

# **Toeholds and Information Precision in Common Value Takeover Auctions**

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This paper analyses the effect of information precision in common value takeover auctions with toeholds. It shows that information precision does not affect the equilibrium when bidders have equal toeholds but has a significant effect when toeholds are different. It demonstrates that increasing relative information precision of the bidder with a lower toehold makes both bidders to bid more aggressively and leads to a higher price. It also analyses the combined effect of toeholds and information precision on equilibrium bidding strategies and discusses the ways the target can increase the expected final price.

Keywords: takeover; toehold; common value auction; information precision

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## **1. Introduction**

When the medium of exchange in a takeover is limited to cash, the takeover contest is often modelled as an English outcry auction. There are, however, several features that distinguish takeover auctions from other types of English auctions. First, as Fishman (1988) noticed, the evaluation of the potential benefit of the successful takeover is a costly process. As a result, the first bidder with high valuation of the target firm may want to place a high jump bid to signal her value and deter potential competition. Such preemptive bidding explains significant initial bid premium in takeover auctions documented by Bradley (1980) and Eckbo (2009). It is also consistent with empirical studies of Jennings and Mazzeo (1993) and Betton and Eckbo (2000) who found that high initial bid makes single-bidder takeover contests more likely, and with experimental study of signalling in auctions with entry fees by Khoroshilov and Dodonova (2014). Arguing that not only value investigation, but also the bidding process itself may be costly, Hirshleifer and P'ng (1989) developed a model that explains a sequence of jump bids in takeover contests.

Another distinctive feature of takeover auctions is the ability of the target firm to reject all offers and enter into negotiation with the highest bidder. Such behavior is equivalent to a shill bidding, a situation when the seller bids on her own object (Chakraborty and Kosmopoulou, 2004), which is usually not allowed in many auctions but perfectly legal in takeover contests. Dodonova and Khoroshilov (2006) show that such ability of the target firm to reject all offers and demand an even higher price from the highest bidder makes bidders unwilling to place preemptive jump bids to signal their values. To restore signalling arguments for observed jump bidding in takeover auctions, Khoroshilov (2015) developed a model of 2-step information acquisition process that

limits the target's ability to use the information revealed during the course of the auction to rip all the benefits from the winning bidder.

Finally, one of the most important features of takeover auctions is the diverse ownership of the target and the ability of the potential acquirer to accumulate a certain percentage of the target's shares, a toehold, before making a tender offer. Burkart (1995) considers a private value takeover auctions with two strategic bidders in which one or both bidders own a fraction of the target firm. He shows that a bidder with a toehold bids above her valuation of the target firm, but such overbidding has no effect on the strategy of the bidder without toehold who still bids up to her valuation. In general, such overbidding benefits the target firm although the increase in the final price is very moderate. Singh (1998) also investigates the private value auctions with toeholds, but allows the winning bidder to renege on her bid. Given such possibility, the bidder with larger toeholds never quits the auction while the bidder without the toehold shades her bids, which, in turn, leads to lower expected price. Dodonova (2012) and Hounwanou (2018) analyze the combined effect of both toeholds and preemptive jump bidding on takeover auction process.

Bulow, Huang, and Klemperer (1999) consider common value takeover auctions with toeholds, which are more appropriate for takeover contests with financial bidders or when strategic bidders have similar plans for the target firm. Similar to private value auctions, they show that an increase in a toehold makes bidder to bid more aggressively. However, contrary to the case of private value auctions, Bulow, Huang, and Klemperer (1999) show that even small toeholds may have a large effect on bidding strategies and the auction's outcome. In particular, the bidder with the larger toehold bids more aggressively while the bidder with the lower toeholds usually

shades her bids. Given symmetric value function that satisfies increasing marginal revenue requirement, Bullock, Huang, and Klemperer (1999) show that unequal toeholds hurt the seller's expected revenue. As one of the remedies to increase the final price, they suggest the target firm to offer some of its equity to the bidder with the lower toehold at a substantial discount or even for free.

Empirical finding (see, e.g., Asquis and Kieschnick, 1999; Betton and Eckbo, 2000) supports the hypothesis of Bullock, Huang, and Klemperer (1999) that unequal toeholds lead to lower takeover premium. The hypothesis that the size of the toehold positively affects the probability of winning the auction (same for both private and common values) is also supported by Choi (1991). Finally, Le and Schultz (2007) show that the bidder's share price reaction on the takeover offer positively depends on her toehold.

In this paper we analyze the effect of information precision in common value takeover auctions with toeholds. Namely, we extend the Bullock, Huang, and Klemperer (1999) model to allow asymmetric value function, and, in particular, to allow one of the bidders to have more precise information about the target's value. While the effect of information precision in takeover contests have been previously studied, such studies usually focus on the information about the private value component in the bidder's valuation (Dionne, St-Amour, and Vencatachellum, 2009; Dionne, La Haye, and Bergeres, 2015; Povel and Singh, 2006), or, when combined with the effect of toeholds, to the effect that larger toeholds may have on the information precision (Povel and Sertsios, 2014).

To add the information precision to the common value takeover auction model, we assume that the bidders' signals may be weighted differently in determining the common value of the target firm, so that one of the bidders has more important (or, alternatively, more precise) information about the target's value than the other bidder does. We show that when bidders have no toeholds or have identical toeholds, such relative information precision does not affect the bidding strategies and the expected price<sup>1</sup>. Indeed, when both bidders have the same signal, the relative information precision has no effect on the object's value. As a result, if in an auction without toeholds a bidder knows that her opponent bids up to the value of her signal, she will use the same strategy too. An equal increase in both bidders' toeholds has identical effect on their bidding functions, keeping their bidding functions independent on information precision.

When bidders' toeholds are different, a bidder with larger toehold has more incentives to overbid and one can no longer assume that the auction ends up with a tie when both bidders have identical signals. As a result, the target's expected value at the time when one of the bidders drops out of the auction depends on whether the dropped out bidder has a more precise or less precise information and we show that in the case of unequal toeholds relative information precision has a significant effect on equilibrium bidding strategies and the expected final price of the target firm. In particular, we show that strategically releasing information to the bidder with the lower toehold, and, consequently, increasing her relative information precision, increases the selling price. Indeed, in equilibrium the bidder with larger toehold overbids while the bidder with lower toehold usually shades her bids. Such bidding behavior makes the latter bidder subject to a winner's curse while the former bidder faces a loser's curse. An increase in relative information precision of the "lower toehold" bidder makes the final price more sensitive to her own signal

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<sup>1</sup> This finding may be the reason why such information asymmetry has not been studied yet.

and less sensitive to the signal of her opponent. The latter reduces her exposure to the winner's curse and leads to an increase in her bidding function. The former makes the loser's curse more severe for the bidder with larger toehold and, as a result, leads to an increase in her bidding function too. An increase in both bidders' bidding functions leads to higher expected final price.

