a product implies changes in virtually all endogenous variables of the model, that is, prices and market shares of all products in the market.[4] Intuitively, when the brand equity of a product increases the market share of the same product will increase, if prices stay unchanged, but the price of the product is also expected to increase. Now, even if the prices of the other products change only insignificantly, the effect on the market share of the product will be ambiguous because the price increase lowers the market share. The literature has not clarified unambiguously that price really increases, if the corresponding brand equity increases. Here we use a simpler version of the model presented in Section 2 in order to clarify how price, market share, revenue, and profit change when the corresponding brand equity increases.

The simpler version of the model considered in this section is the simple logit, which maintains all the variables but eliminates the random coefficients.[5] This yields a model that is analytically tractable in certain respects. Exploiting this analytical tractability, below we present comparative statics results on the signs of the derivatives of price, market share, revenue, and profit. Section 3.1 treats the case when brands are firm-specific, while Section 3.2 studies the case when brands are product-specific. The simulation results for the random coefficient logit in Section 4 suggest that the results derived for the simple logit are likely to hold for the random coefficient logit as well.

3.1 The case of firm-specific brands

As mentioned, we consider the special case when consumer preferences are not heterogenous, that is, we assume that all random coefficients are deterministic (i.e., $\alpha_i=\alpha,\ \beta_i=\beta,\ \delta_i=\delta$). Then, by denoting $d_j=x_j\beta+\delta M_j+\xi_j$, we obtain that the market share of product j is the logit expression

$$s_j=\frac{\exp\left(\beta_f-\alpha p_j+d_j\right)}{1+\sum_{g=1}^{F}\sum_{r\in\mathcal{G}_g}\exp\left(\beta_g-\alpha p_r+d_r\right)}. \tag{4}$$

[4]In reality brand equity is endogenous, as modelled for example by Borkovsky et al. (2017) in a dynamic context. However, in the literature it is not uncommon that for analytical purposes marketing activities devoted for creating brand equity or brand equity itself are treated as exogenous. For example, Sriram et al. (2007) treat advertising as exogenous; Stahl et al. (2012) study the effect of exogenous changes in brand equity.

[5]This has the consequence that the simple logit cannot model consumer heterogeneity regarding the observed product attributes including price, and thus it generates restrictive substitution patterns. Due to this drawback the simple logit is not used in empirical work, but still it has proved useful due to its analytical tractability (e.g., Berry 1994).

In the simple logit the first order condition for profit maximization (3) can be written in a closed form as

$$p_j - c_j = \frac{1}{\alpha}\frac{1}{1-\sum_{r\in\mathcal{G}_j} s_r} \quad \text{for } \ j \in \mathcal{G}_f, \ \ f = 1, ..., F.$$

We know that a unique Nash equilibrium of the prices exists under the assumption $\alpha > 0$ (Konovalov and Sándor 2010), and therefore, we maintain this assumption throughout the paper. Note that for products j belonging to the same firm the equilibrium markups $p_j - c_j$ are the same. Denote the common equilibrium markup of the products belonging to firm f by m_f. Then the first order condition for profit maximization can be rewritten as

$$m_f = \frac{1}{\alpha}\frac{1}{1-\overline{s}_f}, \tag{5}$$

where $\overline{s}_f = \sum_{r\in\mathcal{G}_j} s_r$ is the market share of firm f. In this case the profit of firm f is $\pi_f = m_f \overline{s}_f$.

Note that the market shares can be rewritten to depend on the markup $m_f = p_j - c_j, \ f = 1, ..., F$ instead of $p_j, \ j = 1, ..., J$, that is,

$$s_j = \frac{\exp\left(\beta_f - \alpha m_f - \alpha c_j + d_j\right)}{1 + \sum_{g=1}^{F}\sum_{r\in\mathcal{G}_g}\exp\left(\beta_g - \alpha m_g - \alpha c_r + d_r\right)}.$$

Consequently, the market share of firm f is

$$\overline{s}_f = \frac{\exp\left(\rho_f\beta_f - \alpha m_f + \ell_f\right)}{1 + \sum_{g=1}^{F}\exp\left(\rho_g\beta_g - \alpha m_g + \ell_g\right)}, \tag{6}$$

where $\rho_f = 1 - \alpha\phi_f$ and $\ell_f = \ln\left[\sum_{j\in\mathcal{G}_j}\exp\left(-\alpha\left[w_j\gamma + \omega_j\right] + d_j\right)\right]$ for $f = 1, ..., F$.[6]

In Appendix A.1, equation (29), we derive that

$$\frac{d\pi_f}{d\beta_f} = \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E - \overline{s}_f\right) + \overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f - \rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g}\right) \tag{7}$$

for any firm f and any product j of firm f, where

$$E = 1 - \sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g}, \ \ D = \left[\left(1-\overline{s}_f\right)^2 + \overline{s}_f\right]\left(1 - \sum_{g=1}^{F}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g}\right). \tag{8}$$

Here and throughout the paper $\sum_{g\neq f}$ denotes summation with respect to $g = 1, ..., F$ with $g \neq f$. Note that

$$E > 1 - \sum_{g\neq f}\overline{s}_g = s_0 + \overline{s}_f > \overline{s}_f \ \text{ and} \tag{9}$$

$$D > \left[\left(1-\overline{s}_f\right)^2 + \overline{s}_f\right]\left(1 - \sum_{g=1}^{F}\overline{s}_g\right) > 0. \tag{10}$$

We obtain the following result.

[6] The details of this derivation:

$$\begin{aligned}\overline{s}_f &= \frac{\sum_{j\in\mathcal{G}_j}\exp\left(\left[1-\alpha\varphi_f\right]\beta_f - \alpha m_f - \alpha\left[w_j\gamma + \omega_j\right] + d_j\right)}{1 + \sum_{g=1}^{F}\sum_{r\in\mathcal{G}_g}\exp\left(\left[1-\alpha\varphi_g\right]\beta_g - \alpha m_g - \alpha\left[w_r\gamma + \omega_r\right] + d_r\right)} \\ &= \frac{\exp\left(\left[1-\alpha\varphi_f\right]\beta_f - \alpha m_f\right)\sum_{j\in\mathcal{G}_j}\exp\left(-\alpha\left[w_j\gamma + \omega_j\right] + d_j\right)}{1 + \sum_{g=1}^{F}\exp\left(\left[1-\alpha\varphi_g\right]\beta_g - \alpha m_g\right)\sum_{r\in\mathcal{G}_g}\exp\left(-\alpha\left[w_r\gamma + \omega_r\right] + d_r\right)} \\ &= \frac{\exp\left(\left[1-\alpha\varphi_f\right]\beta_f - \alpha m_f + \ell_f\right)}{1 + \sum_{g=1}^{F}\exp\left(\left[1-\alpha\varphi_g\right]\beta_g - \alpha m_g + \ell_g\right)}.\end{aligned}$$

Proposition 1 *The following statements hold for any firm f.*

1. *If $\phi_f = \min\{\phi_g : g = 1, ..., F\}$ and $\rho_f = 1 - \alpha\phi_f > 0$, the derivative $d\pi_f/d\beta_f$ is positive.*

2. *If $\phi_f = \max\{\phi_g : g = 1, ..., F\}$ and $\rho_f = 1 - \alpha\phi_f < 0$, the derivative $d\pi_f/d\beta_f$ is negative.*

Proof. 1. We have $\rho_f - \rho_g \geq 0$ for all g and $\rho_f > 0$, so (29) implies that $d\pi_f/d\beta_f$ is positive.

2. In this case $\rho_f - \rho_g \leq 0$ for all g and $\rho_f < 0$, so (29) implies that $d\pi_f/d\beta_f$ is negative. ■

Part 1 of this result states that if firm f has the lowest effect of the experience attributes on the marginal cost (i.e., the lowest ϕ_f) and $\rho_f = 1 - \alpha\phi_f > 0$ then the profit of firm f is increasing in the common brand equity of the firm's products. Part 2 of the result states that if firm f has the highest effect of the experience attributes on the marginal cost and $\rho_f = 1 - \alpha\phi_f < 0$ then the profit of firm f is decreasing in the brand equity of the firm's products. The latter result is remarkable because it means that the profit of a firm cannot reflect the marketing effort of the firm for increasing the equity of its brand.

We note that in practice the condition $\rho_f = 1 - \alpha\phi_f < 0$ implies that the profit of firm f decreases as its brand equity increases. This is not unrealistic because firms are willing to give up profitability over a short period at the expense of offering consumers brands with higher utility. Our result in part 2 of Proposition 1 shows that in such periods profit premium does not follow closely variation in brand equity.

The following result treats the case when the effect of experience attributes on marginal cost is the same across brands.

Proposition 2 *Suppose that $\phi_f = \phi \geq 0$ for all $f = 1, ..., F$. Then if $\alpha\phi < 1$ (> 1) the derivative $d\pi_f/d\beta_f$ is positive (negative).*

Proof. $\phi_g = \phi$ for any firm g implies that $\rho_g = \rho \equiv 1 - \alpha\phi$ for any g, so $\rho_f - \rho_g = 0$. The expression that determines the sign of $d\pi_f/d\beta_f$ is

$$\rho_f (E - \overline{s}_f) + \overline{s}_f \sum_{g \neq f} \frac{(\rho_f - \rho_g)\, \overline{s}_g^2}{(1 - \overline{s}_g)^2 + \overline{s}_g} = \rho (E - \overline{s}_f) = (1 - \alpha\phi)(E - \overline{s}_f).$$

By (9) we know that $E - \overline{s}_f > 0$, so the conclusion follows. ■

This proposition states that when experience attributes do not affect marginal cost significantly (i.e., $\alpha\phi < 1$ or $\rho = 1 - \alpha\phi < 0$), then the firm's profit is increasing in the brand equity. However, when marginal cost is highly affected by experience attributes (i.e., $\alpha\phi > 1$ or $\rho = 1 - \alpha\phi < 0$), then the firm's profit depends negatively on brand equity.

3.1.1 The case when brand equity affects fixed costs

In this case we assume that the profit of firm f is

$$\pi_f = N \sum_{h \in \mathcal{G}_f} (p_h - c_h)\, s_h - \kappa_f,$$

where N is the number of consumers in the market and κ_f is the fixed cost of producing the products of firm f. The fixed cost includes expenditure on advertising and product development in order to enhance the brand equity β_f. Therefore, one expects that the fixed cost contains a component that is correlated with the experience attributes and hence also with brand equity.[7] Specifically, we find it reasonable to assume that $\kappa_f = \eta_f \beta_f + \varpi_f$, where η_f is a parameter and ϖ_f is a fixed cost component that is not related to experience attributes.

