Information Spillover in Multi-good Adverse Selection*

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Abstract

This paper analyzes the effect of information spillover in a multi-good adverse selection model where a privately informed seller simultaneously trades two different goods with different buyers. In this setting, buyers not only learn the seller's information from past trading outcomes in the market they participate, they may also learn from transactions in the other market, which is referred to as "information spillover". We characterize the equilibria and identify a sufficient negative correlation condition under which the efficiency loss due to adverse selection diminishes. We also discover a novel coordination friction between the seller and buyers that leads to multiple equilibria. Nonetheless, we show that the equilibria can be welfare-ranked by the number of initial no-trade periods, which ranges from zero to some upper bound. Notably, in the most efficient equilibrium, if one good is sold first then the other good will be traded immediately in the next period. When the sufficient negative correlation condition fails, the efficiency loss is the same as in the case without information spillover.

Keywords: adverse selection, multiple goods, information spillover

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1 Introduction

Akerlof's (1970) seminal work on the "lemons" problem establishes that adverse selection can lead to severe market failure, even complete market unraveling. When sellers have private information about the quality of the good in question, sellers of peaches (that is, high quality goods) will likely stay out of the market, as the market price is likely to be not high enough, but this forces the market price to be even lower, as only sellers of lemons (that is, low quality goods) will likely stay in the market. In the extreme case, only the lowest quality goods are brought to the market, and there is a severe market failure. While Akerlof's results are derived in a competitive market setting and hence do not explain the determination of market prices, the subsequent literature confirms his findings using specific models of price formation (mostly non-cooperative bargaining) to study the price dynamics under adverse selection (e.g., Evans (1989), Deneckere and Liang (2006), Hörner and Vieille (2009), and Daley and Green (2012)). There is also a vast literature providing ways sellers could work around the adverse selection problem (e.g., engaging in costly signaling activities as in Spence (1973), Milgrom and Roberts (1986), etc.). In all these papers, the main focus is on sellers of a single good.

Strictly speaking, because typical goods have several attributes, and as such even a single good is a bundle of attributes, the assumption of a single good is a simplification. This paper relaxes this assumption and studies the adverse selection problem in the context of multi-good sellers. The basic insight is that buyers, while still not able to observe the quality of the goods, can make inferences about the quality of a good by observing the trading activities of the other goods. That is, we consider the possibility of cross-market information spillover as a way around the adverse selection problem. To fix ideas, consider a situation with many sellers and buyers, where sellers sell two goods, A and B, whose qualities are the seller's private information, and can be either high or low. Assume that buyers are divided into two groups, A and B, where buyers from group i only demand good i, for $i = A, B$. Assume that sellers are specialized in the sense that if one of the goods is of high quality, the other must be of low quality, and conversely. The following market arrangement works around the adverse selection problem: the sellers segment the market into two separate markets, 1 and 2; the sellers of high quality good A and low quality good B go to market 1 and the other sellers go to market 2; half of buyers A and half of buyers B go to market 1 and the other buyers go to market 2; the price of good A in market 1 is high, and that of good B is low, the opposite is true for market 2. That is, there's no market failure, and this is achieved solely because of cross-market information spillover: buyers who value good A are willing to go to market 1 as they can see that the prices of both goods in both markets and correctly infer that the quality of good A is high in market 1 and low in market 2. In other words, the quality of a certain good in a given market can be signaled by the price in the other market.

As a more concrete example, let sellers be car mechanics, and buyers be divided into one group that values regular service such as repair and maintenance, and another group that values performance service such as "souping-up" the engines. A seller specialized in regular service is probably not particularly good in performance service, and conversely.[1] Also, the prices of services provided by car mechanics are likely to be consistent with the story of the previous paragraph: in equilibrium, a regular car mechanic will provide performance service at a low price to the residual demand of buyers that end up not served by the performance ones; likewise, performance car mechanics will provide regular work at a low price to the residual demand of buyers that end up not served by regular ones. Buyers are then willing to pay a higher price to the service they value, as they correctly infer that the quality of the service must be good, given the underlying specialization of the sellers. This example is one of many instances in which economic agents with multidimensional private information face multiple decisions and trade off the endogenous information contents revealed by different choices, such as final intermediaries trading multiple correlated assets (He, 2009) and manufactures bargaining with both suppliers and retailers (Leider and Lovejoy, 2016). While the above argument intuitively suggests that information correlation plays an important role in determining the trading patterns (e.g., prices and timing), to our limited knowledge, previous studies have not thoroughly examined the relationship between equilibrium outcomes and information correlation in non-cooperative settings.

Motivated by the above examples, we build a game-theoretic model to examine price formation in a multi-good setting where prices serve as signals of product qualities. Formally, we study a dynamic trading game with multidimensional adverse selection. A long-lived seller has two heterogeneous goods for sale. The qualities of the goods, which are the seller's private information, are either low or high and can be correlated. The seller trades sequentially with potential buyers until an agreement is reached, if ever, and delay is costly. In each period, trade of each good takes place in a competitive market as long as that good is still available; buyers in each market make offers and the seller then decides to accept or reject each offer.

To analyze the effect of cross-market information spillover, we assume that buyers can observe all the previous (non-)transactions in *both* markets, and contrast the findings with the benchmark case where buyers only see the histories in their own market (i.e., without information spillover). The key implication of information spillover is that transactions of one of the goods may provide

[1] In particular, it is not atypical to encounter mechanics almost fully specialized in either regular or performance service.

information about the quality of the other good, which depends on both the underlying correlation of the qualities perceived by the buyers and the seller's equilibrium strategies. Since adverse selection dictates that buyers' initial offers are low, sellers with high quality goods are less likely to trade early. For sellers with low quality goods, the incentive to wait, trade one or both goods is mixed: on the one hand, since waiting is costly, rejecting the current low offers signals the quality of the good and may lead to higher offers in the future; on the other hand, for certain correlation structures trading one of the goods at a low price may make buyers revise their beliefs about the quality of the remaining good, resulting in a favorable offer in the future. The former incentive to wait, which is standard in dynamic adverse selection, causes delay and inefficiency; the latter incentive to trade sequentially, which is a key ingredient of our model, may improve efficiency.

We fully characterize the equilibria of this game and identify a sufficient negative correlation condition on the qualities under which inefficiency is mitigated (Theorem 1). The reason is that an early trade in one market serves as endogenous good news about the quality of the other good, which leads to faster trades and thus a higher total welfare than the case without information spillover. However, multiple equilibria exist in which there is a "trade impasse" at the early stage of the game (Theorem 2), which leads to inefficiency. We discover a novel coordination friction between the seller and buyers in the multiple-good setting, which is the key to the inefficiency and equilibrium multiplicity. During the trade impasse, buyers hold pessimistic off-equilibrium-path beliefs about the quality of the remaining good if one good is traded first. Consequently, without being rewarded by the cross-market information spillover from trading one good first, the seller would rather postpone trade of both goods until buyers regain confidence in the future. In addition, the equilibria are ranked in terms of the social surplus and the length of trade impasse: the longer is the trade impasse, the lower is the social surplus. Interestingly, in any equilibrium, the efficiency loss due to trade impasse is dominated by the efficiency gain from cross-market information spillover. To put it differently, in any equilibrium, there is net efficiency improvement comparing to the case without cross-market observability (Proposition 2). We also establish comparative statics regarding the impact of information structure (negative correlation) on the maximal equilibrium welfare (Proposition 3).

To the contrary, when the sufficient negative correlation condition fails, sequential trade may reveal negative information to buyers, which is reminiscent of the ratchet effect in repeated contracting with limited commitment.[2] Thus, the sellers' incentive to avoid such a ratchet effect nullifies cross-market information spillover in equilibrium; as a result, buyers in one market

[2] See for example Laffont and Tirole (1993) and Schmidt (1993).

update their beliefs as if they do not observe transactions in the other market, and sellers obtain the same surplus as in the case without information spillover (Theorem 3). Notably, even though delay occurs, the seller is never worse off with cross-market information spillover, which is in sharp contrast with the main finding in the existing literature that the ratchet effect hurts the seller. This distinction mainly stems from the difference in the timing of the games: standard ratchet effect models assume that trades occur sequentially and future buyers learn from past trading outcomes, whereas in our setting multiple trades can occur at once. Moreover, in our setting both buyers and the seller would like to trade early: buyers are short-lived; the seller discounts the future and takes into account the ratchet effect. Therefore, in equilibrium multiple trades happen simultaneously, which counters the loss due to the ratchet effect.

In the baseline model discussed above, the seller has all the bargaining power as the demand side is perfectly competitive. In order to examine the robustness of our results, in the Online Appendix, we study an alternative setting where buyers have the bargaining power. Specifically, in each period and for each good, one short-lived buyer arrives and makes an offer to the seller. We characterize the equilibria of this game and show that similar to the equilibria in the baseline model, adverse selection is mitigated by information spillover if and only if the same condition of sufficient negative correlation holds.

Our analysis relates to three aspects of dynamic adverse selection literature: market inefficiency, the arrival of news, and transparency.

Market inefficiency. Standard adverse selection models à la Akerlof (1970) consider almost exclusively the case in which an informed seller trades one good with uninformed buyers. In a dynamic environment, adverse selection leads to market inefficiency, which typically takes the form of delay and, therefore, a central question is how quickly gains from trade are realized. See for example Evans (1989), Vincent (1989, 1990), Janssen and Roy (2002), Deneckere and Liang (2006), Hörner and Vieille (2009), Moreno and Wooders (2002, 2010, 2015), Fuchs and Skrzypacz (2013, 2015), Kim (2015a) and Gerardi and Maestri (2015) for contributions. Our closest precursor is Hörner and Vieille (2009). They study an interdependent-value bargaining model with a single long-run seller and a sequence of short-run buyers. They find that inefficiencies take different forms in the two opposing information structures. While highlighting market inefficiencies caused by information asymmetry, these models have largely overlooked the possibility that having multiple goods for sale could mitigate such inefficiencies.[3]

Arrival of news. There is a strand of literature that considers markets with adverse selection

[3] Gerardi and Maestri (2015) consider adverse selection with multiple goods. However, in their model the qualities of the goods are the same.

where seller's private information is gradually revealed to the uninformed buyers by the arrival of exogenous news. For example, Daley and Green (2012) show that exogenous news with Brownian noises leads to a unique equilibrium with "no-trade region", in which there are periods in which trade occurs with probability zero and the quality of the assets drifts up and down. In the version of Poisson arrival of news, the no-trade region exists if no news is good news.[4]

Different from the above papers, we study information spillover in a setting where a seller sells two lemons. Thus, information is endogenously conveyed by the trading activities in both markets. Importantly, whether the information from the other market is good, bad or neutral about the quality of the good depends on the initial correlation of the two qualities, as well as the seller's equilibrium behavior. Intuitively, if the trade of one good is informative about the quality of the other good, negative (positive) correlation corresponds to the case where no news is bad (good) news. However, this argument is incomplete as the seller's strategic behavior also plays an important role in determining the informativeness of the news. The sufficient negative correlation condition that we identify gives a tight characterization of when information spillover from the trade of one good can improve the overall surplus. Furthermore, we also present a bargaining impasse in the multi-good setting, since the a coordination failure between the seller and buyers makes news contain no information; consequently, the seller is waiting for the news to become informative in the future.

Transparency. There are also several papers that study the impact of information about past rejected offers (transparency) on the efficiency of trade in dynamic markets with asymmetric information. For instance, Hörner and Vieille (2009) show that the observability of past price offers unambiguously reduces market efficiency.[5] Fuchs, Öry, and Skrzypacz (2016) reach similar results in a finite-horizon model with intra-temporal competition. Kim (2015b) demonstrates that market efficiency is not monotone in the amount of information available to buyers in a model with search friction.

In a closely related paper, Asriyan, Fuchs, and Green (2017) consider a two-period multi-good model where there are multiple seller-buyer pairs. They assume that each seller sells one good and the quality of different goods are positively correlated. They show that multiple equilibria exist, as in equilibrium one seller's trading decision is informative about the values of other goods. In addition, they parametrize the degree of observability of trading activities in the other market and show that total welfare is higher when markets are fully transparent than when the market

[4]See also Kremer and Skrzypacz (2007), Zryumov (2015), Kaya and Kim (2018), Lauermann and Wolinsky (2013), and Zhu (2012) for exogenous arrivals of news.

[5]See Kaya and Liu (2015) for the study of price transparency in private-value settings.

is fully opaque. The main difference between this paper and Asriyan, Fuchs and Green (2017) is that news from other markets is controlled by the same seller instead of different sellers. In other words, in our setting the seller endogenizes the informational externalities from the trading activities of all markets. As a result, the coordination problem among different sellers in Asriyan, Fuchs and Green (2017), which leads to multiple equilibria in their model, is not present in our setting. This distinct feature leads to qualitatively different conclusions comparing to those in Asriyan, Fuchs and Green (2017). Most prominently, we find that there is a unique equilibrium when a single seller trades two goods with positively correlated qualities.

The rest of the paper is organized as follows. In Section 2 we set up the baseline model. Section 3 contains the formal analysis: in Section 3.1, we present the equilibrium outcome of the one-good case as a benchmark; in Section 3.2, we give a complete characterization of the equilibria. In Section 4, we compare the equilibrium welfare in different information settings in order to quantify the impact of information spillover and study the impact of information structure (negative correlation) on the total welfare. Finally, Section 5 concludes with a discussion of several extensions pursued in the Online Appendix. Proofs of the results are relegated to Appendices A and B.

2 Model

A long-run seller has one unit for each good, 1 and 2, for sale. The quality of each good is either high (H) or low (L), which is the seller's private information. There are four types of sellers: HH, LL, HL and LH. For seller type xy, the quality of good 1 is $x \in \{H, L\}$ and the quality of good 2 is $y \in \{H, L\}$. There are two kinds of buyers: buyers 1 and 2. Buyer i ($i = 1, 2$) only buys good i. For each good i, the seller's cost and buyer i's valuation are interdependent, indicated by the following table (Table 1). The seller's total cost is the sum of the costs for both goods. We assume that it is common knowledge that there are gains from trade: $v_L > c_L$ and $v_H > c_H$. We focus on the case that $v_L < c_H$, since otherwise there is not adverse selection.

i's quality	seller's value	buyer i's value
H	c_H	v_H
L	c_L	v_L

Table 1: Seller's and buyers' valuations for good $i = 1, 2$.

Time is discrete and infinite, $t \in \{1, 2 \ldots\}$. The seller bargains sequentially with two sequences of potential buyers until agreements are reached, if ever, and delay is costly: the seller's discount factor is $\delta \in (0,1)$. In each period $t \geq 1$, if both goods are still left untraded, then in each market, two or more buyers arrive and make take-it-or-leave-it offers simultaneously. Note that in this setting the bargaining power is on the seller side even though the buyers are the ones making take-or-leave-it offers. Denote the offer of good $i = 1, 2$ as $p_{i,t}$.[6] After observing two offers $(p_{1,t}, p_{2,t})$, the seller decides whether to accept each of the two offers or not. There are four choices for the seller: rr (rejecting both offers), aa (accepting both offers), ar (accepting $p_{1,t}$ and rejecting $p_{2,t}$) and ra (rejecting $p_{1,t}$ and accepting $p_{2,t}$). If both offers are accepted (aa), the game is over. If both offers are rejected (rr), the seller stays with two goods and waits for another two offers in the next period. If one of the offers is accepted (ar or ra), the seller is left with the other good; and in the next period, only buyers for the remaining good make offers, and the seller decides to accept or reject. If the offer is accepted, the game is over. Otherwise, the game repeats with the seller selling the remaining good in the next period. We assume that buyers can observe (non-)transactions for both goods.

