Revenue Maximizing Mechanisms for Interruptible Services*

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January 15, 2019

Abstract

We study selling mechanisms by a monopolist for imperfectly durable, interruptible and homogeneous goods. Having the infrastructure as a service public cloud computing market as a motivating example, we show that when the seller can commit over pricing strategies, she can exploit the time that buyers of different private valuations want to consume the good. Under certain conditions, by offering a randomized mechanism that incorporates the risk of interruption, the seller can improve her revenue in comparison to the standard deterministic mechanisms proposed by Myerson (1981) and Maskin and Riley (1989). Risk of interruption can lead to effective and profitable price discrimination. The mechanism can be implemented by simultaneously allocating the good through a posted price and an auction where buyers face the risk of interruption. Auctioning the goods can be designed so as to incorporate the risk for the winners of losing access to their service while it is still in operation. The posted price mechanism can by construction eliminate that risk. Buyers of high valuations prefer to pay a risk premium and get the service through the posted price mechanism while buyers of low valuations unable to meet the price level of the risk premium enter the auction.

Keywords: Auctions, Price Discrimination, Optimal Mechanism.
JEL-codes: D44, L81, D47.

*I am very thankful to Renato Gomes, Toomas Hinnosaar, Sergei Izmalkov, Paul Klemperer, Wynne Lam, Joana Resende, Patrick Rey and the participants of the lunch seminars at Carlos III, CEMFI and Compass Lexecon as well as of the Ninth IDEI-TSE-IAST conference on the Economics of Intellectual Property, Software and the Internet in Toulouse, the Bruegel Research Meeting in Brussels, the 5th World Congress of Game Theory in Maastricht, the EARIE meeting in Lisbon and the European Meeting of the Econometric Society in Cologne for their useful comments and suggestions.

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1 Introduction

This paper inspired by the Infrastructure as a Service (IaaS) cloud computing market studies selling mechanisms of durable, homogeneous goods that can be interrupted during their usage. We develop a seller's revenue maximizing mechanism which allows the monopolist to price discriminate between buyers of different private valuations through the imposed risk of interruption. The mechanism can be designed in a way that high valuation buyers prefer to pay a risk premium in order to avoid the risk of loosing their purchased goods while the low valuation buyers prefer face the risk of interruption instead of paying the premium. The risk of interruption is essential for more effective price discrimination and can maximize seller's revenue. Crucial for this is that the duration for which a buyer wishes to consume the good or service depends on his valuation through a function that is common knowledge. Due to the duration dimension, the seller has more flexibility in offering pricing options that allows her to extract more rents out of buyers of different valuations. An example of how such a mechanism can be implemented is the IaaS public cloud computing pricing schedules offered by Amazon.

1.1 A Motivating Example: The Infrastructure as a Service Public Cloud Computing Market

IaaS public cloud computing is a fast growing market, leaping up 33% in 2015 to become an estimated $16.5 billion market, according to research firm Gartner[1]. In IaaS market, providers offer physical or virtualized hardware in the form of storage, servers, network, firewalls and load balancers. This is very useful and popular for small scale businesses and startups as they cannot afford to buy much costlier hardware components (built on premise) when they enter the market.

The dominant provider of IaaS services is Amazon[2] which enjoyed in the past extraordinary high market shares of more than 80% of the IaaS market (due to its first mover advantage). Amazon's Elastic Compute Cloud (EC2) has hosted numerous well-known internet companies and websites, such as Expedia, Airbnb, Lyft, Netfix and Adope Systems. The basic unit of computation on EC2 is a virtual machine, known as an instance. Users can specify certain parameters about the hardware and location where their instances will run, and also have several available purchasing options.

Initially, EC2 only offered a posted price selling mechanism, so that buyers could have guaranteed access to the virtual machines by paying a fixed non-discriminatory hourly rate. With only a posted price mechanism , Amazon clearly had frequent slack capacity, and to utilize this in December 2009 introduced spot instances.

Spot instances allow Amazon to auction of excess capacity. To use spot instances, buyers place a spot instance request, specifying, the number of spot instances they want to

[1] http://www.gartner.com/newsroom/id/3055225

[2] Amazon was the first to initiate its Cloud services in August 2006 offering access on a first-come, first-served basis.

run, and the maximum price they are willing to pay per instance hour. Amazon changes the spot price periodically based on supply and demand. When a user's bid is above the current spot price, her instances get scheduled, and run until either they complete, or until the spot price rises above the bid, in which case the instances are automatically interrupted[3].

Amazons description of How Spot Instances Work[4] reveals that spot prices are set through a $(q+1)$th uniform price, sealed-bid auction, in which q is the number of available units in the auction. The bidders are neither aware about the number q nor the total number of bidders or their identity. So, demand conditions and capacity constraints at each point of time are only known to the seller. Bidders only observe the equilibrium price of the auction (as well as the evolution of auction prices for a period up to 90 days).

