Repeat Consumer Search*

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Abstract

Consumers are usually interested in buying a product repeatedly over time, but need to search for information on firms' prices and product features. We characterize the optimal search rule in this environment, showing that under certain conditions consumers 'stagger' their search across periods i.e. they buy several different products early on before settling on one which they then buy repeatedly. We also solve for equilibrium prices, examine how they change as the market matures, and compare them to the usual one-shot search problem. Finally we investigate how the market evolves when firms may offer discounts to past consumers.

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1 Introduction

Consumers are often poorly informed about what products are available in the marketplace, and can only obtain this information through costly search. At the same time, consumers are usually interested in buying a product repeatedly over time, such that information gathered today is valuable in the future. We incorporate this latter feature into an otherwise standard consumer search model, and show that it has several interesting implications, ranging from consumer learning to industry price dynamics.

In more detail, we begin in Section 2 with a benchmark model set in discrete time with overlapping generations of consumers. Products are differentiated and when consumers first enter the market they are uninformed about which product fits them best. However they can learn this information by searching sequentially as in Wolinsky (1986) and Anderson and Renault (1999). Since a consumer's preferences are time-invariant, any information gathered on product valuations in one period is retained and can be used in subsequent periods. Specifically, in each period a consumer can search 'new' products at a cost $s > 0$, which allows her to discover both her match valuation and the current price. At the same time a consumer can also return to a product she searched in an earlier period (and so for which she already knows her valuation) at a cost $r \in (0, s)$, whereupon she learns the current price and has the opportunity to purchase. Consumers randomly exit the market and are replaced by new consumers who again initially have no information about products. Firms are infinitely-lived and cannot commit to future prices, although we relax this later on in the paper.

The optimal consumer search rule is characterized in Section 3. When search costs are relatively small consumers search intensively early on in their lifetimes, and buy different products in each period. Eventually during the course of one of these searches they find a product with which they match very strongly, and then they stop search and return to buy this product for the rest of their lifetime. Intuitively a consumer is willing to buy a product which she knows she will never buy again, provided its match valuation is reasonably high, because she has already sunk her search cost. On the other hand, in the next period she has to pay a return cost to buy the product again, and so prefers to search for a better product. When instead search costs are intermediate, consumers only search one firm in any given period, but do not necessarily return to products they bought in the past. Finally, as one would expect, when search costs are sufficiently high consumers do not search for better matches and instead remain loyal to the same product throughout their lifetime.

Section 4 of the paper then studies the implications of repeated purchases for equilibrium pricing. In general a firm's demand consists of two different segments. Some consumers are 'searchers' and are visiting the firm for the first time in search of a good match. These consumers are price-elastic and give firms an incentive to lower their price. Other consumers are 'return' demand, who searched the firm in the past and come back because they have a good match with its product. These consumers will remain with the firm even if it slightly raises its price, because they have sunk the return cost and so would rather buy than search again. In cases where search costs are relatively high only the second demand component is relevant, and firms charge the monopoly price. However when search costs are relatively small both demand components matter. In such cases we prove that as the market matures, equilibrium prices increase and converge to a steady state. Specifically, in early periods most consumers have not found a very good match and so are searchers, implying that firms' demand is relatively elastic. However as the market matures, more and more consumers return to a previous firm, such that demand becomes less elastic and prices increase. Notice that relative to a standard one-shot search model, there are two important differences. The first is that since consumers are long-lived, they have more incentive to search and so 'searchers' are choosier. The second though is that firms also have return demand whose decision of whether or not to buy is (locally) inelastic. Although it depends on parameters, this tends to imply that in early periods firms charge less than in an equivalent one-shot game, whereas in later periods they charge more.

Finally, in practice firms may offer coupons or special advertised discounts to past customers. We therefore introduce this feature into our benchmark model in Section 5. In particular, we assume that firms can offer existing consumers a different price if they return and buy again in the future. We show that firms optimally give discounts to their existing customers i.e. they 'pay-to-stay' (see Chen (1997)), in order to partially offset the negative effect of the return cost on future demand. Interestingly we find that even though each firm individually has an incentive to offer such discounts, in equilibrium all firms are made worse off by doing so, and consumers pay unambiguously lower prices than they would in a standard one-shot match search game.

Literature review: to be completed later.

2 Model

Time is discrete and indexed by $t = 0, 1, 2, ...$ There is a countably infinite number of products, each of which is sold by $n > 1$ single-product firms. Firms enter the industry at $t = 0$ and are infinitely-lived, and their marginal cost is normalized to zero. There are overlapping generations of consumers: at time $t = 0$ a unit mass of consumers enters the market, and at the start of every subsequent period a fraction $1 - \gamma$ of the consumers exit and are replaced by new consumers. In each period consumers wish to buy one product. Following Anderson and Renault (2000) a consumer's utility from buying product i in period t at price p_t is

$$u(p_t) + \epsilon_i \ , \tag{1}$$

where $u(p_t)$ is an indirect utility function and ϵ_i is a firm-specific match value. In particular consumers have a continuously differentiable, downward-sloping and logconcave demand $Q(p)$ for a product, which gives rise to an indirect utility function $u(p) \equiv \int_p Q(z)\,dz$ and a per-consumer profit function $\pi(p) \equiv pQ(p)$. In addition ϵ_i is distributed independently and identically (across both consumers and products) according to a distribution function $F(\epsilon)$ over the support $[\underline{\epsilon}, \overline{\epsilon}]$. The associated density function $f(\epsilon)$ is strictly positive and continuously differentiable everywhere on the support. We assume that a consumer's match value for each product is the same in every period of her lifetime, and two firms selling the same product are completely undifferentiated.