Let $\mu_k \in (0,1)$ denote buyers' prior belief that the seller is of type $k \in \{HH, HL, LH, LL\}$. Thus, the prior belief of good 1 to be high quality is $\mu_{HL} + \mu_{HH}$ and the prior belief of good 2 to be high quality is $\mu_{LH} + \mu_{HH}$. Define μ^* as $\mu^* v_H + (1 - \mu^*) v_L = c_H$. That is, if the probability of high quality good $i = 1, 2$ is μ^*, then buyer i's expected value is equal to the reservation value of high quality good i for the seller.

3 Analysis

3.1 The One-Good Benchmark

We first consider a benchmark model where the seller sells one good. This benchmark setting can also be viewed as the two-good model without cross-market information spillover, that is, buyers in each market do not know what happens in the market for the other good, and thus we can analyze the trade of each good separately in the one-good benchmark model. Assume the quality of the good is either high (H) or low (L), which is the seller's private information. The initial belief of H is $\mu_1 < \mu^*$. In each period, two or more buyers make simultaneous offers to the seller, the seller then decides whether to reject (r) or accept (a). The seller discounts payoffs according to a discount factor $\delta \in (0,1)$. Define μ_t as the probability of high type in period t. Assume that $\mu_1 < \mu^*$, that is, adverse selection is so severe that no buyer will offer a

[6] We also assume that in each market buyers can use a public randomization device to make random offers.

price that attracts all sellers initially.

The following lemma (Lemma 1) shows that in the one-good benchmark model, the one-dimensional skimming property holds. That is, H type seller is more willing to delay trade for a high future price than L type seller, and L type seller has an incentive to mimic H type seller.

Lemma 1. ***(One-dimensional skimming property)****: In any period $t \geq 1$, if L type weakly prefers rejecting to accepting the offer, then H type strictly prefers rejecting to accepting the offer.*

The above skimming property has the following four implications. First, if $\mu_t > \mu^*$, then trade happens immediately in period t.[7] The proof contains two arguments: (i) if H seller mixes between accepting and rejecting, then by Lemma 1, L seller accepts for sure, and thus it is a profitable deviation for the buyer to increase the current offer to attract H type to accept the new offer; (ii) if H seller rejects the offer for sure, then in the period before H seller decides to accept (since H cannot reject forever), L seller also rejects the offer and thus there is no belief updating, however, it is a profitable deviation for the buyer to make an offer slightly lower than the buyer's expected valuation, in order for both H and L sellers to accept immediately, instead of waiting for a period and getting an offer that is equal to the buyer's expected valuation.

Second, if $\mu_1 < \mu^*$, H seller rejects the offer for sure in period 1, due to the assumption of severe adverse selection. By Lemma 1, if H seller accepts with positive probability in period 1, then low type seller accepts for sure in period 1. Therefore, the price in period 1, which is the expected valuation of the good for buyers, is less than high seller's reservation value c_H, a contradiction to the assumption that H seller accepts with positive probability in period 1. Consequently, in period 1, the only possible serious offer is v_L, since only L seller accepts the offer and buyers get zero profit.

Third, $\mu_2 = \mu^*$. If $\mu_2 > \mu^*$, a winning offer is made by the buyers and the game is over. However, L seller would deviate to reject the offer in period 1 to get a higher offer in period 2, a contradiction. If $\mu_2 < \mu^*$, L seller would deviate to accepting the offer v_L for sure in period 1, instead of waiting for one period to get an offer at most v_L in period 2, a contradiction. A corollary is that L seller randomizes in period 1.

Finally, in period $t \geq 2$, buyers mix between two offers: a winning offer c_H and a losing offer, in a way that L seller is willing to randomize between accepting and rejecting the offer v_L in period 1.

[7] This result is first proved by Swinkels (1999).

The next result (Proposition 1) characterizes the equilibrium outcome in the one-good benchmark.

Proposition 1. *If $\mu_1 < \mu^*$ and $\delta > \frac{v_L - c_L}{c_H - c_L}$, there is a unique perfect Bayesian equilibrium outcome as follows:*

In period 1, the offer is v_L, L seller rejects the offer with probability $\frac{1-\mu^*}{\mu^*}\frac{\mu_1}{1-\mu_1} \in (0,1)$ and H seller rejects the offer.

In period $t \geq 2$, the belief is updated to μ^*, and there is a winning offer c_H with probability λ and a losing offer with probability $1-\lambda$, where λ satisfies $v_L - c_L = \delta(\lambda c_H + (1-\lambda)v_L - c_L)$.

Note that in this one-good benchmark setting, there is efficiency loss due to adverse selection: there is only partial trading for low type seller in period 1, and there is real delay of trading in the long run: the discounted probability of trade is $\frac{v_L - c_L}{c_H - c_L} < 1$. Moreover, since low type seller gets a payoff $v_L - c_L$ and high type seller gets zero surplus, then the expected discounted gain from trade is exactly the seller's expected surplus: $(1-\mu)(v_L - c_L)$, which is less than the efficient gain from trade: $\mu(v_H - c_H) + (1-\mu)(v_L - c_L)$.

3.2 The Two-Good Model

Now we analyze the two-good model. Recall that in any period, when good i ($i = 1, 2$) is available, two or more buyers make simultaneous offers to the seller for the good i. Define μ_t^i as the probability of high quality good i in period t, and $\mu_{t+1}^i(h)$ as the probability of high quality for the remaining good i in period $t+1$, if the choice of the seller in period t is $h \in \{rr, ar, ra\}$ and the outcome in any period before t is always rr. We first establish the following two-dimensional skimming properties (Lemma 2) that are essential for the subsequent analysis.

Lemma 2. ***(Two-dimensional skimming properties):*** *In any period $t \geq 1$,*

1. *If xL weakly prefers yr to za, then xH strictly prefers yr to za, where $x \in \{H, L\}$, $y \in \{a, r\}$, $z \in \{a, r\}$.*

2. *If xL weakly prefers ar to aa, then yH strictly prefers ar to aa, where $x, y \in \{H, L\}$.*

3. *If xH weakly prefers ar to rr and $\mu_{t+1}^2(ar) > \mu^*$, where $x \in \{H, L\}$, then xL weakly prefers ar to rr. Suppose, in addition, xL rejects the offer for good 2 with positive probability in period $t+1$, then xL strictly prefers ar to rr in period t.*

4. *If HL weakly prefers ar to ra, then LH strictly prefers ar to ra. Similarly, if LH weakly prefers ra to ar, then HL strictly prefers ra to ar.*

5. *If HL chooses ar with positive probability, then LH strictly prefers ar to rr. Similarly, if LH chooses ra with positive probability, then HL strictly prefers ra to rr.*

The first two properties are intuitive and implied by the one-dimensional skimming property. That is, the seller with high quality good is more willing to wait for future offers, relative to low quality seller, when the quality of the other good is the same (skimming property 1) or the other good is traded immediately (skimming property 2). Skimming property 3 reveals a cross-market spillover effect: the seller's choice of accepting or rejecting good i is influenced by the quality of the other good. To be specific, LL is more willing to accept offer 1 than LH, conditioning on the fact that the remaining good 2 does not suffer from severe adverse selection. Skimming properties 4 and 5 are two local conditions between two types of sellers that cannot be ranked by product quality. Skimming property 4 shows that between ar and ra, LH favors ar and HL favors ra. Skimming property 5 asserts that for good i, one-dimensional skimming property holds, given that the offer of the other good is rejected.

Next we introduce an equilibrium refinement that will be adopted throughout the characterization. Note that comparing to the one-good benchmark, there could be many more off-equilibrium paths in the two-good model as the seller has more choices, which complicates the analysis. In order to eliminate unreasonable off-equilibrium path beliefs, we consider the following refinement of weak perfect Bayesian equilibrium that is in spirit of the D1 criterion introduced by Cho and Kreps (1987).

Definition 1. *(Refinement D1)*

*Fix two seller types $i, j \in \{HH, HL, LH, LL\}$. Suppose action $A \in \{ar, ra, rr\}$ is off equilibrium path in period t, if type i weakly prefers action A to the equilibrium action for type i in period t and type $j \neq i$ strictly prefers A to the the equilibrium action for type i in period t, regardless of any continuation play from period $t+1$ on, then given action A in period t, **Refinement D1** requires that the updated belief in period $t+1$ is to attach probability zero to type i.*

We begin the analysis by looking at situations in which adverse selection is not severe for both goods: the probability of high quality good i ($i = 1, 2$) is larger than μ^*.[8] Define $V(\mu)$ as the expected buyer's valuation of the good with probability μ on high quality: $V(\mu) \equiv \mu v_H + (1-\mu) v_L$. The following lemma (Lemma 3) implies that trade happens immediately for both goods if the beliefs of both goods are larger than μ^*.

[8]Note that in a static model, good i ($i = 1, 2$) is traded if the probability of high quality good i are larger than μ^*.

Lemma 3. *Under refinement D1, if in equilibrium $\mu_t^1 > \mu^*$ and $\mu_t^2 > \mu^*$, then trade happens immediately for both goods in period t. Moreover, the offer of good i in period t is $V(\mu_t^i)$.*

Although Lemma 3 is parallel to the existing result in the one-good model,[9] the proof is rather different. Specifically, the proof of the one-good model does not extend to the two-good setting, for the following two reasons. First, seller types cannot be ranked by a linear order, so the one-dimensional skimming property does not directly generalize. Second, even if all seller types of good i reject offer i, the belief of good i in the next period may change due to the fact that the seller's choice for the other good also has an impact on the belief updating of good i, and thus it may not be a profitable deviation for buyer i to make a new offer, which is a key step in the one-good case.

Instead, we prove Lemma 3 by repeatedly applying the two-dimensional skimming properties in Lemma 2, with the following four steps. First, we show that if the seller with high quality good i accepts offer i with positive probability in some period $t+k$ for the first time (HH cannot reject both offers forever), then the seller with high quality good j ($j \neq i$) also accepts offer j with positive probability in period $t+k$. Second, we show that in period $t+k$, the seller with low quality good $i = 1, 2$ accepts offer i with probability one. Third, we prove that in period $t+k$, all four seller types accept both offers. Finally, we argue by contradiction that $k = 0$. Assume the contrary that $k \geq 1$. In period $t+k-1$, if buyer 1 makes an offer that is slightly less than buyer 1's expected valuation $V(\mu_{t+k-1}^1)$, and then all four types deviates to choosing ar, for two reasons: for good 1, all four types get a higher payoff by accepting the new offer instead of waiting for one period and getting $V(\mu_{t+k-1}^1)$; for good 2, refinement D1 implies that the remaining good 2 has high quality in period $t+k$, so it is beneficial for all four seller types to choose ar in period $t+k-1$. This new offer brings buyer 1 a positive profit in period $t+k-1$, a contradiction to buyer 1's zero profit condition in the equilibrium.

We now study situations in which adverse selection is severe for both goods in the sense that for each good, the probability of high quality is less than the cutoff level μ^*.

Assumption 1 (Severe adverse selection): $\mu_{HL} + \mu_{HH} < \mu^*$ and $\mu_{LH} + \mu_{HH} < \mu^*$.

Define the following belief set $\mathcal{B} \equiv \{(\mu^1, \mu^2) : \mu^1 < \mu^*, \mu^2 \leq \mu^*\} \cup \{(\mu^1, \mu^2) : \mu^1 \leq \mu^*, \mu^2 < \mu^*\}$.

Lemma 4. *Under refinement D1 and $\delta > \frac{v_L - c_L}{v_H - c_L}$, if the belief in period 1 satisfies Assumption 1 or the belief in period $t \geq 2$ satisfies $(\mu_t^1, \mu_t^2) \in \mathcal{B}$, then in period t, the seller of high quality good i rejects the offer of good i, which equals v_L.*

[9] See Proposition 1 or Swinkels (1999).

Lemma 4 is a preliminary result for the seller's equilibrium behavior if the beliefs for both goods are no larger than μ^* and at least the belief of one good is strictly less than μ^*: the seller with a high quality good always rejects the offer. In the one-good model, a similar result follows directly from the one-dimensional skimming property; however, in the two-good model, there are two difficulties to prove such a result. Specifically, the seller with high quality good i may have extra incentives to accept the offer for good i due to the possibility that the seller with low quality good i may not be willing to accept the offer for good i and the fact that the seller has another good for sale. For example, consider seller HL. Since the seller with low quality good 1 may reject offer 1, then the expected valuation of good 1 may be higher than c_H, an thus HL may get positive profit from good 1 by accepting offer 1. Moreover, even if HL may get negative profit from accepting offer 1 due to severe adverse selection, HL would get a high compensation from good 2. In all, HL may accept the offer for good 1. Nevertheless, we get around the difficulties by applying the skimming properties in Lemma 2. We first show that HH chooses rr for sure. Then, the incentive of HL to mimic HH dominates cross-good compensation described above, which implies that rr dominates ar for HL. Furthermore, we prove that HL would rather choose ar instead of aa to enjoy a higher payoff in the future. Therefore, we reach the conclusion that both HH and HL reject the offer of good 1.

Recall that the main purpose of this paper is to investigate how cross-market information spillover affects equilibrium behavior and welfare. To this end, we introduce the following condition of sufficient negative correlation (Assumption 2).

Assumption 2 (Sufficient negative correlation): $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} < 1$.

Assumption 2 implies that μ_{LL} and μ_{HH} are low enough. Therefore, this condition is effectively that the correlation is negative and strong enough. It is easy to show that Assumption 2 does not hold if the qualities of two goods are independent.[10] To better understand this condition, we can consider the following symmetric distribution as a special case, in which (i) the unconditional probability that either good has a high quality is one half, and (ii) given the quality of one good being high (low), the probability that the other good has a high (low) quality is $\rho \in (0,1)$. In this case, Assumption 2 implies that ρ is low enough: $\rho < \frac{2(2\mu^*-1)(1-\mu^*)}{\mu^*} < \frac{1}{2}$. Therefore, Assumption 2 is stronger than any negative correlation ($\rho < \frac{1}{2}$).

Assumption 2 is closely related to the belief updating formulas. To see this, define p_{ra} and p_{ar} as the probability of choosing ra and ar by LL in period 1, where $p_{ra} + p_{ar} = 1$. Let p_{HL}

[10]With independent qualities, $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} = \frac{\mu^1\mu^2}{2\mu^*-1} + \frac{(1-\mu^1)(1-\mu^2)}{1-\mu^*} > 1$ if $\mu^1 < \mu^*$, $\mu^2 < \mu^*$ (Assumption 1) and $\mu^* > \frac{1}{2}$.

and $1-p_{HL}$ be the probabilities of choosing rr and ra by HL in period 1, respectively. Let p_{LH} and $1-p_{LH}$ be the probabilities of choosing rr and ar by LH, in period 1 respectively. If the updated belief in period 2 satisfies $\mu_2^1(rr) = \mu_2^2(rr) = \mu^*$, then

$$\frac{\mu_{HH} + \mu_{HL}p_{HL}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} = \frac{\mu_{HH} + \mu_{LH}p_{LH}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} = \mu^*.$$

Note that the above expression holds only if $\mu^* > \frac{1}{2}$. Moreover, in order to make LL indifferent between ar and ra in period 1, we need $\mu_2^1(ra) = \mu_2^2(ar) = \hat{\mu}$, i.e.,

$$\frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + \mu_{LL}p_{ra}} = \frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + \mu_{LL}p_{ar}} = \hat{\mu}.$$

It then follows that $\hat{\mu} = 1 - \frac{(2\mu^*-1)\mu_{LL}}{2\mu^*-1-\mu_{HH}}$. Therefore, the condition $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} < 1$ in Assumption 2 is equivalent to $\hat{\mu} > \mu^*$.

From the above intuitive explanation, we can see that Assumption 2 mainly imposes restrictions on μ_{LL}. A higher μ^* requires a lower μ_{LL} to satisfy $\hat{\mu} > \mu^*$ (Assumption 2), which explains why the weight of μ_{LL} in Assumption 2 is increasing in μ^*. To the contrary, a lower μ_{HH} is related to severe adverse selection (Assumption 1). In particular, in the extreme case where $\mu_{LL} = 0$, Assumption 2 is implied by Assumption 1.Note that a lower μ^* requires a lower μ_{HH} to satisfy $\mu_{HH} + \mu_{LH} < \mu^*$ and $\mu_{HH} + \mu_{HL} < \mu^*$ (Assumption 1), which explains why the weight of μ_{HH} in Assumption 2 is decreasing in μ^*.