As in a price auction of multiple goods, each client bids for the desired number of goods (spot instances). The seller/provider chooses the top q bids. She may set q up-front on the basis of available capacity, or, she might retroactively set q after receiving the bids,to maximize revenue. In any case, q cannot exceed the available capacity. The provider sets the uniform price to the price declared by the highest bidder who did not win the auction ($q+1$th bid) and publishes it. The bidders of the q winning bids pay the published price and their instances start running. In this case, the published price is a price-bid by an actual client.

The provider may also decide to ignore bids below a a hidden reserve price or equivalently to reduce the units q offered in the spot market, to prevent the goods from being sold cheaply, or to give the impression of increased demand. The number of auctioned units are in strict relationship with the price at which they are sold and therefore determine the seller's (hidden) reserve participation price. So, the number of units q in the auction can be considered as a choice variable of the auctioneer.

Note an important difference between the posted price selling mechanism and the spot market: If the buyer chooses the posted price, she enjoys a constant price over time, and its instance(s) will never be terminated against his/her will. In contrast, spot users bear the risk of price fluctuations and having their running instances terminated whenever the spot price rises above their bids.

While the motivation for the introduction of the spot market was to eliminate the waste of slack resources, the combination of a simultaneous posted price mechanism with the spot market given that each buyer knows only privately her valuation and does not observe the bids of the other buyers in the (sealed-bid) auction raise the question how the provider can design the selling mechanism that accommodates both selling options in an incentive-compatible and revenue maximizing way.

Note that the Amazon's EC2 is currently the only provider of IaaS cloud services that use a hybrid of posted price and spot market pricing scheme. The other providers including the fast growing Google's Compute Engine and Microsoft's Azure who entered the market

[3]Users are informed few seconds before the interruption takes place so that they can remove their data from the virtual machine before the interruption.

[4]http://aws.amazon.com/ec2/spot-instances/

in 2013[5] adopt a posted price mechanism only[6]. So, a natural question to ask is why the other providers did not adopt the spot market pricing option and whether the large market share of Amazon can explain this difference in pricing options.

1.2 Motivation and Contribution

As it has been pointed out by the literature, for homogeneous goods, the optimal mechanism for their allocation is deterministic[7]. The risk of interruption introduces randomization in the selling mechanism. So, our results can be interpreted as a indication that when goods to sell are durable and can be interrupted, under certain conditions, stochastic mechanisms can dominate deterministic ones (evaluated at the optimal values of parameters).

Damaged good literature[8] provides motives for the monopolist to "damage" units of the good and sell two varieties of it (one damaged and one non-damaged) at two different posted prices. As we show below, even if the good to sell is homogeneous and there is no any damaged units of it, the seller can still price discriminate in a profitable way.

When a service is suddenly interrupted while it is still in use by the buyer, there is a termination cost which decreases the valuation of the buyer for the service. For example, buyers can be considered as downstream firms that use upstream services for their transactions with final consumers. Due to interruption of services[9], buyers may not be in the position to serve efficiently the market and they will incur some loses (e.g. damaged reputation, inability to meet commitments and deadlines which sometimes are enforced by contracts with the final consumers, inconvenience and working cost of transferring the data from virtual machines when they are about to shut down). Since the duration of consuming the good or service depends on the valuation a buyer, it also affects the expected risk of interruption. It is exactly this channel through which a static randomized mechanism can do better than the deterministic one proposed by Myerson (1981).

The implementation of this mechanism can justify the use of a posted price and auction for the simultaneous allocation of goods and services as for example happens in the IaaS market Amazon's pricing schedule. The auctioneer can link the uncertainty over the usage of the service in the auction with the choice of the number of auctioned units. Fewer available units makes interruption more probable and leads to higher expected cost of interruption. The probability of interruption is decreasing in bids and the bidders with bids

[5]Long after the introduction of the spot market by Amazon, where the term "long" is stated with respect to the very dynamic nature and growth of the market.

[6]Google introduced in May 2015 a second pricing mechanism buy selling "preemptive virtual machines". These machines are sold in a posted price that may be interrupted by their provider at any point of time.

[7]See for example, Maskin and Riley (1989), Myerson (1981), Riley and Zeckhauser (1983) and Skreta (2006).

[8]See for example, Varian (1997), Mussa and Rosen (1978), Deneckere and McAfee (1996) and McAfee (2007).

