Optimal Licensing in Market with Quality Innovation

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Abstract

We study an innovator's optimal licensing strategy of a quality-improving innovation. Before the quality innovation, firms produce homogeneous goods with low quality and compete in quantity. Consumers have unit demand and are heterogeneous in their tastes for quality. The innovator chooses the number of licenses for auction to maximize revenue. Firms who obtain the license upgrade the quality of their products and those who do not obtain the license can continue to produce the low-quality good. We characterize the optimal number of licenses the innovator puts up in the auction and the equilibrium in the product markets. When the low-quality good market allows for free entry, as the quality improvement of the new technology increases, the innovator sells fewer licenses, yielding the licensees greater market power, and the equilibrium exhibit excessive product variety.

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1 Introduction

Quality innovation is the key to success in many industries ranging from software, smartphone, personal computer, and pharmacy. It is common practice that quality innovations are developed by companies who do not manufacture the final products but instead license the technologies to firms in the product markets. For instance, Synaptics is a leading company in human interface. It developed the pattern recognition techniques to build the world's first touchpad which can make laptops thinner and dramatically improves user experience. Synaptics licensed the technology to many leading computer manufacturers of the time, including Apple, Compaq, and Dell; Standford's endowment licensed the PageRank algorithm to Google, and ARM licenses chip technology to some of the largest companies in the world.

The innovator's licensing strategy not only affects its revenue but also determines the product variety and the structure of the final product market. For instance, if the innovator licenses its technology to all the firms in the product market, it upgrades the quality of all the products in the industry, rendering firms competing on the same ground. Alternatively, if the innovator licenses the technology to a small number of firms, the licensees will produce the high-quality good and sell in the high-end market while firms without the license continue to produce the low-quality good and sell in the low-end markets.

This paper studies a revenue-maximizing innovator's optimal licensing strategies and explores its welfare implications. An innovator licenses a quality-improving technology to a product market with homogenous firms. In the pre-innovation market, firms sell the same low-quality good and compete in quantity. Consumers have unit demand and are heterogeneous in their tastes for quality. The innovator first announces the number of licenses it puts up for auction. Then, firms bid for the licenses. Finally, firms compete in the product markets. If a firm obtains the license, it produces the high-quality good and sells in the high-end market. Otherwise, the firm can continue to produce the low-quality good and sells in the low-end market.

The first best depends on the comparison of the quality-cost ratios between the high-quality and the low-quality goods. If the quality-cost ratio of the high-quality good is higher than the low-quality good, the first best prescribes that consumers do not consume any products when their tastes for quality are low and consume the high-quality good when their tastes for quality are high. In other words, the high-quality good should replace the low-quality good. If the quality-cost ratio of the high-quality good is lower than the low-quality good, consumers should not consume any products when their taste parameters are low, should consume the low-quality good when their taste parameters are in an intermediate range, and consume the high-quality good when their taste parameters are high. So, it is efficient to have both types of products produced in the market place.

The innovator maximizes it revenue by choosing the number of licenses to sell in the auction. A firm's willingness to pay for the license is the difference between its profit in the high-end market and its profit if it does not have the license. Two possible cases arise when the firm does not have the license. If the price in the low-end market is at least the marginal cost of the low-quality good, the firm continues to produce the low-quality good. Otherwise, the firm exits the low-end market and makes zero profit. When the innovator sells more licenses, it intensifies the competition in the high-end market and reduces firms' profit from selling the high-quality good. Firms' profits in the low-end market are also affected by the number of licenses. The demand for the low-quality good decreases when the number of licenses increases because the high-quality good is cheaper. However, the competition in the low-end market is softened because fewer firms remain in this market. While the demand effect reduces firms' profits in the low-end market, the competition effect increases their profits, rendering the net effect ambiguous.

We fully characterize the equilibrium when the low-end market allows for free entry and is perfectly competitive. The optimal number of licenses decreases when the quality improvement of the new technology is more significant, which gives firms in the high-end market greater market power. When it is more costly to produce the high-quality good than the low-quality good, three cases emerge in equilibrium. Both types of goods are produced in equilibrium when the low-quality good has a high quality-cost ratio, or when both goods have low quality-cost ratios. The high-quality good replaces the low-quality good and is produced by more than one firm if the low-quality good has a low quality-cost ratio whereas the high-quality good has an intermediate quality-cost ratio. Finally, the high-quality good replaces the low-quality good and is sold by a monopoly if the low-quality good has a low quality-cost ratio while the high-quality good has a high quality-cost ratio.

This equilibrium implies that when the quality-cost ratio of the high-quality good increases, it is harder for the low-end market to survive and the firms in the high-end market have more market power. The equilibrium exhibits excessive product variety when the high-quality good's quality-cost ratio is in an intermediate range. In this parameter range, the first best prescribes that the high-quality good should replace the low-quality good but both products are produced in equilibrium.

Patent licensing is a long-standing topic in the literature of Industrial Organization. The existing literature extensively studies how innovators should license cost-reducing innovations to homogenous consumers. In this context, the innovator's licensing activities have no impact on product variety. This strand of the literature includes Katz and Shapiro (1985, 1986), Kamien and Tauman (1984, 1986) and Kamien et al. (1992). One exception is Stamatopoulos and Tauman (2008) who study quality innovation. Our paper differs from Stamatopoulos and Tauman (2008) in many aspects. First, we consider a general model with more than two firms whereas Stamatopoulos and Tauman (2008) study a duopoly model. Second, we focus on quantity competition in markets with vertically differentiated products whereas Stamatopouos and Tauman (2008) consider price competition in markets with both vertically and horizontally differentiated products.

2 Model

Consider a market with N firms selling homogeneous products with quality v_L at unit cost c_L. Firms compete in quantities. There is a mass one of consumers who have unit demand and are indexed by the taste parameter θ. It is common knowledge that the parameter θ is distributed according to the cumulative distribution function $F(\theta)$ in the range of $[0, \overline{\theta}]$. Consumer-θ's utility from consuming the good at price p is $\theta v_L - p$.

A research lab develops a quality innovation which can improve the quality of the good. If a firm obtains a license from the lab, the quality of its product is increased to v_H, with $v_H > v_L$, and its production cost becomes c_H, with $c_H \geq c_L$. The research lab auctions off the licenses of the quality innovation to maximize its revenue.

The timeline of the game is summarized as follows:

Stage 1 The research lab announces to sell n, $n = 0, 1, 2, ..N$, licenses to all firms by the first price auction.

Stage 2 Firms simultaneously bid for the licenses. The n licenses are won by the firms who submit the highest bids.

Stage 3 Firms compete in quantity simultaneously.

The research lab's strategy is $n \in [0, 1, ..., N]$. Firm i's strategy consists of a bidding function $b_i(n)$ and a quantity function $q_i(n, b)$, where $b = (b_1(n), ..., b_N(n))$. Consumer-$\theta$'s strategy is $a(\theta) \in \{H, L, \emptyset\}$, where H means that the consumer buys the high-quality good, L means that the consumer buys the high-quality good, and $\emptyset$ means that he does not buy any goods.

The equilibrium concept is subgame perfect Nash equilibrium $\{n, (b_i(n), q_i(n, b)), a(\theta)\ \}$ for all i and θ.

Assume $\min\left\{\frac{v_H}{c_H}, \frac{v_L}{c_L}, \frac{\Delta v}{\Delta c}\right\} > 1$. Under this assumption, both types of products should be consumed in the first best.