We also show that, when the target firm has a full control over the information precision, it is always able to achieve a higher expected selling price when toeholds are different than when the toeholds are the same or there are no toeholds at all. In addition, when the target has no control over the relative information precision but can offer free shares to one of the bidders, we show that, when the bidder with larger toehold has less precise information, offering her free shares (and, hence, increasing the difference between toeholds even further), increases the expected price<sup>2</sup>.

The rest of the paper is organized as follows. Part 2 presents a model of a common value takeover auction with two bidders who own a part of the target firm (a toehold) and whose private signals about the common value of the target may have different informativeness. It also solves the model and described equilibrium bidding strategies. Part 3 analyzes how the information precision and the relative size of toeholds (as well as interaction between these two parameters) affect the equilibrium bidding strategies and expected target's price. In Part 4 we conclude. All technical proofs and derivations are delegated to the Appendix.

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<sup>2</sup> Note that in the case of symmetric value function, i.e., when both bidders have equal information precision, Bullock, Huang, and Klemperer (1999) shows that the seller should offer free shares to the bidder with lower toehold and decrease the difference between toeholds.

## 2. The Model

Consider a common value takeover auction in which two potential acquirers (“bidders”) bid for a single target firm (“object”). The value of the object is given by  $V(S_1, S_2) = \alpha f(S_1) + (1 - \alpha) f(S_2)$ , where  $S_i \in [0, 1]$  is the private information (“signal”) observed by bidder  $i$ ,  $f(S_i)$  is an increasing differentiable function such that  $f(0) = 0$  and  $f(1) = 1$ , and  $\alpha$  is a parameter that measures the relative importance (or precision) of the first bidder’s information and satisfies  $0 < \alpha < 1$ . Since any distribution of  $S_i$  can be converted into uniform distribution by appropriate change in  $f(S_i)$ , assume, without loss of generality, that  $S_i$  is uniformly distributed on  $[0, 1]$  interval. The stand-alone value of the target firm is normalized to zero (or, alternatively, one can assume that the value function and all bids are measured as a premium in excess of the stand-alone target’s value). In addition, assume that bidder  $i$  owns a fraction  $\theta_i$  (a “toehold”) of the target firm with  $\theta_1 \geq \theta_2 > 0$ . The auction design is a clock-style English auction and, once one of the bidders drops out of the auction, the target firm’s shareholders (the “seller”) accept the offer of the remaining bidder at a price at which the losing bidder has dropped out. In case both bidders drop out simultaneously, assume the item is allocated at random, although such assumption is not important since, with continuous bidding functions, probability that both bidders drop out simultaneously is zero. We will be looking for a Nash Equilibrium in which the strategy of bidder  $i$  is given by a strictly increasing differentiable bidding function  $B_i(S_i)$ .

The above auction set-up is very closely related to the model of Bulow, Huang, and Klemperer (1999) with the exception that we do not require the value function to be symmetric. In contrast, our goal is to study how the asymmetry of the value function, and, in particular, the relative precision  $\alpha$  of the bidders' signals affects the equilibrium behavior and profit allocation, and to investigate any possible measures (such as strategic information disclosure or offering discounted shares to one of the bidders) that the target firm can implement to increase the expected final price.

To find the equilibrium bidding function  $B_i(S_i)$ , denote  $\Phi_j(S_i) = B_j^{-1}(B_i(S_i))$  to be such function that bidder  $i$  with signal  $S_i$  will win the auction if and only if  $S_j < \Phi_j(S_i)$ . Therefore, when bidder  $i$  with signal  $S_i$  chooses at which price level to drop out, she effectively chooses the signal level of the other bidder  $S_j$  such that she will win against a bidder with signal below  $S_j$ .

Therefore, the expected profit of the first bidder with signal  $S_1$  can be written as:

$$\pi_1(S_1) = \max_{S_2} \left\{ \int_0^{S_2} (\alpha f(S_1) + (1-\alpha)f(x) - (1-\theta_1)B_2(x)) dx + \theta_1(1-S_2)B_2(S_2) \right\} \quad (1)$$

Similarly, the expected profit of the second bidder with signal  $S_2$  can be written as:

$$\pi_2(S_2) = \max_{S_1} \left\{ \int_0^{S_1} (\alpha f(x) + (1-\alpha)f(S_2) - (1-\theta_2)B_1(x)) dx + \theta_2(1-S_1)B_1(S_1) \right\} \quad (2)$$

Solving optimization problems (1) and (2) yields

$$\frac{dB_2(S_2)}{dS_2} = \frac{1}{\theta_1(1-S_2)} (B_1(S_2) - \alpha f(S_1) - (1-\alpha)f(S_2)) \quad (3)$$

$$\frac{dB_1(S_1)}{dS_1} = \frac{1}{\theta_2(1-S_1)} (B_1(S_1) - \alpha f(S_1) - (1-\alpha)f(S_2)) \quad (4)$$

Noticing that since in equilibrium one must have  $B_1(1) = B_2(1) = 1$ , equations (3) and (4) have a unique solution and the equilibrium bidding strategies are given by the following Theorem.

Theorem 1: In a unique Nash Equilibrium the equilibrium bidding strategies are given by

$$B_1(S_1) = \frac{1}{\theta_2(1-S_1)^{1/\theta_2}} \int_{S_1}^1 (\alpha f(x) + (1-\alpha)f(\Phi_2(x))) (1-x)^{1/\theta_2-1} dx \quad (5)$$

$$B_2(S_2) = \frac{1}{\theta_1(1-S_2)^{1/\theta_1}} \int_{S_2}^1 (\alpha f(\Phi_1(x)) + (1-\alpha)f(x)) (1-x)^{1/\theta_1-1} dx \quad (6)$$

where

$$\Phi_i(S_j) \equiv B_i^{-1}(B_j(S_j)) = 1 - (1-S_j)^{\theta_j/\theta_i} \quad (7)$$

*Proof*: see appendix.

