Vertical Separation Revisited*

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Abstract

Manufacturers might sell their products under their own brand names or vertically separate the distribution and retail operation and sell products under the distributors'/retailers' private label brand products. We draw the following findings. First, with having double marginalization of upstream and downstream firms, vertical separation is optimal to upstream firms if and only if the degree of product differentiation is below a certain threshold. Second, without having double marginalization, vertical separation is optimal unless upstream firms' products are perfect substitutes. Third, social welfare is strictly greater under vertical separation without double marginalization unless products are perfect substitutes.

Key words: Vertical separation; vertical integration; product differentiation; private label brand; manufacturers' brand.

JEL: L22; L13.

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1 Introduction

We are surrounded by brand names, some of which are retailers' or wholesalers' "private label brands" or "store brands." We rely on retailers' private label brands of commoditized articles for daily use, such as soap, detergent, toiletries, and table wines. We also often prefer distributors' ("*négociant*") brands to vineyard's bottled at the chateau ("*mis en bouteille au chateau*") brands for daily drink.

Since Kaven and Call (1967) who dealt with the ice cream industry, choice of distribution channel between manufacturers' own brands and private label brands has attracted attention of scholars in managerial science. As the significance of private label brands is growing particularly in the last three decades, literature on the issue is also growing (Hyman, Kopf and Dongdae (2010)).

In industrial organization, since Bonanno and Vickers (1988), the issue has been treated as a choice between vertical integration (national brands) and separation (private label brands). Here, we attempt to explain the phenomenon by extending previous works from literature of industrial organization. We first show that low enough degree of product differentiation, alternatively, a high enough substitutability, is the necessary and sufficient condition for vertical separation to be the optimal response of duopolistic upstream firms engaged in price competition.

We next show that, if upstream firm maximizes total profits of itself and its retailer, total gain of vertical separation is greater and the lack of complete commoditization (substitutability) becomes the necessary and sufficient condition for vertical separation to be optimal. This occurs because maximization of total profits prevents double marginalization and prices are not raised too much.

We next examine welfare implications of vertical separation. In essence, gain of vertical separation comes from a rise in products' price. It inevitably subdues consumer surplus. Thus, we compare social welfare under vertical separation with and without double marginalization.

Social welfare is greater under vertical separation without double marginalization than under that with double marginalization.

Regarding the choice between vertical separation and integration, Bonanno and Vickers (1988) provided a foundational theory. In duopolistic price competition setting, they found a sufficient condition for vertical separation to be optimal to upstream firms.

Since their work on vertical separation/integration choice, we have four strands of rich literature on this issue. One approach, represented by Matsushima (2004, 2009) and Matsushima and Mizuno (2012), introduced product differentiation in a Hotelling model. They found that vertical separation is effectively always optimal for upstream firms.

Another approach focuses on the information structure of wholesale contracts that molds strategic interactions. Bonanno and Vickers (1988) implicitly assumed that wholesale contracts are publicly observable, such that both retailers respond to each other symmetrically. However, it has been pointed out that once contacts are assumed to be private, a symmetric response cannot be expected on an equilibrium path. In this case, manufacturers charge wholesale prices at the marginal cost, so vertical separation and integration turn out to be indifferent (Ray and Stiglitz (1988, 1995)). Pagnozzi and Piccolo (2012) then show, by reasonably assuming that retailers have symmetric beliefs with a positive probability, that even if the wholesale contract is private, there exists an equilibrium in which both manufacturers vertically separate and charge wholesale prices higher than the marginal cost. Retailers symmetrically increase their retail prices, and manufacturers earn higher profits than in the case of vertical integration. The results of Bonanno and Vickers (1988) were reestablished in a broader context.

Meanwhile, Jansen (2003) based on Gal-Or (1990) dealt with the transaction cost between upstream and downstream firms and showed that a higher transaction cost induces vertical integration. This strand of literature is quite persuasive concerning emerging economies, which are often dominated by vertically integrated conglomerates. This tendency toward vertical integration might be due to less-transparent markets or higher transaction costs in emerging economies.

The fourth approach has mined the demand schedule even deeper. Cyrenne (1994) considered a demand structure that allows both firms' goods to be substitutes or complements by assuming the consumers' utility function presented by Dixit (1979). He showed that if both products are substitutive enough, vertical separation is a unique equilibrium. That is, Cyrenne (1994) presented another sufficient condition for vertical separation to be optimal to upstream firms.

Pagnozzi and Piccolo (2012) also discusses the demand structure as an extension and showed that vertical separation and integration become indifferent if the two firms' products become perfect substitutes. Gal-Or (1999) also established that the more substitutive the two products are, the more likely vertical separation is and vice versa.

We follow the fourth strand because this is a direct extension of the argument presented initially by Bonanno and Vickers (1988). A strong assumption made by Bonanno and Vickers (1988) was that the market prices are decided by downstream firms regardless of whether or not upstream firms choose to separate the retail operation vertically. The assumption is not entirely consistent with the context of upstream firms' oligopoly they assumed.

In contrast, we begin by asking whether upstream firms choose their retail pricing under vertical integration or wholesale pricing under vertical separation. On this point, our point of departure is a ramification of Cyrenne (1994).

Thus we first pin down the necessary and sufficient condition for vertical separation to be optimal. A broad sufficient condition presented by Bonanno and Vickers (1988) was substantially focused by Cyrenne (1994). We finally furnish the necessary and sufficient condition.

Meanwhile, another question is why Cyrenne (1994) and Matsushima (2004, 2009) and Matsushima and Mizuno (2012) delivered contrasting results. The former found a certain degree of substitutability of products as a sufficient condition for vertical separation to be optimal. The latter found that vertical integration is effectively always optimal.

Technically, the discrepancy is related to the difference in their settings; Dixit (1979)'s utility function in the former and the Hotelling model in the latter. With Dixit (1979)' utility function, degree of product differentiation (commoditization) is given, while the Hotelling model allows players to choose the degree of product differentiation as their choices of locations.

However, the crucial source of the discrepancy is whether to address the double marginalization or not. Either in Bonanno and Vickers (1988) or Cyrenne (1994), double marginalization was not addressed.

Therefore, we second examine a case where double marginalization is excluded and establish that vertical separation is optimal unless products are perfect substitutes. In this case, the lack of differentiation is the necessary and sufficient condition for vertical separation.

Third, we evaluate welfare implications of vertical separation. Since the gain of vertical separation comes from higher prices than those under vertical integration. Therefore, vertical separation always lowers consumer surplus. Hence, comparison between vertical separation and integration is trivial. Comparison between vertical separation without double marginalization and that with double marginalization is not.

