Information Asymmetry and Bidding Behavior in Common Value English Auction

Guoxuan Ma*

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Abstract

In common value English auction, studying the information asymmetry needs to get proper estimate of the value distribution. However, rather than one to one mapping from value distribution to bidding distribution, multiple equilibria and model incompleteness lead to the "many to many" correspondence issue. The paper utilizes the bidder's bidding history and develops a structural econometric model to estimate the value distributions. Regarding the information asymmetry, the model provides a tractable way to analyze the agent's bidding behavior. In addition, the paper proposes a parametric approach for testing the information structure. The paper finds that the information premium mainly comes from the informed bidder's screening effect and is independent of the number of participants. Applying the data from Chinese Justice Auction, I find that the noisy part in private signal is very large. The large noisy part reduces the selling revenue and , combined with asymmetric information, blocks the auction efficiency.

Keywords: English auction, information asymmetry, moment inequalities

*Department of Economics, Pennsylvania State University. Email: gum27@psu.edu. I am very grateful for the endless help and suggestions from Prof. Karl Schurter, Prof. Joris Pinkse, Prof. Peter Newberry, and Prof. Michael Gechter.

1 Introduction

Consider an English auction for an unknown object with common value, V. Each bidder privately observes a noisy signal of V and must decide when and how much to bid. When making this decision, she should consider that her bid (if any) will be interpreted as good news about her private signal and therefore encourage further bidding by her rivals. This is also called learning process in common value English auction. On the other hand, she should also consider that her rivals' signals must be relatively weak if her bid is the last submitted, i.e. she must anticipate the so-called winner's curse. The winner's curse will discourage further bidding activities. The two effects jointly characterize the bidder's bidding behavior. But as emphasized in Bikhchandani, Haile and Riley (2002), the equilibrium bidding strategy involves multiple equilibria problem. Because the bidding price only indicates the lower bound of bidder's evaluation on the selling item, one bidding price could results from many different bidding strategies, reflecting a wide range of possible private signals. Moreover, As pointed out in Haile and Tamer (2003), in English auctions, prices may rise in jumps of varying sizes and active bidding by the rivals may discourage the bidder to make a bid close to her evaluation. Thus, it is difficult to observe bidders' dropout prices, which refers to the highest possible bidding price under certain bidding strategy. The unobservability of dropout price means that we can only build a correspondence from bidding price to the bidder's private signal, mapping the private signal into a region of bidding prices. In ideal situation of an auction model, we bridge the bidding distribution with value distribution under one to one mapping. However, the multiple equilibrium bidding strategies imply that many different value distributions can map to one bidding distribution. And the model incompleteness further implies that many bidding distributions correspond to one value distribution. Therefore, the new challenge now becomes that many value distributions map to many bidding distributions, which requires a new identification and estimation approach to deal with.

In terms of information asymmetry, bidders have different level of information toward the value of the item. Some bidders could receive more precise signal about the V, named as the informed bidder. Since informed bidder knows more about the selling item than others, they can get higher expected payoff. This net payoff between the informed and uninformed bidder is called information premium or information rent in the literature. But because of the learning process in common value English auction, bidders can learn from each other's bidding history to update their evaluation for the selling item. In particular, when the identity of the informed bidder is revealed to the public, other bidders will put higher weight on informed bidder's bidding behavior to update their evaluation. This can induce more aggressive bidding outcomes (Hernando-Veciana (2009)). Then how does the information rent redistribute among different agents? Is more information really good for the informed bidders? Those questions thus far remain inconclusive when the Information asymmetry encounters the learning process. Since bidders interact with each other quite differently under symmetric or asymmetric information structure, specifying the information structure is fundamental for analyzing the bidding behavior and estimating value distribution. But in practice, the information structure is not easy to pin down. For example, we can obtain the auction data, but have no idea about each bidder's private information. if we want to analyze the relation between information asymmetry and bidding behavior, we have to justify information structure first. I propose to address above questions in the context of Chinese Justice Auctions.

Chinese Justice Auction origins from the fact that local courts in China have to deal with 600 billion RMB worth of confiscated assets and property rights every year. To speed up the disposal procedure and avoid corruption, the Supreme People's Court of China began to allow local courts to hold Online auctions to deal with confiscated assets in 2010. Since then, a large proportion of the confiscated assets have been disposed through Online platform. In 2015, there were more than 124,000 Online Justice Auctions held across the country. 84 percent of them were sold successfully. Analyzing the information asymmetry and agent's bidding behavior in Chinese Justice auctions is relevant for a number of reasons. First, mechanism in the Chinese Justice auction satisfies all the key features of the common value English auction that we discussed before. The auction data provides a ideal laboratory to investigate the new method for recovering value distribution in common value English auction. Second, in Chinese Justice Auction, there exists one typical type of bidder in the data called "priority bidder", which, by definition, refers to the person who has some connections to the selling item. Examples are like the tenants of the house for sale or the banks that lend the mortgage. Without loss of generality, the priority bidder in the data is equivalent to the informed bidder. The more interesting thing is that the priority bidders can choose to self-identify themselves or not. With the help of the auction data, we can study the bidder's bidding behavior under different information structure.

This paper aims at analyzing how bidders react to information asymmetry and estimating the value distribution from the data. This work departs from the previous analysis in many ways. First, The paper constructs a new model to analyze the bidding behavior in common value English auction. To overcome the multiple equilibria and incompleteness of the model, the new model in this paper combines Hong and Shum (2003) and Haile and Tamer (2003) to estimate the value distribution in common value English auction. Rather than selecting one equilibrium bidding strategy, This paper characterizes the decision rules that encompass all the potential bidding strategies. Specifically, I use the bidding history within an auction to construct lower and upper bounds on the bidders' private signals. I then estimate the parameters of the model in a moment inequality framework. Second, in terms of asymmetric information, The discussion will focus on how information premium is distributed across bidders. Since uninformed bidders put more weights on the informed bidder's posting price, as a trade-off, the weights of other bidder's bidding activities and bidder's own private signal will be reduced. The paper finds that the positive effect and negative effect on the learning process can be canceled out each other. The existence of informed bidder will screen out the low valued bidders and select high valued bidders entering into the auction. As a result, the information premium mainly comes from the screening effect not from the learning process. Similarly, this model predicts that uninformed bidder's aggressive bidding behavior comes from nothing but more high valued bidders entering into the auction. Regarding the interactions among informed and uninformed bidders, the simulation test find that informed bidder can significantly affect the auction outcome by strategically choosing certain bidding behavior. Hence, knowing more may be not good for the uninformed bidder.

Third, the paper provides a test for the information structure. In the empirical literature, researchers usually build the auction model from presumed information structure to match the data. This paper, however, starts from reverse side using information from the data to test the information structure. Specifically, The test focus on whether the information structure is under symmetric or asymmetric information when the data provides little information about the existence of an informed bidder. To identify the information structure, I utilize the bidder's entry decision which depends on the beliefs of the information structure. In addition, under the same environment, the underlying value distribution should be the same across information structures. Then we can first recover value distribution. After recovering the value distribution, we can make use the probability of bidder's entry decision and construct the hypothesis test for the information structure.

This work relates to both theoretical and empirical auction literature. The bidding behavior also involves asymmetric information problem. Asymmetric information in affiliated value auctions usually induces both competition-enhancing and competition-dampening two contrasting effects. Dionne et al. (2009) analyze how information asymmetry among bidders affects the auction outcome. For competition-enhancing effects, the Hernando-Veciana (2009) shows that a bidder acquiring better information on the common value induces more aggressive bidding by uninformed bidders in open than in sealed bid auctions. On the other hand, the classical winner's curse would discourage uninformed bidder's participation, which is marked as competition-dampening effects. Milgrom and Webber (1982b) prove that the informed bidder earns strictly positive expected revenue while the uninformed bidders earn zero expected revenue. Empirical evidence in Hendricks and Porter (1988) shows that a strong winner's curse will have a detrimental impact on drainage lease prices when better-informed bidders are present, discouraging uninformed bidder's bidding motivation. Regarding the estimation of the common value English auction, Bikhchandani, Haile and Riley (2002) argues that even under button auction format, we still have the multiple biding equilibrium. Haile and tamer (2003) explain why the English auction model could involve the incompleteness problem and propose a non-parametrically identification strategy on the distribution of valuation in independent private value (IPV) settings. Hong and Shum (2003) estimate a asymmetric affiliated English auction with a trackable econometric model. Haile et al (2018) consider the endogenous entry and propose a test for the common value in the first-price auction. Aradillas-López et al. (2013) also relax the IPV setting and investigate the identification of the ascending auctions with correlated private value. Regarding the test of information structure, Hendricks, Porter et al. (2018) use U.S. Offshore Oil and Gas Lease Auctions to do the competition and collusion test under first price auction. In this paper, to recover the bidder's private information, I construct the lower and upper bounds from the bidding price a la Haile and Tamer (2003). Since the lower bound and upper bounds construction involves the moment inequality. The estimation procedure follows Chernozhukov, Hong and Tamer (2007) to construct the confidence regions.

The rest of the paper is organized as follows. Section 2 builds the model and derives the basic results of the bidding behavior. Section 3 introduces the dataset and provides the empirical evidence for the informational asymmetry. Section 4 discusses identification strategies. Section 5 set up the estimation procedure and describes the simulation test and empirical findings. Section 6 introduces the test for the information structure. Section 7 concludes.

2 Model

The paper considers a model of common value English auction[1]. Each auction τ is associated with observed characteristics $M_\tau \in \mathbb{M}$. M_τ includes the reservation price (γ), the minimum bidding increment (Δ), the description of the item, and etc. Each risk neutral bidder i values the object at V_i, but only receives a private and noisy signal X_i of the valuation $V_{i\tau}$. Let $V_\tau = (V_{1\tau}, ..., V_{N_\tau \tau})$, $X_\tau = (X_{1\tau}, ..., X_{N_\tau \tau})$ and $X_{-i\tau} = X_\tau \backslash X_{i\tau}$. The number of bidders entering auction τ is denoted by N_τ. In the biding stage, the realizations of (M_τ, N_τ)

[1] Later I will introduce the endogenous entry for testing the information structure (Section 6)

are common knowledge among bidders, as is the distribution of $(X_\tau, V_\tau)|(N_\tau, M_\tau)$. I make two main assumptions on this conditional distribution.

