# Showrooming in a market of tangible goods with heterogeneous agents[*]
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**Abstract**

Showrooming is a situation where consumers try products at brick-and-mortar stores before purchasing them online at a lower price. One way to prevent showrooming is to use a price matching policy, whereby price is the same in both the physical store and the online channel. We show that for small search costs, a price matching policy is indeed optimal. However for higher search costs price matching is suboptimal, and online and offline purchases coexist with showrooming. The firm apllies price-matching policy then it has the lack of commitment to different online and store prices. A firm, which faces online competition from a foreign multichannel retailer, has incentive togeo-block, i.e. refuse to serve foreign customers, even though it leads to a decrease in potential demand. Geo-blocking relaxes online competition and leads to higher prices both online and in brick-and-mortar stores. A legal price parity requirement helps to eliminate incentives to geo-block and thus, restores online competition in the presence of possibility to showroom.

**JEL Classification**: D83; L12; L13; L81

**Keywords**: showrooming; retailing; price discrimination; geo-blocking; search

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# 1 Introduction

Nowadays we observe a stable growth of the E-commerce sector - the web's share of total retail increased by about 4 % in the last five years and continues to grow. The future of traditional brick-and-mortar stores has been widely discussed in the press. Many experts see e-commerce as a substantial threat to traditional brick-and-mortar stores (BMS), while others remain skeptical about BMSs being replaced by online retailers. In 2012 the chief executive of Gilt Groupe, an online retailer selling women's clothing and accessories, expressed his concerns about the future of traditional retail in an interview with the Economist. According to his opinion, there was "no evidence that there were big opportunities for traditional retailers in online retail" as "bricks-and-mortar shops were gravely threatened by Amazon and other online-only retailers".[1] However, four years later in 2016 Gilt Groupe announced its acquisition by Hudson's Bay Company, owner of luxury department store chains. The following year later Amazon opened its first brick-and-mortar store. Another article published in The Economist [2] provides more examples of online retailers which decided to open a BMS or a showroom, as they realized that "customers wanted shops too". The European commission reports that about 59% of retailers selling online make a choice in favor of multichannel retail distribution(see Report (2017)).

An important problem related to the digital economy is geo-blocking, whereby retailers can refuse to sell to consumers from a "foreign country". It can be implemented by preventing customers from accessing the website and refusing payment or delivery.[3] According to the Report (2017) about 36% of online retailers do not sell cross-border for at least one of the relevant product categories. The median proportion is about 47% across the 28 EU Member States. Moreover, a retailer from a large online market is less likely to geo-block than retailers from small markets which mostly focus on domestic sales. Sometimes the choice of geo-blocking strategy is dictated by vertical restraints imposed by a manufacturer. However, this is not the case for most retailers. It is not obvious why firms may prefer to voluntarily concede a part of the online market to their competitors. We build a competition model in order to analyze multichannel retailers' incentives to geo-block.

[1] "Making it click", *The Economist*, 25th of February 2012.

[2] "Shops to showrooms", *The Economist*, 10 of March 2016.

[3] The retailer can also introduce geographical restraints by re-routing customers to a foreign web page based on their location. This strategy is known as geo-filtering and it is outside the scope of the paper.

This paper makes two main contributions to the existing literature. First, it theoretically explains how the magnitude of consumer search costs may explain observed variations in price differences across retail channels, and when price parity is enforced by a retailer. It discusses the role of showrooming as a way to discriminate between different types of consumers, and in the resolution of the hold-up problem which may potentially occur. Second, it provides an explanation of multichannel retailers' incentives to restrict cross-border sales, shows that geo-blocking is likely to happen in equilibrium, and examines policy responses.

In more detail,we develop a model where firms decide on selling through two distribution channels - an online store and a BMS. BM stores provide customer service, which is costly for the firm, but allow consumers to try the product and learn product characteristics. Typically consumers can find out some information about a product on a website, get an impression about a design, colors and a price, but they are able to figure out whether the product fits only after trying it. However, consumers bear a positive search cost to visit the BMS[4]. Online stores allow consumers to buy goods directly without visiting the physical store. However, online shopping is also associated with additional shopping costs, which are subjective and heterogeneous across customers. The literature reports many factors which determine this heterogeneity (see Swinyard and Smith (2003), Keen et al. (2004)). Most of them are related to psychological reasons such as unwillingness to wait for the product to be delivered, uncertainty of delivery dates, reluctance to make online transactions and so forth. Forman et al. (2009) provide empirical evidence that online shopping creates some additional disutility for customers compared to shopping in BM stores, which cannot be explained by monetary costs.

In the first part of the paper, we analyze a monopoly case with observable retail prices and study how the presence of search and online shopping costs affects the firm's pricing strategies. Cavallo (2017) finds that in most cases (about 72 % on average) online and offline prices are identical. The choice of price matching policy is typically explained by the retailers' attempt to increase profit by preventing competition between its own distribution channels (see Kireyev

[4]In this paper we interpret search costs mostly as costs related to the time of visiting the store. Hence the magnitude of the search cost is mostly determined by factors such as locations, quality of public transport connection, waiting time to be served in a store and so forth. In the appendix we discuss in more details what happens if this assumption is relaxed. We provide some intuition why only expected search costs matter, and thus assumption on search cost homogeneity is not restrictive and still allows to capture all main effects.

et al. (2017))). However, the percentage of price matching strategies varies a lot across different geographical locations, sectors, and retailers,[5] which can be explained by firms' incentives to price discriminate. We find that for low search frictions, the monopolist prefers to set an online price equal or higher than a price in a brick-and-mortar store. As search frictions are low, there is widespread search - all consumers prefer to try the product in BMS. Thus, there is no efficient way to price discriminate between different types, and the online channel is redundant. All sales are redirected to a BMS store. For moderately high search frictions showrooming co-exists with online and offline purchases. We observe lower online prices than those in a store. For high search frictions all consumers prefer either to purchase online without search, or buy in a BMS. Showrooming is not a part of the equilibrium. We may observe online prices which are higher than store prices. For example, the online tariff for the Louvre museum access exceeds the tariff for tickets sold inside the museum. Both prices are precisely posted on the webpage. Even the museum ticket is not a typical tangible good, which consumers would potentially like to try in the store, the example is still a good illustration of our result. An online ticket assumes that consumers make a commitment for a visit at a certain day and a time slot. The purchase should be done at least one day in advance. In this case the probability of a "bad fit" reflects the uncertainty about whether consumers will visit the museum on that day, and at the time they buy a ticket for. Search costs are associated with the waiting time in the queue which can be high.

We provide a model extension for the monopoly case, where we consider an unobservable BMS price. In this situation consumers are potentially exposed to a hold-up problem: the marginal consumer should be indifferent between visiting the store and abstaining from purchase, but as soon as he is in the store, because the search cost is sunk, the firm has incentives to raise the price. Thus, the market collapses (see Diamond (1971)). This problem, however, can be mitigated by the introduction of an online retail channel. By opening an online shop the monopolist not only simply sells to some consumers online, but also creates artificial competition between the two distribution channels, which drives down prices in the BMS, simultaneously increasing the firm's profits. The firm can increase its profits by committing to the same price online and offline, and increase it even further, when search frictions are high, by publicly and credibly disclosing the offline price. The

[5] Cavallo (2017) shows that in some countries, i.e. Argentine and Australia, consumers typically face higher online prices.

price matching policy becomes an important instrument for price advertising when the firm has a lack of commitment power. When search frictions are low, the firm earns the same profit as in the situation where both prices are publicly observed. However, for moderate search cost the firm may be indifferent between advertising or obfuscating offline price. Thus, in presence of any additional costs are associated with advertising offline prices the firm may prefer post only online prices.

The second part of the paper is devoted to competition between retailers. We focus on firms' decisions which retail channels to use and their incentives to introduce restrictions on cross-border sales. We consider competition between two retailers. Consumers are divided geographically: they can only buy from a BMS of their own retailer, as BM stores naturally face some geographical market restrictions. Online stores make it possible for consumers to shop outside of their location. Firms, however, can commit to geo-blocking. We assume that decisions on retail channels and geo-blocking are long-run and precede the price-setting phase of the game. Geo-blocking unambiguously lowers potential demand for a firm imposing this strategy. However, at the later stage competition is much weaker: the competitor understands that its online shoppers are not threatened and raises its prices. This allows the firm which geo-blocks to raise its own BMS price and improve profits compared to the case without geo-blocking. When decisions about geo-blocking are taken simultaneously by both firms, we observe a symmetric equilibrium with two monopolists operating only in their local markets. In this case competition authorities may want to introduce legal restrictions on geo-blocking to restore competition, and increase consumer surplus. If firms' decisions are sequential, we observe an asymmetric market structure in equilibrium, where the follower either geo-blocks or maintains only a BMS, while the leader sells online cross-border. The competition policy authority can use a non-discrimination price policy, which obliges firms to charge the same price in both distribution channels, as an instrument to restore online competition when the market search cost is sufficiently low. It weakens online competition in the absence of geo-blocking, and thus eliminates incentives to ban cross-border sales.

Wang and Wright (2017) is one of the first papers which study the effect of showrooming and price coherence in the presence of small firms and an online platform. In this context consumers search for lower prices and price observability is the way to eliminate potential showrooming. In our paper, we assume that consumers search for unobservable product characteristics. Consequently, showrooming may exist even with full price transparency. This is quite typical in markets of tangible goods. During the last few years there has been a substantial growth in literature discussing showrooming in the context of tangible goods and competition between online retailers and BMSs.

Mehra et al. (2017)'s framework is the most similar to those considered in this paper. They provide a model of competition between BMS and an online shop, and discuss possible strategies of a BMS store to prevent showrooming, as it has an unambiguously negative effect on BMS profit. The positive effect of showrooming on BMS profit is discussed in the paper by Kuksov and Liao (2016), where there is a strategic manufacturer. As the manufacturer is interested in a BMS providing customer service, it can propose to it a better wholesale tariff. The manufacturer's strategy to open an online channel in order to motivate a BMS to improve the quality of the customer service is explored in Yan and Pei (2009). In our paper we don't consider vertical contracts and restraints, but we exploit the same idea of the multichannel retailer being interested in maintaining a BMS in order to provide additional customer service, which results in better matches for customers.

Kireyev et al. (2017) focus on the analysis of multichannel retailers' strategies and consider a price matching policy as the main tool to prevent showrooming. However, they do not take into account the possibility to learn product characteristics and to increase expected utility through the search process. We show that due to the fact that willingness to learn product characteristics may disclose information about consumers' types, it leads to the possibility of price discrimination for a firm. Therefore, a multichannel retailer does not necessarily want to prevent showrooming in a market. Moreover, both the firm and consumers can benefit from higher search costs in presence of the showrooming. This result is related to **?**, where the author shows that higher search costs may allow to screen consumers better and to target those who are more interested in buying the product. The intuition behind our result is similar. The difference is that in our consumers benefit not from the higher resulting level of customer service but from better prices proposed by the firm.

The rest of the paper is organized as follows. Section 2 presents the general model description; Section 3 analyzes consumers' behavior. Section 4 provides the solution for the monopoly case and discusses the role of price observability. Section 5 is devoted to the analysis of competition between multichannel retailers. Section 6 analyzes the model extension with unobservable BMS prices. Section 7 concludes with a discussion of the main findings.

## 2 Model

In this section we discuss the general setup of the model. Assume that there are homogenous goods sold in the market. The product fits a consumer with probability $\pi$, and realizations of successful matches are independent across customers. A consumer has a product valuation normalized to one in the case of successful match and to zero otherwise. Firms present in the market can have two distribution channels - a brick-and-mortar store and an online shop. Consumers are unaware whether the product fits before they try it. BMS provide customer service, which allows consumers to try the product and thus to learn whether they will get a successful match if they buy it. Visiting the BMS is costly for consumers, and they have to pay search cost $s$ when they come to the BMS. At the same time, the firm has positive cost per visit $\eta$ of providing customer service, because a store with a higher number of visits needs a higher number of consultants, more capacity and a higher number of provided samples.

Let $p_w^i$ be the online and $p_s^i$ be the store price set by retailer $i$. Prices are observable by consumers. The multichannel retailer can inform consumers about both prices by posting them directly online, by proposing special tariffs and discounts on online or offline sales or by committing to price parity in different distribution channels. Production cost is normalized to zero. Consumers observe prices and decide whether to search or not and then make a decision about a purchase. Once a consumer buys the product, he leaves the market.

Consumers who decide to buy online bear additional cost of online shopping $\mu_i$, which is distributed according to $F(\mu_i)$. This distribution function has continuous support. $F(0) = 0$ and $F(\pi) = 1$, which means that all consumers want to buy online at zero price, and for any online price below $\pi$, there are consumers who have a positive expected utility of buying online. We make the standard assumption that $F(\cdot)$ is log-concave (see Bagnoli and Bergstorm (2005)), which means that $\frac{F'(\mu_i)}{F(\mu_i)}$ is decreasing in $\mu_i$. To simplify the exposition we assume that consumers cannot return the product but we discuss how to relax this assumption in footnote 6 later in the text.

We consider two different market structures:

- <u>Monopoly:</u>

  There is a unique firm in the market. It makes a decision on distribution channel and then sets prices. Consumers make a decision on their buying/searching strategies after observing prices.

- Duopoly with geographical restrictions:

  There are two markets $A$ and $B$. Consumers are split equally across them. There is one firm in each market. First, both retailers simultaneously decide whether to open an online shop or not. Second, observing each others' decision on a retail channel they simultaneously decide whether to commit to geo-blocking and, thus, refuse to sell abroad and serve only the local market. Third, retailers set prices simultaneously knowing each others' decisions on geo-blocking.

# 3 Consumers' strategies

Let's $p_s$ be the consumer's local store price and $p_w$ be the lowest price available online. Consumers may follow one of three searching and buying strategies. First, they may buy in the BMS. In this case they pay search cost to visit the store and buy the product only if it fits. The expected payoff is $\pi(1-p_s)-s$. We can see that for any price $p_s > \frac{\pi - s}{\pi}$ consumers have a negative payoff from buying in the store and thus do not purchase offline. Second, they can buy online without trying the product beforehand in the BMS. Then they pay the online price plus the additional cost of online shopping, but can learn whether the product fits only after purchasing. As there is no product return[6] the expected payoff is $\pi - p_w - \mu_i$. If the online price is higher than $\pi$, consumers never buy online without first searching in a BMS, because they get a negative expected payoff. Third, consumers can showroom, which means that they visit a BMS, try the product and then buy online in case of a successful match. The expected payoff equals $\pi(1-p_w-\mu_i)-s$. Notice that consumers who do showrooming may want to buy online even if the price is above $\pi$, but it must be below $1-p_w-\frac{s}{\pi}$.

[6]Consider now the case with product returns (see for example Petrikaite (2017)). Suppose that the consumer, who buys online, can return the product, and get reimbursed by $\alpha p$, where $\alpha < 1$. Then we can rewrite the expected utility as $\pi(1-p)-\mu_i-(1-\pi)(1-\alpha)p = \pi 1-\mu_i-p(1-\alpha(1-\pi))$. This we can scale by $1-\alpha(1-\pi)$, and introduce $\tilde{\pi} = \frac{\pi}{1-\alpha(1-\pi)}$. So we are still in the framework of our model, where matching probability is $\tilde{\pi}$ and online shopping cost is distributed between 0 and $\tilde{\pi}$. In other words, ex-ante higher probability to be reimbursed is equivalent to higher probability of matching for online sales.

A choice of the strategy depends on prices, search cost, and online shopping cost. The following Lemma establishes the result.