### 3. The Analysis

In a case of a symmetric value function ( $\alpha = 0.5$ ), Bulow, Huang, and Klemperer (1999) show that the bidder with larger toehold is more likely to win, bids up to a higher price than her

competitor with the same signal but a lower toehold, and that a bidder's bidding function increases with the size of her toehold for any value of her signal  $S_i < 1$ . As one can see from equations (5)-(7), an introduction of the asymmetry does not affect these results, i.e.,

Proposition 1: The bidder with the larger toehold bids more than the bidder with the same signal but lower toehold and is expected to win the auction more often, i.e, if  $\theta_1 > \theta_2$ , then  $B_1(x) > B_2(x)$  for any  $x \in (0,1)$  and  $\Pr(1st\ bidder\ wins) > 1/2$ . Also, an increase in the bidder's toehold increases her bidding function, i.e.  $\frac{\partial B_i(S_i)}{\partial \theta_i} > 0$  for any  $S_i \in [0,1)$ .

*Proof:* see appendix.

Intuitively, the bidder with larger toehold has more incentives to bid up the price since she has more to gain from the price increase if she loses and less to pay for the remaining shares if she wins. As a result, not only she bids more given the same signal, but also she is more likely to win the auction. An increase in a bidder's toehold always makes that bidder to bid more aggressively since she benefits both from selling a larger toehold at a higher price when she loses and from having to buy fewer shares if she wins. The effect of bidder's  $j$  toehold  $\theta_j$  on the other bidder's bidding function  $B_i(S_i)$  is ambiguous and may depend on the relative information precision  $\alpha$ , the component of the value function  $f(\cdot)$  and the bidder's own signal  $S_i$ . In particular, while the desire to sell non-zero toehold incentivise the bidder to bid up the price, the fact that the other bidder may bid above her expected value of the object results in a winner's curse and the desire to bid lower. The higher the toehold is, the more important the first effect is. As a result, the

bidder with a lower toehold faces a stronger winner's curse and may end up bidding below her expected value, especially when she has a high signal. Such underbidding makes the bidder with the higher toehold subject to the loser's curse making her to bid even more and leading to a spiral effect.

While many people's first impulse may be to believe that the bidder with more precise information has an advantage and is more likely to win the auction while the bidder with less precise information is more likely to bid more cautiously, equations (5)-(7) show that it is not true. In fact, the effect of relative information precision on one's bidding behavior (and not just the magnitude but also the direction of the change in one's bidding function) depends on which bidder has larger toehold. Furthermore, as equation (7) shows, the winner of the auction is determined only by the bidders' signals  $S_1$  and  $S_2$  and the ratio of their toeholds  $\theta_2/\theta_1$ , and does not depend on the relative information precision at all. In addition, when bidders have equal toeholds  $\theta \equiv \theta_1 = \theta_2$ , bidders' bidding functions become independent on  $\alpha$  and are given by

$$B_i(S_i) = \frac{1}{\theta(1-S_i)^{1/\theta}} \int_{S_i}^1 f(x)(1-x)^{1/\theta-1} dx.$$

The latter also implies that the final price is not affected by the relative information precision when  $\theta_1 = \theta_2$ . The following proposition summarizes the discussion above:

Proposition 2: The relative information precision  $\alpha$  does not affect the identity of the winning bidder given both bidders' signals. Furthermore, if both bidders have equal toeholds  $\theta_1 = \theta_2$ , then the relative information precision does not affect equilibrium bidding strategies  $B_i(S_i)$  and the expected final price.

To understand the intuition behind Proposition 2, consider an auction with no toeholds. In such an auction, the equilibrium strategy of any bidder is to bid up to the value of her own signal  $f(S_i)$ . The relative informativeness of the signal does not affect the bidding decision since it does not affect  $V$  at the critical point where  $S_1 = S_2$ . Since equal increase in toeholds have the same effect on both bidders' strategies, in equilibrium bidder  $i$  wins if and only if  $S_i > S_j$ , and, therefore,  $\Phi_i(S_j) = S_j$  for  $\theta_1 = \theta_2$ . The latter implies that  $\alpha$  still has no effect on  $V$  at the critical point  $S_1 = S_2$ , making the equilibrium bidding strategy and the expected final price independent on  $\alpha$ .

When bidders have different toeholds, the effect of information precision on bidding functions depends on whose toehold is the largest. Substitution (7) into (5) and (6) while noting that  $\Phi_1(x) < x < \Phi_2(x)$  for  $\theta_1 > \theta_2$  leads to the following result:

Proposition 3: An increase in bidder's relative information precision increases her bidding function when she has smaller toehold than the other bidder and decreases her bidding function

when her toehold is larger, i.e., if  $\theta_1 > \theta_2$  then  $\frac{\partial B_i(S_i)}{\partial \alpha} < 0$  for  $i \in \{1, 2\}$  and  $S_i \neq 1$

Proposition 3 states that higher information quality does not necessary lead to more aggressive bidding while bidder with less precise information does not always shade her bids. Intuitively, there are two things that affect bidding functions in opposite directions: the desire to bid up the

price to be able to sell the existing toehold more expensively and the winner's curse that the winner may experience when her opponent bids above her expected value of the object. The larger the toehold is, the more important the first effect is and, consequently, the more the bidder is willing to bid above her expected value. Such overbidding makes winner's curse more severe for the bidder with the lower toehold, making her often to shade her bids. A decrease in relative information precision of the bidder with lower toehold (who suffers from the winner's curse due to aggressive bidding by her opponent) makes the winner's curse problem more severe to her, leading for additional shading of her bids. Such lower bidding reduces the possible gain that the bidder with larger toehold can receive from bidding up the price as well as decrease her exposure to the loser's curse, making her to reduce her bids too.