We note that the addition of fixed costs to the profit specification affects the comparative statics of profit at least to some extent, as we argue below. By (29) and (27) we have

$$\begin{aligned}\frac{d\pi_f}{d\beta_f} &= \frac{N\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)-\eta_f \\ &= \frac{N\overline{s}_f}{\alpha D}\left(\left(\rho_f-\eta_f/N\right)\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)-\frac{\eta_f\left(1-\overline{s}_f\right)^2E}{D}.\end{aligned}$$

Clearly, this $d\pi_f/d\beta_f$ is smaller than in the case when fixed costs are not taken into account in (29). How much smaller depends on the magnitude of the parameter η_f relative to the number of consumers N. Since this is an empirical question, and since due to lack of appropriate data it is usually difficult to include fixed cost in an empirical analysis, we do not pursue this issue further. However, from the above considerations we can conclude that due to fixed costs profit may be even less related to brand equity, while price, sales, and revenue are not affected.

3.2 The case of product-specific brands

Here we discuss the case when each product of a firm has a different brand name. In this case the utility of consumer i for product $j \in \mathcal{G}_f$ is

$$u_{ij} = \beta_j - \alpha_i p_j + x_j\beta_i + \delta_i M_j + \xi_j + \varepsilon_{ij}.$$

As before, we consider the special case when consumer preferences are not heterogenous, that is, we assume that all random coefficients are deterministic (i.e., $\alpha_i = \alpha,\ \beta_i = \beta,\ \delta_i = \delta$). Then, by denoting $d_j = x_j\beta + \delta M_j + \xi_j$, we obtain that the market share of product j is the simple logit expression

$$s_j = \frac{\exp\left(\beta_j - \alpha p_j + d_j\right)}{1+\sum_{r=1}^{J}\exp\left(\beta_r - \alpha p_r + d_r\right)}.$$

Define the marginal cost of product j as

$$c_j = \phi\beta_j + w_j\gamma + \omega_j,$$

which implies that the demand and supply brand-specific parameters are perfectly correlated; here we maintain the assumption $\phi \geq 0$. The market shares can be rewritten to depend on $m_f = p_j - c_j,\ f = 1, ..., F$ instead of $p_j,\ j = 1, ..., J$, that is,

$$s_j = \frac{\exp\left(\rho\beta_j - \alpha m_f + \ell_j\right)}{1+\sum_{g=1}^{F}\sum_{r\in\mathcal{G}_g}\exp\left(\rho\beta_r - \alpha m_g + \ell_r\right)}, \tag{11}$$

where $\rho = 1 - \alpha\phi$ and $\ell_j = d_j - \alpha\left(w_j\gamma + \omega_j\right)$ for all j. The market share of firm f is

$$\overline{s}_f = \frac{\sum_{j\in\mathcal{G}_f}\exp\left(\rho\beta_j - \alpha m_f + \ell_j\right)}{1+\sum_{g=1}^{F}\sum_{r\in\mathcal{G}_g}\exp\left(\rho\beta_r - \alpha m_g + \ell_r\right)}. \tag{12}$$

In Appendix A.2 in equations (38) and (37), we derive that

$$\begin{aligned}\frac{d\pi_j}{d\beta_j} &= \frac{\rho s_j}{\alpha D}\left(\frac{1-s_j}{1-\overline{s}_f}\overline{s}_f\left(E-\overline{s}_f\right)+\left(1-\overline{s}_f\right)\left(E-s_j\right)\right) \quad \text{and} \\ \frac{d\pi_f}{d\beta_j} &= \frac{\rho s_j}{\alpha D}\left(E-\overline{s}_f\right),\end{aligned}$$

[7] Borkovsky et al. (2017, Table 3) find an explicit empirical relationship between changes in brand equity and advertising.

where $\pi_j = (p_j - c_j) s_j \equiv m_f s_j$ is the profit obtained from product j of firm f. Since by (9) the inequalities $E - s_j \geq E - \bar{s}_f > 0$ and $1 - s_j \geq 1 - \bar{s}_f > 0$ hold, the derivatives $d\pi_j/d\beta_j$, $d\pi_f/d\beta_j$ have signs identical to the sign of $\rho = 1 - \alpha\phi$. So we can state the following result.

Proposition 3 *For any firm f and any product j of firm f, if $\alpha\phi < 1$ (> 1) the derivatives $d\pi_j/d\beta_j$, $d\pi_f/d\beta_j$ are positive (negative).*

Proposition 3 shows that when experience attributes have a small effect on marginal cost (i.e., $\alpha\phi < 1$ or $\rho = 1 - \alpha\phi < 0$), then the firm's profit and the product's profit are increasing in the corresponding brand equity. However, when the marginal cost is highly affected by the experience attributes (i.e., $\alpha\phi > 1$, or $\rho = 1 - \alpha\phi < 0$), then both firm- and product-level profit are decreasing in brand equity. Therefore, we can conclude that when marginal cost is highly affected by experience attributes, profits cannot reflect properly changes in brand equity.

4 Simulations in the random coefficient logit

This section presents Monte Carlo simulation results in the random coefficient logit model. These simulations are meant to study the statistical relationship between brand equity and various brand value measures. These are useful because they show if the results derived in Section 3 for the simple logit model are valid for the random coefficient logit model. We study the statistical relationship via two indicators estimated based on simulated data. The first indicator is the slope coefficient of a linear regression of a given brand value measure on a constant and brand equity. This can be regarded as an estimate of the derivative of a brand value measure with respect to the corresponding brand equity, which we define in Section 3.[8] The second indicator is the Kendall correlation coefficient between a given brand value measure and brand equity, which is expected to capture statistical monotonicity between two variables.[9] Correlation has often been used in the literature in order to validate brand equity or brand value measures, and in addition, it can be regarded as a complement to the other indicator.

First we generate the exogenous variables of this model. Then we generate 100 random replications of the brand equities and their coefficients in the marginal cost from the distributions specified below. Then we determine the market shares and prices of the products as well as the brand value measures for each replication. Finally, for each brand we determine the OLS estimates when regressing the brand value measures on brand equity over the replications as well as the correlations between the dependent and independent variables in these regressions.

First we consider the case when brands are firm-specific. In the simulations we consider cases when the number of products in the market ranges from small to large and so does the number of products per firm. Specifically, the number of products varies as $J = 10,\ 50$ and 100. When $J = 10$ we consider cases of $F = 10$ single product firms, $F = 4$ firms with 3 – 3 – 3 – 1 products and $F = 2$ firms with 5 products each. When $J = 50$ we consider cases with $F = 10$ firms with 5 products each and $F = 5$ firms with 10 products each. When $J = 100$ we consider the case of $F = 10$ firms with 10 products each.

The three attributes x_{jk}, ($k = 2, 3, 4$) and the marketing activities variable M_j are generated as uniform on $[1, 2]$; the marginal cost variables are computed as $w_{jk} = \ln(x_{jk})$. The utility parameters used are $\alpha = 4$, $\sigma_\alpha = 1$,

[8] Due to the way we define brand value measures, the derivative of a brand value measure is equal to the derivative of the corresponding quantity. For example, the derivative of the price premium with respect to the brand equity is equal to the derivative of price.

[9] We have also computed Pearson correlations, which are supposed to capure linearity, although we do not report them in the paper. These Pearson correlations tend to be numerically higher for the positive correlations, but their relative magnitudes are qualitatively similar.

$\beta = (2, 1.5, 2, 2.5)'$, $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = (2.5, 2, 2, 2)$, $\delta = 2$, $\sigma_\delta = 2$.[10] The marginal cost parameters are $\gamma = (0.5, 1, 1-, 1.5)$. We generate the unobserved demand and marginal cost characteristics ξ_j, ω_j as standard normal with $corr(\xi_j, \omega_j) = 0.6$, the firm-specific constants β_f as uniform on $[0, 3]$. Regarding the coefficients ϕ_f in the marginal cost we consider two situations. In the first we assume that ϕ_f is uniform on $[0, 0-]$; therefore, $\alpha\phi_f < 1$ for any brand f. In the paper we refer to this as the case when the effect of the experience attributes on the marginal cost is small. In the second situation we assume that ϕ_f is uniform on $[0, 0.8]$, where it can happen that $\alpha\phi_f > 1$ for some f. We refer to this as the case when the effect of the experience attributes on the marginal cost is large.

We also present simulation results for the case when brands are product-specific, which is studied in Section 3.2. In order to simplify the exposition, here we consider only markets with 10 products and two different market structures, namely, when there are 5 firms with 2 products each and when there are 2 firms with 5 products each. We present slope coefficients and correlations corresponding to the price of the product (i.e., brand) and with both product- and firm-level market share, revenue and profit.

We present the results corresponding to different cases in Tables 1 and 2. The left hand side panels in the tables contain the OLS estimates of the slope coefficients while the right hand side panels the Kendall correlations. For example, the first entry in Table 1, 0.02, is the estimate of the slope coefficient in the linear regression of the profit premium on the equity of brand 1 over the 100 replications. In this case there are 10 firms in the market each having 10 products; this appears in the table as case A. Each column correspond to a different case as follows. B: 50 products/10, C: 10 products/10 firms, D: 50 products/5 firms, E: 10 products/4 firms, F: 10 products/2 firms. We do not report the standard errors of the estimates in order to save space, but we selectively mention those estimates that are statistically significant at 5% significance level.

The upper parts of Tables 1 and 2 present results when experience attributes have a small effect on marginal cost. We notice that in this case slope coefficients are positive and in fact the vast majority of them are statistically different from zero. All Kendall correlations in the upper parts of Tables 1 and 2 are positive and rather high.[11] These results are in line with Proposition 2, and part 1 of Proposition 1 as well as Proposition 3. In conclusion, in the random coefficient logit model considered in the Monte Carlo simulations, when experience attributes have a small effect on marginal cost, profit premiums are positively related to brand equity, which suggests that the results established in Section 3 also hold for the random coefficient logit model.

The outcome of the simulations is quite different when the experience attributes have a large effect on marginal cost; see the lower parts of Tables 1 and 2. We notice that some slope coefficients are negative in both tables. Specifically, in Table 1 in the case B: 50 products/10 firms the slope coefficients for brands 2 and 7 corresponding to profit premium are negative and significantly different from zero. Moreover, there are brands for which the slope coefficients corresponding to profit premium are negative and significantly different from zero in the C: 10 products/10 firms, D: 50 products/5 firms, E: 10 products/4 firms and F: 10 products/2 firms cases as well. Similarly, in the case of product-specific brands reported in Table 2, in both the 5-firm and 2-firm cases the slope coefficients corresponding to profit premiums are negative and significantly different from zero. Further, the Kendall correlations reported in the lower parts of Tables 1 and 2 are mostly negative. These results are in line with part 2 of Proposition 1 and Proposition 3 obtained for the simple logit model.

To conclude this section we note that the results obtained in both the firm-specific and product-specific brand

[10] We have also experimented with different values for the price coefficient, for example $\alpha = -1$, $\sigma_\alpha = 0.2$, while keeping the rest of the data generating process unchanged. The results were qualitatively similar.

[11] The corresponding Pearson correlations (not reported) imply R^2's for the estimated linear regressions in the range $0.4 - 0.9$, which suggests that there is an approximate linear relationships between the profit premiums and brand equity.

