The Impact of Ad-Avoidance Technologies in the Market for Video Streaming

Guofu Tan* Yejia Xu†

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Abstract

To study the impact of Ad-Avoidance Technologies (AATs) usage in the market for video streaming, we construct a game-theoretic model of platform competition: A continuum of audience receives heterogeneous random valuations from watching videos across different platforms where the valuations are identically independently distributed (IID). Each platform provides two types of memberships: the audience could either watch the video bundled with the ads for free (Free Membership), or pay a subscription fee to get rid of ads (Premium Membership). A certain share of the audience, independently from their valuations, has access to AATs which could reduce the disutility from watching ads. Platforms compete by setting subscription fees, and given the subscription fees, the valuation vector and the status of access to AATs, the audience chooses which platform to join and which membership to maximize the final utility, or not watching at all. We further allow the platforms to drop Free Membership. Theoretically, we show that both the equilibrium subscription fee and platform revenue decreases in the penetration rate of AATs. By calibrating the parameters of our model with actual data from the Chinese video streaming market, we are able to simulate counter-factual outcomes under various penetration rates, and confirm that an increase in the usage of AATs would reduce the platforms' revenues, force the platforms to drop Free Membership, and hurt the social welfare in the long run.

Keywords: Video Streaming, Ad-Avoidance Techonologies (AATs), Calibration

*University of Southern California, Department of Economics, Professor of Economics; Email: guofutan@usc.edu

†University of Southern California, Department of Economics, Ph.D. student in Economics; Email: yejiaxu@usc.edu

1 Introduction

Video streaming has become an important form of entertainment in recent years and the market size of the video streaming industry has been increasing dramatically. The more mature U.S. video steaming market earned a total revenue of $10,443 million U.S. dollars in 2017 and is expected to have an annual growth rate of 3.1% in coming years.[1] Meanwhile, the less-developed video streaming market in China had witnessed an annual growth of around 50% in the past years and the estimated total revenue in 2017 reached $95.23 billion CNY (approximately $14,068 million U.S. dollars).[2]

The video streaming platforms usually provide the audience with two options: the Free Membership allows the audience to watch the videos for free, but has to watch the ads bundled with the videos; the Premium Membership requires the audience to pay a subscription fee before they watch videos ads-free. Such a business model implies that there are two major sources of revenues for the video streaming platforms: payments from the audience, mostly subscription fees, and advertising fees from the advertisers. Although there exist platforms that rely on one of the sources only (e.g., Netflix), most platforms rely on both. The latter source is especially important for video streaming platforms in China, where the advertising revenue accounts for almost half of the total revenue in the industry.

Ad-Avodiance Technologies (AATs) targeted at the video streaming platforms emerge along with the prospering of the industry. These AATs are usually in the form of third-party softwares or browser plug-ins, and some new browsers even have ad-blocking as a built-in feature to attract more users. The overall penetration rate was reported to be 18% for the United States and 13% for China, and the annual global growth rate stood at 30% in 2016. [3]

A recent lawsuit in China, *Tencent Video v.s. The World Browser*, provides a handy real-life example to our study. Tencent Video is one of the three major video streaming platforms in China, and it provides the two types of memberships, i.e., Free Membership and Premium Membership, mentioned above. In order to attract users, The World Browser, a software company, provides its users with a built-in function of blocking ads on major video streaming platforms, including Tencent Videos. The users of the World Browser can skip the ads even under Free Membership. The browser operator argued that users could benefit from not watching the ads, which was supported in the trial of first instance on January 26th, 2018. In the court decision, the judge regarded the AATs as technological advancements which the audience has the right to enjoy, and claimed that such practices should be encouraged as long as it is beneficial to the society as a whole.

However, such claim obviously obscured the difference between consumer

[1] Statista. Available at https://www.statista.com/outlook/206/109/video-streaming–svod-/united-states

[2] iResearch. Available at http://report.iresearch.cn/report/201805/3216.shtml

[3] PageFair. Available at https://pagefair.com/downloads/2017/01/PageFair-2017-Adblock-Report.pdf

welfare and social welfare, neglecting the possible harm to the platforms' profits and business models. Moreover, in the long run, the existence of AATs jeopardizes both sources of the platforms' revenues, and if the platforms decide not to provide the videos at all, or at least not for free, or invest less in the video contents, the audience who seeks to use AATs in the first place would be harmed as well. In the trial of second instance on December 28th, 2018, the appeal court took this stand, and supported Tencent Video's claim that the built-in ad-blocking feature of The World Browser is a business foul play.

To examine the impact of AATs more thoroughly, we construct a game-theoretic model of platform competition. A continuum of audience receives heterogeneous random valuations from watching videos across different platforms where the valuations are identically independently distributed (IID). Each platform provides the two types of memberships. We assume that a certain share (i.e., the penetration rate of AATs) of the audience, independently from their valuations, has access to AATs which could reduce the disutility from watching ads. Therefore, it is weakly dominant for these audience to choose Free Membership.

Knowing the existence of AATs, platforms compete by setting subscription fees, and given the subscription fees, the valuation vector and the status of access to AATs, the audience chooses which platform to join and which membership to maximize the final utility, or not watching any videos at all. Each platform earns the subscription income from those who chooses its Premium Membership, and the advertisement income from the advertisers which is exogenously proportional to the mass of audience choosing its Free Membership, yet the proportion coefficient is negatively correlated with the penetration rate.

The platforms are assumed to be ex-ante symmetric, and therefore we are interested in the symmetric pricing equilibrium of the game. We also explore the possibility of platforms dropping the Free Membership by adding a membership selection stage before the pricing game.

Theoretically, we show that the equilibrium subscription fee decreases in the penetration rate and the number of platforms, which is rather intuitive: As the penetration rate increases, the advertising fee per Free Member would decrease as well, and the platforms would reduce the subscription fee to transform more Free Members into Premium Members to maximize the profits. And if there are more platforms in the market, the competition should drive down the subscription fee as well. These results further helps us to show that the platforms' revenue decreases in the penetration rate as well.

However, it's mathematically challenging to derive more detailed analysis on the platform performance, and we turn to numerical analysis for help. We calibrate the parameters in the model with actual data from the Chinese video streaming market, and simulate the equilibrium outcomes under various AATs penetration rates. The simulation results confirm that an increase in the penetration rate would hurt the platforms' total revenue, and may force the platforms to drop Free Membership, yet the total welfare may increase in the short run since a higher valuation is realized for most audience.

We are able to show that audience always benefit from the AATs in the short run, but the society may suffer a loss in total surplus if the platforms should adapt to the change by reducing video resources investment. To explore this long run effect, we examine the "equivalent quality shock", i.e., the minimal quality of the video resources (reflected in the shape parameter of valuation distributions) such that the consumer surplus or total surplus is to be sustained. The results show that even if the platforms have small incentives to reduce investment, the total surplus would still be very likely to drop, despite that the audience gains from the change.

The rest of this paper is organized as follows: Section 2 reviews the related literature in advertising and platform economics. Section 3 provides the basic model setups for audience, platforms, AATs and respective objectives. We derive the analytical results for the equilibrium solution and some comparative statics in Section 4. In Section 5 we construct the measure of welfare and briefly discuss the short run and long run impact of increasing penetration rate of AATs. Section 6 presents our calibration results with actual data from the Chinese video streaming market and simulation under various penetration rates of AATs. Section 7 discusses some alternative specifications of the good market, and Section 8 concludes our work. All proofs are provided in the Appendix.

2 Literature Review

As early as in 1950, economist Kaldor has already pointed out that ads are often jointly supplied with entertaining content over media platforms (Kaldor, 1950). The media platforms cross-subsidize the audience by providing the bundle of media contents or services with ads at a price lower than the marginal cost, and get compensated by the advertising revenue. Such a cross-subsidy is a typical feature of a two-sided market (Armstrong, 2006; Tan & Zhou, 2019). However, it was not until the recent fifteen years that such a relationship between ads and media was addressed as a two-sided market. Some pioneer works include Dukes (2004), Anderson and Coate (2005) and Anderson and Gabszewicz (2006).

Dukes (2004) divided the market into three parts, each of which contains two of the three players in the market, i.e., the platform, the producer (and advertiser), and the consumer (and audience). Nevertheless, we could combine the two parts where the platform lies in and regard it as a two-sided market. The author was interested in how the differences among goods and platforms would affect the advertising intensity: when there is little difference in goods, the producers would compete more fiercely and put out more ads; while if there is little difference in platforms, the platform would lower the advertising intensity to attract audience. Anderson and Coate (2005) provided a more standard two-sided market framework for the commercial TV market. The platforms has to decide the optimal bundles of programs and a certain level of ads for audience with heterogeneous tastes. The framework enabled the authors to perform a welfare analysis to the market performances.

On the basis of this framework, Anderson and Gans (2011) provides a theoretical analysis on AATs. The authors pointed out that AATs in the digital era, such as TiVo and ad-blocking softwares, are completely different from the traditional ways of avoiding ads, in that the marginal cost of using AATs is very low or essentially zero. Moreover, the audience using AATs would be those who dislike ads the most, and as the penetration rate continues to rise, the remaining audience would be more tolerant, which allows the platform to increase the advertising intensity, driving more audience to use AATs.

We shall see that the previous frameworks are not specifically designed for the video streaming industry, and thus our model would be different in the following aspects: First of all, Anderson and Coate (2005) assumed that audience always has to pay the entrance fee, while in our model, the subscription fee is only paid for the Premium Membership. Secondly, since the video streaming platforms have little or no bargaining power on the advertising market, they are usually price-takers, and the places or lengths of the ads are also determined independently before the platforms decide the subscription fee. In other words, the advertising intensity is exogenously given. For the sake of simplicity, we also assume that whether the audience has access to AATs is independent from their valuations, and therefore, no conclusions can be drawn for the marginal valuation as the penetration rate of AATs increases.

Using the US commercial TV market data, Wilbur (2008) estimated that a decrease of 10% in ads would result in an increase of 25% in the number of audience. Moreover, he estimated with simulation that as the penetration rate of AATs increases, the TV network would increase the level of ads, but the effective views of these ads decrease, as well as the advertising revenue of the TV network. Despite the fact that we are not modeling the advertising intensity in our model, we come to a similar result that less audience would be watching ads and thus the advertising revenue should decline.

