# Homophily in Social Media and News Polarization*

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**Abstract**

There is large evidence about the tendency of people to interact more with similar others than with disparate others. This is also well documented in the context of news sharing on online social networks. In this paper, we study how the choice of ideological bias of news media is affected by the degree of homophily of readers. For this purpose, in the baseline model, we consider a media firm which has to choose the ideological bias of its news article to maximize its demand in an environment where, after reading the news, direct consumers of the news have an opportunity to share it with their followers on a online social network. We characterize the conditions under which the homophily in a social network induces the firm to provide biased content.

**Keywords**: Social network, homophily, media slant.

**JEL Codes**:

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# 1 Introduction

Social media has become a very important source for news consumption. According to Shearer and Gottfried (2017), as of August 2017, two-thirds (67%) of Americans report consuming at least some of their news on social media. Facebook remains the most relevant social media site, being a news source for 45% of Americans.

This success of social media, however, is viewed under suspicion as some fear that the tendency that users of social networks end up consuming only the kinds of content they like will prevent them from digesting diverse viewpoints about socially important issues. Although people express such concerns by pointing out problems such as filter bubbles (Pariser, 2011) or echo chambers (Sunstein, 2017), one might think that *homophily* — the tendency of individuals to interact with people like them — is a more fundamental source of problem. In fact, homophily is widely observed in the diverse domains of society, involving race, gender, age, interests, social positions, and convictions (McPherson, Smith-Lovin and Cook, 2001). As a consequence of the homophily in social interactions, anything people experience gets reinforced, affecting among other things the information they receive on social networks; precisely, homophily substantially limits exposure to cross-cutting news in social networks. For instance, by analyzing data from 10.1 million U.S. Facebook users and 7 million distinct URLs shared by them between July 7, 2014 and January 7, 2015, Bakshy, Messing and Adamic (2015) found that homophily is the most important factor limiting their exposure to attitude-challenging content. Similarly, Halberstam and Knight (2016) analyzed information from 2.2 million Twitter users the day before the 2012 U.S. general elections and found that, due to homophily, people are disproportionately exposed to tweets from like-minded others.[1]

This paper addresses the following questions. If homophily induces like-minded people to disproportionately share like-minded content on social networks, how does this affect the incentive for a media firm to choose the media content, in particular in terms of ideological bias? Does homophily induce it to provide biased news? If yes, under which circumstances, does it?

Our questions are partly motivated by anecdotal evidence that this overexposure of people to like-minded content on social networks may affect the content and bias of the news

[1]After classifying users either as "conservative" or "liberal" they found that 91% of retweets of tweets by Democratic candidates were transmitted by liberal voters, and almost 99% of retweets of tweets by Republican candidates were transmitted by conservative voters.

disseminated on social networks. For instance, during the US 2016 presidential election, over a hundred of websites with made-up content were created by teenagers from a small town in Macedonia, seeking for advertising revenues propelled by the large sharing of its news on Facebook (Silverman and Alexander, 2016). As Craig Silverman, one of the reporters that revealed the story, explained in an interview to NPR, the teenagers were using Facebook to drive the traffic to the website where they had ads from Google. Their sites were producing misleading content directed to the extreme of partisanship and getting more engagement than op-eds and commentary pieces from major media (NPR, 2016).

To address our questions, in the baseline model, we develop a variation of a Hotelling model where a media firm chooses the ideological location of its news content in order to maximize its advertising revenue, which is equivalent to maximizing the demand. The novelty is that every direct reader of the news, who is distributed over the Hotelling line, may share it through social media with similar others (her followers) if she finds the news relevant enough to her followers. We adopt a parameter of maximum dissimilarity between a user and her followers as an inverse measure of the homophily in the social network. Every direct reader is assumed to have the same total number of followers, who are uniformly distributed over the interval defined by the inverse measure of homophily. This implies that a direct reader located near the center (a moderate) has as many followers more left-aligned than her as those more right-aligned than her, while a direct reader located near the extremes (an extremist) has, naturally, mostly followers who are more moderate than herself. This asymmetry in the composition of followers plays an important role in the answers to our questions. However, a priori, it is not obvious why more homophily would induce the media firm to choose more extreme content instead of a moderate one as a moderate direct reader is willing to share a moderate news with her moderate followers like an extreme right reader is willing to share an extreme right news with her extreme right friends.

Another important parameter in our model is the level of attention tax that a direct reader imposes on her followers by sharing a news. We assume that a direct reader shares the news only if she reads it *and* she finds that the benefit to her followers is larger than the attention tax. The attention tax captures opportunity cost of attention as sharing a news with a friends induces the latter to pay attention to the news, which does not necessarily mean that the friend reads it. In a world of information overload, the attention tax can be high that a direct reader wants to share a news only if it is relevant enough to her followers.

We first study the *breadth*-maximizing strategy in the baseline model. By the breadth, we mean the interval of the direct readers who share the news. We find that if the social network has a high degree of homophily and the attention tax is not small, the media firm maximizes the breadth by producing biased content. More precisely, the firm uses a *limit polarization policy* such that the bias in the news is just enough to induce the news sharing from the most extreme consumer. This is because the fact that the profiles of followers of extremists are relatively more homogeneous than those of moderates makes it easier for the firm to induce sharing from a group of extremists than from a group of moderates. If the indirect demand (generated by the news sharing) is relatively more important to the media firm than the direct demand, then our result implies that maximizing total demand will also involve some bias. By contrast, if the attention tax is small enough, all direct readers will share the news and both the direct demand and the indirect demand will be maximized when the media firm locates itself at the center.

We provide two extensions. The first allows the firm to target an interval of direct consumers to show the news. Then, we find that if the media firm maximizes the breadth, it is optimal to choose the same location as in the baseline model and to make the left end of the target interval coincide with the most extreme left consumer. In the second extension, we consider resharing the shared news along the hierarchical layers of communication and characterize the strategy that maximizes the depth of sharing, the number of times the news is shared following down the layers of communication. We find that the optimal strategy leads to news polarization.

Our work relates to the literature on demand-driven media bias. Mullainathan and Shleifer (2005) and Gabszewicz, Laussel and Sonnac (2001) use a Hotelling-style model to explain a possible ideological bias of news but in a different context. Mullainathan and Shleifer (2005) consider a duopoly price competition, and media bias emerges as each firm adopts a maximal differentiation strategy in order to soften price competition. By contrast, Gabszewicz, Laussel and Sonnac (2001) obtain minimal differentiation as the firms' main revenue sources are advertising. To the best of our knowledge, we are the first to embed social network into the Hotelling model to study news sharing and its impact on the ideological differentiation of news media. In our model, the news is free and the media firm maximizes its advertising revenue, which is proportionate to its demand. We assume that the surplus generated by the news is such that without sharing, the unique location that maximizes the demand is the center. In this way, we isolate the bias that emerges because of the concern to maximize the indirect demand.

Empirical papers on the topic include Gentzkow and Shapiro (2010) and Larcinese, Puglisi and Snyder (2011). Gentzkow and Shapiro (2010) find that readers have an economically significant preference for like-minded news and that firms respond strongly to consumer preferences. Larcinese, Puglisi and Snyder (2011) find evidence that newspapers cater to readers' partisan tastes on news about unemployment, trade deficit and budget deficit.

In Section 2, we present the baseline model. In Section 3 we show that the depth-maximization strategy leads to news bias. Section 4 provides the two extensions: targeting and depth-maximization. Section 5 concludes. All proofs are placed in the Appendix.