Table 1. OLS estimates of slope coefficients and corresponding correlations when brands are firm-specific

Brands	A	B	C	D	E	F	A	B	C	D	E	F
Experience attributes have a small effect on marginal cost												
	Slope coefficients						Correlations					
1	0.02	0.01	0.04	0.03	0.05	0.06	0.63	0.68	0.63	0.66	0.64	0.60
2	0.00	0.02	0.00	0.01	0.05	0.03	0.72	0.67	0.75	0.69	0.64	0.65
3	0.01	0.00	0.01	0.01	0.03	–	0.70	0.76	0.71	0.73	0.66	–
4	0.02	0.01	0.02	0.09	0.00	–	0.70	0.68	0.63	0.64	0.64	–
5	0.01	0.01	0.03	0.01	–	–	0.70	0.70	0.62	0.69	–	–
6	0.03	0.00	0.01	–	–	–	0.65	0.74	0.67	–	–	–
7	0.01	0.09	0.01	–	–	–	0.66	0.64	0.70	–	–	–
8	0.04	0.00	0.02	–	–	–	0.63	0.76	0.64	–	–	–
9	0.02	0.00	0.00	–	–	–	0.63	0.72	0.73	–	–	–
10	0.00	0.01	0.00	–	–	–	0.72	0.64	0.64	–	–	–
Experience attributes have a large effect on marginal cost												
	Slope coefficients						Correlations					
1	-0.01	-0.00	-0.02	-0.02	-0.04	-0.06	-0.19	-0.14	-0.23	-0.18	-0.23	-0.21
2	0.00	-0.02	0.00	-0.00	-0.04	-0.04	-0.11	-0.20	-0.08	-0.16	-0.23	-0.22
3	0.01	0.00	0.01	0.01	-0.00	–	-0.06	-0.03	-0.05	-0.04	-0.12	–
4	-0.00	-0.00	-0.01	-0.07	-0.00	–	-0.10	-0.12	-0.17	-0.19	-0.10	–
5	0.00	0.00	0.00	0.00	–	–	-0.13	-0.09	-0.16	-0.09	–	–
6	-0.00	0.00	0.00	–	–	–	-0.13	-0.03	-0.12	–	–	–
7	0.00	-0.08	0.00	–	–	–	-0.15	-0.24	-0.13	–	–	–
8	-0.02	0.00	-0.00	–	–	–	-0.16	-0.01	-0.12	–	–	–
9	-0.00	0.00	0.00	–	–	–	-0.17	-0.04	0.01	–	–	–
10	0.00	-0.00	-0.00	–	–	–	-0.07	-0.11	-0.14	–	–	–

Notes. The columns correspond to A:100 products/10 firms, B: 50 products/10, C: 10 products/10 firms, D: 50 products/5 firms, E: 10 products/4 firms, F: 10 products/2 firms. Products are distributed evenly across firms, except for the case of 10 products/4 firms, where the distribution is 3-3-3-1. A small effect of experience attributes on marginal cost corresponds to ϕ_f uniform on [0,0.2]. A large effect of experience attributes on marginal cost corresponds to ϕ_f uniform on [0,0.8].

cases are similar to those obtained for the simple logit in Section 3. Consequently, the specific feature of the random coefficient logit model that it captures substitution of products with similar characteristics does not alter essentially the behavior of the brand value measures.

5 Profit premium in the Dutch car market

In this section we present profit premium estimates for new cars sold in the Netherlands in 2008. First we provide a brief description of the data used, then describe estimation and present the results.

5.1 Data

The data sets we use contain prices, sales, car characteristics, and advertising expenditure of cars sold in the Netherlands between 2003 and 2008. We define a market to consist of the car models that appear in a given year. A car model in a given year is included if its sales exceed fifty. This leads to a total of 320 different car models that were sold during this period; this corresponds to about 230 different models on average per year. We regard each model-year combination as one observation, which results in 1,382 observations in total.

We collected the prices, sales, and car characteristics data from Autoweek Carbase, which is an open online database on all cars sold in the Netherlands.[12] Car characteristics include among others engine power, fuel consumption (as kilometers per liter), weight, size, and dummy variables for whether the car's standard equipment includes cruise control as well as whether the car belongs to some specified car class. All prices available are listed (post-tax) prices; although transaction prices would be more desirable, they are not available. We have normalized

[12] See http://www.autoweek.nl/carbase.php.

Table 2. OLS estimates of slope coefficients and corresponding correlations when brands are product-specific

	A		B		A		B	
Brands	$prpp$	prp	$prpp$	prp	$prpp$	prp	$prpp$	prp
Experience attributes have a small effect on marginal cost								
	Slope coefficients				Correlations			
1	0.04	0.04	0.05	0.03	0.63	0.63	0.62	0.61
2	0.00	0.00	0.00	0.00	0.74	0.75	0.74	0.38
3	0.01	0.01	0.02	0.01	0.71	0.71	0.71	0.70
4	0.03	0.02	0.04	0.01	0.63	0.62	0.63	0.62
5	0.03	0.03	0.05	0.02	0.62	0.61	0.61	0.59
6	0.02	0.01	0.03	0.01	0.67	0.65	0.67	0.67
7	0.01	0.01	0.01	0.00	0.69	0.69	0.66	0.70
8	0.02	0.02	0.03	0.02	0.64	0.65	0.64	0.63
9	0.00	0.00	0.00	0.00	0.73	0.74	0.72	0.67
10	0.00	0.00	0.00	0.00	0.63	0.63	0.61	0.63
Experience attributes have a large effect on marginal cost								
	Slope coefficients				Correlations			
1	-0.02	-0.02	-0.03	-0.02	-0.23	-0.23	-0.23	-0.21
2	0.00	0.00	0.00	0.00	-0.08	-0.07	-0.06	0.04
3	0.01	0.01	0.01	0.01	-0.05	-0.04	-0.05	-0.03
4	-0.01	-0.01	-0.01	-0.00	-0.16	-0.16	-0.17	-0.08
5	-0.00	0.00	0.00	0.01	-0.15	-0.17	-0.16	-0.11
6	0.00	0.00	-0.00	0.00	-0.12	-0.11	-0.13	-0.10
7	0.00	0.00	-0.00	0.00	-0.13	-0.12	-0.14	-0.07
8	-0.00	-0.00	-0.01	0.00	-0.12	-0.12	-0.14	-0.08
9	0.00	0.00	0.00	0.00	0.01	0.01	-0.02	0.22
10	-0.00	-0.00	-0.00	0.00	-0.14	-0.14	-0.12	-0.09

Notes. A: 5 firms, 2 products per firm, B: 2 firms, 5 products per firm prp and $prpp$ refer to profit premium measured at the firm and product level, respectively. Products are distributed evenly across firms. A small effect of experience attributes on marginal cost corresponds to ϕ_f uniform on [0,0.2]. A large effect of experience attributes on marginal cost corresponds to ϕ_f uniform on [0,0.8].

all prices to 2006 euros by using the Consumer Price Index in the corresponding years.

In order to create variables that are needed in the estimation we have supplemented the data set with several variables from the Dutch statistical office (i.e., Statistics Netherlands).[13] For example, we use the total number of households to construct market shares; we use average gasoline prices per year to construct the commonly used fuel consumption variable kilometers per euro, as kilometers per liter divided by the average price of gasoline per liter. In addition we also use data on the distribution of household disposable income.

We use information from 2007 on brand ownership structure to specify which car brands belong to the same parent car producer. There are 39 different brands in our sample over the 2003-2008 period that are owned by 16 different companies. For instance, in 2007 the Volkswagen Group owned Volkswagen, Audi, Porsche, Seat, and Škoda. We use data on advertising expenditure obtained from Nielsen.

Table 3 presents the means weighted by sales for the main variables used in the estimation. The number of different car models sold increased from 213 in 2003 to 241 in 2008. The lowest sales are observed in 2005 while the highest in 2007. Prices had an increasing tendency in real terms with a remarkable drop in 2008, which is probably the result of the upcoming economic crisis. Regarding the share of European cars sold, the period 2003-2008 witnessed a downward trend, which was accompanied by an upward trend of brands that originate from Eastern Asia. Acceleration of cars measured by the ratio of horsepower to weight (denoted HP/weight) showed a steady increase. The size of cars measured as length times width times height increased slightly on average during the sampling period. The share of cars sold with cruise control as standard equipment went up significantly in 2004, but afterwards showed a slight decrease. Fuel efficiency improved on average during the sampling period as shown by the kilometers per liter (denoted KPL) variable. This improvement, however, was not matched by the number of

[13] See http://www.cbs.nl.

kilometers per euro (denoted KP€), which is a fuel efficiency variable relevant to consumers. This means that the improvement in fuel efficiency was not sufficient to offset the increase in gasoline prices over the sampling period. The proportion of family cars sold is rather high in each year and shows a slight decrease. The proportion of luxury cars has a similar trend, although their proportions are rather low. The share of sport cars is extremely low and does not change much during the sampling period. The proportion of MPV's shows a decreasing trend while the proportion of SUV's shows an increasing trend. Finally, we observe that advertising expenditures peaked in 2004 after which they dropped considerably reaching the half of the mean observed in 2005.

Table 3. Summary statistics

Year	No. of Models	Sales	Price	HP/ Weight	Size	Cruise Control	KPL	KP€	Family Car	Luxury	Sport	MPV	SUV	Adver- tising
2003	213	481,913	19,562	0.787	7.153	0.229	14.480	12.497	0.426	0.032	0.006	0.184	0.032	1.681
2004	228	476,581	19,950	0.788	7.184	0.308	14.696	11.737	0.408	0.029	0.008	0.197	0.038	2.145
2005	233	457,897	20,540	0.794	7.270	0.301	14.861	10.987	0.403	0.030	0.007	0.188	0.054	1.799
2006	231	475,636	20,367	0.804	7.271	0.308	15.120	10.707	0.361	0.030	0.006	0.171	0.063	0.842
2007	236	495,091	20,509	0.810	7.330	0.281	15.112	10.356	0.363	0.026	0.007	0.169	0.071	0.863
2008	241	489,584	18,613	0.813	7.271	0.293	15.813	10.290	0.381	0.021	0.006	0.126	0.062	0.845
All	1,382	479,450	19,916	0.799	7.247	0.286	15.018	11.091	0.390	0.028	0.007	0.172	0.054	1.355

Notes: Prices are in 2006 euros. Advertising is in million euros. All variables are sales weighted means, except for the number of models and sales.

5.2 Estimation

We estimate a version of the model presented in Section 2. In the demand model the price coefficient is defined as $\alpha_i = \alpha / y_i$, where y_i is the income of household i and only the constant is specified as random. The supply side is specified slightly differently from the model in Section 2 in that marginal costs are specified in logarithm as in Berry et al. (1995):

$$\ln c_j = \gamma_f + \mathbf{w}_j \gamma + \omega_j, \tag{13}$$

where $\mathbf{w}_j$ is a vector of supply side characteristics of product j specified in the lower part of Table 4.