Shiller, Waldfogel, and Ryan (2018) performed empirical studies on ad-supported websites. The regression analysis shows that, for each additional percentage point of site visitors blocking ads, the website traffic reduces by 0.67% over 35 months, along with a decline in content qualities. The authors argue that the usage of AATs has a compound effect on reducing the website revenues both through reduced actual views of ads directly and reduced visits due to quality decline indirectly. In our paper, we follow the same idea and consider the possibility in the long run welfare analysis that the platforms may respond to the revenue loss due to increasing penetration rates of AATs by reducing video resources investment, and the decline in video qualities may eventually lead to a loss in social welfare.

3 Model

3.1 Audience

A continuum of audience of mass 1 has a valuation of v for the videos on a certain platform. The valuation follows a distribution $F(v)$ with density function $f(v)$ with full support on $\mathbb{R}^+ = [0, \infty)$. When there are multiple platforms, the audience has valuation v_i for platform i, where every v_i follows the identical distribution $F(v)$ independently.

3.2 Platforms

Every platform provides two memberships: Free Membership allows the audience to watch the videos for free as long as they watch the ads bundled to the videos. Premium Membership allows the audience to pay a subscription fee in order to watch the videos ads-free. Later we would allow the platform to provide only the Premium Membership but not the Free Membership, which is a strategy used by major video streaming platforms Netflix and Hulu in the United States.

For audience with valuation v_i for platform i, watching ads would incur a disutility of $kv_i + D$, where $0 < k < 1$ and $D > 0$, and the final utility of choosing Free Membership from platform i is $v_i - kv_i - D$. The disutility comes from the interruption of viewing experience and possible mental cost.[4] In our symmetric setting, different platforms share the same k and D.

The subscription fee for platform i is denoted as p_i, which is chosen by platform i, and the final utility of choosing Premium Membership from platform i is $v_i - p_i$.

The audience has yet an outside option of Not Watching with a normalized utility 0. As a rational agent, the audience always chooses the option with the highest final utility.

3.3 Ad-Avoidance Technology

Independent from their valuations, a mass of δ audience has access to the Ad-Avoidance Technologies (AATs), which allow them to completely skip the ads.[5] In other words, they get a final utility of v_i when using Free Membership from platform i, which always dominates Not Watching or Premium Membership from the same platform.

3.4 Cost and Revenue

Although the video streaming platforms should have invested heavily on their video pools and website constructions, these costs are regarded as sunk cost in the pricing decisions. The marginal cost of providing the videos is rather small and is also ignored in the model. Therefore, maximizing profit is equivalent to maximizing revenue for the platforms.

When there are multiple platforms, all of them are involved in a pricing game, where they choose the subscription fee p_i simultaneously to maximize their own revenue.

[4] See section 7 for a more specific explanation on D.

[5] Alternatively, it could be interpreted as every audience has access to AATs with probability δ.

The platforms' revenue can be divided into two parts: The subscription revenue R_{i1} from Premium Members and the advertising revenue R_{i2} from Free Members who actually watch the ads. Audience with access to AATs would always choose Free Membership but they never watch the ads.

For every Premium Member, the platform earns the subscription fee p_i. The total subscription revenue can thus be denoted as $R_{i1} = p_i Q_{i1}$ where Q_{i1} is the mass of audience choosing Premium Membership from platform i.

For every Free Member that is not an AATs user, the platform earns ϕ from the advertiser. Similarly we denote Q_{i2} as the mass of audience choosing Free Membership from platform i, and moreover Q_{i2}^{δ} as the mass of audience choosing Free Membership from platform i and meanwhile using AATs. Then we can denote $R_{i2} = \phi(Q_{i2} - Q_{i2}^{\delta})$.

To understand ϕ better, we need to know that ads are usually paid in a cost-per-view manner. Suppose that every Free Member who does not use AATs would view a similar average number of ads. Then the mass of audience watching the ads is proportional to the number of times ads being viewed, which in turn is proportional to the advertising revenue. This proportionality is thus reflected in the linear relationship with coefficient ϕ between ads revenue and the mass of Free Members without access to AATs.

We further assume that ϕ is an exogenous function of $\delta \in [0, 1]$ with $\phi'(\delta) < 0$ and $\phi(1) = 0$. In reality, advertisers always have larger bargaining power over the ads pricing issue, and they would not be willing to pay for the ads if they know ads can be easily blocked. Later in the calibration exercise, we focus on the family of power functions for ϕ:

$$\phi(\delta) = \eta(1 - \delta)^c \tag{1}$$

Note that ϕ decreases in δ and $\phi(1) = 0$ as we desired. We also have some flexibility on the choice of c: ϕ is convex in δ when $c > 1$ and concave when $c < 1$.

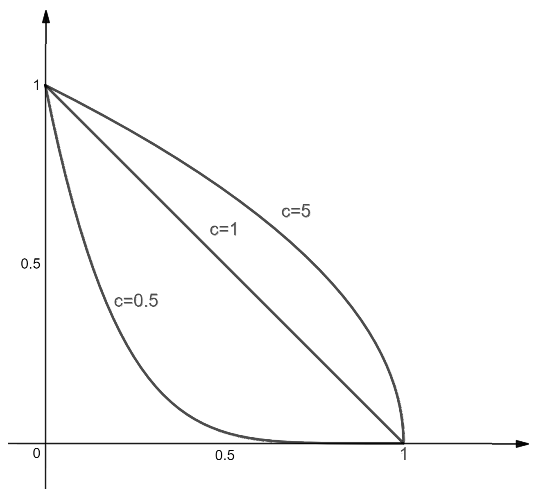


Figure 1: Different ϕ functions with $\eta = 1$ and $c = 0.5, 1, 5$

3.5 Timeline

1. (If applicable) Platforms choose to provide both memberships or only the Premium Membership.

2. Platforms choose subscription fees p_i simultaneously.

3. Audience chooses the best membership (or the outside option) according to their valuations v_i and whether they have access to AATs.

4. Platforms earn the revenue.

4 Equilibrium Analysis

4.1 Consumer's Choice of Membership

For audience without access to AATs, we can equivalently view the industry as a both vertically and horizontally differentiated market. The valuation vector v depicts the horizontal differentiation among different platforms, while the three options correspond to the vertically differentiated "quality-price" packages offered by the platforms: Not Watching has the quality and price both equal to 0; Free Membership has a quality of $1 - k$ and a "price" D not controlled by the platforms; and Premium Membership has a quality of 1 and a price p_i determined by the platforms.

The quoted "price" in the Free Membership scenario represents the two main differences in our model from the classical vertical differentiated multi-product firm model: The "price" D is exogenously given and not controlled by the platforms; and it has nothing to do with the actual payment received by the platform from the Free Members, instead the platforms receive ϕ from the advertiser.

Nevertheless, the demand for each type of membership can be determined as in the classical model. For example, if there is only one monopoly platform in the market (so the audience has only one valuation v), a possible segmentation of the audience is illustrated as follows:

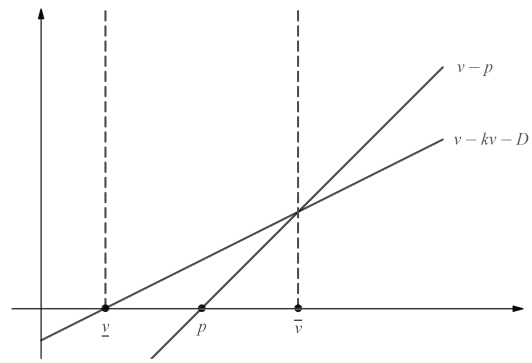


Figure 2: Market Segmentation in Monopoly Case

We can see from the figure that there are two thresholds

$$\underline{v} = \frac{D}{1-k} \tag{2}$$

$$\bar{v} = \frac{p-D}{k} \tag{3}$$

such that for audience not using AATs and with valuation v:

- If $v < \underline{v}$, the audience would prefer Not Watching;
- If $\underline{v} \leq v \leq \bar{v}$, the audience would prefer Free Membership;
- If $v > \bar{v}$, the audience would prefer Premium Membership.

Since $k \in (0,1)$ and $D > 0$, there would always be audience choosing Not Watching; yet to ensure that some audience would choose Free Membership, we need to require

$$\bar{v} > \underline{v} \Leftrightarrow p > \frac{D}{1-k} \tag{4}$$

in the equilibrium. The same requirement goes through to the oligopoly scenario, where we restrict the platforms to price within the interval $[\frac{D}{1-k}, \infty)$ since every option is chosen by at least some audience in reality.

4.2 Both Membership Provided

4.2.1 Equilibrium Pricing

We start with the symmetric scenario where all firms provide both memberships, and we are interested in the symmetric equilibrium where $p_i^* = p^*$ for every platform i.

Proposition 1. *The equilibrium subscription fee p^* is determined by*

$$\frac{1-F^n(\frac{p^*-D}{k})}{n} = \frac{p^*-\phi}{k}F^{n-1}(\frac{p^*-D}{k})f(\frac{p^*-D}{k})+p^*\int_{\frac{p^*-D}{k}}^{\infty}(n-1)F^{n-2}(v_1)[f(v_1)]^2dv_1 \tag{5}$$

Since platforms share the same k, D and p^* in the symmetric equilibrium, the audience only cares about the platform of which she has the highest valuation. Denote

$$u = \max_{i=1}^{n}\{v_i\} \tag{6}$$

then the audience faces a similar choice problem as in the monopoly case, where the only difference is the valuation $u \sim F^n(u)$.