**Lemma 1.** *For any prices $p_s$ and $p_w$ consumers' best response strategies are following:*

*(i) If $s < (1-\pi)p_w$ then consumers search and buy online if $\mu_i < \min\left\{p_s - p_w, \frac{\pi - s}{\pi} - p_w\right\}$, and buy in the BMS if $\mu_i > p_s - p_w$ and $p_s \leq \frac{\pi - s}{\pi}$.*

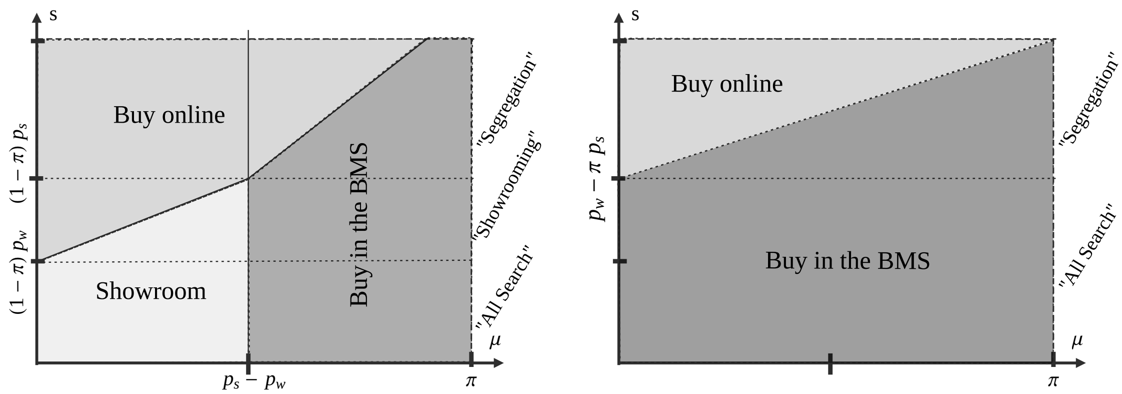
*(ii) If $(1-\pi)p_w \leq s \leq (1-\pi)p_s$ then consumers with cost $\mu_i < \min\left\{\frac{s}{1-\pi} - p_w, \pi - p_w\right\}$ buy directly online, consumers with cost $\frac{s}{1-\pi} - p_w < \mu_i < \min\left\{p_s - p_w, \frac{\pi - s}{\pi} - p_w\right\}$ showroom, consumers with $\mu_i > p_s - p_w$ buy in the store if $p_s \leq \frac{\pi - s}{\pi}$.*

*(iii) If $s > (1-\pi)\max\{p_s, p_w\}$ then consumers with $\mu_i < \{\pi p_s - p_w + s, \pi - p_w\}$ buy directly online, and consumers with $\mu_i > \pi p_s - p_w + s$ buy in the store if $p_s \leq \frac{\pi - s}{\pi}$.*

The proof of Lemma 1 and all other omitted proofs are provided in the appendix.

The choice of prices determine types of consumers' behavior which we observe in the equilibrium. First consider the case where $p_s > p_w$. If search cost is small enough such that all consumers prefer to search in the BMS, then they buy in the BMS. If search cost $s$ is high enough compared to the store price then consumers make a choice between buying directly online and buying in the BMS depending on their online shopping cost. For a moderate search cost there are three types of consumers: consumers with low $\mu_i$ buy directly online, consumers with high $\mu_i$ showroom and consumers with very high $\mu_i$ buy in the store. Second consider the case where $p_w > p_s$, so the online price is higher than the BMS price. This means that consumers never showroom. Thus either everybody buys in the store, when the online price exceeds the threshold $\frac{s}{1-\pi}$, or some consumers buy directly online and another consumers buy in the BMS otherwise. The threshold $\frac{s}{1-\pi}$ is the ratio of the search cost to the uncertainty of a good match. So, intuitively, it is clear that consumers prefer to visit the BMS when the search cost and the probability that the product fits are low.

Consumers' strategies are illustrated in Figure 1. The thresholds which separate the regions are not exogenous but defined by the choice of prices. If the online price is equal or higher than the store price, then there are only two regions, where consumers either buy online or in the store. These regions are separated by the decision line defined as $s = p_w + \mu_i - \pi p_s$.




(a) $p_w < p_s$

(b) $p_w > p_s$

Figure 1: Consumers' strategies

# 4 Monopoly

## 4.1 Observable prices

We start our analysis with the monopoly case. The results of this section will give us some insights regarding the optimal pricing strategy of the monopolist in the absence of competition and the importance of the possibility to commit to prices in different distribution channels. Later we will use some results for the analysis of geo-blocking in the presence of online competition.

We consider the optimal strategy of the firm which opens both online and BMS and sells to consumers through different channels. Monopoly sets prices $p_w$ in the web store and $p_s$ in BMS. Cost of selling is normalized to zero. The monopolists's profit equals

$$\Pi(p_w, p_s) = D_s p_s + D_w p_w - \eta T V_s, \tag{1}$$

where $D_s$ is the demand from consumers who buy in the BMS, $D_w$ is the demand from consumers who buy in the web store, and $TV_s$ is a total number of consumers who visit the store. If per-visit cost $\eta$ is high then for the firm it is not profitable to maintain the BMS. This gives us a necessary condition for in-store sales.

**Remark 1.** *The monopoly maintains the brick-and-mortar store only if $s + \eta < \pi$.*

*Proof.* Consumers visit the store and buy there only if $p_s \leq \frac{\pi - s}{\pi}$. The firm makes a positive profit on in-store sales only if $\pi p_s - \eta > 0$. These two conditions together imply that $\eta \leq \pi - s$ is a necessary condition for positive profit from selling in the BMS. A consumer with online shopping cost $\mu_i$ showrooms only if $\pi(1 - p_w - \mu_i) - s > 0$. Thus if $p_w > \frac{\pi - s}{\pi}$, no consumers showroom. The firm makes a positive profit on consumers who showroom only if $\pi p_w - \eta > 0$. Thus, $\pi - s - \eta > 0$ is also a necessary condition for the firm to make a positive profit on selling to consumers who showroom. □

We will focus on the case when the BMS store is viable, and, therefore, assume that $\pi - s - \eta > 0$. Under this assumption we can show that the firm weakly prefers to charge the BMS price equals $\frac{\pi - s}{\pi}$ to any price above this threshold.

**Remark 2.** *The choice of the BMS price* $\frac{\pi - s}{\pi}$ *weakly dominates all higher prices.*

The logic behind this result is straightforward. Since consumers have heterogeneous online shopping costs and the distribution function is log-concave, for any given online price the measure of consumers who buy online and have zero expected payoff is zero. Thus the firm has exactly the same online demand for any BMS price equal or above $\frac{\pi - s}{\pi}$. At the same time consumers never buy in the BMS if $p_s > \frac{\pi - s}{\pi}$ and might buy there if $p_s = \frac{\pi - s}{\pi}$. So the firm always prefers to charge $p_s \leq \frac{\pi - s}{\pi}$. We will use this results later in the profit maximization problem of the firm.

In order to analyze the optimal choice of prices we have to split our analysis in three cases which correspond to the best response of consumers to prices (Lemma 1). We divide consumer behavior in three classes (all search, showrooming, segregation), compute optimal prices for each class and check when consumers' searching strategies are indeed consistent with these optimal prices.

These classes are defined as follows:

**Case 1** "All search", where all consumers visit the BMS before purchasing online or offline(Lemma 2);

**Case 2** "Segregation", where consumers either search and buy in the BMS or buy directly online, and they never showroom(Lemma 3);

**Case 3** "Showrooming", where some showroom, while others either buy directly online or in the BMS (Lemma 4).

First, let us consider the case of "all search", where all consumers visit the BMS before purchasing either online or in the store (**Case 1**). From Lemma 1 we know that this happens when

$p_w > \frac{s}{1-\pi}$. Consumers with cost $\mu_i < p_s - p_w$ buy online, while consumers with $\mu_i \geq p_s - p_w$ buy in the store if they have non-negative expected utility of buying and the product fits.

$$D_s = \pi(1 - F(p_s - p_w)) \text{ and } D_w = \pi F(p_s - p_w)$$

As long as $p_s \leq \frac{\pi - s}{\pi}$ all consumers are searching in the store and $TV_s = 1$. If the online price is higher than the BMS price, then $D_w = 0$. Therefore, for any price $p_s$ the choice of online price strictly above $p_s$ gives exactly the same profit as the choice of the online price equal to the BMS price. If the firm charges $p_s \leq \frac{\pi - s}{\pi}$ and $p_w \geq \pi$, then all consumers with positive online shopping cost do not buy online, while they still want to visit the BMS. Thus, the firm can also induce "all search" by charging $p_w \geq \pi$. The firm maximizes the expected profit with respect to prices $p_s$ and $p_w$.

$$\max_{p_w, p_s} \pi(1 - F(p_s - p_w))p_s + \pi F(p_s - p_w)p_w - \eta, \text{s.t. } p_s \leq \frac{\pi - s}{\pi}, \max\left\{\frac{s}{1-\pi}, \pi\right\} < p_w \leq 1$$

We can show that optimally the firm sets the online price equal or above the BMS price.

**Lemma 2.** *If the firm wishes to induce "all search", it optimally charges* $p_s = \frac{\pi - s}{\pi}$ *and* $p_w \geq \max\left\{\frac{\pi - s}{\pi}, \pi\right\}$.

As the firm sells only in the BMS, its profit equals $\pi - s - \eta$ which is positive under our assumption on the parameters of the model. In this candidate equilibrium all consumers search in the store, but there is no showrooming. The online shop is therefore redundant.

Second, consider the case which we refer to as "segregation" (**Case 2**). Consumers decide where they buy the product immediately after observing the price. There is no showrooming in the market. From Lemma 1 we know that this requires $p_w < \frac{s}{1-\pi}$ (otherwise everybody visits the BMS) and $p_s < \frac{s}{1-\pi}$ (otherwise some consumers showroom).

Demand functions are

$$D_s = \pi(1 - F(\pi p_s - p_w + s)) \text{ and } D_w = F(\pi p_s - p_w + s)$$

As there is no showrooming $TV_s = 1 - F(\pi p_s - p_w + s)$. The firm maximization problem in this case is

$$\max_{p_w, p_s} (\pi p_s - \eta)(1 - F(\pi p_s - p_w + s)) + F(\pi p_s - p_w + s)p_w,$$
$$\text{s.t. } p_s \leq \min\left\{\frac{\pi - s}{\pi}, \frac{s}{1-\pi}\right\}, p_w \leq \min\left\{\pi, \frac{s}{1-\pi}\right\}$$

**Lemma 3.** *If the firm wants to induce "segregation", it optimally charges* $p_s = \min\left\{\frac{\pi - s}{\pi}, \frac{s}{1-\pi}\right\}$ *and* $p_w$*, which satisfies*

$$\frac{F[\pi p_s + s - p_w^*]}{F'[\pi p_s + s - p_w^*]} = (\eta + p_w^* - \pi p_s), \tag{2}$$

*with* $p_w^* \in [0, \pi]$.

This implies that for any search cost there exists a candidate equilibrium where consumers either buy directly online without searching in the BMS or search and buy in the store, so they are segregated in two groups in their searching/buying strategies. We have a boundary solution for the store price and interior solution for the online price.

Now let's consider a candidate equilibrium where consumers showroom with positive probability and buy directly online with positive probability (**Case 3**). Here we don't put any restrictions on consumers' behavior, except that the firm charges prices such that $(1-\pi)p_w \leq s \leq (1-\pi)p_s$ in order to induce this type of consumer behavior as follows from Lemma 1. Thus, the candidate equilibrium exists only if the online price is below the store price. Demand functions of the firm are

$$D_s = \pi\left(1 - F\left[p_s - p_w\right]\right) \text{ and } D_w = \pi\left(F\left[p_s - p_w\right] - F\left[\frac{s}{1-\pi} - p_w\right]\right) + F\left[\frac{s}{1-\pi} - p_w\right]$$

and the cost of providing customer service is

$$TV_s = 1 - F\left[\frac{s}{1-\pi} - p_w\right].$$

The profit maximization problem is

$$\max_{p_s, p_w} \pi\left(1 - F\left[p_s - p_w\right]\right)p_s + \pi\left(F\left[p_s - p_w\right] - F\left[\frac{s}{1-\pi} - p_w\right]\right)p_w + F\left[\frac{s}{1-\pi} - p_w\right]p_w - \eta TV_s, \tag{3}$$

$$\text{s.t. } p_w \leq \frac{s}{1-\pi}, \frac{s}{1-\pi} \leq p_s \leq \frac{\pi - s}{\pi}$$

We should consider this candidate equilibrium only for search costs below $\pi(1-\pi)$, because otherwise $\frac{\pi - s}{\pi} < \frac{s}{1-\pi}$ and the constraint in equation (3) cannot hold. The profit maximization problem always has a solution as the support of prices is bounded and $F[\cdot]$ is log-concave. We don't solve for the explicit solution. It depends on the shape of the online shopping cost distribution function and we can have either an interior solution (implicitly determined by first order conditions) or we can get a boundary solution. The next lemma proves that showrooming strictly dominates "all search" when the search cost is just below $\pi(1-\pi)$.

**Lemma 4.** *There exists $\varepsilon > 0$, such that for search cost $s \in [\pi(1-\pi)-\varepsilon, \pi(1-\pi))$, there is showrooming in equilibrium.*

The firm may prefer that consumers showroom even if the cost of providing customer service is zero. The reason is that under "showrooming" the firm gets a higher online demand due to additional sales to consumers who buy directly online. These consumers do not pay the search cost and are so unaware about the product fit, and thus may generate higher demand. Therefore, the firm can get a higher profit.

Now we do a pairwise comparison of the firm's profit in all candidate equilibria to derive the equilibrium of the game.

First, consider the interval where $s > \pi(1-\pi)$. We can show that the firm always prefers to induce "segregation". The pair of prices $p_s = \frac{\pi - s}{\pi}$ and $p_w = \pi$ in the "segregation" case gives the same profit as in the "all search" case (where nobody buys online). As we have an interior solution for the online price, which is lower than $\pi$, we know that on the interval of search costs $s > \pi(1-\pi)$ the firm gets higher profit in the candidate equilibrium with "segregation" than in the candidate equilibrium with "all search". At the same time, the firm can not induce showrooming for this interval of search costs.

Second, notice that for the search cost $s < \pi(1-\pi)$ the firm also can induce "segregation" by setting both prices below or equal to $\frac{s}{1-\pi}$. We showed in the proof of Lemma 4 that then the monopoly can always reach higher profit if consumers showroom. Thus inducing "segregation" is never the optimal strategy for search costs below $\pi(1-\pi)$.

Third, consider $s < \pi(1-\pi)$. We compare two cases - when the firm prefers to choose $p_w \geq p_s = \frac{\pi - s}{\pi}$ and when it prefers to charge $p_w$ below $\frac{s}{1-\pi}$, so there is a showrooming in the market. If the cost of search $s$ equals zero, the firm charges $p_w \leq \frac{s}{1-\pi} = 0$ only if it gets higher profit when consumers showroom. Hence the following condition must be satisfied:

$$\underbrace{\pi - \eta}_{\Pi,\ \text{“all search”}} < \underbrace{\pi p_s^*(1 - F[p_s^*]) - \eta}_{\Pi,\ \text{showrooming}},$$

where

$$p_s^* = argmax_{p_s}(\pi p_s)(1 - F[p_s]) - \eta.$$

However, $\pi p_s^*(1 - F[p_s^*]) - \eta < \pi p_s^* - \eta \leq \pi - \eta$. Therefore when the search cost is close to zero, there is "all search" in the equilibrium.

Fourth, we can show that there exists some threshold $\tilde{s} > 0$, such that for $s < \tilde{s}$ the firm sets matching prices, and for $s > \tilde{s}$ it prefers to charge different prices online and in the store. We know that for $s$ close to zero there is "all search" in the equilibrium, for $s$ close to $\pi(1-\pi)$ there is showrooming. Hence in order to prove the result it is sufficient to show the following property:

**Lemma 5.** *If for some search cost $s' < \pi(1-\pi)$ there is showrooming in the equilibrium, then for any search cost $s \in [s', \pi(1-\pi)]$ there is showrooming in the equilibrium.*

We have showed that there exists a threshold $\tilde{s}$, such that it separates two regions where all consumers buy in the store and where there is showrooming in the equilibrium.

The following proposition summarizes these results.