Consumers are imperfectly informed about prices and match values, and can only learn them via search. In particular, a consumer must pay a search cost $s > 0$ to visit a firm which sells a product she has not previously inspected. After paying this search cost s she learns her valuation for that product, and also the price charged by this specific seller. At the same time, a consumer must pay a 'return' cost $r > 0$ to visit a firm which sells a product which she has previously inspected. After paying this return cost r she learns the price currently being charged by this specific seller.[1] We can thus interpret r as the cost of travelling to a store, and $s - r$ as the cost of inspecting the good the first time and hence learning its valuation. In this spirit we assume that a consumer, if she wishes, can visit in the current round another store of the same brand at cost r. Consumers cannot stockpile products. Throughout the paper we focus on symmetric Markov Perfect pricing strategies on the part of firms, and assume that $\underline{\epsilon} > s$ which trivially ensures that consumers search or return in every period. Finally, consumers and firms have respective discount factors $\delta_c \in (0,1)$ and $\delta_f \in (0,1)$.

3 Optimal search rule

We first solve for an optimal consumer search rule which is stationary across time periods. Specifically, suppose the market is currently in period t, and that an arbitrary sequence of equilibrium prices $p_t^*, p_{t+1}^*, p_{t+2}^*, \ldots$ is expected by consumers. A stationary search rule is characterized by a pair of reservation values (a, b), where consumers stop and buy immediately a product whose match value satisfies $\epsilon \geq a$, and return to a product searched in an earlier period if and only if its match value satisfies $\epsilon \geq b$. It is straightforward to argue that $a < b$, and a short proof is provided in the appendix.

The optimal stationary search rule is determined as follows. Since a consumer who draws a match $\epsilon = a$ is indifferent between searching again immediately, or buying now but then searching next period, we can write

$$u\left(p_t^*\right) + a + \gamma \delta_c V_{t+1} = V_t \ , \tag{2}$$

where V_t (derived below) denotes the value from searching a new product in period t. At the same time, a consumer whose best match up to date t is $\epsilon = b$, is indifferent between returning and buying it in every future period, or restarting search. Hence we can also write

[1] We also make the standard assumption that within a period, consumers can costlessly recall any firm that they searched earlier that period. This assumption is innocuous, however, since with an infinite number of firms consumers never recall on the equilibrium path (see Janssen and Parakhonyak (2014)). Moreover, consumers' reservation rule is the same for all values of the recall cost.

that

$$\sum_{i=0}^{\infty} (\gamma \delta_c)^i \left[u\left(p^*_{t+i}\right) + b - r\right] = V_t \ . \tag{3}$$

The value of search can be written in the following recursive form

$$V_t = -s + F(a) V_t + \int_a^b \left[u(p^*_t) + \epsilon + \gamma \delta_c V_{t+1}\right] dF(\epsilon) \\ + \int_b^{\bar{\epsilon}} \left[u(p^*_t) + \epsilon + \sum_{i=1}^{\infty} (\gamma \delta_c)^i \left[u\left(p^*_{t+i}\right) + \epsilon - r\right]\right] dF(\epsilon) \ . \tag{4}$$

To interpret this, notice that after a consumer incurs s and learns her valuation for a new product, one of three things can happen. Firstly, it may be that $\epsilon \leq a$ in which case the consumer prefers to search again in period t. Secondly, it may be that $\epsilon \in (a, b)$ in which case the consumer buys the product in period t and gets $u(p^*_t) + \epsilon$, but then next period prefers to search for a better product. Finally it may be that $\epsilon \geq b$ in which case the consumer buys the product in period t and all subsequent periods as well. Combining these equations we obtain the following result:

Lemma 1. *There is a unique stationary search rule, which solves equations (2) - (4), and which satisfies $a = b - r$. There exist two thresholds*

$$s_0 = \mathbb{E}\epsilon - \underline{\epsilon} + \frac{\gamma \delta_c}{1 - \gamma \delta_c} \int_{\underline{\epsilon}+r}^{\bar{\epsilon}} (\epsilon - r - \underline{\epsilon})\, dF(\epsilon) \quad \text{and} \quad s_1 = r + \frac{\mathbb{E}\epsilon - \underline{\epsilon}}{1 - \gamma \delta_c} \tag{5}$$

such that: i) $a < b < \underline{\epsilon}$ when $s > s_1$, ii) $a < \underline{\epsilon} < b < \bar{\epsilon}$ when $s \in (s_0, s_1)$, and iii) $\underline{\epsilon} < a < b < \bar{\epsilon}$ when $s < s_0$.

According to Lemma 1 a stationary search rule always exists, even when the anticipated equilibrium prices are non-stationary. Intuitively this is because the benefits of search are measured relative to other firms - and we focus on a symmetric equilibrium in which all firms charge the same price in a given time period. (Of course the absolute value of search V_t is non-stationary if equilibrium prices are non-stationary.) The optimal search rule itself differs qualitatively according to the magnitude of the search cost s. Firstly, when $s > s_1$ the cost of searching for new product matches is prohibitively high. Consequently, consumers who are new to the market buy from the first firm they search (since $a < \underline{\epsilon}$), and then return to that firm in every future period (since $b < \underline{\epsilon}$). Secondly, when $s \in (s_0, s_1)$ new consumers again buy the first product they search because the cost of finding a better alternative is too high. However now we have $b > \underline{\epsilon}$ and so in the next period, if the consumer's first match was too low she searches and buys a new product; this continues until either the consumer exits the markets, or she finds a sufficiently good product. Thirdly, when $s < s_0$ the value of search is relatively high and so $\underline{\epsilon} < a < b < \bar{\epsilon}$. Compared with the previous case, the difference is that now consumers who are searching for a better product (either because they are new to the market, or only found low matches in the past) may search multiple times in a given period.[2]