3.2.1 When the sufficient negative correlation condition holds

When Assumption 2 holds, we shall identify two classes of equilibria, depending on whether LL (type seller) rejects both offers with positive probability in period 1. We first characterize all equilibria in which LL does not choose rr in period 1. Define the following outcomes as the *no delay equilibrium*:

- In period 1, the offers of both goods are v_L. HH rejects both offers; HL mixes between rr and ra; LH mixes between rr and ar; LL mixes between ar and ra.

- In period 2, if neither good is sold, $\mu_2^1(rr) = \mu_2^2(rr) = \mu^*$, and the offer of good $i = 1, 2$ is c_H (which is accepted) with probability λ^* and a losing offer with probability $1-\lambda^*$, where $\lambda^* \in (0,1)$ satisfies $v_L - c_L + \delta(V(\hat{\mu}) - c_H) = \delta(\lambda^* c_H + (1-\lambda^*)v_L - c_L)$. If only good i remains unsold, then the updated beliefs satisfy $\mu_2^1(ra) = \mu_2^2(ar) = \hat{\mu} > \mu^*$, and the offer in period 2 is $V(\hat{\mu}) > c_H$, which is accepted for sure.

- In period $t \geq 3$, $\mu_t^1(ra) = \mu_t^2(ar) = \mu_t^1(rr) = \mu_t^2(rr) = \mu^*$. The offer of each good $i = 1, 2$ is c_H (which is accepted) with probability λ and a losing offer with probability $1 - \lambda$, where $\lambda \in (0, 1)$ satisfies $v_L - c_L = \delta(\lambda c_H + (1 - \lambda)v_L - c_L)$.

Theorem 1. *Suppose Assumptions 1 and 2 hold. If the discount factor satisfies*

$$\delta > \max\left\{ \left(\frac{v_L - c_L + \delta(v_H - c_H)}{c_H - c_L} \right)^{\frac{1}{2}}, \frac{v_L - c_L + \delta(V(\hat{\mu}) - c_H)}{v_L - c_L + \delta(v_H - c_H)} \right\},$$

and LL does not choose rr in period 1, then any weak perfect Bayesian equilibrium that satisfies refinement D1 is characterized by the no delay equilibrium.[11]

The key observation of the no delay equilibrium is that cross-market information spillover improves efficiency. There are two effects of information spillover. First, adverse selection is mitigated through a "direct" channel: if only one of the two offers is rejected in period 1, then trade happens immediately for the remaining good in the next period. This follows because Assumption 2 implies that if one good has low quality, it is more likely that the other good has high quality. Since only low quality good can be traded in period 1 due to severe adverse selection, it is likely that the quality of the unsold good is high, and thus trade happens immediately. Second, adverse selection is mitigated though an "indirect" channel: if both goods are left unsold in period 2, then trade happens with higher probability for both goods than in the case without information spillover ($\lambda^* > \lambda$).

Another feature of the no delay equilibrium is that the seller with low quality good i does not necessarily mimic the seller with high quality good i. To be specific, although LH and HL mimics HH by rejecting both offers in period 1, LL instead mimics HL or LH by choosing ra or ar in period 1. The intuition is that LL benefits more from taking advantage of the information spillover through "partial agreement" (i.e., choosing ar or ra) in order to enjoy a high continuation surplus from the unsold good. Formally, define V_k as the continuation payoff of type $k = HH, HL, LH, LL$ in period 1. We can show that LH (HL) is indifferent between ar (ra) and rr in period 1:

$$V_{LH} = V_{HL} = v_L - c_L + \delta(V(\hat{\mu}) - c_H) = \delta(\lambda^* c_H + (1 - \lambda^*)v_L - c_L).$$

Consequently, LL strictly prefers ar (ra) to rr in period 1:

$$V_{LL} = v_L - c_L + \delta(V(\hat{\mu}) - c_L) > 2\delta(\lambda^* c_H + (1 - \lambda^*)v_L - c_L).$$

[11] $\mu^* > \frac{1}{2}$ guarantees that $(\frac{v_L - c_L + \delta(v_H - c_H)}{c_H - c_L})^{\frac{1}{2}} < 1$.

Next, we characterize all equilibria in which LL chooses rr with positive probability in period 1. Let $n+1$ ($n \geq 1$) be the first period in which LL does not choose rr.[12] Define the following outcomes as *delay equilibrium n*: for any n that satisfies $1 \leq n \leq N$, where $N = \min\{n: \ v_L - c_L \geq \delta^n(v_L - c_L + \delta(v_H - c_H))\}$,

- in period 1, the offer of good $i = 1, 2$ is v_L. HH rejects both offers; HL mixes between rr and ra; LH mixes between rr and ar; LL mixes among rr, ar and ra;
- in period $2 \leq t \leq n$, if only good i remains unsold, then $\mu_t^1(ra) = \mu_t^2(ar) = \mu^*$, and buyers mix between an offer c_H (which is accepted) and a losing offer;[13]
- in period $2 \leq t \leq n$, if both goods remain unsold, then $(\mu_t^1(rr), \mu_t^2(rr)) \in \mathcal{B}$, and all four types of sellers choose rr;
- from period $n+1$ onward, the equilibrium is described by the *no delay equilibrium* with an updated belief in period $n+1$ that satisfies Assumption 2.

Theorem 2. *Under the same assumptions as in Theorem 1, if LL chooses rr with positive probability in period 1, then any weak perfect Bayesian equilibrium that satisfies refinement $D1$ is characterized by delay equilibrium n, for $1 \leq n \leq N$.*

Delay equilibrium n features an initial trade impasse for n periods (except in period 1), in which all sellers reject both offers. Consider a particular period t ($2 \leq t \leq n$). If ar is on the equilibrium path, then $\mu_{t+1}^2(ar) > \mu^*$.[14] Consequently, LH chooses ar with positive probability and by skimming property 3 in Lemma 2, LL strictly prefers ar to rr, a contradiction to the definition of n. Similarly, ra is also off the equilibrium path. Finally, we show that if ar (ra) is dominated by rr for LH (HL), then aa is also dominated by rr for LL, and thus aa is also off the equilibrium path.

This trade impasse is a self-fulfilling market failure that may prevent information spillover from mitigating adverse selection. On the one hand, there is an extra benefit for LH to choose rr in period t ($1 \leq t \leq n$), since LH gets $v_L - c_L + \delta(V(\mu_{n+2}^2(ar) - c_H)$ in period $n+1$ due to the information spillover effect in period $n+1$. On the other hand, LH gets only $v_L - c_L$ by choosing ar in period t since $\mu_{t+1}^2(ar) = \mu^*$ under refinement D1. Hence, if the length of the trade impasse n satisfies $v_L - c_L \leq \delta^n(v_L - c_L + \delta(V(\mu_{n+2}^2(ar) - c_H))$, then LH is

[12] In Lemma A.5, we show that $n < +\infty$.

[13] Notice that if $n = 1$, the set of period t described above is empty.

[14] Otherwise LH would rather choose ar for sure in period 1, so only HL and HH are left unsold in period $n+1$, a contradiction to Lemma A.2 in Appendix A.

willing to choose rr with positive probability in period t. Similarly, HL is also willing to choose rr with positive probability in period t. This discussion also indicates that the existence of multiple equilibria and trade impasse relies on the fact that LH is waiting for future benefits from information spillover, which is an implication of the assumption that the demand side is perfectly competitive. In the Online Appendix, we show that if there is a single buyer for each good in each period, instead of competitive buyers, then LH loses the incentive of waiting for higher future payoff and there is no trade impasse, since a high type seller always gets zero profit as now buyers have all the bargaining power.

Next, we further show that there are two types of delay equilibrium n, depending on LL's strategy in period 1. Define p_{rr} as the probability that LL chooses rr in period 1. If $p_{rr} = 1$, then there is no trade for the initial n periods. Thus, the posterior belief in period $n+1$ is the same as the prior belief. If $p_{rr} \in (0,1)$, then there is trade in period 1, which is followed by no-trade for $n-1$ periods. We find that the updated belief of LL in period $n+1$ is less than the prior belief of LL. Let n_1 be the length of impasse if $p_{rr} = 1$ and n_2 the length of impasse if $p_{rr} \in (0,1)$. Define $N_1 \equiv \max\{n: \ v_L - c_L \leq \delta^n(v_L - c_L + \delta(V(\hat{\mu}) - c_H))\}$. Delay equilibrium n can be further divided into two types of equilibria as follows.

Corollary 1. *In Theorem 2, there are two types of delay equilibrium n,*

1. *If $p_{rr} = 1$, then there is no trade in period 1 and $1 \leq n_1 \leq N_1$, which is equivalent to*

$$v_L - c_L \leq \delta^{n_1}(v_L - c_L + \delta(V(\hat{\mu}) - c_H)).$$

2. *If $p_{rr} \in (0,1)$, then $\mu_{LL}^{n_2+1} < \mu_{LL}$ and $N_1 < n_2 \leq N$. Moreover, p_{rr} satisfies*

$$v_L - c_L = \delta^{n_2}(v_L - c_L + \delta(V(\hat{\mu}') - c_H)),$$

where $\hat{\mu}' = 1 - \frac{\mu_{LL}p_{rr}}{1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*} - \frac{\mu_{HH}}{2\mu^*-1}} \in [\hat{\mu}, 1)$.

Note that there is a longer period of trade impasse if there is trade in period 1: $n_2 > n_1$. The reason is the following. If LL trades with positive probability in period 1, then there will be a smaller mass of LL at the end of trade impasse. Consequently, there will be a stronger cross-market information spillover effect ($\hat{\mu}' > \hat{\mu}$), and thus LH (HL) gets a higher continuation payoff at the end of trade impasse. Therefore, a longer period of impasse is required so that LH (HL) is willing to chooses ar (ra) and get a payoff $v_L - c_L$ in period 1. Note that as $p_{rr} \to 0$, the maximal length of impasse approaches its maximal level: $n_2 \to N$. Interestingly, there is no impasse in equilibrium if $p_{rr} = 0$ (see Theorem 1).

3.2.2 When the sufficient negative correlation condition fails

When Assumption 2 fails, either (1) $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$ or (2) $\mu^* \leq \frac{1}{2}$ holds. Intuitively, both cases imply that the initial probability of LL is large. This is immediate in the former case. For the latter case, combining with Assumption 1, we have $\mu_{LL} > 1 - 2\mu^* + \mu_{HH} > \mu_{HH}$, which gives a lower bound on μ_{LL}. The next result shows that in both cases, information spillover does not improve the seller's surplus when the seller is patient enough. Specifically, if $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$, then define $\bar{\delta} \equiv (\frac{v_L - c_L + \delta(v_H - c_H)}{c_H - c_L})^{\frac{1}{2}}$; if $\mu^* \leq \frac{1}{2}$, let $\bar{\delta} \equiv \max\{\frac{c_H - v_L}{v_H - c_H}, \frac{v_L - c_L}{c_H - c_L}, (1 + \frac{2\mu^* - 1 - \mu_{HH} + \mu_{LL}}{\mu^* - \mu_{HH}} \frac{c_H - v_L}{v_L - c_L})^{-1}\}$.[15]

Theorem 3. *Suppose Assumptions 1 holds but Assumption 2 fails. If $\delta > \bar{\delta}$, then there is a unique weak perfect Bayesian equilibrium outcome: the trade of each good is the same as that in the one-good benchmark (in Proposition 1).*

Theorem 3 characterizes the equilibrium outcome when there is not sufficient negative correlation. The key feature is that cross-market information spillover does not mitigate adverse selection. Intuitively, if the initial probability of LL is not low enough, then Bayes rule implies that given ar or ra in period 1, subsequent buyers believe that the probability of high quality for the remaining good is not larger than μ^*, consequently trade does not happen immediately for the remaining good. Perhaps also surprisingly, although information spillover does not improve the seller's surplus, it does no harm either. The intuition is as follows. When Assumption 2 fails, information spillover may lead to the harmful ratchet effect as in repeated bargaining with persistent types. In equilibrium, the seller avoids the ratchet effect "as if" by trading the goods independently. Moreover, there is no trade impasse in this case, since LH and HL have no incentives to reject both offers with probability 1 in early periods to enjoy the future information spillover effect.

4 Welfare Analysis

In this section, we compare the seller's equilibrium surpluses in different information environments.[16] First, as a benchmark, let $\underline{V}$ be the seller's expected surplus without cross-market information spillover. Next, if there is sufficient negative correlation (Assumption 2), let V_0 be the seller's surplus in the *no delay equilibrium*, and V_n the seller's surplus in *delay equilibrium n*, where $n \geq 1$. Finally, let $\tilde{V}$ be the seller's surplus when the sufficient negative correlation condition fails.

[15] It is straightforward to check that $\bar{\delta} \in (0, 1)$.

[16] Since buyers obtain zero surplus, the seller's surplus is also the social welfare.

Proposition 2 shows that cross-market information spillover is surplus (and hence welfare) improving if and only if there is sufficient negative correlation. Furthermore, recall that under sufficient negative correlation, there are multiple equilibria that are parametrized by the length of impasses. We find that the equilibrium surplus is negatively related to the length of impasse, with the *no delay equilibrium* delivering the highest gain from trade.

Proposition 2. *(Welfare Comparison) The seller's equilibrium surpluses can be ranked as follows:*

(i) $\tilde{V} = \underline{V}$.

(ii) V_n *is decreasing in* n, *for any* $1 \leq n \leq N$.

(iii) $V_0 > V_n > \underline{V}$, *for any* $1 \leq n \leq N$.

Next, we study how the correlation of the two qualities influences the highest gain from trade V_0, under sufficient negative correlation. It follows from Theorem 1 that

$$V_0 = \mu_{LL}(v_L - c_L + \delta(V(\hat{\mu}) - c_L)) + (1 - \mu_{HH} - \mu_{LL})(v_L - c_L + \delta(V(\hat{\mu}) - c_H)).$$

The next result (Proposition 3) shows that a higher degree of negative correlation, which is effectively measured by a decrease of μ_{HH} and μ_{LL}, improves efficiency.

Proposition 3. *Suppose that* $v_H - c_H > v_L - c_L$, *then* V_0 *is decreasing in* μ_{LL} *and* μ_{HH}.

Note that μ_{LL} and μ_{HH} affect the surplus through the following two different channels: first, smaller μ_{HH} and μ_{LL} imply that trading one good is a more informative signal of the high quality of the other good (higher $\hat{\mu}$), and thus a higher gain from trade for the remaining good; second, since HH (LL) benefits the least (the most) from information spillover, then the lower μ_{HH} is or the higher μ_{LL} is, the higher is the expected expected surplus. Therefore, both channels conclude that a lower μ_{HH} improves surplus. With respect to μ_{LL}, the two channels work in opposite directions, Nevertheless, we prove that, if the high quality good generates a a higher surplus than the low quality good, i.e., $v_H - c_H > v_L - c_L$, then the first effect dominates the second one and thus a lower μ_{LL} improves surplus.

5 Conclusion

This paper studies the role of cross-market information spillover in markets with adverse selection where the seller trades two heterogeneous goods. We find that adverse selection can be mitigated if and only if there is sufficient negative correlation.

While the baseline model is stylistic, in the Online Appendix we show that the main result holds in several extensions. In particular, we show that cross-market information spillover helps to improve efficiency under other bargaining protocols. For example, in each period and for each good, one short-lived buyer arrives and makes an offer to the seller. Similar to the equilibria in the baseline model, adverse selection is mitigated by information spillover if and only if the condition of sufficiently negative correlation holds. In addition, we also provide a new rationale for specialization in markets with adverse selection in the Online Appendix. Specifically, if the seller can choose the qualities of different goods in a pre-trading stage, in equilibrium she will choose in a way such that the initial beliefs of the buyers satisfy sufficient negative correlation in order to enjoy the information rents in the subsequently trading game.