**Proposition 1.** *There exists $\tilde{s} \in (0, \pi(1-\pi))$ such that in equilibrium the monopolist sets prices $p_s$ and $p_w$ such that:*

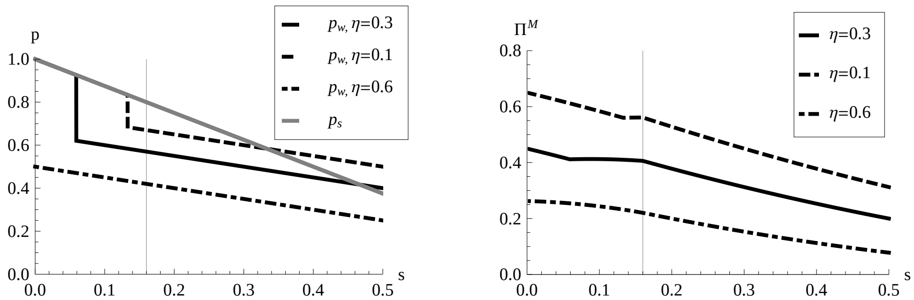
*(i) if $s < \tilde{s}$ then $p_w^* \geq p_s^* = \frac{\pi - s}{\pi}$ and consumers buy in the store if product fits;*

*(ii) if $\tilde{s} \leq s < \pi(1-\pi)$ then $p_w^* \leq \frac{s}{1-\pi} < p_s^* \leq \frac{\pi - s}{\pi}$ and consumers either buy directly online, showroom or buy in the store;*

*(iii) if $s \geq \pi(1-\pi)$ then $p_s = \frac{\pi - s}{\pi}$ and $p_w$ is defined by equation (2), consumers buy either directly online without searching or in the store.*

Figure 2 illustrates equilibrium prices and profits as a function of the search cost, when the online shopping cost is uniformly distributed on $[0, \pi]$.

We see that the firm chooses $p_w \geq p_s$ for a low search cost. The size of this region of the low search cost is decreasing in the customer service marginal cost. As any choice of online price above the store price delivers the same profit to the firm, it would be natural to assume that the firm either closes online shop and posts the information about the store price on the webpage or just sets matching prices $p_s = p_w$. As the firm expects that everybody searches in the store, it cannot effectively price discriminate between different types. The fact that a consumer visits the store does not give any signal about his type. The pricing strategy of the firm is to choose equal prices in the store and online. Sales are directed to the store.




(a) Equilibrium prices (b) Equilibrium profit

Figure 2: Monopoly

This result is robust to search cost heterogeneity, when the upperbound of the search cost distribution is sufficiently low.[7] Thus multichannel retailers with many stores and easy access to them will tend to set the same prices online and in the store, if they maintain both distribution channels. In this case online shops mostly play the role of the information source, where consumers can get informed about prices.

The assumption on common product valuation is not crucial for this result either. Visiting the store reveals to the firm some information about the willingness to buy the product, but does not provide any additional information on preferences of consumers towards online or in-store shopping. Thus heterogeneity in the product valuation will affect the magnitude of the equilibrium prices, but will not affect the optimal choice to set equal prices both online and in the store. We can notice that higher cost of providing customer service leads to a decrease in online prices, as the firm prefers that a bigger proportion of consumers buy directly online.

For high search costs the firm discriminates between two types. The price in the online shop can be both lower and higher than the price in the BMS. This happens due to the fact that the firm has to compensate customers' high search cost if it wants to enhance in-store sales on one side. On the other side higher search costs allow to discriminate better between different types, and thus,

[7] We can think about the search cost as a common component $s$ plus an individual heterogeneous component $\varepsilon_i$: $s_i = s + \varepsilon_i$. The common component $s$ characterizes how easily most of consumers can reach the store. As an example, a chain with multiple stores will have a lower common search cost component than a single store. Consumers pay a lower search cost to reach a shop which is located in a city center than a store located out of the city. Longer working hours also facilitate store visits for consumers. In this case, if magnitude and variation of $\varepsilon$ is small compared to the common component of search cost, we still find that the firm prefers to set equal prices in both retail channels.

the firm can charge a higher online price compared to the store price.

It is less likely to observe $p_w > p_s$ when the customer service cost is high, as the firm has more incentive to decrease the online price. A higher cost of customer service creates incentives for the firm to sell more directly online. Thus the firm has to propose a better online price to prevent showrooming, as benefits of search are increasing in the online price. One can think that if the search cost is sufficiently high it could be a good decision for the firm to shut down the BMS and sell online, as benefits of selling to conservative consumers get outweighed by losses of preventing showrooming. However this is not the case. If there is only an online store, then the optimal choice of online price is such that the expected utility of buying online is positive. Thus, by proposing in the BMS price $p_s = \frac{\pi - s}{\pi}$ the firm can sell to a fraction of consumers who have negative utility of buying online. This will not affect the demand of the online store, and thus the profit of the firm will increase. Later we will show, that this is not the case when the BMS price is not observable.

For moderate search frictions we observe showrooming in equilibrium. On one side, the firm discriminates between different types, on the other side it remains too costly to prevent showrooming in the market. We see that in the equilibrium with showrooming the profit of the firm may increase in the search cost, as it is getting less costly for the firm to discriminate between different types.

#### 4.1.1 Consumer Surplus and Social Welfare

In this section we consider how equilibrium consumer surplus depends on search frictions. When the firm charges price $p_s = \frac{\pi - s}{\pi}$ in the BMS it extracts full consumer surplus from those who buy there. Therefore if the firm does not open an online store and sells only in the BMS, consumer surplus equals zero. This is equivalent to the situation where the search cost is low and the firm sets prices such that everybody visits the store, so consumer surplus is $\pi(1 - \frac{\pi - s}{\pi}) - s = 0$. When the search cost is sufficiently high some consumers buy directly online, and consumer surplus is equal to

$$CS = \int_0^{\pi - p_w} (\pi - p_w - \mu_i) \, dF(\mu_i)$$

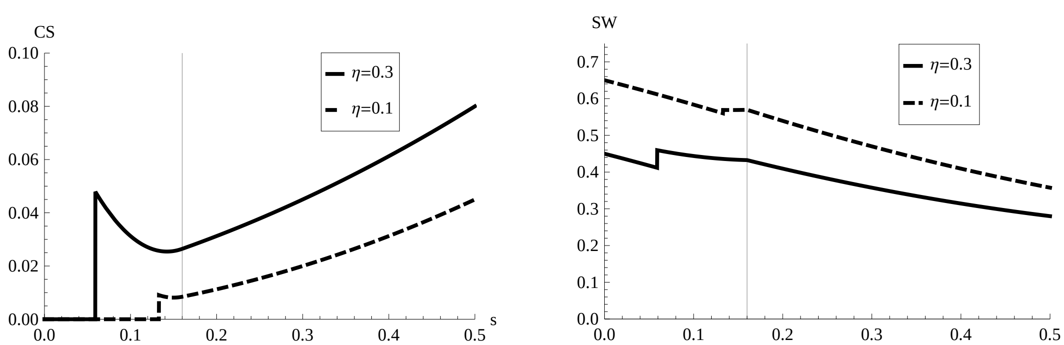
For the moderate search cost $\tilde{s} < s < \pi(1 - \pi)$ the equilibrium store price may be below $\frac{\pi - s}{\pi}$, which means that consumers, who buy in the store, also get positive expected utility. In this case consumer surplus is

$$CS = \int_0^{\frac{s}{1-\pi}-p_w} (\pi - p_w - \mu_i)\, dF(\mu_i) + \int_{\frac{s}{1-\pi}-p_w}^{p_s-p_w} (\pi(1 - p_w - \mu_i) - s)\, dF(\mu_i) +$$
$$+ \int_{p_s-p_w}^{\pi} (\pi(1 - p_s) - s)\, dF(\mu_i)$$

As long as $p_w$ is decreasing in search cost, consumers, who buy online, benefit from a lower online price. At the same time the share of online purchases is increasing. So, the total consumer surplus is increasing if there is no showrooming.

For a moderate search cost the online price offered by the firm is quite high. Consumers have incentives to search in the store and pay an additional search cost, which on one side has a negative effect on consumer surplus. On the other side, the equilibrium prices offered by the firm are decreasing in the search cost, which has a positive effect on consumer surplus. The total effect is ambiguous and $CS'_s$ can be both positive and negative when $\tilde{s} < s < \pi(1-\pi)$. When the search cost is above $\tilde{s}$ maintaining an online store positively affects both the firm's profit and consumer surplus, and thus it is socially desirable.

These results are illustrated on Figure 3.




(a) Consumer surplus (b) Social Welfare

Figure 3: Monopoly

On the interval where there is a full segregation with an interior solution for price $p_w$ the consumer surplus is increasing in the search cost, but decreasing on the interval where there is boundary solution in the equilibrium. At the same time higher cost of customer service affects consumer surplus in the positive way. This happens due to the incentives of the firm to increase share of direct online sales and, thus, to propose better online price.

## 4.2 Unobservable store prices

As a benchmark we analyze a monopoly case with an unobservable BMS price. Typically consumers can easily observe online prices on the webpage of a firm, however they need to visit a store to learn a store price. If the firm does not provide any information about store prices on its webpage, then consumers have to form some expectations about them.

In this section we suppose that the firm can not credibly commit to different an online and a BMS prices (or decides not to do so). The online price $p_w$ is easy to observe at no cost on the webpage of the online shop. Store price $p_s$ is observable only in the store. After observing the online price consumers expect to find price $\hat{p}_s$ if they visit the BMS. If consumers are rational then in the equilibrium they should form correct expectations such that $p_s = \hat{p}_s$, where $p_s$ is the actual BMS price.

When search cost is positive a single BMS suffers from consumers facing a hold-up problem. One way to avoid it is to advertise to consumers the BMS price (see Janssen and Non (2008) for price advertising incentives). We will show that opening an online store and hence creating a downward-slopping demand due to artificial competition between different distribution channels is another way to resolve the hold-up problem. The multichannel retailer can use the online price as an instrument, which affects beliefs of consumers about the store price.

First we can show that there is no equilibrium with "segregation", where the firm has positive sales in the BMS. As we know from Lemma 1, this candidate equilibrium requires the online price bbeing sufficiently low, $p_w \leq \frac{s}{1-\pi}$. Now, suppose that consumers anticipate some price $\hat{p}_s$ in the BMS and some consumers visit the BMS. Consumers visit the store only if they have an intention to buy there in the case of a good match. It means that $\pi(1-\hat{p}_s) - s > \max\{0, \pi - p_w - \mu_i\}$ for the consumer $i$ who prefers to buy in the store. So two conditions should be satisfied: i) a consumer has higher surplus of buying in the BMS than online, or $\mu_i > \pi\hat{p}_s + s - p_w$; ii) this surplus is positive, or $\hat{p}_s < \frac{\pi - s}{\pi}$. Now we can show that the firm always want to charge $p_s > \hat{p}_s$. Suppose that consumers come to the BMS and find the price $p_s = \hat{p}_s + \varepsilon < 1$. If the product fits and they buy it in the store, they get utility $1 - \hat{p}_s + \varepsilon > 0$. So the surplus is positive and consumers still want to buy the product. After observing actual store price they prefer to buy the product online if $\mu_i + p_w < p_s$. As we consider the case where $\hat{p}_s < \frac{s}{1-\pi}$, we can notice that there always exists $\varepsilon > 0$, such that the set $\{\mu : \mu_i \in (\pi\hat{p}_s + s - p_w, \hat{p}_s + \varepsilon - p_w)\}$ is empty. So the firm has incentives to slightly increase the store price because more conservative consumers will still buy in the store after observing $\hat{p}_s + \varepsilon$. The rational consumer should anticipate that, and thus, he does not go to the BMS. The firm sells only online.

Second, we consider cases of "all search" and "showrooming". Notice that in the case where there is no segregation in the market and some consumers may showroom, the final decision about purchasing in the BMS is made by consumers who observe both prices. They prefer to purchase in the BMS if $\mu_i > p_s - p_w$. So the decision whether to buy in the BMS or online is affected by the actual BMS price, while the anticipated price affects the decision to visit the BMS in the first place. As the equilibrium condition requires that the anticipated BMS store price equals the actual BMS price, the following equality must be satisfied:

$$\Pi'_{p_s}|_{p_s^*} = 0.$$

There is no boundary solution for the BMS price. If $p_s = \hat{p}_s \leq \frac{\pi - s}{\pi}$ and $\Pi'_{p_s}|_{\hat{p}_s} > 0$, then the firm has a profitable deviation to some price above $\hat{p}_s$. In "all search" if the online price is equal or higher than the offline price, the firm always has incentives to charge $p_s$ above consumers' expectations.

$$\Pi^{AS} = \pi p_s(1 - F[p_s - p_w]) + \pi p_w F[p_s - p_w]$$

$$\{\Pi^{AS}\}'_{p_s} = \pi F[p_s - p_w] + \pi p_s F'[p_s - p_w] - \pi p_w F'[p_s - p_w]$$

$$\{\Pi^{AS}\}'_{p_s}|_{p_w \geq p_s} = \pi > 0$$

As we know from the previous analysis existence of showroming requires that the online price being lower that the offline price. Thus both situations may happen in the equilibrium only if the online price is below the store price. This means that the firm keeps always online price lower than the BMS price in order to affect beliefs of consumers that the BMS price is not too high.

In the appendix we provide the derivation of the equilibrium structure in the case of unobservable prices. Here show that the firm always weakly prefers to dislose the store price and therefore has incentives to advertise it.

**Proposition 2.** *For any possible set of parameters $s$, $\pi$ and distribution function $F(\cdot)$, it is always profitable for the firm to dislose the offline price.*

We can make two main conclusions. First, by introducing the online channel the firm can create a downward-slopping demand on BMS purchase and thus to avoid a hold-up problem when search cost is not too high. The possibility of showrooming plays an essential role for this result. However, for sufficiently high search cost the cost for avoiding the hold-up problem and inducing offline sales are too higher. The firm prefers to shut down the BMS and purely concentrate on online sales.[8] For search cost above $\pi(1-\pi)$ this is the only possibility because consumers don't want to showroom for any online price below $\pi$.

Second, there may exist the range of search cost such that the firm reaches exactly the same outcome when offline price is advertised or not. For this range of parameters showrooming takes place in the equilibrium in both cases. Therefore, we can conclude that if price advertising is associated with some additional cost which firm has to bear to provide information to consumers, it can prefer not to disclose the store price.

### 4.3 Price matching policy

From the previous analysis, we can conclude that the multichannel retailer has incentives to commit to the BMS price, when consumers are rational. The simplest way to make a credible commitment to the BMS price is to declare the price matching policy, which requires price parity, $p_s = p_w = p$. This should immediately remove the possibility of the showrooming and would imply that there is either "all search" in the market, when search cost is low, or full segregation when search cost is high.

When search cost is equal or above $\pi(1-\pi)$ the monopolist has two possible strategies as we know from Lemma 1. It can either charge price $p \leq \frac{\pi-s}{\pi}$ in order to sell both online and offline, or to set price $p > \frac{\pi-s}{\pi}$, so there will be only online sales. He never charges price $p > \pi$, because otherwise he would have zero sales. Intuitively, we have to expect, that for very high search cost, it is not profitable to enhance in-store sales, as the monopolist has to compensate high search cost to consumers. For example, when search cost approaches to $\pi$ price matching policy requires prices being set close to zero in order to induce positive instore sales. So we can calculate the threshold such that the firm prefers to shutdown the store and sell only online.

[8] If we introduce to the model a fraction of consumers who has zero search cost, then obviously the frim will still support the BMS store to serve these consumers. However, those with positive seach cost will buy only online.

First, suppose that the monopolists decides to sell in both retail channels, which requires that $p \leq \frac{\pi - s}{\pi}$. He maximizes the profit as follows:

$$\max_p pF[s-(1-\pi)p] + \pi p(1-F[s-(1-\pi)p]) - \eta(1-F[s-(1-\pi)p]), \text{ s.t. } p \leq \frac{\pi - s}{\pi}.$$

Notice that if $p < \frac{\pi - s}{\pi}$ and $p < \pi$, and the firm has positive sales both online and offline, then for any $s > \pi(1-\pi)$ the optimal choice of price $p$ is such that $p < \frac{s}{1-\pi}$. The solution of the profit maximization problem is

$$p = \min\{\frac{\pi - s}{\pi}, p'\}, \tag{4}$$

where $p'$ satisfies

$$\pi + (1-\pi)F[s-(1-\pi)p'] - (1-\pi)\left(p'(1-\pi) + \eta F'[s-(1-\pi)p']\right) = 0 \tag{5}$$

Alternatively the firm can decide to sell only online instead of restricting online price by $\frac{\pi - s}{\pi}$. Then it charges price above this threshold. The profit maximization problem is

$$\max_p pF[\pi - p], \text{ s.t. } p < \pi,$$

which has the solution $\tilde{p}$ satisfying

$$F[\pi - \tilde{p}] - \tilde{p}F'[\pi - \tilde{p}] = 0 \tag{6}$$

Log-concavity of function $F(\cdot)$ guarantees that $\tilde{p} \leq \pi$, so there are always positive online sales.