[9]An example from the IaaS market: Consider a downstream firm that needs to have access to a cloud server (virtual machine) for running a service. If it looses the access to the server, it is unable to run the service anymore.

close to the auction equilibrium price are more likely to see their virtual machines to be interrupted. At the same time, the provider, by decreasing the number of auctioned units can make the auction less attractive option for the high valuation buyers who prefer the posted price mechanism even if the posted price is relatively high. The selling mechanism with high posted price and sufficiently low number of auctioned units is incentive compatible in that all the buyers with valuations higher than the posted price, prefer to buy from the posted mechanism avoiding the risky auction (even under risk neutrality) and all the buyers with valuations lower than the posted price participate in the auction, since they do not find profitable to buy the good using the posted price option. Note, that following this reasoning, the explanation of the use of a hybrid of posted price and auction with the risk of interruption can be justified for reasons that go beyond issues related with capacity constraints.

The comparison between different selling mechanisms and in particular between posted prices and auctions has been studied in the literature in various settings both by economic and computer science literature. Harris and Ravin (1981b) consider a multi-unit selling procedure under a uniform distribution of buyers' valuations and concludes that the optimal mechanism is a posted price selling procedure. Maskin and Riley (1989) generalizes this result for general distribution functions under very mind conditions over the buyers' valuations. Riley and Zeckhauser (1983) finds that sequential search (or posted selling) mechanism under commitment is optimal. This result is generalized by Skreta (2006) in the case that the seller cannot commit to a particular selling mechanism. In contrast, Wang (1993) compares the seller's revenue from auctions and posted price mechanisms by considering a seller who meets buyers with exogenously given (Poisson arrival) probabilities and finds that when there are no auctioning costs, auctioning is always optimal. If auctioning costs are present, the steepness of the marginal-revenue curve associated with the distribution of buyers' valuations determines the optimal selling option. When this steepness is large, auctioning is still preferable (for the seller) to the posted price mechanism. Kultti (1999) considers agents who choose whether to participate in markets where goods are sold in auctions or markets where goods are sold through a posted price mechanism and concludes that both mechanisms are totally equivalent. Julien et al. (2002) develop a model with two buyers and sellers who offer homogeneous goods and consider the choice of sales mechanism from three possibilities: posted prices, and auctions with and without reserve prices. They find that sellers' expected revenues are highest when both sellers use auctions with reserve prices.

Hammond (2010) motivated by the finding of Harris and Raviv (1981a) that if the number of goods in a monopolist's inventory exceeds potential demand, a posted price is optimal as well as the analysis of Zeithammer and Liu (2008) who consider the possibility that the inventory is heterogeneous and conclude that a monopolist with a heterogeneous inventory prefers the auction mechanism while a monopolist with a homogeneous inventory prefers the posted price, investigates empirically these theoretical claims based on data collected on compact disc sales. While he finds that the size of the inventory has a significant impact on the choice of the selling procedure by compact-disc sellers, he does not find any sufficient support for the impact of heterogeneity of the inventory on the mechanism's choice. The empirical study of Vakrat and Seidmann (1999) compares prices paid through online auctions and catalogs for the same product. They observe that auctions result in average prices 25% below the catalog ones.

As far as studies that consider the simultaneous use of posted price selling and online auctions are concerned, Budish and Takeyama(2001)consider a single seller and two types of risk-averse buyers and show that the English auction with a buy price can raise the revenue of the seller. Similar result is found by Reynolds and Wooders (2009). Etzion et al.(2006) show that the simultaneous use of posted price selling and online auctions leads to a significant increase in sellers revenue. The buyers in this study takes into account the fact that the level of competition is higher in the online auction market. By taking buyers' discounting of the expected utility of auctions into account, Sun (2008) shows that the achievement of market segmentation is a rationale for the simultaneous use of posted price selling and online auctions. Sun argues that, in the case of a posted price sale, there is little or no uncertainty regarding the price of the product but the number of units sold is subject to uncertainty. The reverse is true in the case of auctions and hence there is no dominant selling mechanism. Sun further argues that the choice of selling mechanism depends on factors such as the sellers inventory cost and the buyers discount factor. Suns analysis is based on near-optimal approximation of the sellers profits. Hammond(2013)argues that differences across buyers do not explain the simultaneous use of auctions and posted price selling. He finds that the simultaneous use of auction and posted price sale decreases the level of competition among the sellers. Sellers with high value items prefer posted price sale, even though it leads to fewer sales, because the items can be sold at a higher price. Celis et al.(2014) further present an analysis of a randomized mechanism that they call buy-it-now or take-a-chance in which bidders have the option of first buying an object at a posted price, and if nobody buys the object at a posted price, the object is then sold at random to one of the top d bidders. They conclude that this mechanism, when only two different types (of valuations) of buyers are available, outperforms a second price auction with optimal reserve price. However, when we move to an environment where the distribution of buyers' valuations is continuous, this is not generally true, but, it depends on the specific values of the parameters of their model. In the infinite-horizon model of van Ryzin and Vulcano (2004), the seller operates auctions and posted prices simultaneously, and replenishes her stock in every period. However, the streams of consumers for both channels are independent, and the seller decides how many units to allocate to each of the channels separately. Etzion et al. (2006) study the profitability of selling consumer goods on-line using posted price and open ascending-bid uniform-price auction simultaneously. They develop a model of consumer behavior when faced with the choice between the two channels. The model is simulated in order to identify the best designs of the dual channel regime and compare its performance with that of the only posted price regime. They find that the best designs of dual channels with open-bid auctions differ from those of dual channels with sealed-bid auctions previously studied. In addition, when optimally designed, the dual channel regime outperforms the posted price regime.