A Additional Lemmas

In this section, we introduce and prove a series of lemmas (Lemmas A.1-A.5), which are used in the proofs of Theorems 1–3 in Appendix B. The proofs of these lemmas are in the Online Appendix.

We denote μ_t^i as the probability of high quality for good $i = 1, 2$ in period k; denote $\mu_t^i(h)$ as the probability of high quality for the remaining good $i = 1, 2$ in period t if the seller's action is $h \in \{ra, ar, rr\}$ in period $t-1$; denote μ_k^t as the probability of the seller type $k \in \{HH, HL, LH, LL\}$ in period t.

Define $(p_{rr}, p_{ar}, p_{ra}, p_{aa})$ as the probability of choosing rr, ar, ra, aa by LL in period 1. The following expressions describe the belief updating in period 2.

$$\frac{\mu_{HL}p_{HL} + \mu_{HH}}{\mu_{HL}p_{HL} + \mu_{LH}p_{LH} + p_{rr}\mu_{LL} + \mu_{HH}} = \frac{\mu_{LH}p_{LH} + \mu_{HH}}{\mu_{LH}p_{LH} + \mu_{HL}p_{HL} + p_{rr}\mu_{LL} + \mu_{HH}} = \mu^*. \quad \text{(A.1)}$$

.

$$\frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + p_{ra}\mu_{LL}} = \frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + p_{ar}\mu_{LL}} = \mu^*. \quad \text{(A.2)}$$

$$\frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + p_{ra}\mu_{LL}} \geq \mu^*, \ \frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + p_{ar}\mu_{LL}} \geq \mu^*. \quad \text{(A.3)}$$

Lemma A.1. *(i) If $\mu^* \leq \frac{1}{2}$, there are solutions to (A.1) and (A.2). (ii) If $\mu^* > \frac{1}{2}$, then there are solutions to (A.1) and (A.2) if and only if $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$. (iii) If $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} < 1$, there is a solution to (A.1) and (A.3), in which $p_{rr} = 0$.*

Lemma A.1 formally analyzes the link between Assumption 2 and the belief updating of the buyers.

Lemma A.2. *Under refinement D1,* $\mu^* > \frac{1}{2}$ *and* $\delta > (\frac{v_L - c_L + \delta(v_H - c_H)}{c_H - c_L})^{\frac{1}{2}}$*, if the belief in period* $t = 1$ *satisfies Assumption 1 or the belief in period* $t \geq 2$ *satisfies* $(\mu_t^1, \mu_2^2) \in \mathcal{B}$*, then* $\mu_{t+1}^1(rr) \leq \mu^*$ *and* $\mu_{t+1}^2(rr) \leq \mu^*$*.*

Lemma A.2 gives a characterization of Bayes updating in any equilibrium under $\mu^* > \frac{1}{2}$: the updated belief of each good never jumps to a level that is larger than the cutoff level μ^*. The idea of proof is as follows. Assume by contradiction that $\mu_{t+1}^1(rr) > \mu^*$, then we can prove that LH can guarantee a payoff at least $\delta^2(c_H - c_L)$ by choosing rr in period t, which is higher than the highest possible payoff from choosing ar: $v_L - c_L + \delta(v_H - c_H)$ if δ is high enough. Consequently, only rr and ra are on the equilibrium path in period t. Moreover, $\mu_{t+1}^1(ra) \geq \mu^*$. However, it is against the Bayes' rule that $\mu_{t+1}^1(rr) > \mu^*$ and $\mu_{t+1}^1(ra) \geq \mu^*$.

Lemma A.3. *Under refinement D1,* $\mu^* \leq \frac{1}{2}$ *and* $\delta > \bar{\delta}$*, if the initial belief in period* $t = 1$ *satisfies Assumption 1 or the belief in period* $t \geq 2$ *satisfies* $(\mu_t^1, \mu_2^2) \in \mathcal{B}$*, then* $\mu_{t+1}^1(rr) \leq \mu^*$ *and* $\mu_{t+1}^2(rr) \leq \mu^*$*.*

Lemma A.3 presents the belief updating under $\mu^* \leq \frac{1}{2}$, which is paralleled with Lemma A.2 with a different proof. Assume by contradiction that $\mu_{t+1}^1(rr) > \mu^*$, then we can prove that LH can guarantee a payoff at least $\delta(c_H - c_L)$ by choosing rr in period t, and LH weakly prefers ar to rr in period t. As a result, LH shall get a payoff at least $\delta(c_H - c_L)$ by choosing ar in period t. However, if $\mu^* \leq \frac{1}{2}$, then μ_{LL} is high enough, and consequently the updated belief of the remaining good given ar is so low that the payoff of LH by choosing ar is lower than $\delta(c_H - c_L)$ in period t (under $\delta > \bar{\delta}$), a contradiction.

Lemma A.4. *Under* $\delta > \frac{v_L - c_L}{c_H - c_L}$*, if there are three types:* HH*,* HL *and* LH *and the belief satisfies* $(\mu_t^1, \mu_t^2) \in \mathcal{B}$ *in period* $t \geq 2$*, then the equilibrium continuation payoff of* LH *and* HL *in period* t *is* $v_L - c_L + \delta(v_H - c_H)$*.*

Lemma A.4 describes the equilibrium outcome when there are only three types, namely HH, HL and LH that are untraded in the market. We prove that information spillover enhances the efficiency to the highest degree: the gain from trade is maximized for type LH and HL. Lemma A.4 is also crucial for the further analysis in the four types case, since the three type case describes a possible continuation play.

Lemma A.5. *Let* $n+1$ *be the first period in which* LL *does not choose* rr*. Suppose Assumptions 1-2 hold,* $\delta > (\frac{v_L - c_L + \delta(v_H - c_H)}{c_H - c_L})^{\frac{1}{2}}$ *and* $n \geq 1$*,*

1. $n < +\infty$.

2. If $n \geq 2$, all types of the seller choose rr in period $2 \leq t \leq n$.

3. $(\mu^1_{t+1}(rr), \mu^2_{t+1}(rr)) \in \mathcal{B}$ for $1 \leq t \leq n$.

4. Under refinement D1, $\mu^1_{t+1}(ra) = \mu^2_{t+1}(ar) = \mu^*$ for $1 \leq t \leq n$.

5. LL does not choose aa in period 1.

Lemma A.5 points out that there could be a delay of trade with limited length n at the beginning of the trading process in which all seller types choose to reject both offers, mainly because the high type seller is waiting for future benefits from information spillover.

B Proofs of the Results in Sections 3 and 4

In this section, recall some notations as follows: denote μ^i_t as the probability of high quality for good $i = 1, 2$ in period k; Denote $\mu^i_t(h)$ as the probability of high quality for the remaining good $i = 1, 2$ in period t if the seller's action is $h \in \{ra, ar, rr\}$ in period $t-1$; denote μ^t_k as the probability of the seller type $k \in \{HH, HL, LH, LL\}$ in period t.

Proof of Lemma 1:

Proof. Define $V^t_s(k)$ as the continuation payoff of type $s \in \{H, L\}$ by choosing $k \in \{a, r\}$ in period t. It is trivial that $V^t_L(a) - V^t_H(a) = c_H - c_L$. By mimicking L's strategy from period $t+1$ on, then if L accepts the offer in period $t+k$, where $k \geq 1$, then $V^t_L(r) - V^t_H(r) \leq \delta^k(c_H - c_L) < V^t_L(a) - V^t_H(a)$. Therefore, if L weakly prefers r to a, then H strictly prefers r to a. □

Proof of Proposition 1:

Proof. Define μ_t as the probability of high type seller in period t.

First, we prove that if $\mu_t > \mu^*$, then buyers offer $V(\mu_t)$, which the seller accepts for sure. Define $m > t$ is the last period that is reached with positive probability. That means that in period m, both types accept. It is trivial to show that $m < +\infty$, since otherwise H gets zero profit in the equilibrium, but a buyer in period n can offer $c_H + \epsilon$ to attract H and the buyer makes a positive profit, a contradiction. Since period m is reached in the equilibrium, H type seller rejects the equilibrium offer with positive probability in period $m-1$. There are two cases:

(1) H seller accepts with positive probability in any period $m-1$. Then, by skimming property, L seller accepts for sure in period $m-1$. Therefore, in period m, the buyer believes that there is only H type in the market, then the offer in period m is 1. In period $m-1$, L rejects for sure to get the higher offer in period m, a contradiction. (2) H rejects for sure in period $m-1$. We will show that L rejects in period $m-1$. Assume by contradiction that L accepts with positive probability, then the offer is v_L. However, by Bayes' rule, $\mu_m > \mu_{m-1} > \mu^*$ and in period m, the offer is $V(\mu_m) > c_H$. In period $m-1$, L type strictly prefers rejecting to accepting: $v_L - c_L < \delta(V(\mu_m) - c_L)$, a contradiction. In all, both types reject in period $m-1$, thus $\mu_{m-1} = \mu_m$. Therefore, in period $m-1$, a buyer can make an offer $V(\mu_m) - \epsilon$, where $\epsilon > 0$ satisfies $V(\mu_m) - \epsilon - c_L > \delta(V(\mu_m) - c_L)$ to attract both H and L seller in period $m-1$ and makes a profit, a contradiction to the fact that buyers make zero profit in any period. We have proved that only period t is reached in the equilibrium, which means that both types accepts the offer in period t. As a consequence, the offer in period t is $V(\mu_t)$.

Second, if $\mu_t < \mu^*$, H seller rejects, and consequently the equilibrium offer in period t is v_L. Assume by contradiction that if the high type trades with positive probability in period t then the low type will trade with probability one in period t, and thus the bid in period t is bounded above by the asset's unconditional expected value $\mu_t v_H + (1-\mu_t)v_L$, which is lower than H seller's reservation value c_H, a contradiction. This implies that the equilibrium offer in period t must be v_L, since the high type seller will not trade in period t.

We argue by contradiction that if $\mu_t < \mu^*$, then $\mu_{t+1} \geq \mu^*$. Assume the contrary that $\mu_{t+1} < \mu^*$, then in period t, the low type accepts for sure since by accepting, the low type gets $v_L - c_L$, and by rejecting, he gets at most $\delta(v_L - c_L)$. Therefore, the updated belief is $\mu_{t+1} = 1$, a contradiction.

Next, we show that if $\mu_t \leq \mu^*$, then $\mu_{t+1} \leq \mu^*$. Assume the contrary that $\mu_{t+1} > \mu^*$, so in order to support the belief updating, the seller's low type must accept with positive probability, but not the high type. We have shown that the offer in period t will then be $V(\mu_{t+1}) \geq c_H$, which means that the offer must be $p = \delta V(\mu_{t+1}) + (1-\delta)c_L$, since by accepting, the seller's low type gets $p - c_L$ and by rejecting, he gets $\delta(V(\mu_{t+1}) - c_L)$. However, by assumption, $\delta V(\mu_{t+1}) + (1-\delta)c_L > v_L$, which means that this offer would be unprofitable for buyers.

We thus have shown that $\mu_1 < \mu^*$ implies that $\mu_2 = \mu^*$. In order to support the belief updating in period 1, the low quality seller rejects the offer with probability $\frac{1-\mu^*}{\mu^*}\frac{\mu_H}{1-\mu_H}$. In period $t \geq 2$, there is a winning offer c_H with probability λ and a losing offer with probability $1-\lambda$ so that the low quality seller is indifferent between accepting and rejecting in period $t-1$: $v_L - c_L = \delta(\lambda c_H + (1-\lambda)v_L - c_L)$.

□

Proof of Lemma 2:

Proof. Define the continuation value of seller of type $s \in \{HH, HL, LH, LL\}$ to choose action $k \in \{rr, aa, ar, ra\}$ in period t as $V_s^t(k)$. We prove five skimming properties as follows.

Property 1: We prove that $V_{xL}^t(za) - V_{xH}^t(za) = c_H - c_L$. By mimicking the strategy of xL from period $t+1$ on, then if xL accepts the offer of good 2 in period $t+k$, where $k \geq 1$, then $V_{xL}^t(yr) - V_{xH}^t(yr) \leq \delta^k(c_H - c_L) < V_{xL}^t(za) - V_{xH}^t(za)$. Therefore, if xL weakly prefers yr to za, then xH strictly prefers yr to za.

Property 2: This is equivalent to prove the one-dimensional skimming property.

Property 3: If $\mu_{t+1}^2(ar) > \mu^*$, then $V_{xL}^t(ar) - V_{xH}^t(ar) = \delta(c_H - c_L)$. By mimicking the strategy of xL from period $t+1$ on, then if xL accepts the offer of good 2 for sure in period $t+k$, where $k \geq 1$, then $V_{xL}^t(rr) - V_{xH}^t(rr) \leq \delta^k(c_H - c_L) \leq V_{xL}^t(ar) - V_{xH}^t(ar)$. Therefore, if xH weakly prefers ar to rr, then xL weakly prefers ar to rr.

If xL reject offer 2 with positive probability in period $t+1$, then $k \geq 2$ and $V_{xL}^t(rr) - V_{xH}^t(rr) \leq \delta^k(c_H - c_L) < \delta(c_H - c_L) = V_{xL}^t(ar) - V_{xH}^t(ar)$. Therefore, if xH weakly prefers ar to rr, then xL strictly prefers ar to rr.

Property 4: It is trivial to show that $V_{LH}^t(ar) - V_{HL}^t(ar) \geq \delta(1-\delta)(c_H - c_L)$. Similarly, $V_{HL}^t(ra) - V_{LH}^t(ra) \geq \delta(1-\delta)(c_H - c_L)$. Therefore, $V_{LH}^t(ar) - V_{HL}^t(ar) \geq \delta(1-\delta)(c_H - c_L) > -\delta(1-\delta)(c_H - c_L) \geq V_{LH}^t(ra) - V_{HL}^t(ra)$. Therefore, if HL weakly prefers ar to ra, then LH strictly prefers ar to ra. Also, if LH weakly prefers ra to ar, then HL strictly prefers ra to ar.

Property 5: If not specified, the action of the seller and the buyers are taken in period t. Assume by contradiction that HL chooses ar with positive probability, but LH weakly prefers rr to ar. There are four observations: (1) HH chooses ar or rr, and LL chooses ar or aa, by skimming property 1. (2) HL mixes between ar and rr. Otherwise, only HH and LH could choose rr in period t, and consequently, $\mu_{t+1}^2(rr) = 1$ and there is a winning offer v_H in period $t+1$, and thus LH strictly prefers ar to rr in period t, a contradiction to the assumption that LH weakly prefers rr to ar. (3) LH is indifferent between ar and rr. Assume the contrary that LH strictly prefers rr to ar. Then HL chooses ar for sure, since otherwise buyer 1 can increase the offer of good 1 in period t to attract HL to accept the new offer for sure, without influencing the decision of low quality good 1. This is a profitable deviation for buyer 1, a contradiction. However, we have shown that HL mixes between ar and rr, a contradiction. (4) HH strictly prefers rr to ar, by skimming property 1 and the fact that LH is indifferent between ar and rr.

We proceed by check two cases: $\mu_t^2 \leq \mu^*$ and $\mu_t^2 > \mu^*$.

Case 1: $\mu_t^2 \leq \mu^*$.