Assumption 1. *(i) For all $n \in suppN_\tau|(X_\tau, M_\tau)$, $F_{XV}(X_\tau, V_\tau|n, M_\tau)$ has a continuously differentiable joint density that is affiliated, exchangeable in the indices $i = 1, ..., n$, and positive the support of X and V; (ii) $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, M_\tau]$ exists and is strictly increasing in $X_{i\tau}$.*

Because the object has common value part, $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, M_\tau]$ depends on $X_{-i\tau}$. In the bidding stage, there exist multiple bidding periods $t = 1, ..., T$, in which at most one bid will recorded. The auction begins whenever a bidder starts to submit the first bid. The bidding price follows the reserve price plus integer multiple of the minimum bidding increment. I refer to the bidder whose bid is selected in period t will be the posting bidder. To simplify the mechanism, I model each period t as a sealed bid auction in which the auctioneer collects all the submissions and select highest bidding price as the posting bidder. If more than two bidders submit the same bid, the auctioneer randomly picks one of the bidders as the posting bidder. The auctioneer ends current period after announcing the posting bidder and corresponding posting price. At the beginning of the next period, the bidding history is updated. In the mean time, All bidders update their beliefs about their coompetitors' signals, make bidding decisions and submit the new bids again. If there is no new bid, the posting bidder wins the item and the auction ends (Ending period is denoted by T). Without loss of generality, bidders are indexed by $i = 1, ...N_\tau$ where the ordering $1, ..., N_\tau$ indicates the order of highest bidding price of each bidder, so that bidder N's highest bidding price is the lowest among all the bidders, and bidder $i = 1$ wins the auction. Since it is a typical common value English auction, every new bid will reveal some information to the public. Here I introduce the second assumption that regulates bidding behavior:

Assumption 2. *Bidders do not allow an opponent to win at a price they are willing to bid and they will never bid higher than $\mathbb{E}[V_{i\tau}|X_{i\tau}, X_{-i\tau}; N_\tau, M_\tau]$*

Assumption 2 is motivated by "the essential feature of the English auction" (McAfee and McMillan 1987): the ability of bidders to observe and respond to the current best bid with higher bids of their own. From the rules in the auction model, it is a dominated strategy to let others win at price that is below their own evaluation of the item. However, In this model, because the auctioneer only posts one bidder in each period, we do not know whether it is the case that all the other bidders have dropped out before that period or that there have been multiple submission but only one is selected. Hence, not just econometricians, even bidders no longer observe rivals' dropout prices. This is the major distinction from the previous button auction literature. If the dropout price is observable, remaining bidders can infer the private information possessed by the bidders who have dropped out (Milgrom and Weber (1982),Hong and Shum (2003)) under certain selection rule (Bikhchandani, Haile and Riley (2002)). But due to the unobservability of dropout price, we face the incompleteness problem. Still, the bidding history provides a lower and upper bound on the corresponding bidder's private signal so that we can get partial identification of the value distribution. Fortunately, with the help of the learning process in the auction, we can construct a tighter bound for identification.

2.1 Equilibrium Bidding Strategies

Let us take the number of bidders N_τ as given and discuss equilibrium bidding function first. The bidding functions for each bidder i for each period t are defined as $\beta_i^t(X_i;\Omega_t)$,i.e. $\{\beta_i^1(X_i;\Omega_t),...,\beta_i^T(X_i;\Omega_T)\}$, where X_i denotes bidder i's private signal and Ω_t denotes the public information set at period t. Ω_t includes the bidding history up to period t, the observed characteristics M_τ, the number of bidders N_τ and the setup of information structure which will be discussed later. $\beta_i^t(X_i;\Omega_t)$ tells bidder i at which price she should stop bidding at period t. Within the interval between P_{t-1}, the posting price in $t-1$, and $\beta_i^t(X_i;\Omega_t)$, bidder i can submit any price equal to $P_{t-1}+k\Delta, k\in\mathbb{N}_+$, Δ represents the minimum bidding increment. $\beta_i^t(X_i;\Omega_t)$ actually reflects bidder i's real evaluation, which is called pivotal bidding function. The collections of pivotal bidding functions $\left\{\beta_i^1(X_i;\Omega_t),..,\beta_i^T(X_i;\Omega_t)\right\}$ for bidders $i=1,...,N$ are common knowledge.

Up to period t, we have the past bidding path $\{P_{t-1},...P_1\}$ of the corresponding bidders. Since $\left\{\beta_i^1(X_i;\Omega_t),..,\beta_i^T(X_i;\Omega_t)\right\}$ are common knowledge, bidder i can use the the bidding history to infer the lower bound of private signals $\{X_N, ..., X_1\}$ by inverting the bidding functions: i.e., $\underline{X}_j = (\beta_j^t)^{-1}(P_j^{t*}; \Omega_t)$, where P_j^{t*} represent the highest bidding price of bidder j up to period t. In what follows, I focus on increasing bidding strategies (i.e. $\beta_i^t(X_i; \Omega_t)$ is increasing in X_i for $t = 1, 2, ..., T$) as described in Hong and Shum (2003) and Haile and Tamer (2003). Let $i = 1, ..., N_\tau$ denote the highest bidding order. For any period t, the lower bounds of the bidders' private signals can be constructed by system equations of N conditional expectations:

$$\begin{aligned} \mathbb{E}[V_1|\underline{X}_{1t}, \underline{X}_{2t} ..., \underline{X}_{Nt}] &= P_1^{t*}, \\ \mathbb{E}[V_2|\underline{X}_{1t}, \underline{X}_{2t} ..., \underline{X}_{Nt}] &= P_2^{t*}, \\ \mathbb{E}[V_N|\underline{X}_{1t}, \underline{X}_{2t} ..., \underline{X}_{Nt}] &= P_N^{t*}, \end{aligned} \tag{1}$$

where $\underline{X}_{1t}, ..., \underline{X}_{Nt}$ are the unknown variables and P_i^{t*}, $i = 1, ..., N$ represents the highest bidding price for bidder i up to period t. If bidder i has not submitted the bid yet, $P_i^{t*} = \gamma$, where γ is the reserve price. The system of equations in 1 is justified and we can get the unique solutions for $\underline{X}_{1t}, ..., \underline{X}_{Nt}$, which is summarized in the following lemma.

Lemma 1. *The solution of the N unknown variables in equations. 1 are unique and strictly increasing in $P^{t*} = \{P_1^{t*}, ..., P_N^{t*}\}$, for all possible realizations of the lower bounds of $X_{1t}, ...X_{Nt}$.*

Lemma 1 relates the existence of a monotonic equilibrium to the existence of a monotonic solution to the nonlinear system of equations 1. After getting the lower bounds of private signals for each bidder, we can plug the lower bound $\{\underline{X}_1, ..., \underline{X}_N\}$ into the conditional expected value of the bidder i: $\mathbb{E}[V_i|X_i, X_j \geq \underline{X}_j; \Omega_t]$. Regarding the upper bounds, we introduce a high order assumption:

Assumption 3. *bidder is willing to bid only when she expects all her rivals will dropout and she will win in the next period.*

Since whenever submitting a new bid, bidders will reveal part of their private signals to the public. A rational bidder will choose to keep the information revelation frequency as low as possible. Thus, bidder chooses to bid only when she must bid, for instance, a situation that she expects all her rivals will dropout. Otherwise, the winner would be the current posting bidder not herself. Then we apply this condition to construct the system of equations that pins down the upper bound for the private signals:

$$\begin{aligned}\mathbb{E}[V_1|\overline{X}_{1t},\overline{X}_{2t}...,\overline{X}_{Nt}] &= P_i^t+\Delta,\\ \mathbb{E}[V_i|\overline{X}_{1t},\overline{X}_{2t}...,\overline{X}_{Nt}] &= P_i^t+\Delta,\\ \mathbb{E}[V_N|\overline{X}_{1t},\overline{X}_{2t}...,\overline{X}_{Nt}] &= P_i^t+\Delta,\end{aligned} \tag{2}$$

where $\overline{X}_{1t},\overline{X}_{2t}...,\overline{X}_{Nt}$ are the unknown upper bounds , and $P_i^t+\Delta$ represents the bidder i's next period bidding price. Similarly, we get the lemma 2.

Lemma 2. *The solution of the N unknown variables in equations. 2 are unique and strictly increasing in $P_i^t+\Delta$, for all possible realizations of the upper bounds of $X_{1t},...X_{Nt}$.*

In the symmetric case, it is not hard to see $\overline{X}_j = \overline{X}_t$. Upper bound is more or less easy to compute compared with the lower bound. The following proposition states that three assumptions plus two lemmas are sufficient to ensure the existence of equilibrium bidding strategies.

Proposition 1. *For the auction τ with N_τ number of bidders, given assumptions 1,2,3 and lemma 1,2, there exists pivotal bidding functions in period t,*

$$\beta_i^t(X_i,\Omega_t)=\mathbb{E}[V_i|X_i,\underline{X}_{tj}\le X_j\le\overline{X}_t,\ j=1,...,N,j\neq i;\Omega_t], \tag{3}$$

for the bidders $i=1,...N$, where lower bound,$\underline{X}_j$, upper bound, $\overline{X}$, and public information, Ω_t, are defined above. Moreover, there exists a Bayesian Nash Equilibrium in strictly increasing bidding strategies when

$$\beta_i^t(X_i,\Omega_t)\ge P_{t-1}+\Delta,\ i=1,2,...,N,$$

where posting price, P_{t-1}, and minimal bidding increment, Δ, are defined above.

Proof. In the appendix □

Unlike an unique bidding strategy in button auction literature, proposition 1 defines an equilibrium bidding rules which bidders could have multiple bidding strategies, but all the bidding strategies should satisfy the pivotal bidding functions. This provides a convenient way to deal with the multiple equilibrium bidding strategies in English auction model.

2.2 Log-normal auction model

In auction, A structural econometric model would utilize the mapping between unobserved private signals and observed bidding history.To characterize the conditional joint distribution of $F_{XV}(X_\tau, V_\tau|N_\tau, M_\tau)$, I take a parametric approach by restricting attention to a family of joint distribution parameterized by a finite-dimensional vector θ, and use the bounded conditions to derive the moment conditions for identification and estimation. As emphasized in Hong and Shum (2003), the learning process in the common value ascending auction introduces a large amount of recursive computation into the bidding function. And the computational burden is even higher in this paper. For example, let us assume four bidders (A, B, C, D) attend the auction and assume the first 4 periods bidding prices are $\left\{P_A^1, P_C^2, P_D^3, P_A^4\right\}$. At the beginning of period 5, All the bidders have to calculate the lower bounds as well as upper bounds of the signals of their rivals from the price set $\{P_A^4, P_B^0, P_C^2, P_D^3\}$,where bidder B's bidding price (P_B^0) equals to the reserve price. In the first step, we get $\left\{\underline{X}_A, \underline{X}_B, \underline{X}_C, \underline{X}_D\right\}$ and $\left\{\overline{X}_A, \overline{X}_B, \overline{X}_C, \overline{X}_D\right\}$ by solving the system of equations. In the second step, bidder i will take her rivals' lower and upper-bound signals to calculate pivotal bidding function 3. By comparing the updated conditional expected value with the bidding price, bidder i then decides whether to bid or not in this period. If the auctioneer selects bidder B as the posting bidder at the end of period 5. In period 6, we would use new bidding path $\left\{P_A^4, P_B^5, P_C^2, P_D^3\right\}$ and calculate the bounds again to get$\left\{\underline{X}'_A, \underline{X}'_B, \underline{X}'_C, \underline{X}'_D\right\}$ and $\left\{\overline{X}'_A, \overline{X}'_B, \overline{X}'_C, \overline{X}'_D\right\}$. Hence, in each period, every bidder will use the updated lower and upper bounds to calculate their updated conditional expected value. Therefore, the feasibility of structural estimation lies in choosing a parametric family of joint distributions $F_{XV}(X_\tau, V_\tau|n, M_\tau; \theta)$ for the latent valuations and signals such that the resulting conditional expectation functions have closed-form expressions which are easy to invert. Among the limited choice of parameterizations that satisfy the criterion, I assume that bidders' valuations are log-normally distributed a la Hong and Shum (2003).