Now we can find threshold $\tilde{s}$, such that for search cost below this threshold the firm prefers to sell through both chanells, while for search cost above the threshold it chooses to sell only online. Obviuosly, if there exists interior solution for $p'$ such that $p' < \frac{\pi - s}{\pi}$, the firm optimally sells through both chanells. When it is getting too costly to keep both prices sufficiently small to induce instore sales, the firm prefers to increase the online price and switch to online sales only. We derive threshold $\tilde{s}$ from the following profits' equality condition:

$$\Pi_{p=\frac{\pi-s}{\pi}, s=\tilde{s}} = \Pi_{\tilde{p}, s=\tilde{s}}$$

We can easily show that there always exists $\tilde{s}$ such that the last equality is satisfied. Suppose that $s$ is close to $\pi$, than the profit of the firm selling both online and offline goes to zero, while the profit of the firm selling only online is positive. Now suppose that $s$ is aproaching $\pi(1-\pi)$. Then the existence of the interior solution for equation (6) guarantees $\tilde{p} < \pi = \frac{\pi - s}{\pi}$, and thus optimally the firm sells in both online and BM stores. The profit of the firm selling through both channels is decreasing in $s$, while the profit of the firm selling only online remains the same for any search cost. Thus, we there exists unique $\tilde{s} > \pi(1-\pi)$, such that for any search cost above this threshold the firm prefers to sell only online, and for search cost below it sells also in the BMS. Hence, when search cost is sufficiently high, price matching policy leads to an exclusion of a fraction of consumers from the market. If we compare these results to the case of unobservable prices we can see that *i):* the monopolist cannot increase its profit by committing to price parity when search cost is above $\tilde{s}$; *ii)* for search frictions $s \in (\pi(1-\pi), \tilde{s})$ the price matching policy is a profitable strategy for the firm.

If search cost is below $\pi(1-\pi)$ then the multichannel retailer can charge price $p_w > \frac{s}{1-\pi}$, so everybody buys in the store. Optimal choice of prices is $p_w = p_s = \frac{\pi - s}{\pi}$ and profit is equal to $\pi - s - \eta$. We can show that there exists a threshold $s'$, such that for any search cost below $0 \leq s' \leq \pi(1-\pi)$ the firm charges $p_s = p_w = \frac{\pi-s}{\pi}$ and for search cost above the firm charges $p = \min\left\{\frac{s}{1-\pi}, p'\right\}$[9]. First of all, when search cost goes to zero the profit goes to $\pi - \eta$ if all consumers buy in the store (prices above $\frac{s}{1-\pi}$), and to zero, otherwise. At the same time, when search cost approaches to $\pi(1-\pi)$, for any positive $\eta$ the firm can reach higher profit by charging $p_w = p_s = \frac{s}{1-\pi}$ than by charging $p_s = p_w = \frac{\pi-s}{\pi}$. This happens as $\frac{s}{1-\pi}$ approaches to $\frac{\pi-s}{\pi}$ and the firm gets strictly positive and bounded below savings on customer service if it keeps prices such that a fraction of consumers buy directly online. The difference in profits $\Pi^{Seg} - \Pi^{AS}$[10] is increasing in $s$. Thus, there exists a unique threshold $s'$, such that for search cost above the firm sets prices to segregate consumers, and for search cost below everybody goes to the BM store in the equilibrium. The following proposition summarizes results of this section.

**Proposition 3.** *There exist $s'$ and $\tilde{s}$, such that in the equilibrium the monopolists, who commit to price matching policy, sets prices $p_w$ and $p_s$ such that*

*(i) if $s < s'$ then $p_w = p_s = \frac{\pi-s}{\pi}$, consumers buy in the BMS;*

[9]We assume that condition from 2 is satisfied when s goes to zero, so $\eta < \pi$. Otherwise, it is obviously never profitable for the monopolist to maintain the BMS

[10]Here $\Pi^{Seg}$ is the highest profit in the candidate equilibrium with segregation and $\Pi^{AS}$ is the highest profit in the candidate equilibrium with "all search". $\frac{\partial(\Pi^{Seg} - \Pi^{AS})}{\partial s} = 1 + (\tilde{p}(1-\pi) + \eta)F'[(-1+\pi)\tilde{p} + s] > 0$, where $\tilde{p} = \min\left\{p', \frac{s}{1-\pi}\right\}$

*(ii) if $s' \leq s < \tilde{s}$ then $p_w = p_s = \min\left\{\frac{\pi - s}{\pi}, \frac{s}{1-\pi}, p'_w\right\}$, where $p'_w$ is defined as in equation (5), consumers either buy directly online or in the BMS;*

*(iii) if $s \geq \tilde{s}$ then $p_w = \tilde{p}_w$ and there are no sales in the BMS, consumers buy online.*

So we can see that for sufficiently low search cost $s$ and service cost $\eta$ the price matching policy allows to reach the equilibrium profit which is the same as in the case of observable prices. It allows as well to ensure in-store sales, when the firm can not induce showrooming, and its BMS faces a hold-up problem.

**Corollary 1.** *If the store price is unobservable, then there exists a range of search cost, such that the monopolist strictly prefers to commit to the price matching policy.*

Notice that this range of search cost is not necessarily convex. The firm may prefer to commit to the price matching policy for search cost close to zero and slightly above $\pi(1-\pi)$, but at the same time for search cost slightly below $\pi(1-\pi)$ it may reach the exactly same equilibrium outcome independently of whether the store price is observable. Figure 4 illustrates profits' comparison for the uniform distribution of online shopping cost. On Figure 5 we illustrate the comparison of firms

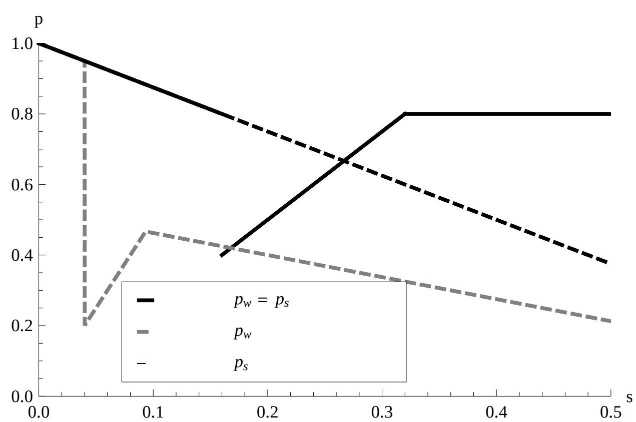



Figure 4: Price matching policy: Equilibrium Prices.

profits in there cases assuming uniform distribution of online shopping cost. Obviously, the firm can achieve the highest profit when prices are observable. However, if it cannot credibly commit to different online and the BMS prices, then it increases the profit by applying the price parity commitment, which is efficient for sufficiently low search frictions. Otherwise, the optimal choice

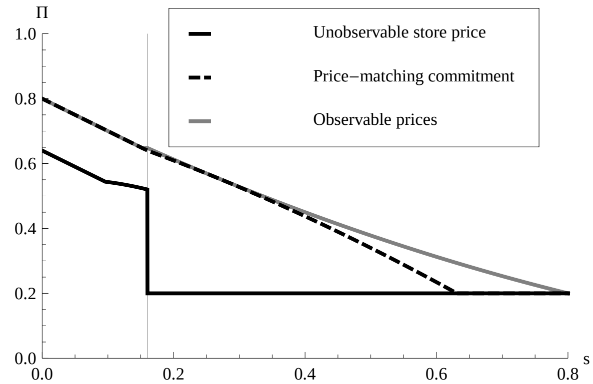



Figure 5: Monopoly profit and the store price observability.

of prices is such that all sales are redirected to online shop under price matching policy. In order to induce in-store sales, the firm has to compensate the high search cost to consumers, and, therefore, it loses the profit from online sales. In this situation, the optimal decision is to sacrifice in-store sales and to charge higher online price.

## 5 Cross-border competition and geo-blocking

In this section we focus on competition between two retailers which are geographically separated. They can have local BMSs, where only consumers from the local market can buy. At the same time they compete online à la Bertrand since goods are homogenous. For simplification we assume that the cost of providing customer service is equal to zero. We analyze decisions of firms on retail channels and their incentives to geo-block. Geo-blocking assumes that consumers from one geographical zone have no access to online stores operating in another geographical zone. First, we provide some insights explaining why firms may want to voluntarily refuse to sell in foreign markets. Second, we explain why banning geo-blocking may be not a good decision for the competition authorities, and how a non-discrimination price policy can help to eliminate incentives of firms to refuse to sell abroad.

As decisions on retail channels and geo-blocking are long-run, it is naturally to assume that they precede competition stage. Hence we assume that the game consists of three stages: i) firms decide simultaneously[11] on retail channels; ii) firms decide simultaneously on geo-blocking strategy;

[11] Later in this section we will discuss how it changes if firms take decisions on retail channel and geo-blocking

iii) firms decide simultaneously on price, competition takes place, and firms' profits are realized. As in the monopoly case we will focus on situations where $s < \pi$, so firms maintain brick-and-mortar stores.

We solve for subgame perfect equilibrium, and thus use backward induction approach. First, we should consider all possible market structures and derive equilibrium profits in each case. Overall we have six possibilities - three cases with a symmetric market structure and three cases with an asymmetric market structure:

- Firms $A$ and $B$ do not open online stores;
- Firms $A$ and $B$ open online stores and geo-block;
- Firms $A$ and $B$ open online stores and do not geo-block;
- Firm $A$ opens online store and does not geo-block, firm B has only the BMS store;
- Firm $A$ opens online store and geo-block, firm B has only the BMS;
- Firms $A$ and $B$ open online stores, firm $A$ does not geo-block and firm $B$ does.

We have already solved for the first two symmetric market structures. The simplest case is when both retailers decide to maintain only BMSs. Both firms are monopolists at the local markets. They charge prices equal $\frac{\pi - s}{\pi}$ in the local stores, as it is the highest price which consumers are ready to pay. The equilibrium profit of each firm is $\Pi^{BMS}(s) = \pi p_s = \pi - s$. If retailers decide to open online shops and both geo-block, then they get monopoly profits derived in the previous section, which are not lower then $\pi - s$ for any search frictions $s$ as we have already shown before. We will denote these profits as $\Pi^M(s)$.

The asymmetric market structure, when firm $A$ opens an online store and geo-blocks and firm $B$ has only the BMS, never takes place in the equilibrium. Obviously in this case there is no competition between firms, and thus in equilibrium firm $A$ gets the monopoly profit $\Pi^M(s)$ and firm $B$ gets profit $\Pi^{BMS}(s)$. Firm $A$ can get at least not lower profit if it sells also in market $B$ online. Thus it does not have any incentives to geo-block.

The remaining symmetric market structure to consider has competition between two multi-channel retailers which open online shops and do not geo-block. Since goods are homogeneous

sequentially.

consumers who buy online buy from the firm which offers the lowest price. Firms compete online à la Betrand, and therefore each firm has incentives to slightly undercut the online price of the competitor as then it serves all online sales and thus increases its profit. [12] Thus in the equilibrium it should be that $p_w^A = p_w^B = 0$, i.e. firms charge online prices at marginal cost. Store prices should be strictly below $\frac{\pi - s}{\pi}$, as otherwise everybody prefers to buy online.

Consumers in market $j$ prefer buying directly online to showrooming if $\mu_i < \frac{s}{1-\pi}$. Therefore, if $s < \pi(1-\pi)$ and $p_s^j \geq \frac{s}{1-\pi}$, then there is a positive proportion of consumers who showroom. If $s > \pi(1-\pi)$ or $p_s^j < \frac{s}{1-\pi}$, then nobody showrooms. So we can observe either showrooming or "segregation" in equilibrium. The following Lemma establishes the equilibrium outcome.

**Lemma 6.** *If two multichannel retailers compete online à la Bertrand, then there exists threshold* $0 < \tilde{s} < \pi(1-\pi)$*, such that the equilibrium prices are*

$$p_w^A = p_w^B = 0,$$

$$p_s^A = p_s^B = \begin{cases} p^* & \text{if } s \geq \tilde{s}, \\ \max\left\{p', \frac{s}{1-\pi}\right\}, & \text{if } s < \tilde{s}, \end{cases}$$

*where* $p^*$ *satisfies*

$$1 - F[\pi p^* + s] - \pi p^* F'[\pi p^* + s] = 0,$$

*and* $p'$ *satisfies*

$$1 - F[p'] - p' F'[p'] = 0.$$

We can see that equilibrium online prices are at marginal production costs, which are normalized to zero. At the same time store prices are below the optimal price of a single BMS which does not face an online competitor. Thus, we can conclude that in this case each firm gets the profit which is lower than the profit of a single BMS which does not compete with an online store. Let's denote equilibrium profits in this subgame as $\Pi^C(s)$. So, we showed that $\Pi^C(s) < \Pi^{BMS}(s)$.

[12] This is guaranteed by the continuity of the monopoly profit in both prices.

**Competition between a BMS and a multichannel retailer:** Now we consider the case of asymmetric competition where the multichannel retailer $A$, which can sell online in two markets and at the local store, competes with the BMS $B$, which sells only in the local store in market $B$. We do not fully derive firms' equilibrium strategies, but we will show that (i) the equilibrium exists and (ii) equilibrium profits satisfy some inequalities. The following Lemma establishes the result:

**Lemma 7.** *When there is competition between multichannel retailer A selling in markets A and B online and a single BM store B the equilibirum profits satisfy the following conditions:*

*(i) profit of firm A is not lower than* $\Pi^M(s)$ *for any search cost s;*

*(ii) the equilibrium profit of firm B is strictly higher than* $\Pi^C(s)$ *and strictly lower than* $\Pi^{BMS}(s)$.

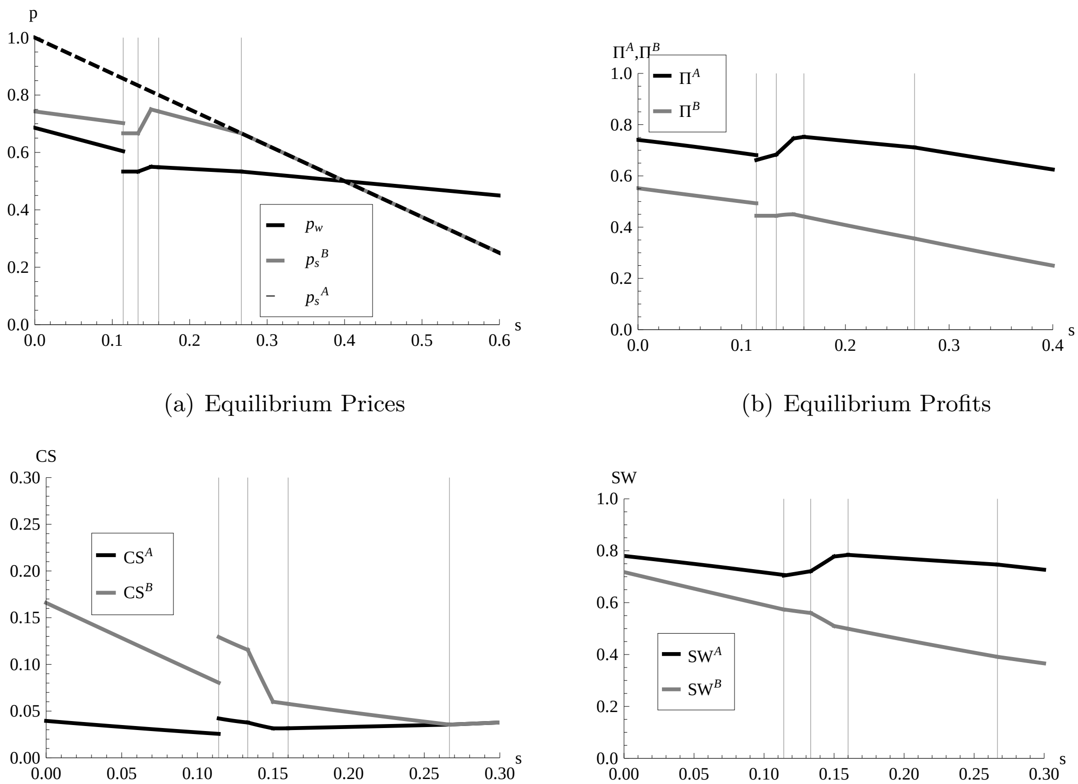
Therefore, single BMS store can reach higher profit than a multichannel retailer who competes with another multichannel retailer online. Decision of irm $B$ not to sell online allows to relax competition and leads two higher prices in both stores. Even if firm $B$ looses a part of the market, it gets higher profit from selling to local consumers who have high online shopping cost and thus prefer to buy in the brick-and-mortar store.