**Figure 1:** The effect of information precision  $\alpha$  on bidding functions when  $\theta_1 = 10\%$ ,  $\theta_2 = 2\%$  and the value function is linear  $V(S_1, S_2) = \alpha S_1 + (1 - \alpha) S_2$

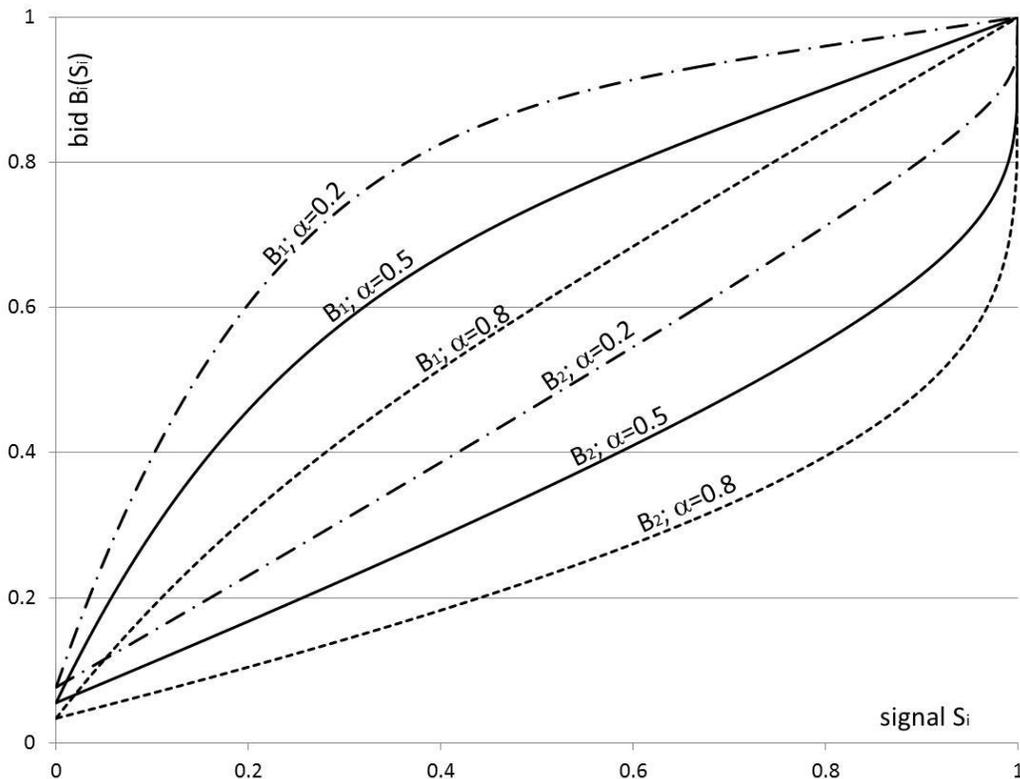


Figure 1 presents bidding functions  $B_i(S_i)$  for a linear value function  $V(S_1, S_2) = \alpha S_1 + (1 - \alpha) S_2$  when  $\theta_1 = 10\%$ ,  $\theta_2 = 2\%$  and three different relative information precision values:  $\alpha = 0.2$ ,  $\alpha = 0.5$ , and  $\alpha = 0.8$ . As it can be seen from this Figure, the effect of information precision is economically significant for the chosen toehold values. Note that when bidders have equal toeholds  $\theta \equiv \theta_1 = \theta_2$  and the value function is linear, both bidding functions are given by  $B_i(S_i) = \theta/(1 + \theta) + S_i/(1 + \theta)$ , i.e., a straight line connecting  $B_i(0) = \theta/(1 + \theta)$  and  $B_i(1) = 1$ , and they are independent of  $\alpha$ .

If shareholders of the target firm are able to affect the relative information precision by strategically releasing information to one of the bidders, Proposition 3 will lead to an important implication:

Proposition 4: If  $\theta_1 > \theta_2$ , then the final price negatively depends on the first bidder's relative information precision  $\alpha$  for any realized signals  $S_1$  and  $S_2$ , and, consequently, the expected final price decreases with  $\alpha$ .

Bullow, Huang, and Klemperer (1999) show that, when the value function satisfies the increasing marginal revenue requirement and the size of the toeholds are sufficiently small, the expected price become smaller as the difference between toeholds become larger. As one of the possible way to increase the expected price, Bullow, Huang, and Klemperer (1999) suggest the target firm to offer its shares to the bidder with the lower toehold at a discount of even for free.

As Proposition 4 shows, a cheaper way to increase the expected price would be to provide additional information to such bidder.

Using Theorem 1 and equations (1) and (2), the expected bidders' profits  $\pi_i(S_i)$  and the expected price  $P$  can be written as (see appendix for the proof):

$$\pi_1(S_1) = \theta_1 B_1(0) + \alpha \int_0^{S_1} \left(1 - (1-x)^{\theta_1/\theta_2}\right) f'(x) dx \quad (8)$$

$$\pi_2(S_2) = \theta_2 B_2(0) + (1-\alpha) \int_0^{S_2} \left(1 - (1-x)^{\theta_2/\theta_1}\right) f'(x) dx \quad (9)$$

$$P = \frac{E(f(x)) - E(\pi_1(S_1) + \pi_2(S_2))}{1 - \theta_1 - \theta_2} = \quad (10)$$

$$= \frac{E(f(x)) - \theta_1 B_1(0) - \theta_2 B_2(0) - \int_0^1 \int_0^y \left(1 - (1-x)^{\theta_2/\theta_1}\right) f'(x) dx dy - \alpha \int_0^1 \int_0^y \left((1-x)^{\theta_2/\theta_1} - (1-x)^{\theta_1/\theta_2}\right) f'(x) dx dy}{1 - \theta_1 - \theta_2}$$

where

$$B_1(0) = B_2(0) = \frac{1}{\theta_1} \int_0^1 \left( \alpha f \left(1 - (1-x)^{\theta_2/\theta_1}\right) + (1-\alpha) f(x) \right) (1-x)^{1/\theta_1-1} dx \quad (11)$$

When  $\theta_1 > \theta_2$ , Propositions 3 and 4 show that both the bidding functions and the expected price negatively depend on the first bidder's relative information precision  $\alpha$ . As it can be seen from Equation (9), the expected profit of the second bidder also negatively depends on her rival's information precision. Indeed, an increase in  $\alpha$  reduces both the price the second bidder receives

for her toehold when she loses and the gain she receives when she wins. The effect of  $\alpha$  on the profit of the first bidder is ambiguous. On one hand, since increase in  $\alpha$  leads to lower bidding functions, she receives less for her toehold when she loses. On the other hand, her profit from winning positively depends on her information precision. When her signal  $S_1$  is small, the probability of her losing the auction is high and the first effect dominates the second one. When  $S_1$  is high, the second effect dominates the first one and her expected profit increases with  $\alpha$ . Ex-ante (before the first bidder observes her signal), however, she is more likely to win the auction and her ex-ante expected profit positively depends on her relative information precision. Saying it differently, while the first bidder, on average, benefits when her information precision improves, she does not do so all the time and better information precision hurts her when her signal is low and she is bidding up the price in hope to lose and sell her toehold. The following proposition summarizes the discussion above:

Proposition 5: If  $\theta_1 > \theta_2$ , then the expected profit of the second bidder positively depends on her

own information precision  $(1-\alpha)$  for any signal  $S_2$ , i.e.,  $\frac{d\pi_2(S_2)}{d\alpha} < 0$ . An increase in the first

bidder's information precision  $\alpha$  positively affects her own profit when her signal  $S_1$  is high

and negatively when it is low, i.e., there is such  $\bar{S}_1 \in (0,1)$  that  $\frac{d\pi_1(S_1)}{d\alpha} < 0$  for any  $S_1 < \bar{S}_1$  and

$\frac{d\pi_1(S_1)}{d\alpha} > 0$  for any  $S_1 > \bar{S}_1$ . Furthermore, an increase in the first bidder's information precision

$\alpha$  increases her ex-ante expected profit, i.e.,  $\frac{dE(\pi_1(S_1))}{d\alpha} > 0$ .

*Proof:* see appendix.

Propositions 2 and 4 state that the ability of the seller to affect the relative information precision  $\alpha$  by strategically releasing information to one of the bidders has no effect on the final price when both bidders have equal toeholds, but increases the price when the information is released to the bidder with the lower toehold. Furthermore, as equation (10) confirms, the benefit of such information release is higher when the toeholds are substantially different from each other than when toeholds are relatively similar. In particular, the following result is true:

Proposition 6: Let  $\lambda = \theta_2/\theta_1 < 1$ . Then a positive effect that an increase in the second bidders information precision  $(1-\alpha)$  has on the expected price becomes smaller when  $\lambda$  increases

through the increase in the first bidder's toehold, i.e.,  $\frac{\partial^2 P}{\partial(1-\alpha)\partial\lambda}\Big|_{\theta_2=\text{constant}} < 0$ . Furthermore, if

toeholds are sufficiently small then such effect also becomes smaller when  $\lambda$  increases through

the decrease in the second bidder's toehold, i.e., there is such  $\bar{\theta}_1$  that  $\frac{\partial^2 P}{\partial(1-\alpha)\partial\lambda}\Big|_{\theta_1=\text{constant}} < 0$  for

any  $\theta_1 < \bar{\theta}_1$ .

*Proof:* see appendix.

How effective the strategic information release can be and can it overcome any possible damage to the selling price that unequal toeholds can bring? For  $\theta_1 > \theta_2$ , Proposition (4) and equation (10) imply that when the target firms shareholders have a full control over the relative information precision they must make the second bidder's information as precise as possible, i.e, make  $\alpha$  as small as possible. Furthermore, such strategy is more effective when the toeholds are

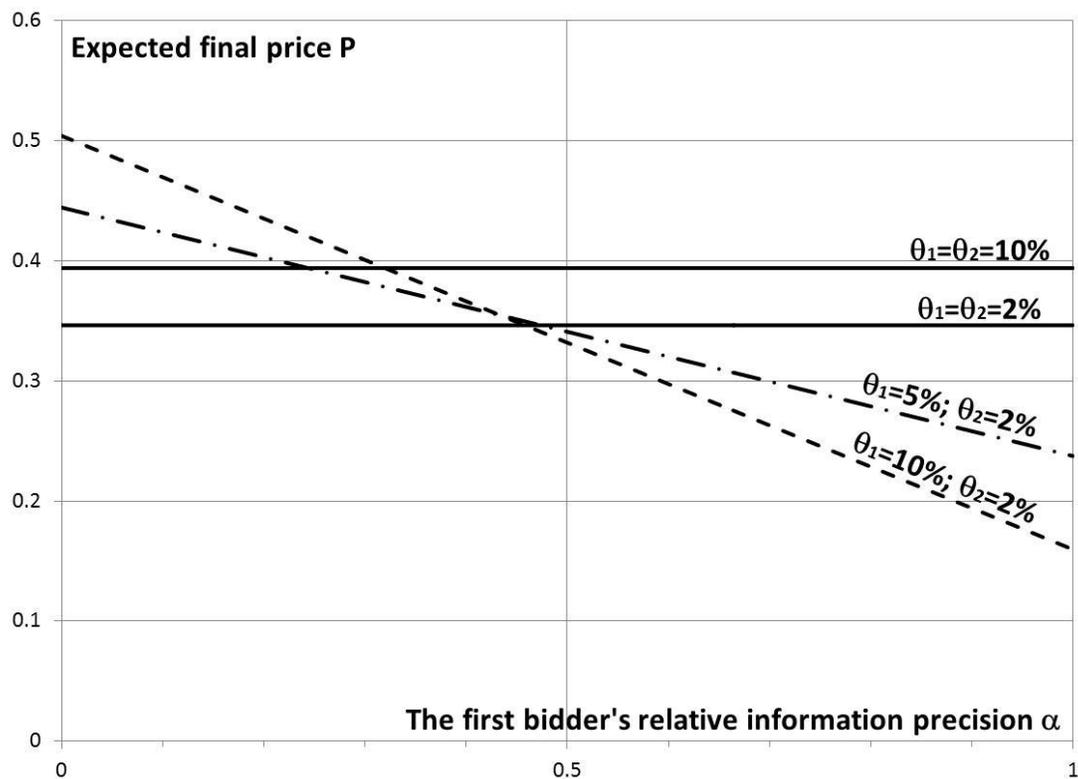
substantially different from each other, i.e., when the toeholds ratio  $\lambda = \theta_2/\theta_1$  is small. When both toeholds are small, the incentive to bid up the price to sell the toeholds more expensively is small and an increase in expected selling price when both bidders have low signals is overweighed by the decrease in price when bidders have high signals. Hence, when both toeholds are small and the target firm is able to control the information precision it prefers unequal toeholds to equal or no toeholds. Proposition 7 summarizes this result:

Proposition 7: If the target firm's shareholders have a full control over the relative information precision  $\alpha$ , then for any  $\lambda < 1$  there is a  $\bar{\theta}$  such that for any  $\theta_1 < \bar{\theta}$  the expected price is higher when  $\theta_2 = \lambda\theta_1 < \theta_1$  than when  $\theta_2 = \theta_1$

*Proof*: see appendix

Figure 2 presents the effect of the first bidders' relative information precision  $\alpha$  on the expected final price when the bidders' value function is linear  $V(S_1, S_2) = \alpha S_1 + (1 - \alpha) S_2$  for the case of equal large toeholds  $\theta_1 = \theta_2 = 10\%$ , equal small toeholds  $\theta_1 = \theta_2 = 2\%$ , unequal toeholds with high toehold ratio  $\theta_1 = 10\%$ ,  $\theta_2 = 2\%$ , and unequal toeholds with small toehold ratio  $\theta_1 = 5\%$ ,  $\theta_2 = 2\%$ . It illustrates the results of Propositions 2 and 4 that  $\alpha$  does not affect the expected price in case of equal toeholds but has a negative effect on it when the first bidder has a larger toehold. It also illustrates the results of Propositions 6 and 7 that the effect of  $\alpha$  is larger when toeholds are less equal and that when the seller has a full power over  $\alpha$  she prefers unequal toeholds. Furthermore, for a given set of parameters Figure 2 shows that information precision has greater effect on the expected price than the toehold ratio does.