An interesting implication of Lemma 1 is therefore that when s is relatively small, consumers may 'stagger' their search across multiple periods. Specifically, when $s < s_1$ some consumers end up buying a different product in each of the early periods of their lifetime, before finally settling on a particular product and repeatedly buying it thereafter. Put differently, consumer learning is gradual. This behavior is optimal because after searching a new product the search cost is sunk and so immediate purchase is relatively attractive, but buying the same product in a future period requires payment of $r > 0$ and so is less attractive. Notice also that when $s < s_0$ some consumers search multiple times during the early period of their lifetime, but that since $b - a = r$ the fraction who repeat purchase shrinks to zero as $r \to 0$.

Figure 1 plots the three different search regions described in Lemma 1 as a function of s and r. (Note that since by assumption $r < s$, only the upper triangle is relevant.) Observe that s_1 is linearly increasing in r, whilst s_0 is decreasing and convex in r with $\lim_{r \to 0} s_0 = \lim_{r \to 0} s_1$. Hence the third possibility with $\underline{\epsilon} < a < b < \bar{\epsilon}$ only arises when both r and s are relatively small. Finally, the search rule has some intuitive comparative statics properties:

Remark 1. *An increase in s, or a decrease in $\gamma\delta_c$, leads to a reduction in both a and b. An increase in r reduces a but increases b.*

An increase in the search cost s reduces a consumer's value from seeking out new matches.

[2]Notice that by shifting up the support of the entire ϵ distribution, the condition $\underline{\epsilon} > s$ from Section 2 becomes easier to meet, but none of the search regions described in Lemma 1 change.

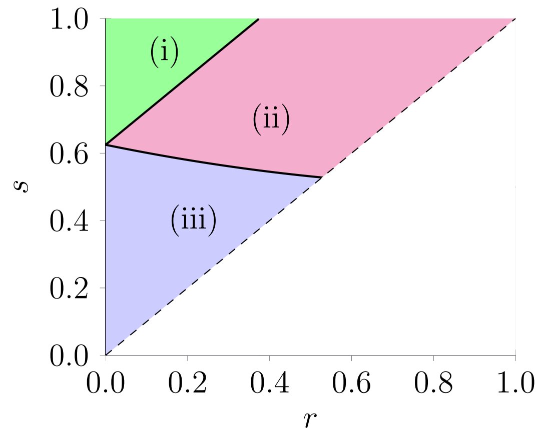

Figure 1: Regions of Search Rule

Hence as s increases, consumers become less choosy both in terms of what to buy and what offers to return to. Similarly, an increase in the return cost r reduces the value of search and therefore makes consumers less choosy when buying, but also makes them more willing to search afresh rather than return to the firm they last purchased from. As one would expect, as $\gamma\delta_c$ increases consumers place more weight on future surplus, and so have more incentive to search in the present and find a good match, hence a and b both increase.

4 Equilibrium prices

We now solve for equilibrium prices when consumers follow the search rule in Lemma 1.

In order to derive an expression for profit, it is useful to introduce some additional notation. Let X_t be the fraction of consumers who (on the equilibrium path) search for a match in period t. The remaining fraction $1 - X_t$ of consumers immediately return to (and buy) a product which they searched in a previous period. We then have the following difference equation:

$$X_t = 1 - \gamma + \gamma \frac{F(b) - F(a)}{1 - F(a)} X_{t-1} \ . \tag{6}$$

To understand this, note that $1 - \gamma$ new consumers enter the market in period t, and by definition they need to search for a match. In addition, consumers who were searching for a match in period $t - 1$ will have stopped upon finding a product with $\epsilon \geq a$. However conditional on surviving to period t, those for whom their last match satisfied $\epsilon \in [a, b)$

prefer to restart search. Solving equation (6) and using $X_0 = 1$ as a boundary condition, we obtain

$$X_t = \frac{[1 - F(a)]\,(1-\gamma)}{1 - F(a) - \gamma\,[F(b) - F(a)]} \left[1 - \left(\gamma \frac{F(b) - F(a)}{1 - F(a)}\right)^t\right] + \left(\gamma \frac{F(b) - F(a)}{1 - F(a)}\right)^t . \tag{7}$$

It is straightforward to check that X_t is monotonically decreasing with t. This is because as time progresses, more of the consumers who have survived from previous periods have found a strong match and so have no further need to search.

We now look for a subgame perfect equilibrium where regardless of its past history of prices, a firm charges price p_t^* in period t. We also make an additional assumption that if (off the equilibrium path) a consumer wishes to buy a product and she searched two different sellers of it in the preceding period, she will return to the one she searched first. Given this, we can write firm j's profit function in period t when it charges a price $p_{j,t}$ as follows:

Lemma 2. *Given any history of prices, firm j's discounted profit in period t is as follows:*

$$X_t \left(\frac{\pi\left(p_{j,t}\right)\left[1 - F\left(a + u\left(p_t^*\right) - u\left(p_{j,t}\right)\right)\right] + \left[1 - F(b)\right] \sum_{i=1}^{\infty} (\gamma\delta_f)^i \pi\left(p_{t+i}^*\right)}{1 - F\left(a\right)} \right)$$
$$+ \left(1 - X_t\right) \left(\pi\left(p_{j,t}\right) + \sum_{i=1}^{\infty} (\gamma\delta_f)^i \pi\left(p_{t+i}^*\right) \right) \qquad \text{if } p_{j,t} \leq u^{-1}\left[u\left(p_t^*\right) - r\right] , \tag{8}$$

and

$$\left(X_t \frac{1 - F(b)}{1 - F\left(a\right)} + 1 - X_t \right) \sum_{i=1}^{\infty} (\gamma\delta_f)^i \pi\left(p_{t+i}^*\right) \qquad \text{if } p_{j,t} > u^{-1}\left[u\left(p_t^*\right) - r\right] . \tag{9}$$

A key step in the proof of Lemma 2 is to show that regardless of what prices firm j charged in the past, in period t it receives the same share of searchers X_t and return consumers $1 - X_t$ as any other firm. The derivation of demand is then straightforward. The lemma distinguishes between whether $u\left(p_{j,t}\right)$ is above or below $u\left(p_t^*\right) - r$. In the former case, a consumer who has searched firm j prefers to buy its product there rather than pay r and buy it from another (otherwise identical) seller. The reverse is true in the latter case.

Equation (8) considers the case $u\left(p_{j,t}\right) \geq u\left(p_t^*\right) - r$. The first term in (8) is the discounted present value of profits earned on the X_t consumers who are looking for a match. As is standard, the measure of these consumers which search firm j is proportional to $1/[1-F(a)]$. When such a consumer searches j, she buys the product if $\epsilon_j \geq a + u(p_t^*) - u(p_{j,t})$, and otherwise keeps searching and never comes back. Moreover since (8) is written for prices which satisfy $a + u(p_t^*) - u(p_{j,t}) \leq b$, in the following periods only those consumers with $\epsilon_j \geq b$ who have survived will return and buy from firm j. The second expression in (8) is the discounted present value of profits earned on consumers who return to firm j in period t. Given the prices considered in (8), these consumers prefer to buy from firm j rather than visit another seller of the same product, and also prefer to buy rather than search for a new product because $u(p_{j,t}) + \epsilon_j \geq u(p_t^*) + a$. Equation (9) considers the case $u(p_{j,t}) < u(p_t^*) - r$. No consumer will buy from firm j in period t because it is cheaper to pay r and visit another seller of the same good. However given our tie-break rule, consumers with $\epsilon_j \geq b$ who survive to later periods will come back and buy from firms j.

Using the profit expressions in Lemma 2 we can then solve for equilibrium prices. To this end it is convenient to define for all $p < p^m$ the function

$$H(p) = -\frac{\pi(p)\, u'(p)}{\pi'(p)} \; , \tag{10}$$

which due to logconcavity of $Q(p)$ is monotonically increasing in all $p < p^m$ and satisfies $\lim_{p \to p^m} H(p) = \infty$. We can then state:

Proposition 1. *Suppose either that $f(\epsilon)$ is weakly increasing, or that $1 - F(\epsilon)$ is strictly logconcave and γ is sufficiently small. There is a subgame perfect equilibrium in which after any history a firm charges:*
i) If $s > s_0$ then $p_t^* = p^m$, and
ii) If $s < s_0$ then $p_t^* < p^m$ is strictly increasing over time and satisfies

$$H(p_t^*) = \frac{1 - F(a)}{X_t f(a)} \; . \tag{11}$$

According to Proposition 1 our model has the novel prediction that prices should weakly rise as the market matures.[3] Intuitively, as we showed in Lemma 2, each firm faces a mix of searchers and return consumers. The return consumers are locally price-insensitive because they revisit a firm knowing its match is relatively high, and so in the spirit of Diamond (1971) once they have sunk the return cost r these consumers will buy even if the price is somewhat higher than expected. The searchers are also locally price-insensitive if $s > s_0$ (such that $a < \underline{\epsilon}$) because the search cost is too high; in this case firms sell to every consumer who comes to them, and so each acts like a local monopolist charging p^m. However when $s < s_0$ (such that $a > \underline{\epsilon}$) the searchers are price sensitive, and increases in price induce some of them to leave in search of a better match. The equilibrium price trades off the incentive to price low in order to sell to more searchers, but raise price towards p^m in order to exploit return consumers. Since the ratio of searchers falls over time, demand becomes less elastic and therefore the equilibrium price increases.

An interesting feature of our model is that when a firm slightly changes its price p_t away from the equilibrium level p_t^*, it only affects flow profit earned in period t i.e. there is no effect on future profits. This contrasts with switching costs models, in which one motive for firms to moderate their price is to invest in future market share (see e.g. Klemperer (1987)). Such an investment motive is absent from our model, due to the wedge between the two reservation values a and b. Precisely, marginal fresh consumers in period t who are just indifferent about buying will not return to the firm in the future. Therefore even if the firm slightly reduces its price in period t and induces more fresh consumers to buy, this has no effect on its future profitability.