# 2 The Baseline Model

We consider a media firm whose objective is to maximize a weighted sum of the readership from direct readers and the one from indirect readers. As consumers can consume on social media multiple news provided by different media firms, we here focus on one media firm. In the baseline model, we limit attention to a single layer of indirect readers: we study resharing of shared news in section 4.

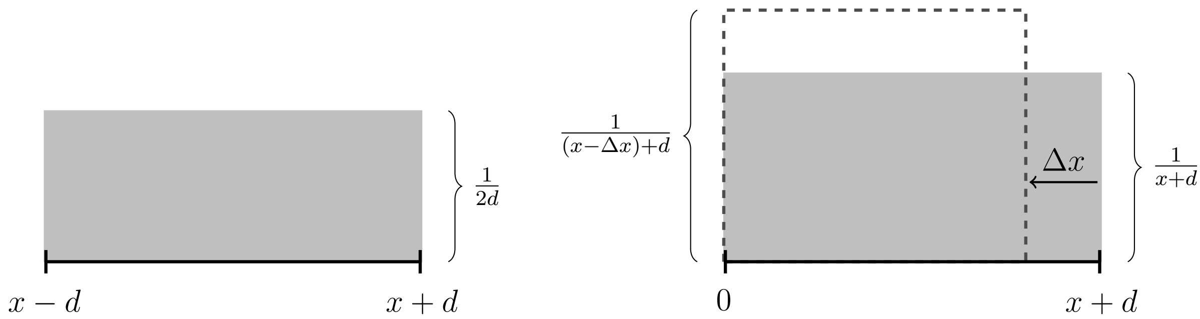
The media firm has to choose its political leaning, $y \in [0, 1]$. There is a continuum of consumers. Each consumer is located on the interval $[0, 1]$. A consumer's location, say $x$, represents the ideal news she would like to read. By reading the news located at $y$, a consumer gets a surplus equal to $u$ and incurs a disutility $(x - y)^2$ from the mismatch between the news opinion location, $y$, and her ideal one, $x$. That is, the utility that a consumer located at $x$ obtains from reading a news located at $y$ is

$$U(x, y) = u - (x - y)^2.$$

We consider only free news.

**Network structure.** Each consumer belongs to one of two different groups: *direct consumers* — those who *follow* the media firm and have direct access to the news — and *indirect consumers* — those who follow a direct consumer and have access to the news only if it is *shared* by the consumer they follow. We assume there is a unit mass of direct consumers whose locations are uniformly distributed over the interval $[0, 1]$. Each direct consumer has a distinct group of followers (i.e. indirect consumers). We assume that every direct consumer has the same measure of followers, which is normalized to one. Given a direct consumer located at $x$, the distribution of her followers depends on the degree of homophily as is explained below.

**Homophily.** To capture the degree of homophily in the network, we introduce a parameter $d \in [0, 1]$, which is an inverse measure of homophily. Consider a direct consumer located at $x$ and her follower located at $x'$. Then, the *inverse degree of homophily*, $d$, represents the maximum distance between $x$ and $x'$. We assume that every direct consumer has the same maximum distance with respect to her followers and that every direct consumer has the same number of followers, of which the mass is normalized at one. This mass one of followers is uniformly distributed over the interval $[0, 1] \cap [x - d, x + d]$ (see Figure 1). In one extreme case of $d = 0$ all followers have the same preference as the direct consumer they follow; at the opposite extreme case of $d = 1$, all direct consumers have their followers uniformly distributed over the unit interval no matter their location.




(a) Density of followers of a moderate (b) Density of followers of an extremist

Figure 1: Density of mass of followers.

**Reading and sharing.** Consumers read the news whenever they have access to it and get a non-negative utility from reading it. Each direct consumer who *reads* the news has the opportunity to *share* it with her followers and she is assumed to share the news whenever the average benefit that her followers obtain from the sharing of the news is large enough.[2] Let

[2]The altruistic behavior of consumers when deciding to share the news can be interpreted as an appreciation for receiving positive reviews (as *likes* on Facebook or *tweethearts* on Twitter) for their shared

$B(x,y,d)$ denote the average benefit that the followers of the direct consumer located at $x$ obtain when the latter shares the news located at $y$ given the (inverse) degree of homophily $d$, that is,

$$B(x,y,d) = \int_{\underline{z}}^{\overline{z}} \frac{\max\{U(z,y),0\}}{\overline{z}-\underline{z}}\, dz, \tag{1}$$

with $\underline{z} = \max\{x-d,0\}$ and $\overline{z} = \min\{x+d,1\}$.

In order to capture that news is shared only when it is considered sufficiently relevant, we assume that a direct reader's sharing a news imposes some exogenous *attention tax* on each of her followers, which represents a cost of sharing $c \geq 0$. The attention tax $c$ represents the opportunity cost of attention as the news sharing requires each follower to pay attention to the news, which does not necessarily mean reading it. Therefore, a direct consumer $x$, who reads the news, shares it if and only if $B(x,y,d) > c$.

**Profit.** The firm makes profits from traffic from direct consumers and traffic from indirect consumers. Given the location of its news at $y$, the profit of the firm is given by

$$\Pi(y) = (1-\alpha)\, D_0(y) + \alpha\, D_1(y),$$

where $D_0(y)$ and $D_1(y)$ are, respectively, the direct and indirect demands attracted by the firm, and $\alpha$ captures the importance the firm gives to indirect demand relative to direct demand.

**Direct demand.** Let $\mathcal{R}_0(y)$ denote the set of locations of all direct consumers with non-negative utility of reading the news located at $y$. Hence, the direct demand for the news is given by the measure of $\mathcal{R}_0(y)$, that is, $D_0(y) = \mu\left(\mathcal{R}_0(y)\right)$.

From now on we set $u = 1/4$. This is in order to reduce the number of parameters. In addition, $u = 1/4$ generates the worst scenario in terms of news polarization as the direct demand of the media firm has a unique maximum at $y = 1/2$. In fact, for $u = 1/4$, it follows that

$$D_0(y) = \begin{cases} 1/2 + y & \text{if } 0 \leq y \leq 1/2 \\ 3/2 - y & \text{if } 1/2 \leq y \leq 1. \end{cases}$$

Thus, the media firm will move away the center only if the consequent loss in the direct demand is compensated by an increase in its indirect demand. Note also that for all other

content.

values of $u \geq 0$, there exist a continuum of values (containing 1/2) that maximize firm's direct demand, $D_0$.

**Indirect demand.** Due to the symmetry of the problem, if $y^*$ maximizes the overall demand (i.e. the weighted sum of the direct demand and the indirect demand), so does $1 - y^*$. Therefore, it is sufficient to analyze only a half of the interval. From now on we restrict attention to $y \in [0, 1/2]$, w.l.o.g.. This implies $\mathcal{R}_0(y) = [0, y + 1/2]$.