3 First best

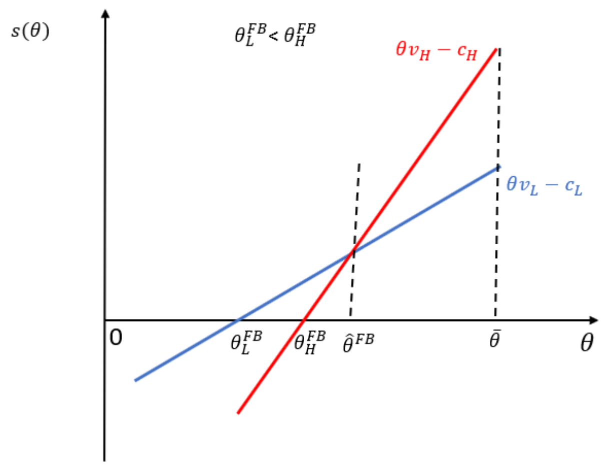
Let $S(\theta)$ denote the social surplus generated by consumer-θ's consumption. The first best assignment prescribes that

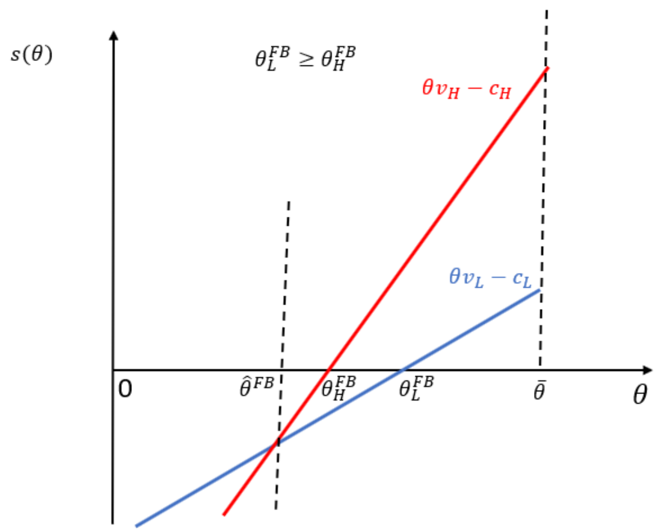
$$S(\theta) = \max\{\theta v_L - c_L, \theta v_H - c_H, 0\}.$$

Denote $\triangle v \equiv v_H - v_L$ and $\Delta c = c_H - c_L$. Define $\theta_L^{FB} \equiv \frac{c_L}{v_L}$, $\theta_H^{FB} \equiv \frac{c_H}{v_H}$, and $\widehat{\theta}^{FB} = \frac{\Delta c}{\Delta v}$. If $\theta \geq \theta_i^{FB}$, $i = H, L$, $\theta v_i - c_i \geq 0$. Moreover, if $\theta \geq \widehat{\theta}^{FB}$, $\theta v_H - c_H \geq \theta v_L - c_L$.

Lemma 1 *If $\theta_L^{FB} < \theta_H^{FB}$, the first best prescribes consumers to exit the markets for $\theta \in [0, \theta_L^{FB})$, buy the low-quality good for $\theta \in [\theta_L^{FB}, \widehat{\theta}^{FB})$ and the high-quality good for $\theta \in [\widehat{\theta}^{FB}, 1]$. If $\theta_L^{FB} \geq \theta_H^{FB}$, the first best prescribes consumers to exit the market for $\theta \in [0, \theta_H^{FB})$ and buy the high-quality good for $\theta \in (\theta_H^{FB}, 1]$.*

The first best can be illustrated in the following figures:





Proof: We first show that $\theta_H^{FB} < \widehat{\theta}^{FB}$ if $\theta_L^{FB} < \theta_H^{FB}$.. Given $\theta_L^{FB} < \theta_H^{FB}$, $v_L c_H > v_H c_L$. Subtract both sides of the inequality by $v_H c_H$:

$$
\begin{aligned}
v_L c_H - v_H c_H &> v_H c_L - v_H c_H \qquad (1)\\
(v_L - v_H) c_H &> v_H (c_L - c_H)\\
\frac{c_H}{v_H} &< \frac{c_H - c_L}{v_H - v_L}.
\end{aligned}
$$

Therefore, $\theta_L^{FB} < \theta_H^{FB} < \widehat{\theta}^{FB}$. So, $S(\theta) = 0$ for $\theta \in [0, \theta_L^{FB})$, $S(\theta) = \theta v_L - c_L$ for $\theta \in [\theta_L^{FB}, \widehat{\theta}^{FB})$, and $S(\theta) = \theta v_H - c_H$ for $\theta \in [\widehat{\theta}^{FB}, 1]$.

It can be verified that $\widehat{\theta}^{FB} \leq \theta_H^{FB}$ if $\theta_L^{FB} \geq \theta_H^{FB}$ by an analysis analogous to (1). So, $\widehat{\theta}^{FB} \leq \theta_H^{FB} \leq \theta_L^{FB}$. This implies $S(\theta) = 0$ for $\theta \in [0, \theta_H^{FB})$ and $S(\theta) = \theta v_H - c_H$ for $\theta \in (\theta_H^{FB}, 1]$. Q.E.D.

3.1 Pre-innovation

We begin by analyzing the market without the quality innovation. Given price p, a consumer θ will buy the low-quality product if and only if

$$\begin{aligned} \theta v_L &\geq p \\ \theta &\geq \theta_L^* \equiv \frac{p}{v_L}, \end{aligned}$$

provided that $p \leq v_L$. So, the demand for the low-quality good is $Q_d(p) = \Pr(\theta \geq \theta_L^*) = 1 - F(\theta_L^*)$.

Now, we describe how the market price depends on the total output Q_L^s. If $Q_L^s < 1$, the price is determined by the market clear condition

$$\begin{aligned} Q_L^s &= 1 - F(\frac{p}{v_L}) \\ p &= v_L F^{-1}(1 - Q_L^s). \end{aligned} \tag{2}$$

If $Q_L^s > 1$, supply exceeds the maximum demand and hence $p = 0$.

We focus on the equilibrium in which the market is cleared at the price determined by (2). For a given price p, $Q_L^s(p) \equiv \sum_{i=1}^{N} q_L^i$ Firm i solves the following profit maximization problem:

$$\underset{q_L^i}{Max} \left(v_L F^{-1}(1 - \sum_{i=1}^{N} q_L^i) - c_L \right) q_L^i.$$

Take the first order condition:

$$v_L F^{-1}(1 - \sum_{i=1}^{N} q_L^i) - c_L - \frac{v_L q_L^i}{f(\theta_L^*)} = 0 \Leftrightarrow \tag{3}$$

$$v_L \theta_L^* - c_L - \frac{v_L q_L^i}{f(\theta_L^*)} = 0 \tag{4}$$

The second order condition is satisfied when

$$-\frac{v_L}{f(\theta_L^*)} + v_L \left(1 + \frac{q_L^i f'(\theta_L^*)}{(f(\theta_L^*))^2}\right) \frac{\partial \theta_L^*}{\partial q_L^i} \leq 0, \tag{5}$$

which is satisfied if $\frac{f'(\theta)}{(f(\theta))^2} \geq -1$.