The assumption used for identification is that the demand and supply side unobserved characteristics corresponding to a given product are mean independent of the observed characteristics of all products, which is the assumption used by Berry et al. (1995). In a first stage we estimate the demand parameters by GMM with moments based on the unobserved demand characteristics and instruments based on predicted prices and differentiation instruments (Gandhi and Houde 2016) constructed from the observed characteristics. The estimates are shown in the upper part of Table 4. The estimates of brand equities obtained in the same estimation as brand-specific intercepts are shown in Table 5. In the second stage we estimate the supply side parameters based on the first order conditions for profit maximization (3) while we make use of the demand parameter estimates. We do so by substituting the c_j's from (3) into (13) and using OLS estimation.

Regarding the experience attribute effect on marginal cost γ_f we consider three specifications. Specification (A) assumes no heterogeneity of the experience attribute effect on marginal cost, which implies perfect correlation between the brand equity β_f and γ_f, that is, $\gamma_f = \phi \beta_f$. Specification (B) allows for some (but not complete) heterogeneity by assuming $\gamma_f = \phi_b \beta_f$, where ϕ_b is the experience attribute effect on marginal cost in bin b, $b = 1, ..., 5$. These five bins are determined based on the magnitudes of brand equities; for example, bin 1 corresponds

Table 4. Estimation results

Variable	(A) Coeff.	(A) Std. Err.	(B) Coeff.	(B) Std. Err.	(C) Coeff.	(C) Std. Err.
Base coefficients						
constant	-18.367	$(6.405)^{***}$	-18.367	$(6.405)^{***}$	-18.367	$(6.405)^{***}$
HP/weight	2.414	$(0.551)^{***}$	2.414	$(0.551)^{***}$	2.414	$(0.551)^{***}$
cruise control	0.468	$(0.107)^{***}$	0.468	$(0.107)^{***}$	0.468	$(0.107)^{***}$
km per euro	0.770	$(0.279)^{***}$	0.770	$(0.279)^{***}$	0.770	$(0.279)^{***}$
size	10.863	$(1.464)^{***}$	10.863	$(1.464)^{***}$	10.863	$(1.464)^{***}$
advertising	0.559	$(0.040)^{***}$	0.559	$(0.040)^{***}$	0.559	$(0.040)^{***}$
family car	-0.750	$(0.172)^{***}$	-0.750	$(0.172)^{***}$	-0.750	$(0.172)^{***}$
luxury	-0.174	(0.211)	-0.174	(0.211)	-0.174	(0.211)
sport	-0.673	$(0.251)^{***}$	-0.673	$(0.251)^{***}$	-0.673	$(0.251)^{***}$
MPV	-0.229	(0.157)	-0.229	(0.157)	-0.229	(0.157)
SUV	0.563	$(0.220)^{**}$	0.563	$(0.220)^{**}$	0.563	$(0.220)^{**}$
Random coefficients						
price/income	-6.976	$(2.404)^{***}$	-6.976	$(2.404)^{***}$	-6.976	$(2.404)^{***}$
constant	2.489	(4.973)	2.489	(4.973)	2.489	(4.973)
Marginal cost parameters						
constant	2.637	$(0.029)^{***}$	2.483	$(0.032)^{***}$	1.961	$(0.076)^{***}$
log(HP/weight)	0.550	$(0.032)^{***}$	0.547	$(0.032)^{***}$	0.403	$(0.033)^{***}$
cruise control	0.087	$(0.012)^{***}$	0.095	$(0.012)^{***}$	0.093	$(0.012)^{***}$
log(km per liter)	-0.691	$(0.041)^{***}$	-0.670	$(0.040)^{***}$	-0.669	$(0.038)^{***}$
log(size)	1.288	$(0.068)^{***}$	1.303	$(0.065)^{***}$	1.612	$(0.064)^{***}$
family car	0.127	$(0.018)^{***}$	0.128	$(0.018)^{***}$	0.099	$(0.016)^{***}$
luxury	0.335	$(0.028)^{***}$	0.335	$(0.027)^{***}$	0.268	$(0.024)^{***}$
sport	0.423	$(0.027)^{***}$	0.409	$(0.026)^{***}$	0.414	$(0.024)^{***}$
MPV	0.144	$(0.019)^{***}$	0.149	$(0.019)^{***}$	0.118	$(0.017)^{***}$
SUV	0.265	$(0.024)^{***}$	0.271	$(0.023)^{***}$	0.226	$(0.021)^{***}$
brand equity	0.133	$(0.005)^{***}$	—		—	
brand equity × bin 1	—		0.742	$(0.067)^{***}$	—	
brand equity × bin 2	—		0.337	$(0.026)^{***}$	—	
brand equity × bin 3	—		0.267	$(0.017)^{***}$	—	
brand equity × bin 4	—		0.189	$(0.011)^{***}$	—	
brand equity × bin 5	—		0.179	$(0.007)^{***}$	—	
R^2 supply side	0.930		0.935		0.954	
Brand fixed effects supply side	no		no		yes	

Notes: * significant at 10%; ** significant at 5%; *** significant at 1%. The number of observations is 1,382. The number of simulated consumers used for the aggregate moments is 2,209. Standard errors are in parenthesis. Instruments used: predicted price and differentiation instruments for all car segments. The same demand side estimates are used for the marginal cost specifications (A), (B), (C).

to the lowest and bin 5 to the highest equities. The estimates of specifications (A) and (B) are shown in the lower part of Table 4. Specification (C) allows for full heterogeneity of the experience attribute effect on marginal cost, which means that γ_f is allowed to be different for each brand f. Overall, the three specifications correspond to the following situations. The ratio of brand-specific intercepts corresponding to marginal cost and demand γ_f/β_f is constant (specification A); $\gamma_f/\beta_f = \phi_b$ for the specified five bins (specification B); $\gamma_f/\beta_f = \phi_f$ (specification C). The estimates of the constant and slope coefficients are presented in the lower part of Table 4 while the brand effects on marginal cost are presented in Table 5.

5.3 Results

The upper part of Table 4 contains the demand estimates obtained in the first stage. *Base coefficients* and *Random coefficients* refer to estimates of the coefficients of observed demand characteristics and price over income as well as the random constant, respectively. We include a relatively large number of characteristics in order to exploit

Table 5. Brand-specific intercepts and profit premiums

	Brand-specific intercepts				Profit premiums					
	Brand equity (A-C)		Marginal cost (C)							
Variable	Coeff.	Std. Err.	Coeff.	Std. Err.	Specification (A)		Specification (B)		Specification (C)	
bmw	3.522	(0.547)***	1.251	(0.076)***	50.065		39.141		-96.826	
mini	2.791	(0.562)***	1.365	(0.094)***	9.354		8.899		4.590	
rover	0.209	(0.510)	1.001	(0.080)***	–		–		–	
chrysler	0.428	(0.468)	0.847	(0.078)***	0.945		-6.305		-50.790	
dodge	-0.149	(0.483)	0.670	(0.093)***	-0.368		0.925		-27.287	
jeep	1.088	(0.560)*	0.983	(0.080)***	1.246		-0.344		-30.856	
mercedes-benz	3.623	(0.604)***	1.307	(0.075)***	36.885		28.924		-96.358	
smart	1.477	(0.677)**	1.497	(0.083)***	1.685		1.486		0.371	
alfa romeo	1.310	(0.465)***	0.902	(0.077)***	5.600		1.968		-21.993	
fiat	0.725	(0.424)*	0.858	(0.075)***	25.247		17.514		-22.043	
lancia	0.318	(0.477)	1.074	(0.079)***	0.450		-0.656		-9.731	
ford	1.890	(0.415)***	0.854	(0.075)***	99.440		87.498		-3.748	
jaguar	3.074	(0.657)***	1.274	(0.080)***	3.152		0.891		-76.332	
land rover	3.079	(0.733)***	1.181	(0.079)***	7.299		3.020		-114.534	
mazda	0.965	(0.447)**	0.917	(0.075)***	11.843		2.205		-53.008	
volvo	3.053	(0.516)***	1.120	(0.075)***	54.024		44.343		-52.485	
subaru	0.416	(0.453)	1.002	(0.077)***	1.036		-2.720		-29.281	
cadillac	0.100	(0.513)	0.974	(0.086)***	0.034		-0.243		-26.224	
chervrolet	0.340	(0.409)	0.725	(0.075)***	4.783		-5.633		-50.883	
opel	1.893	(0.462)***	0.974	(0.075)***	92.887		83.947		3.031	
saab	1.917	(0.481)***	0.972	(0.082)***	4.697		3.455		-12.888	
honda	0.963	(0.449)**	1.052	(0.077)***	12.437		2.588		-73.437	
hyundai	0.531	(0.407)	0.766	(0.074)***	16.146		7.798		-69.910	
kia	0.063	(0.408)	0.657	(0.075)***	1.558		-1.610		-88.550	
mitsubishi	0.995	(0.441)**	0.922	(0.075)***	11.759		4.912		-33.947	
porsche	4.241	(0.743)***	1.414	(0.082)***	3.132		1.031		-44.463	
citroen	1.304	(0.433)***	0.933	(0.075)***	44.543		26.498		-82.514	
peugeot	1.298	(0.443)***	0.944	(0.075)***	74.361		53.209		-53.860	
dacia	0.000		0.000		0.000		0.000		0.000	
nissan	1.200	(0.459)***	0.946	(0.075)***	18.259		10.631		-45.082	
renault	1.759	(0.441)***	0.916	(0.075)***	81.785		73.329		-11.517	
suzuki	1.005	(0.412)**	0.813	(0.076)***	21.889		17.388		-4.989	
daihatsu	0.831	(0.466)	0.998	(0.078)***	12.588		9.833		-3.158	
lexus	2.019	(0.563)***	1.168	(0.080)***	2.797		1.024		-88.698	
toyota	1.916	(0.460)***	1.071	(0.075)***	87.022		76.533		-78.608	
audi	3.263	(0.536)***	1.190	(0.076)***	57.283		47.559		-61.154	
seat	1.172	(0.454)**	0.894	(0.076)***	23.405		16.378		-22.179	
skoda	1.396	(0.490)***	0.920	(0.079)***	18.936		13.984		-6.587	
volkswagen	2.337	(0.488)***	1.017	(0.074)***	119.929		107.750		16.984	
Correlations (p-values)										
Pearson			0.737	(0.000)	0.304	(0.071)	0.328	(0.051)	-0.256	(0.131)
Kendall			0.491	(0.000)	0.298	(0.010)	0.327	(0.005)	-0.063	(0.586)

Notes: * significant at 10%; ** significant at 5%; *** significant at 1%. The number of observations is 1,382. The number of simulated consumers used for the aggregate moments is 2,209. Standard errors are in parenthesis. Numbers are based on the estimates in Table 4. Profits are measured in € mln. Profit premium is calculated as profits minus counterfactual profits (setting brand to the minimum brand found).

as much product-level variation as possible so that to obtain more precise brand equity estimates. This explains our decision for including a large number of characteristics and only a few random coefficients. As our simulations from Section 4 show, including more random coefficients does not have qualitative implications on profit premiums.