Till now, I have not introduced the information structure yet. The information structure in this paper refers to what kinds of private signal that each bidder can receive. Different information structures (symmetric or asymmetric) will have different impacts on the learning process, affecting the bidding behavior and auction outcome. The value of the object to bidder i ,V_i, is assumed to take a multiplicative form $V_i = A_i \times V$, where A_i is a private value for bidders and V is a common value component unknown to all bidders. I assume that V and A_i are independently log normally distributed. Let $v \equiv \ln V$, and $a_i \equiv \ln A_i$:

$$
\begin{aligned}
v &= \mu_v + \epsilon_v \sim N(\mu_v, \sigma_v^2), \\
a_i &= \mu_a + \epsilon_a \sim N(\mu_a, \sigma_a^2), \\
v_i &= a_i + v. \qquad (4)
\end{aligned}
$$

Each bidder is assumed to have a single noisy signal of the value of the object, X_i, which has the form $X_i = V_i \times E_i$ where $E_i = V \cdot \exp\{\sigma_\epsilon \xi_i\}$ and ξ_i is an (unobserved) error term that has a normal distribution with mean 0 and variance 1. If we let $v_i \equiv \ln V_i$ and $x_i \equiv \ln X_i$, then conditional on v_i, $x_i = v_i + \sigma_i \xi_i \sim N(v_i, \sigma_\epsilon^2)$, the joint distribution of $(V_i, X_i, i = 1..., N) = \exp(v_i, x_i, i = 1, .., N)$ is fully characterized by $\{\mu_v, \mu_a, \sigma_v, \sigma_a, \sigma_\epsilon\}$. These parameters are all common knowledge among the bidders and denoted by θ in the model.

If there is an informed bidder, $i = I$, her signal could be thought of precise compared to others:

$$\xi_i = 0, \ if \ i = I, \ and \ \xi_i > 0 \ if \ i \neq I.$$

The above expression implies that only the informed bidder I is able to observe her own valuation directly, i.e. $x_I = v_I$. To simplify the model, the paper assumes that there exists at most one informed bidder in the auction. Still, I is unable to observe the common value part, as a_i and v are not individually distinguishable in equation 4. To represent the information structure, the model assumes that every bidder have homogenous prior beliefs (π) about the existence of an informed bidder in the auction. When $\pi = 1$, we say that every bidder in the auction knows that there exists an informed bidder and the identity is revealed in the auction. When $\pi = 0$, everyone believes that there is no informed bidder attending the auction. When $0 < \pi < 1$, the uninformed bidders believe that each of her rivals has π probability to be the informed bidder. But in this scenario, we do not know who is the priority bidder. Hence, the prior belief, π, determines information structures. Because the bidding function under different information structures is similar to each other, I will take $\pi = 0$ to derive the equilibrium bidding functions.

Because of log-normality assumption, the system of equations in 1,2 and 3 is log-linear in the signals, allowing us to derive the equilibrium bidding functions for each period in closed form. Let us begin with the system of equations 1 which defines the equilibrium inverse bidding strategies (lower bounds) for N bidders in period t:

$$
\begin{aligned}
P_1^{t*} &= \mathbb{E}[V_1 | \underline{X}_1 = (\bar{\beta}_1^t)^{-1}(P_1^{t*}),\ \underline{X}_2 = (\bar{\beta}_2^t)^{-1}(P_2^{t*}), ..., \underline{X}_N = (\bar{\beta}_N^t)^{-1}(P_N^{t*}); \theta],\\
P_2^{t*} &= \mathbb{E}[V_1 | \underline{X}_1 = (\bar{\beta}_1^t)^{-1}(P_1^{t*}),\ \underline{X}_2 = (\bar{\beta}_2^t)^{-1}(P_2^{t*}), ..., \underline{X}_N = (\bar{\beta}_N^t)^{-1}(P_N^{t*}); \theta],\\
&\vdots\vdots\\
P_N^{t*} &= \mathbb{E}[V_1 | \underline{X}_1 = (\bar{\beta}_1^t)^{-1}(P_1^{t*}),\ \underline{X}_2 = (\bar{\beta}_2^t)^{-1}(P_2^{t*}), ..., \underline{X}_N = (\bar{\beta}_N^t)^{-1}(P_N^{t*}); \theta],
\end{aligned}
$$

where $\bar{\beta}_i^t(\underline{X})$ represents the mapping from the lower-bound signal to the bidding price. Given the log-normality assumption, the conditional expectation functions for V_i take the form:

$$
\mathbb{E}[V_i | X_1, ..., X_N; \theta] = \exp\left(\mathbb{E}(v_i | x_1, .., x_N; \theta) + \frac{1}{2} Var(v_i | x_1, ..., v_N; \theta)\right), i = 1, ..., N. \tag{5}
$$

Furthermore, we denote the marginal mean-vector and variance–covariance matrix of $(v_i, x_1, ...x_N)$ by $\mu_i \equiv (\mu_i, \mu^*)$ and $\Sigma_i \equiv \begin{bmatrix} \sigma_i^2 & \sigma_i^{*'} \\ \sigma_i^* & \Sigma^* \end{bmatrix}$. Then, the conditional mean and variance of jointly normal random variables are:

$$\mathbb{E}[v_i|\underline{x} \equiv (\underline{x}_1, ..., \underline{x}_N)'; \theta] = (\mu_i - \mu^{*\prime}\Sigma^{*-1}\sigma_i^*) + \underline{x}'\Sigma^{*-1}\sigma_i^*, \tag{6}$$

and

$$Var(v_i|\underline{x}; \theta) = \sigma_i^2 - \sigma_i^{*\prime}\Sigma^{*-1}\sigma_i^*. \tag{7}$$

By plugging 6and7into 5, and noting that conditional variance expression is not a function of x, we see that the conditional expectation function in 6 are log-linear in $\underline{x}_i$. The derivation of the lower bound follows the Hong and Shum (2003). After obtaining the lower bound, $\underline{x}$, at period t, we do the same operation for the upper bound $\overline{x}$:

$$\begin{aligned}
\mathbb{E}[V_1|\overline{X}_{1t} = (\bar{\beta}_1^t)^{-1}(P_i^t + \Delta), \overline{X}_{2t} = (\bar{\beta}_2^t)^{-1}(P_i^t + \Delta), ..., \overline{X}_{Nt} = (\bar{\beta}_N^t)^{-1}(P_i^t + \Delta)] = P_i^t + \Delta,\\
\mathbb{E}[V_2|\overline{X}_{1t} = (\bar{\beta}_1^t)^{-1}(P_i^t + \Delta), \overline{X}_{2t} = (\bar{\beta}_2^t)^{-1}(P_i^t + \Delta), ..., \overline{X}_{Nt} = (\bar{\beta}_N^t)^{-1}(P_i^t + \Delta)] = P_i^t + \Delta,\\
\vdots\vdots\\
\mathbb{E}[V_N|\overline{X}_{1t} = (\bar{\beta}_1^t)^{-1}(P_i^t + \Delta), \overline{X}_{2t} = (\bar{\beta}_2^t)^{-1}(P_i^t + \Delta), ..., \overline{X}_{Nt} = (\bar{\beta}_N^t)^{-1}(P_i^t + \Delta)] = P_i^t + \Delta,
\end{aligned}$$

where $\bar{\beta}_i^t(\overline{X})$ represents the mapping from the upper-bound signal to the bidding price. similar to 5,we can get $\overline{x}$.Now we can finally recover the pivotal bidding function 3 by plugging the lower bound, $\underline{x}$, and upper bound, $\overline{x}$, into the equations:

$$\mathbb{E}[V_i|X_1, X_j \geq \underline{X}_j, j \neq i, j = 1, 2, ...N; \theta] = \exp\begin{pmatrix} \mathbb{E}(v_i|x_i, \overline{x} \geq x_j \geq \underline{x}_j, j \neq i, j = 1, 2, ...N; \theta) + \\ \frac{1}{2}Var(v_i|x_i, \overline{x} \geq x_j \geq \underline{x}_j, j \neq i, j = 1, 2, ...N; \theta) \end{pmatrix}, \ i = 1, ..., N$$

where we need the truncated normal property to recover the pivotal bidding functions. So far, we get the pivotal bidding functions by fixing the prior belief $\pi = 0$. When $\pi = 1$, we follow the similar derivation except for the identity of the informed bidder. Since informed bidder knows exactly her evaluation of the objective, she does not need to consider others' private signals. The bidding function for the informed bidder is:

$$\exp(\mathbb{E}[v_I|x_I; \theta]) \geq P_I^t.$$

Apart from informed bidders, uninformed bidders learn from each other through the correlation across the private signals: $\sigma_i^{*\prime}\Sigma^{*-1}$. Signal recovered from the informed bidder has higher weight, which is more valuable to the uninformed bidders. If $\pi = 1$, we know the identity of the informed bidder and the pivotal function of the uninformed bidders becomes:

$$\mathbb{E}[V_i|X_i, \underline{X}_j \leq X_j \leq \overline{X},\ j = 1,...,N, j \neq i, I;$$
$$\underline{X}_I \leq X_I \leq \overline{X}_I; \Omega_t\ ; \theta \quad] \geq P_i^t,$$

where $\underline{X} = \{X_1,...,X_N\}$ and $\overline{X}$ represents lower and upper-bound signals respectively. If $\pi \in (0,1)$, uninformed bidders need to consider the possibilities that each of her rival could be the informed bidder. And the pivotal function of the uninformed bidders at period t becomes:

$$\bar{\mathbb{E}}_t[V_i|X_i, \bar{X}_{-i} \geq X_{-i} \geq \underline{X}_{-i}; \pi, \Omega_t, \theta] = \sum_{I\in\{j:,\ j\neq i\}} \mathbb{E}[V_i|X_i, \overline{X}_{-I} \geq X_{-I} \geq \underline{X}_{-I},\ \overline{X}_I \geq X_I \geq \underline{X}_I; \theta]Pr(\pi)$$
$$+\mathbb{E}[V_i|X_i,\ \overline{X}_{-I} \geq X_{-I} \geq \underline{X}_{-I}, ; \theta](1 - \sum_{I\in\{j:,\ j\neq i\}} Pr\{\pi\}), \quad (8)$$
$$Pr(\pi) = \frac{\pi(1-\pi)^{N-2}}{(N-1)\pi(1-\pi)^{N-2} + (1-\pi)^{N-1}},$$
$$\bar{\mathbb{E}}_t[V_i|X_i, \overline{X}_{-i} \geq X_{-i} \geq \underline{X}_{-i}; \pi, \Omega_t; \theta] \geq P_i^t. \quad (9)$$

In this case, we have to compute non-trivial expectation of the prior belief π. Since $Pr(\pi)$ is increasing in π, conditional expectation$\bar{\mathbb{E}}_t[V_i|X_i,\ \overline{X}_{-i} \geq X_{-i} \geq \underline{X}_{-i}; \pi, \Omega_t]$ is also monotonic increasing with π. So far, we have characterized the pivotal bidding functions under different information structures. In the next part, we will discuss the relation between prior belief π and entry choice.