The equilibrium price in our model also depends upon parameters in an intuitive way:

Lemma 3. *Suppose the hypotheses of Proposition 1 hold and that $a > \underline{\epsilon}$. The equilibrium price is increasing in s, decreasing in δ_c, and independent of δ_f.*

The search cost s and consumer discount factor δ_c affect equilibrium pricing through two complementary channels. As s increases or δ_c decreases, the value of search falls and according to Remark 1 consumers become less choosy. In turn this reduces the fraction X_t of searchers in the market, and also makes those who do search less price elastic – rendering

[3] In the knife-edge case of $s = s_0$ (which means $a = \underline{\epsilon}$) there is generally a continuum of equilibria in any period (of which $p_t^* = p^m$ is one) because a firm's profit function is kinked.

firms' overall demand less elastic, and prompting them to charge a higher price at each point in time. However δ_f has no effect on equilibrium pricing because, as discussed above, local changes in δ_f have no effect on the measure of consumers who return and buy in future periods, and therefore at each point in time a firm is effectively maximizing the profit it earns just in that one period. On the other hand, both the return cost r and the survival probability γ have ambiguous effects on demand elasticity, and numerical simulations show that both can either increase or decrease equilibrium price depending on other parameters. To illustrate this, consider the effect of a small change in r. When $t = 0$ all consumers are searchers (i.e. $X_0 = 1$) and so since an increase in r makes searchers less choosy the equilibrium price is increasing in r. However when $t \geq 1$ there is an additional effect, namely that an increase in r also dissuades consumers from returning to products with low matches, thereby raising the fraction of consumers are searchers. Hence an increase in r shifts demand towards more elastic searchers, but simultaneously reduces the elasticity of those searchers, resulting in an ambiguous comparative static.

Finally, it is also possible to provide a partial comparison with the standard one-shot search model that is commonly used in the literature:

Lemma 4. *Suppose the hypotheses of Proposition 1 hold and that $a > \underline{\epsilon}$. There exists a $\tau \geq 0$ such that for all periods $t = 0, 1, ..., \tau$ the equilibrium price in our model is strictly lower than in the standard model where consumers buy only once.*

There are two key ways in which prices in our model differ from the standard one. On the one hand, ceteris paribus prices are higher because a portion of each firm's demand come from return consumers whose decision of whether or not to buy is perfectly inelastic (even though the quantity they buy is price sensitive). On the other hand, since consumers are long-lived they have more incentives to search harder at the beginning of their lifetime, which leads to increased competition amongst firms for searchers. Lemma 4 shows that in earlier periods, when searchers are relatively more numerous, the second effect dominates and prices are lower than in the standard model (which can equivalently be thought of as a special case of our model with $\gamma = 0$).

5 Extension: price commitment

We now extend our benchmark model by allowing firms to have some commitment power over the prices they will charge in future periods. Specifically, in each period firms choose two different prices $p_{f,t}$ and $p_{r,t}$. Fresh consumers who search a firm for the first time in period t pay s and then learn both of these prices: $p_{f,t}$ denotes the price they pay if they buy in period t, and $p_{r,t}$ denotes the personalized price which the firm commits to charge them in the future if they buy today. (For simplicity we assume that this committed price is the same in all future periods $t+1, t+2, ...$)

Following our earlier analysis, we start by solving for consumers' optimal stationary search rule. We look for two thresholds a and b such that consumers buy if $\epsilon \geq a$, return if $\epsilon \geq b$, and where not all consumers return to firms from whom they previously purchased. (After deriving the equilibrium we check that the latter is satisfied.) The analysis closely follows that from Section 3. In particular closely mimicking equations (2) and (3) the two thresholds satisfy

$$a + u\left(p_{f,t}^{*}\right) + \gamma\delta_c V_{t+1} = V_t \ , \tag{12}$$

$$\frac{b + u\left(p_{r,t}^{*}\right) - r}{1-\gamma\delta_c} = V_{t+1} \ , \tag{13}$$

where V_t denotes the value of search in period t and satisfies

$$V_t = -s + F(a) V_t + \int_a^b \left[u\left(p_{f,t}^{*}\right) + \epsilon + \gamma\delta_c V_{t+1}\right] dF(\epsilon) + \int_b^{\bar{u}} \left[u\left(p_{f,t}^{*}\right) + \epsilon + \frac{\gamma\delta_c}{1-\gamma\delta_c}\left[u\left(p_{r,t}^{*}\right) + \epsilon - r\right]\right] dF(\epsilon) \ . \tag{14}$$

Exactly as in Lemma 1 we can prove existence of a unique pair of time-independent thresholds (a, b) which satisfy $\underline{\epsilon} < a < b < \bar{\epsilon}$ provided search costs are sufficiently small.

Now consider a firm's optimal pricing problem given that search costs lie in this part of the parameter space. In any given period t the firm has some return demand, but the price paid by those consumers has already been determined and is no longer under the firm's control. The profit earned by searchers in period t can be written as

$$\frac{\tilde{X}_t}{1-F(a)}\left(\pi\left(p_{f,t}\right)\left[1-F(a+u\left(p_{f,t}^*\right)-u\left(p_{f,t}\right))\right]+\right.$$
$$\left.\frac{\gamma\delta_f}{1-\gamma\delta_f}\pi\left(p_{r,t}\right)\left[1-F(b+u\left(p_{r,t}^*\right)-u\left(p_{r,t}\right))\right]\right) . \quad (15)$$

where $\tilde{X}_t$ is defined in a similar way as in equations (6) and (7) from earlier. Note that in writing down this profit function we are focusing on small deviations from $p_{f,t}^*$ and $p_{r,t}^*$ such that searchers who are just indifferent about buying in period t will indeed prefer not to return to the same firm in the following period. Recalling the definition of $H(p)$ in equation (10), it is then straightforward to prove that:

Proposition 2. *Suppose that $1-F(u)$ is strictly logconcave and that s and r are relatively small. When firms can commit to future prices, there is a stationary subgame perfect equilibrium in which firms charge $p_{r,t}^* < p_{f,t}^*$ which are independent of time and solve*

$$H\left(p_{f,t}^*\right)=\frac{1-F(a)}{f(a)} \quad \text{and} \quad H\left(p_{r,t}^*\right)=\frac{1-F(b)}{f(b)} . \quad (16)$$