Let $\mathcal{S}_0(y, d, c)$ denote the set of locations of all direct consumers who share the news, that is,

$$\mathcal{S}_0(y, d, c) = \{x \in \mathcal{R}_0(y) \text{ such that } B(x, y, d) \geq c\}.$$

Given $x_0 \in \mathcal{S}_0(y, d, c)$, let $\mathcal{R}_1(x_0, y, d)$ denote the set of locations of $x_0$'s followers that have positive utility of reading the news when it is shared by $x_0$, that is,

$$\begin{aligned}\mathcal{R}_1(x_0, y, d) &= \{x_1 \in [0, 1] \cap [x_0 - d, x_0 + d] \text{ such that } U(x_1, y) \geq 0\} \\ &= [0, 1] \cap [x_0 - d, x_0 + d] \cap [0, y + 1/2].\end{aligned}$$

Thus, the indirect demand of the media firm, as a function of its location $y$, is

$$D_1(y) = \int_{x \in \mathcal{S}_0(y,d,c)} \frac{\mu\left(R_1(x, y, d)\right)}{\mu\left([0, 1] \cap [x - d, x + d]\right)}\, dx,$$

where the denominator represents the density of the followers of $x \in \mathcal{S}_0(y, d, c)$ — who are uniformly distributed over $[0, 1] \cap [x - d, x + d]$.

# 3 Breadth-maximizing location

Here, we analyze how consumers' homophily and propensity for sharing news (captured, respectively, by the parameters $d$ and $c$) impact the indirect demand of the firm and thereby the location of its news. Both parameters affect the decision of direct consumers to share the news — the homophily parameter by changing the shape of the benefit function and the attention tax by restricting the news to be shared only by those consumers whose followers greatly benefit from reading it.

From equation (1), whenever $U(x, y) \geq 0$ for all $x \in [\underline{z}, \overline{z}]$, with $\underline{z} = \max\{x - d, 0\}$ and

$\overline{z} = \min\{x + d, 1\}$, follows that

$$\begin{aligned} B(x, y, d) &= \int_{\underline{z}}^{\overline{z}} \frac{1/4 - (z - y)^2}{\overline{z} - \underline{z}} \, dz \\ &= \frac{1}{4} - \frac{(\overline{z} - \underline{z})^2}{12} - \left( y - \frac{\underline{z} + \overline{z}}{2} \right)^2 \\ &= \frac{1}{4} - \text{variance}(Z) - (y - \text{mean}(Z))^2, \qquad Z \sim \text{Uniform}(\underline{z}, \overline{z}). \end{aligned}$$

Therefore, to each direct consumer all whose followers have positive utility of reading, the benefit of sharing decreases with an increase in the ideological dispersion of her followers and with the mismatch between the location of the news and the ideological location of her average follower.

Particularly, when $y = 1/2$ all consumers have positive utility of reading and the benefit function has the following functional form

$$B(x, 1/2, d) = \begin{cases} (x + d)\left(\frac{1}{2} - \frac{1}{3}(x + d)\right) & \text{if } x \in [0, \min\{d, 1 - d\}] \\ \left(-x^2 + x - \frac{d^2}{3}\right) \mathbf{1}\{d \leq \frac{1}{2}\} + \left(\frac{1}{6}\right) \mathbf{1}\{d \geq \frac{1}{2}\} & \text{if } x \in [\min\{d, 1 - d\}, \max\{d, 1 - d\}], \\ (1 - x + d)\left(\frac{1}{2} - \frac{1}{3}(1 - x + d)\right) & \text{if } x \in [\max\{d, 1 - d\}, 1]. \end{cases}$$

Figure 2 shows how the graph of the benefit function conditional on $y = 1/2$ changes depending on the value of $d$. As it is easily seen, when the firm locates its news at the center, all consumers have a positive benefit of sharing it. Hence, if the attention tax, $c$, is "low enough" all consumers will share the news and, as every consumer have a positive utility of reading it, both the direct demand and the indirect demand are maximized when the firm chooses the central location $y = 1/2$.

**Proposition 1** (Zero Polarization)**.** *If the attention tax is low enough (i.e.* $c < \min\{1/6, d/2 - d^2/3\}$*),the media firm's profit is maximized by the central location (i.e.* $y^* = 1/2$*), which generates* $\Pi(1/2) = 1$*.*

The following result highlights that the benefit function is not always unimodal with respect to direct consumers' location $x$. This occurs as for some values of $c$, even if two direct consumers share the news, a third and intermediate one does not share; that is, to some values of $c$, the set of consumers willing to share may not constitute an interval.

**Lemma 1.** *For* $y \in (d, 4d/3)$*,* $B(x, y, d)$ *as a function of* $x$ *is bimodal with local maximums at* $x = 3y/2 - d$ *and* $x = y$*, and a local minimum at* $x = d$*.*

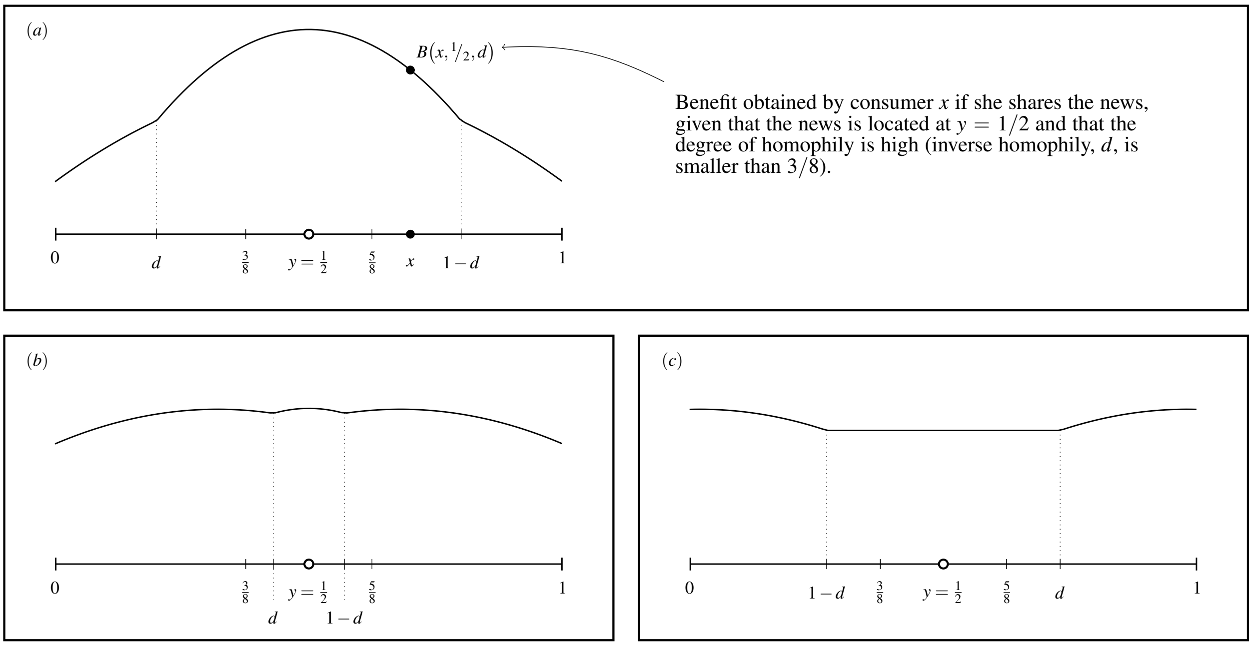



Figure 2: Format of the benefit function according to the network's degree of homophily. (a) $d \in [0, 3/8]$; (b) $d \in (3/8, 1/2]$; (c) $d \in (1/2, 1]$.

From now on we focus on the case in which the benefit function is unimodal when the firm is located at the center, that is, we assume $d < 3/8$ (i.e., $4d/3 < 1/2$), which means a high degree of homophily. Additionally, we discard cases where $c$ is too high such that either (i) no direct consumer is ever willing to share the news ($c > 1/4 - d^2/12$), or (ii) only extreme consumers are possibly willing to share ($c > 1/4 - d^2/3$).