On Figure 6 we illustrate the equilibrium solution for the linear cumulative distribution function $F[\cdot]$. We can see, that there is a range of search cost where firm $A$ is mixing between two online prices below and above $\frac{s}{1-\pi}$.

The general structure of the equilibrium depending on search cost is similar to the solution which we got for the monoply problem. However, we can see that there is no range of search cost[13] where the multichannel retailer prefers to choose equal prices online and offline. When search cost is low firm $A$ prefers to charge lower price online due to presence of competition.

**Competition between two multichannel retailers, where one retailer geo-blocks:** The last possible case is an asymmetric market structure, when both firms open online shops and one firm (let's say $B$) commits to geo-blocking. First we show that there is no pure strategy Nash equilibrium. We can notice that if price $p_w^B > p_w^A$ then firm $B$ does not sell online to local customers. If it decreases the online price down to $p_w^A$ that will not affect demand for BMS $B$, plus half of online customers will buy from $B$. Thus this deviation is strictly profitable. Therefore, firm $B$

[13] Apart of one single point




(a) Equilibrium Prices

(b) Equilibrium Profits

(c) Consumer Surplus

(d) Social Welfare

Figure 6: Competition: $F(\mu_i) = \frac{\mu_i}{\pi}$

always prefers to charge $p_w^B \leq p_w^A$. If $p_w^B = p_w^A$ then both firms have incentives to undercut the price as they double the demand from the customers in market $B$ who buy online. Price $p_w^A$ should be strictly positive, otherwise there is a profitable deviation for $A$ to prices which guarantee at least the monopoly profit $\Pi^M$. Thus, the existence of pure strategy equilibrium requires that $p_w^B < p_w^A$, which means that firm $A$ does not sell online on market $B$. If firm $A$ sells online only in the local market it gets profit which is not higher than $\Pi^M(s)$. However, in the equilibrium firm $A$ should get the profit which is not lower than $\Pi^M(s)$ as well. So it should be that firm $A$ gets exactly $\Pi^M(s)$.

As markets are symmetric, another necessary condition, which should be satisfied, is profits' equality $\Pi^A = \Pi^B$, otherwise firm $B$ always has a profitable deviation. If $\Pi^A > \Pi^B$, then firm $B$ can charge price $p_s^B = p_s^A$ and $p_w^B$ slightly below $p_w^A$, and thus it increases its profit. At the same time the profit of firm $B$ is not higher than $\Pi^M(s)$. Thus, the existence of the pure strategy equilibrium requires that $\Pi^A = \Pi^B = \Pi^M$ and $p_w^B < p_w^A$. That means that monopolistic problem should have multiple equilibria, which contradicts to Proposition 1. So, there is no equilibrium in pure strategies where one multichannel retailer commits to geo-blocking and the other does not. In the equilibrium firms play mixed strategies. The existence of mixed strategy equilibrium is guaranteed by standard results of auction theory applied to the duopoly framework with Bertrand competition (see Dasgupta and Maskin (1986)).

Firms should mix on some interval of prices $p_w^B, p_w^A \in [\underline{p}_w, \bar{p}_w]$, and stores prices are the optimal prices $p_s^B(p_w^B, E(p_w^A))$, $p_s^A(p_w^A)$. We know, that firm $A$ can guarantee at least the monopoly profit $\Pi^M$. Firm $B$ never plays $p_w^B = \bar{p}_w$ with strictly positive probability in the equilibrium. If it does, then firm $A$ also should charge the online price at the upper bound with strictly positive probability, but then firm $B$ should charge online price at the upperbound with probability 0. So we have a contradiction. This means that $\Pi^A(p_w^A = \bar{p}_w, p_s^A(\bar{p}_w)) = \Pi^M$. The mixed strategy equilibrium requires that for all prices in the support firms get the same expected profit. So, the lowerbound of the equilibrium price distribution comes from the profit's equality condition. Profit of firm $A$ charging $p_w^A = \underline{p}_w$ and $p_s^A(\underline{p}_w)$ is equal to the monopoly profit $\Pi^M$. Thus, if firm $B$ geo-blocks than its expected profit is increasing as online prices charged by the competitor with positive probabilities are strictly above 0. Let's denote the equilibrium profit of firm $B$ in this subgame as $\Pi^G$. We showed that $\Pi^C(s) < \Pi^G(s) < \Pi^M(s)$.

We see that if both firms decide to open online stores at the first round, then each firm has an incentive to commit to geo-blocking at the second stage of the game in order to prevent the tough competition at the stage of competition. The matrix of payoffs of two multichannel retailers is illustrated on Figure 7.

As geo-blocking is a weakly dominant strategy for both firms, we get three possible weak subgame Nash equilibria. One is symmetric, where both firms geo-block and become monopolists in the local markets. This is a unique trembling hand perfect equilibrium. Both firms get higher profits than in the case of running only brick-and-mortar store. Other two are asymmetric equilibria, where one firm geo-blocks and another bans cross-border sales. They are weak Nash equilibria.

Now we analyze decision of firms at the first stage, when they decide whether to open an online shop. If firm $A$ opens an online store, then

- if firm $B$ opens an online store, both firms geo-block and get profit $\Pi^M$ at the last stage;

**Retailer B**

| **Retailer A** | Geo-blocking | No |
|---|---|---|
| Geo-blocking | $(\underline{\Pi^M}, \underline{\Pi^M})$ | $(\Pi^G, \Pi^M)$ |
| No | $(\Pi^M, \Pi^G)$ | $(\Pi^C, \Pi^C)$ |

Figure 7: Geo-blocking: Profits of Multichannel Retailers

- if firm $B$ does not open an online store, then firm $A$ does not geo-block at the second stage and gets the profit $\Pi^{MC}(s) \geq \Pi^M(s)$.

If firm $A$ decides not to open an online shop at the first stage then

- if firm $B$ opens an online store, it does not geo-block at the second stage, and firm $A$ gets profit $\Pi^{BC}(s)$ at the last stage;
- if firm $B$ does not open an online store, then firm $A$ gets the profit $\Pi^{BMS}(s)$ at the stage of competition.

Thus, the matrix of firms' payoffs is

So, we can see that if decisions are taken simultaneously, then both firms should open online stores at the first stage and commit to geo-blocking at the second stage. So there are two monopolists at the local markets not selling abroad in the equilibrium.

Thus, we can formulate the following result:

**Proposition 4.** *Both retailers open online stores and commit to geo-blocking in the equilibrium.*

In this situation competition authorities may be interested in restoring the market competition, and thus they take a decision to forbid geo-blocking strategy and require that firms do not discriminate consumers based on their geographical location. Let's consider how the market equilibrium changes if firms cannot geo-block at the second stage. The matrix of payoffs is illustrated

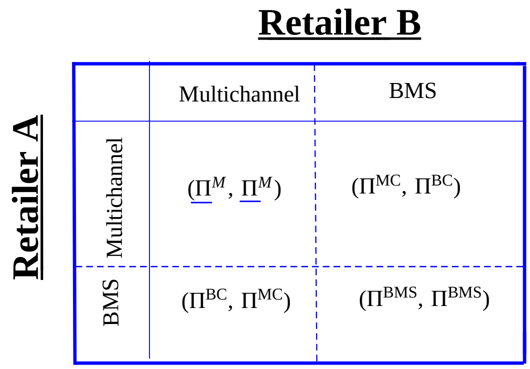



Figure 8: Decisions on Distribution Channels: Profits of Retailers

on Figure 9. Since firms get profit $\Pi^C s$ if both open online stores, at least one firm does not open an online shop in the equilibrium. We should observe asymmetric market structure with only one multichannel retailer. Consumers surplus increases as firms charge lower prices in the equilibrium, compared with the case when there are two multichannel retailers which geo-block. However, this policy leads to the exclusion of one multichannel retailer from the online market.

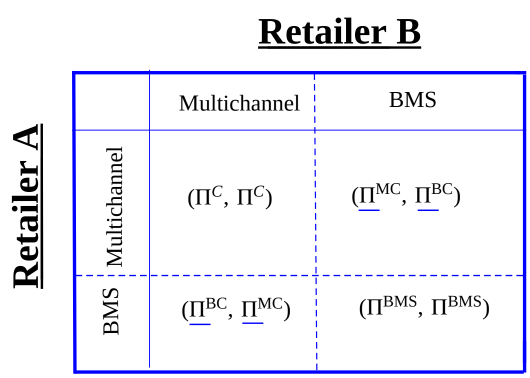



Figure 9: Legal Restrictions on Geo-blocking: Profits of Retailers

## 5.1 Sequential decisions on retail channels and geo-blocking

While legal restrictions on geo-blocking lead to unambiguously higher consumer surplus when firms take decisions simultaneously, they may potentially lead to higher expected market prices when decisions are taken sequentially. Suppose that there is a market leader, which decides first on the retail channel and geo-blocking, and the market follower who takes decision on its retailer channel and geo-blocking after observing decisions of the competitor. So the timing of the game is following: i) firm A decides whether to open an online shop and whether to geo-block ii) firm B takes its decisions on a retail channel and geo-blocking iii) competition occurs, and profits realize.

Let's focus on optimal strategies of firm $B$. Suppose that firm $A$ does not open an online shop, then firm $B$ should open an online store and sell in both markets online, as we have already shown above. If firm $A$ opens an online store and geo-blocks, then the optimal decision of firm $B$ is to open an online store, and it is indifferent between geo-blocking and not geo-blocking. If firm $A$ opens an online store and does not geo-block, then firm $B$ may optimally either open an online store and geo-block or maintain only its BMS. The optimal decision comes from the profit comparison. If $\Pi^{BC}(s) > \Pi^{G}(s)$ then firm $B$ does not open an online store. If $\Pi^{G}(s) \geq \Pi^{BC}(s)$ then firm $B$ opens an online stores, but geo-blocks in equilibrium.

Now we focus on optimal decision of firm $A$ at the first stage of the game. If firm $A$ does not open an online shop it gets profit $\Pi^{BC}$. If firm $A$ opens an online shop and geo-blocks then it gets either $\Pi^{M}$ or $\Pi^{G}$ depending on geo-blocking decision of firm $B$. As firm $B$ is indifferent between two options, we can see that if there is some small positive probability $\varepsilon$ that it decides not to geo-block, then expected profit of firm $A$ is strictly below $\Pi^{M}$. If firm $A$ opens an online store and does not geo-block, then its expected profit is greater or equal $\Pi^{M}$. Therefore we can conclude that the optimal decisions of firm $A$ at the first stage is to open an online store and sell in both markets online.

We observe an asymmetric market structure in the equilibrium. The market leader sells online in both markets, and the market follower either does not sell online at all, or sells only in the local market. Suppose that competition policy authorities want to prevent geo-blocking in the second case and introduce legal restrictions on it. Then in equilibrium firm $B$ does not maintain the online shop.

The total effect of geo-blocking restrictions on consumer surplus can differ depending on function

$F[\mu_i]$ and market parameters. On one side, we can observe that firm $A$ may charge higher expected online price when firm $B$ maintains an online shop.

**Lemma 8.** *If $F[\mu_i]$ is convex then firm $B$ opens an online shop and geo-blocks only if firm $A$ charges higher expected price in the case of online competition.*

This result comes from the fact that the multichannel retailer, which faces tougher online competition, would prefer to concentrate more on its local market and thus to charge higher prices. The same idea is well established in Rosenthal (1980), where the author shows that the increasing number of competing firms leads to higher expected price in the equilibrium. Hence the decision to ban geo-blocking will not lead to a decrease of consumer surplus in market $A$ if function $F[\mu_i]$ is convex.

On the other side consumers in market $B$ buy at the lowest available online price. In the presence of competition firms charge with positive probabilities lower prices than in the case when firm $A$ sells online. Thus, the competition can lead to the lower expected minimum online price in market $B$. Since the store price $B$ is increasing in the expected minimum online price, in this case we should observe that the expected prices in both channels decrease, and therefore consumer surplus in market $B$ is higher in the presence of online competition.

## 5.2 Non-discrimination price policy

Non-discrimination price policy can be used as an instrument to decrease incentives of multichannel retailers to geo-block. The requirements to charge equal prices online and in a store should weaken a competition between firms when they do not geo-block.

We start our analysis with the case when two multichannel retailers do not geo-block. First of all, we can show that we can not have an asymmetric equilibrium, where $p^A < p^B$. Suppose it exists and prices are $\tilde{p}^A < \tilde{p}^B$.

Firms should have equal profits in the equilibrium because of the same reason which we discussed before in the symmetric competition section. In addition to the profit equality condition there should be that ${\Pi^{A}}'_{p^A}|_{\tilde{p}^A} = 0$ if $\tilde{p}^A \neq \frac{s}{1-\pi}$, so the firm $A$ does not have incentives to deviate. Analogously, ${\Pi^{B}}'_{p^B}|_{\tilde{p}^B} = 0$ if $\tilde{p}^B \neq \frac{\pi - s}{\pi}$. At the same time we can notice, that the profit of the firm $B$ is non-decreasing in price $p^A$, as higher online price of the competitor cannot negatively affect firm's profit, or $\frac{\partial \Pi^B}{\partial p^A} \geq 0$, and this inequality is strict for online price below store prices.

First, suppose that $\tilde{p}^B < \frac{s}{1-\pi}$. The firm $A$ can deviate and charge $p^A = p^B - \varepsilon > \tilde{p}^A$, where $\varepsilon > 0$. Then $\Pi^A(p^A = \tilde{p}^B - \varepsilon, p^B = \tilde{p}^B) > \Pi^B(p^A = \tilde{p}^B - \varepsilon, p^B = \tilde{p}^B) \geq \Pi^B(p^B = \tilde{p}^B, p^A = \tilde{p}^A)$ for sufficiently small $\varepsilon$. Thus, there is a profitable deviation for firm $A$. Second, suppose that $\tilde{p}^B > \frac{s}{1-\pi}$ then $\Pi^A(p^A = \tilde{p}^B, p^B = \tilde{p}^B) = \Pi^B(p^A = \tilde{p}^B, p^B = \tilde{p}^B) > \Pi^B(p^B = \tilde{p}^B, p^A = \tilde{p}^A)$. Therefore, we can see that there is also a profitable deviation for the firm $A$ in this case. We conclude that there is no asymmetric equilibrium.

Now let us consider a symmetric equilibrium. If consumers buy with positive probability online (search cost is high), or $p^j \leq \frac{s}{1-\pi}$, then both firms have incentives to slightly undercut the price, as consumers buy online from the firm which charges the lowest price. At the same time firms do not charge prices equal to zero, as they can always get the positive profit charging positive prices even if price of the competitor is equal to zero due to the monopoly power of local BM stores. So we can conclude that in this case there should be a mixed strategy equilibrium where the lower bound of the price support is positive.

When search cost is low (close to zero), consumers prefer to search in the store, profit is equal to $\pi p^j$. Firms do not have incentives to slightly undercut price if $1 - \pi p^j F'[0] \geq 0$. At the same time they do not have incentives to charge a bit higher prices if $1 - p^j F'[0] \leq 0$, so the equilibrium price is $p^j = p^j = \min\{\frac{1}{F'[0]}, \frac{\pi - s}{\pi}\}$, subject to $\frac{1}{F'[0]} > \frac{s}{1-\pi}$. Otherwise, there is no pure strategy equilibrium and firm should mix on the interval of prices. So we can conclude that in equilibrium, when nobody geo-blocks, firms charge strictly positive prices. When search cost is sufficiently low, there exists pure strategy equilibrium $p^j = p^j = \min\{\frac{1}{F'[0]}, \frac{\pi - s}{\pi}\}$ if $\frac{1}{F'[0]} > \frac{s}{1-\pi}$, otherwise there is a mixed strategy equilibrium.

Now, let's consider what happens when one firm makes a decision to geo-block. Suppose that firm $B$ decides to ban cross-border sales. First of all, if $s$ is close to zero, then consumers search before purchasing in both markets. Thus, firm $B$ never charges price below $p^A$. So, it must be that $p^B \geq p^A$.