4.2.2 Platform Revenue

We can thus write out the industry total revenue as

$$nR = (1-\delta)[p^*[1-F^n(\frac{p^*-D}{k})] + \phi[F^n(\frac{p^*-D}{k}) - F^n(\frac{D}{1-k})]] \tag{7}$$

and the total revenue for each platform is thus

$$R = \frac{1-\delta}{n}[p^*[1-F^n(\frac{p^*-D}{k})] + \phi[F^n(\frac{p^*-D}{k}) - F^n(\frac{D}{1-k})]] \tag{8}$$

4.2.3 Monopoly as a Special Case

We can look at the monopoly scenario as a special case:

Corollary 1. *Under monopoly, the optimal subscription fee p^* is determined by*

$$I(\frac{p^*-D}{k}) = \frac{\phi-D}{k} \tag{9}$$

where $I(v)$ denotes the "virtual valuation function"

$$I(v) \triangleq v - \frac{1-F(v)}{f(v)} \tag{10}$$

The appearance of the virtual valuation function $I(v)$ is not surprising: when a seller is selling a single good to a buyer with unknown valuation, it's a classic result that the optimal subscription fee should be such that the buyer at the margin has a virtual valuation (i.e., actual valuation less the information rent) equal to 0.

Our model has two features generating the extra term on the RHS. Since the audience has an option of Free Membership, any marginal change in price induces $\frac{1}{k}$ of that change in the valuation of the marginal audience. Besides, the marginal audience is not paying the "price" D as in the option of Free Membership, but instead indirectly "pays" the platform ϕ through the advertiser.

The following comparative statics results can be obtained under monopoly:

Proposition 2. *Under monopoly, suppose that $I'(v) > 0$, then:*

- *R decreases in δ;*
- *p^* decreases in δ;*
- *If $\phi'' > 0$, then R is convex in δ.*

Recall that a sufficient condition for $I(v)$ to increase in v is that the distribution $F(v)$ has monotone increasing hazard rate. Many distributions, including the exponential distribution which we would focus on later, have this property.

These results follow our intuition and daily observations to some extent: When more audience has access to AATs, the platform usually suffers a loss in revenue. Meanwhile, since ϕ decreases in δ, i.e., advertisers are paying less for every audience watching ads, the platform has an incentive to lower the subscription fee to attract more Free Members to subscribe, as long as there aren't many Premium Members, or equivalently a heavy tail in the distribution $F(v)$.

4.2.4 Comparative Statics

Unfortunately, it's a mathematically challenging problem to derive comparative statics results under oligopoly for arbitrary distributions $F(v)$. Our best results focus on the exponential distribution $Exp(\lambda)$.

Lemma 1. *If $v_i \sim Exp(\lambda)$, then $p^* > (<) \frac{1}{\lambda}$ if and only if $\phi > (<) \frac{1-k}{\lambda}$.*

Proposition 3. *If $v_i \sim Exp(\lambda)$, then p^* increases in ϕ and decreases in δ. Further if $\phi > \frac{1-k}{\lambda}$, then p^* decreases in n and R decreases in δ.*

The explanations for that p^* and R would decrease as more audience uses AATs are similar to those in the monopoly case; on the other hand, it's also very intuitive that when the number of platforms increases, the fiercer competition would drive down the equilibrium subscription fee as well.

4.3 Premium Membership Only

4.3.1 Equilibrium Pricing

If all platforms are forced to drop the Free Membership simultaneously, we are interested in the symmetric equilibrium where $p_{ci} = p_c$ for every platform i.

Proposition 4. *The equilibrium subscription fee p_c is determined by*

$$\frac{1 - F^n(p_c)}{n} = p_c F^{n-1}(p_c) f(p_c) + p_c \int_{p_c}^{\infty} (n-1) F^{n-2}(v_1) [f(v_1)]^2 dv_1 \quad (11)$$

4.3.2 Platform Revenue

When we derive the equilibrium subscription fee p_c, the total revenue would be

$$nR_c = p_c[1 - F^n(p_c)] \quad (12)$$

and for each platform

$$R_c = \frac{p_c}{n}[1 - F^n(p_c)] \quad (13)$$

4.3.3 Monopoly as a Special Case

Again we take a look at the special case of monopoly:

Corollary 2. *Under monopoly, the optimal subscription fee p_c is determined by*

$$I(p_c) = 0 \quad (14)$$

Here we obtain the classic result that the virtual valuation at the margin equals 0.

4.3.4 Perloff-Salop Model with Outside Option

When all firms drop their Free Membership, our model essentially reduces to the Perloff-Salop model (as in Perloff and Salop (1985)) with outside options, and therefore we arrive at the same pricing equilibrium as the previous literature. For example, rewriting the condition above into

$$p_c = \frac{[1 - F^n(p_c)]/n}{F(p_c)^{n-1} f(p_c) + \int_{p_c}^{\infty} f(x) dF(x)^{n-1}} \quad (15)$$

we recover Equation (13) in Zhou (2017).

The interpretation of the equilibrium pricing rule (Equation (11)) is as usual matching marginal benefit and marginal cost. If platform i unilaterally raises its subscription fee, the marginal benefit is proportional to the size of its Premium Members, while the marginal cost consists of two parts: some audience leaves the platform and prefers Not Watching (”exclusion effect” as summarized in Zhou (2017)) while some audience joins the other platforms (”competition effect”).

The similarity in the form of Equation (11) and Equation (5) suggests that, we may also relate our results in the previous sub-section where all firms provide both memberships to the same model by a slight modification.

Note first that we restrict the platforms to price above the threshold $\frac{D}{1-k}$. Provided that the equilibrium pricing are interior, the audience at the margin always chooses between Free Membership and Premium Membership, and essentially we are replacing the outside option of 0 in the original model with a new varying outside option of $v_i - kv_i - D$, which in addition provides an advertising revenue of ϕ per audience.

Under the modification, when the platform unilaterally raises its subscription fee, the marginal benefit remains the same, yet the marginal cost will be slightly different: some audience leaves the platform and prefers Free Membership ("exclusion effect"), resulting in a loss of $p^* - \phi$ per audience (and since the marginal audience has a valuation of $\frac{p^*-D}{k}$, the size of audience leaving would be $\frac{1}{k}$ times the price change); while some audience switches to another platforms' Premium Membership ("competition effect"), resulting in a loss of p^* per audience.[6]

4.4 Asymmetric Membership Provision

Finally, we relax the restriction that platforms provides memberships symmetrically, and allow that some platforms provide both memberships while some provide Premium Membership only. We focus on the exponential distribution $Exp(\lambda)$ for tractability in this sub-section.

The following lemma would help us solve the asymmetric pricing problem under exponential distribution:

Lemma 2. *If $v \sim Exp(\lambda)$, for any platform i that provides Premium Membership only, it's a dominant strategy to set $p_{ci} = p_c^* = \frac{1}{\lambda}$ in the pricing game.*

Not surprisingly, Lemma 2 should coincide with Proposition 4 and Corollary 2 if we plug in the expression for exponential distribution $Exp(\lambda)$ into the required conditions.

4.4.1 Duopoly

We first consider the duopoly case and mark the platform which drops the Free Membership with subscript p and the platform which provides both with subscript b.

Proposition 5. *If $v \sim Exp(\lambda)$, the asymmetric equilibrium subscription fees under duopoly are determined by:*

$$p_p = p_c^* = \frac{1}{\lambda} \tag{16}$$

$$\frac{2\lambda(\phi - p_b) + 2k}{2\lambda(\phi - p_b) + k + kp_b\lambda} = e^{-\lambda(\frac{p_b - D}{k} - p_b) - 1} \tag{17}$$

The revenues of each firm are denoted as R_p and R_b respectively. We can see directly from Proposition 5 that if $\frac{D}{1-k} \leq p_b \leq \phi + \frac{\lambda}{k}$ in equilibrium, then

$$p_b > \frac{1}{\lambda} = p_p \tag{18}$$

[6]Some audience may switch to another platforms' Free Membership, but these audience has a mass of 0 at the margin and thus can be neglected.

4.4.2 Triopoly

Since we are going to focus on the Chinese market with three major video streaming platforms in the calibration exercise, we derive the analytical pricing results for the asymmetric triology case as well.

Define p_{21}^* such that

$$\frac{\phi - p_{21}^*}{k} F(\tilde{v}_{21}) F(\tilde{v}_{21}) f(\bar{v}_{21}) + \frac{1}{2}[1 + \kappa_{21} - (2\kappa_{21} + 1)e^{1-\kappa_{21}}]e^{1-\kappa_{21}}[1 - F^2(\tilde{v}_{21})] + \frac{1}{3}(2\kappa_{21} + 1)e^{2-2\kappa_{21}}[1 - F^3(\tilde{v}_{21})] - \frac{1}{2}\kappa_{21}[1 - F^2(\bar{v}_{21})] = 0 \quad (19)$$

where

$$\bar{v}_{21} = \frac{p_{21}^* - D}{k} \quad (20)$$

$$\tilde{v}_{21} = \bar{v}_{21} - p_{21}^* + \frac{1}{\lambda} \quad (21)$$

$$\kappa_{21} = p_{21}^* \lambda \quad (22)$$

and p_{12}^* such that

$$\frac{2\kappa_{12} + 1}{3}[1 - F^3(\tilde{v}_{12})] + \frac{\phi - p_{12}^*}{k} F^2(\tilde{v}_{12}) f(\tilde{v}_{12}) = \kappa_{12}[1 - F^2(\tilde{v}_{12})] \quad (23)$$

where

$$\bar{v}_{12} = \frac{p_{12}^* - D}{k} \quad (24)$$

$$\tilde{v}_{12} = \bar{v}_{12} - p_{12}^* + \frac{1}{\lambda} \quad (25)$$

$$\kappa_{12} = p_{12}^* \lambda \quad (26)$$

Proposition 6. *If $v \sim Exp(\lambda)$, the asymmetric equilibrium subscription fees under triopoly are determined by:*

1. *If two platforms provide both memberships while the other provides Premium Membership only, the latter would price at p_c^*, while the former two price at p_{21}^*.*

2. *If two platforms provide Premium Membership only while the other provides both memberships, the former two would price at p_c^*, while the latter prices at p_{12}^*.*

Denote the revenues in the first case as

$$R_{21p} = (1 - \delta)p_c[\int_{p_c}^{\tilde{v}_{21}} F^2(\frac{v - p_c + D}{1 - k}) f(v)dv + \int_{\tilde{v}_{21}}^{\infty} F^2(v - p_c + p_{21}^*) f(v)dv] \quad (27)$$

$$R_{21b} = (1 - \delta)[\phi \int_{\underline{v}}^{\bar{v}_{21}} F(v - kv - D + p_c) F(v) f(v)dv + p_{21}^* \int_{\bar{v}_{21}}^{\infty} F(v - p_{21}^* + p_c) F(v) f(v)dv] \quad (28)$$

and the revenues in the second case as

$$R_{12p} = (1-\delta)p_c[\int_{p_c}^{\tilde{v}_{12}} F(\frac{v-p_c+D}{1-k})F(v)f(v)dv + \int_{\tilde{v}_{12}}^{\infty} F(v-p_c+p_{12}^*)F(v)f(v)dv] \tag{29}$$

$$R_{12b} = (1-\delta)[\phi\int_{\underline{v}}^{\tilde{v}_{12}} F^2(v-kv-D+p_c)f(v)dv + p_{12}^*\int_{\tilde{v}_{12}}^{\infty} F^2(v-p_{12}^*+p_c)f(v)dv] \tag{30}$$

where

$$\underline{v} = \frac{D}{1-k} \tag{31}$$

4.5 Membership Selection Game

Now that we have solved the pricing equilibrium under both symmetric and asymmetric cases, we are able to consider the membership selection game as a stage preceding the pricing game.