The base coefficient estimates have the expected signs. The constant is large and negative reflecting that only a small proportion of households buy a new car in a given year. The characteristics HP/weight, cruise control, kilometers per euro, and size affect utility positively and statistically significantly, as one would expect. Advertising expenditure has a positive significant effect on utility. The special car class dummy variables have effects with mixed signs on utility depending on the popularity of the respective class. In this regard, family cars beyond the fact that have larger size do not seem to be popular; sport cars are not popular either. SUV's are relatively popular while the luxury car and MPV dummies do not affect utility significantly. The coefficient of price/income is estimated to be highly negative and statistically significant reflecting the negative utility of price. The random constant (i.e., the standard deviation parameter corresponding to the random constant parameter) estimate is not significant statistically.

The brand-specific intercept estimates presented in Table 5 are obtained by assuming Dacia to be the base brand for both the demand and supply sides. We recall that we define brand equity as the brand-specific intercepts in the demand model. We note that most estimates of brand equity are statistically significant at 5%. These estimates show several patterns. First, German luxury (Porsche, Mercedes, BMW, Audi) brands tend to have the highest brand equities (above 3) followed by luxury brands from Great Britain (Jaguar, Land Rover), so European luxury brands tend to have higher equities. Second, some American luxury brands (e.g., Cadillac, Jeep) tend to have low equities. Third, highly popular brands (Ford, Opel, Volkswagen, Toyota, Renault) have equities around 2. Overall one can say that these brand equity estimates are reasonable.

The lower part of Table 4 contains the marginal cost estimates obtained in the second stage. In order to capture brand-specific effects as precisely as possible, we include a large number of covariates in the marginal cost specification. The estimates across the three specifications are rather similar qualitatively and similar quantitatively across specifications (A) and (B). Apart from the sign of log(km per liter), all estimates have the expected signs; all characteristics that increase utility shift marginal costs upwards according to our estimates.[14] Recall that in specification (A) we assume no heterogeneity of the experience attribute effect on marginal cost, that is, we study the effect of the brand equities estimated in the first stage on marginal cost. We find this effect to be positive and statistically significant, so in the Dutch new car market experience attributes increase marginal cost significantly. Specification (B) allows the effect of experience attributes to differ across five bins. All five estimates are positive and statistically significant; they are decreasing as the brand equities increase. Specification (C) allows for full heterogeneity of the experience attribute effect on marginal cost, so all brand-specific intercepts are estimated. These estimates are all statistically significant and positive. We also report the R^2's for the three specifications. These are rather high for each specification and rather similar for specifications (A) and (B). Specification (C) has a somewhat higher R^2 than the other two, which shows that the brand-specific intercepts capture variation in marginal costs in addition to that of brand equities.

The estimates of the brand-specific intercepts are related to brand equities. For example, luxury brands tend to

[14] Berry et al. (1995) obtained a similar result with respect to fuel efficiency for their main specification. Their explanation is that fuel efficiency is positively correlated with sales, so if sales is negatively correlated with marginal costs (which happens if there are increasing returns to scale), the fuel efficiency parameter may be biased downward. By adding a log(sales) variable to one of their specifications in order to proxy for log(production), the sign on fuel efficiency got reversed. However, with our data this is not feasible because almost none of the cars is produced in the Netherlands, so domestic sales are not a good predictor of total production.

have higher brand-specific marginal costs. The correlation coefficients between the brand equities and the marginal cost intercepts reported at the bottom of Table 5 confirm the strong positive relationship between these two variables. Both the Pearson and Kendall correlations are rather large and statistically significant (0.737 and 0.491, respectively, with both p-values equal to 0.000). It is important to mention that some brands with low equities have disproportionately high marginal cost brand-specific intercepts; examples for these are the above-mentioned American luxury brands. This is intuitively plausible: brand equities depend on consumer preferences but marginal cost brand-specific intercepts reflect the technology used, so they are supposed to be high for luxury brands.

The profit premium estimates based on data from 2008 are presented in Table 5. Those corresponding to specification (A) are all positive, while some of them corresponding to specifications (B) are negative. It is remarkable that the majority of the profit premiums estimated for specification (C) are negative. According to specifications (A) and (B) the brands with the highest profit premiums are the highly popular brands Volkswagen, Toyota, Ford, Opel, Renault, although their equities are clearly lower than those of the European luxury brands (Porsche, Mercedes, BMW, Audi, Jaguar, Land Rover). Profit premiums computed for specification (C) show a pattern quite different from those corresponding to the other two specifications. In this regard, several brands with high profit premium obtained for specifications (A) and (B) have relatively low profit premiums for specification (C) (e.g., BMW, Mercedes, Volvo, Toyota, Audi).

Regarding the correlations of profit premiums and brand equities one can notice that the Kendall correlation coefficients corresponding to specifications (A) and (B) are relatively low but positive and statistically significant while the Pearson correlation coefficients, although positive, are low and not significant at 5%. Further, neither of the correlation coefficients corresponding to specification (C) is significant, and in fact both are negative. This result is in line with the theoretical findings from part 2 of Proposition 1 and the simulation results reported in Table 1 where in the case when experience attributes have a large effect on marginal cost (Kendall) correlations are negative. The profit premium results in specification (A) appear to be related to that from Proposition 2 for the case when the effect of experience attributes on marginal cost is low.

5.4 Discussion

Depending on the marginal cost specification we have obtained results for profit premiums that are qualitatively different. Recall that the three marginal cost specifications depend on whether the ratios of brand-specific intercepts corresponding to marginal cost and demand γ_f/β_f are either assumed to be constant (specification A) or allowed to take five different values (specification B) or allowed to be brand-specific (specification C). The marginal cost specification having the highest fit according to R^2 is the latter one. For this specification almost all profit premiums are negative and their Kendall correlation with brand equity is negative and statistically not significant. This suggests that profit premium is not increasing in brand equity and it can occur for some brands that it is decreasing. Below we offer two arguments why this can occur.

One possible explanation to this phenomenon is that the effects of brand equity on marginal cost are overestimated due to the fact that brand equity depreciation is not taken into account since we can only estimate one equity per brand during the whole sampling period. What happens in fact is that the costs involved in building brand equity are not properly reflected in the brand equities measured. Had brand equity depreciation been taken into account, the brand equity values (β_f) used in the marginal cost would have been higher, so in order to correspond to the same total effect the ϕ_f estimates would have been lower. According to Propositions 1 and 2, if ϕ_f is lower then it is more likely that profit premiums are positive.

Profit premium is also likely to be negative when the brand equity of a luxury brand is relatively low (e.g., Cadillac, Jeep). This can occur in many situations, for example when a luxury brand enters a new market and its brand equity has not yet been built up due to little experience of consumers with the brand. In this situation its marginal cost brand-specific intercept is still expected to be high (as it happens with Cadillac and Jeep), so the coefficient of brand equity ϕ_f will be large. In this case part 2 of Proposition 1 predicts negative profit premium.

6 Conclusions

This paper studies the relationship between brand equity and profit premium in the framework of an equilibrium model. In the literature it is commonly perceived that a proper brand value measure reflects changes in brand equity, that is, it is monotonically increasing in brand equity. Employing the explicit structure of the equilibrium model, we study the monotonicity of profit premium in different situations. In the paper we derive comparative statics results for the simple logit model and we verify these results by Monte Carlo simulations for the random coefficient logit model. The two sets of result are qualitatively similar, which suggests that the more realistic substitution patterns of the random coefficient logit does not significantly influence the behavior of profit premium with respect to brand equity. We have also obtained that profit premium has a similar behavior irrespective of whether each product of a firm represents a different brand or all products of a firm belong to the same brand.

Our results imply that a factor that drives the behavior of profit premium with respect to brand equity is the magnitude of brand-specific effects on marginal cost. One set of results claims that when these effects are small profit premium is highly positively correlated with brand equity. Another set of results states that when these effects are large then profit premium does not preserve this behavior. This finding is practically relevant because many firms are willing to accept lower profits over a short period at the expense of offering consumers brands with higher utility. Our findings have implications to brand managers on how they should interpret numerical realizations of profit premium. When, as a consequence of investing in brand equity, short term profits may fall, managers should be aware that this phenomenon may be accompanied by mostly unchanged or even decreased profit premium.

A Appendix

Notation. Vectors appear as boldface characters; $\mathbf{v}'$ denotes the transpose of the vector $\mathbf{v}$. For $\mathbf{m} = (m_1, ..., m_F)'$ and $\beta = (\beta_1, ..., \beta_F)'$ we use the notation for the derivative

$$\frac{d\mathbf{m}}{d\beta'} = \begin{pmatrix} \frac{dm_1}{d\beta_1} & \cdots & \frac{dm_1}{d\beta_F} \\ & \ddots & \\ \frac{dm_F}{d\beta_1} & \cdots & \frac{dm_F}{d\beta_F} \end{pmatrix}.$$

A.1 The case of firm-specific brands

The derivative of the profit $\pi_f = m_f \bar{s}_f$ of firm f with respect to brand equity β_f is

$$\frac{d\pi_f}{d\beta_f} = \frac{dm_f}{d\beta_f}\bar{s}_f + m_f \frac{d\bar{s}_f}{d\beta_f}. \tag{14}$$

In order to calculate this first we calculate $dm_f/d\beta_f$. Let $\mathbf{m} = (m_1, ..., m_F)'$ and $\beta = (\beta_1, ..., \beta_F)'$ It will be easier to compute first $d\mathbf{m}/d\beta'$. We have

$$\frac{d\mathbf{m}}{d\beta'} = \frac{d\mu(\mathbf{m}, \beta)}{d\beta'} = \frac{\partial\mu(\mathbf{m}, \beta)}{\partial\mathbf{m}'}\frac{d\mathbf{m}}{d\beta'} + \frac{\partial\mu(\mathbf{m}, \beta)}{\partial\beta'},$$

where $\mu(\mathbf{m}, \beta) = (\mu_1(\mathbf{m}, \beta), ..., \mu_F(\mathbf{m}, \beta))'$ with $\mu_f(\mathbf{m}, \beta) = 1/[\alpha(1 - \bar{s}_f)]$. Here

$$\frac{\partial\mu(\mathbf{m}, \beta)}{\partial\mathbf{m}'} = \mathbf{a}\bar{\mathbf{s}}' - A \quad \text{and} \quad \frac{\partial\mu(\mathbf{m}, \beta)}{\partial\beta'} = \frac{1}{\alpha}(B - \mathbf{b}\bar{\mathbf{s}}'),$$

where

$$\mathbf{a} = \left(\frac{\bar{s}_1}{(1-\bar{s}_1)^2}, ..., \frac{\bar{s}_F}{(1-\bar{s}_F)^2}\right)', \quad \bar{\mathbf{s}} = (\bar{s}_1, ..., \bar{s}_F)',$$

$$\mathbf{b} = \left(\frac{\rho_1\bar{s}_1}{(1-\bar{s}_1)^2}, ..., \frac{\rho_F\bar{s}_F}{(1-\bar{s}_F)^2}\right)', \quad \rho_f = 1 - \alpha\phi_f,$$

and A and B are the diagonal matrix with main diagonal equal to $\mathbf{a}$ and $\mathbf{b}$, respectively.