Proposition 2 shows that with price commitment firms offer a discount to returning consumers. This may seem counter-intuitive, given that an important implication of the main model was that return demand was less elastic than demand coming from searchers. The difference arises because in the main model return consumers could not observe the price before paying r, whereas with commitment they do observe the price. Moreover since the presence of a return cost disincentivizes consumers from coming back, firms respond to this by offering them a discount. Hence in the parlance of the literature on behavior-based price discrimination, firms 'pay-to-stay'. Mirroring Lemma 4 from earlier, it is also straightforward to prove the following result:

Lemma 5. *Suppose that $1-F(u)$ is strictly logconcave and that s and r are relatively small. Both equilibrium prices with commitment are strictly lower than in the standard model where consumers buy only once.*

Interestingly, with commitment and behavior-based price discrimination, all consumers pay less than they would in the standard Wolinsky-Anderson-Renault model. Intuitively this derives from the fact that due to the return cost $r > 0$, marginal buyers in period t will not return in period $t + 1$, and therefore firms face separable problems when choosing $p^*_{f,t}$ and $p^*_{r,t}$. In particular $p^*_{f,t}$ is chosen to maximize a profit function which is identical to the standard (one-shot) model with one important difference - in our model fresh consumers may live for many periods, and so are choosier which leads to fiercer competition. Therefore fresh consumers pay less than in the standard model. Return consumers benefit from a further discount, and so also pay less than in the standard model. Consequently behavior-based price discrimination is unambiguously good for consumers and bad for firms, even though each firm individually has an incentive to use it when possible.

Finally, all our analysis in this section is based on the assumption that firms commit to the future *price* they will charge returning customers. An alternative policy would be to commit to the size of a discount (if any) that return consumers could receive off the price paid in future periods by fresh consumers. Qualitatively the analysis is unchanged: the search rule is the same, and firms continue to offer a strictly positive discount to their existing customers. One difference however is that now a small increase in the fresh price partially holds up returning customers, and so the solution is a mixture of the analysis in the benchmark model and the analysis from this section.[4]

6 Conclusion

Many markets have the feature that consumers wish to buy repeatedly and need to search in order to learn about what products are available in the market. Unfortunately much of the existing search literature has ignored the importance of repeat purchases. In this paper we show that accounting for this leads to several novel predictions. For instance consumers may optimally choose to spread their search for new products over several periods, even though doing so leads them to purchase relatively low-value products when they are new to the market. Our paper also makes the novel prediction that mature industries should have larger prices, ceteris paribus, than newer markets because over time the match between

[4]Further details are available on request.

consumers and sellers improves, and this induces sellers to partially 'hold up' consumers. Finally we considered the implications for price commitment, and showed that the ability to offer discounts to repeat consumers completely undoes the hold up problem, and actually leads to fiercer competition and lower industry profits.

7 Appendix

Proof of Lemma 1 First note that if a stationary search rule exists, it must have $b < \bar{\epsilon}$. This is because if $b \geq \bar{\epsilon}$ then consumers search every period, but then their payoff in any period τ cannot exceed $u(p^*_\tau) + \bar{\epsilon} - s$, and so we infer that

$$V_t \leq \sum_{i=0}^{\infty} (\gamma\delta_c)^i \left[u\left(p^*_{t+i}\right) + \bar{\epsilon} - s\right] .$$

However using (3) this contradicts the supposition that consumers with best match $\epsilon = \bar{\epsilon}$ prefer to search rather than return.

Second we prove existence and uniqueness of a stationary search rule with $a < b$. Use equation (2) to substitute out for $\gamma\delta_c V_{t+1}$ in equation (4), which yields

$$V_t = \frac{1}{1 - F(b)} \left[-s + \int_a^{\bar{\epsilon}} \epsilon dF(\epsilon) - a\left[F(b) - F(a)\right] + \frac{\gamma\delta_c}{1 - \gamma\delta_c} \int_b^{\bar{\epsilon}} (\epsilon - r)\, dF(\epsilon)\right] + \sum_{i=0}^{\infty} (\gamma\delta_c)^i u\left(p^*_{t+i}\right) . \quad (17)$$

Next use (17) to substitute out V_t and (moving everything forward one period) V_{t+1} from equation (2), which yields

$$a = \frac{1 - \gamma\delta_c}{1 - F(b)} \left[-s + \int_a^{\bar{\epsilon}} \epsilon dF(\epsilon) - a\left[F(b) - F(a)\right] + \frac{\gamma\delta_c}{1 - \gamma\delta_c} \int_b^{\bar{\epsilon}} (\epsilon - r)\, dF(\epsilon)\right] . \quad (18)$$

Similarly, use (17) to substitute V_t out of equation (3), which yields

$$b - r = \frac{1 - \gamma\delta_c}{1 - F(b)} \left[-s + \int_a^{\bar{\epsilon}} \epsilon dF(\epsilon) - a\left[F(b) - F(a)\right] + \frac{\gamma\delta_c}{1 - \gamma\delta_c} \int_b^{\bar{\epsilon}} (\epsilon - r)\, dF(\epsilon)\right] . \quad (19)$$

Notice that (18) and (19) imply that $a = b - r < b$, as claimed in the lemma. Substituting

$a = b - r$ into (18) we obtain that a solves $G(a) = 0$ where

$$G(a) = a\left[1 - F(a) - \gamma\delta_c F(a+r) + \gamma\delta_c F(a)\right] + s(1 - \gamma\delta_c) \\ - (1 - \gamma\delta_c)\int_a^{\bar{\epsilon}} \epsilon dF(\epsilon) - \gamma\delta_c \int_{a+r}^{\bar{\epsilon}} (\epsilon - r)\, dF(\epsilon) \quad (20)$$