Adding up the first order conditions for all firms, we obtain

$$N(v_L\theta_L^* - c_L) = \frac{v_L \sum_{i=1}^{N} q_L^i}{f(\theta_L^*)} = \frac{v_L\,(1 - F(\theta_L^*))}{f(\theta_L^*)}, \tag{6}$$

which determines the marginal consumer and can be rewritten as the modified Lerner index

$$\frac{p - c_L}{p} = -\frac{1}{N\varepsilon_d}. \tag{7}$$

Example: **uniform distribution**. Consider the case where θ is uniformly distributed in $[0, 1]$. Substitute $f(\theta) = 1$ and $F(\theta) = \theta$ into (6), we have

$$\theta_L^* = \frac{Nc_L + v_L}{(N+1)v_L} \Rightarrow \tag{8}$$

$$p_L^* = v_L\widehat{\theta} = \frac{Nc_L + v_L}{N+1} \tag{9}$$

$$Q_L^* = 1 - F(\widehat{\theta}) = \frac{N}{N+1}\left(1 - \frac{c_L}{v_L}\right) \tag{10}$$

$$q_L^* = \frac{Q_L^S}{N} = \frac{1}{N+1}\left(1 - \frac{c_L}{v_L}\right) \tag{11}$$

$$\pi_L^* = (p_L^S - c_L)q_L^S = \left(\frac{1}{N+1}\right)^2 \frac{(v_L - c_L)^2}{v_L}. \tag{12}$$

The industry profit is

$$\Pi_L = N\pi_L^* = \frac{N}{(N+1)^2}\frac{(v_L - c_L)^2}{v_L}$$

Note that as $N \to \infty$, $\theta_L^* \to \frac{c_L}{v_L}$, $p_L^* \to c_L$, $Q_L^* \to \frac{v_L - c_L}{v_L}$, $q_L^* \to 0, \pi_L^* \to 0$. So, as $N \to \infty$, the market converges to the competitive equilibrium.

4 Equilibrium

Now, we derive the subgame perfect Nash equilibrium of the game. We proceed with backward induction and begin the analysis from the third stage. Suppose that n firms obtain the license and produce the high-quality good and $N - n$ firms do not have the license and produce the low-quality good. Let p_i, $i = H, L$, denote the price of the i-quality good. We focus on the price range $p_H \leq v_H$ and $p_L \leq v_L$. A consumer-θ's payoff from purchasing good i, $i = L, H$, is $u(i, \theta) = \theta v_i - p_i$. So, the consumer's maximum utility is

$\max\{\theta v_L - p_L, \theta v_H - p_H, 0\}$. Define $\theta_L \equiv \frac{p_L}{v_L}$, $\theta_H \equiv \frac{p_H}{v_H}$ and $\widehat{\theta} \equiv \frac{p_H - p_L}{\triangle v}$. It is easy to verify that $u(L,\theta) \geq 0$ if and only if $\theta \geq \theta_L$, $u(H,\theta) \geq 0$ if and only if $\theta \geq \theta_H$, and $u(H,\theta) \geq u(L,\theta)$ if and only if $\theta \geq \widehat{\theta}$.

When $\frac{p_L v_H}{v_L} < p_H < p_L + \triangle v$, $\theta_L < \theta_H < \widehat{\theta} < 1$. So, consumer-$\theta$ will not buy any goods if $\theta \in [0, \theta_L)$, buy the low-quality good if $\theta \in [\theta_L, \widehat{\theta})$, and buy the high-quality good if $\theta \in [\widehat{\theta}, 1]$. When $p_L + \triangle v \leq p_H$, $\theta_L < \theta_H < 1 < \widehat{\theta}$. In this case, the consumer will not buy any goods if $\theta \in [0, \theta_L)$ and will buy the low-quality good if $\theta \in [\theta_L, 1]$. Finally, when $p_H \leq \frac{p_L v_H}{v_L}$, $\widehat{\theta} \leq \theta_H \leq \theta_L$. So, the consumer will not buy any goods if $\theta \in [0, \theta_H)$ and will buy the high-quality good if $\theta \in [\theta_H, 1]$. Based on this analysis, we can derive the demand for each good for any price configurations:

We summarize the demand for each good in all price configurations in the following lemma:

Lemma 2 *The demand for the high quality good is*

$$Q_H^d(p_H, p_L) = \begin{cases} 1 - F(\theta_H) & p_H \leq \frac{p_L v_H}{v_L} \\ 1 - F(\widehat{\theta}) & \frac{p_L v_H}{v_L} < p_H < p_L + \triangle v \\ 0 & p_H \geq p_L + \triangle v \end{cases}. \tag{13}$$

The demand for the low quality good is

$$Q_L^d(p_H, p_L) = \begin{cases} 0 & p_H \leq \frac{p_L v_H}{v_L} \\ F(\widehat{\theta}) - F(\theta_L) & \frac{p_L v_H}{v_L} < p_H < p_L + \triangle v \\ 1 - F(\theta_L) & p_H \geq p_L + \triangle v \end{cases}. \tag{14}$$

Next, we describe how the market price p_H and p_L are determined by the total supply Q_H^s and Q_L^s. If $Q_i^s > 0$, $i = L, H$, and $Q_H^s + Q_L^s < 1$, p_H and p_L are determined by the market clear conditions:

$$Q_H^d(p_H, p_L) = Q_H^s,\ Q_L^d(p_H, p_L) = Q_L^s. \tag{15}$$

If both markets exist and the total supply is less than the maximum demand, prices will adjust to clear each market. If only one product is produced, i.e. $Q_i^s = 0$, $i = L, H$, and $Q_j^s > 0$, $j \neq i$, then the benchmark case applies. If $Q_H^s + Q_L^s \geq 1$ and $Q_H^s < 1$, $p_L = 0$ and p_H is determined by $Q_H^s = 1 - F\left(\frac{p_H}{\triangle v}\right)$. If $Q_H^s + Q_L^s \geq 1$ and $Q_H^s > 1$, $p_L = p_H = 0$. The last two cases specify that when the total supply exceeds the total demand, the high-quality market will clear first.

We begin the analysis when both products are produced and sold; that is, $p_i > c_i$, $i = H, L$. Firm i in the high-quality market solves the following problem:

$$\max_{q_H^i} \left(p_H(Q_H^s, Q_L^s) - c_H\right) q_H^i$$

The F.O.C. is

$$p_H(Q_H^s, Q_L^s) - c_H + \frac{\partial p_H(Q_H^s, Q_L^s)}{\partial Q_H^s} q_H^i = 0. \tag{16}$$

Adding up the first order conditions for n firms in the high-quality market, we obtain

$$n\left(p_H(Q_H^s, Q_L^s) - c_H\right) + \frac{\partial p_H(Q_H^s, Q_L^s)}{\partial Q_H^s} Q_H^s = 0 \tag{17}$$

Similarly, Firm j in the low-quality market solves:

$$\max_{q_L^j} \left(p_L(Q_H^s, Q_L^s) - c_L\right) q_L^j$$

and the F.O.C is

$$p_L(Q_H^s, Q_L^s) - c_L + \frac{\partial p_L(Q_H^s, Q_L^s)}{\partial Q_L^s} q_L^j = 0 \tag{18}$$

Adding up the first order conditions for $N - n$ firms in the low-quality market, we have

$$(N-n)\left(p_L(Q_H^s, Q_L^s) - c_L\right) + \frac{\partial p_L(Q_H^s, Q_L^s)}{\partial Q_L^s} Q_L^s = 0. \tag{19}$$

The equilibrium Q_H^s, p_H, Q_L^s and p_L are jointly determined by (17), (19), and the market clear conditions

$$\begin{aligned} Q_H^d(p_H, p_L) &= Q_H^s \\ 1 - F(\widehat{\theta}) &= Q_H^s \\ 1 - F\left(\frac{p_H - p_L}{\triangle v}\right) &= Q_H^s, \end{aligned} \tag{20}$$

and

$$\begin{aligned} Q_L^d(p_H, p_L) &= Q_L^s \\ F(\widehat{\theta}) - F(\theta_L) &= Q_L^s \\ F\left(\frac{p_H - p_L}{\triangle v}\right) - F\left(\frac{p_L}{v_L}\right) &= Q_L^s, \end{aligned} \tag{21}$$

where $Q_H^d(p_H, p_L)$ and $Q_L^d(p_H, p_L)$ are the demand in the high and the low markets, respectively, in the price range $\frac{p_L v_H}{v_L} < p_H < p_L + \triangle v$ in Lemma 2.