**Condition 1** (High degree of homophily and non-negligible attention tax). $d \in (0, \frac{3}{10})$ and $c \in (d - \frac{4}{3}d^2, \frac{1}{4} - \frac{7}{9}d^2)$.

Under Condition 1 and our assumption of $u = 1/4$, maximizing the indirect demand is equivalent to maximizing the measure of sharers. This is because under Condition 1, the attention tax is high enough that as long as a direct sender shares the news (as it has such an incentive), all her followers read it.[3] Since maximizing the indirect demand is equiv-

[3]Given $y$, under our assumption of $u = 1/4$, a consumer having access to the news reads it as long as her location belongs to $[0, y + 1/2]$. Hence, as long as the location of the marginal right-side sharer is on the left of $y + 1/2 - d$, all followers who get the news shared will read it. Condition 1 guarantees that this happens as the attention tax is high enough, which moves the location of the marginal right-side sharer toward the

alent to maximizing the measure of sharers, we define the breadth of news sharing as follows:

**Definition 1**: We define *the breadth* of news sharing as the measure of direct consumers who shares the news.

In what follows, we study the breadth-maximizing strategy.

**Proposition 2** (Limit Polarization). *Under Condition 1, the breadth of news sharing is maximized by the limit polarization location which makes the most extreme direct reader indifferent between sharing and not sharing the news. Namely, the firm chooses* $y^* = d/2 + \sqrt{1/4 - d^2/12 - c}$ *such that it solves* $B(0, y^*, d) = c$*. And the firm gets a profit of*

$$\Pi(y^*) = (1-\alpha)\left(\frac{1}{2} + y^*\right) + \alpha\left(y^* + \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}\right), \tag{2}$$

*while*

$$\Pi(1/2) = (1-\alpha)\,1 + \alpha\left(2\sqrt{\frac{1}{4} - c - \frac{d^2}{3}}\right). \tag{3}$$

The effect of polarization is given by

$$\Pi(y^*) - \Pi\left(\frac{1}{2}\right) = \alpha\underbrace{\left(y^* - \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}\right)}_{\text{gain of indirect demand}} - (1-\alpha)\underbrace{\left(\frac{1}{2} - y^*\right)}_{\text{loss of direct demand}}.$$

To reach out to indirect consumers, the media firm needs to get direct consumers to first read the news and then share it with their followers. When the attention tax is too low, any direct reader is very likely to share what they read. So in order to maximize the breadth, all the media firm has to do is to increase the number of direct readers and it does so by positioning itself at the center. Incentives to maximize both direct demand and indirect demand coincide, jointly calling for $y^* = 1/2$. On the other hand, when the attention tax is not negligible, the media firm faces a trade-off between direct demand and indirect demand as we explain below.

To induce a direct reader to share the news, the media firm must increase the benefit of her followers. What is important to notice is that the followers of an extremist reader are less dispersed than the followers of a moderate one, thus making it easier for the firm to provide

left extreme.

a greater benefit to the former than to the latter. To be precise, let $x_{\text{ext}}$ be the location of an extremist and $x_{\text{mod}}$ the location of a moderate. Due to the lower dispersion of their followers, we have that $\max_y B(x_{\text{ext}}, y, d) > \max_y B(x_{\text{mod}}, y, d)$, meaning that inducing extremists to share the news is easier than inducing moderates to share the news. More importantly, given two extremists $x_{\text{ext}}$ and $x'_{\text{ext}}$ with $x'_{\text{ext}} - x_{\text{ext}} = \epsilon > 0$ and two moderates $x_{\text{mod}}$ and $x'_{\text{mod}}$ with $x'_{\text{mod}} - x_{\text{mod}} = \epsilon$, the difference in the composition of the followers of $x_{\text{ext}}$ and $x'_{\text{ext}}$ is smaller than the difference in the composition of the followers of $x_{\text{mod}}$ and $x'_{\text{mod}}$,[4] making it easier for the media firm to simultaneously induce sharing from the extremists, $x_{\text{ext}}$ and $x'_{\text{ext}}$, than from the moderates, $x_{\text{mod}}$ and $x'_{\text{mod}}$. Therefore, when the degree of homophily is high and the attention tax is not negligible (Condition 1), in order to maximize the indirect demand, the media firm will always be willing to lose some moderate senders to induce more extremists to share its news.

In the optimum, the media firm chooses the limit polarization location $y^*$ such that the most extremist consumer is just indifferent between sharing and not sharing the news, that is $B(0, y^*, d) = c$. It is easy to see that $B(0, y, d) > c$ is not optimal as there is some slack benefit enjoyed by the most extreme reader: then by moving $y$ a bit toward the center, the media firm can induce some additional moderate readers to share the news. It is not optimal to have $B(0, y, d) < c$. $B(0, y, d) < c$ means that the most extreme reader does not share the news. However, as we previously explained, increasing the benefit of extremists readers is more efficient to boost the indirect demand than increasing the benefit of moderates readers.

**Proposition 3** (Comparative Statics)**.** *Under Condition 1, the location of the news that maximizes the breadth of news sharing gets more polarized as the attention tax increases and/or the degree of homophily increases (i.e. as $d$ decreases).*

Consider $d$ and $c$ satisfying Condition 1. If initially the attention tax is $c$ and the firm is located at the point $y^*$ that maximizes the breadth, then a slight increase in the attention tax from $c$ to $\tilde{c}$ leads to $B(0, y^*, d) < \tilde{c}$, causing the most extremist consumer to stop sharing. To recover the extremist, the media firm has to move towards the extreme, at the cost of losing some moderate senders. Consequently, the new optimal location, $\tilde{y}^*$, would be such that $\tilde{y}^* < y^*$ and $B(0, \tilde{y}^*, d) = \tilde{c}$.

[4]The difference in the composition of followers of two (left-side) extremists occurs only on the right side, whereas the difference in the composition of followers of two moderates occurs on both sides.

With a decrease in the degree of homophily, the composition of followers of a direct consumer becomes more dispersed and the moderate direct consumers are more affected than the extremists, since the effect on the composition occurs only on the right side of an extremist direct consumer whereas the effect occurs on both sides of a moderate. Thereafter, such change would intensify the incentives of the media firm to trade moderate senders for extremists ones. In other words, let $x_{\text{ext}}$ and $x_{\text{mod}}$ be the locations of an extremist direct consumer and a moderate direct consumer, respectively. If the dissimilarity of followers change from $d$ to $\hat{d}$ with $\hat{d} - d = \epsilon > 0$ we have that

$$\max_y B(x_{\text{ext}}, y, \hat{d}) - \max_y B(x_{\text{mod}}, y, \hat{d}) > \max_y B(x_{\text{ext}}, y, d) - \max_y B(x_{\text{mod}}, y, d)$$

and the new location maximizing indirect demand, $\hat{y}^*$, will again satisfy $B(0, \hat{y}^*, \hat{d}) = c$. That being so, the direction of change from $y^*$ to $\hat{y}^*$ will be such that

$$\text{sign}\left(\hat{y}^* - y^*\right) = \text{sign}\left(U(d, y^*) - c\right)$$

**Remark 1.** *$U(d, y^*) > c$ is equivalent to $c < 1/4 - d^2/9$ (condition presented in the proof of Proposition 3), which is satisfied under Condition 1.*

The maximization of the media firm's profit is discussed in Proposition 4. The firm will provide biased content only when indirect demand is sufficiently valuable relative to direct demand, what is captured by the parameter $\alpha$.