Total profits of upstream firms and downstream firms is also greater under the vertical separation without double marginalization if products are differentiated and vice versa. Double marginalization is avoidable usually if upstream and downstream firms are under the same ownership. If we differentiate integration and separation by ownership, hence, it is a type of vertically integrated ownership. These are the key factor by which Matsushima (2004, 2009) and Matsushima and Mizuno (2012) deliver their result. They evaluate the vertical integration as a mechanism to avoid double marginalization. Furthermore, in a Hotelling setting, different from that of Dixit (1979)' setting, the degree of product differentiation is endogenous. We show that vertical separation without double marginalization provides a greater total profit unless their products are perfect substitutes. Matsushima (2004, 2009) and Matsushima and Mizuno (2012) found that endogenizing the product differentiation raises the value of vertically integrated ownership because firms' ability to differentiate their products more. Our result is consistent with theirs.

Social welfare is greater under vertical separation without double marginalization unless products are perfect substitutes. The gain is increasing in the degree of product differentiation, alternatively, decreasing in the product commoditization, and becomes zero when products are perfectly commoditized, alternatively, perfect substitutes.

After all, our contribution is first to furnish the necessary and sufficient condition on the shoulders of Bonanno and Vickers (1988), who found a broad sufficient condition, and Cyrenne (1994), who substantially narrowed it. We conclude the literature.

Second, we integrate apparently inconsistent results Cyrenne (1994) and Matsushima (2004, 2009) and Matsushima and Mizuno (2012) under our framework. By differentiating the relationship between upstream and downstream firms by the one with double marginalization and the one without double marginalization, we can reproduce both results in our framework.

Lastly, we evaluate welfare impacts of vertical separation without double marginalization and with double marginalization. The former's social welfare dominates the latter's unless products are perfect substitutes and the discrepancy is greater if products are differentiated. Total profits of upstream and downstream firms are also greater under vertical separation without double marginalization. Thus, a company will prefer vertical separation between upstream subsidiary and downstream subsidiary to maximize total profits if products are differentiated. It then is preferred in terms of social welfare.

The remaining part of the paper is organized as follows. Section 2 presents the model with price competition, and section 3 deducts predictions. Subsection 1 analyzes when upstream firms choose vertical separation of the retail operation with double marginalization. Subsection 2 shows a case where double marginalization is avoided and upstream firms maximize total of own profit and downstream retailer's profit jointly. Subsection 3 compare vertical separations with and without double marginalization, and examines their welfare implications.

Subsection 4 extends the argument to quantity competition, being motivated by Singh and

Vives (1984). Section 5 concludes with a policy implication.

2 Model

Suppose that there are two manufacturing firms, i and j, which produce potentially differentiated products. We follow the standard model by Dixit (1979) to specify the utility function U of the representative consumer such that

$$U(q_i, q_j) = \alpha(q_i + q_j) - \frac{\beta}{2}\left(q_i^2 + 2\gamma q_i q_j + q_j^2\right),$$

where q_i and q_j denote products of firm i and j, respectively, and $\gamma \in [0, 1]$ captures the degree of product commoditization, alternatively, $1/\gamma \in (0, 1]$ is the degree of product differentiation. That is, $\gamma = 0$ means that products are perfectly differentiated and $\gamma = 1$ indicates that products are perfectly commoditized, that is, perfect substitutes. The first-order condition for utility maximization gives the inverse demand functions for products of firm i and j such that

$$\begin{aligned} p_i &= \alpha - \beta q_i - \beta\gamma q_j, \\ p_j &= \alpha - \beta q_j - \beta\gamma q_i, \end{aligned} \tag{1}$$

which imply the demand functions are

$$\begin{aligned} q_i &= \frac{\alpha}{\beta(1+\gamma)} - \frac{1}{\beta(1-\gamma)(1+\gamma)}p_i + \frac{\gamma}{\beta(1-\gamma)(1+\gamma)}p_j, \\ q_j &= \frac{\alpha}{\beta(1+\gamma)} - \frac{1}{\beta(1-\gamma)(1+\gamma)}p_j + \frac{\gamma}{\beta(1-\gamma)(1+\gamma)}p_i. \end{aligned}$$

Regarding the production technology, we assume identical marginal cost $c \leq \alpha$. When the manufacturing firms choose vertical separation, each firm consigns its product to exclusive retailers i and j. We assume that upstream manufacturing firms can extract all profit by arbitrarily imposing the wholesale prices $w_{S,i}$ and $w_{S,j}$ on the downstream retailers.

In line with the literature, we focus on price competition. As an extension, we analyze quantity competition in section 4.

We consider two cases of the timing of game. First case is the one with double marginalization.

First, the manufacturing firms i and j decide whether to integrate or separate the retail operation vertically.

In case of vertical integration, second, firms i and j simultaneously decide prices p_i and p_j to maximize their profits, $\Pi_{I,i}$ and $\Pi_{I,i}$.

In case of vertical separation, second, firm i's downstream retailer and firm j's downstream retailer decide p_i and p_j to maximize their profits $R_{S,i}$ and $R_{S,j}$.

Third, in case of vertical separation with double marginalization, upstream manufacturing firms i and j simultaneously decide wholesale prices $w_{S,i}$ and $w_{S,j}$ to maximize their profits $\Pi_{S,i}$ and $\Pi_{S,j}$.

The second case is the one without double marginalization. In this case, in the third stage of vertical separation, manufacturing firms simultaneously decide wholesale prices w_i^T and w_j^T to maximize total profit of upstream manufacturers and downstream retailers, $\Pi_{T,i} + R_{S,i}$ and $\Pi_{T,j} + R_{S,j}$.