2.3 Information Premium Discussion

Based on the previous construction, the model has several new features. First of all, from pivotal bidding function, bidders may bid higher in this auction than that of in button auction model (Hong and Shum (2003)). Since in button auction model, a bidder will update the information only if someone drops. However, in this model updating process happens whenever bidding price is updated, resulting in a higher expectation of the item value. learning effect will have bigger impact on the updating process. Second, under the same conditional expected value, the higher the lower bounds of rivals' private signals are, the lower bidder i's private signals will be. This is because increased lower bounds of the rivals' signals lead to a higher learning effect. Thus, given other things unchanged, bidder i still gets higher evaluation of the item even though her own private signal is not high.

Conclusion 1. In pivotal functions ($\mathbb{E}[V_i|X_i, \underline{X}_j \leq X_j \leq \overline{X},\ j = 1, ..., N, j \neq i; \Omega_t, N; \theta]$), X_i is non-increasing with respect to $\underline{X}_j$.

Besides above mentioned learning effects, the information asymmetry will introduce another channel of learning process. If an informed bidder enters the auction, other uninformed bidders know the informed bidder has more precise private signal to the value of the item. They will put more weight to the bid submitted by the informed bidders. Such difference between an informed bidder and uninformed bidders reflects the information premium. There are two ways to formulate the measurement of information premium. First, by calculating premium of uninformed bidder given the existence of the informed bidder, we can get the learning effect ϖ_2 induced by the informed bidder:

$$\begin{aligned}\varpi_1 = &\mathbb{E}_t[V_i|X_i; \overline{X}_I \geq X_I \geq \underline{X}_i;\ \overline{X}_j \geq X_j \geq \underline{X}_j,\ j \neq i, I, j = 1, 2, ...N; \Omega_t; \theta] \\ &- \mathbb{E}_t[V_i|X_i; \overline{X}_j \geq X_j \geq \underline{X}_j,\ j \neq i, j = 1, 2, ...N; \Omega_t; \theta]\,. \qquad (10)\end{aligned}$$

The first term in 10 indicates the conditional expectation of the uninformed bidder i with an informed Ias one of his/her rivals. While the second term indicates the identical situation without the informed bidder I. The premium will depend on the bidding history and the number of bidders. In particular, if more bidders attend the auction, the influence of the informed bidder will be diluted. As more rivals attend the auction, bidder i will have more information sources to update the information. On the other hand, a higher bidding price submitted by the informed bidder will have a bigger effect on bidder i's conditional expectation. The uninformed bidder is more sensitive to the price changes of the informed bidder. The simulated results show the variations of the information premium that correlates with the bidding history and number of bidders.

From the simulated results in figure 1, we see that as the lower bound of the informed opponent increases, bidder i's updated expected value is rising quickly. but when the updating process only comes from the uninformed bidder, the marginal increment of information value is decreasing (second order derivative is negative). This is because the higher bidding price of the uninformed bidders may come from a noisy or private signal rather than a higher common value. In addition, figure 2 shows the impact of the number of bidders on the information premium. As we can see, when more bidders attend the auction, the impact of the informed bidder on the updating process is decreasing.

Another measurement of the information premium can be the the difference between the winning bid of informed bidder and uninformed bidder, which is defined by:

$$\varpi_2 = b(I=1;N_\tau,M_\tau) - b(i=1;N_\tau,M_\tau).$$

where, $b(i;N_\tau,M_\tau)$ indicates the bidder i's highest posting price[2] given the N_τ participants and M_τ observable characteristics in auction τ. ϖ_2 directly measures the information premium paid from the informed bidder when she wins the item. The important thing is that ϖ_2 is independent of the number of participants N_τ. (Dionne et al 2009). As the uninformed bidders will put more weights on informed bidder's private information, the uninformed bidder will correspondingly reduce the weight from other uninformed rivals as well as his own private signals. It can be proved that the positive effect exactly offset the negative effect. Thus, as the number of bidders increase, it does not affect the information premium.

[2] i represents the rank order of the bidders within the auction τ.

Proposition 2. [3]*The information premium defined by*

$$\varpi_2 = b(I=1;N_\tau,M_\tau) - b(i=1;N_\tau,M_\tau).$$

is independent of number of participants N_τ.

Proof. In the appendix C. □

Regarding the common value auction, the most commonly used indices of the information premium are the winner's curse and loser's curse. Usually, the winner's curse is defined as the difference between the conditional expectation when bidder$i$ realizes that others have lower private signals than hers:

$$\mathbb{E}[V_i|X_i = x_i, X_j < x_i, j \neq i; \theta] - \mathbb{E}[V_i|X_i = x_i; \theta].$$

From the property of the conditional expectation, the result is negative, meaning that winning brings bad news (Krishna (2002)). Moreover, the larger the x_i or N_τ , the worse the curse will be. On the other side, the common value auction may also induces the loser's curse(Pesendorfer and Swinkels(1997)), indicating that early dropout may also be bad news. In this model, the loser's curse is defined as the difference between the conditional expectation when bidder$i$ realizes that she drops out too early. Similar to the winner's curse, the information premium for the loser's curse can be expressed as:

$$\mathbb{E}[V_i|X_i = x_i; \theta] - \mathbb{E}[V_i|X_i = x_i, X_j > x_i,\ j \neq i; \theta].$$

As the x_i increases, the marginal premium is increasing as well. The winner's curse and loser's curse are like mirror image in which the pivotal value x_i determines the relative strength of the two offsetting effects. Under the symmetric value distribution settings, winner's curse is equivalent to loser's curse when $x_i = \mu_x$. When $x_i > \mu_x$, although both winner's curse and loser's curse increase, winner's curse effect dominates, vice versa.

[3]Dionne et al. (2009) also propose this claim. But they do not give rigorous proof. they summarize the fact from the simulation experiments.

Due to the learning process, every period bidder i will update their information of the lower bound signal from the rivals. And the information premium becomes:

$$\mathbb{E}[V_i|X_i = x_i; x_i \geq X_j \geq \underline{X}_j,\ j \neq i; \theta] - \mathbb{E}[V_i|X_i; \overline{X}_j \geq X_j \geq \underline{X}_j,\ j \neq i; \theta].$$

Given the upper bound x_i fixed, as the lower bound $\underline{X}_j$ increases, the curse will be alleviated, which is explained in the appendix. If the lower bounds of other bidders are high, the item tends to have higher common value, alleviating the curse of winning the item. Information revelation usually reduces the information premium of the high valued bidder. As the price gradually rises up, the (uninformed) bidder will realize that someone may have a higher signal and start following the high valued bidder. This will induce more aggressive competition which in turn decreases the expected revenue of winning the auction.

3 Data Statistics and Empirical Evidence

3.1 Chinese Justice Online Auction

The primary objective of the Chinese Justice Auction is to process confiscated properties transparently and efficiently. In general, the courts need to deal with 600 billion RMB worth of assets and property rights every year. The item for selling includes furniture, car, house, land, and etc.The auction is organized as a typical online English auction. Before the auction begins, the local court will publish the announcement Online one month earlier. The announcement includes important information about the item to the public, such as evaluation value, reservation price, the minimum bidding increment, location, the existence of the priority bidder, usage information, photos, documentation, plus the third-party evaluation report. Anyone who wants to participate can submit the E-form application with the required margin (approximately 10% of the item value) in the registration period. The margin will be refunded if the bidder fails to win the auction. During the auction, anyone who submits the bid successfully at the current bidding period is the posting bidder. And her price is the current highest bidding price, which is called posting price. Whoever wants to win the item has to submit a bid no less than the posting price plus the integral multiple of minimum bidding increment. the number of bidders is publicly known at the end of registration period. However, the bidder's identity is anonymous. In the data, there is one typical type of bidder called "priority bidder", who, by definition, has the correlation with the selling property. For example, the priority bidder could be the tenant of the house for selling, or the banks that lend the mortgage. Many auctions will identify the bid submitted by the priority bidders. Some priority bidders will bid very actively, and finally win the auction. But some priority bidders bid very inactively. The heterogeneous priority bidder's bidding behavior is one of the distinctive features in the data, and later section will focus on this discussion. After the auction, the website will publish the winning bidder's information if the auction is sold successfully. If the item fails to be sold out at first time, the item has extra chance to organize a new auction again. So each item will have two chances to hold a normal auction. In the first-time auction, the reservation price should be no less than 80% (70% after 2017) of the evaluation rice of the item. In the second-time auction, the reserve price can be no less than 64% of the evaluation price. If the item can not be sold successfully after the second-time auction, the item will turn to other disposal methods. In this research, I focus on first-time housing auction and the data time ranges from 2015Q1 to 2017Q4. For the time being, I select auction data from 4 cities with 2211 successful first-time real estate auctions to run the test.

3.2 Data Statistics

The online platform provides detailed information for each auction: the description of the item; the date for the auction; whether the item was sold successfully or not; who won the auction; the reservation price; the evaluation price; the location of the item if the item was a real estate, whether a priority agent attended the auction or not; the local court who organized the auction; the number of bidders in the auction; the number of bidding period. More importantly, we can observe the bidding history for each bidders per auction.

The data sample excludes one bidder auction in which the bidder wins the auction at the reservation price. From the summary statistics in table 1, we see that only 166 auctions have the priority bidders, which is a very small portion of the total data sample. The reserve ratio, which is defined by the reservation price over the evaluation price, ranges from 0.7 to 1, satisfying the reservation price regulation on the first-time auction. Regarding the number of participants, around 80% of the successful sold first-time auctions have no more than 11 bidders. The normalized winning price, defined as $\frac{win\ price}{reserve\ price}$, has relatively narrow range: 75% auctions has the value less than 1.537. But the highest winning price is around 3 times than the reservation price. This implies a relatively large variance across the auctions. Another key variable is the bidding increments. Larger increment will increase the bidding cost, reduce the bidding activity and depress the information revelation process. Notice that although the bidding increment is large in terms of absolute value, compared to the total values of the item, the minimum bidding ladder is, in average, less than 0.4% of the reserve price (ladder ratio in the table). This implies that minimum bidding increment is not a problem in the analysis of this open ascending auction. Apart from summary statistics, there are also several other features that worth to mention. From figure 3, we see that there exist a fat tail in the first time auction. And the winning bid is positively correlated with the number of bidders. Because priority bidder involves information asymmetry problem, I will compare the bidding outcomes in detail between the two subsets grouped by the priority bidder in the next part.

3.3 Priority Bidders and Information Asymmetry

The priority bidder in the auction refers to the one who has the correlation with the selling property. We can treat the priority bidder as the informed bidder. Since the priority bidder could be, for example, either the tenant of the house or the bank that lends the mortgage, there exists large heterogeneity across different types of bidders. Being a priority bidder, the agent wins the item whenever there is a tie during the bidding stage. While others need to add minimum bidding requirement to bid against posting bidder. To become a priority bidder, a bidder needs to fulfill an E-application form claiming the priority bidder privilege. If the informed bidder wants to hide her identity, she can also choose not to apply. Because the being a priority bidder is not a mandatory requirement, for those auctions without priority bidder, we are not sure about whether there exists an informed bidder or not. Compared with the low benefits of being a priority bidder, if the priority bidder really wants the item, it is better to hide her identity. However, in the auction, we actually observe many priority bid actively and win the item. On the other hand, uninformed bidder could also have the belief that some informed bidder may hide their information and attend the auction. Actually, absence knowledge of existence and identity of the informed bidder are the common issue for the auctions in the real world.