We start with the monopoly case, where the only platform decides whether to provide both memberships or the Premium Membership only (although technically speaking, this is a decision problem instead of a game). As in the previous section, if we focus on the exponential distribution $Exp(\lambda)$, we can actually write out the revenues explicitly as

$$R = (1-\delta)[\frac{k}{\lambda}e^{-1-\lambda\frac{\phi-D}{k}} + \phi e^{-\lambda\frac{D}{1-k}}] \tag{32}$$

and

$$R_c = \frac{1}{\lambda e} \tag{33}$$

We know from Proposition 2 that R decreases in δ, and obviously R_c is a constant independent from δ, and the following corollary follows immediately

Corollary 3. *Under monopoly, if $v \sim Exp(\lambda)$ and*

$$\frac{k}{\lambda}e^{-1-\lambda\frac{\eta-D}{k}} + \eta e^{-\lambda\frac{D}{1-k}} > \frac{1}{\lambda e} \tag{34}$$

then there exists $\delta^* \in (0,1)$ such that the monopoly platform prefers providing both memberships if $\delta < \delta^*$ and Premium Membership only if $\delta > \delta^*$.

The duopoly and triopoly cases are more complicated and can be summarized in the following two tables (due to the ex-ante symmetry of the platforms, we list only the payoffs of the row player)

	Both	P.O.
Both	R	R_b
P.O.	R_p	R_c

Table 1: Payoff Matrix of the Duopoly Game

Both	Both	P.O.		P.O.	Both	P.O.
Both	R	R_{21b}		Both	R_{21b}	R_{12b}
P.O.	R_{21p}	R_{12p}		P.O.	R_{12p}	R_c

Table 2: Payoff Matrix of the Triopoly Game

To determine the possible Nash equilibria in the membership selection game, we are interested in the relationship between each pair of payoffs in the same column. Unfortunately, these revenues and the subscription fees are all too complicated to be determined analytically, and we turn to the calibration exercise for help.

5 Welfare Analysis

5.1 Consumer Surplus and Total Surplus

Consider a benchmark case where the audience has complete access to the videos with no restrictions. Since all the valuations v are realized, the consumer surplus is maximized at the benchmark.

Compared to the benchmark, we have to make the following adjustment to work out the consumer surplus in our model.

- For audience with access to AATs, the valuation is always realized;
- For audience without access to AATs, choosing Not Watching would result in a loss of all the valuation, choosing Free Membership would result in a disutility of $kv + D$, and choosing Premium Membership would result in a payment of p^*.

To sum up, the consumer surplus in our model would thus be:

$$\begin{aligned} CS = &\int_0^{\infty} udF^n(u) \\ &- (1-\delta)[\int_0^{\frac{D}{1-k}} udF^n(u) + \int_{\frac{D}{1-k}}^{\frac{p^*-D}{k}} (ku+D)dF^n(u) + \int_{\frac{p^*-D}{k}}^{\infty} p^* dF^n(u)] \end{aligned} \tag{35}$$

where $u \sim F^n(u)$ is the audience's maximum valuation among platforms.

The total surplus would thus be the sum of consumer welfare and the platforms' total revenue, provided that the advertising fee ϕ is exogenously determined such that the advertiser and producers are competitive and receive no profits.

$$TS = \int_0^{\infty} udF^n(u) - (1-\delta)[\int_0^{\frac{D}{1-k}} udF^n(u) + \int_{\frac{D}{1-k}}^{\frac{p^*-D}{k}} (ku + D - \phi)dF^n(u)] \tag{36}$$

5.2 Short Run

We define the "short run" as the scenario where the platforms passively change their subscription fees to address the change in δ (and thus ϕ).

Proposition 7. *In the short run, consumer surplus CS always increases in δ.*

When δ increases in the short run, there are two direct effects that both promote the consumer surplus: more audience has access to AATs and their valuations are fully realized; meanwhile, the platforms lowers the subscription fee p^*, and overall the audience pays (either the disutility or the subscription fee) less.

In fact, when $\delta \to 1$, all the audience has access to AATs and the maximum consumer surplus in the benchmark case would be realized.

As for the total surplus, since the platforms' total revenue decreases in δ, we are unable to draw the same conclusion analytically.

5.3 Long Run

We define the "long run" as the scenario where, in addition to the pricing decisions, the platforms can further transfer the loss in revenue into the reduction in video resource investment. The actual mechanism for such investment decisions may be complicated, so instead we consider the "equivalent quality shock", i.e., a change in the parameter λ such that it fully counters the impact of the change in δ to the consumer surplus CS and total surplus TS back to the status quo.

Again we are unable to provide analytical expressions; it's even challenging to derive the comparative statics results for the equilibrium subscription fees p^* with respect to λ (except for the monopoly case where p^* obviously decreases in λ). Nevertheless, our intuition suggests that, as λ increases, the valuation distribution would be more left-skewed and the audience is more likely to get lower valuations, which would incur a loss in both consumer surplus and total surplus (and can hardly be compensated by subscription fee changes alone).

6 Calibration and Simulation

To evaluate the impact of AATs, we have to construct counter-factual results based on the model from previous sections. As a starting point, we use actual data from the Chinese video streaming market to calibrate the parameters. Combining these parameters with various penetration rates δ would allow us to simulate the possible outcome of membership percentages, subscription fee and revenues if the altered δ is realized. The calibration and simulation exercise not only checks our theoretical results from the model, but also allows us to have a better understanding of the interests involved in the Chinese lawsuit case mentioned above.

6.1 Parametric Setting

For the purpose of calibration, we need a more specific parametric setting. As is mentioned above, we would use the functional form

$$\phi(\delta) = \eta(1-\delta)^c \tag{37}$$

for the advertising fee, while for the distribution $F(v)$, we would prefer the exponential distribution $Exp(\lambda)$, which has a constant hazard rate of λ. The exponential distribution is also left-skewed, which corresponds to the fact that the density of audience decreases in valuation.

$$F(v) = 1 - e^{-\lambda v} \tag{38}$$

$$f(v) = \lambda e^{-\lambda v} \tag{39}$$

6.2 Industry Data

The following industry data of video streaming platforms in China are used for calibration:

$$n = 3 \tag{40}$$

$$p^* = 15 \tag{41}$$

$$\delta = 13\% \tag{42}$$

$$Q_1 + Q_2 = 75\% \tag{43}$$

$$\frac{Q_1}{Q_1 + Q_2} = 22.5\% \tag{44}$$

$$\frac{R_1}{R_2} = \frac{24.8\%}{48.6\%} \tag{45}$$

where Q_1, Q_2, R_1 and R_2 are the total mass of Premium Members, the total mass of Free Members, subscription revenue and advertising revenue of the industry respectively.

$$Q_1 = (1-\delta)[1 - F^n(\frac{p^* - D}{k})] \tag{46}$$

$$Q_2 = \delta + (1-\delta)[F^n(\frac{p^* - D}{k}) - F^n(\frac{D}{1-k})] \tag{47}$$

$$R_1 = \frac{(1-\delta)p^*}{n}[1 - F^n(\frac{p^* - D}{k})] \tag{48}$$

$$R_2 = \frac{(1-\delta)\phi}{n}[F^n(\frac{p^* - D}{k}) - F^n(\frac{D}{1-k})] \tag{49}$$

Although there are more than three video streaming platforms in China, the three platforms in the top tier (iQiyi, YoukuTudou and Tencent Videos) cover most of the markets while the rest have little say in the industry, and hence we set $n = 3$.

All three platforms have set their monthly subscription fee at 15 Chinese Yuan, and hence our description of consumer values are monthly based as well.

The AATs penetration rate δ can be found in "2017 Global Adblock Report" by PageFair.[7] The rest can be found in "The Report of China's Business Situation in Online Video 2018" by iReserach.[8]

6.3 Parameters Calibration

There are 5 parameters that need to be calibrated: k, D, δ, λ, ϕ (or equivalently η provided the choice of c).

$$k \approx 0.5773 \tag{50}$$

$$D \approx 3.425 \tag{51}$$

$$\delta = 0.13 \tag{52}$$

$$\lambda \approx 0.1331 \tag{53}$$

$$\phi \approx 10.99 \Rightarrow \begin{cases} c = 5, & \eta \approx 22.06 \\ c = 1, & \eta \approx 12.64 \\ c = 0.5, & \eta \approx 11.79 \end{cases} \tag{54}$$

6.4 Impact on Platforms' Performance

Using the calibrated parameters we can now construct counter-factual results with altered δ in the short run. More specifically, the platforms are not allowed to change λ or drop the Free Membership.

We have three choices of parameter c: $c = 5$(convex ϕ), $c = 1$(linear ϕ) and $c = 0.5$(concave ϕ). For each c, we investigate the platforms' performances at 5 different values of δ: 13%(status quo), 8%(status quo −5%), 18%(status quo +5%), 23%(status quo +10%), 0(completely forbidden).