**Proposition 4** (Profit Maximizing Strategy)**.** *Under Condition 1, there exists $\alpha^* \in (0, 1)$ such that*

*i. for $\alpha < \alpha^*$, the profit of the firm is maximized by the central location (i.e., at $y^{**} = 1/2$);*

*ii. for $\alpha > \alpha^*$, the profit of the firm is maximized by $y^{**}(\alpha) \in [y^*, \overline{y})$, where $y^*$ is the location that maximizes indirect demand and $\overline{y} = d + \sqrt{1/4 - c - d^2/3}$. Moreover, $y^{**}(\alpha)$ is decreasing in $\alpha$ (i.e., news polarization increases as the indirect demand become more profit relevant to the firm).*

# 4 Extensions

## 4.1 Targeting

We here extend the baseline model by enabling the media firm to target an interval of direct readers. For instance, the Macedonian teenagers, mentioned in the introduction, purchased bogus Facebook accounts and used them to target certain profiles of users to spread their fake news. In the baseline model, all direct consumers on the Hotelling line receive the news, which can be interpreted as the case of no targeting since it is conceptually similar to the case in which the direct consumers who receive the news is randomly selected from the Hotelling line.

Suppose that the media firm can target direct readers belonging to an interval of length $l \in [0, 1)$ to send its news. Hence, the media firm now should choose not only the location of its news $y$ and the target interval $[a, a + l] \subset [0, 1]$. We show that when the media firm maximizes the breadth, it is a weakly dominant strategy to mimic the limit polarization strategy by choosing $y = y^*$ and $a = 0$.

**Proposition 5** (Targeting Strategy)**.** *Suppose that the media firm can choose $a$ to target direct consumers belonging to $[a, a + l] \subset [0, 1]$ with $l \in [0, 1)$ in addition to choosing the news location $y$. Under Condition 1, the indirect demand is maximized by the limit polarization strategy identified in Proposition 2: it is a weakly dominant strategy to choose $y = y^*$ and $a = 0$.*

The proposition shows that the limit polarization strategy is still optimal even if we allow for targeting. This result is very consistent with the practice of the Macedonian teenagers and suggests that it would be optimal for them to use their bogus Facebook accounts to target the most extreme segment of consumers.

## 4.2 Resharing and the depth-maximization strategy

We can consider the possibility for indirect readers to have their own followers and to share the news. We can go down different layers of sharing starting from sharing of the news by direct consumers, resharing of the news shared, resharing of the reshared news and so on. Here, we provide an extension which, focuses on *the depth of sharing.*

For instance, suppose that the constant $u$ depreciates over time such that it also depreciates when the layer of sharing increases. That is, the utility that a consumer located at $x$ obtains from reading a news located at $y$ after it has been shared $n = 1, 2, \ldots$ times would be

$$U_n(x, y) = \delta^{n-1} u - (x - y)^2, \quad \text{with } \delta \in (0, 1),$$

and the benefit generated by sharing the news one extra time given by

$$B_{n+1}(x, y, d) = \int_{\underline{z}}^{\overline{z}} \frac{\max\{U_{n+1}(z, y), 0\}}{\overline{z} - \underline{z}}\, dz$$

with $\underline{z} = \max\{x - d, 0\}$ and $\overline{z} = \{x + d, 1\}$.

But for the sake of computational simplicity, we adopt a continuous version of the problem where sharing and consumption of the news occur instantaneously, in other words, lets $t \in [0, +\infty)$ be the time at which a news located at $y$ reaches an indirect consumer located at $x$, then the consumer's utility of reading the news and her benefit of resharing it will be given, respectively, by

$$U_t(x, y) = \delta^t u - (x - y)^2 \quad \text{and} \quad B_t(x, y, d) = \int_{\underline{z}}^{\overline{z}} \frac{\max\{U_t(z, y), 0\}}{\overline{z} - \underline{z}}\, dz$$

with $\underline{z} = \max\{x - d, 0\}$ and $\overline{z} = \{x + d, 1\}$. Then, we can check how long the communication keeps going and take it as a measure of the number of reshares a news will receive.

**Definition 2**: We define *the dept* of news sharing as the maximum number of times the news is shared following down the hierarchical layers of communications.

The next result states that in order to maximize the depth, the firm must produce biased content.

**Proposition 6** (Depth Maximizing Strategy)**.** *The media firm's optimal strategy to maximize the depth of news sharing is characterized as follows.*

*(i) If the attention cost is high ($c \geq d^2/6$), it is optimal to set its location at $y^*_{depth} = d/2$ and the maximum depth reached, $t^*$, is such that $\delta^{t^*} u = c + \frac{d^2}{12}$.*

*(ii) Otherwise, the firm's depth maximizing strategy and the maximum depth reached are determined by the following equations:*

$$\delta^{t^*} u = (d - y^*_{depth})^2 \quad \text{and} \quad \frac{2}{3} \frac{(d - y^*_{depth})^3}{y^*_{depth}} = c,$$

*with* $y^*_{depth} \in (d/2, d]$.

The higher is the benefit of sharing of a direct consumer, the longer it will take to for the news to depreciate to the level where the benefit equals the attention tax. Because the firm is able to provide higher benefit of sharing to direct consumers with less dispersed followers, it would like to maximize the benefit of sharing of a consumer located at $x = 0$, which is done by locating its news at $y = d/2$. However, as it also happens that, by assumption, only consumers who read the news are able to share it, and since the further away consumers are from the news location the lesser is their utility of reading it, unless the cost of sharing is high enough such that the net benefit of sharing equals zero before there is substantial depreciation in the utility of reading the news, the firm chooses to locate its news in such way that it provides maximum utility to consumers that are at the same time located near the extreme (where followers are less disperse) and not far from the news location (where the utility of reading is higher).

**Proposition 7** (Comparative Statics on Depth)**.**

*(i) As the attention tax increases, the location of news that maximizes the depth gets more polarized (when* $c < d^2/6$*) and the maximum depth decreases.*

*(ii) As the degree of homophily increases, the location of news that maximizes the depth gets more polarized and the maximum depth increases.*

### 4.3 Impact of polarized readers (To be done)

We can also study how the fact readers and their followers get more ideologically polarized affects the news content polarization. For instance, in U.S, polarization of partisan preferences has dramatically increased over the past 40 years (Lazer et al., 2018). How does it affect the media polarization?

## 5 Conclusion

We show that homophily of social networks and the propensity of users to share news with others can create incentives for a profit-maximizing media firm to focus on a segment of the market by creating partisan content. This occurs when social networks have a high degree of homophily and the attention tax imposed on followers by news sharing is not small. Then, a media firm valuing enough indirect demand prefers to show partisan content. We show that both the strategy maximizing the breadth of sharing and the strategy maximizing the depth of sharing lead to polarization.

If one considers that false news about politics tend to be hyperpartisan (Silverman et al., 2016), our results would be consistent with the findings of Vosoughi, Roy and Aral (2018) that by analyzing approximately 126,000 news stories distributed on Twitter from 2006 to 2017, conclude that false news diffused deeper and more broadly than the truth, specially for false political news.