3 Analysis

3.1 Vertical integration

Let us begin with the case of vertical integration. Both firms profits' are given by

$$\Pi_{I,i} =(p_i - c)q_i = (p_i - c)\left[\frac{\alpha}{\beta(1+\gamma)} - \frac{1}{\beta(1-\gamma)(1+\gamma)}p_i + \frac{\gamma}{\beta(1-\gamma)(1+\gamma)}p_j\right],$$
$$\Pi_{I,j} =(p_j - c)q_j = (p_j - c)\left[\frac{\alpha}{\beta(1+\gamma)} - \frac{1}{\beta(1-\gamma)(1+\gamma)}p_j + \frac{\gamma}{\beta(1-\gamma)(1+\gamma)}p_i\right].a$$

The first-order condition gives the equilibrium prices

$$p_{I,i}^* = p_{I,i}^* = \frac{c + \alpha(1-\gamma)}{2-\gamma}. \tag{2}$$

Given the prices, the equilibrium quantities and profits are

$$q_{I,i}^* = q_{I,j}^* = \frac{\alpha - c}{\beta(1+\gamma)(2-\gamma)}, \tag{3}$$

$$\Pi_{I,i}^* = \Pi_{I,j}^* = \frac{(1-\gamma)(\alpha-c)^2}{\beta(1+\gamma)(2-\gamma)^2}. \tag{4}$$

Note that $p_{I,i}^* = p_{I,j}^* = c \Leftrightarrow \alpha = c$. When the price competition results in a price equal to the marginal cost, it implies that $\alpha = c$ and $\Pi_{I,i}^* = \Pi_{I,j}^* = 0$.

3.2 Downstream retailers' maximization under vertical separation

If vertical separation is chosen, in the second stage, downstream retailers i and j simultaneously decide the prices to maximize their own profit

$$\begin{aligned} R_{S,i} =& (p_{S,i} - w_{S,i})q_{S,i} \\ =& (p_{S,i} - w_{S,i})\left[\frac{\alpha}{\beta(1+\gamma)} - \frac{1}{\beta(1-\gamma)(1+\gamma)}p_i + \frac{\gamma}{\beta(1-\gamma)(1+\gamma)}p_j\right], \\ R_{S,j} =& (p_{S,j} - w_{S,j})q_{S,j} \\ =& (p_{S,j} - w_{S,j})\left[\frac{\alpha}{\beta(1+\gamma)} - \frac{1}{\beta(1-\gamma)(1+\gamma)}p_{S,j} + \frac{\gamma}{\beta(1-\gamma)(1+\gamma)}p_{S,i}\right]. \end{aligned} \tag{5}$$

The first-order conditions for both downstream retailers give the equilibrium prices

$$\begin{aligned} p_{S,i} =& \frac{2w_{S,i} + \gamma w_{S,j} + \alpha\left[2 - \gamma(1+\gamma)\right]}{(2-\gamma)(2+\gamma)}, \\ p_{S,i} =& \frac{2w_{S,j} + \gamma w_{S,i} + \alpha\left[2 - \gamma(1+\gamma)\right]}{(2-\gamma)(2+\gamma)}. \end{aligned}$$

Demand functions imply

$$\begin{aligned} q_{S,i} =& \frac{(a - w_{S,i})(2-\gamma^2) - \gamma(\alpha - w_{S,j})}{\beta(1-\gamma)(1+\gamma)(2-\gamma)(2+\gamma)}, \\ q_{S,j} =& \frac{(a - w_{S,j})(2-\gamma^2) - \gamma(\alpha - w_{S,j})}{\beta(1-\gamma)(1+\gamma)(2-\gamma)(2+\gamma)}. \end{aligned}$$

3.3 Vertical separation with double marginalization

Let us begin with vertical separation with double marginalization. In this case, in the third stage, upstream manufacturing firms simultaneously decide wholesale prices to maximize only their own profits,

$$\begin{aligned} \mathit{\Pi}_{S,i} =& (w_{S,i} - c) q_{S,i} \\ =& \frac{(w_{S,i} - c)\left[(\alpha - w_i)\left(2-\gamma^2\right) - \gamma\left(\alpha - w_j\right)\right]}{\beta\left(1-\gamma\right)\left(1+\gamma\right)\left(2-\gamma\right)\left(2+\gamma\right)}, \\ \mathit{\Pi}_{S,j} =& \frac{(w_{S,j} - c)\left[(\alpha - w_j)\left(2-\gamma^2\right) - \gamma\left(\alpha - w_i\right)\right]}{\beta\left(1-\gamma\right)\left(1+\gamma\right)\left(2-\gamma\right)\left(2+\gamma\right)}. \end{aligned}$$

Note that the cross derivatives of $\mathit{\Pi}_{S,i}$ and $\mathit{\Pi}_{S,j}$ with respect to $w_{S,i}$ and $w_{S,j}$ are

$$\frac{\partial^2 \mathit{\Pi}_{S,i}}{\partial w_{S,i} \partial w_{S,j}} = \frac{\partial^2 \mathit{\Pi}_{S,j}}{\partial w_{S,i} \partial w_{S,j}} = \frac{\gamma}{\beta\left(1-\gamma\right)\left(1+\gamma\right)\left(2-\gamma\right)\left(2+\gamma\right)} > 0.$$

This is because a rise in one's own wholesale price is accompanied by a rise in the other's price, which implies that competition to cut the wholesale price never occurs on the equilibrium path.

The first-order condition for profit maximization gives the equilibrium wholesale prices, retail prices, quantities, and profits,

$$w^*_{S,i} = w^*_{S,j} = \frac{(c+\alpha)\left(2-\gamma^2\right) - \alpha\gamma}{4-2\gamma^2-\gamma}, \tag{6}$$

$$p^*_{S,i} = p^*_{S,j} = \frac{c\left(2-\gamma^2\right) + 2\alpha\left(1-\gamma\right)\left(3-\gamma^2\right)}{\left(2-\gamma\right)\left(4-2\gamma^2-\gamma\right)}, \tag{7}$$

$$q^*_{S,i} = q^*_{S,j} = \frac{\left(2-\gamma^2\right)\left(\alpha-c\right)}{\beta\left(1+\gamma\right)\left(2-\gamma\right)\left(4-2\gamma^2-\gamma\right)}, \tag{8}$$

and

$$\mathit{\Pi}^*_{S,i} = \mathit{\Pi}^*_{S,j} = \frac{\left(2-\gamma^2\right)\left(2+\gamma\right)\left(1-\gamma\right)\left(\alpha-c\right)^2}{\beta\left(4-2\gamma^2-\gamma\right)^2\left(1+\gamma\right)\left(2-\gamma\right)}. \tag{9}$$

We first examine when vertical separation is the optimal response to upstream manufacturing firms.