Table 3: Platform Performances Under Altered δ, $c = 5$

δ	p^*	Q_1	Q_2	nR_1	nR_2	nR
0%	25.00	2.06%	69.20%	0.52	7.61	8.12
8%	18.67	7.96%	65.60%	1.49	6.33	7.82
13%	15.00	16.88%	58.13%	2.53	4.96	7.49
18%	12.08	29.12%	47.32%	3.52	3.22	6.74
23%	9.87	41.34%	36.54%	4.08	1.49	5.57

[7] Available at https://pagefair.com/downloads/2017/01/PageFair-2017-Adblock-Report.pdf

[8] Available at http://report.iresearch.cn/report_pdf.aspx?id=3216

Table 4: Platform Performances Under Altered δ, $c = 1$

δ	p^*	Q_1	Q_2	nR_1	nR_2	nR
0%	16.71	13.39%	57.88%	2.24	6.36	8.60
8%	15.66	15.49%	58.08%	2.43	5.50	7.93
13%	15.00	16.88%	58.13%	2.53	4.96	7.49
18%	14.34	18.30%	58.14%	2.62	4.41	7.04
23%	13.68	19.72%	58.15%	2.70	3.86	6.56

Table 5: Platform Performances Under Altered δ, $c = 0.5$

δ	p^*	Q_1	Q_2	nR_1	nR_2	nR
0%	15.83	16.23%	55.03%	2.57	6.05	8.62
8%	15.32	16.64%	56.92%	2.55	5.38	7.93
13%	15.00	16.88%	58.13%	2.53	4.96	7.49
18%	14.67	17.08%	59.36%	2.51	4.55	7.05
23%	14.32	17.25%	60.62%	2.47	4.14	6.61

These results confirm our analytical results from Proposition 3 that p^* and R always decreases in δ. Here are some other findings summarized from the three tables:

- The mass of Premium Members Q_1 always increases in δ;
- The mass of Free Members Q_2 increases in δ for small c, while decreases in δ for large c; yet the mass of Free Members who actually watch the ads $Q_2 - \delta$ always decreases in δ;
- The industry subscription revenue nR_1 increases in δ for large c while decreases in δ for small c;
- The industry advertising revenue nR_2 and total revenue nR always decreases in δ;

For Q_1, as we've discussed in the monopoly case, the platforms have incentive to attract more Free Members to subscribe when δ increases. Note that a larger c makes ϕ drops faster around status quo, and Q_1 fluctuates more violently at the same time.

For Q_2, it's obvious that $Q_2 - \delta$ would always decrease in δ: more Free Members get access to AATs, and some of the rest are attracted by the lower p^* to subscribe for Premium Membership. Similar to Q_1, the fluctuation of $Q_2 - \delta$ is more violent with large c and thus the decreasing trend dominates; with small c, $Q_2 - \delta$ is rather stable and the increasing trend of δ makes Q_2 increase as well.

For nR_2, note that it equals to the product of $Q_2 - \delta$ and ϕ. Since both decrease in δ, nR_2 must be decreasing in δ as well.

Finally for nR, since nR_2 usually accounts for a larger part in revenue than nR_1, the decreasing trend of nR_2 dominates and passes along to nR.

Recall that $\delta = 0$ represents the scenario where AATs are completely forbidden. By trial and error we can determine that the total revenue nR evaluated at $\delta = 0$ has a maximum of approximately 8.62 with a choice of $c \approx 0.375$. In 2017, the total revenue of the video streaming industry was $95.23 billion CNY (from iResearch report). Therefore, we can claim that the existence of AATs result in a maximum of 13.11% loss in revenue to the industry, or equivalently $14.37 billion CNY in 2017.

A by-product of our simulation is to examine the impact of changes in the number of platforms, as shown in the table below.

Table 6: Platform Performances Under Altered n

n	p^*	Q_1	Q_2	nR_1	nR_2	nR	R_1	R_2	R
1	15.33	5.59%	37.00%	0.86	2.64	3.50	0.86	2.64	3.50
2	15.17	11.21%	50.90%	1.70	4.17	5.87	0.85	2.08	2.93
3	15.00	16.88%	58.13%	2.53	4.96	7.49	0.84	1.65	2.50
4	14.81	22.60%	60.90%	3.35	5.27	8.61	0.84	1.32	2.15
5	14.60	28.43%	60.69%	4.15	5.24	9.39	0.83	1.05	1.88
6	14.36	34.39%	58.43%	4.94	4.99	9.93	0.82	0.83	1.66
7	14.08	40.55%	54.71%	5.71	4.59	10.29	0.82	0.66	1.47
8	13.75	47.01%	49.86%	6.46	4.05	10.51	0.81	0.51	1.31
9	13.32	54.00%	43.94%	7.19	3.40	10.59	0.80	0.38	1.18
10	12.72	62.01%	36.63%	7.89	2.60	10.49	0.79	0.26	1.05

As is predicted in Proposition 3, p^* decreases in n. There are two effects playing crucial roles in determining the platforms' other performances as n goes up: On the one hand, since the valuations for videos on different platforms are IID, the distribution of the maximal valuation u skews to the right as n increases. In other words, the audience is more likely to get a higher u. The fact that the probability of getting a high u is always increasing but the probability of getting a medium u would increase first and decrease later, is reflected in the increasing trend of Q_1 and nR_1, and the inverted-U shape of Q_2 and nR_2, which in turn leads to the little drop of nR at $n = 10$. On the other hand, the more platforms there are, the fiercer competition would be. Therefore, it's no wonder that the revenue for each platform R_1, R_2 and R are always decreasing in n.

6.5 Impact on Membership Selection Game

We can now use the simulated parameters to numerically calculate the payoff matrices for the membership selection game to find its Nash equilibrium.

6.5.1 Monopoly

As is suggested in Corollary 3, the revenue from providing both memberships starts higher than that from providing Premium Membership only, and the gap narrows down as δ increases and eventually reversed at $\delta \approx 0.25$. In other words, it's the monopoly platform's best choice to provide both memberships when δ is rather small and drop the Free Membership when δ becomes large enough.

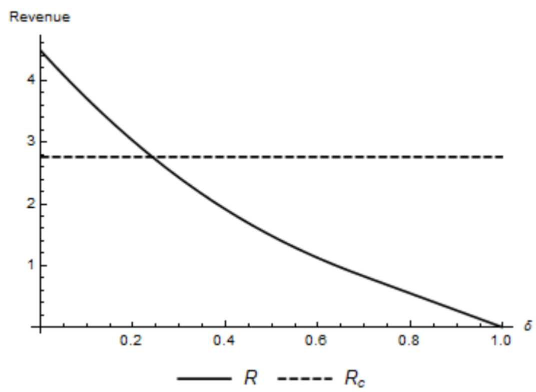


Figure 3: Revenue Comparison for Monopoly

6.5.2 Duopoly

The following diagram shows the different revenues for varying δ.

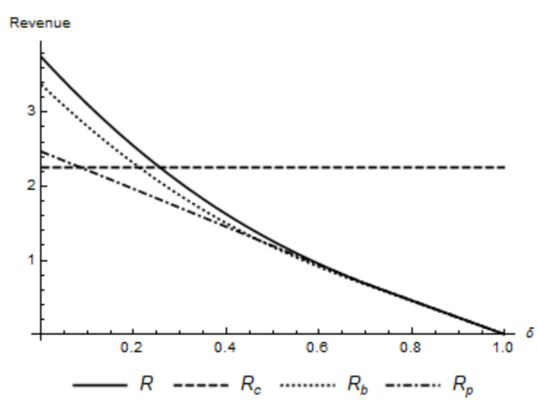
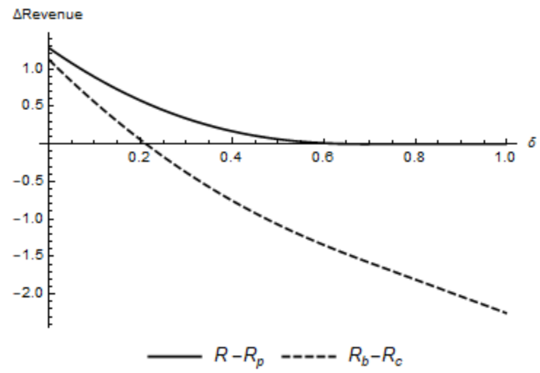
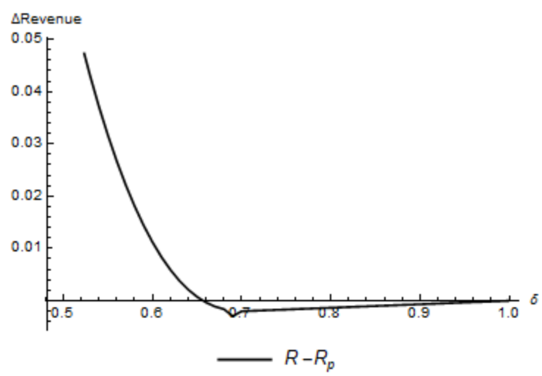


Figure 4: Revenue Comparison for Duopoly

Since the Nash equilibrium of the membership selection game depends only on the relationship between certain pairs of payoffs, we can reproduce the diagram as follows:



Obviously, $R_b - R_c$ starts positive when $\delta = 0$ and turns negative after around $\delta_1 \approx 0.2$. If we take a closer look at $R - R_p$ for $\delta > 0.5$ in the following figure, we notice that at around $\delta_2 \approx 0.66$, the sign is reversed as well.



We can now conclude that there are three possible scenarios for the duopoly membership selection game:

- For $\delta < \delta_1$, both $R > R_p$ and $R_b > R_c$ hold, and it's a dominant strategy equilibrium for both platforms to choose Both.
- For $\delta_1 < \delta < \delta_2$, $R > R_p$ but $R_b < R_c$. The game now has two pure strategy Nash equilibria that the platforms coordinate either on Both or on Premium Only.
- For $\delta > \delta_2$, both $R < R_p$ and $R_b < R_c$ holds, and it's a dominant strategy equilibrium for both platforms to choose Premium Only.