Let's check first order conditions for the interior solution:

$$\Pi^{B\prime}_{p^B} = \pi(1 - F[p^B - p^A] - p^B F'[p^B - p^A]) = 0, \text{ s.t. } p^B < \frac{\pi - s}{\pi}$$

$$\Pi^{A\prime}_{p^A} = \pi(1 + F[(p^B - p^A)] - p^A F'[p^B - p^A]) = 0, \text{ s.t. } p^A < p^B$$

We can notice that if the first condition for firm $B$ is satisfied as an equality, or derivative $\Pi^{B\prime}_{p^B}$

is positive, then the $\Pi^{A'}_{p^A}$ is positive for any price $p^A$ which is less than $p^B$. Thus, we should look at the candidate equilibrium where $p^A = p^B$. In this case firm $A$ does not want to undercut the price of the competitor or to charge higher price if

$$\Pi^{A'}_{p^A}|_{p^A=p^B} = \pi - \pi p^A F'[0] = 0 \Rightarrow p^A = \frac{1}{F'[0]}$$

and firm $B$ does not want to charge higher price if

$$\Pi^{B'}_{p^B}|_{p^A=p^B} = \pi - \pi p^B F'[0] = 0 \Rightarrow p^B = \frac{1}{F'[0]}$$

Obviously if $\frac{1}{F'[0]} > \frac{\pi-s}{\pi}$, then both firms charge prices $\frac{\pi-s}{\pi}$ and this is an equilibrium. Thus, equilibrium prices are $p^{A*} = p^{B*} = \min\{\frac{1}{F'[0]}, \frac{\pi-s}{\pi}\}$. Therefore, for small search cost close to 0 the equilibrium prices with one-side geo-blocking are not different from those without geo-blocking. Consumers buy in stores, profits are $\pi p^{j*}$. This equilibrium does not exist if $\frac{s}{1-\pi} > \min\{\frac{1}{F'[0]}, \frac{\pi-s}{\pi}\}$, or if one of the firms wants to deviate to online price such that online shoppers do not search. There is profitable deviation for any $s > s'$, where $s'$ comes from the equation

$$\underbrace{\pi p^{j*}}_{\text{equil. profit}} = \underbrace{\frac{s\left(\pi + \pi F\left[\left(p^{j*} - \frac{s}{1-\pi}\right)\right]\right)}{1-\pi}}_{\text{deviation profit}}$$

As a result, given search cost being sufficiently low (below $s'$), firms do not have strict incentives to geo-block. Equilibrium prices are $\min\left\{\frac{1}{F'[0]}, \frac{\pi-s}{\pi}\right\}$. If there proportion of consumers with low $\mu$ is higher, or $F'[0]$ is high, then equilibrium prices are below $\frac{\pi-s}{\pi}$, which is lower than in the equilibrium without non-discrimination price policy, when both firms prefer to geo-block.

If search cost is high and consumers do not search before buying online, there is no equilibrium in pure strategies with one-side geo-blocking, where prices are such that $p^A = p^B$. So we look for candidate equilibria with asymmetric pricing. If $p^A > p^B$, then each firm sells online to consumers only from the local market. Using the profit equality argument and the solution of the monopolistic problem, we can show that it can not happen in the equilibrium as one of two firms prefers to deviate. So we should look at the candidate equilibrium such that $p^B > p^A$. First order conditions are

$$\Pi^{B'}_{p^B} = \pi(1 - F[\pi p^B - p^A + s] - \pi p^B F'[\pi p^B - p^A + s]) = 0, s.t. p^B < \frac{\pi - s}{\pi},$$

$$\Pi^{A'} = \pi + F[\pi p^B - p^A + s] + (1-\pi)F[s - (1-\pi)p^A] - p^A F'[\pi p^B - p^A + s] -$$
$$(1-\pi)^2 p^A F'[s - (1-\pi)p^A] = 0.$$

The additional condition is that firm $B$ does not want to deviate and slightly undercut price $p^A$

$$\pi p^B(1 - F[\pi p^B - p^A + s]) > \pi p^A + (p^A - \pi p^A)F[-p^A - \pi p^A + s].$$

Obviously, if search cost is too high, then firm $A$ has incentives to charge price close to or above $\frac{\pi - s}{\pi}$, thus firms will play mixed strategies. In Lemma 9 we provide the formal proof of this result.

**Lemma 9.** *If both firms commit to price matching policy and one firm commits to geo-blocking at the preliminary stage of the competition, then there exists a threshold $\tilde{s}$ such that firms play mixed strategies in equilibrium if $s > \tilde{s}$ .*

We have to compare profit of the firm $A$ in the case when it geo-blocks and when it does not. If firms play mixed strategies on some support of prices $[\underline{p}, \bar{p}]$, then if firm $A$ charges the price at the upper bound, in the worst case (there is no atom in the price distribution played by firm $B$), consumers buy online from the firm $B$ with probability 1. Then firm $A$ has to get the profit which is equal to the monopolistic profit, otherwise it can deviate to the optimal monopolistic price. Thus, in the mixed strategy equilibrium firm $A$ gets at least the monopolistic profit. Depending, on the exact shape of online cost distribution function $F(\mu)$ it can get high profit if firm $B$ charges price at the upper bound with positive probability.

For the middle range of search cost there may exist an asymmetric equilibrium in pure strategies, where firm $A$ serves all online sales. Here we can notice that the firm $A$ can always guarantee to itself at least a monopoly profit. The question is whether it gets more when the other firm geo-blocks.

Thus, if one firm geo-blocks another firm in general weakly prefers not to geo-block, and depending on particular parameters and shape of distribution function $F(\mu)$ it may also strictly prefer not to geo-block. Therefore, we can conclude that non-discrimination price policy which imposes price parity in different retailing channels of the same firm, will eliminate incentives of firms to geo-block[14].

Figure 10 illustrates how one-side geo-blocking affects profit of firm $B$ for high search cost in case of the uniform distribution of online purchasing costs.

[14] Supopse that consumers never showroom. They either buy online directly or go to a store. Our general analysis on incentives to geo-block does not change if there is no possibility to showroom for consumers. However, we can see that non-dsicrimination price policy should not be applied to eleminate incentives to geo-block for sufficiently small search cost $s < \pi(1 - \pi)$, because it would be harmful for consumers. In this case firms would prefer to sell in stores only at price equal to $\frac{\pi - s}{\pi}$. Thus consumer surplus at each market would be lower than even in the monopoly case.

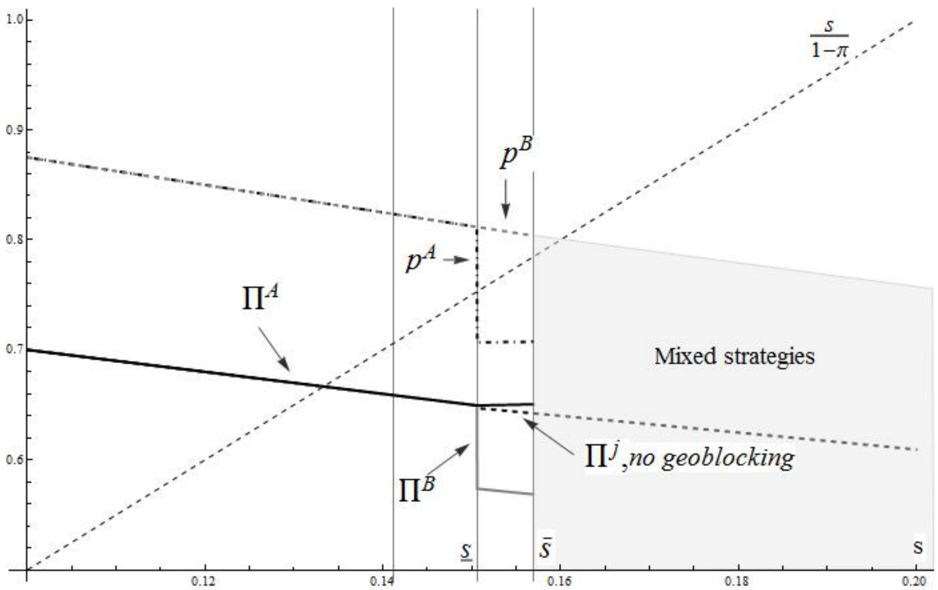



Figure 10: Equilibrium with one-side geo-blocking and non-discrimination price policy.

# 6 Discussion

This paper analyzed pricing strategies of multichannel retailers in a market of tangible goods, where consumers are heterogeneous in their attitude towards online shopping and have some search cost of visiting a BMS. We show that the choice of price strategies depends on the search frictions present in the market. The monopoly retailer prefers to choose equal prices when search frictions are low, as it can not effectively discriminate between different types, but wants to prevent a competition between its own distribution channels. In this situation, the online shop mostly plays the role of informative source, where consumers can find the information about products and prices. The situation changes for higher search cost. When store visits become substantially costly for consumers, the firm gets an opportunity to screen for consumers' types and, therefore, to price discriminate. By offering a lower online price it creates incentives to showroom for some consumers in the market. For very high search cost, they are split in two groups consisting of more conservative consumers who buy in the store, and consumers who prefer to buy directly online without pre-purchasing search in the store. In this situation we may find higher prices in an online shop than in a BMS.

We show that by opening an online store and becoming a multichannel retailer, the BMS can avoid a hold-up problem, which appears when there are positive search frictions in the market, and the firm can not credibly commit to the store price. The online price is usually easily observable, and thus can be a tool to send a signal about the store price, and to ensure consumers that they will not encounter excessively high prices after coming to a BMS. However, this strategy only works for low or moderate search frictions. In this case the firm can increase its profit by committing to a price matching policy and guarantee consumers that they can buy the product in the store at the price posted online. Thus we can see that price matching plays an important role as a price commitment device, while it can be inefficient if the firm has another possibility to commit to the store price.

The other result of this paper is related to the international competition of online retailers. When two retailers have stores at their local geographical markets (where they are monopolists) and compete in prices online, they have incentives to ban cross-border sales. The firm prefers to concede a part of the international online market in order to prevent severe online competition, which results in low in-store prices as well. Thus, the retailer can obtain higher expected profit at the local market. There exist two types of the weak Nash equilibria. The first one is symmetric, where both firms decide to commit to geo-blocking and operate as monopolists at the local market. The second one is asymmetric, where one firm is present online in both markets, while the other one operates only at the local market. If decisions to geo-block are not taken simultaneously, then only the follower commits to geo-blocking while the leader serves all markets. This can explain while newcomers to the online market are more likely to focus on their home country sales.

In the situation when geo-blocking has an anticompetitive effect and can lead to the monopolization of local markets both online and offline, the competition authorities may want to prevent firms from geo-blocking. However, the full exclusion of a possibility to geo-block creates incentives to shut down online stores for retailers. This leads to the asymmetric market structure where only one firm is present online. First, this relaxes online competition between firms. Second, it is harmful in terms of the growth of e-commerce sector. We show that in this situation authorities can use a non-discrimination price policy, which obliges firms to set equal prices in different distribution channels, in order to prevent geo-blocking and to restore the online competition.

# 7 Appendix

## 7.1 Search cost heterogeneity

We briefly discuss how relaxing the assumption of search cost homogeneity would affect main results of this section.

Let's consider monopoly case with observable prices and $\eta = 0$. Suppose that we have a fraction of consumers $\lambda$ who have search cost $\underline{s}$ and a fraction of consumer $1-\lambda$ who have search cost $\bar{s} > \underline{s}$. For simplicity we will call consumers with lower search cost "shoppers" and consumers with higher search cost "non-shoppers". Let's consider observable store prices. Shoppers and non-shoppers may exert all types of behaviour considered above depending on price choice. Let's consider what is the optimal prices choice of the firm depending on $\underline{s}$, $\bar{s}$ and $\lambda$. We will not provide the formal solution but consider some examples and discuss the general intuition.

Again if search cost is sufficiently small for both types, then for the firm is optimal to induce "'all search". The profit function of the firm remains mainly the same as in the previous analysis as both categories of consumers follow the same rule when they decide where to buy. The only difference is a boundary solution for the store price. Now, if the firm charges any price slightly above $\frac{\pi-\bar{s}}{\pi}$, there is still proportion $\lambda$ of consumers who buy in the store. Thus, if $\lambda$ is large, then the firm will sell only to shoppers and charge price equal to

$$\frac{\pi - \underline{s}}{\pi}$$

. As soon as $\underline{s}$ goes to $\bar{s}$, $\lambda$ should go to one, otherwise the firm prefers to charge $p_s = \frac{\pi-\bar{s}}{\pi}$. In both case it chooses $p_w \geq p_w$ to induce all search.

Now suppose that search cost is high, $\bar{s} > \underline{s} > \pi(1-\pi)$. As we remember from the previous analysis both types of consumers will not showroom. So again if $\underline{s}$ is sufficiently high, it is too costly for consumers to visit the store, and thus the firm can charge higher online price.

In both cases analyzed above we assumed that heterogeneity of search cost is small. Now let's consider what happens if it is large. Suppose $\bar{s} > \pi(1-\pi)$ and $\underline{s} = 0$.

1. If the firm charges $p_w > \pi$ and $p_w > p_s$, then nobody buys online. It is not optimal as by reducing price to online price to $p_w$ the firm does not lose any in-store sales, but can gain some profit from online sales if $p_s < \pi$. So it never happens in the equilibrium.

2. If the firm charges $p_s > \frac{\pi - s}{\pi}$ and $p_s > p_w$, then there are no in-store sales. Again, this never happens in the equilibrium, as the firm can reduce $p_s$ to $p_w$ and gain some additional in-store sales without decreasing its profit from online sales.

3. We can conclude that if $p_w > \pi$ or $p_s > \frac{\pi - s}{\pi}$ in the equilibrium then $p_s = p_w$.

4. Now suppose that $p_w \leq \pi$ and $p_s \leq \frac{\pi - s}{\pi}$. Then the profit function of the firm is

$$\Pi(p_s, p_w) = \begin{cases} \lambda \pi p_s + (1-\lambda)\left(\pi p_s \left(1 - F[\pi p_s - p_w + s]\right) + p_w F[\pi p_s - p_w + s]\right), & \text{if } p_s \leq p_w \\ \lambda \pi \left(p_s (1 - F[p_s - p_w]) + p_w F[p_s - p_w]\right) + & \\ +(1-\lambda)\left(\pi p_s \left(1 - F[\pi p_s - p_w + s]\right) + p_w F[\pi p_s - p_w + s]\right), & \text{if } p_s > p_w \end{cases}$$

There could be two possible cases: $p_w \leq p_s$ or $p_w > p_s$. Let's look at the derivatives of profit function with respect to both prices in both cases.

First, we consider $p_w \leq p_s$:

$$\Pi'_{p_s} = \pi(1-\lambda)\big(1 - F[\pi p_s - p_w + s] - (\pi p_s - p_w)F'[\pi p_s - p_w + s]\big) +$$
$$+ \pi \lambda \big(1 - F[p_s - p_w] - (p_s - p_w)F'[p_s - p_w]\big)$$

$$\Pi'_{p_w} = \pi \lambda \big(F[p_s - p_w] + (p_s - p_w)F'[p_s - p_w]\big) +$$
$$+ (1-\lambda)\big(F[\pi p_s - p_w + s] + (\pi p_s - p_w)F'[\pi p_s - p_w + s]\big)$$

Second, we consider $p_w > p_s$:

$$\Pi'_{p_s} = \pi \left(\lambda + (-1+\lambda)\big(-1 + F[\pi p_s - p_w + s] + (\pi p_s - p_w)F'[\pi p_s - p_w + s]\big)\right)$$

$$\Pi'_{p_w} = (1-\lambda)\big(F[\pi p_s - p_w + s] + (\pi p_s - p_w)F'[\pi p_s - p_w + s]\big)$$

Suppose that $\lambda$ is close to zero. We know the solution for $\lambda = 0$, which is such that in-store price equals $\frac{\pi - s}{\pi}$, and there is an interior solution for online price being lower than $\pi$. Thus, if for given search cost and $\lambda = 0$ the firm optimally charges $p_w < p_s = \frac{\pi - s}{\pi}$, then by slightly increasing $\lambda$ we will increase incentives of the firm to slightly increase the online price (which comes from the comparison of derivatives). If for given search cost and $\lambda = 0$ the firm optimally charges $p_w > p_s = \frac{\pi - s}{\pi}$, then by slightly increasing $\lambda$ we will increase incentives of the firm to slightly increase the offline price (which comes from the comparison of derivatives and the fact that $\pi p_s + s > p_s$ for $s > \pi(1-\pi)$). However, this derivative $\Pi'_{p_s}$ is already positive, so the optimal choice of prices won't be affected.