Example: uniform distribution We use the uniform distribution example to solve for the model analytically. The demand for the high-quality goods is

$$Q_H^d(p_H, p_L) = \begin{cases} 1 - \dfrac{p_H}{v_H} & p_H \leq \frac{p_L v_H}{v_L} \\ 1 - \dfrac{p_H - p_L}{\triangle v} & \frac{p_L v_H}{v_L} < p_H < p_L + \triangle v \\ 0 & p_H \geq p_L + \triangle v \end{cases}, \tag{22}$$

and the demand for the low-quality good is

$$Q_L^d(p_H, p_L) = \begin{cases} 0 & p_H \leq \frac{p_L v_H}{v_L} \\ \frac{p_H - p_L}{\triangle v} - \frac{p_L}{v_L} & \frac{p_L v_H}{v_L} < p_H < p_L + \triangle v \\ 1 - \frac{p_L}{v_L} & p_H \geq p_L + \triangle v \end{cases}. \tag{23}$$

When both markets exist, using (22) and (23), we can express the inverse demand function in each market

$$\begin{aligned} p_L(Q_H^s, Q_L^s) &= v_L\left(1 - Q_H^s - Q_L^s\right), & (24) \\ p_H(Q_H^s, Q_L^s) &= p_L + \triangle v\left(1 - Q_H^s\right) & (25) \\ &= v_L\left(1 - Q_H^s - Q_L^s\right) + \triangle v\left(1 - Q_H^s\right) \\ &= \left(1 - Q_H^s\right) v_H - Q_L^s v_L, \\ p_H(Q_H^s, Q_L^s) - p_L(Q_H^s, Q_L^s) &= \triangle v\left(1 - Q_H^s\right) > 0, & (26) \end{aligned}$$

which holds when $\frac{p_L v_H}{v_L} \leq p_H \leq p_L + \triangle v$.

Proposition 1 *i) If* $\frac{v_L}{c_L} \leq \frac{2v_H}{v_H + c_H}$*, the high-quality good is produced but the low quality good is not produced for a given* $n \in [1, N]$*. The equilibrium in the high-quality good market is* $p_H^e = \frac{nc_H + v_H}{n+1}$*,*$Q_H^e = \frac{n}{n+1}\left(1 - \frac{c_H}{v_H}\right)$*,* $q_H^e = \frac{1}{n+1}\left(1 - \frac{c_H}{v_H}\right)$*, and* $\pi_H^e = \left(\frac{1}{n+1}\right)^2 \frac{(v_H - c_H)^2}{v_H}$*.*

ii) If $\frac{2v_H}{v_H + c_H} < \dfrac{v_L}{c_L} < \dfrac{v_H}{c_H}$*, there exists* $n^* \equiv \dfrac{v_H(v_L - c_L)}{v_H c_L - v_L c_H}$ *such that both types of goods are produced when* $n < n^*$ *and only the high-quality good is produced when* $n \in [n^*, N]$*. When both types of goods are produced,* *the equilibrium is*

$$
\begin{aligned}
p_H^e(n) &= \frac{v_H\left[(N-n+1)(v_H+nc_H)-(N-n)(v_L-c_L)\right]-v_Lc_Hn(N-n)}{v_H(n+1)(N-n+1)-v_Ln(N-n)} &(27)\\
p_L^e(n) &= \frac{v_H\left[v_L+c_L(n+1)(N-n)\right]+v_Ln\left[c_H-c_L(N-n)\right]}{v_H(n+1)\left(N-n+1\right)-v_Ln(N-n)} &(28)\\
q_H^e(n) &= \frac{(N-n+1)(v_H-c_H)-(N-n)(v_L-c_L)}{v_H(n+1)(N-n+1)-v_L(N-n)n} &(29)\\
q_L^e(n) &= \frac{v_H(v_L-c_L)-n(v_Hc_L-v_Lc_H)}{v_L\left[v_H(n+1)(N-n+1)-v_L(N-n)n\right]} &(30)\\
\pi_H^e(n) &= v_H(q_H^e(n))^2 &(31)\\
\pi_L^e(n) &= v_L(q_L^e(n))^2 &(32)\\
Q_L^e(n) &= \frac{(N-n)\left[v_H(v_L-c_L)-n(v_Hc_L-v_Lc_H)\right]}{v_L\left[v_H(n+1)(N-n+1)-v_Ln(N-n)\right]} &(33)\\
Q_H^e(n) &= \frac{n\left[(v_H-c_H)(N-n+1)-(v_L-c_L)(N-n)\right]}{v_H(n+1)(N-n+1)-v_L(N-n)n}. &(34)
\end{aligned}
$$

When only the high-quality good is produced, the equilibrium is characterized in i).

iii) If $\frac{v_H}{c_H} \leq \frac{v_L}{c_L}$*, both types of goods are produced for* $n \in [1, N]$*. The equilibrium is characterized by (27)-(34)*

Proof: Substitute (25) and $\frac{\partial p_H(Q_H^s,Q_L^s)}{\partial Q_H} = -v_H$ into (17), we obtain

$$Q_H^s = \frac{n}{n+1}\left[1-\frac{c_H}{v_H}-\frac{v_L}{v_H}Q_L^s\right] \tag{35}$$

Substitute (24) and $\frac{\partial p_L(Q_H^s,Q_L^s)}{\partial Q_L} = -v_L$ into (19), we obtain

$$Q_H^s = (1-\frac{c_L}{v_L}) - \frac{N-n+1}{N-n}Q_L^s. \tag{36}$$

We can solve the equilibrium outputs Q_H and Q_L from (35) and (36). It follows that

$$Q_L(n) = \frac{(N-n)\left[v_H(v_L-c_L)-n(v_Hc_L-v_Lc_H)\right]}{v_L\left[v_H(n+1)(N-n+1)-v_Ln(N-n)\right]} \tag{37}$$

The equilibrium aggregate output in the market of the low-quality good is $Q_L^e(n) = \max\{0, Q_L(n)\}$. Note that the denominator of $Q_L(n)$ is positive, so the sign of $Q_L(n)$ is determined by its numerator. If $\frac{v_H}{c_H} \leq \frac{v_L}{c_L}$ (or equivalently $\theta_H^{FB} \geq \theta_L^{FB}$), $Q_L(n) \geq 0$ for all n. If $\frac{v_H}{c_H} > \frac{v_L}{c_L}$ (or equivalently $\theta_H^{FB} < \theta_L^{FB}$), the numerator of (37) is decreasing in n and reaches zero at $n^* \equiv \frac{v_H(v_L-c_L)}{v_Hc_L-v_Lc_H}$. So, $Q_L(n) > 0$ for $n < n^*$ and $Q_L(n) \leq 0$ for $n \geq n^*$. It can be verified that n^* increases in $\frac{v_L}{c_L}$, and decreases in $\frac{v_H}{c_H}$. In what follows, we summarize the market equilibrium for a given $n \leq N$.