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# A Proofs

**Proposition 1** (Zero Polarization)**.** *If the attention tax is low enough (i.e.* $c < \min\{1/6, d/2 - d^2/3\}$*),the media firm's profit is maximized by the central location (i.e.* $y^* = 1/2$*), which generates* $\Pi(1/2) = 1$*.*

*Proof.* Follows immediately from

$$\min_x B(x, 1/2, d) = \min\{1/6, d/2 - d^2/3\}.$$

□

**Lemma 1.** *For* $y \in (d, 4d/3)$*, follows that* $B(x, y, d)$ *as a function of* $x$ *is bimodal with local maximums at* $x = 3y/2 - d$ *and* $x = y$*, and a local minimum at* $x = d$*.*

*Proof.* Note that for $y \leq 1/2$ we have from equation (1) that

$$\max_{x \in (0,d]} B(x, y, d) = \max_{x \in (0,d]} \frac{1}{4} - \frac{(x+d)^2}{12} - \left(y - \frac{x+d}{2}\right)^2$$

has an interior solution only if $y < 4d/3$. And, analogously, the problem

$$\max_{x \in [d, 1/2+y-d)} B(x, y, d) = \max_{x \in [d, 1/2+y-d)} \frac{1}{4} - \frac{d^2}{3} - (y - x)^2$$

has an interior solution only if $y > d$.

The fact that $x = d$ is a local minimum for $y \in (d, 4d/3)$ follows directly from the fact that the benefit as a function of $x$ when restricted to either $[0, d]$ or $[d, 1/2 + y - d]$ is quadratic. □

**Lemma 2.** *For* $y, d \leq 1/2$*, follows that* $B(x, y, d)$ *as a function of* $x$ *is strictly decreasing for* $x \leq 1/2 + y - d$*.*

*Proof.* Once for $x \geq 1/2 + y - d$ the benefit function $B(x, y, d)$ is either equal to

$$\int_{x-d}^{1/2+y} \frac{U(z, y)}{2d}\, dz \quad \text{or} \quad \int_{x-d}^{1/2+y} \frac{U(z, y)}{1 - x + d}\, dz,$$

it is sufficient to show that both functions are decreasing for $x \geq 1/2 + y - d$.

Claim 1. $\int_{x-d}^{1/2+y} \frac{U(z,y)}{2d}\, dz$ is decreasing for $x \geq 1/2 + y - d$.

$$\begin{aligned} \frac{\partial}{\partial x}\left(\int_{x-d}^{1/2+y} \frac{U(z,y)}{2d}\, dz\right) &= \frac{\partial}{\partial x}\left(\frac{1}{6d}(1-y-d+x)\left(\frac{1}{2}+y+d-x\right)^2\right) \\ &= \frac{1}{2d}\left((y+d-x)^2 - \frac{1}{4}\right) \end{aligned}$$

and since $x > 1/2 + y - d \Leftrightarrow y + d - x < 2d - 1/2$ follows that

$$\begin{aligned} \frac{\partial}{\partial x}\left(\int_{x-d}^{1/2+y} \frac{U(z,y)}{2d}\, dz\right) &< \frac{1}{2d}\left(\left(2d - \frac{1}{2}\right)^2 - \frac{1}{4}\right) \\ &= 2d - 1 < 0, \end{aligned}$$

as claimed.

Claim 2. $\int_{x-d}^{1/2+y} \frac{U(z,y)}{1+x-d}\, dz$ is decreasing for $x \geq 1/2 + y - d$.

Observing that

$$\int_{x-d}^{1/2+y} \frac{U(z,y)}{1-x+d}\, dz = \left(\frac{2d}{1-x+d}\right)\left(\int_{x-d}^{1/2+y} \frac{U(z,y)}{2d}\, dz\right)$$

is a product of two functions decreasing in $x$, the result follows and completes the proof.

□

**Proposition 2** (Limit Polarization). *When the degree of homophily is sufficiently high and the attention tax is not negligible (i.e. under Condition 1), the indirect demand is maximized at by the limit polarization location which makes the most extreme direct reader indifferent between sharing and not sharing the news. Namely, the firm chooses* $y^* = d/2 + \sqrt{1/4 - d^2/12 - c}$ *such that it solves* $B(0, y^*, d) = c$*. And the firm gets a profit of*

$$\Pi(y^*) = (1-\alpha)\left(\frac{1}{2} + y^*\right) + \alpha\left(y^* + \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}\right), \tag{4}$$

*while*

$$\Pi(1/2) = (1-\alpha)\, 1 + \alpha\left(2\sqrt{\frac{1}{4} - c - \frac{d^2}{3}}\right). \tag{5}$$

*Proof.* From Lemmas 1 and 2 it may be concluded that under Condition 1 we have $\mathcal{R}_1(x, y, d) = [0, 1] \cap [x - d, x + d]$ for all $x \in S_0$, resulting that $D_1(y) = \mu(S_0(y))$.

Assume $y \in (4d/3, 1/2)$ such that $B(x, y, d)$ is unimodal in $x$, and write

$$D_1(y) = D_1^L(y) + D_1^R(y)$$

where $D_1^L(y)$ (resp. $D_1^R(y)$) denote the indirect demand reached by the sharing of direct consumers located at the left (resp. right) of the firm's location, $y$ and let $MS^L = MS^L(y, c, d)$ and $MS^R = MS^R(y, c, d)$ denote, respectively, the marginal sender on the left side and the marginal sender on the right side. In other words,

$$\begin{aligned} MS^L &= \min\{x \text{ such that } x \in \mathcal{S}_0(y, c, d)\}, \text{ and} \\ MS^R &= \max\{x \text{ such that } x \in \mathcal{S}_0(y, c, d)\}. \end{aligned}$$

Thus, $\mathcal{S}_0(y, c, d)$ is an interval and we have

$$D_1^L(y) = y - MS^L \quad \text{and} \quad D_1^R(y) = MS^R - y. \tag{6}$$

Solving for $MS^L$ and $MS^R$ we have that

i. $MS^R$ is the largest solution of

$$c = \frac{1}{4} - \frac{d^2}{3} - (y - MS^R)^2.$$

Therefore, from equation (6), we have

$$c = \frac{1}{4} - \frac{d^2}{3} - (D_1^R)^2$$

resulting that

$$D_1^R = \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}.$$

So, $D_1^R$ does not depend on $y$ and the maximization of the indirect demand depends only on the maximization of $D_1^L$.

ii. To solve for $MS^L$ we have three cases to consider,

Case 1: If $MS^L = 0$, we have $D_1^L = y$.

Case 2: If $MS^L \in (0, d)$, then it is the smallest solution of

$$c = \frac{1}{4} - \frac{(MS^L + d)^2}{12} - \left(y - \frac{MS^L + d}{2}\right)^2 = \frac{1 - y^2}{4} - \frac{1}{3}\left(\frac{y}{2} - d + D_1^L\right)^2$$

Therefore,

$$D_1^L = d - \frac{y}{2} + \sqrt{3\left(\frac{1 - y^2}{4} - c\right)}$$

is strictly decreasing in $y$.

Case 3: If $MS^L \in [d, 1/2]$, then it is the smallest solution of

$$c = \frac{1}{4} - \frac{d^2}{3} - (y - MS^L)^2$$

and we have $D_1^L = D_1^R$.