Regarding the existence of priority bidder[4], the data displays many distinct features of the bidding outcomes (figure 5). First, auctions with priority bidder will have higher winning price while fewer participants. The result from endogenous entry choice in the previous section is consistent with this data feature. As mentioned before, the identity of the informed bidder will have two offsetting effects on uninformed bidder's decision. On the one hand, it reveals that the item has a higher value, encouraging uninformed bidders to participate. On the other hand, the existence of informed bidder indicates the item has higher value in ex ante, discouraging other uninformed bidder's incentive to participant. From the unexpected lower participation rate in the data, the discouraging effect dominates. Second, auctions with the priority bidder have higher bidding frequency, where the bidding frequency is defined as the total number of bids submitted within an auction. The number indicates that bidders tend to behave more aggressively in the auctions with the priority bidder. To specify the effect of priority bidder on the winning price, number of bidders and bidding frequency, I conduct the reduced form econometric analysis:

$$y_\tau = \delta 1_\tau\{priority\} + \beta x_\tau + \varepsilon_\tau,$$

where the y_τ indicates the dependent variables such as winning index, the number of bid-

[4]Since most priority bidders attend the first time auction, the following analysis is focused on first time auction.

ders or the bidding frequency. Notice that I define the winning price index as the net increase from reserve price, $\frac{win\ price}{reserve\ pricce}$. The indicator $1_\tau\{priority\}$ represents whether the current auction τ has the priority bidder or not. The x_τ represents the control variables, such as the reserve price, the location fixed effect and etc. The error term ε_τ is independent of other covariates. The column 1, 3, 5 in table 3 are the simple regression while column 2, 4, 6 controls for the location and year specific. The reduced form regression shows that the identity of priority has a significant positive effect on the winning price and bidding frequency while the negative effect on the number of bidders. For the winning price, the existence of priority bidder will drive the winning bid up 5% relative to the reserve price. Meanwhile, one more bidder attending the auction will also drive the winning price up to 3% relative to the reserve price. In addition, the priority bidder's effect can more or less be mitigated by encouraging more bidder's attending the auction. Both the number of bidders and the existence of priority bidder tell the same story. the existence of the priority bidder indicates the item must have higher value to attract the informed bidder. Similarly, more bidders attending the same auction implies people have a consistent belief about the value of the item. Comparing first 4 columns, we see that the existence of the priority bidder and the number of bidders have strategic substitute effect: the existence of the priority will discourage nearly 1.5 bidders to attend the auction, which is consistent with the endogenous entry prediction. The existence of the priority bidder not only attract more bidders attending the auction but also increase the entry threshold for other bidders, resulting in less but high valued bidders attendance. Regarding the bidding frequency, the effect from the priority bidder equals approximately two more attendance of uninformed bidders. In term of absolute measure, the existence of the priority bidder will induce 7 more submissions while one extra bidder will induce 3 more submissions. All the above discussion is under the assumption that all priority bidder would like to win the item. If we further group the priority bidder with the active level, we see clearly that the inactive group has less mean value of winning price than that of in auctions without the priority bidder. The divergent bidding behavior across priority bidders may imply that priority bidder have some strategic behaviors in different situation. Later in the simulation test, we will see more clearly how the priority bidder affect the auction outcome.

4 Identification

4.1 Identification strategy

The identification challenge for the set of distribution parameters, θ, comes from many to many correspondence between value distribution and bidding distribution. Although there is no moment equality for us to point identify the parameters, we can still get partial identification by the moment inequalities. Similar to Haile and Tamer (2003), the bidding price helps to construct the lower bound of the dropout price and the winning bid helps to establish the upper bounds except for the winner. If we order the bidder based on highest posting price, we have the following conditional moment inequalities:

$$\begin{aligned} P_i^{t_i} &\leq \beta(X_i, \Omega_{t_i}; \theta) \leq P_{win} + \Delta \; i > 2, \\ P_i^{t_i} &\leq \beta(X_i, \Omega_{t_i}; \theta) \leq P_{win} + \Delta \; i = 2, \\ P_{win} &\leq \beta(X_i, \Omega_T; \theta) \qquad\qquad i = 1, \end{aligned} \tag{11}$$

where $P_i^{t_i}$ indicates the highest bidding price of bidder i at period t_i, Ω_{t_i}indicates information needed up to t_i and θ indicates the set of parameters that we care about, Δ represents the minimum bidding increment. The identified feature of the moment inequality model is the set of parameter values that obey these restrictions for X and represents the set of auction models that are consistent with the empirical evidence. I define identified set:

Definition 1. Let Θ_I be such that

$$\Theta_I = \{\theta \in \Theta, \; s.t. \text{ inequalities 11 are satisfied at } \theta \; \forall X \; a.s.\}.$$

We say that Θ_I is the identified set.

Regarding the model setup, the parameters set for identification contains $\{\mu_v, r, \sigma_v^2, \sigma_\alpha^2, \sigma_\epsilon^2\}$. The mean value of private signal has $X_i = V_i = \mu_v + r \cdot \log(\gamma)$, where I restrict mean value of the private part (μ_a) and noisy part (μ_ϵ) to zero and controls for the heterogeneity in reserve price.

4.2 Estimation Setup

The estimation problem for θ is based on the moment inequality 11. The objective function is

$$Q(\theta) = \int [\Big|\Big|\{P^{lower} - \beta(x, \Omega_{t_i}; \theta)\}_+\Big|\Big| + \Big|\Big|\{P^{upper} - \beta(x, \Omega_{t_i}; \theta)\}_-\Big|\Big|]dF_x,$$

where $\{A\}_+ = [a_1 1(a_1 \geq 0), .., a_N 1(a_N \geq 0)]$ and $\{A\}_- == [a_1 1(a_1 \leq 0), .., a_N 1(a_N \leq 0)]$, and where $||\cdot||$ is the Euclidean norm. It is straightforward to see that $Q(\theta) \geq 0$ for all $\theta \in \Theta$ and that $Q(\theta) = 0$ if and only if $\theta \in \Theta_I$, the identified set in Definition 1. Following Chernozhukov, Hong and Tamer (2007) (CHT hereafter), I construct the consistent estimation for the identified set, which contains parameters that cannot be rejected as the truth. To estimate Θ_I, I need to define a sample analog of $Q(\theta)$ and estimated set $\hat{\Theta}_I$:

$$\hat{\Theta}_I = \{\theta \in \Theta | nQ_n(\theta) \leq \nu_n\},$$

where $\nu_n \to \infty$ and $\nu_n/n \to 0$ and the sample analog of the objective function is

$$Q_n(\theta) = \frac{1}{T}\sum_{\tau=1}^{T}\frac{1}{J}\sum_{j=1}^{J}[\Big|\Big|\{P_\tau^{lower} - \beta(x_j, \Omega; \theta)\}_+\Big|\Big| + \Big|\Big|\{P_\tau^{upper} - \beta(x_j, \Omega; \theta)\}_-\Big|\Big|].$$

where $||\cdot||$ represents the Euclidean norm, T represents the number of auctions, P_τ^{lower} and P_τ^{upper} represent the lower and upper bounds respectively for each auction τ, $\beta(\cdot, \cdot; \theta)$ represents the bidder's bidding functions given the parameters θ. Inside summation in $Q_n(\theta)$ represents the empirical integration of x. The model supported by the estimated parameters should be consistent with the empirical evidence. To do the inference, similar to HT and CHT, I also construct the Hausdorff-consistent estimator of the set Θ_I. The key is to find the confidence set(region) C_n such that $\lim_{n\to\infty} P(\theta_I \in C_n) \geq \alpha$ for a pre-specified $\alpha \in (0, 1)$ for any $\theta_i \in \Theta_I$. This confidence region is based on the principle of collecting all of the parameters that cannot be rejected, which is constructed as follows:

$$C_n(c) = \{\theta \in \Theta : \; n(Q_n(\theta) - \min_t Q_n(t)) \le c\}.$$

The detailed construction procedure follows CHT. Since we get a set, $\hat{\Theta}_I$, for the parameter values of the value distribution. It is not easy to construct the hypothesis testing for π by $\hat{\Theta}_I$. Ideally, across all the set of parameters, we can get consistent results for rejecting or not. But we may also encounter the case that part of the parameters results in rejecting, while the rest parameters lead to no rejecting the hypothesis. In order to get robust testing results, first of all, I will use the lower and upper bounds of the set of parameters to apply the hypothesis test. Then I will randomly pick a different combination of parameters to apply the test again and compare the difference between the two results.

5 Estimation

5.1 Empirical Results

The parameter set of value distribution waiting for estimation consists of $\theta_I = \{\mu_v, \; r \;, \sigma_v^2, \; \sigma_\alpha^2, \; \sigma_\epsilon^2\}$, where r represent the coefficient of the control variable, reserve price. The mean value of private signal is $E[X_i] = E[V_i] = \mu_v + r \log(\gamma)$. Because there are more auctions without the priority bidder in the data, I assume the the auctions without the priority bidder have the symmetric information structure first and conduct the estimation procedure. The estimation results are shown in table 5.

From the estimation results, we see that each bidder indeed has private value for the item, although σ_α^2 is very small compared to the common value part. The common value part actually determines the learning effect among the participants. But based on the estimation results, we see that the learning effect is largely blocked by the noisy part. The huge variance in noisy part indicates if someone owns information advantages, he/she will have much larger information advantage. Moreover, from the model, we know that If σ_ϵ^2 is large, uninformed bidders will put much higher weight on the bid submitted by the informed bidder. If the informed bidder bids actively in the auction, every other uninformed bidders would learn that the item should worth high and everyone will bid actively. However, if the informed bidder bids very inactively or never show up during the bidding stage, uninformed bidders would expect the item probably has much less value. Because the uninformed bidder puts much higher weight on the informed bidder's bid, even if his/her own private signal may be high enough, the uninformed bidder would dropout quickly. And the auction revenue could be seriously affected by the informed bidder's bidding behavior. In the next part we will employ counterfactual experiment to test for the impact of priority bidder's behavior on winning price.

5.2 Simulation Test

To evaluate the performance of the model, I apply Monte Carlo Simulation to do the Experiment. I generate the simulated auction data under the model assumptions. To generate the data, we have to specify those key value distribution parameters first. The mean and variance for the signal v_i, x_i follow the table setup. Notice that a_i and ϵ_i are independent with each other. Based on the parameters, we generate random private signals x_i for each bidder. For each auction, I also randomize the reservation price,γ, and bidding ladder,Δ. At the very beginning, each bidder only knows their own private signal x_i. Once someone starts to bid, bidders can obtain more and more price information from the bidding process. They will combine their own private signals and others' signals recovered from the bidding activity to update their conditional expectation. After simulating S auctions, we can calculate the moment conditions and distributions that we are interested in. To simulate the data, I let the number of participants range from 2 to 10 and draw $S = 5000$ replications with randomized parameter set[5]. The preliminary results are shown in figure 6.