Therefore,

$$\frac{\partial\mathbf{m}}{\partial\beta'} = \frac{1}{\alpha}(I_F + A - \mathbf{a}\bar{\mathbf{s}}')^{-1}(B - \mathbf{b}\bar{\mathbf{s}}'), \tag{15}$$

where I_F is the identity matrix of dimension F. We know that (Dhrymes 1984, p.40)

$$(I_F + A - \mathbf{a}\bar{\mathbf{s}}')^{-1} = (I_F + A)^{-1} + \frac{1}{1 - \bar{\mathbf{s}}'(I_F + A)^{-1}\mathbf{a}}(I_F + A)^{-1}\mathbf{a}\bar{\mathbf{s}}'(I_F + A)^{-1}, \tag{16}$$

where

$$
(I_F + A)^{-1} = \begin{pmatrix} \frac{(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix},
$$

$$
1 - \overline{\mathbf{s}}' (I_F + A)^{-1} \mathbf{a} = 1 - \sum_{f=1}^{F} \frac{\overline{s}_f^2}{(1-\overline{s}_f)^2 + \overline{s}_f} > 0,
$$

$$
\mathbf{a}\overline{\mathbf{s}}' = \begin{pmatrix} \frac{\overline{s}_1^2}{(1-\overline{s}_1)^2} & \cdots & \frac{\overline{s}_1\overline{s}_F}{(1-\overline{s}_1)^2} \\ \vdots & \ddots & \vdots \\ \frac{\overline{s}_1\overline{s}_F}{(1-\overline{s}_F)^2} & \cdots & \frac{\overline{s}_F^2}{(1-\overline{s}_F)^2} \end{pmatrix},
$$

$$
(I_F + A)^{-1} \mathbf{a}\overline{\mathbf{s}}' (I_F + A)^{-1} = \begin{pmatrix} \frac{\overline{s}_1^2(1-\overline{s}_1)^2}{\left[(1-\overline{s}_1)^2+\overline{s}_1\right]^2} & \cdots & \frac{\overline{s}_1\overline{s}_F(1-\overline{s}_F)^2}{\left[(1-\overline{s}_1)^2+\overline{s}_1\right]\left[(1-\overline{s}_F)^2+\overline{s}_F\right]} \\ \vdots & \ddots & \vdots \\ \frac{\overline{s}_1\overline{s}_F(1-\overline{s}_1)^2}{\left[(1-\overline{s}_1)^2+\overline{s}_1\right]\left[(1-\overline{s}_F)^2+\overline{s}_F\right]} & \cdots & \frac{\overline{s}_F^2(1-\overline{s}_F)^2}{\left[(1-\overline{s}_F)^2+\overline{s}_F\right]^2} \end{pmatrix}.
$$

Therefore,

$$
\frac{dm_f}{d\beta_f} = \mathbf{e}_f' \frac{d\mathbf{m}}{d\beta'} \mathbf{e}_f,
$$

where $\mathbf{e}_f$ is an $F \times 1$ vector having components equal to 0 except for component f, which is 1. Using (15) and (16) we get

$$
\begin{aligned}
\frac{dm_f}{d\beta_f} &= \frac{1}{\alpha} \mathbf{e}_f' \left((I_F + A)^{-1} + \frac{1}{1 - \overline{\mathbf{s}}' (I_F + A)^{-1} \mathbf{a}} (I_F + A)^{-1} \mathbf{a}\overline{\mathbf{s}}' (I_F + A)^{-1} \right) (B - \mathbf{b}\overline{\mathbf{s}}') \mathbf{e}_f \\
&= \frac{1}{\alpha} \mathbf{e}_f' (I_F + A)^{-1} (B - \mathbf{b}\overline{\mathbf{s}}') \mathbf{e}_f \\
&+ \frac{1}{\alpha} \frac{1}{1 - \overline{\mathbf{s}}' (I_F + A)^{-1} \mathbf{a}} \mathbf{e}_f' (I_F + A)^{-1} \mathbf{a}\overline{\mathbf{s}}' (I_F + A)^{-1} (B - \mathbf{b}\overline{\mathbf{s}}') \mathbf{e}_f. \qquad (17)
\end{aligned}
$$

Note that

$$
\mathbf{e}_f' (I_F + A)^{-1} = \begin{pmatrix} 0 & \cdots & \frac{(1-\overline{s}_f)^2}{(1-\overline{s}_f)^2+\overline{s}_f} & \cdots & 0 \end{pmatrix}, \qquad (18)
$$

$$
\mathbf{e}_f' (I_F + A)^{-1} \mathbf{a} = \frac{\overline{s}_f}{(1-\overline{s}_f)^2 + \overline{s}_f}, \qquad (19)
$$

$$
\overline{\mathbf{s}}' (I_F + A)^{-1} = \begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \cdots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix}, \qquad (20)
$$

$$
(B - \mathbf{b}\overline{\mathbf{s}}') \mathbf{e}_f = \overline{s}_f \left(-\frac{\rho_1 \overline{s}_1}{(1-\overline{s}_1)^2}, \ldots, \frac{\rho_f}{1-\overline{s}_f}, \ldots, -\frac{\rho_F \overline{s}_F}{(1-\overline{s}_F)^2} \right)', \qquad (21)
$$

$$
\begin{aligned}
\overline{\mathbf{s}}' (I_F + A)^{-1} (B - \mathbf{b}\overline{\mathbf{s}}') \mathbf{e}_f &= \begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \cdots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix} \\
&\quad \cdot \left(-\frac{\rho_1 \overline{s}_1}{(1-\overline{s}_1)^2}, \ldots, \frac{\rho_f}{1-\overline{s}_f}, \ldots, -\frac{\rho_F \overline{s}_F}{(1-\overline{s}_F)^2} \right)' \\
&= \overline{s}_f \left(\frac{\rho_f \overline{s}_f (1-\overline{s}_f)}{(1-\overline{s}_f)^2 + \overline{s}_f} - \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{(1-\overline{s}_g)^2 + \overline{s}_g} \right). \qquad (22)
\end{aligned}
$$

Collecting these expressions we obtain

$$\frac{dm_f}{d\beta_f} = \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right). \tag{23}$$

This is because by (17) and straightforward calculations we obtain

$$\begin{aligned}
\frac{dm_f}{d\beta_f} &= \frac{1}{\alpha}\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}+\frac{1}{\alpha}\frac{1}{1-\overline{s}'\left(I_F+A\right)^{-1}a}\frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\\
&= \frac{\overline{s}_f}{\alpha}\left[\frac{\rho_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}+\frac{1}{1-\sum_{g=1}^{F}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}}\frac{\overline{s}_f}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right]\\
&= \frac{\overline{s}_f}{\alpha D}\left[\rho_f\left(1-\overline{s}_f\right)\left(1-\sum_{g=1}^{F}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)+\overline{s}_f\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right]\\
&= \frac{\overline{s}_f}{\alpha D}\left[\rho_f\left(1-\overline{s}_f\right)\left(\frac{1-\overline{s}_f}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right.\\
&\left.+\overline{s}_f\left(\frac{\rho_f\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{\overline{s}_f}{\alpha D}\left[\frac{\rho_f\left(1-\overline{s}_f\right)^2}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\rho_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}+\frac{\rho_f\overline{s}_f^2\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right]\\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(1-\overline{s}_f\right)-\rho_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\rho_f\overline{s}_f\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}-\overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)\\
&= \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right)+\overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right).
\end{aligned}$$

We turn now to the derivation of $d\overline{s}_f/d\beta_f$. We start with the derivative of (6), that is,

$$\frac{d\overline{s}_f}{d\beta_f} = \overline{s}_f\left[\rho_f\left(1-\overline{s}_f\right)-\alpha\frac{dm_f}{d\beta_f}\left(1-\overline{s}_f\right)+\alpha\sum_{g\neq f}\frac{dm_g}{d\beta_f}\overline{s}_g\right]. \tag{24}$$

The (g, f) off-diagonal element of the matrix $d\mathbf{m}/d\beta'$ is

$$\begin{aligned}
\frac{dm_g}{d\beta_f} &= \mathbf{e}_g'\frac{d\mathbf{m}}{d\beta'}\mathbf{e}_f\\
&= \frac{1}{\alpha}\mathbf{e}_g'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f+\frac{1}{\alpha}\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\mathbf{e}_g'\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{b}\overline{\mathbf{s}}'\right)\mathbf{e}_f.
\end{aligned}$$

Using the formulas (18)-(22) we obtain

$$\frac{dm_g}{d\beta_f} = -\frac{1}{\alpha}\frac{\rho_g \overline{s}_f \overline{s}_g}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} + \frac{1}{\alpha}\frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\frac{\overline{s}_f \overline{s}_g}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\left(\frac{\rho_f \overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f} - \sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)$$

$$= \frac{\overline{s}_f \overline{s}_g}{\alpha\left[\left(1-\overline{s}_g\right)^2+\overline{s}_g\right]}\left(-\rho_g + \frac{1}{1-\sum_{h=1}^{F}\frac{\overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}}\left(\frac{\rho_f \overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2+\overline{s}_f} - \sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)\right)$$

$$= \frac{\overline{s}_f \overline{s}_g}{\alpha\left[\left(1-\overline{s}_g\right)^2+\overline{s}_g\right]D}\left(\rho_f \overline{s}_f\left(1-\overline{s}_f\right) - \rho_g D - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right). \tag{25}$$

Further, from (25) we get

$$\sum_{g\neq f}\frac{dm_g}{d\beta_f}\overline{s}_g = \sum_{g\neq f}\frac{\overline{s}_f \overline{s}_g^2}{\alpha\left[\left(1-\overline{s}_g\right)^2+\overline{s}_g\right]D}\left(\rho_f \overline{s}_f\left(1-\overline{s}_f\right) - \rho_g D - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)$$

$$= \frac{\overline{s}_f}{\alpha D}\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\left(\rho_f \overline{s}_f\left(1-\overline{s}_f\right) - \rho_g D - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)$$

$$= \frac{\overline{s}_f}{\alpha D}\left(\rho_f \overline{s}_f\left(1-\overline{s}_f\right) - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}$$

$$- \frac{\overline{s}_f}{\alpha}\sum_{g\neq f}\frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}$$

$$= \frac{\overline{s}_f}{\alpha D}\left(\rho_f \overline{s}_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\sum_{h\neq f}\frac{\rho_h \overline{s}_h^2}{\left(1-\overline{s}_h\right)^2+\overline{s}_h}\right)$$

$$- \frac{\overline{s}_f}{\alpha}\sum_{g\neq f}\frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}. \tag{26}$$