Proposition 1. *There is a certain threshold of the degree of product commoditization γ^*, such that, if and only if $\gamma^* < \gamma < 1$, vertical separation strictly dominates vertical integration.*

Proof. From (4) and (9), we have

$$\mathit{\Gamma}_S\left(\gamma\right) \equiv \mathit{\Pi}^*_{S,i} - \mathit{\Pi}^*_{I,i} = \frac{\left(\alpha-c\right)^2\left(1-\gamma\right)\left(-8-3\gamma^4-4\gamma^3+9\gamma^2+8\gamma\right)}{\beta\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-2\gamma^2-\gamma\right)^2}.$$

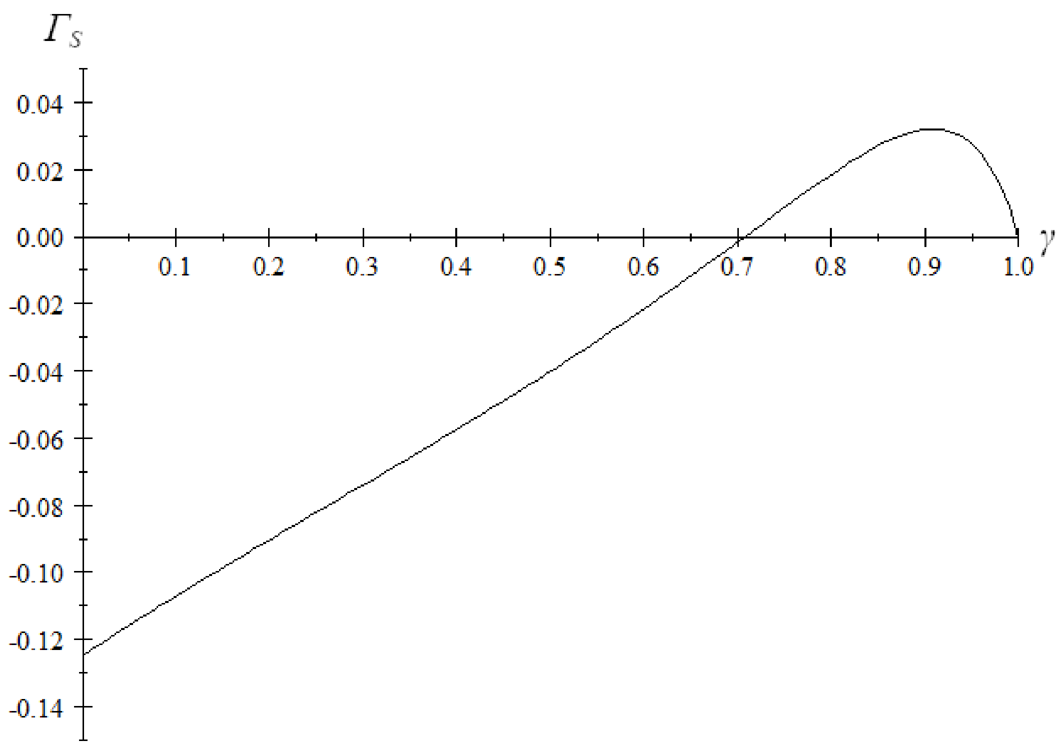
$\mathit{\Gamma}_S\left(1\right) = 0$, and, for $\gamma \in [0,1)$, $\mathit{\Gamma}_S\left(\gamma\right) > 0$ if and only if $f_S\left(\gamma\right) \equiv -8-3\gamma^4-4\gamma^3+9\gamma^2+8\gamma > 0$. Since, $f_S\left(0\right) = -8$, $f_S\left(1\right) = 2$, and

$$\frac{\partial f_S\left(\gamma\right)}{\partial\gamma} = 2\left[\left(1-\gamma\right)\left(2\gamma+\left(1+\gamma\right)\left(6\gamma+4\right)\right)+\gamma\right] > 0$$

for $\gamma \in (0,1)$, we have γ^* such that $\Gamma_S(\gamma) > 0$ if and only if $\gamma^* < \gamma < 1$. □

A numerical example of $\Gamma_S(\gamma)$ when $(\alpha - c)^2 = \beta$ is depicted in Figure 1.

Figure 1: Upstream firms' gain of vertical separation $\Gamma(\gamma)$ when $(\alpha - c)^2 = \beta$.



Note: γ is the degree of product commoditization. Alternatively, $1/\gamma$ is the degree of product differentiation.

Regarding the gain of total profits of upstream firms and downstream retailers, we have the qualitatively same argument.

Proposition 2. *There is a certain threshold of the degree of product commoditization γ^*, such that, if and only if $\gamma^* < \gamma < 1$, vertical separation strictly dominates vertical integration in terms of total profits of upstream and downstream firms.*

Proof. From (7) and (8), we have

$$\begin{aligned} \Pi_{S,i} + R_{S,i} &= (w_i - c)q_{S,i} + \left(p_{S,i} - w_{S,i}\right) q_{S,i} = \left(p_{S,i} - c\right) q_{S,i} \\ &= \frac{2\left(c-\alpha\right)^2\left(1-\gamma\right)\left(3-\gamma^2\right)\left(2-\gamma^2\right)}{\beta\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-2\gamma^2-\gamma\right)^2} \end{aligned} \tag{10}$$

From (4) and (10), we have

$$\Gamma_{S^T}(\gamma) \equiv \left(\Pi_S + R_{S,i}\right) - \Pi_I = \frac{\left(\alpha-c\right)^2\left(1-\gamma\right)\left(2+\gamma\right)\left(5\gamma-2\gamma^3-2\right)}{\beta\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-2\gamma^2-\gamma\right)^2}. \tag{11}$$