There are several other studies that try to explain the pricing options of eBay and related markets and specifically, to provide an economic rationale for the buy-it-know option that is followed by an efficient auction (which can be viewed as a hybrid of a posted price and an auction offered simultaneously)[10]. The majority of the relevant theoretical studies find that such kind of mechanism price discriminates between high and low valuation buyers based on the fact that the allocation of the good is not simultaneous. Buyers that prefer the buy-it-know option get the object immediately by paying the respective posted price while buyers that go to the auction are subject to delays until they get the goods on sale (if they win the auction). These delays create opportunity costs (especially as some of the buyers may be very impatience). High valuation buyers that find the buy-it-know price affordable they prefer this option in order to avoid the opportunity costs involved with the auction options. The buyers with lower valuation find preferable the auction selling mechanism due to the higher buy-it-know price. A similar reasoning holds if buyers are differentiated with respect to how impatient or how risk averse (as already pointed above) they are.

Our novel approach provides a rationale for using posted price and auction selling mechanisms for goods that are not only simultaneously offered but also simultaneously allocated (regardless the choice of the mechanism the buyer chooses), sharing in this way a main feature of IaaS market. Hence, opportunity costs or arguments about buyers' impatience cannot be relevant for the justification of the co-existence of the two selling procedures. Moreover, we consider that the good sold is a durable and interruptible. As we will show below, when such a good is sold to risk neutral buyers, the seller can adopt an optimal mechanism which maximize the seller's revenue under the simultaneous use of posted price and auctions by introducing sufficient risk of interruption in the auction.

The rest of the paper is organized as follows: In section 2 we discuss the environment we are interested by using an one buyer-one seller approach. Section 3 derives a randomized mechanism and describe the conditions that it can be revenue improving in comparison to deterministic mechanisms. It also discusses the case of multiple buyers with unit demand. Section 4 discusses how a hybrid of posted prices and auctions can generate higher revenue than optimal standalone mechanisms.

2 One-buyer and One-seller

To begin with, consider one seller and one buyer. The seller has a durable indivisible and interruptible good for sale. She produces it as zero cost. The buyer has a private valuation for the good which is drawn randomly from distribution $F(v)$ with finite probability density function $f(v)$ which is positive in the support $[\underline{v}, \overline{v}]$, with $\overline{v} > \underline{v} \geq 0$ from which the valuation of the buyer is drawn. The distribution of valuations is common knowledge. The buyer with valuation v wants to consume the good for duration of $b(v)$. The function $b(\cdot)$ is also common knowledge. The valuation v refers to consumption of the good for a period

[10]See for example Wang et. al. (2004), Hummel (2015), Einav et. al. (2015), Mathews (2004), Anwar and Zheng (2015), Onur and Tomak (2009), Kirkegaard and Overgaard (2008), Caldentey and Vulcano (2007), Chen et al. (2013), Peters and Severinov (2006), Hidvegi et al. (2006), Ackerberg et al. (2006), Ambrus et al. (2014), Ockenfels and Roth (2006) and Roth and Ockenfels (2002) to name a few.

$b(v)$ without any interruption. If the good is interrupted before the end of the period $b(v)$, then the buyer incurs cost c[11].

So, the buyer's utility is:

$$U(v) = \begin{cases} v & \text{if interruption does not occur within the period } b(v) \\ v - c & \text{otherwise and if } v \geq c \\ 0 & \text{if interruption occurs within a period } b(v) \text{ and } v < c \end{cases} \quad (1)$$

Let the seller offer:

- an initial allocation rule $x(\cdot)$ which assigns the object to type v with probability $x(v)$
- probabilities $\{\lambda_t(\cdot)\}_{t=1}^{b(v)}$ that the consumption of good will not be interrupted for a type v and at each unit t of the time period that equals duration $b(v)$. So, at each point of time t the risk of interruption $1 - \lambda_t(v)$ can have an impact only when type v gets the object with positive probability $x(v)$ and the good has not been interrupted at $t' < t$.
- a payment $p(\cdot)$ from the buyer of type v to the seller for the allocation of the good under the rules $x(v)$ and $\{\lambda_t(v)\}_{t=1}^{b(v)}$