- If $\frac{v_H}{c_H} \leq \frac{v_L}{c_L}$(or equivalently $\theta_H^{FB} \geq \theta_L^{FB}$), $Q_L^e(n) = Q_L(n) > 0$ and is determined by (37), whereas

$$Q_H^e(n) = \frac{n\left[(v_H - c_H)(N-n+1) - (v_L - c_L)(N-n)\right]}{v_H(n+1)(N-n+1) - v_L(N-n)n} > 0.$$

  Note that the assumption $\frac{\Delta v}{\Delta c} > 1$ implies $Q_H^e(n) > 0$. The equilibrium is (27) to (34).

- $\frac{v_L}{c_L} < \frac{v_H}{c_H}$ (or equivalently $\theta_H^{FB} < \theta_L^{FB}$).

  - For $n < n^*$, both markets exist and the equilibrium is (27)-(34).

  - For $n \geq n^*$, the low-quality firms are driven out of the market and firms with the license operate in the high-quality good market. The equilibrium is the baseline model after replacing c_H for c_L, v_H for v_L, and n for N. We obtain

$$p_H^e = \frac{nc_H + v_H}{n+1} \tag{38}$$

$$Q_H^e = \frac{n}{n+1}\left(1 - \frac{c_H}{v_H}\right) \tag{39}$$

$$q_H^e = \frac{1}{n+1}\left(1 - \frac{c_H}{v_H}\right) \tag{40}$$

$$\pi_H^e = \left(\frac{1}{n+1}\right)^2 \frac{(v_H - c_H)^2}{v_H} \tag{41}$$

$$\pi_L^e = 0 \tag{42}$$

$$p_L^e = 0 \tag{43}$$

$$Q_L^e = 0 \tag{44}$$

$$q_L^e = 0 \tag{45}$$

Next, we consider a special case in which one license is enough to eliminate the low-quality good market. This is called drastic innovation. For the innovation to be drastic, it is necessary to have $\frac{v_H}{c_H} > \frac{v_L}{c_L}$ and $n^* \leq 1$. The condition $n^* \leq 1$ requires

$$\begin{aligned} \frac{v_H(v_L - c_L)}{v_H c_L - v_L c_H} &\leq 1 \\ \frac{v_L}{c_L} &\leq \frac{2v_H}{v_H + c_H}. \end{aligned}$$

So, the innovation is drastic if and only if

$$\frac{v_L}{c_L} \leq \min\{\frac{2v_H}{v_H + c_H}, \frac{v_H}{c_H}\} = \frac{2v_H}{v_H + c_H}$$

where the equality holds under the assumption $v_H > c_H$. Q.E.D.

Next, we solve Stage 1 of the game. The research lab chooses the number of licenses to auction off to maximize the revenue. So, it solves the following problem:

$$\max_n n(\pi_H^e(n) - \pi_L^e(n))$$

subject to $\pi_H^e(n)$ and $\pi_L^e(n)$ are determined in Stage 2 of the game. The innovator's optimal number of licenses for auction is summarized in the following proposition:

Proposition 2 *i) If $\frac{v_L}{c_L} \leq \frac{2v_H}{v_H+c_H}$, the optimal number of license is $n^e = 1$; the firm who obtains the license becomes the monopoly in the high-quality good market and all other firms exit the low-quality good market.*

ii) If $\frac{2v_H}{v_H+c_H} < \frac{v_L}{c_L} < \frac{v_H}{c_H}$, the optimal number of licence is $n^e \in [1, n^*]$. Moreover, n^e satisfies

$$\begin{aligned}
v_H(q_H^e(n^e))^2 - v_L(q_L^e(n^e))^2 + 2n^e\left[v_H.q_H^e(n^e).\frac{\partial q_H^e(n^e)}{\partial n} - v_L.q_L^e(n^e).\frac{\partial q_L^e(n^e)}{\partial n}\right] &= 0, \textit{ if } n^e \in (1, n^*) \\
v_H(q_H^e(1))^2 - v_L(q_L^e(1))^2 + 2\left[v_H.q_H^e(1).\frac{\partial q_H^e(1)}{\partial n} - v_L.q_L^e(1).\frac{\partial q_L^e(1)}{\partial n}\right] &\leq 0, \textit{ if } n^e = 1 \\
q_H^e(n^*) + 2n^*\frac{\partial q_H^e(n^*)}{\partial n} &\geq 0, \textit{ if } n^e = n^*.
\end{aligned}$$

Both type of products are produced when $n^e \in [1, n^*)$, and only the high-quality good is produced when $n^e = n^*$.

iii) If $\frac{v_H}{c_H} \leq \frac{v_L}{c_L}$, the optimal number of license is $n^e \in [1, N]$ and both types of products are produced. The solution n^e satisfies

$$\begin{aligned}
v_H(q_H^e(n^e))^2 - v_L(q_L^e(n^e))^2 + 2n^e\left[v_H.q_H^e(n^e).\frac{\partial q_H^e(n^e)}{\partial n} - v_L.q_L^e(n^e).\frac{\partial q_L^e(n^e)}{\partial n}\right] &= 0, \textit{ if } n^e \in (1, N) \\
v_H(q_H^e(1))^2 - v_L(q_L^e(1))^2 + 2\left[v_H.q_H^e(1).\frac{\partial q_H^e(1)}{\partial n} - v_L.q_L^e(1).\frac{\partial q_L^e(1)}{\partial n}\right] &\leq 0, \textit{ if } n^e = 1 \\
q_H^e(N) + 2N\frac{\partial q_H^e(N)}{\partial n} &\geq 0, \textit{ if } n^e = N
\end{aligned}$$

Proof: **Case 1:** $\frac{v_L}{c_L} \leq \frac{2v_H}{v_H+c_H}$. In this parameter range, the innovation is drastic. So, the research lab's revenue from selling $n \in [1, N]$ licenses is

$$\begin{aligned}
R(n) &= n\pi_H^e(n) \\
&= \frac{n}{(n+1)^2}\frac{(v_H - c_H)^2}{v_H}.
\end{aligned}$$

Take the derivative

$$\frac{dR(n)}{dn} = \frac{1-n^2}{(n+1)^4}\frac{(v_H-c_H)^2}{v_H} < 0.$$

So, the research lab will sell $n^e = 1$ license and the licensee becomes the monopoly. Note that the research lab fully extracts the monopoly profit. The output is inefficiently low compared with the first best.

Case 2: $\frac{2v_H}{v_H+c_H} < \frac{v_L}{c_L} < \frac{v_H}{c_H}$. In this parameter range, $1 < n^*$. The analysis in case 1 shows that $n^e \le n^*$. So, the optimal number of license is $n^e \in [1, n^*]$ and solves

$$\max_n n(v_H(q_H^e)^2 - v_L(q_L^e)^2).$$

The first order condition is

$$\begin{aligned}
v_H(q_H^e(n^e))^2 - v_L(q_L^e(n^e))^2 + 2n^e\left[v_H.q_H^e(n^e).\frac{\partial q_H^e(n^e)}{\partial n} - v_L.q_L^e(n^e).\frac{\partial q_L^e(n^e)}{\partial n}\right] &= 0, \text{ if } n^e \in (1, n^*)\\
v_H(q_H^e(1))^2 - v_L(q_L^e(1))^2 + 2\left[v_H.q_H^e(1).\frac{\partial q_H^e(1)}{\partial n} - v_L.q_L^e(1).\frac{\partial q_L^e(1)}{\partial n}\right] &\le 0, \text{ if } n^e = 1\\
q_H^e(n^*) + 2n^*\frac{\partial q_H^e(n^*)}{\partial n} &\ge 0, \text{ if } n^e = n^*.
\end{aligned}$$

The first equality is the interior solution whereas the last two inequalities are the corner solutions. Note that in the last inequality, we have substituted $q_L^e(n^*) = 0$ which is the definition of n^*.