Note that $G(a)$ is globally increasing in a since

$$G'(a) = 1 - F(a) - \gamma\delta_c\left[F(a+r) - F(a)\right] > 1 - F(a+r) > 0 \ ,$$

and also satisfies

$$\lim_{a \to \bar{\epsilon} - r} G(a) = (1 - \gamma\delta_c)\left\{s - r\left[1 - F(\bar{\epsilon} - r)\right] + \int_{\bar{\epsilon} - r}^{\bar{\epsilon}} \left[\bar{\epsilon} - \epsilon\right] dF(\epsilon)\right\} > 0 \ ,$$

and $\lim_{a \to -\infty} G(a) = -\infty$. Consequently for any set of parameters there is a unique a and b. Clearly (18) and (19) also imply that these a and b are time-independent and therefore stationary.

Third we prove that a and b follow the characterization in the lemma. (i) $b < \underline{\epsilon}$ if and only if $G(\underline{\epsilon} - r) > 0$ which is equivalent to $s > s_1$ where s_1 is defined in the lemma. (ii) $a < \underline{\epsilon}$ if and only if $G(\underline{\epsilon}) > 0$ which is equivalent to $s > s_0$ where s_0 is defined in the lemma.

Finally we prove there is no stationary search rule with $a \geq b$. Suppose to the contrary that there is, and let W_t denote the value of search in this case (to distinguish it from the value V_t which we defined for the case $a < b$). Since a consumer who draws $\epsilon = a$ is indifferent between searching again or buying it now and in all future periods, we have

$$\sum_{i=0}^{\infty} (\gamma\delta_c)^i \left[u\left(p_{t+i}^*\right) + a - r\right] + r = W_t \ .$$

Since a consumer who previously drew $\epsilon = a$ weakly prefers to return and buy it from period t onwards we also have

$$\sum_{i=0}^{\infty} (\gamma\delta_c)^i \left[u\left(p_{t+i}^*\right) + a - r\right] \geq W_t \ .$$

However these two conditions are incompatible since $r > 0$, yielding a contradiction. ■

Proof of Remark 1 Recall from the proof of Lemma 1 that a satisfies $G(a) = 0$ where $G(a)$ was defined earlier in equation (20). Recall also that $G'(a) > 0$. First, notice that $\partial G(a)/\partial s = 1 - \gamma\delta_c > 0$ such that $\partial a/\partial s < 0$. Moreover since $b = a + r$ this also implies that $\partial b/\partial s < 0$. Second, notice that $\partial G(a)/\partial r = \gamma\delta_c[1 - F(a+r)] > 0$ such that $\partial a/\partial r < 0$. Moreover since $b = a + r$ we have

$$\frac{\partial b}{\partial r} = 1 - \frac{\gamma\delta_c[1 - F(a+r)]}{1 - F(a) - \gamma\delta_c[F(a+r) - F(a)]} = \frac{[1 - F(a)](1 - \gamma\delta_c)}{1 - F(a) - \gamma\delta_c[F(a+r) - F(a)]} > 0 \ .$$

Third, notice that

$$\frac{\partial G(a)}{\partial(\gamma\delta_c)} = -s + r[1 - F(a+r)] + \int_a^{a+r} (\epsilon - a)\, dF(\epsilon) < -s + r[1 - F(a)] < 0 \ ,$$

where the first inequality uses $\int_a^{a+r} (\epsilon - a)\, dF(\epsilon) < r[F(a+r) - F(a)]$, and the second uses $r < s$. Hence we conclude that $\partial a/\partial(\gamma\delta_c) > 0$, and since $b = a + r$ we also have $\partial b/\partial(\gamma\delta_c) > 0$. ■

Proof of Lemma 2

Step 1: We prove that irrespective of its past prices, firm j is visited by an equal share of the searchers X_t and of the return consumers $1 - X_t$ in period t. This clearly holds for $t = 0$, so we prove by induction that it is true for any t.

Suppose that firm j was visited by an equal share in period $t-1$. Firstly, in period t the firm must receive an equal share of searchers because they cannot observe any firm's past history of prices. Secondly then, consider returning consumers in period t. (a) Given our tie-break rule and consumer beliefs about prices, all those who returned in period $t-1$ and have survived will return in period t. (b) Consider consumers who searched firm j in period $t-1$. If $u(p_{j,t-1}) \geq u(p^*_{t-1}) - r$ then all those with $\epsilon_j \geq b$ bought from firm j in period t and will return (given their price beliefs, if they survive) in period t. If instead $u(p_{j,t-1}) \leq u(p^*_{t-1}) - r$ then those with $\epsilon_j \geq b$ bought j's product from another seller, but given their beliefs and our tie-break rule will return and buy from firm j in period t.

Step 2: The remainder of the lemma is explained in the text, except for the $1/[1 - F(a)]$ which multiplies the first term in equation (8). Each consumer is randomly drawn a list of products which she has not yet searched. Conditional on not having searched firm j previously, firm j might be first in the list and so searched for sure, or it might be second in the list and so searched with probability $F(a)$, and so forth. Since the list is infinitely long, we obtain $1/[1-F(a)]$. ■

Proof of Proposition 1 Firstly, note that any $p_{j,t} > u^{-1}[u(p_t^*) - r]$ is strictly dominated. Similarly from (8) it is clear that prices strictly above p^m are strictly dominated.