When $\lambda$ is increasing the firm has more and more incentives to increase online price if $p_w < p_s$, or to charge $p_s > \frac{\pi - s}{\pi}$ if $p_s < p_w$. So for sufficiently high $\lambda$ (equivalently, sufficiently low expected search cost in the population )there is a moment when the firm prefers to switch to price matching policy.

If $\lambda$ is close to one. Then the firm would prefer to charge the same online and offline prices and to sell only to shoppers in the store.

Therefore, we can see that when search cost are heterogeneous the equilibrium type basically depends on expected search cost in the population. These results are in line with our solution for homogeneous search cost.

### 7.2 Proofs Consumers:

*Proof of Lemma 1.* For deriving the optimal buying behavior, we have to do the pairwise comparison of strategies.

**Step 1** First of all, we can see that consumers prefer showrooming to outright online purchase when

$$\pi(1 - p_w - \mu_i) - s > \pi - p_w - \mu_i \Rightarrow s < (1 - \pi)(p_w + \mu_i)$$

If the online price $p_w$ is above $\frac{s}{1-\pi}$, then all consumers prefer to visit the BMS independently of $\mu_i$. Thus, we have to consider two cases: $p_w \leq \frac{s}{1-\pi}$ and $p_w > \frac{s}{1-\pi}$.

**Step 2** Suppose $p_w > \frac{s}{1-\pi}$, then from **Step 1** it follows that consumers always visit the BMS before buying if they have positive expected utility. After visiting the store they buy in the BMS if

$$\pi(1 - p_s) - s > \pi(1 - p_w - \mu_i) - s \Rightarrow \mu_i > p_s - p_w,$$

and $p_s < \frac{\pi - s}{\pi}$. Otherwise they buy online if $\mu_i < 1 - p_w - \frac{s}{\pi}$. This explains part (i) of the Lemma.

**Step 3** Suppose that $p_w \leq \frac{s}{1-\pi}$, such that consumers may potentially buy directly online without visiting the BMS, buy in the BMS and showroom. Consumers prefer to buy directly online when $\mu_i \leq \pi - p_w$ and **i)** outright online purchase is better than showrooming $\pi(1-p_w-\mu_i)-s < \pi - p_w - \mu_i \Rightarrow \mu_i < \frac{s}{1-\pi} - p_w$, and **ii)** outright online purchase is better than shopping in the BMS $\pi(1-p_s)-s < \pi - p_w - \mu_i \Rightarrow \mu_i < \pi p_s - p_w + s$.

**Step 4** Suppose that $p_s > \frac{s}{1-\pi}$ and $p_w < \frac{s}{1-\pi}$, then $\frac{s}{1-\pi} - p_w < \pi p_s - p_w + s$ and $\frac{s}{1-\pi} - p_w < p_s - p_w$. Form **Step 3** we get that consumers with online shopping cost $\mu_i < \frac{s}{1-\pi} - p_w, \pi - p_w$ prefer buying directly online to showrooming and buying in the BMS. From **Step 1** we know that consumers with $\mu_i > p_s - p_w$ prefer the BMS purchase to showrooming. Hence, we get that consumers **i)** buy online if $\mu_i < \min\left\{\frac{s}{1-\pi} - p_w, \pi - p_w\right\}$, **ii)** showroom if $\frac{s}{1-\pi} - p_w \leq \mu_i < \min\left\{p_s - p_w, 1 - \frac{s}{\pi} - p_w\right\}$, **iii)** buy in the BMS if $\mu_i \geq p_s - p_w$ and $p_s < \frac{\pi - s}{\pi}$. This explains part (ii) of the Lemma.

**Step 5** Suppose $p_s < \frac{s}{1-\pi}$ and $p_w < \frac{s}{1-\pi}$, then consumers consumers prefer showrooming if $\frac{s}{1-\pi} - p_w < \mu_i < p_s - p_w$ (from **Step 1** and**Step 3**). We can see that if $p_s < \frac{s}{1-\pi}$, then the set of $\{\mu_i\}$, such that $\frac{s}{1-\pi} - p_w < \mu_i < p_s - p_w$ is an empty set. Hence, consumers buy directly online if $\mu_i < \min\{\pi p_s - p_w + s, \pi\}$ (from **Step 3**) or consumers buy in the BMS if $p_s \leq \frac{\pi - s}{\pi}$ otherwise. This explains part (iii) of the Lemma.

We considered all possible combinations of online and store price, and therefore the analysis is complete. □

## 7.3 Proofs Monopoly:

*Proof of Lemma 2.* We can rewrite the profit function in the case of "all search" as

$$\Pi^{AS} = \pi p_s(1 - F[p_s - p_w]) + \pi p_w F[p_s - p_w] - \eta.$$

It is a weighted sum of $\pi p_s$ and $\pi p_w$, where weights sum up to 1. Thus, the optimal solution of the problem, if no constraints bind, is such that $p_s = p_w$. Depending on which constraint binds first we get that

1. if $s \leq \pi(1-\pi)$, then $\frac{\pi - s}{\pi} \geq \frac{s}{1-\pi}$, $\pi > \frac{s}{1-\pi}$ and $\frac{\pi - s}{\pi} \geq \pi$, and the solution is $p_s = \frac{\pi - s}{\pi}$, $p_w \geq \frac{\pi - s}{\pi}$;

2. if $s > \pi(1-\pi)$, then $\frac{\pi - s}{\pi} < \frac{s}{1-\pi}$ and $\frac{s}{1-\pi} > \pi$, the solution is $p_s = \frac{\pi - s}{\pi}$, $p_w > \pi$;

□

*Proof of Lemma 3.* The derivatives of the profit function with respect to online price in the case of "segregation" is

$$\Pi'_{p_w} = F[\pi p_s - p_w + s] + (\pi p_s - p_w - \eta)F'[\pi p_s - p_w + s] \tag{7}$$

The profit function can be rewritten as a weighted sum of $\pi p_s - \eta$ and $p_w$:

$$\Pi^{Seg} = (\pi p_s - \eta)(1 - F[\pi p_s - p_w + s]) + p_w F[\pi p_s - p_w + s].$$

Thus, if no constraints bind then optimally $p_w = \pi p_s - \eta$. Depending on which constraint binds first we get that

1. If $s \geq \pi(1-\pi)$, then $\frac{\pi - s}{\pi} \leq \frac{s}{1-\pi}$, $\frac{\pi - s}{\pi} \leq \pi$ and $\pi \leq \frac{s}{1-\pi}$, and thus the first binding constraint is $p_s = \frac{\pi - s}{\pi}$. The store price is equal to $\frac{\pi - s}{\pi}$ and online price $p_w^*$ is defined by the first order condition from equation (7):

$$F[\pi - p_w^*] + (\pi - p_w^* - s - \eta)F'[\pi - p_w^*] = 0$$

   This equation has a unique solution $p_w^* > 0$ due to log-concavity of function $F[\cdot]$. When $p_w$ goes to $\pi$ LHS of the equation is negative, so the optimal price satisfies $p_w^* < \pi$.

2. If $s < \pi(1-\pi)$, then $\frac{\pi - s}{\pi} > \frac{s}{1-\pi}$ and $\pi > \frac{s}{1-\pi}$. The first binding constraint is $p_s = \frac{s}{1-\pi}$. Thus the optimal BMS price is $\frac{s}{1-\pi}$. The optimal online price comes from the first order condition

$$F[\frac{s}{1-\pi} - p'_w] + (\frac{\pi s}{1-\pi} - p'_w - \eta)F'[\frac{s}{1-\pi} - p'_w] = 0.$$

   It has a unique solution $p'_w > 0$ due to the log-concavity of the function $F(\cdot)$. As LHS of the last equation is negative when $p_w$ approaches to $\frac{s}{1-\pi}$, so $p'_w < \frac{s}{1-\pi}$.

□

*Proof of Lemma 4.* First of all, we can show that the solution of profit maximization problem when the firm induces showrooming gives higher profit than the solution which induces "all search". Suppose that $s$ is close to $\pi(1-\pi)$, then the firm's profit in the case of "all search" goes to

$$\lim_{s \to \pi(1-\pi)} \Pi\left(p_s = p_w = \frac{\pi - s}{\pi}\right) = \pi^2 - \eta,$$

We can show that there always exists $\varepsilon$ such that by choosing prices $p_w = \frac{s}{1-\pi} - \varepsilon$ and $p_s = \frac{\pi - s}{\pi}$, the firm gets higher profit than by setting matching prices.

$$\lim_{s \to \pi(1-\pi)} \Pi \left( p_s = \frac{\pi - s}{\pi}, p_w = \frac{s}{1-\pi} - \varepsilon \right) = \tag{8}$$
$$= \pi^2 - \eta + \left( \pi - \pi^2 - \epsilon + \eta \right) F[\epsilon],$$

So, we can see that we can always choose $\varepsilon > 0$, such that firm's profit is higher when consumers showroom and online price is lower than the store price.

At the same time from the proof of Lemma 3 it follows that if the firm wants to induce "segregation" for the range of search cost $s < \pi(1-\pi)$, it has to charge some prices $p_s^* = \frac{s}{1-\pi}$ and $p_w^* < \frac{s}{1-\pi}$, which coincides with the boundary solution for the showrooming case.

□

*Proof of Lemma 5.* We know that in the case of "all search" the profit of the firm is equal to $\pi - s - \eta$. Suppose that for $s'$ the firm prefers that consumers showroom, so it gets the profit higher than $\pi - s' - \eta$, for some prices $p_w(s') < \frac{s}{1-\pi} < p_s(s')$.

**Step 1** If the optimal choice of prices $p_w(s')$ and $p_s(s')$ is such that $p_s(s') = \frac{\pi - s'}{\pi} - \delta_1 = \frac{s}{1-\pi} + \delta_2$, where $\delta_1, \delta_2 > 0$, then for any search cost $s' + \varepsilon$, where $\varepsilon \in (0, \min\{\delta_1 \pi, \delta_2\})$, the firm can still induce showrooming and get the same profit as for search cost $s'$ by charging $p_s(s' + \varepsilon) = p_s(s') < \frac{\pi - (s' + \varepsilon)}{\pi}$ and $p_w(s' + \varepsilon) = p_w(s') < \frac{s'}{1-\pi} < \frac{s' + \varepsilon}{1-\pi}$.

**Step 2** If the optimal choice of prices is such that $p_s(s') = \frac{\pi - s'}{\pi}$, then the profit at search cost $s'$ is equal to

$$\Pi^{Show}(s') = \pi - s' - \eta + (p_w - \pi p_w) F[-p_w + \frac{s'}{1-\pi}] + (\pi(-1 + p_w) + s' + \eta) F[1 - p_w - \frac{s'}{\pi}]$$

We assumed that this profit is higher than $\pi + \varepsilon - s'$, then we can immediately check that for $s = s' + \varepsilon$, the firm can get the profit higher than $\pi - s' - \eta - \varepsilon$ by charging $p_w(s' + \varepsilon) = p_w(s')$ and $p_s(s' + \varepsilon) = \frac{\pi - s'}{\pi}$.

$$\Pi^{Show}(s' + \varepsilon) = \pi - s' - \varepsilon - \eta + (\pi(-1 + p_w) + s' + \varepsilon + \eta) F[-\frac{(\pi(-1 + p_w) + s' + \varepsilon]}{\pi}] +$$
$$+ (p_w - \pi p_w) F[-p_w + \frac{s + \varepsilon}{1-\pi}] > \pi - s' - \varepsilon - \eta$$

Thus, if for some search cost $s' < \pi(1-\pi)$ there is a showrooming in the equilibrium, then there is exists $\varepsilon > 0$, such that the firm induces showrooming in the equilibrium for search cost $s' + \varepsilon < \pi(1-\pi)$, where $\varepsilon > 0$.

□

*Proof of Proposition* 2. In order to prove the result we will show that for any candidate equilibrium with unobservable $p_s$ the firm can do at least not worse and in some situations strictly better by disclosing the offline price.

First, it has been already shown that in "segregation" candidate equilibrium the firm never sells to rational consumers in the BMS. Thus, for any $s > \pi(1-\pi)$ the firm sells only online if the store price is unobservable. By disclosing the store price and setting it equal to $\frac{\pi-s}{\pi}$ the firm can induce additional instore sales without affecting any online sales (as a surplus of consumers who by in the BMS is equal to 0 in this case). Thus, it can strictly improve its profit by disclosing $p_s$.

Second, we consider a candidate equilibrium with "all search". As we have shown before the following condition must be satisfied in equilibrium when the offline price is unobsarvable:

$$\Pi'_{p_s}|_{p_s^*} = 0.$$

From Lemma 2 it follows that in the case of "all search" the firm can not satisfy the first order condition for the store price if $p_w \geq p_s$. Hence if the firm wants to sell in the BMS it has to charge $p_w < p_s \leq \frac{\pi-s}{\pi-s}$. If the firm dislose the offline price, it doesn't need to keep lower online price to induce instore sales. Thus, it can simply increase online price $p_w$ up to $p_s$ and the profit will unambiguosly increase.

Third, we consider a candidate equilibrium with "showrooming". In this candidate euqilibrium there is a part of consumers who visit the BMS before buiyng online. The marginal consumer is indiffernt between buing in the BMS and buying online after visiting the store. Obviously by disclosing the offline price the firm can achive at least the same profit as in the case of unobservable prices as an equilibrium condition requiers that consumers anticipate the correct offline price. However, if the optimal solution in the case of observable prices is such that $\Pi'_{p_s} = 0$, then the firm can achieve the same outcome when prices are unobservable. The existence of this solution depends on parameters of the model and distribution function $F$. □

## 7.4 Proofs Competition:

*Proof of Lemma 6.* We have proved that online prices are equal to zero in the equilibrium. Both firms get zero profits by setting zero store prices and by setting store prices equal to $\frac{\pi-s}{\pi}$, as zero measure of consumers buys in the store in this case. As firms are symmetric, they should charge the same store prices in the equilibrium, so we will derive the optimal store price for some firm $j$, where $j = \overline{\{A, B\}}$.

If firm $j$ sets price $p_s^j < \frac{s}{1-\pi}$, then some consumers will buy directly online. So profit of the firm equals $\pi p_s^j(1 - F[\pi p_s^j + s])$. The first order condition of the profit maximization problem is

$$\pi(1 - F[\pi p_s^j + s]) - \pi^2 p_s^j F'[\pi p_s^j + s] = 0. \tag{9}$$

If $p_s^j = \frac{\pi-s}{\pi}$, then the derivative of profit function is negative. Hence, if the firm wants to induce "segregation" it charges $\min\left\{\frac{s}{1-\pi}, p^*\right\}$, where $p^*$ is the store price which satisfies equation (9).

Firm $j$ can also induce showrooming if it charges $p_s > \frac{s}{1-\pi}$. Profit of the firm is equal to $\pi p_s^j(1 - F[p_s^j])$. The first order condition of the profit maximization problem is

$$1 - F[p_s^j] - \pi p_s^j F'[p_s^j] = 0. \tag{10}$$

If $p_s^j = \frac{\pi-s}{\pi}$ and $\frac{\pi-s}{\pi} > \frac{s}{1-\pi}$, then the derivative of profit function is negative. So the interior solution of equation (10) is always below $\frac{\pi-s}{\pi}$. Therefore, if the firm wants to induce showrooming, then it charges $\max\left\{\frac{s}{1-\pi}, p'\right\}$, where $p'$ is an interior solution of equation (10).

For any search cost $s > \pi(1-\pi)$ the firm cannot induce showrooming as $\frac{\pi-s}{\pi} < \frac{s}{1-\pi}$. So in this case there is a segregation and $p_s^j = p^*$.

When search cost equals zero, firm $j$ prefers induce showrooming as $\frac{s}{1-\pi} = 0$, so it charges price $p_s^j = p'$.