A type of Myerson mechanism is to define a threshold type $\tilde{v} \in [\underline{v}, \overline{v}]$ which is defined by the equation $v - \frac{1-F(v)}{f(v)} = 0$. In addition, $\lambda_t(v) = 1, \forall t, v \in [\underline{v}, \overline{v}]$. Under the monotone likelihood property that $\frac{1-F(v)}{f(v)}$ is monotonously decreasing in $v, \forall v \in [\underline{v}, \overline{v}]$, buyer's types with $v \geq \tilde{v}$ get the object with probability $x(v) = 1$, while types $v < \tilde{v}$ get the object with probability $x(v) = 0$. The expected payment for the seller is $T(v) = \int_{\underline{v}}^{\overline{v}} x(v)(v - \frac{1-F(v)}{f(v)})dF(v)$.

We investigate whether the seller can do better by randomizing over probabilities $x(\cdot)$ and $\lambda(\cdot)$ in environments as the one described above. Without loss of generality we restrict our attention to direct mechanisms following the revelation principle. Then, the mechanism we are looking for should be selected from the pool of mechanisms that satisfy the conditions of individual rationality and incentive compatibility. Namely, buyer of type v should be willing to participate in the trade, so the benefit $r(v)$ from trade should be positive:

$$r(v) = x(v)\left[v - \left(1 - \tilde{\lambda}(v, b(v))\right)c\right] - p(v) \geq 0$$

where $\tilde{\lambda}(v, b(v)) = \prod_{t=1}^{t=b(v)} \lambda_t(v)$ is the overall probability of continuation within period $b(v)$ taking into account probabilities set by the seller at each point of time $\{\lambda_t(v)\}_{t=1}^{b(v)}$. Without a loss of generality, we can restrict ourselves to the case that $\tilde{\lambda}(v, b(v)) = \lambda^{b(v)}(v)$.

[11]For simplicity we assume that the cost c does not depend on the specific time of interruption within the period $b(v)$. It is the event of interruption that have an impact on buyer's utility and not the specific time that the interruption occurs.

Any path of probabilities $\{\lambda_t(v)\}_{t=1}^{b(v)}$ that result an overall probability $\tilde{\lambda}(v, b(v))$ can be replicated by adjusting the value of $\lambda_t(v)$ such that $\lambda_{t+1}(v) = \lambda_t(v), \forall t$. So, the individual rationality constraint becomes

$$x(v)\left[v - \left(1 - \lambda^{b(v)}(v)\right)c\right] - p(v) \geq 0 \tag{2}$$

The mechanism should also certify that the buyer of any true type v will not have any incentive to misrepresent himself by reporting a type v' and this should be true for every $v' \in [\underline{v}, \overline{v}] \& v' \neq v$:

$$x(v)\left[v - \left(1 - \lambda^{b(v)}(v)\right)c\right] - p(v) \geq x(v')\left[v - \left(1 - \lambda^{b(v)}(v')\right)c\right] - p(v') \tag{3}$$

Incentive compatibility requires that

$$r(v) = \int_{\underline{v}}^{v} x(s)\left[1 + \lambda^{b(s)}(s)\ln(\lambda)b'(s)c\right]ds$$

So, the payment to seller will be

$$p(v) = x(v)\left[v - \left(1 - \lambda^{b(v)}(v)\right)c\right] - \int_{\underline{v}}^{v} x(s)\left[1 + \lambda^{b(s)}(s)\ln(\lambda)b'(s)c\right]ds$$

so the expected profit for the seller from the transaction will be:

$$\begin{aligned}
\Pi_s &= \int_{\underline{v}}^{\overline{v}} p(v)dF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \left[x(v)\left[v - \left(1 - \lambda^{b(v)}(v)\right)c\right] - \int_{\underline{v}}^{v} x(s)\left[1 + \lambda^{b(s)}(s)\ln(\lambda)b'(s)c\right]ds\right]dF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \left[x(v)\left[v - \left(1 - \lambda^{b(v)}(v)\right)c\right] - \frac{1 - F(v)}{f(v)}x(v)\left[1 + \lambda^{b(v)}(v)\ln(\lambda(v))b'(v)c\right]\right]dF(v)
\end{aligned} \tag{4}$$

The maximization of seller's profit requires to set optimally the choice variables $x(\dot{)}$, $\lambda(\cdot)$ The optimal value $\lambda^*(v)$ is:

$$\lambda^*(v) = e^{\frac{b(v) - \frac{1-F(v)}{f(v)}}{\frac{1-F(v)}{f(v)}b(v)b'(v)}} \tag{5}$$

Note that $\lambda^*(v) < 1$ only if $\frac{b(v) - \frac{1-F(v)}{f(v)}}{\frac{1-F(v)}{f(v)}b(v)b'(v)} < 0$.