Note that

$$\left[\left(1-\overline{s}_f\right)^2+\overline{s}_f\right]\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} = 1-\overline{s}_f - D,$$

so by further straightforward calculations we obtain that

$$\sum_{g\neq f}\frac{dm_g}{d\beta_f}\overline{s}_g = \frac{\overline{s}_f}{\alpha D}\left(\rho_f \overline{s}_f\left(1-\overline{s}_f\right)\sum_{g\neq f}\frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \left(1-\overline{s}_f - D\right)\sum_{g\neq f}\frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)$$

$$- \frac{\overline{s}_f}{\alpha}\sum_{g\neq f}\frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}$$

$$
\begin{aligned}
&= \frac{\rho_f \overline{s}_f^2 \left(1-\overline{s}_f\right)}{\alpha D} \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \frac{\overline{s}_f}{\alpha D}\left(1-\overline{s}_f-D\right) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \\
&- \frac{\overline{s}_f}{\alpha} \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \\
&= \frac{\rho_f \overline{s}_f^2 \left(1-\overline{s}_f\right)}{\alpha D} \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{\alpha D} \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{\alpha D} \left(\rho_f \overline{s}_f \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right).
\end{aligned}
$$

Substituting this and (23) into (24), we obtain

$$
\begin{aligned}
\frac{d\overline{s}_f}{d\beta_f} &= \overline{s}_f \left[\rho_f \left(1-\overline{s}_f\right) - \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left(\rho_f \left(E-\overline{s}_f\right) + \overline{s}_f \sum_{g \neq f} \frac{\left(\rho_f-\rho_g\right) \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \right. \\
&\left. + \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left(\rho_f \overline{s}_f \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \right] \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \\
&\times \left[\rho_f D - \overline{s}_f \left(\rho_f \left(E-\overline{s}_f\right) + \overline{s}_f \sum_{g \neq f} \frac{\left(\rho_f-\rho_g\right) \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \rho_f \overline{s}_f \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} + \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \right] \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left[\rho_f D - \overline{s}_f \left(\rho_f \left(E-\overline{s}_f\right) - \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} + \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \right] \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left[\rho_f D - \overline{s}_f \left(\rho_f \left(E-\overline{s}_f\right) + \left(1-\overline{s}_f\right) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \right] \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left[\rho_f D - \rho_f \overline{s}_f \left(E-\overline{s}_f\right) - \overline{s}_f \left(1-\overline{s}_f\right) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right] \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left[\rho_f \left[D - \overline{s}_f \left(E-\overline{s}_f\right)\right] - \overline{s}_f \left(1-\overline{s}_f\right) \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right].
\end{aligned}
$$

Now we use

$$
D - \overline{s}_f \left(E-\overline{s}_f\right) = \left[\left(1-\overline{s}_f\right)^2 + \overline{s}_f\right] E - \overline{s}_f^2 - \overline{s}_f E + \overline{s}_f^2 = \left(1-\overline{s}_f\right)^2 E, \tag{27}
$$

so

$$
\begin{aligned}
\frac{d\overline{s}_f}{d\beta_f} &= \frac{\overline{s}_f \left(1-\overline{s}_f\right)}{D} \left(\rho_f \left(1-\overline{s}_f\right)^2 E - \left(1-\overline{s}_f\right) \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)^2}{D} \left(\rho_f \left(1-\overline{s}_f\right) E - \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right) \\
&= \frac{\overline{s}_f \left(1-\overline{s}_f\right)^2}{D} \left(\rho_f \left(1-\overline{s}_f\right) E - \overline{s}_f \sum_{g \neq f} \frac{\rho_f \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} + \overline{s}_f \sum_{g \neq f} \frac{\rho_f \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \overline{s}_f \sum_{g \neq f} \frac{\rho_g \overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} \right)
\end{aligned}
$$

$$= \frac{\overline{s}_f\left(1-\overline{s}_f\right)^2}{D}\left(\rho_f\left(1-\overline{s}_f\right)E - \rho_f\overline{s}_f\left(1-E\right) + \overline{s}_f\sum_{g\neq f}\frac{\rho_f\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g} - \overline{s}_f\sum_{g\neq f}\frac{\rho_g\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right)$$

$$= \frac{\overline{s}_f\left(1-\overline{s}_f\right)^2}{D}\left(\rho_f\left(E-\overline{s}_f\right) + \overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right). \tag{28}$$

Substituting the derivatives from (23) and (28) and the equilibrium markup expression (5) we get

$$\frac{d\pi_f}{d\beta_f} = \frac{\overline{s}_f}{\alpha D}\left(\rho_f\left(E-\overline{s}_f\right) + \overline{s}_f\sum_{g\neq f}\frac{\left(\rho_f-\rho_g\right)\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2+\overline{s}_g}\right). \tag{29}$$

A.2 The case of product-specific brands

The derivative of the profit of firm f with respect to the brand equity β_j of product j that belongs to firm f is

$$\frac{d\pi_f}{d\beta_j} = \frac{dm_f}{d\beta_j}\overline{s}_f + m_f\frac{d\overline{s}_f}{d\beta_j}.$$

so we need to calculate $\partial m_f/\partial\beta_j$ and $\partial\overline{s}_f/\partial\beta_j$. The former will follow from $\partial\mathbf{m}/\partial\beta'$, which is equal to

$$\frac{d\mathbf{m}}{d\beta'} = \frac{d\mu\left(\mathbf{m},\beta\right)}{d\beta'} = \frac{\partial\mu\left(\mathbf{m},\beta\right)}{\partial\mathbf{m}'}\frac{d\mathbf{m}}{d\beta'} + \frac{\partial\mu\left(\mathbf{m},\beta\right)}{\partial\beta'},$$

where $\mu\left(\mathbf{m},\beta\right) = \left(\mu_1\left(\mathbf{m},\beta\right),...,\mu_F\left(\mathbf{m},\beta\right)\right)'$ with $\mu_f\left(\mathbf{m},\beta\right) = 1/\left[\alpha\left(1-\overline{s}_f\right)\right]$. Here

$$\frac{\partial\mu\left(\mathbf{m},\beta\right)}{\partial\mathbf{m}'} = \mathbf{a}\overline{\mathbf{s}}' - A \quad\text{and}\quad \frac{\partial\mu\left(\mathbf{m},\beta\right)}{\partial\beta'} = \frac{\rho}{\alpha}\left(B-\mathbf{a}\mathbf{s}'\right),$$

where

$$\mathbf{a} = \left(\frac{\overline{s}_1}{\left(1-\overline{s}_1\right)^2},...,\frac{\overline{s}_F}{\left(1-\overline{s}_F\right)^2}\right)', \quad \mathbf{s} = \left(s_1,...,s_J\right)',$$

$$B = \begin{pmatrix} \frac{\mathbf{s}_1'}{\left(1-\overline{s}_1\right)^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\mathbf{s}_F'}{\left(1-\overline{s}_F\right)^2} \end{pmatrix}.$$

Therefore,

$$\frac{d\mathbf{m}}{d\beta'} = \frac{\rho}{\alpha}\left(I_F + A - \mathbf{a}\overline{\mathbf{s}}'\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right),$$

where I_F is the identity matrix of dimension F. We know from Dhrymes (1984) that

$$\left(I_F + A - \mathbf{a}\overline{\mathbf{s}}'\right)^{-1} = \left(I_F+A\right)^{-1} + \frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1},$$

so

$$\frac{d\mathbf{m}}{d\beta'} = \frac{\rho}{\alpha}\left(\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right) + \frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)\right). \tag{30}$$

Element (f,j) of the matrix $d\mathbf{m}/d\beta'$ for $j\in\mathcal{G}_f$ is

$$\frac{dm_f}{d\beta_j} = \frac{\rho}{\alpha}\left(\mathbf{e}_f'\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)\mathbf{e}_j + \frac{1}{1-\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\mathbf{a}}\mathbf{e}_f'\left(I_F+A\right)^{-1}\mathbf{a}\overline{\mathbf{s}}'\left(I_F+A\right)^{-1}\left(B-\mathbf{a}\mathbf{s}'\right)\mathbf{e}_j\right),$$

where $\mathbf{e}_f$ and $\mathbf{e}_j$ are column vectors of conformable size having 1 on position f and j, respectively, and 0 elsewhere.

It holds that

$$\begin{aligned}
\mathbf{e}_f' \left(I_F + A\right)^{-1} &= \begin{pmatrix} 0 & \dots & \frac{(1-\overline{s}_f)^2}{(1-\overline{s}_f)^2+\overline{s}_f} & \dots & 0 \end{pmatrix}, \\
\mathbf{e}_f' \left(I_F + A\right)^{-1} \mathbf{a} &= \frac{\overline{s}_f}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f}, \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} &= \begin{pmatrix} \frac{\overline{s}_1(1-\overline{s}_1)^2}{(1-\overline{s}_1)^2+\overline{s}_1} & \dots & \frac{\overline{s}_F(1-\overline{s}_F)^2}{(1-\overline{s}_F)^2+\overline{s}_F} \end{pmatrix}, \\
\left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \begin{pmatrix} -\frac{\overline{s}_1}{(1-\overline{s}_1)^2} & \dots & \frac{1}{1-\overline{s}_f} & \dots & -\frac{\overline{s}_F}{(1-\overline{s}_F)^2} \end{pmatrix}', \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \left(\frac{\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} - \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g} \right).
\end{aligned}$$

So

$$\frac{dm_f}{d\beta_j} = \frac{\rho s_j}{\alpha D} \left(E - \overline{s}_f\right). \tag{31}$$

This is because

$$\begin{aligned}
\frac{dm_f}{d\beta_j} &= \frac{\rho}{\alpha} \left(\frac{s_j\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} + \frac{\frac{s_j \overline{s}_f}{(1-\overline{s}_f)^2+\overline{s}_f} \left(\frac{\overline{s}_f(1-\overline{s}_f)}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{g\neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g} \right)}{1 - \sum_{g=1}^F \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g}} \right) \\
&= \frac{\rho}{\alpha} \frac{s_j}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} \left(\left(1-\overline{s}_f\right) + \frac{\overline{s}_f \left(\frac{\overline{s}_f(1-\overline{s}_f)}{(1-\overline{s}_f)^2+\overline{s}_f} - \sum_{g\neq f} \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g} \right)}{1 - \sum_{g=1}^F \frac{\overline{s}_g^2}{(1-\overline{s}_g)^2+\overline{s}_g}} \right) \\
&= \frac{\rho s_j}{\alpha D} \left[\left(1-\overline{s}_f\right) \left(E - \frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} \right) + \overline{s}_f \left(\frac{\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} - \sum_{g\neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g} \right) \right] \\
&= \frac{\rho s_j}{\alpha D} \left[\left(1-\overline{s}_f\right) \left(E - \frac{\overline{s}_f^2}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} \right) + \overline{s}_f \left(\frac{\overline{s}_f\left(1-\overline{s}_f\right)}{\left(1-\overline{s}_f\right)^2 + \overline{s}_f} - 1 + E \right) \right] \\
&= \frac{\rho s_j}{\alpha D} \left(E - \overline{s}_f\right),
\end{aligned}$$

where for the third equality we use (8).