6.5.3 Triopoly

We now turn to the actual Chinese video streaming market case of triopoly. The different revenues are illustrated as follows:

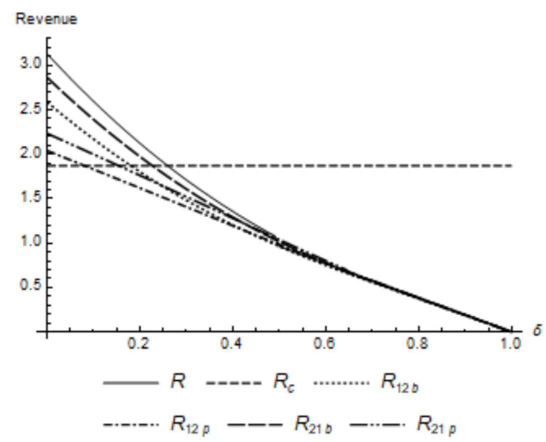
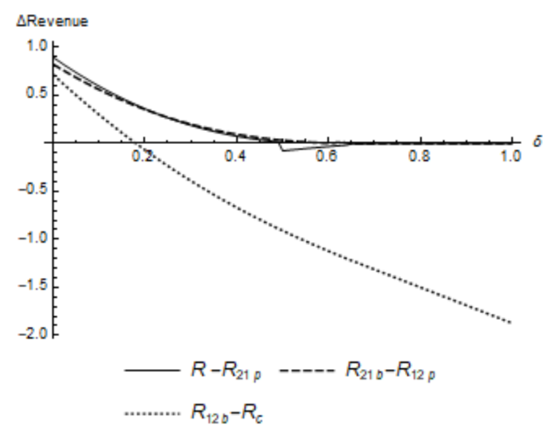
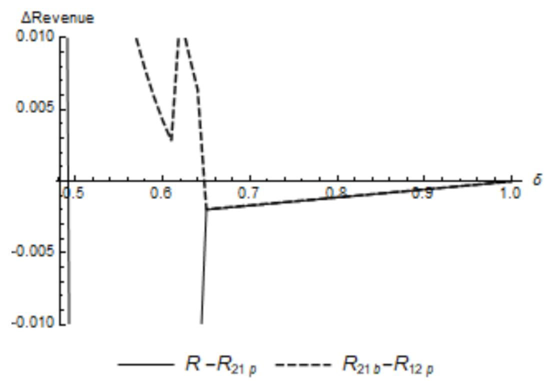


Figure 5: Revenue Comparison for Triopoly

Again we are interested in the relationship between certain pairs:



We see the same trend as in monopoly and duopoly for the curve $R_{12b} - R_c$, reversing the sign at $\delta_1 \approx 0.17$. The more detailed diagram for $\delta > 0.5$ shows that, the reversion for $R - R_{21p}$ takes place at $\delta_2 \approx 0.5$ and the reversion for $R_{21b} - R_{12p}$ at $\delta_3 \approx 0.65$.



To sum up, there are four possible scenarios for the triopoly membership selection game:

- For $\delta < \delta_1$, it's a dominant strategy equilibrium for all three platforms to choose Both.
- For $\delta_1 < \delta < \delta_2$, we have $R > R_{21p}$, $R_{21b} > R_{12p}$, and $R_{12b} < R_c$. The game has two pure strategy Nash equilibria that all platforms coordinate either on Both or on Premium Only.
- For $\delta_2 < \delta < \delta_3$, $R_{21b} > R_{12p}$, $R < R_{21p}$, and $R_{12b} < R_c$. The game has a symmetric pure strategy Nash equilibrium where all platforms coordinate on Premium Only, yet there are three asymmetric pure strategy Nash equilibria where any two platforms choose Both while the other chooses Premium Only.
- For $\delta > \delta_3$, once again it's a dominant strategy equilibrium for all three platforms to choose Premium Only.

6.5.4 Summary

The discussion on membership selection game provides us with three important implications:

- The current penetration rate $\delta = 0.13$ falls in the first scenario of the triopoly case which predicts that all three platforms will provide both memberships, as they do in reality.
- In general, as the penetration rate increases, the platforms are more inclined to achieve equilibria where the Free Membership is dropped.
- Taken into consideration that the market starts at the equilibrium where both memberships are provided by all three platforms, the situation would probably sustain until δ approaches as high as $\delta_2 \approx 0.5$. After that, we shall optimistically expect the equilibrium where only one platform would drop Free Membership. But when δ is so high as it goes beyond $\delta_3 \approx 0.65$, all the platforms would inevitably drop Free Membership.

6.6 Impact on Social Welfare

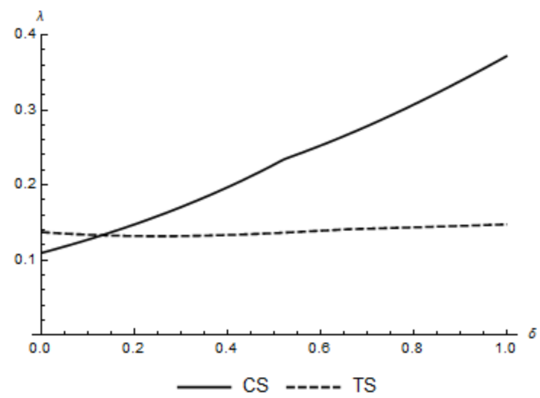


Figure 6: Equivalent Quality Shock for CS and TS

The figure above illustrates, for any given penetration rate δ, the maximum value of λ (i.e., the lowest quality of video resources) to maintain at least the same level of consumer surplus or total surplus respectively.

We've shown analytically that the consumer surplus is always increasing in δ, and our intuition suggests that it should be decreasing in λ, which is confirmed by the upward-sloping curve in the figure. Therefore, as δ increases, it would hurt the consumers only if the platforms have a even stronger incentive to reduce video quality investment and thus increase λ by a lot.

On the other hand, the curve for total surplus is rather flat, meaning that it requires much smaller reduction in video quality investment for the total surplus to drop.

The figure provides us with certain evidences for our argument that, if the long run impact of reduced investment for video quality coming along with the higher AATs penetration rate is incorporated, the society as a whole would suffer a loss in total surplus, even if the magnitude of such impact could be small or the audience could actually be gaining in consumer surplus.

7 Discussion

7.1 Goods Market

In the model above we used a generic parameter D as the fixed disutility of watching ads. To understand it better, we need to think about how ads actually affect the purchasing decisions of the audience.

Suppose that, with probability π, the advertised good is of value α to the audience, and with probability $1 - \pi$ it's of value 0. The good is sold at a competitive price β. The audience does not know the realized value of the good unless they watch the ads, but watching the ads would incur a mental cost of γ.

We further assume that: If the audience has full information, she would purchase the good only if the high value α is realized:

$$\alpha > \beta > 0 \tag{55}$$

Since the audience does not know the realized value ex-ante, she would not purchase the good since the expected value is lower than the price:

$$\pi\alpha < \beta \tag{56}$$

The audience would not watch the ads alone, since the mental cost of watching the ads cannot be covered by the value of information:

$$\pi(\alpha - \beta) < \gamma \tag{57}$$

Let

$$D = \gamma - \pi(\alpha - \beta) > 0 \tag{58}$$

denote the final cost of watching the ads.

To deal with such situations, the advertiser (who is also the producer) can cooperate with the video streaming platforms by bundling the videos with the ads. For those who watch the ads, their viewing experience may be interrupted, resulting in a disutility kv which increases in v. The natural idea is that, if you like the videos more, you are more annoyed to be interrupted. The cost D is also incurred. Nevertheless, the audience is compensated by watching the videos free of charge, which gives them a utility v.

Now, for every audience who watches the ads, they would purchase with probability π and pay β to the advertiser. If the advertiser and the platform can negotiate a certain proportion ψ of the increased revenues as the payment to the platform, we have

$$\phi = \psi\pi\beta \tag{59}$$

All the rest of the revenues would be used to cover the production cost since we assume that the advertiser and producers are competitive.

The discussion above completes the platforms' role in a two-sided market: it cross-subsidized the audience with free videos by selling their eyeballs to the advertiser.

8 Conclusion

This paper looks into the welfare impact of AATs in the market for video streaming. By constructing a game-theoretic model of video platform competition and calibrating the parameters with actual data from the Chinese video streaming market, we are able to simulate the possible outcomes under various penetration rates of AATs. The simulation results suggest that as the penetration rate increases, AATs would indeed do harms to the platforms' revenues. And if the platforms adapt to the changes by reducing video resources investment, the total surplus for the whole society is likely to drop as the video quality gets worse. Moreover, the platforms will be forced to give up the Free Membership should the penetration rate be sufficiently high. We conclude that audience seeking to use AATs would eventually suffer from their short-sighted behavior.

The model and calibration exercise in this paper are very preliminary and there are yet many works to be done. From the modeling perspective, some of the important elements, including the costs of acquiring or investing video contents by the platforms, the decision process of the advertisers, the dependency between video valuations and the access to AATs, and the level of advertising intensity are all assumed away to make our analysis simpler. One would expect the results to change if we put back these elements. Switching to Logit demand may be helpful in both theoretical and empirical work, and more robustness checks with different sources of industry data are needed for the calibration exercise.