We check the case that LL does not choose aa. It is trivial that HL does not choose aa. Then, $\mu_{t+1}^2(ar) \geq \mu^*$, since otherwise aa dominates ar for LL. Denote p_{HL}^k as HL's ($LH's$) probability on $k \in \{rr, ar\}$. Since

$$\mu_{t+1}^2(ar) = \frac{\mu_{LH}^t p_{LH}^{ar}}{\mu_{LH}^t p_{LH}^{ar} + \mu_{HL}^t p_{HL}^{ar} + \mu_{LL}^t} \geq \mu^*,$$

then together with $\mu_{LH}^t + \mu_{HH}^t = \mu_t^2 \leq \mu^*$, the conditional probability of high quality good 1 in period t, conditioning on ar in period t, is

$$\frac{\mu_{HL}^t p_{HL}^{ar}}{\mu_{LH}^t p_{LH}^{ar} + \mu_{HL}^t p_{HL}^{ar} + \mu_{LL}^t} < \frac{\mu_{HL}^t}{\mu_{HL}^t + \mu_{LH}^t + \mu_{LL}^t}.$$

In period t, buyer 1 can increase the offer a little bit to attract HL, LL and LH to accept the higher offer for sure, and buyer 1 makes a profit, a contradiction to buyer 1's zero profit condition.

If LL chooses aa with positive probability, then it is trivial to show that LL and HL are indifferent between aa and ar and $\mu_{t+1}^2(ar) = \mu^*$. Therefore, $V_{LH}^t(ar) - V_{HL}^t(ar) = c_H - c_L - (v_L - c_L)$. It is trivial to show that $V_{LH}^t(rr) - V_{HL}^t(rr) < c_H - c_L - (v_L - c_L)$, which means that LH strictly prefers ar to rr, a contradiction to the fact that LH is indifferent between ar and rr.

Case 2: $\mu_t^2 > \mu^*$.

We will prove that $V_{LH}^t(rr) - V_{HL}^t(rr) < \delta(1-\delta)(c_H - c_L)$. Let us check the history after LH chooses rr in period t.

We start with the case that LH chooses ra with positive probability in some period $t+h \geq t+1$. If HL mimics the strategy of LH by choosing ra in period $t+h$, then the payoff difference between LH and HL in period $t+h$ is bounded above by $-(1-\delta)(c_H - c_L)$. Therefore, $V_{LH}^t(rr) - V_{HL}^t(rr) \leq -\delta^h(1-\delta)(c_H - c_L) < 0$.

We then deal with the case that LH chooses aa with positive probability in some period $t+i \geq t+1$. If HL mimics the strategy of LH by choosing aa in period $t+i$, then the payoff difference between LH and HL in period $t+i$ is 0. Therefore, $V_{LH}^t(rr) - V_{HL}^t(rr) < 0$.

We only need to check the case LH chooses ar or rr from period $t+1$ on. Assume that LH chooses ar with positive probability in period $t+k \geq t+1$. If $\mu_{t+k+1}^2(ar) > \mu^*$, then $V_{LH}^{t+k}(ar) - V_{HL}^{t+k}(ar) = (1-\delta)(c_H - c_L)$. Therefore, $V_{LH}^t(rr) - V_{HL}^t(rr) \leq \delta^k(1-\delta)(c_H - c_L) \leq \delta(1-\delta)(c_H - c_L)$. The proof is done. Therefore, we only deal with the situation that

$\mu^2_{t+k+1}(ar) \le \mu^*$. Actually, we argue that it is impossible $\mu^2_{t+k+1}(ar) \le \mu^*$ in the following four steps.

Firstly, there are only HH, HL and LH in period $t+1$, given rr in period t. In period t, HL weakly prefers ar to rr, then by skimming property 1, we have LL strictly prefers ar to rr in period t. Therefore, LL does not stay in period $t+1$.

Secondly, we prove that $\mu^2_{t+1}(rr) > \mu^*$. Assume the contrary that $\mu^2_{t+1}(rr) \le \mu^*$. We have proved that in period t, HL chooses ar with positive probability, HH chooses rr, LL chooses ar or aa, and LH chooses ar or rr. Assume without loss of generality that aa is off the equilibrium path. Since

$$\mu^2_{t+1}(rr) = \frac{\mu^t_{HH} + \mu^t_{LH} p^{rr}_{LH}}{\mu^t_{HH} + \mu^t_{LH} p^{rr}_{LH} + \mu^t_{HL} p^{rr}_{HL}} \le \mu^*,$$

then the conditional probability of high quality good 1 in period t, conditioning on ar in period t, is

$$\frac{\mu^t_{HL} p^{ar}_{HL}}{\mu^t_{HL} p^{ar}_{HL} + \mu^t_{LH} p^{ar}_{LH} + \mu^t_{LL}} \le \frac{\mu^t_{HL}(1 - p^{rr}_{HL})}{\mu^t_{HL}(1 - p^{rr}_{HL}) + \mu^t_{LH}(1 - p^{rr}_{LH}) + \mu^t_{LL}} < \frac{\mu^t_{HL}}{\mu^t_{HL} + \mu^t_{LH} + \mu^t_{LL}}.$$

The last inequality uses the fact that $\mu^t_{HL} < 1 - \mu^*$, which holds since $1 - \mu^t_{HL} = \mu^t_{LH} + \mu^t_{HH} + \mu^t_{LL} > \mu^2_t > \mu^*$. In period t, it is a profitable deviation for buyer 1 by increasing the offer a little bit to attract HL, LL and LH to accept the higher offer for sure, a contradiction to buyer 1's zero profit condition.

Thirdly, show that $\mu^2_{t+k}(rr) > \mu^*$ for any $k \ge 1$. In period $t+1$, LH chooses rr or ar. If LH chooses rr in period $t+1$, then HH, HL and LH chooses rr in period $t+1$, so $\mu^2_{t+2}(rr) = \mu^2_{t+1} > \mu^*$. We need to check the case that LH chooses ar with positive probability in period $t+1$. Since $\mu^2_{t+2}(ar) \le \mu^*$, then HL chooses ar with positive probability in period $t+1$ to satisfy the belief updating. By the same logic as above, we get $\mu^2_{t+2}(rr) > \mu^*$. By induction, we show that $\mu^2_{t+k}(rr) > \mu^*$, for all $k \ge 1$.

Fourthly, we define that HH chooses ar with positive probability in period $t+k^* \ge t+1$. We will show that there is a winning offer for good 1 in period $t+k^*$. Since HH chooses ar with positive probability in period $t+k^*$, then skimming property 1 implies that LH chooses ar for sure in period $t+k^*$. It is trivial that HL and HH is not indifferent between accepting and rejecting the offer of good 1 in period $t+k^*$, since otherwise buyer 1 can increase the offer of good 1 a little bit in period $t+k^*$ to attract HH or HL to accept offer 1 for sure, which is a profitable deviation for buyer 1. Therefore, HH chooses ar for sure in period $t+k^*$. Also, HL chooses ar with positive probability in period $t+k^*$ to guarantee that $\mu^2_{t+k^*+1}(ar) \le \mu^*$.

Consequently, HL accepts offer of good 1 for sure in period $t+k^*$. We have shown that there is a winning offer for good 1 in period $t+k^*$, then by the result of one-good model and $\mu^2_{t+k*} > \mu^*$ there is a winning offer for good 2, a contradiction to the fact that LH chooses ar in period $t+k^*$.

To summarize our findings so far, we have proved that $V^t_{LH}(rr) - V^t_{HL}(rr) < \delta(1-\delta)(c_H - c_L)$. It is trivial to show that $V^t_{LH}(ar) - V^t_{HL}(ar) \geq \delta(1-\delta)(c_H - c_L)$. Then, $V^t_{LH}(rr) - V^t_{HL}(rr) < \delta(1-\delta)(c_H - c_L) < V^t_{LH}(ar) - V^t_{HL}(ar)$. Therefore, in period t, since HL chooses ar with positive probability, then LH strictly prefer ar to rr, a contradiction to the assumption that LH weakly prefers rr to ar.

□

Proof of Lemma 3:

Proof. In period t, $\mu^1_t > \mu^*$ and $\mu^2_t > \mu^*$. Assume that period $t+k_i$ is the first period such that both goods remain untraded and the offer of good $i = 1, 2$ (good 2) is accepted by a high quality seller with positive probability. Assume without loss generality that $k_1 \leq k_2$. Therefore, in period $t, t+1 \cdots, t+k_1-1$, the beliefs of both goods 1 are larger than μ^*, and there are losing offers for both goods. In the following proof, if not specified, the strategies of all types of sellers and buyers are taken in period $t+k_1$. The proof is broken into the following 5 steps.

Step 1: $k_1 = k_2$.

Assume the contrary: $k_1 < k_2$. Therefore, in period $t+k_1$, there is a losing offer for good 2: all types of seller reject offer 2.

First, we prove that LL chooses ar. Assume the contrary that LL chooses rr with positive probability, then HL chooses rr for sure. By the definition of k_1, then HH chooses ar with positive probability. Skimming property 1 implies that LH chooses ar for sure. If $\mu^2_{t+k_1+1}(ar) > \mu^*$, then skimming property 3 shows that LL strictly prefers ar to rr, a contradiction to the fact that LL chooses rr with probability. If $\mu^2_{t+k_1+1}(ar) \leq \mu^*$, then buyer 2 can make an offer $V(\mu^2_{t+k_1}) - \epsilon$ so that LH and HH is willing to choose aa, instead of ar. In all, buyer 2 makes a positive profit, a contradiction to buyer 2's zero profit condition.

We next show that LH choose ar. Otherwise, LH chooses rr with positive probability, then by skimming property 5, HL and HH chooses rr for sure, a contradiction to the definition of k_1. A corollary is that if HH (HL) chooses ar with positive probability, then HH (HL) strictly prefers ar to rr. Otherwise buyer 1 could increase the offer by a little bit to attract HH and HL to choose ar for sure, without influencing the decision of LL and LH. This is a profitable deviation for buyer 1, a contradiction. There are three cases:

Case 1: HL chooses rr. Then by the definition of k_1, HH chooses ar. Therefore, HL reveals its type by choosing rr, and the payoff is $\delta(v_L - c_L + v_H - c_H)$. If buyer 2 offers $v_L - \epsilon$ (small enough), then HL strictly prefers ra to rr, for the following reasons: HL is the only type that could choose ra, and thus D1 show that $\mu^1_{t+k_1}(ra) = 1$. Then, ra brings HL a payoff $v_L - c_L + \delta(v_H - c_H)$, which is higher than HL's payoff of choosing rr. In all, buyer 2 can make a positive profit by making the offer $v_L - \epsilon$, a contradiction to buyer 2's zero profit condition.

Case 2: HL chooses ar and HH chooses rr. In period $n + k_1 + 1$, only HH remains untraded and two offers are v_H. Since HL strictly prefers ar to rr, then simply calculation shows that HH also strictly prefers ar to rr, a contradiction.

Case 3: Both HH and HL chooses ar. Therefore, Bayes' rule implies that $\mu^2_{t+k_1+1}(ar) = \mu^2_{t+k_1} > \mu^*$. In period $n + k_1 + 1$, only good 2 remains. By the result of the one-good model, there is a winning offer $V(\mu^2_{t+k_1+1}(ar)) = V(\mu^2_{t+k_1})$ in period $n + k_1 + 1$. In period $n + k_1$, buyer 1 can guarantee a positive profit by making an offer $V(\mu^2_{t+k_1}) - \epsilon$, since for small enough $\epsilon > 0$, all types of seller are willing to accept this slightly lower offer instead of waiting for one period to get the offer $V(\mu^2_{t+k_1})$ in period $n + k_1 + 1$. However, this contradicts buyer 2's zero profit condition.

Step 2: In period $t + k_1$, LH accepts offer 1 for sure or HL accepts offer 2 for sure.

Assume by contradiction that LH rejects offer 1 with positive probability and HL rejects offer 2 with positive probability. There are also three cases:

Case 1: HL chooses ar with positive probability. By skimming property 5, LH strictly prefers ar to rr. As a result, LH does not choose rr. Together with the fact that LH rejects offer 1 with positive probability, we have LH chooses ra with positive probability. In all, HL weakly prefers ar to ra and LH weakly prefers ra to ar, a contradiction to skimming property 4.

Case 2: LH chooses ra with positive probability. By the similar logic, it is impossible that LH chooses ra with positive probability.

Case 3: HL chooses rr with positive probability but does not choose ar, and LH chooses rr with positive probability but does not choose ra. By skimming property 1, since HL weakly prefer rr to ra and LH weakly prefer rr to ar, then HH strictly prefers rr to ar and ra. Also by skimming property 1, if HL weakly prefers rr to aa, then HH strictly prefers rr to aa. In all, HH chooses rr for sure. By the definition of k_1 and k_2, both HL and LH choose aa with positive probability. In all, HL weakly prefers aa to ra and LH weakly prefers aa to ar. Therefore, LL strictly prefers aa to ar and ra. If ar is on the equilibrium path, then $\mu^2_{t+k_1+1}(ar) = 1$. If ar is off the equilibrium path, then D1 implies that $\mu^2_{t+k_1+1}(ar) = 1$, since LH prefers ar to aa more than LL and HL, by skimming property 2. Consequently, the fact that LH prefers aa to ar implies that $p_2 - c_H \geq \delta(v_H - c_H)$. Similarly, we can show $p_1 - c_H \geq \delta(v_H - c_H)$. Summing up the two inequalities, we have $p_1 - c_H + p_2 - c_H \geq \delta(v_H - c_H + v_H - c_H)$, which means that for HH weakly prefers aa to rr, a contradiction to the fact that HH strictly prefers rr to aa.

Step 3: In period $t + k_1$, LH accepts offer 1 for sure *and* HL accepts offer 2 for sure.

By step 2, we assume WLOG that LH accepts offer 1 for sure. We need to show that HL also accepts offer 2 for sure. Assume by contradiction that HL rejects offer 2 with positive probability. There are two cases:

Case 1: HL chooses rr with positive probability. Then, by skimming property 1, HH strictly prefers rr to ra and aa, which means that HH rejects offer 2 for sure. By the definition of k_2, LH accepts offer 2 with positive probability. Since LH accepts offer 1 for sure, then LH chooses aa with positive probability. Since LH weakly prefers aa to ar, then HL strictly prefers aa to ar, by skimming property 2. Therefore, HL does not choose ar. Since LH weakly prefers aa to ar, then HH also weakly prefers aa to ar. As a result, HH chooses rr for sure. Since HL does not choose ar, HL chooses aa with positive probability by the definition of k_1. Since HL weakly prefers aa to ra and LH weakly prefers aa to ar, then LL strictly prefers aa to ar and ra, by skimming property 1. In all, LL chooses aa for sure. If ar is on the equilibrium path, then $\mu^2_{t+k_1+1}(ar) = 1$. If ar is off the equilibrium path, then D1 implies that $\mu^2_{t+k_1+1}(ar) = 1$, since LH prefers ar to aa more than LL and HL (by skimming property 1). Consequently, the fact that LH prefers aa to ar implies that $p_2 - c_H \geq \delta(v_H - c_H)$. Similarly, we can show $p_1 - c_H \geq \delta(v_H - c_H)$. Summing up the two inequalities, we have $p_1 - c_H + p_2 - c_H \geq \delta(v_H - c_H + v_H - c_H)$, which means that for HH weakly prefers aa to rr, a contradiction to the fact that HH strictly prefers rr to aa.

Case 2: HL chooses ar with positive probability. Skimming property 1 implies that LL strictly prefers ar to rr and ra. Therefore, LL accepts offer 1 for sure. Since both LH ad LL accepts offer 1 for sure, then neither HH nor HL is indifferent between between accepting and rejecting offer 1. Therefore, HL chooses ar for sure. If HH rejects offer 1, then HH is the only type to reject offer 1, so the best response of HH is to choose ra. However, it is a profitable deviation for LL to choose ra to get $v_H - c_L + \delta(v_H - c_L)$, which is the higher than accepting offer 1. If HH accepts offer 1, then all four types choose to accept offer 1. From the result of one-good model, there is a winning offer of good 2 in period $n + k_1$, a contradiction to HL chooses ar with positive probability.

Step 4: All types of sellers choose aa in period $t + k_1$.