Case 3: $\frac{v_H}{c_H} \le \frac{v_L}{c_L}$ In this case, the low-quality good producers cannot be driven out of the market for any $n < N$. Following the a similar argument in Case 2, the solution n^e solves

$$\begin{aligned}
v_H(q_H^e(n^e))^2 - v_L(q_L^e(n^e))^2 + 2n^e\left[v_H.q_H^e(n^e).\frac{\partial q_H^e(n^e)}{\partial n} - v_L.q_L^e(n^e).\frac{\partial q_L^e(n^e)}{\partial n}\right] &= 0, \text{ if } n^e \in (1, N)\\
v_H(q_H^e(1))^2 - v_L(q_L^e(1))^2 + 2\left[v_H.q_H^e(1).\frac{\partial q_H^e(1)}{\partial n} - v_L.q_L^e(1).\frac{\partial q_L^e(1)}{\partial n}\right] &\le 0, \text{ if } n^e = 1\\
q_H^e(N) + 2N\frac{\partial q_H^e(N)}{\partial n} &\ge 0, \text{ if } n^e = N.
\end{aligned}$$

Q.E.D.

5 Free entry in the low-quality good market

In this section, we consider that the low-quality good market allows for free entry. So, $N \to \infty$ and firms producing the low-quality good make zero profit at $p_L = c_L$. Hence, a firm's willingness to pay for the innovation is its profit in the high-quality good market.

Suppose that both goods are sold in equilibrium.Taking the limit $N \to \infty$ in (27) to (34), the equilibrium is summarized as follows:

$$
\begin{aligned}
p_H^e &= \frac{v_H\left[(v_H + nc_H) - (v_L - c_L)\right] - v_L c_H n}{v_H(n+1) - v_L n} && (46)\\
q_H^e &= \frac{(v_H - c_H) - (v_L - c_L)}{(v_H - v_L)n + v_H} && (47)\\
Q_H^e &= \frac{n\left[(v_H - c_H) - (v_L - c_L)\right]}{v_H(n+1) - v_L n} && (48)\\
\pi_H^e &= v_H (q_H^e)^2 && (49)\\
p_L^e &= c_L && (50)\\
q_L^e &= 0 && (51)\\
\pi_L^e &= 0 && (52)\\
Q_L^e &= \frac{v_H(v_L - c_L) - n(v_H c_L - v_L c_H)}{v_L\left[v_H(n+1) - v_L n\right]}. && (53)
\end{aligned}
$$

Note that $Q_L^e > 0$ for any n if $\frac{v_H}{c_H} \leq \frac{v_L}{c_L}$. In this case, firms producing the low quality good will never be driven out of market. A firm is willing to pay up to its profit in the high quality market.

$$\max_n \; R(n) = n v_H (q_H^e)^2 = \frac{An}{((v_H - v_L)n + v_H)^2} \tag{54}$$

, where $A = v_H \times (v_H - c_H - v_L + c_L)^2$.

Proposition 3 *Suppose $c_H > c_L$. The optimal number of license is*

$$
n^e \begin{cases}
\widehat{n} = \frac{v_H}{v_H - v_L} & \text{if } 2 \leq \frac{v_L}{c_L} \text{ or } 1 < \frac{v_L}{c_L} < 2 \text{ and } 1 < \frac{v_H}{c_H} < \frac{v_L}{2c_L - v_L}\left(1 - \frac{v_L - c_L}{c_H}\right) \\
n^* & \text{if } 1 < \frac{v_L}{c_L} < 2 \text{ and } \frac{v_L}{2c_L - v_L}\left(1 - \frac{v_L - c_L}{c_H}\right) \leq \frac{v_H}{c_H} \leq \frac{v_L}{2c_L - v_L} \\
1 & \text{if } 1 < \frac{v_L}{c_L} < 2 \text{ and } \frac{v_L}{2c_L - v_L} < \frac{v_H}{c_H}
\end{cases}
$$

Proof: First, consider $\frac{v_H}{c_H} \leq \frac{v_L}{c_L}$. In this parameter range, $Q_L^e > 0$ for any n, and hence both types of products are produced. So, the optimal number of license is the solution to the program (54). The first order condition yields

$$\begin{aligned} R'(n) &= 0 \Leftrightarrow \\ \frac{A\left[v_H - n(v_H - v_L)\right]}{((v_H - v_L)n + v_H)^3} &= 0 \\ n &= \frac{v_H}{v_H - v_L}. \end{aligned}$$

Since $R'(n) > 0$ for $n < \frac{v_H}{v_H - v_L}$ and $R'(n) < 0$ for $n > \frac{v_H}{v_H - v_L}$, $R(n)$ is maximized by $\widehat{n} \equiv \frac{v_H}{v_H - v_L}$, which is decreasing in v_H/v_L and converges to 1 as $v_H/v_L \to \infty$.

Next, consider $\frac{v_H}{c_H} > \frac{v_L}{c_L}$. In this case, $Q_L^e = 0$ if and only if $n \geq n^*$. The low-quality good firms are driven out of the market if $n \geq n^* = \dfrac{v_H(v_L - c_L)}{v_H c_L - v_L c_H} > 0$ and both types of goods are produced in equilibrium otherwise. We divide the analysis in 2 cases.

- **Case 1:** $n^* < 1$. In this case, a high-quality good monopoly is sufficient to drive other firms out of the low-quality good market. So, the innovator's revenue from selling n licenses is

$$R(n) = n\pi_H^e = n\left(\frac{1}{n+1}\right)^2 \frac{(v_H - c_H)^2}{v_H},$$

where the second equality follows (41). Since $R'(n) < 0$, the optimal number of license is $n^e = 1$. The condition for Case 1 can be written as

$$\frac{v_H}{c_H}(2 - \frac{v_L}{c_L}) > \frac{v_L}{c_L}. \tag{55}$$

Condition (55) is satisfied if and only if $\frac{v_L}{c_L} < 2$ and $\frac{v_H}{c_H} > \frac{v_L}{2c_L - v_L}$. Note that $\frac{v_L}{2c_L - v_L} > \frac{v_L}{c_L}$.