The optimal value $x^*(v)$ depends on the expression:

$$\begin{aligned}
A(v) &= v - \frac{1 - F(v)}{f(v)} - \left(1 - (\lambda^*)^{b(v)}(v)\right)c - \frac{1 - F(v)}{f(v)}(\lambda^*)^{b(v)}(v)\ln(\lambda^*(v))b'(v)c \\
&= v - \frac{1 - F(v)}{f(v)} - c\left(1 - (\lambda^*)^{b(v)}(v)\frac{\frac{1-F(v)}{f(v)}}{b(v)}\right)
\end{aligned}$$

It is

$$x^*(v) = \begin{cases} 1 & \text{if } A(v) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Note that for $\lambda^* = 1$, the allocation rule converges to the Myerson one.

Clearly, the exact allocation depends on the distribution of types and the duration function $b(v)$. It is meaningful to consider that the virtual valuation $A(v)$ is increasing in v. In this way, according to the optimal rule $x^*(\cdot)$, high valuation types get the good with probability one while low valuation types do not get the good at all. While in the Myerson mechanism, the monotone likelihood property was sufficient for such allocation, here specific assumptions over the duration function are needed.

Let $\frac{1-F(v)}{f(v)}$ be monotonously decreasing in $v, \forall v \in [\underline{v}, \overline{v}]$. Moreover, let $b(v)$ be a monotonously increasing function of v and weakly concave. If $b(\underline{v}) > \frac{1-F(\underline{v})}{f(\underline{v})}$, then, $\lambda^*(v) = 1, \forall v \in [\underline{v}, \overline{v}]$.

If $b(\underline{v}) < \frac{1-F(\underline{v})}{f(\underline{v})}$, then, let $\hat{v}$ be the unique solution of $b(v) = \frac{1-F(v)}{f(v)}$. So, $\lambda^*(v)$ is monotonously increasing in v for every $v < \hat{v}$. Indeed,

$$\frac{d\lambda^*}{dv} = \lambda^*(v)\left(\frac{1}{b''(v)}\left(\frac{1}{\frac{1-F(v)}{f(v)}} - \frac{1}{b(v)}\right) + \frac{1}{b'(v)}\left(-\frac{\frac{d\left(\frac{1-F(v)}{f(v)}\right)}{dv}}{\left(\frac{1-F(v)}{f(v)}\right)^2} + \frac{b'(v)}{b^2(v)}\right)\right) > 0$$

So, if

- $\hat{v} \leq \underline{v}$, the risk of interruption is zero for every $v \in [\underline{v}, \overline{v}]$ as in the Myerson case.
- $\hat{v} \geq \overline{v}$, the risk of interruption is monotonously decreasing for all $v \in [\underline{v}, \overline{v}]$
- $\hat{v} \in (\underline{v}, \overline{v})$, the risk of interruption is decreasing for all $v \in [\underline{v}, \hat{v})$ and equals to zero for all $v \in [\hat{v}, \overline{v}]$

We continue by considering the latter more interesting case.

We now derive some sufficient conditions for $A(v)$ to be monotonously increasing in v to get an idea of the classes of duration functions for which we get the increasing pattern:

Lemma 1. *The virtual valuation $A(v)$ is monotonously increasing in v for all $v \in [\underline{v}, \overline{v}]$ if the following conditions for the duration function $b(v)$ simultaneously hold for every $v \in [\underline{v}, \hat{v})$:*

- *The concavity of $b(v)$ is such that the condition $b''(v) > -b(v)$ is satisfied*
- *$b(v)$ is increasing in v such that $b'(v) < b(v)$*
- *The expected cost of interruption is smaller than $b(v)$, $b(v) > (\lambda^*)^{b(v)}c$*

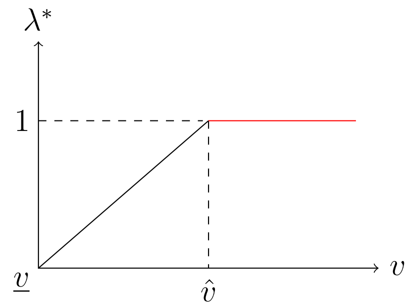

Figure 1: The optimal λ^* as a function of v for $\underline{v} < \hat{v} < \overline{v}$

Proof. Note that for $v \in [\hat{v}, \overline{v}]$, $\lambda^*(v) = 1$, so, since, $\frac{1-F(v)}{f(v)}$ is monotonously decreasing in v, $A(v)$ is increasing. For $v \in [\underline{v}, \hat{v})$ it is $b(v) < \frac{1-F(v)}{f(v)}$. So, we have:

$$\frac{dA(v)}{dv} = 1 - \frac{d\left(\frac{1-F(v)}{f(v)}\right)}{dv} + \frac{d\left((\lambda^*)^{b(v)}(v)\frac{\frac{1-F(v)}{f(v)}}{b(v)}\right)}{dv}$$