If LL accepts offer 1 for sure, then we have shown that both LL and LH choose to accept offer 1. Then, HH and HL need to choose pure strategy. Moreover, if HL chooses ra, then HL needs to strictly prefer ra to aa.Then, HH also strictly prefers ra to aa, a contradiction to the definition of k_1. If HL chooses aa, then HL strictly prefers aa to ra, then HH also strictly prefers aa to ra. Therefore, HH accepts offer 1 for sure and consequently all four types choose to accept offer 1. By the result of one-good model, all four types accepts offer 2.

If LL accepts offer 2 for sure, by the same logic, we can show that all four types choose aa.

The only remaining case is that LL rejects offer 1 with positive probability and rejects offer 2 with positive probability. It is impossible that LL chooses rr for sure, since otherwise by skimming property 1, LH and HL will choose rr for sure, a contradiction to the fact that LH accepts offer 1 and HL accepts offer 2. Therefore, LL chooses ra with positive probability and chooses ar with positive probability. Skimming property 1 shows that HL strictly prefers ra to aa, and thus HL chooses ra for sure. By the same logic, LH chooses ar for sure. Also, HH strictly prefers ra to aa. Also LL does not choose aa, since otherwise LL reveals its type. If rr is on the equilibrium path, then in period $t+k_1+1$, both remaining goods have high quality, If rr is off the equilibrium path, then D1 implies that both remaining goods have high quality in period $t+k_1+1$ (since by skimming property 1, HH favourites rr the most). Define the offer of good $i = 1, 2$ in period $t+k_1$ as p_i and the offer of good i in period $t+k_1+1$ if only good i remains as p'_i. Since LH weakly prefers ar to rr: $p_1 - c_L + \delta(p'_2 - c_H) \geq \delta(v_H - c_L + v_H - c_H)$, then $p_1 - c_L > \delta(v_H - c_L) > \delta(p'_1 - c_L)$, which implies that LL strictly prefers aa to ra, a contradiction to the fact that LL does not chooses aa.

Step 5: $k_1 = k_2 = 0$.

Assume the contrary that $k_1 = k_2 \geq 1$. By the definition of k_1 and k_2, where $k_1 = k_2$, we know that all types choose rr in period $t+k_1-1$. Therefore, Bayes' rule implies that $\mu^1_{t+k_1}(rr) = \mu^1_{t+k_1-1}$. In period $t+k_1-1$, consider the strategy that buyer 1 makes an offer $V(\mu^1_{t+k_1-1}) - \epsilon$ in period $n+k_1-1$. Next, we will prove that all types choose ar in period $t+k_1-1$, faced with the offer $V(\mu^1_{t+k_1-1}) - \epsilon$. We have shown that in period $t+k_1$, all types choose aa. Therefore, in period $t+k_1-1$, $V^{t+k_1-1}_{LH}(rr) - V^{t+k_1-1}_{HL}(rr) = 0 < (1-\delta)(c_H - c_L) \leq V^{t+k_1-1}_{LH}(ar) - V^{t+k_1-1}_{HL}(ar)$. Moreover, $V^{t+k_1-1}_{LH}(rr) - V^{t+k_1-1}_{LL}(rr) = -\delta(c_H - c_L) \leq -\delta(c_H - c_L) \leq V^{t+k_1-1}_{LH}(ar) - V^{t+k_1-1}_{LL}(ar)$.[17] Refinement D1 implies that $\mu^2_{t+k_1}(ar) = 1$. In all, all types of sellers are willing to choose ar instead of rr in period $t+k_1-1$ since they get higher continuation payoff from both goods: for good 1, they get $V(\mu^1_{t+k_1-1}) - \epsilon$ by choosing ar in period $t+k_1-1$ instead of waiting for one period and get

[17] The second inequality is strict if $\mu^2_{t+k_1}(ar) < \mu^*$ or there is not a winning offer for the only remaining good 2 in period $n+k_1$ if $\mu^2_{t+k_1}(ar) = \mu^*$.

$V(\mu^1_{t+k_1-1})$ with a discount; for good 2, they get the highest possible offer v_H in period $t+k_1$ by choosing ar in period $t+k_1-1$. From buyer 1's point of view, the offer $V(\mu^1_{t+k_1-1})-\epsilon$ in period $t+k_1-1$ brings buyer 1 a positive profit, a contradiction to buyer 1's zero profit condition.

□

Proof of Lemma 4:

Proof. If not specified, the action of the seller is taken in period t. The proof is broken into 4 steps.

Step 1: HH chooses rr in period 1. If $(\mu^1_t, \mu^2_t) \in \mathcal{B}$ in period $t \geq 2$, HH chooses rr in period t.

Assume by contradiction that HH chooses aa with positive probability. By skimming property 1, all three other types chooses aa for sure. However, in market $i = 1, 2$, buyer i's expected valuation is bounded above by $V(\mu^i_t)$, which is also the offer of good i, by buyer i's zero profit condition. Since $(\mu^1_t, \mu^2_2) \in \mathcal{B}$, then $V(\mu^1_t) + V(\mu^2_t) < 2c_H$. Therefore, HH gets a negative profit by choosing aa, a contradiction.

Assume by contradiction that HH mixes between ar and ra and does not choose aa. By skimming property 1, HL, LH and LL do not choose rr. Also, LH accepts offer 1 for sure and HL accepts offer 2 for sure. There are two cases: (1) LH or HL chooses aa with positive probability. Skimming property 1 implies that LL chooses aa for sure. Therefore, given ar or ra, the belief of the remaining good in the next period is 1. Therefore, LH chooses ar for sure, and HL chooses ra for sure. However, LL is willing to deviate to ar or ra to get a higher payoff than aa, a contradiction. (2) LH chooses ar and HL chooses ra for sure. If rr is on the equilibrium path, then $\mu^1_{t+1}(rr) = \mu^2_{t+1}(rr) = 1$. If rr is off the equilibrium path, then D1 implies that $\mu^1_{t+1}(rr) = \mu^2_{t+1}(rr) = 1$, since HH favors rr the most. Denote the offer of good i in period t as p_i, where $i = 1, 2$. Since LH prefers ar to rr and HL prefers ra to rr, then it is trivial that $p_i - c_L > \delta(v_H - c_L)$ for $i = 1, 2$ and thus LL strictly prefers aa to ar and ra. However, for each good i, L seller of good i accepts offer i, then the expected value of the good i for the seller is less than c_H, and thus $p_i < c_H$. As result, $c_H - c_L > p_i - c_L > \delta(v_H - c_L)$, a contradiction to the fact that $\delta > \frac{c_H - c_L}{v_H - c_L}$.

Assume by contradiction that HH chooses ar with positive probability, but not ra and aa. By skimming properties 1 and 2, LH and LL accept offer 1 for sure. Therefore, if rr is on the equilibrium path, then $\mu^1_{t+1}(rr) = 1$. If rr is off the equilibrium path, D1 show that $\mu^1_{t+1}(rr) = 1$, since skimming property 1 shows that HH favourites rr the most. Denote p_1 as the offer 1 in period t. There are two possibilities: (1) $\mu_t^1 < \mu^*$. Then, $p_1 < c_H$. Therefore, by choosing ar, HH gets at most $p_1 - c_H + \delta(v_H - c_H) < \delta(v_H - c_H)$. By choosing rr, HH can guarantee $\delta(v_H - c_H)$. Therefore, HH will deviate to rr, a contradiction. (2) $\mu_t^1 = \mu^*$. If $p_1 < c_H$, then the same argument as (1) reaches a contradiction. If $p_1 = c_H$, then there is a winning offer for good 1 in period t. Show that HL chooses aa with positive probability, since otherwise all types choose ar and Bayes' rule implies that $\mu_{t+1}^2(ar) = \mu_t^2 < \mu^*$, and thus aa dominates ar in period t for HL, a contradiction. If $t \geq 2$, then HL gets $v_L - c_L$ in period $t \geq 2$. However, HL can guarantee $v_L - c_L$ in period 1, instead of getting $v_L - c_L$ in period $t \geq 2$. Therefore, in period t, there are at most two types HH and LH, a contradiction to $\mu_t^2 < \mu^*$.

Step 2: LH rejects offer 2 for sure *or* HL rejects offer 1 for sure in period t.

Assume that contrary that LH accepts offer 2 with positive probability and HL accepts offer 1 with positive probability. Skimming property 1 shows that LL chooses aa for sure. We need to rule out following 3 cases: (1) LH chooses ra with positive probability. Then, LH weakly prefers ra to aa. Skimming property 1 implies that HL strictly prefers ra to aa. Since HL accepts offer 1 with positive probability, then HL chooses ar with positive probability. Skimming property 2 show that LH strictly prefers ar to aa. Since LH accepts offer 2 with positive probability, then LH chooses ra with positive probability. In all, HL weakly prefers ar to ra and LH weakly prefers ra to ar, a contradiction to skimming property 4. (2) HL chooses ar with positive probability (by the same logic as above). (3) LH and HL choose aa with positive probability. By skimming property 1, LL chooses neither ar nor ra; LH does not choose ra; HL does not choose ar. Therefore, given ar (ra), the belief of the remaining good is 1.[18] Therefore, since LH weakly prefers aa to ar, then $p_2 - c_H > \delta(v_H - c_H)$. By symmetry, $p_1 - c_H > \delta(v_H - c_H)$. In all, $p_1 - c_H + p_2 - c_H > 2\delta(v_H - c_H)$, which means that HH strictly prefers aa to rr, a contradiction.

Step 3: LH rejects offer 2 for sure *and* HL rejects offer 1 for sure in period t.

By Step 2, assume WLOG that LH rejects offer 2 for sure. For good 2, the only serious offer is v_L. We will prove that HL rejects offer 1 for sure. Assume the contrary. There are two cases:

Case 1: HL chooses aa with positive probability. Skimming property 1 guarantees that LL chooses to accept offer 1 for sure. There are two sub-cases:

(i) HL does not choose rr. Observe that LH does not choose rr. Assume by contradiction

[18] If ar is off the equilibrium, D1 guarantees that the updated belief of the only remaining good is 1, since LH prefers ar to aa more than HL and LL.

that rr is chosen by LH, by skimming property 5, HL strictly prefers ar to rr and aa, and thus LL chooses aa for sure. Consequently, HL and LL accepts the offer of good 2 for sure. As a result, LH strictly prefers rr to aa, since otherwise it is a profitable deviation for buyer 2 by increasing the offer of good 2 a little bit to attract LH to accept for sure, without influencing the decision of HL and LL. Since $\mu^2_{t+1}(rr) = 1$, then $V^t_{HL}(aa) - V^t_{LH}(aa) = 0 \leq V^t_{HL}(rr) - V^t_{LH}(rr)$. Since LH strictly prefers rr to aa, then HL strictly prefers rr to aa, a contradiction. Since LH does not choose rr, then LH chooses ar for sure. Then, the offer of good 1 is less than c_H, since $\mu^1_t \leq \mu^*$ and HH chooses rr. Therefore, ra dominates aa for HL, a contradiction.

(ii) HL chooses rr with positive probability. We know that LH is indifferent between ar and rr, since otherwise buyer 1 can increase the offer of good 1 a little bit to attract HL to accept offer 1 for sure, without influencing the decision of LH, and buyer 2 makes a profit. By skimming property 5, HL strictly prefers rr to ar, and thus HL and LL strictly prefer aa to ar. Therefore, $\mu^2_{t+1}(ar) = 1$, and thus HL would rather choose ar than aa, since $v_L - c_L < \delta(v_H - c_L)$. We reach a contradiction.

Case 2: HL chooses ar with positive probability. By skimming property 2, LL accepts offer 1 for sure. By skimming property 5, LH chooses ar for sure. We can show that HL strictly prefers accepting offer 1 to rejecting offer 1 and HH strictly prefers rr to ar, since otherwise buyer 1 in period t can increase the offer a little bit to attract HL and HH to accept the new offer for sure, without influencing the decision of the seller with low quality good 1, and this is a profitable deviation for buyer 1. Therefore, HH is the only type to choose rr, and thus it is trivial to show that $V^t_{HL}(ar) - V^t_{HH}(ar) = \delta(c_H - c_L) = V^t_{HL}(rr) - V^t_{HH}(rr)$. Since HH strictly prefers rr to ar, then HL strictly prefers rr to ar, a contradiction to the fact that HL chooses ar with positive probability.

Step 4: Any serious offer of good i is v_L.

Since buyer $i = 1, 2$ believes that only low quality seller accepts the offer i given belief is in the set $\mathcal{B}$ in period $t \geq 2$. By Bertrand competition, the only serious offer of good i is v_L.

□

Proof of Theorem 1:

Proof. In this proof, the actions of the seller is taken in period 1, if not specified. The proof is broken into 4 steps.

Step 1: The updated belief in period 2 satisfies $\mu^1_2(rr) = \mu^2_2(rr) = \mu^*$.

Since we analyze all the equilibria in which LL does not choose rr in period 1, then in period 2, there are at most three types left untraded: HH, HL, LH. Assume by contradiction that the updated belief in period 2 is not (μ^*, μ^*). By Lemma A.4, the equilibrium payoff of LH and HL in period 2 is $v_L - c_L + \delta(v_H - c_H)$. By deviating to rr in period 2, LL can get $\delta(v_L - c_L + \delta(v_H - c_L))$ in period 1.

It is trivial that rr dominates aa for LL in period 1. Therefore, LL chooses ar or ra in period 1. Assume WLOG that LL mixes between ar and ra in period $n+1$. Then, in period 1, LH chooses ar with positive probability and HL chooses ra with positive probability since otherwise LL reveals its type. Also, HL and LH chooses rr with positive probability since otherwise the updated belief in period 2 is that one of the two goods is high type for sure, a contradiction. In all, in period 1, LH mixes between rr and ar, and HL mixed between rr and ra.

Since LH (HL) is indifferent between rr and ar (ra), then $v_L - c_L + \delta(V(\tilde{\mu}) - c_H) = \delta(v_L - c_L + \delta(v_H - c_H))$, where $\mu_2^1(ra) = \mu_2^2(ar) \equiv \tilde{\mu}$. Bayes' rule shows that

$$\frac{\mu_{HH} + \mu_{HL}p_{HL}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} \leq \mu^*, \; \frac{\mu_{HH} + \mu_{LH}p_{LH}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} \leq \mu^*,$$

$$\frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + \mu_{LL}p_{ar}} = \frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + \mu_{LL}p_{ra}} = \tilde{\mu}.$$

where $p_{HL}(p_{HL})$ is the probability of rr chosen by $HL(LH)$, and $p_{ar}(p_{ra})$ is the probability of $ar(ra)$ chosen by LL. Then, $\tilde{\mu} \leq \hat{\mu} \equiv 1 - \frac{(2\mu^*-1)\mu_{LL}}{2\mu^*-1-\mu_{HH}}$. However, by $\delta > \frac{v_L - c_L + \delta(V(\hat{\mu}) - c_H)}{v_L - c_L + \delta(v_H - c_H)}$, we can show that $v_L - c_L + \delta(V(\tilde{\mu}) - c_H) \leq v_L - c_L + \delta(V(\hat{\mu}) - c_H) < \delta(v_L - c_L + \delta(v_H - c_H))$, a contradiction.

Step 2: Find the equilibrium in period 1.

In period 1, HL and LH chooses rr with positive probability, since otherwise given rr in period 1, the updated belief of one of the two goods in period 2 is 1, a contradiction to Lemma A.2.