[5]Based on the fixed parameter value in table 3, I add random shock $\epsilon_\mu \sim U(-0.075, 0.075)$ to the mean of common value μ_v and $\epsilon_{\sigma 1}, \epsilon_{\sigma 2}, \epsilon_{\sigma 3} \sim U(-0.05, 0.05)$ to the σ_v^2, σ_α^2, σ_ϵ^2 in every replication. This ensures that my results are not dependent on a specific parameter set.

As predicted from the model, the presence of the informed bidder will induce more aggressive bidding, resulting in a higher winning bid and more frequent bidding submission. To see the learning effect, I calculate the bidding ladder difference between the highest posting position (winning bid) and the rest. Since the distance between the second highest and the highest is always 1, I drop out the second highest result and calculate the average bidding ladder difference in figure 6(c). In general, if the bidders can learn from each other, given other things equal, higher learning effect will induce lower ladder distance. However, when an informed bidder enters the auction, two more effects will be introduced. Because the presence of the informed bidder brings a higher valued signal to the item, the low valued bidders realize that it is not possible for them to win the item. They will stop bidding quickly. While for those high valued bidders, the presence of informed bidder will give them more confidence to bid aggressively, resulting in a higher winning bid. In the meanwhile, the bidding ladder distance between lower valued and higher valued bidders will enlarge as well. Therefore, we see a fat tail under the informed bidder in figure 6(c) and the left shifted density crest under the informed bidder in figure 6(d).

Regarding the strategic behavior of the informed bidder, I further control the level of bidding activity for the informed bidder under different situations (figure 7). The experiment procedure is described in the appendix. The counterfactual analysis illustrates three scenarios. First, in normal case, the auctioneer randomly picks one of the bid submitted by all the candidate bidders (including the informed bidder's bid). Second, in active case, the auctioneer deliberately chooses the bid submitted by the informed bidder every other period. Third, in inactive case, the auctioneer randomly picks one of the bid submitted by all but the informed bidder. From figure 6, we see that under the inactive case, the wining price distribution has significantly left shift pattern, which implies by strategically depress their bidding activities, the informed bidders would have an big impact on the bidding outcomes.

6 Extension: Endogenous Entry

6.1 Entry Decisions

In this section, I extend the model to include the entry stage. Before the auction τ begins, the bidder learns the reservation price γ from observed characteristics M_τ and their private signals X_i. Bidders simultaneously make entry decisions and remaining bidders enter the bidding stage all together. To characterize the endogenous entry process, I assume that the number of potential bidders follows certain distribution $F(N_\tau|M_\tau)$. When making entry decisions, bidder i will first compare her conditional expected value with the reservation price under distribution of the potential number of attending bidders. This is the feasibility constraint:

$$\sum_n^{\infty} P(N_\tau = n|M_\tau)\mathbb{E}[V_i|X_i = X_1^r, X_j = X_2^r,\ j = 1, 2, ...n] \geq \gamma,$$

where X_1^r and X_2^r denote bidder i and rival's feasible entry threshold. In symmetric case, $X_1^r = X_2^r$. If there exists an informed bidder, The threshold for X_I^r is different from other uninformed biddersX_2^r. Moreover, since the informed bidder perfectly observes the value of the item, the feasibility constraint degenerates into:

$$X_I^r \geq \gamma.$$

Because it is a common value auction, the rival's bidding behavior will also affect bidder i's winning probability. The more rivals attend the auction, the more aggressive competition will be and less likely bidder i will win the item. This is the incentive constraint:

$$\sum_n^{\infty} P(N_\tau = n|M_\tau) \left\{ \begin{array}{c} \mathbb{E}[V_i|X_i = X_1^r,\ X_j = X_2^r, j = 1, 2, ..., n, j \neq i] \\ -\mathbb{E}[V_j|X_i = X_1^r,\ X_j = X_2^r, j = 1, 2, ..., n, j \neq i] \end{array} \right\} \geq 0$$

Therefore, conditional on the distribution of the number of potential bidders (n) and reservation price (γ), bidder i will face the threshold $\underline{X}_i^*$, $i \in \{informed,\ uninformed\}$ that he/she will choose to enter if her signal X_i is above $\underline{X}_i^*$. Based on the above conditions, we can get the equilibrium entry decisions for the English auction:

Proposition 3. *Given assumptions 1 and 2, there exists an Bayesian–Nash Equilibrium of entry decision that a potential bidder will choose to enter at auction* τ *if his/her private signal is no less than threshold* $\underline{X}_i^*$, $i \in \{informed, uninformed\}$, *where* $\underline{X}_i^*(M_\tau, \gamma_\tau, I) = \inf\{X^r \; satifies \; 1213; M_\tau, \gamma, I\}$:

$$\sum_n^\infty P(N_\tau = n|M_\tau)[\mathbb{E}[V_i|X_i = X_1^r, X_{-i} = X_2^r] \geq \gamma \tag{12}$$

$$\sum_n^\infty P(N_\tau = n|M_\tau) \begin{Bmatrix} \mathbb{E}[V_i|X_i = X_1^r, \; X_j = X_2^r, j = 1, 2, ..., n, j \neq i \; ; M, I] \\ -\mathbb{E}[V_j|X_i = X_1^r, \; X_j = X_2^r, j = 1, 2, ..., n, j \neq i \; ; M_\tau, I] \end{Bmatrix} \geq 0 \tag{13}$$

Proof. In the appendix □

From simple calculation, under the symmetric case, only the 12 binds while 13 binds under asymmetric case.

6.2 Distribution of the number of participants

Apart from the observable characteristic, M_τ, and private signal, x, the entry choice also depends on prior belief, π. Different prior beliefs affect both feasible and incentive constraints (12, 13), resulting in different entry thresholds $X^r(\gamma, \pi)$. To simplify the model, the paper again assumes the number of potential participants follows the Poisson process, where λ represents the instantaneous rate of arrival of this process. Hence, the number of potential participants is

$$Pr(N = k|M_\tau) = \frac{e^{-\lambda}(\lambda)^k}{k!}, \; k = 0, 1, 2, ..., n \; .$$

Then conditional on π and $Pr(N = k|M_\tau)$, we can get the threshold $\underline{X}^*(\gamma, \pi)$ from 13 and 12. After getting the threshold $\underline{X}^*(\gamma, \pi)$, the probability of attending the auction for each potential participant is:

$$H(\underline{X}^*(M_\tau, \gamma, \pi); \; \theta) = 1 - \Phi(\underline{X}^*(M_\tau, \gamma, \pi), \; \theta).$$

To simplify the notation, I will use H to represent $H(\underline{X}^*(M_\tau,\gamma,\pi);\ \theta)$. Thus, based on the Bayesian probability, we can derive the distribution of number of bidders across the auctions. Conditioning on the threshold and parameters, the probability of real number of bidders in auction τ is:

$$
\begin{aligned}
Pr(N_\tau = n|\underline{X}^*(M,\gamma,\pi),\theta) &= \sum_{k\geq n}^{\infty}\left(\frac{e^{-\lambda}(\lambda)^k}{k!}\right)\left\{\begin{pmatrix} k \\ n \end{pmatrix} H(\underline{X}^*(M,\gamma,\pi);\ \theta)^n[1-H(\underline{X}^*(M,\gamma,\pi);\ \theta)]^{k-n}\right\} \\
&= \left(\frac{H}{1-H}\right)^n \sum_{k\geq n}^{\infty}\left(\frac{e^{-\lambda}[\lambda(1-H)]^k}{k!}\begin{pmatrix} k \\ n \end{pmatrix}\right) \\
&= \left(\frac{H}{1-H}\right)^n \sum_{k\geq n}^{\infty}\left(\frac{e^{-\lambda}[\lambda(1-H)]^k}{\not{k}!}\frac{\not{k}!}{n!(k-n)!}\right) \\
&= \left(\frac{H}{1-H}\right)^n \frac{1}{n!}[\lambda(1-H)]^n e^{-\lambda H}.
\end{aligned}
$$

As we can see, the distribution of the number of bidders across the auction will depend on the observed characteristics M_τ, value parameter set θ and prior belief π.

6.3 Hypothesis test of the Information Structure (π)

From the equation, $\bar{\mathbb{E}}[V_i|X_i, X_{-i} \geq \underline{X}_{-i}; \pi, \Omega_t, \theta]$, the prior belief π directly affects the conditional expected value in each period. However, the learning process and the π shares the same channel to affect the bidding behavior. Higher π will induce a higher conditional expected value. But it is hard to distinguish the combination between higher π but lower learning effect and lower π but higher learning effect. As the number of bidder increases, this issue becomes more severe. To identify the parameter and apply the test for the information structure, we need the variation across different auctions. In the above discussion, entry decisions correlates with π, parameter set θ and observable characteristics (M). But M can be obtained directly from data and we can separately get estimated $\hat{\theta}$. Through the entry threshold$\underline{X}_*^r(M,\theta,\pi)$, we can build mapping from π to the number of bidders. To construct the identification strategy for π, we need the probability of real number of bidders attending the auction τ:

$$Pr(N_\tau = n|\underline{X}^*(M,\gamma,\pi),\theta) = \left(\frac{H}{1-H}\right)^n \frac{1}{n!}[\lambda(1-H)]^n e^{-\lambda H}. \tag{14}$$

14 is the basis to construct the test for information structure. Since the set of parameters should be identical across different information structure, we can first assume that $\pi = 0$ for the auctions without the priority bidder, and get the estimate of parameter set $\hat{\theta}$. if $\pi = 0$ is true for the auctions without priority bidder and given $\hat{\theta}$, we should get the identical λ under $\pi = 1$. Otherwise, when we estimate the value distribution under $\pi = 1$, we would get different value for λ. Then, we can construct two-sample hypothesis test for λ. The basic logic is to get maximum likelihood estimator of $\hat{\lambda}$ given the $\hat{\theta}$. Then we can use resampling techniques to get the standard errors under two different cases ($\pi = 0$ and $\pi = 1$). Then we can start two-sample T test procedure to check whether $\hat{\lambda}$ in $\pi = 0$ is significantly different from $\hat{\lambda}$ in $\pi = 1$.

6.4 Testing Results

Under the symmetric information structure ($\pi = 0$), the estimated $\hat{\lambda}_0$ is 8.35 with standard error, 0.0431. While under the existence of informed bidder ($\pi = 1$), $\hat{\lambda}_1$ becomes 7.193 with standard error, 0.0392. The null hypothesis for the two-sample t-test for unpaired data can be defined:

$$H_0: \ \lambda_0 = \lambda_1$$

From the Test Statistic, we have :

$$T = \frac{\hat{\lambda}_0 - \hat{\lambda}_1}{\sqrt{s_1^2/N_1 + s_2^2/N_2}},$$

where s_1^2, s_2^2 represent the sample variance and N_1, N_2represent the sample size. The results shows that we reject the null hypothesis. The testing indicates even there is no informed bidder attending the auction, the participants still believe that some informed bidder hides her identity and attends the auction.

7 Conclusion

In common value English auction, recovering the value distribution is non-trivial because mapping from the value distribution to the bidding distribution is no longer one to one. Due to the multiple equilibria issue and incompleteness of econometric structure, we face the many to many correspondence issue. The new challenge requires a new method for the value distribution estimation. On the other hand, the learning process and winner's curse also makes it hard to trace the bidding behavior under information asymmetry. The redistribution effect of information premium still remains inconclusive. Moreover, policy analysis such as auction revenue and efficiency is quite sensitive to whether the information structure is symmetric or not. Thus, testing the asymmetric information is a pre-request for policy analysis.