Since $MS^L$ is increasing in $y$ and by continuity of $D_1^L$, it follows that the maximum of $D_1^L$ in the interval $(4d/3, 1/2]$ is reached at $y^* = \max\{y \text{ such that } MS^L(y, c, d) = 0\}$. Thus, $y^*$ is given by

$$c = \frac{1}{4} - \frac{(0 + d)^2}{12} - \left(y^* - \frac{0 + d}{2}\right)^2,$$

that is,

$$y^* = \frac{d}{2} + \sqrt{\frac{1}{4} - c - \frac{d^2}{12}}.$$

It is straightforward to verify that for $c \in (d/2 - d^2/3, 1/4 - 7d^2/9)$ we have $y^* \in (4d/3, 1/2)$, as assumed.

For $y \leq 4d/3$, the (possibly bimodal) benefit function implies that $\mathcal{S}_0(y, c, d)$ may not be an interval, and consequently

$$D_1^L(y) \leq y \qquad \text{and } D_1^R(y) \leq \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}.$$

Hence, $y^*$ is global maximum and

$$D_1(y^*) = \frac{d}{2} + \sqrt{\frac{1}{4} - c - \frac{d^2}{12}} + \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}$$

while

$$D_1(1/2) = \sqrt{\frac{1}{4} - c - \frac{d^2}{3}} + \sqrt{\frac{1}{4} - c - \frac{d^2}{3}}$$

□

**Proposition 3** (Comparative Statics)**.** *When the degree of homophily is sufficiently high and the attention tax is not negligible (i.e. under Condition 1), the location of the news that maximizes the indirect demand gets more polarized as the attention tax increases and/or the degree of homophily increases (i.e. as d decreases).*

*Proof.* From Proposition 2, we have that $y^* = d/2 + \sqrt{1/4 - d^2/12 - c}$. Therefore,

$$\frac{dy^*}{dc} = -\frac{1}{2\sqrt{1/4 - d^2/12 - c}} < 0.$$

That is, an increase in $c$ causes a decrease in $y^*$, moving it to further away from the center. Thus, increasing news polarization.

On the other hand,

$$\frac{dy^*}{dd} = \frac{1}{2}\left(1 - \frac{d/6}{\sqrt{1/4 - d^2/12 - c}}\right).$$

Consequently,

$$\frac{dy^*}{dd} < 0 \Leftrightarrow \sqrt{\frac{1}{4} - \frac{d^2}{12} - c} < \frac{d}{6} \Leftrightarrow c > \frac{1}{4} - \frac{d^2}{9} > \frac{1}{4} - \frac{d^2}{3},$$

where the last inequality holds for all $d \in (0, 1)$. And since we consider only the case $c < 1/4 - d^2/3$, we have that $dy^*/dd > 0$. That is, an increase in homophily (decrease in $d$) leads to a decrease in $y^*$. Thereafter, increasing news polarization. □

**Proposition 4** (Profit Maximizing Strategy)**.** *Under condition 1, there exists* $\alpha^* \in (0, 1)$ *such that*

*i. for* $\alpha < \alpha^*$*, the profit of the firm is maximized by the central location (i.e., at* $y = 1/2$*);*

*ii. for* $\alpha > \alpha^*$*, the profit of the firm is maximized by* $y^{**}(\alpha) \in [y^*, \overline{y})$*, where* $y^*$ *is the location that maximizes indirect demand and* $\overline{y} = d + \sqrt{1/4 - c - d^2/3}$*. Moreover,* $y^{**}(\alpha)$ *is decreasing in* $\alpha$ *(i.e., news polarization increases as the indirect demand become more profit relevant to the firm).*

*Proof.* We have that $\Pi(y) = (1 - \alpha)\, D_0(y) + \alpha\, D_1(y)$. Suppose there exists $y^{**}$ satisfying the first-order condition of the maximization problem $\max_{y \in [0,1/2]} \Pi(y)$. Then, we have that

$$0 = \Pi'(y^{**}) \Rightarrow D_1'(y^{**}) = -\frac{1-\alpha}{\alpha} < 0.$$

Hence, $y^{**}$ must belong to the interval $[y^*, \hat{y}] = [d/2+\sqrt{1/4 - d^2/12 - c}, d+\sqrt{1/4 - c - d^2/3}]$ since this is the only range of values for which $D_1(y)$ is decreasing.

Additionally, by differentiating implicitly with respect to $\alpha$ the equation

$$D_1'\left(y^{**}(\alpha)\right) = -\frac{1-\alpha}{\alpha}$$

it follows that

$$\frac{dy^{**}}{d\alpha} = \frac{1}{\alpha^2 D''(y^{**})} < 0,$$

since $D''(y) < 0$ for all $y \in [y^*, \hat{y})$.

Finally, observing that

1. as $\alpha \to 0$, we have $\Pi(y) \to D_0(y)$ which has a strict maximum at $y = 1/2$; and
2. as $\alpha \to 1$, we have $\Pi(y) \to D_1(y)$ which has a strict maximum at $y = y^*$,

by continuity, there exists $\alpha^*$ such that for $\alpha > \alpha^*$ the profit is maximized at a location $y \neq 1/2$, leading us to an interior solution for the maximization problem and, consequently, to the optimal being given by a $y^{**}$ satisfying the first-order condition of the maximization problem $\max_{y\in[0,1/2]} \Pi(y)$. What was to be shown. □

**Proposition 5** (Targeting Strategy)**.** *Suppose that the media firm can choose a to target direct consumers belonging to $[a, a+l] \subset [0,1]$ with $l \in [0,1)$ in addition to choosing the news location y. Under Condition 1, the indirect demand is maximized by the limit polarization strategy identified in Proposition 2: it is a weakly dominant strategy to choose $y = y^*$ and $a = 0$.*

*Proof.* Suppose that in the baseline model in which $[a, a+l] = [0,1]$, $y = y^*$ leads to the direct readers belonging to $[0, b]$ to share the news. Now consider the case in which $[a, a+l] = [0, l] \subset [0,1]$ and $y = y^*$.

There are two cases to consider. First, $l \leq b$. Then, all direct readers in $[0, l]$ share the news. The media firm cannot strictly do better. Second, $l > b$. Then, all direct readers in $[0, b]$ share the news. The media firm cannot strictly do better. This ends the proof. □

**Proposition 6** (Depth Maximizing Strategy)**.** *If the attention cost is high ($c \geq d^2/6$), firm's optimal strategy to maximize news depth is to set its location at $y^*_{depth} = d/2$ and the* *maximum depth reached, $t^*$, is such that $\delta^{t^*}u = c + \frac{d^2}{12}$. Otherwise, firm's depth maximizing strategy and the maximum depth reached are determined by the following equations:*

$$\delta^{t^*}u = (d - y^*_{depth})^2 \quad \text{and} \quad \frac{2}{3}\frac{(d - y^*_{depth})^3}{y^*_{depth}} = c,$$

*with $y^*_{depth} \in (d/2, d]$.*

*Proof.* Assume that the firm locates its news at $y = d/2$, then the most extreme direct consumer is the one with the higher utility of sharing the news. Let $\bar{t}$ be the time for which $U_{\bar{t}}(0, d/2) = 0$, i.e., $\delta^{\bar{t}}u = d^2/4$. Hence,

$$\begin{aligned} B_{\bar{t}}(0, d/2, d) &= \int_0^d \frac{\max\{U_{\bar{t}}(z, d/2), 0\}}{d}\, dz \\ &= \int_0^d \frac{\delta^{\bar{t}}u - (z - d/2)^2}{d}\, dz \\ &= \delta^{\bar{t}}u - \frac{d^2}{12} \\ &= \frac{d^2}{4} - \frac{d^2}{12} = \frac{d^2}{6}. \end{aligned}$$

Therefore, for $c \geq d^2/6$, the last consumer to share the news have zero net benefit of sharing it while a positive utility of reading, the firm's depth maximizing strategy is to locate its news at $y^*_{\text{depth}} = d/2$ and $t^*$ is such that $\delta^{t^*}u = c + d^2/12$. In fact, suppose there exist $x'$ and $y'$ such that $B_{t^*}(x', y', d) > c = B_{t^*}(0, d/2, d)$, then we must have $B_0(x', y', d) > B_0(0, d/2, d)$ but that is not possible since $B_0(0, d/2, d) = \max_{(x,y)} B_0(x, y, d)$.