**Figure 2:** The effect of information precision  $\alpha$  on the expected price for different values of toeholds and linear value function  $V(S_1, S_2) = \alpha S_1 + (1 - \alpha) S_2$



Can the target firm's shareholders increase the expected price if they have no control over the relative information precision  $\alpha$ ? Bulow, Huang, and Klemperer (1999) show that when the value function is symmetric ( $\alpha = 0.5$ ) and both toeholds are small, they can do so by offering a discounted or free shares to the bidder with the lower toehold, hence, reducing the difference in toeholds. When one of the bidders has information advantage, such strategy is no longer correct. According to Propositions 6 and 7, shareholders of the target firm prefer a situation when the bidder with larger toehold has an information disadvantage, and such disadvantage has the highest positive effect on the target's price when the toeholds are significantly different from

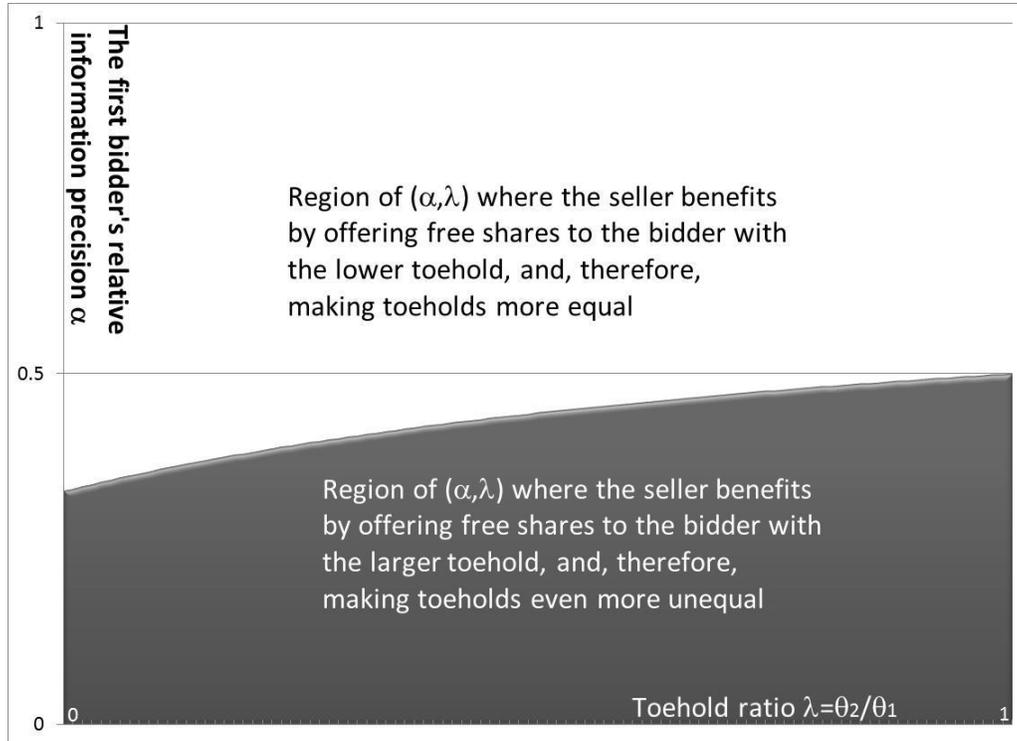
each other. Hence, one can expect that given a sufficient difference in information quality, shareholders of the target firm may benefit from offering free shares to the bidder with larger toehold but information disadvantage and, by doing so, would increase the difference between toeholds even further. The following Proposition provides the exact condition for such strategies:

**Proposition 8:** Let  $\theta_2 = \lambda\theta_1 < \theta_1$ . Then there is  $\bar{\theta}$  such that for any  $\theta_1 < \bar{\theta}$  there is  $\bar{\alpha}(\lambda)$  and  $\varepsilon(\lambda, \bar{\theta})$  which satisfy  $0 < \bar{\alpha}(\lambda) < 1$  and  $\lim_{\bar{\theta} \rightarrow 0} (\varepsilon(\lambda, \bar{\theta})) = 0$  such that  $\frac{d\pi_s}{d\lambda} < 0$  for any  $\alpha < \bar{\alpha}(\lambda) - \varepsilon(\lambda, \bar{\theta})$  and  $\frac{d\pi_s}{d\lambda} > 0$  for any  $\alpha > \bar{\alpha}(\lambda) + \varepsilon(\lambda, \bar{\theta})$ , where  $\lambda = \theta_2/\theta_1$  and  $\pi_s = P \times (1 - \theta_1 - \theta_2)$  is the expected profit of the non-bidding (i.e., excluding both bidders) target firm's shareholders.

*Proof: see appendix*

In a less complicated terms, Proposition 8 states that when the first bidder has larger toehold, i.e.,  $\theta_1 > \theta_2$ , while both toeholds are sufficiently small, there is a critical value of information precision  $\bar{\alpha}(\lambda)$  that may depend on the current toehold ratio  $\lambda = \theta_2/\theta_1$  such that shareholders of the target firm can increase their profit by offering free shares to the first bidder (and, hence, increasing the difference between toeholds even further), if the first bidder's information precision is less than its critical value  $\bar{\alpha}(\lambda)$ , but they should offer free shares to second bidder if the first bidder information precision is above  $\bar{\alpha}(\lambda)$ .