This paper aims to address those issues. The Chinese Justice auction data offers an ideal laboratory to analyze the problems. the paper builds a structural auction model that utilizes the bidding history to construct the upper and lower bound to disentangle the estimation difficulty. To test the asymmetric information, we need the endogenous entry decisions that bridge the variations of number of participants with different information structure. from the model, we see that the existence of the informed bidder will screen out the low valued bidder and select high valued bidder enter into the auction. As a result, the information premium mainly comes from the screening effect not from the learning process. This further leads to the fact that information premium is independent of number of participants in the auction. From the estimation result, the private signal has quite large noisy part which reduces the auction revenue. Also the unproportionally large noisy private signal indicates the informed bidder could have much higher information advantages in the auction. Through the simulation test, informed bidder can significant affect the auction outcome by strategically choosing her bidding behavior. The framework developed in this paper can have broader application. Any auctioneer who holds an open ascending auction format wants to analyze the information structure and the corresponding equilibrium bidding outcome can make use of this model. Indeed, the model is just simple abstraction from the real world that aims to capture the key influence of information structure. A lot of details are not included, which is waited for further research.

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Appendix A. Proofs

Proof of Proposition 1. Similar to the proof of Proposition 1 in Hong and Shum (2003), we show that if all bidders $j \neq i$ follow their own equilibrium strategies $\beta_j^t(\cdot)$, bidder i's best response is to play $\beta_i^t(\cdot)$ because this guarantees that bidder i will win the auction if and only if her expected net payoff is positive conditional on winning.

Given the current bidding price of bidder i, P_i^t, if bidder i wins the auction in period t when all remaining bidders simultaneously exit at a price of P_i^t, her ex-post valuation is:

$$\mathbb{E}[V_i|X_i,\underline{X}_j \leq X_j \leq \overline{X},\ j=1,...,N, j\neq i;\Omega_t].$$

Since this conditional expectation is increasing in X_i (from Assumption 1), bidder i makes a positive expected profit from winning in period t by staying actively in the auction at a price of P if and only if $X_i \geq (\beta_i^t)^{-1}(P_i^t;\Omega_t) \Leftrightarrow \beta_i^t(X_i;\Omega_t) \geq P_i^t$ (Assumption 2). In other words, $\beta_i^t(X_i,\Omega_t)$ specifies the price below which bidder i makes a positive expected profit by staying in the auction and above which bidder i makes a negative expected profit by staying in the auction. Therefore, for every realization of X_i, $\beta_i^t(X_i;\Omega_t)$ specifies a best-response dropout price for bidder i in period t.

Proof of Proposition 3. Suppose that there exists a bidder k whose private signal is X_k. Given the entry threshold$\underline{X}_\tau^*$ at auction τ, bidder k has the following choices:

1. $X_k < \underline{X}_\tau^*$, bidder k decides to enter.
2. $X_k < \underline{X}_\tau^*$, bidder k decides not to enter.
3. $X_k \geq \underline{X}_\tau^*$, bidder k decides to enter.
4. $X_k \geq \underline{X}_\tau^*$, bidder k decides not to enter.

We have to show 1 and 4 are dominated by 2 and 3. Let us focus on 1 and 2 first. Under the scenario $X_k \geq \underline{X}^*$, if bidder k decides to enter, one of her rivals has higher conditional expectation towards the item. The incentive constraints is violated:

$$\sum_{n}^{\infty} P(N_\tau = n|M_\tau) \left\{ \begin{array}{c} \mathbb{E}[V_i|X_i,\ X_j, j = 1,2,...,n, j \neq i] \\ -\mathbb{E}[V_j|X_i,\ X_j, j = 1,2,...,n, j \neq i] \end{array} \right\} < 0$$

$$\rightarrow$$

$$\mathbb{E}[V_i|X_i,\ X_j, j = 1,2,...,n, j \neq i] < \mathbb{E}[V_j|X_i,\ X_j, j = 1,2,...,n, j \neq i].$$

The ex-ante payoff expectation for bidder k attending the auction is non-positive. While if he/she chooses not to enter, bidder k receive zero payoffs, which dominate the enter decision. Similarly, when $X_k \geq \underline{X}_\tau^*$, the payoff for bidder who decides to enter is non-negative. However, if the bidder k decides not to enter, she can only receive negative payoffs. Therefore, The best response of the entry decision for any potential bidder is to enter auction τ whenever $X_k \geq \underline{X}_\tau^*$.

Explanation of Curse Relief. The conjuncture is equivalent to show as $\underline{X}$ increases,

$$\frac{d}{d\underline{X}} \left\{ \mathbb{E}[V_i|X_i; x_i \geq X_j \geq \underline{X}_j,\ j \neq i] - \mathbb{E}[V_i|X_i; X_j \geq \underline{X}_j,\ j \neq i] \right\} < 0.$$

Due to the property of the conditional expectation and truncated normal distribution, we learn that

$$\frac{d}{d\underline{X}} \left\{ \mathbb{E}[V_i|X_i; x_i \geq X_j \geq \underline{X}_j,\ j \neq i] - \mathbb{E}[V_i|X_i; X_j \geq \underline{X}_j,\ j \neq i] \right\} \propto \frac{d}{d\underline{X}} R \left\{ \frac{\phi(\underline{X}_j) - \phi(x_i)}{\Phi(x_i) - \Phi(\underline{X}_j)} - \frac{\phi(\underline{X}_j)}{1 - \Phi(\underline{X}_j)},\ j \neq i \right\}$$

Without loss of generality, I assume that $\alpha = \underline{X}_j$ and $\beta = x_i$. Then we can get

$$\frac{\phi(\alpha) - \phi(\beta)}{\Phi(\beta) - \Phi(\alpha)} - \frac{\phi(\alpha)}{1 - \Phi(\alpha)} = \frac{\phi(\alpha)[1 - \Phi(\beta)] - \phi(\beta)[1 - \Phi(\alpha)]}{(1 - \Phi(\alpha))(\Phi(\beta) - \Phi(\alpha))} \equiv A(\alpha)$$

It is not hard to see that

$$\frac{d}{d\alpha}A(\alpha) = \frac{\phi(\alpha)[-\alpha(1-\Phi(\beta)) + \phi(\beta) + (1-\Phi(\alpha))]}{(1-\Phi(\alpha))^2(\Phi(\beta)-\Phi(\alpha))^2},$$

where $\phi'(\alpha) = -\alpha\phi(\alpha)$. As $\alpha \uparrow$, $-\alpha(1-\Phi(\beta)) + \phi(\beta) + (1-\Phi(\alpha)) < 0$, which indicates the alleviation of the curse.

Algorithm for Priority bidder's strategic behavior counterfactual experiment.

- Draw private signal X based on estimated parameter $\hat{\theta}$
- Generate the identity of informed bidder
- Increase high bid by Δ each period
- Identify the set of bidders willing to bid
- Choose one candidate bid as posting price at each period t
  - normal: no restriction
  - active: choose informed first
  - inactive: never choose informed

Appendix B. Graphs and Tables

Table 1: Summary Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
reserv price(1000 RMB)	2,211	1,801.8	2,375.7	36.98	455	2,211.8	20,707.6
$reserve\ ratio$	2,211	0.841	0.135	0.700	0.700	1.000	1.000
$\frac{win\ price}{reserve\ price}$	2,211	1.406	0.302	1.019	1.188	1.537	2.940
# bidder	2,211	8.735	6.526	2	4	11	54
priority bidder	2,211	0.069	0.254	0	0	0	1
win bid(1000 RMB)	2,211	2,423	3,169	52	650	2,971	24,300
bidding freq	2,211	64.897	50.496	5	28	88	284
bid ladder(1000 RMB)	2,262	8.8	13	0.3	2	10	200
ladder ratio	2,211	0.005	0.004	0.0002	0.002	0.007	0.037

Table 2: Reduced Econometric Analysis

	$\frac{win\ price}{reserve\ price}$		(# of bidder)		bid freq	
	(1)	(2)	(3)	(4)	(5)	(6)
num bidder	0.033^{***}	0.032^{***}			3.546^{***}	3.479^{***}
	(0.001)	(0.001)			(0.147)	(0.153)
reserve ratio			-2.269^{**}	-8.900^{***}	18.007^{**}	14.738
			(1.027)	(1.266)	(7.089)	(9.166)
priority bidder	0.047^{***}	0.048^{***}	-1.436^{***}	-1.245^{**}	7.384^{*}	7.480^{**}
	(0.018)	(0.018)	(0.546)	(0.529)	(3.770)	(3.791)
Constant	0.119^{***}	0.123^{***}	10.742^{***}	16.415^{***}	18.274^{***}	16.848
	(0.008)	(0.043)	(0.875)	(1.839)	(6.237)	(13.402)
Observations	2,211	2,211	2,211	2,211	2,211	2,211
R^2	0.493	0.496	0.005	0.079	0.210	0.212
Adjusted R^2	0.493	0.494	0.004	0.076	0.209	0.209
Note:				*p<0.1; **p<0.05; ***p<0.01		

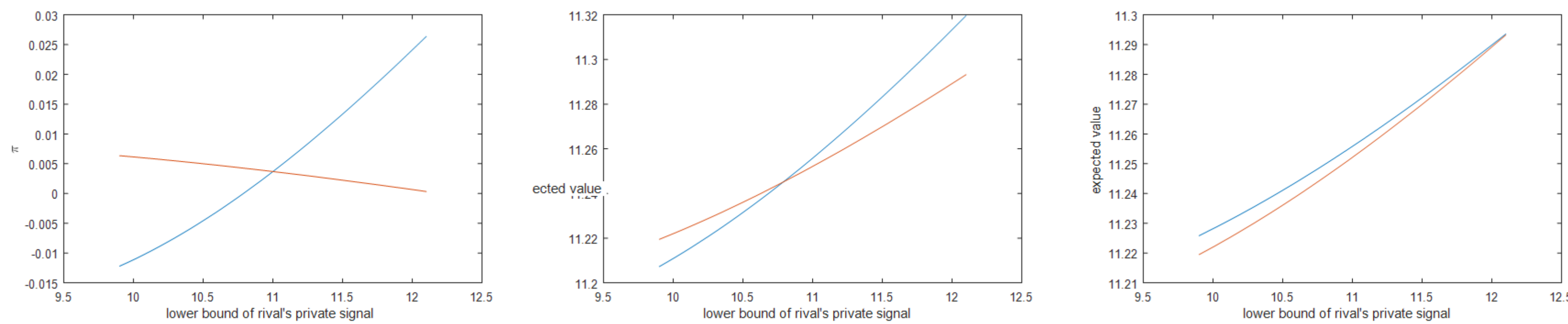
Table 3: Estimation Results

	μ_v	γ	σ_v^2	σ_a^2	σ_ϵ^2
Estimated	0.0080	1.2929	0.1609	0.00135	0.1226
lower	0.00524	1.163	0.1279	0.00129	0.1063
upper	0.0109	1.400	0.1906	0.00144	0.1336

Appendix C Proof for the proposition.2

Since the proof is mainly matrix operation, the detailed steps can be accessed at https://psu.box.com/s/rb