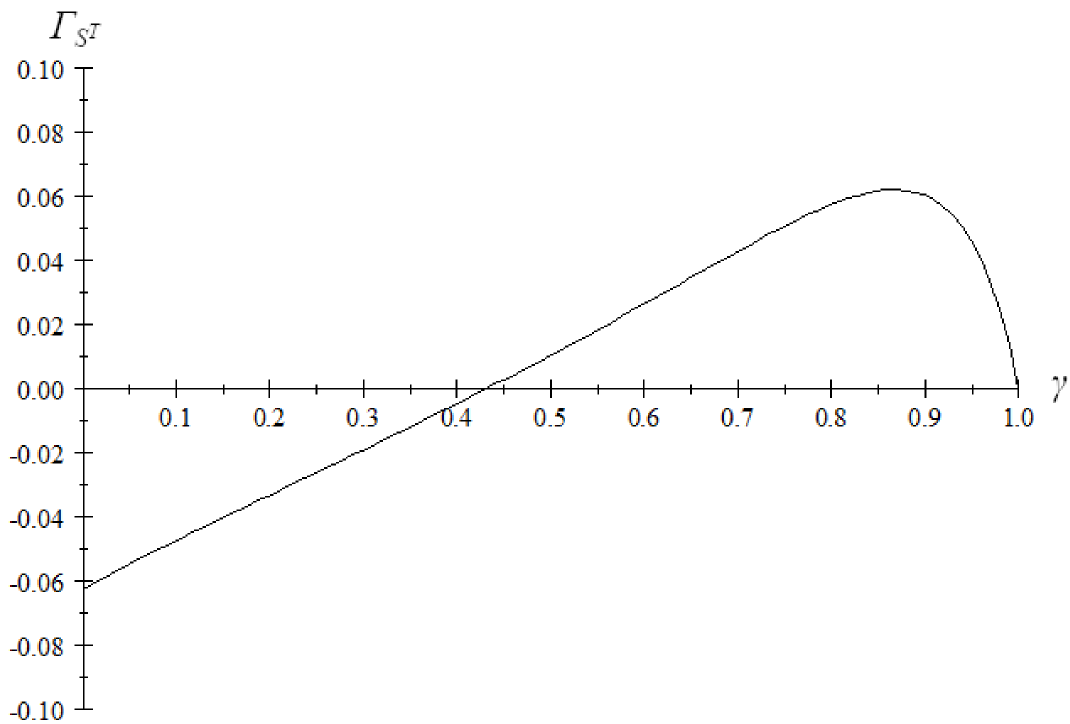
$\Gamma_{S^T}(1) = 0$, and for $\gamma \in [0,1)$, $\Gamma_{S^T} > 0$ if and only if $f_{S^T}(\gamma) \equiv -2\gamma^3 + 5\gamma - 2 > 0$. Since $f_{S^T}(0) = -2$, $f_{S^T}(1) = 1$, and

$$\frac{\partial f_{S^T}(\gamma)}{\partial \gamma} = -\left(6\gamma^2 - 5\right), \quad \left.\frac{\partial f_{S^T}(\gamma)}{\partial \gamma}\right|_{\gamma=0} = 5, \quad \left.\frac{\partial f_{S^T}(\gamma)}{\partial \gamma}\right|_{\gamma=1} = -1,$$

$f_{S^T}(\gamma)$ is increasing at $\gamma = 0$, decreasing at $\gamma = 1$, and has only one extreme at $\sqrt{5}\sqrt{6}/6$ in $\gamma \in [0,1]$. Furthermore, $f_{S^T}\left(\sqrt{5}\sqrt{6}/6\right) = 5\sqrt{5}\sqrt{6}/9 - 2 > 0$. Therefore, there exists γ^* such that $\Gamma_{S^T}(\gamma)$ if and only if $\gamma^* < \gamma < 1$. □

A numerical example when $\left(\alpha - c\right)^2 = \beta$ is shown in Figure 2.

Figure 2: Upstream and downstream firms' total gain of vertical separation $\Gamma_{S^T}(\gamma)$ when $(\alpha - c)^2 = \beta$.



Note: γ is the degree of product commoditization. Alternatively, $1/\gamma$ is the degree of product differentiation.

From (2) and (7), we have

$$p_{S,i} - p_{I,i} = \frac{(\alpha - c)(1-\gamma)(2+\gamma)}{(2-\gamma)(4-2\gamma^2-\gamma)} \begin{cases} > 0 & \text{if } \gamma < 1 \\ = 0 & \text{if } \gamma = 1. \end{cases}$$

From (3) and (8), we have

$$q_{S,i} - q_{I,i} = -\frac{(\alpha - c)(1-\gamma)(2+\gamma)}{\beta(1+\gamma)(2-\gamma)(4-2\gamma^2-\gamma)} \begin{cases} < 0 & \text{if } \gamma < 1 \\ = 0 & \text{if } \gamma = 1. \end{cases}$$

The upstream firms' gain of vertical separation comes from higher retail prices, which decrease demand. Consumer surplus under vertical integration is

$$CS_I \equiv U\left(q^*_{I,i}, q^*_{I,j}\right) - \left(p^*_{I,i}q^*_{I,i} + p^*_{I,j}q^*_{I,j}\right) = \frac{(\alpha - c)^2}{\beta(1+\gamma)(2-\gamma)^2}, \tag{12}$$

and that under vertical integration is,

$$CS_S \equiv U\left(q^*_{S,i}, q^*_{S,j}\right) - \left(p^*_{S,i}q^*_{S,i} + p^*_{S,i}q^*_{S,j}\right) = \frac{(\alpha - c)^2(2-\gamma^2)^2}{\beta(1+\gamma)(2-\gamma)^2(4-2\gamma^2-\gamma)^2}. \tag{13}$$

Thus, we have

$$CS_S - CS_I = -\frac{(\alpha - c)^2(1-\gamma)(2+\gamma)(6-3\gamma^2-\gamma)}{\beta(1+\gamma)(2-\gamma)^2(4-2\gamma^2-\gamma)^2} \begin{cases} < 0 & \text{if } \gamma < 1 \\ = 0 & \text{if } \gamma = 1. \end{cases} \tag{14}$$

The gain from vertical separation intrinsically contains a transfer from consumers to firms.

3.4 Vertical separation without double marginalization

Next, let us consider vertical separation without double marginalization. That is, in the third stage after vertical separation being chosen, upstream manufacturing firms chose wholesale prices to maximize total of its own profit $\mathit{\Pi}_{S^T,i}$ and its retailer's profit $R_{S^T,i}$, $\mathit{\Pi}_{S^T,i} + R_{S^T,i} = \left(w_{S^T,i} - c\right) q_{S^T,i} + \left(p_{S^T,i} - w_{S^T,i}\right) q_{S^T,i} = \left(p_{S^T,i} - c\right) q_{S^T,i}$. First order conditions give optimal wholesale prices, retail prices, quantities, and upstream manufacturing firms' profits;

$$w_{S^T,i}{}^* = w_{S^T,j}{}^* = \frac{(1-\gamma)\left[2c\,(2+\gamma)+\alpha\gamma^2\right]+c\gamma^3}{4-\gamma^2-2\gamma}, \tag{15}$$

$$p_{S^T,i}{}^* = p_{S^T,j}{}^* = \frac{2\alpha\,(1-\gamma)+c\,(2-\gamma^2)}{4-\gamma^2-2\gamma}, \tag{16}$$

$$q_{S^T,i}{}^* = q_{S^T,j}{}^* = \frac{(2-\gamma^2)\,(\alpha-c)}{\beta\,(1+\gamma)\,(4-\gamma^2-2\gamma)}, \tag{17}$$

and

$$\mathit{\Pi}_{S^T,i} = \mathit{\Pi}_{S^T,j} = \left(w_{S^T,i}{}^* - c\right) q_{S^T,i}{}^* = \frac{(\alpha-c)^2\,(1-\gamma)\,\gamma^2\,(2-\gamma^2)}{\beta\,(1+\gamma)\,(4-\gamma^2-2\gamma)^2}. \tag{18}$$

We then have

$$\mathit{\Pi}_{S^T,i} + R_{S^T,i} = (p_{S^T,i}{}^* - c)q_{S^T,i}{}^* = \frac{2\,(\alpha-c)^2\,(1-\gamma)\,(2-\gamma^2)}{\beta\,(1+\gamma)\,(4-\gamma^2-2\gamma)^2}. \tag{19}$$

Here we see that removal of double marginalization relaxes the constraint by the degree of commoditization on vertical separation.