In order to compute $\partial \overline{s}_f / \partial \beta_j$ we compute $\partial s_r / \partial \beta_j$ for $r = j$ and $r \neq j$. The derivative with respect to β_j of the market share s_r of product r different from j is

$$\begin{aligned}
\frac{ds_r}{d\beta_j} &= -s_r \left(\rho s_j + \alpha \frac{\partial m_f}{\partial \beta_j} \left(1-\overline{s}_f\right) - \alpha \sum_{g\neq f} \frac{\partial m_g}{\partial \beta_j} \overline{s}_g \right) \\
&= -s_r \left(\rho s_j + \frac{\rho}{D} s_j \left(E - \overline{s}_f\right)\left(1-\overline{s}_f\right) + \frac{\rho s_j \left(1-\overline{s}_f\right)^2}{D} \sum_{g\neq f} \frac{\overline{s}_g^2}{\left(1-\overline{s}_g\right)^2 + \overline{s}_g} \right) \\
&= -\frac{\rho s_j s_r}{D} \left(D + \left(E - \overline{s}_f\right)\left(1-\overline{s}_f\right) + \left(1-\overline{s}_f\right)^2 \left(1-E\right) \right) \\
&= -\frac{\rho s_j s_r}{D} \left(\overline{s}_f \left(E - \overline{s}_f\right) + \left(1-\overline{s}_f\right)^2 E + \left(E - \overline{s}_f\right)\left(1-\overline{s}_f\right) + \left(1-\overline{s}_f\right)^2 \left(1-E\right) \right) \\
&= -\frac{\rho s_j s_r}{D} \left(E - \overline{s}_f + \left(1-\overline{s}_f\right)^2 \right).
\end{aligned} \tag{32}$$

The derivative of the market share s_j from (11) with respect to its own brand equity β_j is

$$\frac{ds_j}{d\beta_j} = s_j \left(\rho \left(1 - s_j\right) - \alpha \frac{dm_f}{d\beta_j} \left(1 - \overline{s}_f\right) + \alpha \sum_{g \neq f} \frac{dm_g}{d\beta_j} \overline{s}_g \right). \tag{33}$$

The derivative $dm_g/d\beta_j$ is element (g, j) of the matrix $\partial m/\partial \beta'$ for $j \in \mathcal{G}_f$ and $g \neq f$, which by (30) is

$$\frac{dm_g}{d\beta_j} = \frac{\rho}{\alpha} \left(\mathbf{e}_g' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j + \frac{1}{1 - \overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \mathbf{a}} \mathbf{e}_g' \left(I_F + A\right)^{-1} \mathbf{a}\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j \right),$$

where

$$\begin{aligned}
\mathbf{e}_g' \left(I_F + A\right)^{-1} &= \begin{pmatrix} 0 & \ldots & \frac{(1-\overline{s}_g)^2}{(1-\overline{s}_g)^2 + \overline{s}_g} & \ldots & 0 \end{pmatrix}, \\
\mathbf{e}_g' \left(I_F + A\right)^{-1} \mathbf{a} &= \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g}, \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} &= \begin{pmatrix} \frac{\overline{s}_1 (1-\overline{s}_1)^2}{(1-\overline{s}_1)^2 + \overline{s}_1} & \ldots & \frac{\overline{s}_F (1-\overline{s}_F)^2}{(1-\overline{s}_F)^2 + \overline{s}_F} \end{pmatrix}, \\
\left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \begin{pmatrix} -\frac{\overline{s}_1}{(1-\overline{s}_1)^2} & \ldots & \frac{1}{1-\overline{s}_f} & \ldots & -\frac{\overline{s}_F}{(1-\overline{s}_F)^2} \end{pmatrix}', \\
\overline{\mathbf{s}}' \left(I_F + A\right)^{-1} \left(B - \mathbf{a}\mathbf{s}'\right) \mathbf{e}_j &= s_j \left(\frac{\overline{s}_f}{\left(1 - \overline{s}_f\right)^2 + \overline{s}_f} - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{\left(1 - \overline{s}_h\right)^2 + \overline{s}_h} \right).
\end{aligned}$$

Substituting these, we get

$$\frac{dm_g}{d\beta_j} = -\frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \frac{\left(1 - \overline{s}_f\right)^2}{D}. \tag{34}$$

This is because

$$\begin{aligned}
\frac{dm_g}{d\beta_j} &= \frac{\rho}{\alpha} \left(-s_j \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} + \frac{\frac{\overline{s}_g}{(1-\overline{s}_g)^2 + \overline{s}_g} s_j \left(\frac{\overline{s}_f}{(1-\overline{s}_f)^2 + \overline{s}_f} - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h} \right)}{1 - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h}} \right) \\
&= \frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \left(-1 + \frac{\frac{\overline{s}_f}{(1-\overline{s}_f)^2 + \overline{s}_f} - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h}}{1 - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h}} \right) \\
&= \frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \left(\frac{\frac{\overline{s}_f}{(1-\overline{s}_f)^2 + \overline{s}_f} - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h} - 1 + \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h}}{1 - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h}} \right) \\
&= \frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \left(\frac{\frac{\overline{s}_f}{(1-\overline{s}_f)^2 + \overline{s}_f} - 1}{1 - \sum_{h=1}^{F} \frac{\overline{s}_h^2}{(1-\overline{s}_h)^2 + \overline{s}_h}} \right) \\
&= -\frac{\rho s_j}{\alpha} \frac{\overline{s}_g}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \frac{\left(1 - \overline{s}_f\right)^2}{D}.
\end{aligned}$$

Substituting the derivatives (34) and (31) into (33), we get

$$
\begin{aligned}
\frac{ds_j}{d\beta_j} &= s_j \left(\rho \left(1 - s_j\right) - \frac{\rho}{D} s_j \left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) - \frac{\rho s_j \left(1 - \overline{s}_f\right)^2}{D} \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \right) \\
&= \frac{\rho s_j}{D} \Big(\left(1 - s_j\right) D - s_j \left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) - s_j \left(1 - \overline{s}_f\right)^2 \left(1 - E\right) \Big) \\
&= \frac{\rho s_j}{D} \Big(D - s_j \left[\left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) + D\right] - s_j \left(1 - \overline{s}_f\right)^2 \left(1 - E\right) \Big) \\
&= \frac{\rho s_j}{D} \Big(D - s_j \Big[\left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) + \overline{s}_f \left(E - \overline{s}_f\right) + \left(1 - \overline{s}_f\right)^2 E + \left(1 - \overline{s}_f\right)^2 \left(1 - E\right)\Big] \Big) \\
&= \frac{\rho s_j}{D} \Big(\overline{s}_f \left(E - \overline{s}_f\right) + \left(1 - \overline{s}_f\right)^2 E - s_j \Big[E - \overline{s}_f + \left(1 - \overline{s}_f\right)^2\Big] \Big) \\
&= \frac{\rho s_j}{D} \Big(\left(\overline{s}_f - s_j\right)\left(E - \overline{s}_f\right) + \left(1 - \overline{s}_f\right)^2 \left(E - s_j\right) \Big). \qquad (35)
\end{aligned}
$$

The derivative of the market share of firm f (12) with respect to the brand equity β_j of product j is

$$
\frac{d\overline{s}_f}{d\beta_j} = \rho s_j \left(1 - \overline{s}_f\right) - \alpha \overline{s}_f \left(\frac{dm_f}{d\beta_j} - \sum_{g=1}^{F} \frac{dm_g}{d\beta_j} \overline{s}_g \right).
$$

Substituting the derivatives (34) and (31), we get

$$
\begin{aligned}
\frac{d\overline{s}_f}{d\beta_j} &= \rho s_j \left(1 - \overline{s}_f\right) - \alpha \overline{s}_f \frac{dm_f}{d\beta_j} \left(1 - \overline{s}_f\right) + \alpha \overline{s}_f \sum_{g \neq f} \frac{dm_g}{d\beta_j} \overline{s}_g \\
&= \rho s_j \left(1 - \overline{s}_f\right) - \overline{s}_f \rho \frac{s_j}{D} \left(E - \overline{s}_f\right)\left(1 - \overline{s}_f\right) - \rho s_j \overline{s}_f \frac{\left(1 - \overline{s}_f\right)^2}{D} \sum_{g \neq f} \frac{\overline{s}_g^2}{\left(1 - \overline{s}_g\right)^2 + \overline{s}_g} \\
&= \frac{\rho s_j \left(1 - \overline{s}_f\right)}{D} \left(D - \overline{s}_f \left(E - \overline{s}_f\right) - \overline{s}_f \left(1 - \overline{s}_f\right)\left(1 - E\right) \right) \\
&= \frac{\rho s_j \left(1 - \overline{s}_f\right)}{D} \Big(\left(1 - \overline{s}_f\right)^2 E - \overline{s}_f \left(1 - \overline{s}_f\right)\left(1 - E\right) \Big) \\
&= \frac{\rho s_j \left(1 - \overline{s}_f\right)^2}{D} \left(E - \overline{s}_f\right), \qquad (36)
\end{aligned}
$$

where for the fourth equality we use (27).

Substituting the derivatives from (31) and (36) as well as the markup expression (5), we obtain

$$
\frac{d\pi_f}{d\beta_j} = \frac{\rho s_j}{\alpha D} \left(E - \overline{s}_f\right) \overline{s}_f + \frac{1}{\alpha} \frac{1}{1 - \overline{s}_f} \frac{\rho s_j \left(1 - \overline{s}_f\right)^2}{D} \left(E - \overline{s}_f\right) = \frac{\rho s_j \left(E - \overline{s}_f\right)}{\alpha D}. \qquad (37)
$$

The derivative of the profit from product j with respect to the brand equity β_j is

$$
\frac{d\pi_j}{d\beta_j} = \frac{dm_f}{d\beta_j} s_j + m_f \frac{ds_j}{d\beta_j}.
$$

Substituting (31), (35) and (5), we obtain

$$\begin{aligned}
\frac{d\pi_j}{d\beta_j} &= \frac{\rho s_j}{\alpha D}\left[(E-\overline{s}_f)\,s_j + \frac{1}{1-\overline{s}_f}\left((\overline{s}_f - s_j)\,(E-\overline{s}_f) + (1-\overline{s}_f)^2\,(E-s_j)\right)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[(E-\overline{s}_f)\,s_j + \frac{(\overline{s}_f - s_j)\,(E-\overline{s}_f)}{1-\overline{s}_f} + (1-\overline{s}_f)\,(E-s_j)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\frac{(1-\overline{s}_f)\,(E-\overline{s}_f)\,s_j + (\overline{s}_f - s_j)\,(E-\overline{s}_f)}{1-\overline{s}_f} + (1-\overline{s}_f)\,(E-s_j)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\frac{E-\overline{s}_f}{1-\overline{s}_f}\left((1-\overline{s}_f)\,s_j + (\overline{s}_f - s_j)\right) + (1-\overline{s}_f)\,(E-s_j)\right] \\
&= \frac{\rho s_j}{\alpha D}\left[\frac{1-s_j}{1-\overline{s}_f}\overline{s}_f\,(E-\overline{s}_f) + (1-\overline{s}_f)\,(E-s_j)\right]. \qquad (38)
\end{aligned}$$

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