**Figure 3:** Target’s decision to offer free or discounted shares to one of the bidders as a function of the toehold ratio  $\lambda = \theta_2/\theta_1 \in [0,1]$  and the first bidder’s relative information precision  $\alpha$  in a case of a linear value function  $V(S_1, S_2) = \alpha S_1 + (1-\alpha)S_2$  and small toeholds



In case of a linear value function  $V(S_1, S_2) = \alpha S_1 + (1-\alpha)S_2$ , one can use equation (10) for  $\lambda = \theta_2/\theta_1$  and infinitesimal  $\theta_i$  to find the explicit formula for  $\bar{\alpha}(\lambda) = \frac{2\lambda + 1}{3\lambda + 3}$ . For this linear example, Figure 3 illustrates the seller’s decision to provide free shares to either the first (with larger toehold) or the second (with smaller toehold) bidder as a function of the toehold ratio  $\lambda = \theta_2/\theta_1$ . As it can be seen from this Figure, even a small information disadvantage of the bidder with the larger toehold makes non-buying target firm’s shareholders willing to offer free or discounted shares to such bidder and, as a result, to make toeholds even less equal. Furthermore, when the second bidder has at least twice as precise information as the first bidder

does, i.e., when  $\alpha \leq 1/3$ , the target firm's shareholders want to make the toeholds as distinct as possible so that the ratio  $\lambda = \theta_2/\theta_1$  becomes zero. Note, however, that Figure 3 and Proposition 8 assume that both toeholds are sufficiently small, so, the target cannot decrease  $\lambda$  to an arbitrary small number by offering discounted shares to the first bidder since it will make  $\theta_1$  too large to ignore additional effects in (10). An alternative approach to decreasing  $\lambda = \theta_2/\theta_1$  while keeping toeholds sufficiently small would be buying back shares from the second bidder at a premium, hence, reducing  $\theta_2$ . This approach, however, may be hard to implement since the second bidder can always replenish her toehold by buying shares on the open market.

#### **4. Conclusion**

This paper extends the model of Bullow, Huang, and Klemperer (1999) of common value takeover auctions with toehold by incorporating the possible asymmetry in information precision. It shows that information precision has no effect on the equilibrium bidding functions and the expected target's price when both bidders have identical toeholds or no toeholds at all; however, it has a significant effect on both the bidding strategies and expected price when bidders' toeholds are not the same. It shows that an increase in the relative information precision of the signal that bidder with lower toeholds receives makes both bidders to bid more aggressively and leads to higher expected target's price. As a result, the target firm may increase its expected revenue by strategically releasing information to the bidder with lower toeholds. We further show that the large the difference between toeholds is, the more beneficial such strategic information release is. Furthermore, if such information release can lead to a large effect on information precision, the target firm's shareholders always prefer bidders having different

toeholds to having identical toeholds or no toeholds at all. Finally, we analyze how the change in toehold ratio affects the expected target's revenue and show that when the bidder with larger toehold has an information disadvantage, the target firm's shareholders may benefit by offering such bidder discounted shares and increasing her toehold, thus, making the difference between toeholds even larger.

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## **Appendix:**

### Proof of Theorem 1:

To solve (3) and (4), note that dividing (4) by (3) while taking  $S_2 = \Phi_2(S_1)$ , and, dividing (4) by (3) while taking  $S_1 = \Phi_1(S_2)$  yields

$$\frac{B'_i(S_i)}{B'_j(\Phi_j(S_i))} = \frac{\theta_i(1-\Phi_j(S_i))}{\theta_j(1-S_i)} \quad (\text{A1})$$

where  $B'_i(x)$  is the first derivative of  $B_i(x)$  with respect to its argument. At the same time, taking a full differential of  $B_i(S_i) = B_j(\Phi_j(S_i))$  yields

$$\frac{B'_i(S_i)}{B'_j(\Phi_j(S_i))} = \frac{d\Phi_j(S_i)}{dS_i} \quad (\text{A2})$$

Substituting (A2) into (A1) results in a differential equation

$$\frac{d\Phi_j(S_i)}{dS_i} = \frac{\theta_i(1-\Phi_j(S_i))}{\theta_j(1-S_i)} \quad (\text{A3})$$

Since in equilibrium a bidder with signal  $S_i = 1$  never wants to lose the auction, it leads to a boundary condition  $\Phi_j(1) = 1$  and a unique solution to (A3):

$$\Phi_j(S_i) = 1 - (1 - S_i)^{\theta_i/\theta_j} \quad (\text{A4})$$

Substituting (A4) into (3) and (4) and solving the corresponding ordinary differential equations with a boundary condition  $B_i(1) = 1$  results in (5) and (6).

Proof of Proposition 1:

Note that if  $\theta_1 > \theta_2$  then  $\Phi_1(x) < x < \Phi_2(x)$ . Therefore,

$$\begin{aligned} B_1(x) &= \frac{1}{\theta_2(1-x)^{1/\theta_2}} \int_x^1 (\alpha f(y) + (1-\alpha)f(\Phi_2(y))) (1-y)^{1/\theta_2-1} dy > \\ &> \frac{1}{\theta_2(1-x)^{1/\theta_2}} \int_x^1 f(y) (1-y)^{1/\theta_2-1} dy > \frac{1}{\theta_1(1-x)^{1/\theta_1}} \int_x^1 f(y) (1-y)^{1/\theta_1-1} dy > \\ &> \frac{1}{\theta_1(1-x)^{1/\theta_1}} \int_x^1 (\alpha f(\Phi_1(y)) + (1-\alpha)f(y)) (1-y)^{1/\theta_1-1} dy = B_2(x) \end{aligned} \quad (\text{A5})$$

Substituting (7) into (5) and (6) immediately shows that  $B_i(S_i)$  increases with  $\theta_i$  for any  $S_i$ .