- **Case 2:** $n^* > 1$, which is satisfied if $\frac{v_L}{c_L} \geq 2$ or $\frac{v_L}{c_L} < 2$ and $\frac{v_L}{c_L} < \frac{v_H}{c_H} \leq \frac{v_L}{2c_L - v_L}$. Following the argument in Case 1, the innovator's revenue decreases in n for $n > n^*$, so the optimal number of license is at most n^*. The optimal number of license is the interior solution $\widehat{n}$ if $\widehat{n} < n^*$ and is the corner solution n^* if $\widehat{n} \geq n^*$. The condition

$$\begin{aligned} \widehat{n} &< n^* \Leftrightarrow \\ \frac{v_H}{v_H - v_L} &< \frac{v_H(v_L - c_L)}{v_H c_L - v_L c_H} \Leftrightarrow \\ v_H - v_L &> \frac{v_H c_L - v_L c_H}{v_L - c_L} \Leftrightarrow \\ v_H(1 - \frac{c_L}{v_L - c_L}) &> v_L(1 - \frac{c_H}{v_L - c_L}). \end{aligned} \tag{56}$$

Because $1 - \frac{c_L}{v_L - c_L} > 1 - \frac{c_H}{v_L - c_L}$, (56) is satisfied when

$$\begin{aligned} 1 - \frac{c_L}{v_L - c_L} &\geq 0 \Leftrightarrow \\ \frac{v_L}{c_L} &\geq 2 \end{aligned}$$

or when $\frac{v_L}{c_L} < 2$ and

$$\begin{aligned} v_H &< \frac{v_L(1 - \frac{c_H}{v_L - c_L})}{(1 - \frac{c_L}{v_L - c_L})} \\ \frac{v_H}{c_H} &< \frac{v_L}{2c_L - v_L}\left(1 - \frac{v_L - c_L}{c_H}\right). \end{aligned}$$

For Case 2 to be valid, the condition $\frac{v_H}{c_H} > \frac{v_L}{c_L}$ must hold. For the ease of analysis, let $\beta_L \equiv \dfrac{v_L}{c_L}$, $\beta_H \equiv \dfrac{v_H}{c_H}$, $f(\beta_L) \equiv \frac{\beta_L}{2-\beta_L} = \frac{v_L}{2c_L - v_L}$, and $g(\beta_L) \equiv \frac{\beta_L}{2-\beta_L}\left(1 - \frac{\beta_L - 1}{c_H/c_L}\right) = \frac{v_L}{2c_L - v_L}\left(1 - \frac{v_L - c_L}{c_H}\right)$. It follows that

$$\begin{aligned} g(1) &= 1,\ g(2) = \infty \\ g'(\beta_L) &= \frac{c_L(\beta_L)^2 - 4c_L\beta_L + 2(c_H + c_L)}{(2 - \beta_L)^2 c_H} \\ g''(\beta_L) &= \frac{4(c_H - c_L)}{(2 - \beta_L)^3} \\ g'(1) &= 2 - \frac{c_L}{c_H},\ g'(2) = \infty \\ f(1) &= 1, f(2) = \infty \\ f'(\beta_L) &= \frac{2}{(2 - \beta_L)^2} > 0, f''(\beta_L) > 0 \\ f'(1) &= 2,\ f'(2) = \infty \\ f(\beta_L) &> g(\beta_L) \end{aligned}$$

The sign of $g'(\beta_L)$ is determined by its numerator, which is quadratic in β_L and reaches the minimum at $\beta_L = 2$. It can be verified that $g'(2) = \frac{2(c_H - c_L)}{(2-\beta_L)^2 c_H} > 0$ if and only if $c_H > c_L$. So, $g'(\beta_L) > 0$ if $c_H > c_L$. Moreover, $g''(\beta_L) > 0$ and $g'(1) > 1$ if $c_H > c_L$. Hence, $g(\beta_L) > \beta_L$. Because $g(\beta_L) = f(\beta_L)\left(1 - \frac{\beta_L - 1}{c_H/c_L}\right)$, $g(\beta_L) < f(\beta_L)$. In summary, $n^e = 1$ if $1 < \beta_L < 2$ and $\beta_H > f(\beta_L)$; $n^e = \widehat{n}$ if $\beta_L \geq 2$ or $1 < \beta_L < 2$ and $\beta_L < \beta_H < g(\beta_L)$; $\widehat{n} = n^*$ if $1 < \beta_L < 2$ and $g(\beta_L) \leq \beta_H < f(\beta_L)$.

The equilibrium number of licenses is summarized in Table 1:

	$2 \leq \beta_L$	$1 < \beta_L < 2$
$1 < \beta_H \leq \beta_L$	$\widehat{n}$	$\widehat{n}$
$\beta_L < \beta_H < g(\beta_L)$	$\widehat{n}$	$\widehat{n}$
$g(\beta_L) \leq \beta_H \leq f(\beta_L)$	$\widehat{n}$	n^*
$f(\beta_L) < \beta_H$	$\widehat{n}$	1

Table 1

Finally, we need to verify (46)-(53) is indeed the equilibrium. Recall (22) and (23), both markets exist if and only if $\frac{c_L v_H}{v_L} < p_H < c_L + \triangle v$.

Taking the derivative of p_H^e defined in (46) with respect to n, we obtain

$$\frac{\partial p_H^e}{\partial n} = \frac{v_H \triangle v(\triangle c - \triangle v)}{(n \triangle v + v_H)^2} < 0,$$

where the inequality follows from the assumption $\triangle v > \triangle c$. Hence, p_H^e is maximized at $n = 1$ and minimized at $n = n^*$. Therefore, $\frac{c_L v_H}{v_L} < p_H < c_L + \triangle v$ is satisfied when

$$\frac{c_L v_H}{v_L} \leq p_H^e(n^*) \text{ and } p_H^e(1) < c_L + \triangle v$$

$$\begin{aligned} p_H^e(1) &< c_L + \triangle v \Leftrightarrow \\ \frac{v_H\left[(v_H + c_H) - (v_L - c_L)\right] - v_L c_H}{2v_H - v_L} &< c_L + \triangle v \Leftrightarrow \\ \Delta c &< \triangle v, \end{aligned}$$

which is satisfied by assumption.

$$\begin{aligned} \frac{c_L v_H}{v_L} &\leq p_H^e(n^*) \\ \frac{c_L v_H}{v_L} &\leq \frac{(\triangle v + c_L)(v_H c_L - v_L c_H) + \triangle v(v_L - c_L)c_H}{v_H c_L - v_L c_H + \triangle v(v_L - c_L)} \\ v_L c_L &\leq v_L c_L. \end{aligned}$$

Q.E.D.