Figure 1: Information Premium and Variations of lower bounds

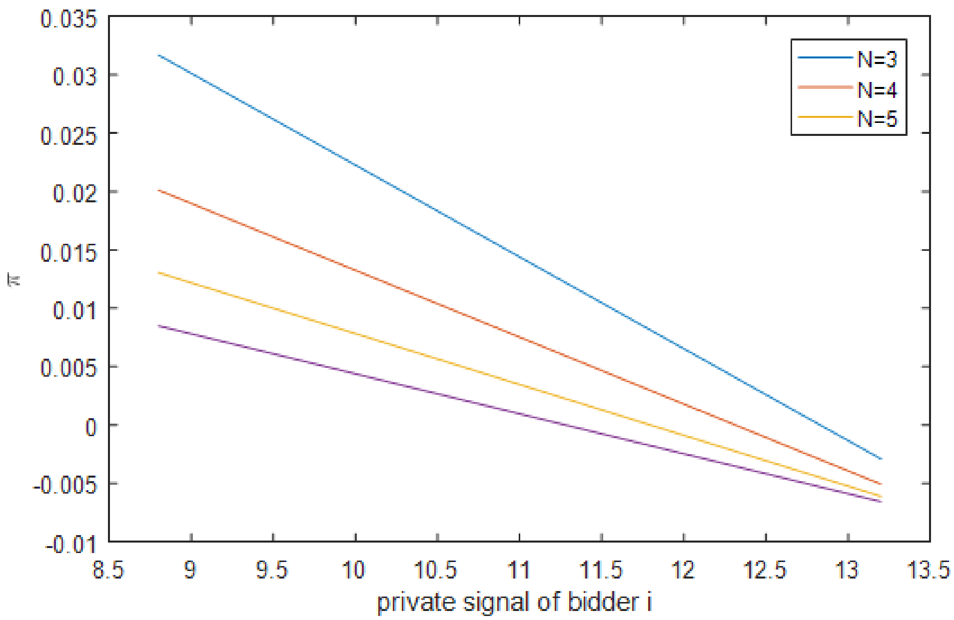


Remark. For all the figures, The settings are $N = 5,\ x_i = 10.95$. The information parameters are $\{\mu_v = 11, r = 1, \sigma_v^2 = 0.8; \sigma_a^2 = 1.2; \sigma_\epsilon^2 = 0.8\}$. The first graph on the left indicates how the changes of information premium respond to the lower bound of rival's private signal. The blue line indicates the case when the informed opponent changes her lower bound. And the red line indicates the case when the uninformed opponent changes the lower bound. The second graph shows how the conditional expected value varies when the informed opponent changes the lower bound. And the third graph shows the situation when the uninformed opponent changes her lower bound.

Table 4: Information Parmeter for Simulation

name	moment	parameter	value
common value v	mean	μ_v	-0.35
	variance	σ_v	0.2
private value a	mean	$\bar{a}$	0
	variance	σ_a^2	0.15
noise	mean ϵ		0
	variance	σ_ϵ^2	0.1
control coefficient r			0

Figure 2: Information Premium and Number of bidders



Remark. The lower bound of the rivals are fixed at $X = 11$. The information parameters are $\{\mu_v = 11, r = 1, \sigma_v^2 = 0.8; \sigma_a^2 = 1.2; \sigma_\epsilon^2 = 0.8\}$.

Figure 3: Information Premium Independent of the Number of Bidders

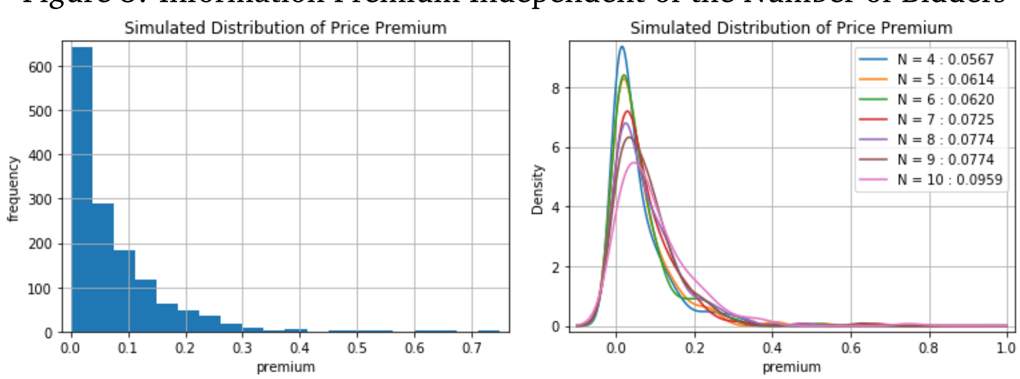
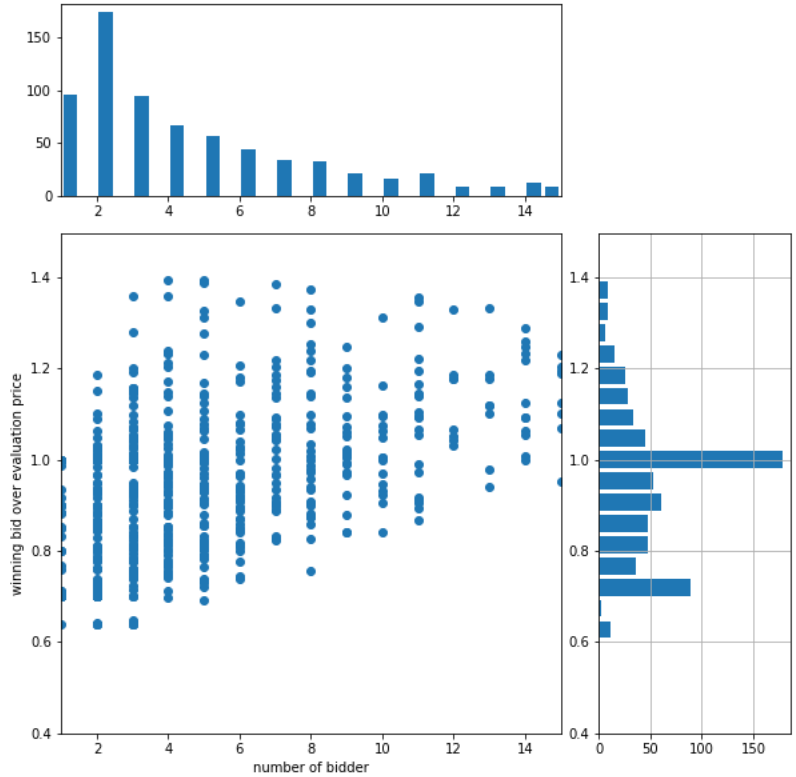


Figure 4: The relation between Winning bid and Number of bidders (First Time Auction)



Remark. The y axis is the ratio of winning bid over reserve price, measuring the total bidding ladders from reservation price.

Figure 5: Bidding Outcomes of the Auction with / without Priority Bidder

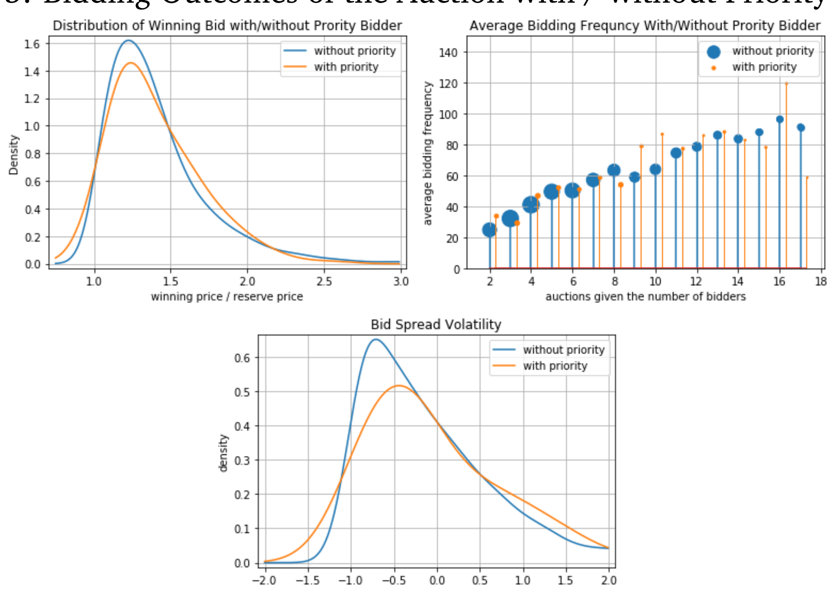
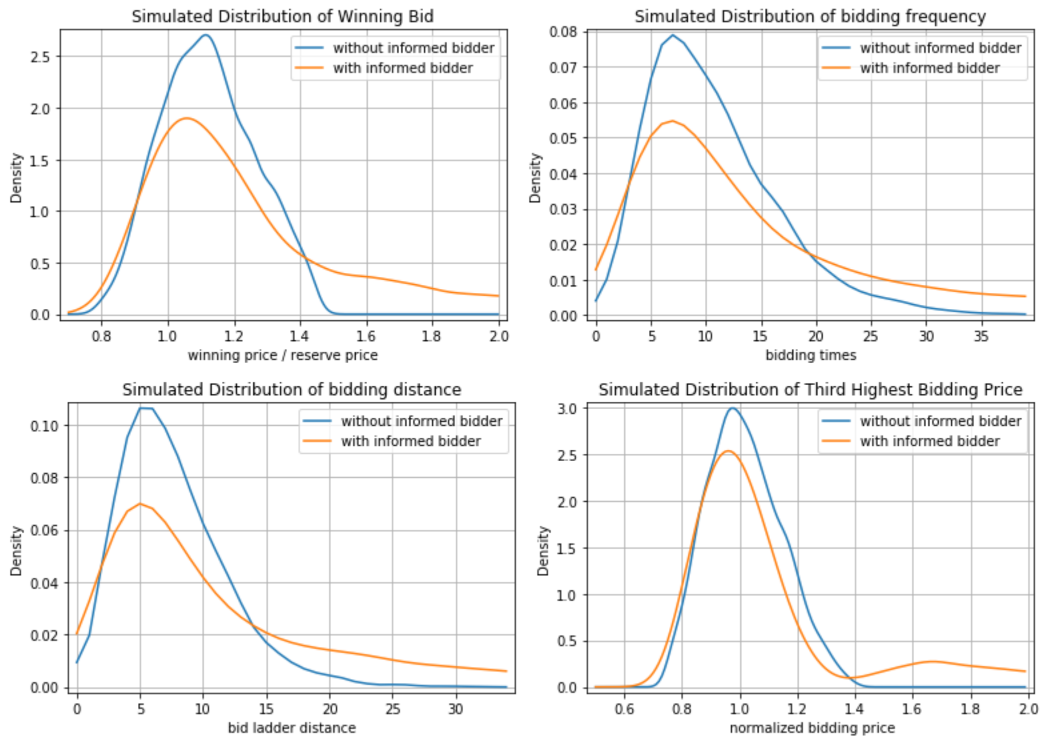


Figure 6: Simulation Results



Remark. Figures from left to right and top to bottom are denoted (a), (b), (c), (d).

Figure 7: Counterfactual analysis of priority bidder's behavior