Finally,  $\Phi_1(x) < \Phi_2(x)$  implies

$$\Pr(1st\ bidder\ wins | S_1 = x) = \Phi_2(x) > \Phi_1(x) = \Pr(2nd\ bidder\ wins | S_2 = x) \quad (A6)$$

And, therefore,  $\Pr(1st\ bidder\ wins) > 1/2 > \Pr(2nd\ bidder\ wins)$

Proof of formulas (8)-(11):

Applying envelope theorem to equation (1) results in

$$\frac{d\pi_1(S_1)}{dS_1} = \int_0^{\Phi_2(S_1)} \alpha \frac{df(S_1)}{dS_1} dx = \alpha f'(S_1) \Phi_2(S_1) \quad (A7)$$

Therefore,

$$\pi_1(S_1) = \pi_1(0) + \alpha \int_0^{S_1} f'(x) \Phi_2(x) dx \quad (A8)$$

Since  $\pi_1(0) = \theta_1 B_1(0)$ , equation (A8) is the same as equation (8). Similarly, equation (9) follows from applying envelope theorem to (2). Equation (10) follows from (9) and (10) by integrating (9) and (10) over all  $S_1$  and  $S_2$  respectively, and noticing that the total profit off all bidders and

the remaining  $(1-\theta_1-\theta_2)$  shareholders is equal to  $E(f(x))$ . Finally, to show that

$B_1(0)=B_2(0)$  one can rewrite the integral in (6) by changing the variable of integration

$$y=1-(1-x)^{\theta_2/\theta_1}.$$

Proof of Proposition 5:

From Proposition 2,  $\frac{\partial B_i(S_i)}{\partial \alpha} < 0$  for  $i \in \{1,2\}$  and  $S_i \neq 1$ . Using this result for  $B_i(0)$ , equation

(9) immediately leads to  $\frac{d\pi_2(S_2)}{d\alpha} < 0$ .

Since  $\frac{d\pi_2(S_2)}{d\alpha} < 0$  for any  $S_2$ , it follows that  $\frac{dE(\pi_2(S_2))}{d\alpha} < 0$ . Furthermore, since

$$E(\pi_1(S_1))+E(\pi_2(S_2))+P \times (1-\theta_1-\theta_2)=E(f(x))$$
 does not depend on  $\alpha$  and  $\frac{dP}{d\alpha} < 0$ , it

follows that  $\frac{dE(\pi_1(S_1))}{d\alpha} > 0$

From (8), it follows that

$$\frac{d\pi_1(S_1)}{d\alpha} = \int_0^1 \left( f\left(1-(1-x)^{\theta_2/\theta_1}\right) - f\left(1-(1-x)^{\theta_1/\theta_2}\right) \right) + \int_0^{S_1} \left(1-(1-x)^{\theta_1/\theta_2}\right) f'(x) dx \tag{A9}$$

Note that  $\frac{d\pi_1(0)}{d\alpha} < 0$  and  $\frac{d\pi_1(S_1)}{d\alpha}$  is a continuous increasing function of  $S_1$ . Furthermore, since

$\frac{dE(\pi_1(1))}{d\alpha} > 0$ , there is such  $\bar{S}_1 \in (0,1)$  that  $\frac{d\pi_1(S_1)}{d\alpha} < 0$  for any  $S_1 < \bar{S}_1$  and  $\frac{d\pi_1(S_1)}{d\alpha} > 0$  for any  $S_1 > \bar{S}_1$ .

### Proof of Proposition 6:

From (10), it follows that

$$\frac{dP}{d(1-\alpha)} = \frac{\int_0^1 \int_0^y \left( (1-x)^\lambda - (1-x)^{1/\lambda} \right) f'(x) dx dy}{1 - \theta_1 - \theta_2} \quad (\text{A10})$$

An increase in  $\lambda = \theta_2/\theta_1$  always reduces  $(1-x)^\lambda - (1-x)^{1/\lambda}$ . If such increase in  $\lambda$  is attributed to the decrease in  $\theta_1$ , then  $1 - \theta_1 - \theta_2$  increases and  $\frac{dP}{d(1-\alpha)}$  decreases. If  $\theta_1$  is fixed, then

$$\begin{aligned} & \lim_{\theta_1 \rightarrow 0} \left( \frac{\partial^2 P}{\partial(1-\alpha)\partial\lambda} \Big|_{\theta_1 = \text{constant}} \right) = \\ & = \lim_{\theta_1 \rightarrow 0} \left( \frac{\frac{d \int_0^1 \int_0^y \left( (1-x)^\lambda - (1-x)^{1/\lambda} \right) f'(x) dx dy}{d\lambda} (1 - (1+\lambda)\theta_1) + \theta_1 \int_0^1 \int_0^y \left( (1-x)^\lambda - (1-x)^{1/\lambda} \right) f'(x) dx dy}{(1 - \theta_1 - \theta_2)^2} \right) = \\ & = \frac{d \int_0^1 \int_0^y \left( (1-x)^\lambda - (1-x)^{1/\lambda} \right) f'(x) dx dy}{d\lambda} < 0 \end{aligned} \quad (\text{A11})$$

Proof of Proposition 7:

The proof follows directly from equation (7) since

$$\lim_{\theta \rightarrow 0} \left( \sup_{\alpha} \left( P \Big|_{\theta_2 = \lambda \theta_1} \right) \right) = \int_0^1 \int_0^y \left( 1 - (1-x)^{\theta_2/\theta_1} \right) f'(x) dx dy > \int_0^1 \int_0^y x f'(x) dx dy = \lim_{\theta \rightarrow 0} \left( P \Big|_{\theta_2 = \theta_1} \right) \quad (\text{A12})$$

Proof of Proposition 8:

Consider a specific value of  $\lambda < 1$  and denote

$$Q(\alpha) \equiv \frac{d \left( \lim_{\bar{\theta} \rightarrow 0} \left( \pi_S \Big|_{\theta_2/\theta_1 = \text{constant}} \right) \right)}{d\lambda} \quad (\text{A13})$$

From equation (10) it follows that

$$Q(\alpha) = \frac{d \left( \int_0^1 \int_0^y (1-x)^{\lambda} f'(x) dx dy \right)}{d\lambda} - \alpha \frac{d \left( \int_0^1 \int_0^y \left( (1-x)^{\lambda} - (1-x)^{1/\lambda} \right) f'(x) dx dy \right)}{d\lambda} \quad (\text{A14})$$

Note that  $Q(0) < 0$ ,  $Q(1) > 0$ , and  $\frac{Q(\alpha)}{d\alpha} > 0$ . Hence, there is a threshold  $\bar{\alpha}$  such that an increase

in  $\lambda$  increases  $\pi_S$  for any  $\alpha > \bar{\alpha} + \varepsilon(\bar{\theta})$  and decreases  $\pi_S$  for any  $\alpha < \bar{\alpha} - \varepsilon(\bar{\theta})$  where  $\varepsilon(\bar{\theta})$

is an infinitesimal function of  $\bar{\theta}$ , i.e.,  $\lim_{\bar{\theta} \rightarrow 0} \left( \varepsilon(\bar{\theta}) \right) = 0$ .