Proposition 3. *Total profit of upstream manufacturing firm and downstream retailer under vertical separation without double marginalization dominates profit under vertical integration if and only if* $\gamma < 1$.

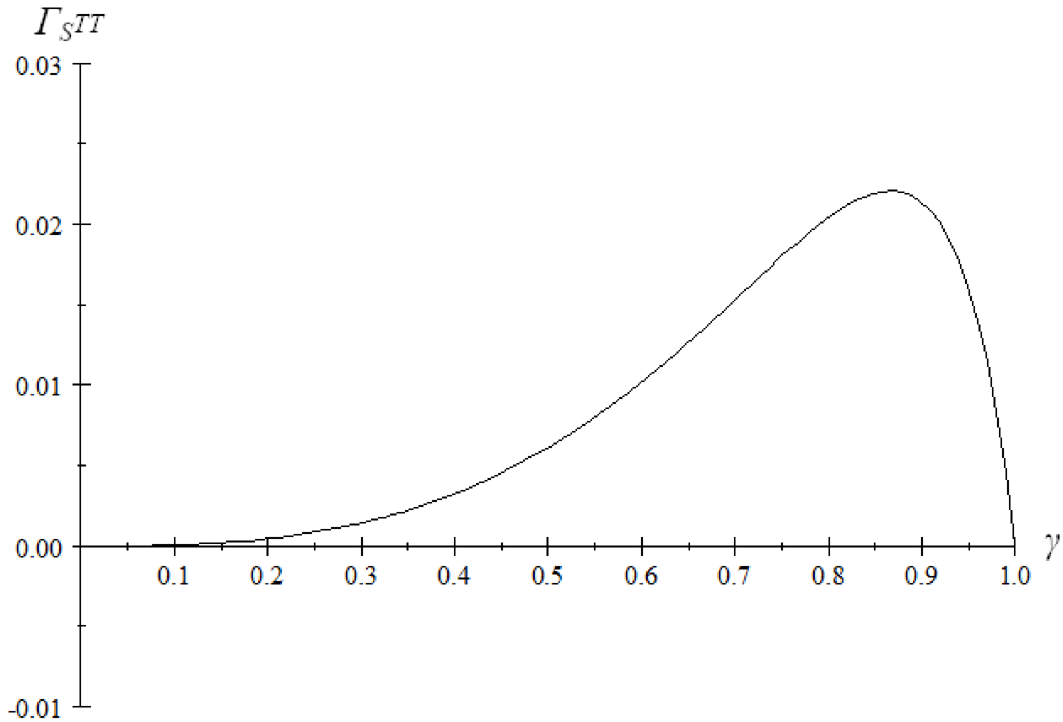
Proof. From (4) and (19), we have

$$\mathit{\Gamma}_{S^{TT}} \equiv \left(\mathit{\Pi}_{S^T,i} + R_{S^T,i}\right) - \mathit{\Pi}_{I,i} = \frac{(\alpha-c)^2\,(1-\gamma)\,(4-3\gamma)\,\gamma^3}{\beta\,(1+\gamma)\,(2-\gamma)^2\,(2\gamma+\gamma^2-4)^2} \begin{cases} >0 & \text{if } \gamma<1 \\ =0 & \text{if } \gamma=1. \end{cases} \tag{20}$$

□

As seen in propositions 2 and 3, removal of double marginalization expands the range of vertical separation dominance. Unless upstream firms' products are perfectly commoditized to be perfect substitutes, vertical separation of operation raises total profit. A numerical example when $(\alpha - c)^2 = \beta$ is shown in Figure 3.

Figure 3: Upstream and downstream firms' total gain of vertical separation $\Gamma_{S^{TT}}(\gamma)$ when $(\alpha - c)^2 = \beta$.



Note: γ is the degree of product commoditization. Alternatively, $1/\gamma$ is the degree of product differentiation.

Proposition 4. *There is a certain threshold of the degree of commoditization γ^* such that total profit of upstream manufacturer and downstream retailer under vertical separation without double marginalization is greater than that under vertical separation with double marginalization if and only if* $0 \leq \gamma < \gamma^*$.

Proof. From (10) and (19), we have

(21)
$$\Gamma_{SS^{TT}}(\gamma) \equiv \left(\Pi_{S^{T},i} + R_{S^{T},i}\right) - \left(\Pi_{S,i} + R_{S,i}\right)$$
$$= \frac{2(\alpha - c)^2 (1-\gamma)(8 - 5\gamma^4 + 8\gamma^3 + 12\gamma^2 - 24\gamma)(2-\gamma^2)^2}{\beta(1+\gamma)(2-\gamma)^2(4-\gamma^2-2\gamma)^2(4-2\gamma^2-\gamma)^2}.$$

$\Gamma_{SS^{TT}}(1) = 0$, and $\Gamma_{SS^{TT}}(\gamma) > 0$ if and only if

(22)
$$f_{SS^{TT}}(\gamma) \equiv 8 - 5\gamma^4 + 8\gamma^3 + 12\gamma^2 - 24\gamma > 0$$

for $\gamma \in [0, 1)$. $f_{SS^{TT}}(0) = 8$ and $f_{SS^{TT}}(1) = -1$. From

$$\frac{\partial f_{SS^{TT}}(\gamma)}{\partial \gamma} = -20\gamma^3 + 24\gamma^2 + 24\gamma - 24,$$

we have

$$\left.\frac{\partial f_{SS^{TT}}(\gamma)}{\partial \gamma}\right|_{\gamma=0} = -24, \quad \left.\frac{\partial f_{SS^{TT}}(\gamma)}{\partial \gamma}\right|_{\gamma=1} = 4.$$