A Appendix: Proofs

A.1 Proof of Proposition 1

Without loss of generality, suppose

$$p_2 = p_3 = \cdots = p_n = p^* \tag{60}$$

and platform 1 is now deciding p_1. Let

$$u_{-1} = \max\{v_2, v_3, \cdots, v_n\} \sim F^{n-1}(u_{-1}) \tag{61}$$

The consumer would choose Free Membership from platform 1 if and only if

$$v_1 - kv_1 - D \geq \max\{0, u_{-1} - ku_{-1} - D, u_{-1} - p^*, v_1 - p_1\} \tag{62}$$

and the consumer would choose Premium Membership from platform 1 if and only if

$$v_1 - p_1 \geq \max\{0, u_{-1} - ku_{-1} - D, u_{-1} - p^*, v_1 - kv_1 - D\} \tag{63}$$

Therefore, the total revenue for platform 1 would be: If $p_1 \geq p^*$

$$\frac{R_1}{1-\delta} = \phi[\int_{\frac{D}{1-k}}^{\frac{p^*-D}{k}} F^{n-1}(v_1)f(v_1)dv_1 + \int_{\frac{p^*-D}{k}}^{\frac{p_1-D}{k}} F^{n-1}((1-k)v_1+p^*-D)f(v_1)dv_1]$$
$$+ p_1 \int_{\frac{p_1-D}{k}}^{\infty} F^{n-1}(v_1 - p_1 + p^*)f(v_1)dv_1 \tag{64}$$

If $p_1 \leq p^*$

$$\frac{R_1}{1-\delta} = \phi[\int_{\frac{D}{1-k}}^{\frac{p_1-D}{k}} F^{n-1}(v_1)f(v_1)dv_1 + \int_{\frac{p_1-D}{k}}^{\frac{p^*-D}{k}} F^{n-1}(\frac{v_1-p_1+D}{1-k})f(v_1)dv_1] + p_1\int_{\frac{p^*-D}{k}}^{\infty} F^{n-1}(v_1-p_1+p^*)f(v_1)dv_1 \quad (65)$$

Taking derivatives with respect to p_1, and let $p_1 \to p^*$, we get

$$\frac{\phi-p^*}{k}F^{n-1}(\frac{p^*-D}{k})f(\frac{p^*-D}{k}) + \frac{1}{n}[1-F^n(\frac{p^*-D}{k})] = p^*\int_{\frac{p^*-D}{k}}^{\infty}(n-1)F^{n-2}(v_1)[f(v_1)]^2dv_1 \quad (66)$$

A.2 Proof of Corollary 1

Setting $n = 1$ in Equation (66):

$$\frac{\phi-p^*}{k}f(\frac{p^*-D}{k}) + 1 - F(\frac{p^*-D}{k}) = 0 \quad (67)$$

$$\frac{\phi-D}{k} = \frac{p^*-D}{k} - \frac{1-F(\frac{p^*-D}{k})}{f(\frac{p^*-D}{k})} = I(\frac{p^*-D}{k}) \quad (68)$$

A.3 Proof of Proposition 2

For the optiaml subscription fee p^*:

$$I'(\frac{p^*-D}{k})\frac{1}{k}\frac{\partial p^*}{\partial \delta} = \frac{\phi'(\delta)}{k} \quad (69)$$

$$\frac{\partial p^*}{\partial \delta} = \frac{\phi'(\delta)}{I'(\frac{p^*-D}{k})} < 0 \quad (70)$$

For the maximal total revenue R: Let

$$R_0 = \frac{R}{1-\delta} = \max_p\{p[1-F(\frac{p-D}{k})] + \phi[F(\frac{p-D}{k}) - F(\frac{D}{1-k})]\} \quad (71)$$

Using the envelope theorem

$$\frac{\partial R_0}{\partial \delta} = \phi'(\delta)[F(\frac{p-D}{k}) - F(\frac{D}{1-k})] < 0 \quad (72)$$

$$\frac{\partial^2 R_0}{\partial \delta^2} = \phi''(\delta)[F(\frac{p-D}{k}) - F(\frac{D}{1-k})] + \frac{[\phi'(\delta)]^2 f(\frac{p^*-D}{k})}{kI'(\frac{p^*-D}{k})} > 0 \quad (73)$$

$$\frac{\partial R}{\partial \delta} = -R_0 + (1-\delta)\frac{\partial R_0}{\partial \delta} < 0 \quad (74)$$

$$\frac{\partial^2 R}{\partial \delta^2} = -2\frac{\partial R_0}{\partial \delta} + (1-\delta)\frac{\partial^2 R_0}{\partial \delta^2} > 0 \quad (75)$$

A.4 Proof of Lemma 1

Denote

$$\kappa(v) = -\frac{f'(v)}{f(v)} \tag{76}$$

$$h(v) = \frac{f(v)}{1 - F(v)} \tag{77}$$

Integration by part

$$\begin{aligned}&\frac{\phi - p^*}{k} F^{n-1}(\frac{p^* - D}{k}) f(\frac{p^* - D}{k}) + \frac{1}{n}[1 - F^n(\frac{p^* - D}{k})] \\ &= -p^* F^{n-1}(\frac{p^* - D}{k}) f(\frac{p^* - D}{k}) - p^* \int_{\frac{p^* - D}{k}}^{\infty} f'(v_1) F^{n-1}(v_1) dv_1\end{aligned} \tag{78}$$

Using the Mean Value Theorem for Integrals, for some $\xi \in (\frac{p^* - D}{k}, \infty)$

$$\begin{aligned}&\frac{\phi - p^*}{k} F^{n-1}(\frac{p^* - D}{k}) f(\frac{p^* - D}{k}) + \frac{1}{n}[1 - F^n(\frac{p^* - D}{k})] \\ &= -p^* F^{n-1}(\frac{p^* - D}{k}) f(\frac{p^* - D}{k}) + \frac{p^* \kappa(\xi)}{n}[1 - F^n(\frac{p^* - D}{k})]\end{aligned} \tag{79}$$

$$\frac{\phi - p^*}{k} - \frac{p^* \kappa(\xi) - 1}{n h(\frac{p^* - D}{k})} \sum_{t=0}^{n-1} F^{-t}(\frac{p^* - D}{k}) + p^* = 0 \tag{80}$$

If $F(v) \sim Exp(\lambda)$

$$\kappa(v) = h(v) = \lambda \tag{81}$$

$$\frac{\phi - p^*}{k} - \frac{p^* - \frac{1}{\lambda}}{n} \sum_{t=0}^{n-1} F^{-t}(\frac{p^* - D}{k}) + p^* = 0 \tag{82}$$

Denote

$$G(p^*; n, \lambda, \phi) = \frac{\phi - p^*}{k} - \frac{p^* - \frac{1}{\lambda}}{n} \sum_{t=0}^{n-1} F^{-t}(\frac{p^* - D}{k}) + p^* \tag{83}$$

If $p^* = \frac{1}{\lambda}$, then

$$\frac{\phi - p^*}{k} + p^* = 0 \Leftrightarrow \phi = \frac{1 - k}{\lambda} \tag{84}$$

If $p^* > \frac{1}{\lambda}$, then

$$\frac{\phi - p^*}{k} + p^* > 0 \Rightarrow \phi > (1 - k)p^* > \frac{1 - k}{\lambda} \tag{85}$$

If $p^* < \frac{1}{\lambda}$, then

$$\frac{\phi - p^*}{k} + p^* < 0 \Rightarrow \phi < (1 - k)p^* < \frac{1 - k}{\lambda} \tag{86}$$

A.5 Proof of Proposition 3

Following the proof for Lemma 1, the S.O.C. requires that

$$\frac{\partial G}{\partial p^*} < 0 \tag{87}$$

From the expression of G, we can also work out that

$$\frac{\partial G}{\partial \phi} > 0 \tag{88}$$

Since $\phi > \frac{1-k}{\lambda}$ and thus $p^* > \frac{1}{\lambda}$ in equilibrium, we also have

$$\frac{\partial G}{\partial n} < 0 \tag{89}$$

Hence

$$\frac{\partial p^*}{\partial \phi} = -\frac{\partial G/\partial \phi}{\partial G/\partial p^*} > 0 \tag{90}$$

$$\frac{\partial p^*}{\partial \delta} = \frac{\partial p^*}{\partial \phi}\frac{\partial \phi}{\partial \delta} < 0 \tag{91}$$

$$\frac{\partial p^*}{\partial n} = -\frac{\partial G/\partial n}{\partial G/\partial p^*} < 0 \tag{92}$$

Now we turn to the equilibrium revenue. Consider

$$\begin{aligned}\frac{\partial}{\partial \delta}(nR) = -\frac{nR}{1-\delta} + (1-\delta)[\frac{\partial \phi}{\partial \delta}(F^n(\frac{p^*-D}{k}) - F^n(\frac{D}{1-k}))+ \\ \frac{\partial p^*}{\partial \delta}(1-F^n(\frac{p^*-D}{k})) + \frac{\phi-p^*}{k}nF^{n-1}(\frac{p^*-D}{k})f(\frac{p^*-D}{k})\frac{\partial p^*}{\partial \delta}]\end{aligned} \tag{93}$$

In order for

$$\frac{\partial}{\partial \delta}(nR) < 0 \tag{94}$$

to hold, a sufficient condition would be that

$$1 - F^n(\frac{p^*-D}{k}) + \frac{\phi-p^*}{k}nF^{n-1}(\frac{p^*-D}{k})\lambda(1-F(\frac{p^*-D}{k})) \geq 0 \tag{95}$$

$$\lambda\frac{\phi-p^*}{k} + \frac{1}{n}\sum_{t=0}^{n-1}F^{-t}(\frac{p^*-D}{k}) \geq 0 \tag{96}$$

Notice that Equation (82) tells us:

$$\frac{1}{n}\sum_{t=0}^{n-1}F^{-t}(\frac{p^*-D}{k}) = \frac{\frac{\phi-p^*}{k}+p^*}{p^*-\frac{1}{\lambda}} \tag{97}$$

Therefore the sufficient condition becomes

$$\lambda \frac{\phi - p^*}{k} + \frac{\frac{\phi - p^*}{k} + p^*}{p^* - \frac{1}{\lambda}} \geq 0 \tag{98}$$

Since we have $p^* > \frac{1}{\lambda}$

$$(\lambda p^* - 1)\frac{\phi - p^*}{k} + \frac{\phi - p^*}{k} + p^* \geq 0 \tag{99}$$

$$\lambda \frac{\phi - p^*}{k} + 1 \geq 0 \tag{100}$$

$$p^* \leq \phi + \frac{k}{\lambda} \tag{101}$$

As the final step, recall from Corollary 1 that the optimal monopoly subscription fee with exponential distribution is determined by

$$I(\frac{p^* - D}{k}) = \frac{p^* - D}{k} - \frac{1}{\lambda} = \frac{\phi - D}{k} \tag{102}$$

$$p^* = \phi + \frac{k}{\lambda} \tag{103}$$

Combined with the comparative statics result that

$$\frac{\partial p^*}{\partial n} < 0 \tag{104}$$

we can see that the sufficient condition in Equation (101) always holds, and thus

$$\frac{\partial R}{\partial \delta} = \frac{1}{n} \frac{\partial}{\partial \delta}(nR) < 0 \tag{105}$$