Notice that the derivative of the profit function at $p_s^j = \frac{s}{1-\pi}$ in the case of showrooming is not higher than in the "segregation" case, and profit is continuous on the whole interval of possible prices. Therefore, the profit function is continuous and single-picked. So we can conclude that there exists a unique threshold $0 < \tilde{s} < \pi(1-\pi)$, such that for any $s < \tilde{s}$ firms charge in the equilibrium $p_s^A = p_s^B = \max\left\{\frac{s}{1-\pi}, p'\right\}$, and for $s > \tilde{s}$ they charge $p_s^A = p_s^B = p^*$. □

*Proof of Lemma 7.* First, we prove equilibirum existence. As firms can set any prices from a continuous support, it is enough to show that firms' profits are continuous[15] in all prices.[16] Suppose firm $A$ sets price $p_s^A$ in BMS $A$ and $p_w^A$ online, and firm $B$ sets price $p_s^B$ in BMS $B$. The profit of firm $B$ is

$$\Pi^B(p_s^B) = \begin{cases} \pi(1 - F[p_s^B - p_w^A])p_s^B, & \text{if } p_w^A \geq \frac{s}{1-\pi} \\ \pi(1 - F[\pi p_s^B - p_w^A + s])p_s^B, & \text{if } p_w^A \leq \frac{s}{1-\pi}, p_s^B \leq \frac{s}{1-\pi} \\ \pi(1 - F[p_s^B - p_w^A])p_s^B, & \text{if } p_w^A \leq \frac{s}{1-\pi}, p_s^B \geq \frac{s}{1-\pi} \end{cases}$$

The profit function $\Pi^B$ is continuous in $p_s^B$ because $\pi\frac{s}{1-\pi} + s = \frac{s}{1-\pi}$. It is obviously continuous in $p_w^A$, when $p_s^B \geq \frac{s}{1-\pi}$. If $p_s^B < \frac{s}{1-\pi}$ then we have $F[\pi p_s^B + s - p_w^A] = F[\pi p_s^B - \frac{\pi s}{1-\pi}] = F[0] = F[p_s^B - p_w^A] = F[p_s^B - \frac{s}{1-\pi}]$ at $p_w^A = \frac{s}{1-\pi}$. So we can conclude that profit of firm $B$ is continuous in both prices $p_s^B$ and $p_w^A$.

Now we look at the profit function of firm $A$. As firm $A$ is the monopolist at market $A$, we can write its profit as $\Pi^A(p_s^A, p_w^A, p_s^B) = \Pi^{Monop}(p_s^A, p_w^A) + \Pi^O(p_s^B, p_w^A)$, where $\Pi^{Monop}$ is the monopoly profit at the local market, and $\Pi^O(p_s^B, p_w^A)$ is the profit which comes from online sales in market $B$. $\Pi^{Monop}$ is continuous in $p_s^A$ and $p_w^A$, so we need to show that $\Pi^O$ is continuous in $p_w^A$ and $p_s^B$.

$$\Pi^O(p_w^A, p_s^B) = \begin{cases} \pi F[p_s^A - p_w^A]p_w^A, & \text{if } p_w^A \geq \frac{s}{1-\pi} \\ F[\pi p_s^B - p_w^A + s]p_w^A, & \text{if } p_w^A \leq \frac{s}{1-\pi}, p_s^B \leq \frac{s}{1-\pi} \\ \left(\pi F[p_s^B - p_w^A] + (1-\pi)F[\frac{s}{1-\pi} - p_w^A]\right)p_w^A, & \text{if } p_w^A \leq \frac{s}{1-\pi}, p_s^B \geq \frac{s}{1-\pi} \end{cases}$$

The profit function $\Pi^O(p_s^B, p_w^A)$ is obviously continuous in price $p_w^A$. When $p_s^B = \frac{s}{1-\pi}$ we have $F[\pi p_s^B + s - p_w^A] = F[\frac{s}{1-\pi} - p_w^A] = \left(\pi F[p_s^B - p_w^A] + (1-\pi)F[\frac{s}{1-\pi} - p_w^A]\right)p_w^A$, so the profit is also continuous in $p_s^B$. We can conclude that function $\Pi^A(p_s^A, p_w^A, p_s^B)$ is continuous in prices $p_s^A$, $p_s^B$, and $p_w^A$. Therefore, there exists a Nash equilibrium.

Second, we show that the equilibrium profit of the firm $B$ (let's denote it as $\Pi^{BC}(s)$[17]) is strictly below $\Pi^{BMS}$ and strictly above $\Pi^C(s)$. We need to show that the best response of firm $A$ to the competitor's price $p_s^B = \frac{\pi - s}{\pi}$ is such that the positive measure of consumers buy online in market $B$. Suppose that search cost is above $\pi(1-\pi)$. Then $\pi\frac{\pi-s}{\pi} + s = \pi$, and thus by charging any price $p_w < \pi$ firm $A$ sells online in market $B$. The proof of Lemma 3 guarantees us that firm $A$ never

[15] See the fixed point theorem of Fan-Glicksberg

[16] However we cannot guarantee the existence of a pure strategy Nash equilibrium as the profit function of firm $A$ is not necessarily single-peaked for all possible values of the search cost.

[17] We use notations $BC$ and $MC$ to refer to competition between a single BMS and a multichannel retailer.

sets $p_w \geq \pi$ when $s > \pi(1-\pi)$. Now suppose that $s \leq \pi(1-\pi)$. Firm $B$ sells in the BMS at price $\frac{\pi-s}{\pi}$ to all consumers in market $B$ only if $p_w^A > \frac{\pi-s}{\pi}$. So we need to show that the best response of firm $A$ is to set price $p_w < \frac{\pi-s}{\pi}$. If search cost is in the range $(\tilde{s}, \pi(1-\pi)]$, where $\tilde{s}$ is defined in Proposition 1, then firm $A$ sets price $p_w^A < \frac{\pi-s}{\pi}$. If search cost belongs to the range $(0, \tilde{s}]$, then firm $A$ can charge prices $p_w^A = \frac{\pi-s}{\pi} - \varepsilon, p_s^A = \frac{\pi-s}{\pi}$, where $\varepsilon > 0$, and get the profit

$$(\pi - s)(1 - F[\varepsilon]) + 2F[\varepsilon]\left(\pi - s - \pi\varepsilon\right) = \pi - s + F[\varepsilon]\left(\pi - s - 2\pi\varepsilon\right) > \pi - s,$$

if $\varepsilon$ is sufficiently small. Therefore, we can conclude that in the equilibrium firm $B$ gets strictly lower profit than $\Pi^{BMS}(s)$ for any search cost. At the same time firm $A$ can always reach at least the monopoly profit, and therefore it charges a strictly positive online price $p_w^A > 0$ (as otherwise firm $A$ gets profit $\Pi^C < \Pi^M$). Thus, the profit of the firm $B$ has to be higher than $\Pi^C$ as it faces less online competition. So we showed that $\Pi^C < \Pi^{BC} < \Pi^{BMS}$.

Finally, we can notice that the equilibrium profit of firm $A$ (let's denote it as $\Pi^{MC}(s)$) is not less than $\Pi^M(s)$. Obviously, firm $A$ can always charge optimal monopoly prices as derived in Proposition 1 and get at least the monopoly profit. Thus the equilibrium profit $\Pi^{MC}(s) \geq \Pi^M(s)$, otherwise firm $A$ has a profitable deviation. □

*Proof of Lemma 8.* Suppose that firm $A$ charges expected online price $\tilde{p}_w^A$ in subgame equilibrium where firm $B$ does not open an online shop. Suppose that if firm $B$ opens an online shop and geo-blocks then firm $A$ draws an online price from support $[\underline{p}, \overline{p}]$ according to cumulative probability distribution function $G(p)$ in subgame equilibrium. We can show that $\int_{\underline{p}}^{\overline{p}} p dG(p) > \tilde{p}_w^A$.

We know that if firm $i$ plays mixed strategy then it gets the same expected profit for any price set with positive probability. Now suppose that firm $B$ opens online shop and geo-blocks. It does not sell online when it charges price $p_w^B$ at the upperbound of the equilibrium price support. Thus it gets the profit only from in-store sales. The profit function of firm $B$ which charges $p_B^w = \overline{p}$ and $p_s^B(\overline{p})$ is

$$\int_{\underline{p}}^{\overline{p}} \pi \left(1 - \tilde{F}\left[p_w^A, p_s^B(\overline{p})\right]\right) dG(p_w^A),$$

where function $\tilde{F}$ is either $F[p_s^B(\overline{p}) - p_w^A]$ or $F[\pi p_s^B(\overline{p}) - p_w^A + s]$ depending on $p_s^B(\bar{(p)})$. We have already shown that function $\tilde{F}[\cdot]$ is continuous in $p_w^A$. If $F[x]$ is convex then $F''[x] > 0$, and

$(1-F[x])''_x = -F''[x] < 0$. So we can apply Jensen's inequality and thus

$$\int_{\underline{p}}^{\overline{p}} \pi\left(1-\tilde{F}\left[p_w^A, p_s^B(\overline{p})\right]\right) dG(p_w^A) < \pi\left(1-\tilde{F}\left[E(p_w^A), p_s^B(\overline{p})\right]\right)$$

If the expected price of the competitor is below $\tilde{p}_w^A$ then it is not profitable for firm $B$ to open the online shop at first place. Thus firm $B$ opens an online shop and geo-blocks only if $\int_{\underline{p}}^{\overline{p}} p dG(p) = E(p_w^A) > \tilde{p}_w^A$.

As market is fully covered in the equilibrium, we can conclude that consumer surplus in market $A$ is higher when firm $B$ does not open an online store(and geo-block). □

*Proof of Lemma 9.* We can show that for sufficiently high search cost firms play mixed strategies. First of all, we can notice that for any price $p^A \in [0, \frac{\pi-s}{\pi}]$ condition $p^A < \frac{s}{1-\pi}$ is satisfied if $s > \pi(1-\pi)$. The firm which charges higher price does not charge price above $\frac{\pi-s}{\pi}$, because otherwise it has zero profit as it does not sell neither online nor in the store. Suppose, that there is an equilibrium in pure strategies $(\tilde{p}^A, \tilde{p}^B)$, then as we have already shown the following condition should be satisfied in the equilibrium:

$$\Pi^{A\prime}_{p^A}|_{\tilde{p}^A,\tilde{p}^B} = \pi + F[\pi\tilde{p}^B - \tilde{p}^A + s] + (1-\pi)F[s-(1-\pi)\tilde{p}^A] - \tilde{p}^A F'[\pi\tilde{p}^B - \tilde{p}^A + s] -$$
$$(1-\pi)^2\tilde{p}^A F'[s-(1-\pi)\tilde{p}^A] = 0,$$

where $\tilde{p}^A < \tilde{p}^B$. At the same derivative of the profit $\Pi^B$ w.r.t. price $p^B$ should be equal to zero if $\tilde{p}^B < \frac{\pi-s}{\pi}$, or not negative if $\tilde{p}^B = \frac{\pi-s}{\pi}$.

$$\Pi^{B\prime}_{p^B}|_{\tilde{p}^A,\tilde{p}^B} = \pi(1-F[\pi p^B - p^A + s]) - \pi p^B F'[\pi p^B - p^A + s]) = 0$$

We can show that for sufficiently high $s$, these two equations can not be satisfied at the same time for $p^A < p^B$. When $s$ goes to $\pi$, profit of the firm $A$ goes to zero, at the same it can get strictly positive profit by selling online at the price $\frac{\pi-s}{\pi} < p^A < \pi$ (obviously, this set is non-empty when $s > \pi(1-\pi)$). Thus firm $A$ has no incentives to play any prices below $\frac{\pi-s}{\pi} \geq p^B$. Thus, we excluded all possible pure strategy candidate equilibria, and firms have to mix in the equilibrium.

□

### 7.5 Asymmetric competition: uniform distribution of online shopping cost

Here we consider linear distribution function, when there is a range of search cost such that the multichannel has incentives to mix. We suppose that $F(\mu) = \frac{\mu}{\pi}$, thus function $F(\mu)$ is concave, and $F(\pi) = 1$, $F(0) = 0$. We fully solve the asymmetric competition problem and derive equilibria for any possible search cost.

1. When search cost if low, we know the optimal solution for the price $p_s^A = \frac{\pi - s}{\pi}$. So we plug in this price into F.O.C.s for firms $A$ and $B$ and get solution
$$p_w^* = \frac{1}{7}\left(4 + \pi - \frac{4s}{\pi}\right)$$
$$p_s^{B*} = \frac{2}{7}\left(1 + 2\pi - \frac{s}{\pi}\right)$$
So we can notice that indeed $p_w < p_s^B$, and condition $s < (1-\pi)p_w$ is satisfied if
$$s < \frac{(1-\pi)\pi(4+\pi)}{4+3\pi} < (1-\pi)\pi \equiv s'.$$
The interior solution for the price in store $B$ is lower than $\frac{\pi - s}{\pi}$ for any $s$ below $\pi(1-\pi)$. We can check as well, that given this best response of firm $B$ there is a profitable deviation for firm $A$ when it prefers to a price below $\frac{s}{1-\pi}$ for the search cost below $\tilde{s} < s'$.
$$\tilde{s} = \frac{2\left((-8+\pi)\pi(4+\pi) + 7\sqrt{\pi^3(4+\pi)^2}\right)}{-64+\pi}$$

2. As the curvature of function $F(\mu)$ is equal to zero, we know that for any search cost there exists a pure strategy equilibrium.

3. For search cost $\tilde{s} < s < \bar{s}$ there exists an equilibrium with showrooming in both markets. The equilibrium prices derived from the first order conditions are
$$p_s^{A*} = \frac{\pi - s}{\pi},$$
so the store price in market $A$ is defined again by the boundary solution.

   The online price is
$$p_w^* = \frac{\pi(4+\pi)}{8-\pi}.$$
   The price in store $B$ is
$$p_s^{B*} = \frac{6\pi}{8-\pi}$$

We can notice that in this case prices $p_w$ and $p_s^B$ do not depend on search cost. Due to the linearity of the distribution function, the drop of the store price $p_s^A$ and repartition of the share of consumers who buy directly online and showroom in market $A$ compensate each other in a such way, that the interior solution for the online price remains the same.

At the same time when search cost is equal to $\bar{s} = \frac{6(1-\pi)\pi}{8-\pi}$ the interior solution for firm $B$ is equal to $\frac{s}{1-\pi}$. Thus, for higher search cost we have to consider another type of the equilibrium.

4. When search cost is $\bar{s} < s \leq \pi(1-\pi)$, in the equilibrium there is a showrooming only on the market $A$. The store price in market $B$ is below or equal to $\frac{s}{1-\pi}$, thus consumers choose whether to buy directly online or in the store.

   The equilibrium prices are

$$p_w^* = \begin{cases} \frac{1}{4}\pi\left(2 - \frac{s}{-1+\pi}\right), & s > \frac{6(1-\pi)\pi}{4+3\pi} \\ \frac{s}{1-\pi}, & s \leq \frac{6(1-\pi)\pi}{4+3\pi} \end{cases}$$

$$p_s^{B*} = \begin{cases} \frac{2(3\pi-2s)}{7\pi}, & s > \frac{6(1-\pi)\pi}{4+3\pi} \\ \frac{s}{1-\pi}, & s \leq \frac{6(1-\pi)\pi}{4+3\pi} \end{cases}$$

$$p_s^{A*} = \frac{\pi - s}{\pi}$$

   Obviously, when search cost is approaching to $\pi(1-\pi)$, consumers in market $A$ are also loosing their incentives to search, as a result we get the equilibrium without showrooming, where consumers on both markets are fully segregated in two subsets both in searching and purchasing strategies.

5. When search cost is high $s > (1-\pi)\pi$, then we have the price in store $A$ should be still equal to $\frac{\pi-s}{\pi}$. Online price and price in store $B$ are

$$p_w = \begin{cases} \frac{1}{7}(5\pi - s), & s < \frac{\pi}{3}, \\ \frac{1}{4}(3\pi - s), & s \geq \frac{\pi}{3}. \end{cases}$$

$$p_s^B = \begin{cases} \frac{2(3\pi - 2s)}{7\pi}, & \text{if } s < \frac{\pi}{3}, \\ \frac{\pi - s}{\pi}, & s \geq \frac{\pi}{3} \end{cases}$$

We can see that for sufficiently high search cost again online price will be higher than prices in stores.

Equilibrium prices and profits are illustrated on the Figure 6.

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