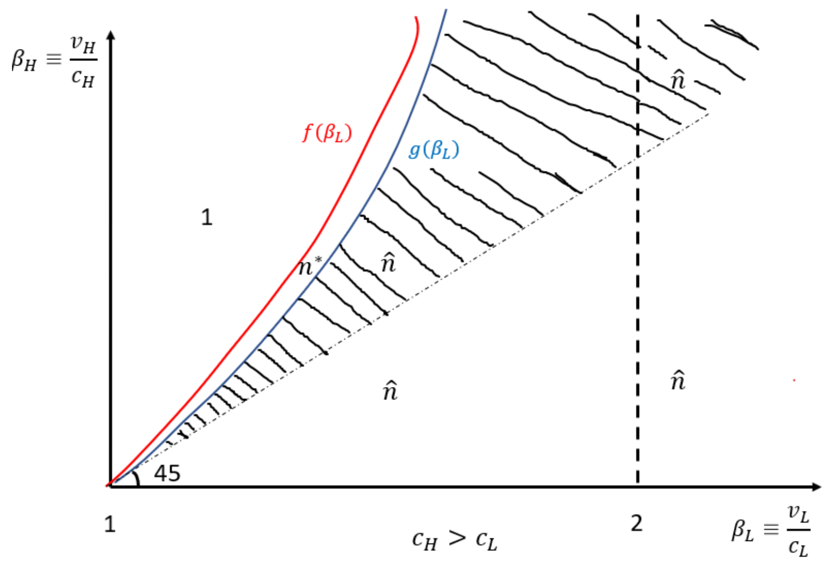


Figure 3

Proposition 3 is illustrated in Figure 3. In the figure, $\beta_H \equiv \frac{v_H}{c_H}$, $\beta_L \equiv \frac{v_L}{c_L}$, $f(\beta_L) \equiv \frac{\beta_L}{2-\beta_L} = \frac{v_L}{2c_L - v_L}$, and $g(\beta_L) \equiv \frac{\beta_L}{2-\beta_L}\left(1 - \frac{\beta_L - 1}{c_H/c_L}\right) = \frac{v_L}{2c_L - v_L}\left(1 - \frac{v_L - c_L}{c_H}\right)$. Because the low-quality good market allows for free entry, the low-quality good is priced at the marginal cost c_L when it is produced and hence low-quality good firms make zero profit. Consequently, firms will bid their profit in the high-quality good market in the auction. When the innovator auctions off more licenses, it intensifies the competition in the high-quality good market, dilutes firm profit from selling the high-quality good, and reduces firms' willingness to pay for the license. The optimal number of license balances the innovator's marginal gain and inframarginal loss. Three cases emerges in equilibrium. When the optimal number of license is $\widehat{n}$, both types of goods are produced. When the optimal number of license is n^* or 1, only the high-quality good is produced in equilibrium. In the former case, more than one firm sell the high-quality good and in the latter, there is a monopoly in the high-quality good market.

Figure 3 shows that both types of good are produced either when the low-quality good is very cost efficient ($\frac{v_L}{c_L} > 2$) or when the high quality good has a small quality improvement upon the low-quality good ($\beta_H < g(\beta_L)$). When the low-quality good is not cost efficient ($\frac{v_L}{c_L} < 2$) and the quality improvement is moderate ($g(\beta_L) < \beta_H < f(\beta_L)$), the innovator sells n^* licenses, which is just enough to drive those firms without the license out of the low-quality good market. Finally, if the low-quality good is not cost efficient ($\frac{v_L}{c_L} < 2$) and the quality improvement is significant ($f(\beta_L) < \beta_H$), the innovator sells only one license. The licensee has the monopoly power and all other firms are driven out of the low-quality good market.

We can use Figure 3 to compare the equilibrium product variety with the first best for all parameter configurations. According to the first best (Lemma 1), only the high-quality good should be produced when $\beta_H > \beta_L$ and both types of goods should be produced when $\beta_H < \beta_L$. In Figure 3, the equilibrium product variety coincides with the first best when the quality improvement is low ($\beta_H < \beta_L$) or when it is high ($\beta_H > g(\beta_L)$). When the quality innovation is moderate (the shaded area), there is excessive product variety because both types of good are produced whereas the first best stipulates that the high-quality good should replace the low-quality good. Moreover, when the low-quality good is more cost efficient, the equilibrium exhibits excessive product variety in a wider range of quality improvement.

Proposition 4 *Suppose $c_H \leq c_L$. The optimal number of license is*

$$n^e = \begin{cases} \widehat{n} = \frac{v_H}{v_H - v_L} & \text{if } 2 \leq \frac{v_L}{c_L} \\ n^* & \text{if } 1 < \frac{v_L}{c_L} < 2 \text{ and } \frac{v_L}{c_L} \leq \frac{v_H}{c_H} \leq \frac{v_L}{2c_L - v_L} \\ 1 & \text{if } 1 < \frac{v_L}{c_L} < 2 \text{ and } \frac{v_L}{2c_L - v_L} < \frac{v_H}{c_H} \end{cases}$$

Proof: Given $c_H \leq c_L$, $\beta_H > \beta_L$ because $v_H > v_L$. The analysis is analogous to the proof of Proposition 3. The optimal number of license is $n^e = 1$ if $1 < \beta_L < 2$ and $f(\beta_L) < \beta_H$. Take the difference

$$\begin{aligned} \beta_L - g(\beta_L) &= \beta_L - \frac{\beta_L}{2 - \beta_L}\left(1 - \frac{\beta_L - 1}{c_H / c_L}\right) \\ &= \frac{\beta_L(\beta_L - 1)(c_L - c_H)}{(2 - \beta_L)c_H}. \end{aligned}$$

So, $\beta_L \geq g(\beta_L)$ for $\beta_L < 2$. Apply the same argument in the proof of Proposition 3, $n^e = \widehat{n}$ if $\beta_L \geq 2$ and $n^e = n^*$ if $1 < \beta_L < 2$ and $\max\{g(\beta_L), \beta_L\} = \beta_L \leq \beta_H < f(\beta_L)$. The solution is summarized in the following table:

	$2 \leq \beta_L$	$1 < \beta_L < 2$
$\beta_L < \beta_H \leq f(\beta_L)$	$\widehat{n}$	n^*
$f(\beta_L) < \beta_H$	$\widehat{n}$	1

Table 2

Q.E.D.

Proposition 4 is illustrated in Figure 2.

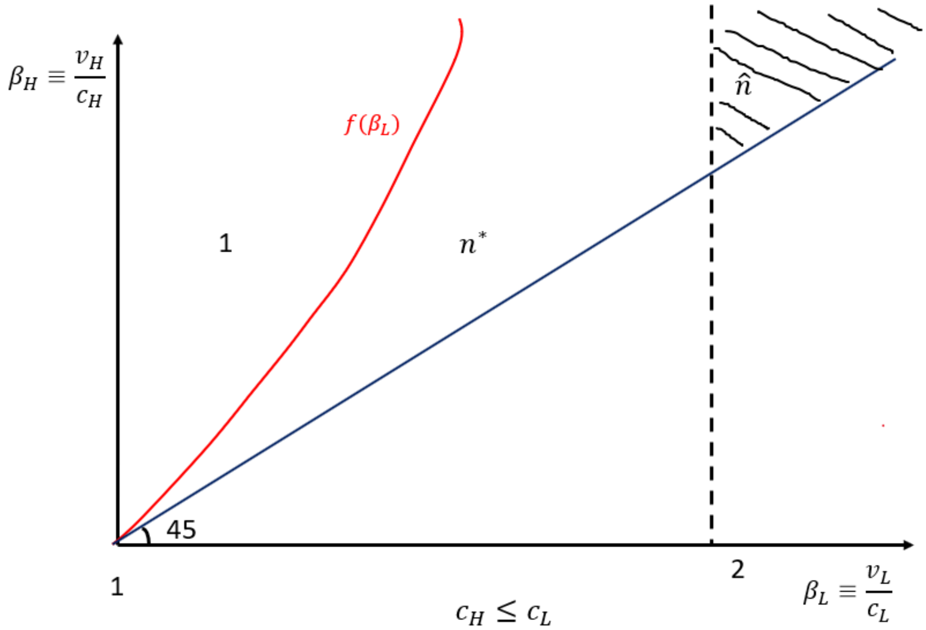


Figure 2

6 Conclusion

We study an innovator's optimal licensing strategy of a quality improving innovation to a market with homogeneous firms. Before the innovation, firms produce a low quality good and compete in quantity. Firms who obtain the license upgrade the quality of their products and those without the license can continue to produce the low-quality good. Consumers are heterogeneous in their tastes in quality and self select into a high-end and a low-end market when both goods are produced. The innovator chooses the number of license for auction to maximize its revenue. Its optimal licensing strategy determines product variety as well as the structure of the final good markets. We fully characterize the equilibrium when the low-quality good market allows for free entry and evaluate the welfare implication of the quality improving innovation. In our future work, we will compare auction with royalties in terms of the innovator's revenue and the social welfare.

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