In period 1, HL chooses ra with positive probability. Assume the contrary that HL chooses rr for sure. Therefore, ra is off the equilibrium path. If $\mu_2^2(ar) > \mu^*$, then LL strictly prefers ar to aa. Therefore, only rr and ar are on the equilibrium path, but it violates Bayes' rule that $\mu_2^2(ar) > \mu^*$ and $\mu_2^2(rr) = \mu^*$. If $\mu_2^2(ar) < \mu^*$, then LL strictly prefers aa to ar. Therefore, HH, LH and HL choose rr for sure and LL chooses aa for sure. Bayes' rule implies that

$\frac{\mu_{HH}}{2\mu^*-1} + \mu_{LL} = 1$, which violates Assumption 2. If $\mu_2^2(ar) = \mu^*$, then

$$\frac{\mu_{HH} + \mu_{HL}}{\mu_{HH} + \mu_{HL} + \mu_{LH}p_{LH}} = \frac{\mu_{HH} + \mu_{LH}p_{LH}}{\mu_{HH} + \mu_{HL} + \mu_{LH}p_{LH}} = \frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + \mu_{LL}p_{ar}} = \mu^*.$$

The above equations and $p_{ar} \le 1$ imply that $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \ge 1$ and $\mu^* > \frac{1}{2}$, a contradiction to Assumption 2. In summary, we have shown that HL chooses ra with positive probability. By symmetry, LH chooses ar with positive probability.

In period 1, LL mixes between ar and ra. Assume the contrary that LL does not choose ar, then LL can deviate to ar to pretend to be LH and gets a payoff $v_L - c_L + \delta(v_H - c_L)$, which is higher than LL's equilibrium payoff, a contradiction.

In order to make LL indifferent between ar and ra in period $n+1$, we have $\mu_2^1(ra) = \mu_2^2(ar) \ge \mu^*$, since otherwise aa dominates ar and ra for LL. However, if $\mu_2^1(ra) = \mu_2^2(ar) = \mu^*$, then Lemma A.1 shows that Assumption 2 does not hold, a contradiction. In all, $\mu_{n+2}^1(ra) = \mu_{n+2}^2(ar) \equiv \hat{\mu} > \mu^*$. A corollary is that LL does not choose aa. Bayes' rule implies that

$$\frac{\mu_{HH} + \mu_{HL}p_{HL}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} = \frac{\mu_{HH} + \mu_{LH}p_{LH}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} = \mu^*,$$

$$\frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + \mu_{LL}p_{ra}} = \frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + \mu_{LL}p_{ar}} \equiv \hat{\mu}.$$

Therefore, we solve for $\hat{\mu}$, p_{LH}, p_{HL}, p_{ar} and p_{ra}:

$$\hat{\mu} = 1 - \frac{(2\mu^*-1)\mu_{LL}}{2\mu^* - 1 - \mu_{HH}} > \mu^*,$$

$$p_{HL} = \frac{(1-\mu^*)\mu_{HH}}{(2\mu^*-1)\mu_{HL}},\ p_{LH} = \frac{(1-\mu^*)\mu_{HH}}{(2\mu^*-1)\mu_{LH}},$$

$$p_{ra} = \frac{(2\mu^*-1)\mu_{HL} - (1-\mu^*)\mu_{HH}}{(2\mu^*-1)(1-\mu_{LL}) - \mu_{HH}},\ p_{ar} = \frac{(2\mu^*-1)\mu_{LH} - (1-\mu^*)\mu_{HH}}{(2\mu^*-1)(1-\mu_{LL}) - \mu_{HH}}.$$

Step 3: If $\mu_t^1 = \mu_t^2 = \mu^*$, then for each good $i = 1, 2$, there can only be a winning offer c_H or a losing offer in period t (in this step, actions are taken in period t, if not specified).

Notice that by Lemma A.5, there are only three types HH, HL and LH that remain untraded in period t.

First, we show that if HH or HL accepts offer 1 with positive probability, then there is a winning offer for good 1. If HH or HL accepts offer 1 with positive probability, by skimming properties in Lemma 2, LH and LL accepts offer 1 for sure. Then, HH and HL choose pure strategy with respect to good 1, since otherwise buyer 1 can increases the offer a little bit to make a profit. Next, show that HH and HL accepts offer 1. There are four cases to eliminate: (1) HH chooses rr with positive probability and HL accepts offer 1. HH is the only type to choose rr, and it is trivial that LH will deviate to rr, instead of accepting offer 1. (2) HH chooses ra and HL accepts offer 1. By skimming properties in Lemma 2, HL chooses aa with positive probability. However, ra dominates aa for HL, since HL gets negative profit from good 1 by choosing aa and gets $\delta(v_H - c_H)$ from good 1 by choosing ra. (3) HH chooses aa with positive probability and HL rejects offer 1. By skimming properties in Lemma 2, HL chooses ra with positive probability. Since HH strictly prefers aa to ra, then HL also strictly prefers aa to ra, a contradiction. (4) HH chooses ar and HL rejects offer 1. HH gets a negative payoff from good 1, so HH gets less than $\delta(v_H - c_H)$ by choosing ar. Since $\mu^1_{t+1}(rr) = 1$, then by choosing rr, HH can guarantee at least $\delta(v_H - c_H)$. Therefore, rr is a profitable deviation for HH. Moreover, the winning offer is $V(\mu^1_t) = V(\mu^*) = c_H$.

We next prove that if HH and HL reject offer 1, then there is a losing offer for good 1. We first deal with the case that HH or LH accepts offer 2. By previous argument, there is a wining offer for good 2. Therefore, we come back to the one-good model for good 1, and by the result of one-good model, there is a losing offer for good 1. We only need to to consider the case that HH and LH rejects offer 2. Assume by contradiction that LH chooses ar with positive probability. Then, if HL chooses rr for sure, then $\mu^1_{t+1}(rr) > \mu^1_t = \mu^*$, a contradiction to Lemma A.2. If HL chooses ra with positive probability, then it is trivial that $(\mu^1_{t+1}(rr), \mu^2_{t+1}(rr)) \in \mathcal{B}$, since it violates Bayes' rule that $\mu^1_t = \mu^2_t = \mu^1_{t+1}(rr) = \mu^2_{t+1}(rr) = \mu^*$. However, if $(\mu^1_{t+1}(rr), \mu^2_{t+1}(rr)) \in \mathcal{B}$ holds, Lemma A.4 shows that LH and HL gets $v_L - c_L + \delta(v_H - c_H)$ in period $t+1$. However, LH prefers ar to rr in period t, since ar brings LH a payoff $v_L - c_L + \delta(v_H - c_H)$ one period earlier than rr. Similarly, HL chooses ra for sure in period t. However, Bayes' rule implies that $\mu^1_{t+1}(rr) = \mu^2_{t+1}(rr) = 1$, a contradiction.

In summary, there is a winning offer or a losing offer for good 1. By symmetry, there is a winning offer or a losing offer for good 2. Moreover, buyer $i = 1, 2$ is indifferent between the winning offer and the losing offer, and both brings buyer i zero profit.

Step 4: Find the equilibrium in period $t \geq 2$.

By Step 3, there is a winning offer c_H or a losing offer in period $t \geq 2$, thus there is no belief updating: $\mu^1_t = \mu^2_t = \mu^*$ in period $t \geq 2$. In period 2, buyers 1 offers c_H with probability λ^* and a losing offer with probability $1 - \lambda^*$, so that LH is indifferent between ar and rr in period 1: $v_L - c_L + \delta(V(\hat{\mu}) - c_H) = \delta(\lambda^* c_H + (1 - \lambda^*) v_L - c_L)$. In period $t \geq 3$, buyers 1 offers c_H with probability λ and a losing offer with probability $1 - \lambda$, so that LH is indifferent between ar and

rr in period $t-1$: $v_L - c_L = \delta(\lambda c_H + (1-\lambda)v_L - c_L)$. By symmetry, in period $t \geq 2$, buyer 2's strategy is the same as buyer 1.

□

Proof of Theorem 2:

Proof. Define $n+1 \geq 2$ as the first period in which LL does not choose rr.

We start with equilibria that LL chooses rr for sure in period 1. First, all four types choose rr for sure in period 1, since other LH chooses ar in period 1 with positive probability, then LL has incentives to mimic LH by choosing ar in period 1, a contradiction. Together with Lemma A.5, all four types choose rr for period $1 \leq t \leq n$. In all, the joint distribution of two goods in period $n+1$ is the same as the initial joint distribution of two goods in period 1. We can treat period $n+1$ exactly as period 1 in Theorem 1, and the result follows. Next, we show that n satisfies $v_L - c_L \leq \delta^n[v_L - c_L + \delta(V(\hat{\mu}) - c_H)]$. We need to verify that in period $t \leq n$, LH chooses rr for sure. In any period $1 \leq t \leq n$, by choosing rr, LH gets $\delta^{n+1-t}[v_L - c_L + \delta(V(\hat{\mu}) - c_H)]$; by choosing ar, LH gets $v_L - c_L$, since $\mu^2_{t+1}(ar) = \mu^*$ by Lemma A.5. Since LH (HL) prefers rr to ar (ra) in any period $1 \leq t \leq n$, then $v_L - c_L \leq \delta^n[v_L - c_L + \delta(V(\hat{\mu}) - c_H)]$.

Next, we proceed to the equilibria that LL chooses rr with probability less than 1. By Lemma A.5, $\mu^1_2(ra) = \mu^2_2(ar) = \mu^*$:

$$\frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + \mu_{LL}p_{ar}} = \frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + \mu_{LL}p_{ra}} = \mu^*.$$

where p_{HL} (p_{LH}) is the probability of rr by HL (LH) in period 1, p_{rr}, p_{ra} and p_{ar} is the probability of choosing rr, ra and ar by LL in period 1.

By Lemma A.5, LL does not choose aa in period 1, then $p_{rr} + p_{ra} + p_{ar} = 1$. Therefore, the probability of rr by the seller in period 1 is $\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH} + \mu_{LL}p_{rr} = 1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*}$. Therefore, the updated belief in period 2 is that

$$\mu^2_{LL} = \frac{\mu_{LL}p_{rr}}{1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*}},\ \mu^2_{HH} = \frac{\mu_{HH}}{1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*}}.$$

It is easy to prove that $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} < 1$ implies $\frac{\mu^2_{HH}}{2\mu^*-1} + \frac{\mu^2_{LL}}{1-\mu^*} < 1$. Lemma A.5 shows that all types choose rr for sure in period $2 \leq t \leq n$, and thus the belief in period $n+1$ is the same as in period 2: $\mu^{n+1}_{HH} = \mu^2_{HH}$ and $\mu^{n+1}_{LL} = \mu^2_{LL}$.

We observe that $\mu^1_{n+2}(rr) = \mu^2_{n+2}(rr) = \mu^*$. Otherwise, Lemma A.4 shows that the equilibrium continuation payoff of LH and HL in period $n+2$ is $v_L - c_L + \delta(v_H - c_H)$. Since

LH gets $v_L - c_L$ in period 1 and LH is indifferent between ar and rr in period 1, we need $v_L - c_L = \delta^{n+1}(v_L - c_L + \delta(v_H - c_H))$, which does not hold generically.

To summarize, period $n+1$ can be treated exactly as period 1 in Theorem 1 with a different initial belief: $(\mu_{HH}^{n+1}, \mu_{HL}^{n+1}, \mu_{LH}^{n+1}, \mu_{LL}^{n+1})$, which satisfies Assumption 2. In period $n+1$, HH chooses rr, HL mixes between rr and ra with probability p_{HL}^{n+1} and $1 - p_{HL}^{n+1}$, LH mixes between rr and ar with probability p_{LH}^{n+1} and $1 - p_{LH}^{n+1}$, and LL mixes between ar and ra with probability p_{ar}^{n+1} and p_{ra}^{n+1}. Define $\mu_{n+2}^2(ar) = \mu_{n+2}^1(ra) \equiv \hat{\mu}'$. Bayes' rule implies that

$$\frac{\mu_{HH}^{n+1} + \mu_{HL}^{n+1} p_{HL}^{n+1}}{\mu_{HH}^{n+1} + \mu_{HL}^{n+1} p_{HL}^{n+1} + \mu_{LH}^{n+1} p_{LH}^{n+1}} = \frac{\mu_{HH}^{n+1} + \mu_{LH}^{n+1} p_{LH}^{n+1}}{\mu_{HH}^{n+1} + \mu_{HL}^{n+1} p_{HL}^{n+1} + \mu_{LH}^{n+1} p_{LH}^{n+1}} = \mu^*,$$

$$\frac{\mu_{LH}^{n+1}(1 - p_{LH}^{n+1})}{\mu_{LH}^{n+1}(1 - p_{LH}^{n+1}) + \mu_{LL}^{n+1} p_{ar}^{n+1}} = \frac{\mu_{HL}^{n+1}(1 - p_{HL}^{n+1})}{\mu_{HL}^{n+1}(1 - p_{HL}^{n+1}) + \mu_{LL}^{n+1} p_{ra}^{n+1}} = \hat{\mu}'.$$

Therefore,

$$p_{HL}^{n+1} = \frac{(1-\mu^*)\mu_{HH}^{n+1}}{(2\mu^* - 1)\mu_{HL}^{n+1}},\ p_{LH}^{n+1} = \frac{(1-\mu^*)\mu_{HH}^{n+1}}{(2\mu^* - 1)\mu_{LH}^{n+1}},$$

$$p_{ra}^{n+1} = \frac{(2\mu^* - 1)\mu_{HL}^{n+1} - (1-\mu^*)\mu_{HH}^{n+1}}{(2\mu^* - 1)(1 - \mu_{LL}^{n+1}) - \mu_{HH}^{n+1}},\ p_{ar}^{n+1} = \frac{(2\mu^* - 1)\mu_{LH}^{n+1} - (1-\mu^*)\mu_{HH}^{n+1}}{(2\mu^* - 1)(1 - \mu_{LL}^{n+1}) - \mu_{HH}^{n+1}}.$$

$$\hat{\mu}' = 1 - \frac{\mu_{LL} p_{rr}}{1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*} - \frac{\mu_{HH}}{2\mu^*-1}} \in [\hat{\mu}, 1),$$

where $\hat{\mu} = 1 - \frac{(2\mu^*-1)\mu_{LL}}{2\mu^*-1-\mu_{HH}}$. In order to guarantee the indifference condition of LH and HL in period 1, we need to choose $p_{rr} \in (0,1)$ such that

$$v_L - c_L = \delta^n(v_L - c_L + \delta(V(\hat{\mu}') - c_H)).$$

As a result, n satisfies $\delta^n \geq \frac{v_L - c_L}{v_L - c_L + \delta(v_H - c_H)}$.

□

Proof of Corollary 1:

Proof. See the proof of Theorem 2. □

Proof of Theorem 3:

Proof. If Assumption 2 is violated, there are two cases: (1) $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$; (2) $\mu^* \leq \frac{1}{2}$. We will deal with these two cases separately.

Case 1: We analyze the equilibrium under $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$.

We start with the case that LL does not choose rr in period 1. First, we shall show that $\mu_2^1(rr) = \mu_2^2(rr) = \mu^*$. Assume the contrary. By Lemma A.2, $(\mu_2^1(rr), \mu_2^2(rr)) \in \mathcal{B}$. In period 2, there are at most three types left untraded: HH, HL, LH. By Lemma A.4, the equilibrium payoffs of LH and HL in period 2 are $v_L - c_L + \delta(v_H - c_H)$. If LL deviates to rr in period 1, then in period 2, LL can get $v_L - c_L + \delta(v_H - c_L)$.