It is

$$\begin{aligned}\frac{d\left((\lambda^*)^{b(v)}\right)}{dv} &= (\lambda^*)^{b(v)}\left(\frac{d\lambda^*}{dv}\frac{b(v)}{\lambda^*} + b'(v)\ln[\lambda^*]\right)\\ &= \left(1 + \frac{b(v)}{b''(v)}\right)(\lambda^*)^{b(v)}\frac{b(v) - \frac{1-F(v)}{f(v)}}{b(v)\frac{1-F(v)}{f(v)}} + (\lambda^*)^{b(v)}\frac{1}{b'(v)}\left(\frac{b'(v)}{b^2(v)} - \frac{\frac{d\left(\frac{1-F(v)}{f(v)}\right)}{dv}}{\left(\frac{1-F(v)}{f(v)}\right)^2}\right)b(v)\end{aligned}$$

which is positive in v if $b''(v) > -b(v)$. So, the $\frac{dA(v)}{dv}$ can be rewritten as:

$$\frac{dA(v)}{dv} = 1 - \frac{d\left(\frac{1-F(v)}{f(v)}\right)}{dv}\left(1 - \frac{c(\lambda^*)^{b(v)}}{b(v)}\right) + c\frac{d\left((\lambda^*)^{b(v)}\right)}{dv}\frac{1-F(v)}{f(v)} - c(\lambda^*)^{b(v)}\frac{1-F(v)}{f(v)}\frac{b'(v)}{b^2(v)}$$

given that $b''(v) > -b(v)$ and consequently $\frac{d\left((\lambda^*)^{b(v)}\right)}{dv} > 0$, $A(v)$ is increasing in v if $b(v) > c(\lambda^*)^{b(v)}$ and if $b'(v) < b(v)$ □

Here after we assume that $A(v)$ is increasing function of v. Then, if v^{cr} is the solution of $A(v) = 0$, it is easy to see that $v^{cr} < \tilde{v}$ if only if $v^{cr} < \hat{v}$, otherwise, $v^{cr} = \tilde{v}$. In fact $v^{cr} < \tilde{v}$ is the case only when $b(v^{cr}) < (\lambda^*)^{b(v^{cr})}(v^{cr})\frac{1-F(v^{cr})}{f(v^{cr})}$. Using a first order approximation of the exponential power series of λ^*, we conclude to the sufficient condition $b'(v) > 1$.

Under this condition, the optimal rules are $x^*(v) = 1$ and $\lambda^*(v) < 1$ for some v, so, more types of buyers are served under the optimal rule than under the Myerson one. We have an expansion of trade to lower valuations because the risk of interruption allows the seller to price discriminate among buyers at an incentive compatible way. The optimal allocation is:

- For $v \in [\underline{v}, v^{cr})$, there is no trade
- For $v \in [v^{cr}, \hat{v})$, the buyer is allocated the good with probability one, but he faces a risk $\lambda^*(v)$ that the consumption of the good will be interrupted
- For $v \in [\hat{v}, \overline{v}]$, the buyer get the good with probability one and without any risk of interruption.

The optimal payment to the seller for each type v when $v^{cr} < \hat{v}$ is:

$$p^*(v) = \begin{cases} \hat{v} - \int_{v^{cr}}^{\hat{v}} \left[1 + (\lambda^*(s))^{b(s)} \ln(\lambda^*) b'(s) c\right] ds & \text{if } v \geq \hat{v} \\ v - \int_{v^{cr}}^{v} \left[1 + (\lambda^*(s))^{b(s)} \ln(\lambda^*) b'(s) c\right] ds & \text{if } \hat{v} > v \geq v^{cr} \\ 0 & \text{if } v < v^{cr} \end{cases}$$

3 N Buyers and Implementation

Let's consider a framework of one seller can produce up to Q units of a homogeneous good or service for sale and N buyers with unit demand. The valuations of the buyers are drawn identically and independently from distribution $F(v)$ as it is defined above. By considering the duration function such that $\lambda^* < 1$ for at least some $v \in [\underline{v}, \overline{v}]$ and $A(v)$ is increasing in v, then the mechanism above maximizes again the profit of the seller. Let $Q > N(F(\overline{v}) - F(\hat{v}))$, then in equilibrium the risk of interruption is strictly positive for some types. Let's summarize the insights of the previous section for the case of N buyers and Q units in the following proposition