A.6 Proof of Proposition 4

As in the proof of Proposition 1, suppose

$$p_2 = p_3 = \cdots = p_n = p_c \tag{106}$$

and platform 1 is now deciding p_1. Again let

$$u_{-1} = \max\{v_2, v_3, \cdots, v_n\} \sim F^{n-1}(u_{-1}) \tag{107}$$

The consumer would choose Premium Membership from platform 1 if and only if

$$v_1 - p_1 \geq \max\{0, u_{-1} - p_c\} \tag{108}$$

The total revenue for platform 1 would be

$$R_1(p_1) = p_1 \int_{p_1}^{\infty} F^{n-1}(v_1 - p_1 + p_c) f(v_1) dv_1 \tag{109}$$

The derivative with respect to p_1 should equal 0 evaluated at $p_1 = p_c$:

$$\int_{p_c}^{\infty} F^{n-1}(v_1) f(v_1) dv_1 - p_c F^{n-1}(p_c) f(p_c) - p_c \int_{p_c}^{\infty} (n-1) F^{n-2}(v_1) [f(v_1)]^2 dv_1 = 0 \tag{110}$$

A.7 Proof of Corollary 2

Setting $n = 1$ in Equation (110):

$$1 - F(p_c) - p_c f(p_c) = 0 \Leftrightarrow I(p_c) = 0 \tag{111}$$

A.8 Proof of Lemma 2

Without loss of generality, assume platform 1 decides to provide Premium Membership only and is now deciding the subscription fee p_1.

Denote $V(v_2, v_3, \cdots, v_n)$ as the utility from the best options among all other platforms (and the outside option). The audience would choose Premium Membership from platform 1 if and only if

$$v_1 - p_1 \geq V(v_2, v_3, \cdots, v_n) \Leftrightarrow v_1 \geq p_1 + V(v_2, v_3, \cdots, v_n) \tag{112}$$

Thus the total revenue for platform 1 would be

$$R_1 = p_1 \int_0^\infty \int_0^\infty \cdots \int_0^\infty [1 - F(p_1 + V(v_2, v_3, \cdots, v_n))] f(v_2) \cdots f(v_n) dv_1 dv_2 \cdots dv_n \tag{113}$$

$$= p_1 \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-\lambda(p_1 + V(v_2, v_3, \cdots, v_n))} f(v_2) \cdots f(v_n) dv_1 dv_2 \cdots dv_n \tag{114}$$

$$= p_1 e^{-\lambda p_1} \int_0^\infty \int_0^\infty \cdots \int_0^\infty e^{-\lambda V(v_2, v_3, \cdots, v_n)} f(v_2) \cdots f(v_n) dv_1 dv_2 \cdots dv_n \tag{115}$$

Therefore, maximizing R_1 is equivalent to maximizing $p_1 e^{-\lambda p_1}$

$$e^{-\lambda p_1} - p_1 \lambda e^{-\lambda p_1} = 0 \Leftrightarrow p_1 = \frac{1}{\lambda} \tag{116}$$

A.9 Proof of Proposition 5

Lemma 2 ensures that

$$p_p = \frac{1}{\lambda} \tag{117}$$

The audience would choose Free Membership from platform b if and only if both

$$\frac{D}{1-k} < v_b < \frac{p_b - D}{k} \tag{118}$$

$$v_p < v_b - k v_b - D + p_p \tag{119}$$

hold.

The audience would choose Premium Membership from platform b if and only if both

$$v_b > \frac{p_2 - D}{k} \tag{120}$$

$$v_p < v_b - p_b + p_p \tag{121}$$

hold.

Therefore, the total revenue for platform b is

$$\frac{R_b}{1-\delta} = \phi(\int_{\frac{D}{1-k}}^{\frac{p_b-D}{k}} F(v_b - kv_b - D + p_p) f(v_b) dv_b) + p_b(\int_{\frac{p_b-D}{k}}^{\infty} F(v_b - p_b + p_p) f(v_b) dv_b) \tag{122}$$

F.O.C.

$$(\phi - p_b)\frac{1}{k}F(\frac{p_b - D}{k} - p_b + p_p)f(\frac{p_b - D}{k}) + \int_{\frac{p_b-D}{k}}^{\infty} F(v_b - p_b + p_p)f(v_b)dv_b$$
$$- p_b \int_{\frac{p_b-D}{k}}^{\infty} f(v_b - p_b + p_p)f(v_b)dv_b = 0 \tag{123}$$

Let

$$\tau = \frac{p_b - D}{k} - p_b + p_p \tag{124}$$

$$\frac{\phi - p_b}{k}F(\tau)\lambda[1-F(\tau)] + \frac{1}{2}[1-F^2(\tau)] - p_b\lambda[1-F(\tau)] + p_b\lambda\frac{1}{2}[1-F^2(\tau)] = 0 \tag{125}$$

$$\frac{\phi - p_b}{k}F(\tau)\lambda + \frac{1}{2}[1 + F(\tau)] - p_b\lambda + p_b\lambda\frac{1}{2}[1 + F(\tau)] = 0 \tag{126}$$

$$\frac{p_b\lambda - \frac{1}{2} - \frac{p_b\lambda}{2}}{\lambda\frac{\phi - p_b}{k} + \frac{1}{2} + \frac{p_b\lambda}{2}} = F(\tau) = 1 - e^{-\lambda(\frac{p_b - D}{k} - p_b + p_p)} \tag{127}$$

$$\frac{2\lambda(\phi - p_b) + 2k}{2\lambda(\phi - p_b) + k + kp_b\lambda} = e^{-\lambda(\frac{p_b - D}{k} - p_b) - 1} \tag{128}$$

A.10 Proof of Proposition 6

If two platforms provide both memberships while the other provides Premium Membership only, we know from Lemma 2 that the third platform would price at p_c^*. We consider the symmetric equilibrium subscription fee for the former two platforms. Again we assume the second firm is already pricing at the equilibrium p^* and the first firm is deciding its subscription fee p_1.

The audience would choose the Free Membership from the first platform if and only if

$$v_1 - kv_1 - D \geq \max\{v_1 - p_1, v_2 - kv_2 - D, v_2 - p^*, v_3 - p_c^*, 0\} \tag{129}$$

and the Premium Membership from the first platform if and only if

$$v_1 - p_1 \geq \max\{v_1 - kv_1 - D, v_2 - kv_2 - D, v_2 - p^*, v_3 - p_c^*, 0\} \tag{130}$$

As in the proof of Proposition 1, the first platform's total revenue should be continuously differentiable at $p_1 = p^*$. Without loss of generality, assume $p_1 <$

p^*, and the total revenue would be

$$R_1(p_1) = \phi \int_{\frac{D}{1-k}}^{\frac{p_1-D}{k}} F(v_1)F((1-k)v_1 - D + p_c^*)f(v_1)dv_1$$
$$+ p_1[\int_{\frac{p_1-D}{k}}^{p_1+\frac{(1-k)p^*-D}{k}} F(\frac{v_1 - p_1 + D}{1-k})F(v_1 - p_1 + p_c^*)f(v_1)dv_1$$
$$+ \int_{p_1+\frac{(1-k)p^*-D}{k}}^{\infty} F(v_1 - p_1 + p^*)F(v_1 - p_1 + p_c^*)f(v_1)dv_1] \quad (131)$$

Taking derivative with respect to p_1 and consider the left limit as $p_1 \to p_-^*$, we find that the symmetric equilibrium subscription fee for the first two platforms should be p_{21}^* as defined above.

If two platforms provide Premium Membership only while the other provides both memberships, we know from Lemma 2 that the former two would price at p_c^*. If the third platform prices at p_3, the audience would choose the Free Membership from the third platform if and only if

$$v_3 - kv_3 - D \geq \max\{v_3 - p_3, v_1 - p_c^*, v_2 - p_c^*, 0\} \quad (132)$$

or equivalently both

$$\frac{D}{1-k} \leq v_3 \leq \frac{p_3 - D}{k} \quad (133)$$

$$\max\{v_1, v_2\} \leq v_3 - kv_3 - D + p_c^* \quad (134)$$

hold. The audience would choose the Premium Membership from the third platform if and only if

$$v_3 - p_3 \geq \max\{v_3 - kv_3 - D, v_1 - p_c^*, v_2 - p_c^*, 0\} \quad (135)$$

or equivalently both

$$v_3 \geq \frac{p_3 - D}{k} \quad (136)$$

$$\max\{v_1, v_2\} \leq v_3 - p_3 + p_c^* \quad (137)$$

hold. Hence the total revenue for the third platform would be

$$R_3(p_3) = (1-\delta)[\phi \int_{\frac{D}{1-k}}^{\frac{p_3-D}{k}} F^2(v_3 - kv_3 - D + p_c^*)f(v_3)dv_3 +$$
$$p_3 \int_{\frac{p_3-D}{k}}^{\infty} F^2(v_3 - p_3 + p_c^*)f(v_3)dv_3] \quad (138)$$

Taking derivative with respect to p_3, we find that the optimal subscription fee should be p_{12}^* as defined above.

A.11 Proof of Corollary 3

The given condition shows that when $\delta \to 0$, $R > R_c$. On the other hand, it's obvious that when $\delta \to 1$, $R = 0 < R_c$. Since we know from Proposition 2 that R decreases in δ, the intermediate value theorem ensures the existence of such δ^*.

A.12 Proof of Proposition 7

The consumer surplus can be rewritten as

$$CS = \int_0^\infty [u - (1-\delta)\min\{u, ku + D, p^*\}]dF^n(u) \tag{139}$$

Since p^* decreases in δ, the minimum value among u, $ku + D$ and p^* would thus be weakly decreasing in δ, and obviously the term $(1-\delta)$ decreases in δ. Therefore, the integrand increases in δ and so does the integral.

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