In period 1, aa is dominated by rr for LL. Since LL does not remain in period 2, then LL does not play rr in period 1. As a result, LL chooses ar or ra in period 1. Assume WLOG that LL mixes between ar and ra in period 1. Then, LH chooses ar with positive probability and HL chooses ra with positive probability since otherwise LL reveals its type. Also, HL and LH chooses rr with positive probability, since otherwise the updated belief in period 2 is that one of the two goods is high type for sure, a contradiction. In all, in period 1, LH mixes between rr and ar, and HL mixed between rr and ra. Since LH is indifferent between rr and ar in period 1, then $v_L - c_L + \delta(V(\tilde{\mu}) - c_H) = \delta(v_L - c_L + \delta(v_H - c_H))$, and thus $\tilde{\mu} > \mu^*$. Bayes' rule implies that

$$\frac{\mu_{HH} + \mu_{HL}p_{HL}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} < \mu^*, \quad \frac{\mu_{HH} + \mu_{LH}p_{LH}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} < \mu^*,$$

$$\frac{\mu_{LH}(1 - p_{LH})}{\mu_{LH}(1 - p_{LH}) + \mu_{LL}p_{ar}} = \frac{\mu_{HL}(1 - p_{HL})}{\mu_{HL}(1 - p_{HL}) + \mu_{LL}p_{ra}} = \tilde{\mu}.$$

where $p_{HL}(p_{HL})$ is the probability of rr chosen by $HL(LH)$ in period 1, and $p_{ar}(p_{ra})$ is the probability of $ar(ra)$ chosen by LL in period 1. If Assumption 3 holds, then $\tilde{\mu} < 1 - \frac{(2\mu^*-1)\mu_{LL}}{2\mu^*-1-\mu_{HH}} \leq \mu^*$, a contradiction to $\tilde{\mu} > \mu^*$.

Next, notice that $\mu_2^1(ra) \leq \mu^*$ and $\mu_2^2(ar) \leq \mu^*$. Assume by contradiction that $\mu_2^2(ar) > \mu^*$. Since $\mu^* > \frac{1}{2}$ and $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$, Bayes' rule implies that $\mu_2^1(ra) < \mu^*$. In period 1, ra strictly dominates ar and aa for LL. If LH chooses ar with positive probability in period 1, then $\mu_2^2(ar) = 1$, a contradiction. Therefore, LH chooses rr for sure in period 1, then ar is off the equilibrium in period 1. Then, only ra and rr are on the equilibrium path in period 1, but it violates Bayes' rule that $\mu_2^1(ra) > \mu^*$ and $\mu_2^1(rr) = \mu^*$.

We will prove that $\mu_2^1(ra) = \mu^*$ ($\mu_2^2(ar) = \mu^*$), if ra (ar) is on the equilibrium path in period 1. Assume the contrary that $\mu_2^1(ra) < \mu^*$, then LL does not choose ra in period 1. If HL chooses ra with positive probability in period 1, then $\mu_2^1(ra) = 1$, a contradiction. Therefore, in period 1, HL chooses rr for sure, which means that ra is off the equilibrium in period 1, a contradiction.

To summarize, in period $t \geq 2$, $\mu_t^1(ra) = \mu_t^2(ar) = \mu_t^1(rr) = \mu_t^2(rr) = \mu^*$. In order to guarantee the correct belief updating in period 1, the probability of accepting offer 1 in period 1 satisfies $1-\frac{\mu_{HH}+\mu_{HL}}{\mu^*}$ and the probability of accepting offer 2 is $1-\frac{\mu_{HH}+\mu_{LH}}{\mu^*}$. Moreover, for good $i=1,2$, there is a winning offer or a losing offer in period $t \geq 2$, by step 3 of Theorem 1. To make LH indifferent between ar and rr in period $t \geq 2$, buyer 1 makes a winning offer c_H with probability λ and a losing offer with probability $1-\lambda$ such that $v_L - c_L = \delta(\lambda_i c_H + (1-\lambda_i)v_L - c_L)$. By symmetry, buyer 2 behaves the same as buyer 1 in period t.

We next study the case that LL chooses rr with positive probability in period 1. Define $n+1 \geq 2$ as the first period in which LL does not choose rr. Define $(\mu_{HH}^{n+1}, \mu_{HL}^{n+1}, \mu_{LH}^{n+1}, \mu_{LL}^{n+1})$ as the updated belief in period $n+1$.

We observe that if $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$, then $\frac{\mu_{HH}^{n+1}}{2\mu^*-1} + \frac{\mu_{LL}^{n+1}}{1-\mu^*} \geq 1$. There are two cases: (1) If LL chooses rr for sure in period 1, then all types choose rr for sure in period 1. Together with Lemma A.5, we have all types choose rr for sure in period $1 \leq t \leq n$, so the joint distribution of two goods in period $n+1$ is the same as the initial joint distribution of two goods in period 1. (2) If LL chooses rr with probability less than 1, then Lemma A.5 shows that $\mu_2^1(ra) = \mu_2^2(ar) = \mu^*$. Belief updating shows that

$$\frac{\mu_{LH}(1-p_{LH})}{\mu_{LH}(1-p_{LH}) + \mu_{LL}p_{ar}} = \frac{\mu_{HL}(1-p_{HL})}{\mu_{HL}(1-p_{HL}) + \mu_{LL}p_{ra}} = \mu^*.$$

Show by calculation the updated belief in period 2 is that

$$\mu_{LL}^2 = \frac{\mu_{LL}p_{rr}}{1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*}},\ \mu_{HH}^2 = \frac{\mu_{HH}}{1 - \frac{\mu_{LL}(1-p_{rr})}{1-\mu^*}}.$$

Moreover, $\frac{\mu_{HH}}{2\mu^*-1} + \frac{\mu_{LL}}{1-\mu^*} \geq 1$ implies that $\frac{\mu_{HH}^2}{2\mu^*-1} + \frac{\mu_{LL}^2}{1-\mu^*} \geq 1$. By Lemma A.5, there is no belief updating in period $2 \leq t \leq n+1$, so $\frac{\mu_{HH}^{n+1}}{2\mu^*-1} + \frac{\mu_{LL}^{n+1}}{1-\mu^*} = \frac{\mu_{HH}^2}{2\mu^*-1} + \frac{\mu_{LL}^2}{1-\mu^*} \geq 1$. Therefore, we can treat period $n+1$ as period 1, in which LL does not choose rr in period 1.

Finally, show that $n=0$. Assume by contradiction that $n \geq 1$. Since $\mu_{n+1}^2(ar) = \mu^*$, then the payoff of LH in period $n+1$ is $v_L - c_L$. In period $t \leq n$, by choosing ar, LH gets $v_L - c_L$; by choosing rr, LH gets $\delta^{n+1-t}(v_L - c_L)$. Therefore, LH strictly prefers ar to rr in period $t \leq n$, a contradiction to the fact that LH chooses rr in period $t \leq n$, as is shown by Lemma A.5.

Case 2: We analyze the equilibrium under $\mu^* \leq \frac{1}{2}$.

By Lemma A.3, $\mu_2^1(rr) \leq \mu^*$ and $\mu_2^2(rr) \leq \mu^*$. Together with $\mu^* \leq \frac{1}{2}$, Bayes' rule implies that LL chooses rr with positive probability in period 1, since otherwise

$$\frac{\mu_{HH} + \mu_{HL}p_{HL}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} \le \mu^*, \ \frac{\mu_{HH} + \mu_{LH}p_{LH}}{\mu_{HH} + \mu_{HL}p_{HL} + \mu_{LH}p_{LH}} \le \mu^*,$$

which is impossible if we sum up the two inequalities.

We first prove that $\mu_2^2(ar) = \mu^*$, if ar is on the equilibrium path in period 1. First, $\mu_2^2(ar) \ge \mu^*$, since otherwise ar is dominated by aa in period 1, a contradiction to the fact that ar is on the equilibrium path in period 1. Second, $\mu_2^2(ar) \le \mu^*$. Assume the contrary, then LL does not choose aa in period 1. Moreover, LL mixes between ar and ra in period 1, since otherwise ra is off the equilibrium path in period 1. It violates Bayes' rule that $\mu_2^2(ar) \le \mu^*$ and $\mu_2^2(rr) = \mu^*$. In all, $\mu_2^2(ar) = \mu_2^1(ra) > \mu^*$. Consequently, in period 1, HL chooses ra with positive probability and LH chooses ar with positive probability. Skimming property 3 implies that LL strictly prefers ra to rr in period 1, a contradiction to the fact that LL chooses rr with positive probability in period 1.

By symmetry, $\mu_2^1(ra) = \mu^*$, if ra is on the equilibrium path in period 1. A corollary is that LH and HL gets at most $v_L - c_L$ in period 1.

Next, we prove that $\mu_2^1(rr) = \mu_2^2(rr) = \mu^*$. If $(\mu_2^1(rr), \mu_2^2(rr)) \in \mathcal{B}$, LH gets at most $v_L - c_L$ in period 2, by the same logic as LH gets at most $v_L - c_L$ in period 1. Therefore, LH strictly prefers ar to rr in period 1, and thus skimming property 3 shows that LL also strictly prefers ar to rr in period 1, a contradiction to the fact that LL chooses rr with positive probability in period 1. By Lemma A.3, the only possibility is $\mu_2^1(rr) = \mu_2^2(rr) = \mu^*$.

We have proved that $\mu_2^1(ra) = \mu_2^2(ar) = \mu_2^1(rr) = \mu_2^2(rr) = \mu^*$. Lemma A.1 shows that there exists a seller's strategy to satisfy the above belief updating. Actually, the probability of accepting offer 1 in period 1 is $1 - \frac{\mu_{HH}+\mu_{HL}}{\mu^*}$ and the probability of accepting offer 2 in period 1 is $1 - \frac{\mu_{HH}+\mu_{LH}}{\mu^*}$.

In period $t \ge 2$, there is a winning offer c_H or a losing offer for good 1. By the same logic as in step 3 of Theorem 2, if HH or HL accepts offer 1 with positive probability, then there is a winning offer for good 1. We only need to show that if HH and HL reject offer 1, then there is a losing offer for good 1. There are two cases: (1) HH or LH accepts offer 2. We know that there is a wining offer for good 2. Therefore, we come back to the one-good model. Since HH and HL reject offer 1, then LH and LL also reject offer 1, since otherwise the probability of high quality of good 1 is larger than μ^* in period $t+1$, LL or LH is willing to reject the offer v_L in period t and gets an offer that is higher than c_H in period $t+1$, a contradiction. (2) HH and LH rejects offer 2. Assume that LH chooses ar with positive probability. Then, if HL chooses rr for sure, then $\mu_{t+1}^1(rr) > \mu_t^1 = \mu^*$, a contradiction to step 2. If HL chooses ra

with positive probability, then it is trivial that LL mixes between ar and ra. Step 2 implies that $\mu^1_{t+1}(rr) \leq \mu^*$ and $\mu^2_{t+1}(rr) \leq \mu^*$. Bayes' rule implies that $\mu^1_{t+1}(ra) = \mu^2_{t+1}(ar) > \mu^*$. Since there cannot be winning offers for both goods in period $t+1$, then skimming property 3 implies that rr is dominated by ar or ra for LL in period t. In all, if there are two goods in period $t+1$, it is not possible that the seller is LL. However, if $\mu^* \leq \frac{1}{2}$, then it violates the Bayes' rule that $\mu^1_{t+1}(rr) \leq \mu^*$ and $\mu^2_{t+1}(rr) \leq \mu^*$.

In period $t \geq 2$, buyer 1 makes a winning offer c_H with probability λ and a losing offer with probability $1-\lambda$, and both choices bring buyer 1 zero profit. To make LH indifferent between ar and rr, $v_L - c_L = \delta(\lambda c_H + (1-\lambda)v_L - c_L)$. By symmetry, buyer 2's behavior is the same as buyer 1 from period 2 on.

□

Proof of Proposition 2:

Proof. Since the buyers always get zero profits, then the expected discounted gain from trade is equivalent to the seller's expected discounted surplus. Define the surplus of seller with type $k \in \{HH, HL, LH, LL\}$ in *delay equilibrium n* as V^k_n.

First, the fact that $\tilde{V} = \underline{V}$ follows directly from Theorem 3. We next prove the monotonicity of V_n. There are three steps:

(i) V_{n_1} is decreasing in n_1 for any $1 \leq n_1 \leq N_I$, which holds since $V_{n_1} = \delta^{n_1} V_0$.

(ii) V_{n_2} is decreasing in n_2 for any $N_I < n_2 \leq N$. First, $V^{HH}_{n_2} = 0$, independent of n_2. Second, by Corollary 1, $V^{HL}_{n_2} = V^{LH}_{n_2} = v_L - c_L = \delta^{n_2}(v_L - c_L + \delta(V(\hat{\mu}') - c_H))$, independent of n_2. Finally, $V^{LL}_{n_2} = \delta^{n_2}(v_L - c_L + \delta(V(\hat{\mu}') - c_L)) = v_L - c_L + \delta^{n_2+1}(c_H - c_L)$, which is decreasing in n_2. In all, V_{n_2} is decreasing in n_2.

(iii) $V_{n_1} > V_{n_2}$ for any $1 \leq n_1 \leq N_I < n_2 \leq N$. First, $V^{HH}_{n_1} = V^{HH}_{n_2} = 0$. Second, $V^{HL}_{n_1} = V^{LH}_{n_1} = \delta^{n_1}(v_L - c_L + \delta(V(\hat{\mu}) - c_H) > v_L - c_L = V^{HL}_{n_2} = V^{LH}_{n_2}$. Finally, $V^{LL}_{n_1} = \delta^{n_1}(v_L - c_L + \delta(V(\hat{\mu}) - c_L)) = V^{HL}_{n_1} + \delta^{n_1+1}(c_H - c_L) > v_L - c_L + \delta^{n_2+1}(c_H - c_L) = V^{LL}_{n_2}$. Therefore, the expected surplus satisfies $V_{n_1} > V_{n_2}$.

Finally, we prove that $V_0 > V_n > \underline{V}$ for any $1 \leq n \leq N$. Based on the argument above, we only need to show that $V_{n_1} < V_0$ for any $1 \leq n_1 \leq N_I$ and $V_{n_2} > \underline{V}$ for any $N_I < n_2 \leq N$. The former is straightforward since $V_{n_1} = \delta^{n_1} V_0$ and the latter is proved as follows: (i) $V^{HH}_{n_2} = 0$ with or without information spillover; (ii) $V^{HL}_{n_2} = V^{LH}_{n_2} = v_L - c_L + \delta(V(\hat{\mu}') - c_H)) > v_L - c_L$, which is exactly the payoff that LH (HL) gets without cross-market information spillover; (3) $V^{LL}_{n_2} > 2(v_L - c_L)$. By $\delta > \frac{v_L - c_L + \delta(v_H - c_H)}{c_H - c_L}$, we have $\delta > \frac{v_L - c_L + \delta(V(\hat{\mu}') - c_H)}{c_H - c_L} = \frac{\delta^{-n_2}(v_L - c_L)}{(c_H - c_L)}$. Therefore, $V^{LL}_{n_2} = v_L - c_L + \delta^{n_2+1}(c_H - c_L) > 2(v_L - c_L)$, which is LL's payoff without cross-market information spillover. To summarize, LL type seller strictly benefits from information spillover, and the other three types' payoffs remain the same, so the seller's expected surplus $V_{n_2} > \underline{V}$.

$\square$

Proof of Proposition 3:

Proof. A corollary of Theorem 1 is that

$$V_0 = \mu_{LL}(v_L - c_L + \delta(V(\hat{\mu}) - c_L)) + (1 - \mu_{HH} - \mu_{LL})(v_L - c_L + \delta(V(\hat{\mu}) - c_H)).$$

$$= (1 - \mu_{HH})(v_L - c_L + \delta(V(\hat{\mu}) - c_H)) + \mu_{LL}\delta(c_H - c_L).$$

First, we prove that V_0 is decreasing in μ_{HH}. By $\hat{\mu} = 1 - \frac{(2\mu^*-1)\mu_{LL}}{2\mu^*-1-\mu_{HH}}$, $\hat{\mu}$ is decreasing in μ_{HH}, and it follows that V_0 is decreasing in μ_{HH}.

Second, we derive that V_0 is decreasing in μ_{LL}, which follows by

$$\frac{dV_0}{d\mu_{LL}} = -\frac{(2\mu^* - 1)(1 - \mu_{HH})}{2\mu^* - 1 - \mu_{HH}}\delta(v_H - v_L) + \delta(c_H - c_L) < -\delta(v_H - v_L) + \delta(c_H - c_L) < 0.$$

$\square$

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