Otherwise, if $c < d^2/6$, let $t^*$ be maximal and take $(x', y')$ such that both $U_{t^*}(x', y') \geq 0$ and $B_{t^*}(x', y', d) \geq c$. Clearly, at least one of the inequalities must be binding.

Suppose $U_{t^*}(x', y') = 0$ and $B_{t^*}(x', y', d) > c$. As the benefit function is continuous in $x$, there exists $\epsilon > 0$ such that for $x'' = (1-\epsilon)x' + \epsilon y$ we have $B_{t^*}(x'', y', d) > c$ and $U_{t^*}(x'', y') > 0$. Contradicting the fact that $t^*$ is maximal. Therefore, we must have $B_{t^*}(x', y', d) = c$.

Suppose, otherwise, that $U_{t^*}(x', y') > 0$,

1. If $||x' - y'|| \geq d/2$, then $U_{t^*}(0, d/2) \geq U_{t^*}(x', y') > 0$ and $B_{t^*}(0, d/2, d) \geq B_{t^*}(x', y', d) = c$ which is impossible. To have $B_{t^*}(0, d/2, d) > c$ implies $t^*$ is not maximal, while $B_{t^*}(0, d/2, d) = c$ implies $c \geq d^2/6$.

2. If $||x' - y'|| < d/2$, let

$$(x'', y'') \in \underset{\substack{(x,y)\in[0,1]\times[0,1/2] \\ ||x-y|| \leq ||x''-y''||}}{\operatorname{argmax}} B_0(x, y, d).$$

Therefore, $y'' = d - ||x' - y'||$ and $x'' = 2y'' - d = d - 2||x' - y'|| > 0$. Moreover, $U_{t^*}(x'', y'') = U_{t^*}(x', y') > 0$ and $B_{t^*}(x'', y'', d) \geq B_{t^*}(x', y', d) = c$. Once more, $t^*$ maximal requires $B_{t^*}(x'', y'', d) = c$.

Then, there exists $\epsilon > 0$ such that $U_{t^*}(x'' - 2\epsilon, y'' - \epsilon) > 0$ and

$$\begin{aligned}
B_{t^*}(x'' - 2\epsilon, y'' - \epsilon, d) &= \int_0^{x''+d-2\epsilon} \frac{\max\{U_{t^*}(z, y'' - \epsilon), 0\}}{x'' + d - 2\epsilon}\, dz \\
&= \int_0^{x''+d-2\epsilon} \frac{\max\{U_{t^*}(z + \epsilon, y''), 0\}}{x'' + d - 2\epsilon}\, dz \\
&= \int_\epsilon^{x''+d-\epsilon} \frac{\max\{U_{t^*}(z, y''), 0\}}{x'' + d - 2\epsilon}\, dz \\
&> \int_0^{x''+d} \frac{\max\{U_{t^*}(z, y''), 0\}}{x'' + d}\, dz \\
&= B_{t^*}(x'' - 2\epsilon, y'' - \epsilon, d) = c,
\end{aligned}$$

which again contradicts the maximality of $t^*$.

Thus, it must holds that both $U_{t^*}(x^*, y^*_{\text{depth}}) = 0$ and $B_{t^*}(x^*, y^*_{\text{depth}}, d) = c$, with $x^* = 2y^*_{\text{depth}} - d$. Yielding to

$$U_{t^*}(2y^*_{\text{depth}} - d, y^*_{\text{depth}}) = 0 \Rightarrow \delta^{t^*} u = (d - y^*_{\text{depth}})^2 \tag{7}$$

and

$$\begin{aligned}
c &= B_{t^*}(2y^*_{\text{depth}} - d, y^*_{\text{depth}}, d) \\
&= \int_0^{2y^*_{\text{depth}}} \frac{\max\{U_{t^*}(z, y^*_{\text{depth}}), 0\}}{2y^*_{\text{depth}}}\, dz \\
&= \frac{d - y^*_{\text{depth}}}{y^*_{\text{depth}}} \int_{2y^*_{\text{depth}}-d}^{d} \frac{U_{t^*}(z, y^*_{\text{depth}})}{2(d - y^*_{\text{depth}})}\, dz \quad \text{(Limits of integration determined by equation (7))} \\
&= \frac{d - y^*_{\text{depth}}}{y^*_{\text{depth}}} \left( \delta^{t^*} u - \frac{(d - y^*_{\text{depth}})^2}{3} \right) \\
&= \frac{2}{3} \frac{(d - y^*_{\text{depth}})^3}{y^*_{\text{depth}}} \quad \text{(Again from equation (7))}
\end{aligned}$$

□

**Proposition 7** (Comparative Statics on Depth)**.**

*(i) As the attention tax increases the location of news that maximizes the depth gets more polarized (when $c < d^2/6$) and the maximum depth decreases.*

*(ii) As the degree of homophily increases the location of news that maximizes the depth gets more polarized and the maximum depth increases.*

*Proof.* For $c \geq d^2/6$, the equilibrium is characterized by

$$y = \frac{d}{2} \quad \text{and} \quad \delta^{t^*} u = c + \frac{d^2}{12}.$$

Following that,

$$\frac{dy}{dc} = 0, \ \frac{dy}{dd} > 0, \ \frac{dt}{dc} < 0, \text{ and } \frac{dt}{dd} < 0.$$

For $c < d^2/6$, the equilibrium is characterized by

$$\delta^t u = (d-y)^2 \quad \text{and} \quad \frac{2}{3}\frac{(d-y)^3}{y} = c.$$

By totally differentiating these equations, we have

$$\ln(\delta)\delta^t u \, dt = 2(d-y)\, dd - 2(d-y)\, dy \tag{8}$$

and

$$\frac{2(d-y)^2}{y}\, dd - \frac{2(3y+1)}{3}\left(\frac{d-y}{y}\right)^2 dy = dc \tag{9}$$

Taking $dd = 0$ on equations (8) and (9), it follows that

$$\ln(\delta)\delta^t u \, dt = -2(d-y)\, dy \quad \text{and} \quad -\frac{2(3y+1)}{3}\left(\frac{d-y}{y}\right)^2 dy = dc.$$

Therefore, $dc > 0$ implies $dy < 0$ and $dt < 0$.

While taking $dc = 0$ on equations (8) and (9) leads to

$$\ln(\delta)\delta^t u \, dt = 2(d-y)\,(dd - dy) \quad \text{and} \quad dd - \left(1 + \frac{1}{3y}\right) dy = 0,$$

that is,

$$\ln(\delta)\delta^t u \, dt = \frac{2(d-y)}{3y}\, dy \quad \text{and} \quad dd = \left(1 + \frac{1}{3y}\right) dy.$$

Thereafter, $dd < 0$ implies $dy < 0$ and $dt > 0$.

**Remark 2.** *As $\delta \in (0,1)$ it follows that $\ln(\delta)$ is negative.*

□