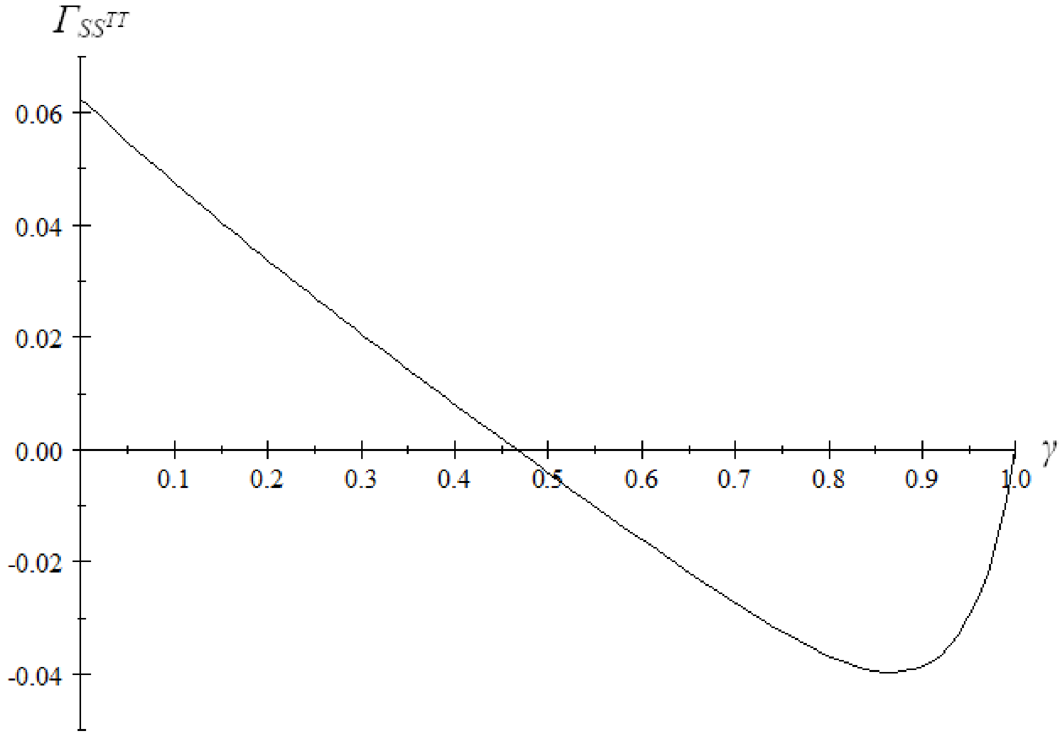
Since

$$\frac{\partial^2 f_{SS^{TT}}(\gamma)}{\partial \gamma^2} = -60\gamma^2 + 49\gamma + 24 > 1$$

for $\gamma \in [0, 1]$, $f_{SS^{TT}}$ is strictly convex for $\gamma \in [0, 1]$. Hence there exists only one γ^* such that $f_{SS^{TT}} > 0$ if and only if $0 \leq \gamma < \gamma^*$. Therefore, there exists γ^* such that if and only if $\Gamma_{SS^{TT}}(\gamma) > 0$ if and only if $\gamma < \gamma^*$. □

A numerical example when $(\alpha - c)^2 = \beta$ is shown in Figure 4.

Figure 4: Upstream and downstream firms' total gain of vertical separation $\Gamma_{SS^{TT}}(\gamma)$ when $(\alpha - c)^2 = \beta$.



Note: γ is the degree of product commoditization. Alternatively, $1/\gamma$ is the degree of product differentiation.

Consumer surplus under vertical separation without double marginalization is

$$CS_{S^T} \equiv \frac{(\alpha - c)^2 (2 - \gamma^2)^2}{\beta (1 + \gamma)(4 - \gamma^2 - 2\gamma)^2} \tag{23}$$

From (12) and (23), we have

$$(24)\qquad CS_I - CS_{S^T} = -\frac{(\alpha - c)^2 (1-\gamma)\gamma^2 (8+\gamma^3 - 3\gamma^2 - 4\gamma)}{\beta (1+\gamma)(2-\gamma)^2 (4-\gamma^2-2\gamma)^2} \begin{cases} < 0 & \text{if } \gamma < 1 \\ = 0 & \text{if } \gamma = 1. \end{cases}$$

Firms' gain from vertical separation without double marginalization also contains a transfer from consumers.

3.5 Welfare implications of double marginalization

From (7) and (16), we have

$$(25)\qquad p_{S,i} - p_{S^T,i} = \frac{2(\alpha - c)(1-\gamma)(2-\gamma^2)^2}{(2-\gamma)(4-\gamma-2\gamma^2)(4-2\gamma-\gamma^2)} \geq 0$$

for $\gamma \in [0,1]$. From (8) and (17), we have

$$(26)\qquad q_{S^T,i} - q_{S,i} = \frac{2(\alpha - c)(1-\gamma)(2-\gamma^2)^2}{\beta(2-\gamma)(1+\gamma)(4-\gamma-2\gamma^2)(4-2\gamma-\gamma^2)} \geq 0$$

for $\gamma \in [0,1]$. From (13) and (23), we have

$$(27)\qquad CS_{S^T} - CS_S = \frac{4(\alpha - c)^2 (1-\gamma)(6+\gamma^3-2\gamma^2-4\gamma)(2-\gamma^2)^3}{\beta(1+\gamma)(2-\gamma)^2(4-2\gamma^2-\gamma)^2(4-\gamma^2-2\gamma)^2} \geq 0$$

for $\gamma \in [0,1]$. Thus, vertical separation without double marginalization delivers a greater quantity at a lower price than that with double marginalization and attains a higher consumer surplus. In total, we have the following.

Proposition 5. *Social welfare under vertical separation without double marginalization dominates that under vertical separation with double marginalization if and only if products are not perfect substitutes.*