Proposition 2. *When $A(v)$ is monotonously increasing in v and $v^{cr} < \hat{v}$, the optimal allocation rules are*

$$x^*(v) = \begin{cases} 1 & \text{if } v \geq v^{cr} \text{ and } Q > N(F(\overline{v}) - F(v)) \\ 0 & \text{if } v < v^{cr} \text{ or if } v \geq v^{cr} \text{ and } Q < N(F(\overline{v}) - F(v)) \end{cases} \tag{7}$$

and

$$\lambda^*(v) = \left\{ \begin{array}{cc} 1 & \text{if } v \geq \hat{v} \\ e^{\frac{b(v) - \frac{1-F(v)}{f(v)}}{\frac{1-F(v)}{f(v)} b(v) b'(v)}} & \text{if } v < \hat{v} \end{array} \right\}. \tag{8}$$

When on the contrary, $v^{cr} > \hat{v}$, then $v^{cr} = \tilde{v}$, so,

$$x^*(v) = \begin{cases} 1 & \text{if } v \geq \tilde{v} \\ 0 & \text{if } v < \tilde{v} \end{cases}$$

and

$$\lambda^*(v) = 0$$

$\forall v \in [\underline{v}, \overline{v}]$.

When the risk of interruption is part of the optimal allocation rule, then it allows the seller to sell the good to more buyers than the Myerson mechanism, Since, $\tilde{v} \geq \hat{v}$, expanding the buyers base also increases the total welfare. The following colorally applies:

Corollary 3. *When the seller finds optimal to allocate the good at non-zero risk of interruption for some types, then this is also total welfare improving in comparison to the Myerson allocation rule.*

The implementation of these allocation rules can accommodate the simultaneous allocation through a posted price and an auction with the risk of interruption. For example, the posted price set at $p = \hat{v}$, and the auction that is efficient and is designed in a way that participants bid their true valuations can implement this mechanism (for example, a uniform price auction). Through the auction, the seller can link each bid (valuation) with a type specific optimal risk of interruption that is decreasing as types increase. So, buyers with valuations above $\hat{v}$ choose the posted price mechanism while the rest of the buyers can go to the auction and face a risk of interruption if the good is allocated to them. Setting the reserve price of the auction at value equal to v^{cr}, buyers with valuations $v \in [v^{cr}, \hat{v})$ can get a unit of the good in the auction (provided that the number of offered units Q is sufficient) implementing the optimal allocation rules.

4 Conclusion

Motivated by the IaaS cloud computing market we provide an economic rationale of its currently using pricing schemes. Specifically, we illustrate under what conditions the seller can increase her revenue by offering simultaneously a posted price scheme and an auction for selling services. What we find is that when the seller introduces a risk of interruption for the holders of the goods, she can price discriminate among buyers of different valuations and generate in this way higher payoff. The combination of the risk of interruption with an auction in which it is a weakly dominant strategy to bid truthfully allows the seller to severely punish the high valuation buyers for choosing the auction. In this way the seller can implement an ideal price discrimination in which all the high valuation buyers prefer the posted mechanism and all the low valuation bidders go to the auction. The principles of this mechanism correspond to the optimal mechanism for selling on-line services.

There are other markets for that our model can be applied. For example the market for transmission of electricity by generators and the risk of congestion rents has some similarities with our environment. However, in such network industries which are heavily regulated, the objective is often the maximization of total welfare and not the profit of the generators. As it is shown in this paper, price discrimination generated by the risk of interruption is total welfare enhancing as it lead to larger market segment coverage.

An interesting extension of our current framework is to study how the optimal pricing is affected in the case of competing sellers. Both in terms of its optimality and welfare implications. In the cloud computing market for example, Amazon is the only firm that has adopted a hybrid selling mechanism with and auction option. Price discrimination and the impact of competition is not a new topic to study[12]. But, finding the optimal mechanism under competition among sellers with different market power could be insightful about how the pricing options and price discrimination motives are affected by market competition.

We decided to focus on a static version of the problem in order to investigate whether and when the hybrid mechanism that facilitates price discrimination is chosen by the seller neglecting the complications that could be in place from a more dynamic considerations. However, we plan to check our idea and main results in more dynamic frameworks to see how robust is our rationale for the pricing options. In fact, a dynamic extension could also help us to study issues related to capacity constraints of the seller and whether they could justify additional reasons for the use of the hybrid mechanism and the market based pricing option. In addition, we could look at the issue of fluctuating demand which is the case in many on-line markets. Demand uncertainty could provide additional motives for a spot market pricing approach as a way to infer buyers valuations. Obviously, the issue of commitment is crucial for the success and the optimality of a selling mechanism. A market based pricing option may be perceived as a way to commit to a risk of interruption that is demand driven and therefore under the control of the seller. For this to be the case, transparency over the auction rules and information about its participants is necessary.

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