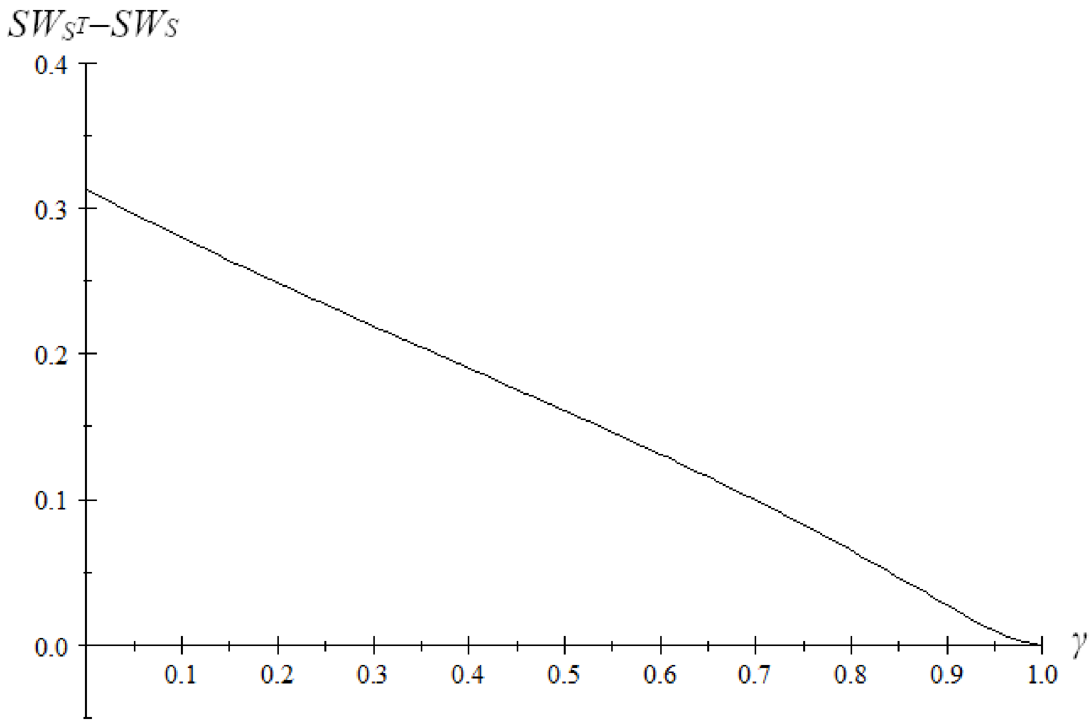
Proof. From (10), (13), (19), and (23), we have

$$
\begin{aligned}
SW_{S^T} - SW_S &\equiv \left[CS_{S^T} + 2\left(\mathit{\Pi}_{S^T,i} + R_{S^T,i}\right)\right] - \left[CS_S + 2\left(\mathit{\Pi}_{S,i} + R_{S,i}\right)\right] \\
&= \frac{4\left(\alpha - c\right)^2\left(1-\gamma\right)^2\left(2-\gamma^2\right)^2\left(20+\gamma^4+4\gamma^3-10\gamma^2-12\gamma\right)}{\beta\left(1+\gamma\right)\left(2-\gamma\right)^2\left(4-2\gamma^2-\gamma\right)^2\left(4-\gamma^2-2\gamma\right)^2} \\
&\begin{cases} > 0 & \text{if } \gamma < 1 \\ = 0 & \text{otherwise.} \end{cases}
\end{aligned} \tag{28}
$$

□

A numerical example when $(\alpha - c)^2 = \beta$ is shown in Figure 5.

Figure 5: Upstream and downstream firms' total gain of vertical separation $\mathit{\Gamma}_{S^T}(\gamma)$ when $(\alpha - c)^2 = \beta$.



Note: γ is the degree of product commoditization. Alternatively, $1/\gamma$ is the degree of product differentiation.

4 Extension to quantity competition

Let us extend our analysis to quantity competition in the retail market. First, manufacturing firms i and j simultaneously decide whether to integrate or separate retail vertically. Second, in case of vertical integration, firms i and j simultaneously decide quantities, $q^q_{I,i}$ and $q^q_{I,j}$ to maximize own profits. In case of vertical separation, firm i's downstream retailer and firm j's downstream retailer simultaneously decide quantities, $q^q_{S,i}$ and $q^q_{S,j}$ to maximize own profits. Third, in case of vertical separation, firms i and j decide wholesale prices w_i and w_j to maximize their own profits.

In case of vertical integration, optimal quantity and profit are,

$$q^q_{I,i}{}^* = q^q_{I,j}{}^* = \frac{\alpha - c}{\beta\left(2+\gamma\right)} \tag{29}$$

and

$$\Pi^q_{I,i}{}^* = \Pi^q_{I,j}{}^* = \frac{\left(\alpha - c\right)^2}{\beta\left(\gamma+2\right)^2} \tag{30}$$

In case of vertical separation, optimal quantity, wholesale price, and profit are,

$$q^q_{S,i}{}^* = q^q_{S,j}{}^* = \frac{2\left(\alpha - c\right)}{\beta\left(2+\gamma\right)\left(4-\gamma\right)}, \tag{31}$$

$$w^q_{S,i}{}^* = w^q_{S,j}{}^* = \frac{2c + \alpha\left(2-\gamma\right)}{4-\gamma}, \tag{32}$$

and

$$\Pi^q_{S,i}{}^* = \Pi^q_{S,j}{}^* = \frac{2\left(\alpha - c\right)^2\left(2-\gamma\right)}{\beta\left(2+\gamma\right)\left(4-\gamma\right)^2} \tag{33}$$

Then we have the following result.

Proposition 6. *Under quantity competition, vertical integration dominates vertical separation.*

Proof. From (30) and (33), we have

$$\Pi^q_{S,i} - \Pi^q_{I,i} = -\frac{(\alpha - c)^2 (8 + 3\gamma^2 - 8\gamma)}{\beta (2+\gamma)^2 (4-\gamma)^2} < 0. \tag{34}$$

□

As seen in (29) and (31), vertical separation under quantity competition accompanies a deep reduction of quantity with retailers' profits being maximized. The effect is large enough to reduce upstream manufacturers' profits as seen in (30) and (33).

5 Discussion

Consumers tend to choose private label brands, or store brands more likely when perceived probability of making a mistake in choice (Batra and Sinha (2000)). In practice, we observe private label products such as the Kirkland Signature by Costco and the 365 Everyday Value owned by Whole Foods Market in markets of commoditized products for daily use. Table wines labeled by distributors instead of vineyards are also another example.

Propositions 1 and 2 explain why upstream manufacturing firms choose private labeling by retailers. If products are not highly differentiated, it is optimal for upstream firms to separate retail operation vertically. Furthermore, proposition 3 demonstrates that removing double marginalization, vertically separated operation of retail process increase total profit of upstream manufacturing firm and downstream retailer unless products are perfect substitutes.

Proposition 4 also shows that the gain of removing double marginalization is greater if products are more differentiated. Proposition 5 establishes that removal of double marginalization improves social welfare more if products are more differentiated.

A straightforward way to remove double marginalization is vertically integrated ownership. The classic option is observed, for instance, in auto market. Another measure is transfer between upstream manufacturers and downstream retailers, probably through competition between national and private brands. Extension to consider the issue is left for future research.

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