Secondly, suppose $a < \underline{\epsilon}$. Notice that if $p_t^* < p^m$ then (8) is strictly increasing in $p_{j,t}$ at the point $p_{j,t} = p_t^*$, yielding a contradiction. However if $p_t^* = p^m$ then (8) is strictly increasing in $p_{j,t} < p^m$ and maximized at $p_{j,t} = p^m$. Hence the equilibrium price is p^m.

Thirdly, suppose that $a > \underline{\epsilon}$. Notice that if $p_t^* = p^m$ then (8) is strictly decreasing in $p_{j,t}$ at the point $p_{j,t} = p^m$, yielding a contradiction. Taking a first order condition of (8) with respect to $p_{j,t}$ and imposing symmetry yields (11). The claim that p_t^* is strictly increasing in t follows because we know from (11) that X_t strictly decreases in t, and as claimed in the text the lefthand side of (11) is strictly increasing in p_t^*. To see the latter, note that we can write

$$H(p_t^*) = \frac{\pi(p_t^*)}{1 + p_t^* \frac{Q'(p_t^*)}{Q(p_t^*)}} \ ,$$

and that $\pi(p_t^*)$ is strictly increasing for all $p_t^* < p^m$, as is $p_t^* Q'(p_t^*)/Q(p_t^*)$ due to the assumption that $Q(p)$ is logconcave.

To complete the proof, we provide sufficient conditions for (8) to be quasiconcave in $u(p_{j,t})$ (and therefore also in $p_{j,t}$). As a first step, notice that $\pi'(u(p)) = -\left[1 + p_t^* \frac{Q'(p_t^*)}{Q(p_t^*)}\right]$ and so $\pi''(u(p)) \leq 0$ due to $Q(p)$ being logconcave. (a) The second derivative of (8) with respect to $u\left(p_j^t\right)$ is easily shown to be strictly negative provided that $f'(\epsilon) \geq 0$. (b) Alternatively, we may write profit as a constant plus

$$\pi(u(p_{j,t})) \times \left[X_t \frac{1 - F(a + u(p_t^*) - u(p_{j,t}))}{1 - F(a)} + 1 - X_t\right] \ ,$$

The first term is concave and so logconcave, whilst the second terms is logconcave provided that $1 - F(\epsilon)$ is strictly logconcave and X_t is sufficiently close to 1 or equivalently γ is sufficiently small. ■

Proof of Lemma 3

Defining $Z(a) = \gamma [F(a+r) - F(a)] / [1 - F(a)]$ we can rewrite (7) as

$$X_t = \frac{(1-\gamma)}{1 - Z(a)} \left[1 - Z(a)^t\right] + Z(a)^t \ .$$

Notice that X_t is strictly increasing in $Z(a)$, and that in turn $Z'(a) > 0$ because

$$\frac{\partial Z(a)}{\partial a} \propto \frac{f(a+r)}{1 - F(a+r)} - \frac{f(a)}{1 - F(a)} > 0 \ .$$

Hence X_t is increasing in a, and therefore so is $X_t f(a) / [1 - F(a)]$. Using Remark 1 an increase in s or a decrease in δ_c reduces a and so increases the righthand side of (11), which therefore increases p_t^* given that we showed in the proof of Proposition 1 that the lefthand side of (11) is strictly increasing in $p_t^* < p^m$. Finally, price is independent of δ_f because δ_f has no effect either on a (c.f. Lemma 1) or on X_t (c.f. equation (7)). ■

Proof of Lemma 4 Note that under the hypotheses of Proposition 1 an equilibrium exists in Anderson and Renault (1999)'s static model. Since from Proposition 1 price is strictly increasing in t, it is sufficient to prove that p_0^* is strictly lower than the price charged in a standard one-shot search model, which from Anderson and Renault (1999) and Anderson and Renault (2000) satisfies

$$H(p^*) = \frac{1 - F(\tilde{a})}{f(\tilde{a})} \quad \text{where} \quad \int_{\tilde{a}}^{\bar{u}} (u - \tilde{a})\, dF(u) = s \ .$$

Given that in our model p_0^* satisfies (11) with $X_0 = 1$, and given that the lefthand side of (11) is increasing in p_t^*, we need to prove that $a > \tilde{a}$. To prove this, notice that we can rewrite $G(a) = 0$ (with $G(a)$ defined in (20)) as

$$s = \int_a^{\bar{\epsilon}} (\epsilon - a)\, dF(\epsilon) + \frac{\gamma\delta_c}{1 - \gamma\delta_c} \int_{a+r}^{\bar{\epsilon}} (\epsilon - a - r)\, dF(\epsilon) \ , \tag{21}$$

and since $\gamma\delta_c > 0$ this implies that

$$\int_a^{\bar{u}} (u - a)\, dF(u) < \int_{\tilde{a}}^{\bar{u}} (u - \tilde{a})\, dF(u) \ ,$$

which in turn implies $a > \tilde{a}$. ■

Proof of Proposition 2 A proof concerning the search rule is available on request. Notice that each of the terms inside the brackets in equation (15) are fully separable, are quasiconcave in $u\left(p^*_{f,t}\right)$ and $u\left(p^*_{r,t}\right)$ given that $Q\left(p\right)$ is logconcave, and that the first order conditions give the expressions in equation (16). Since prices are independent of time we observe from equation (14) that $V_t = V_{t+1} = ...$, and so imposing this on equations (12) and (13) and combining them gives

$$a - p^*_f = b - p^*_r - r \ .$$

Note that by using the prices from (16), the lefthand side is monotonically increasing in a, whilst the righthand side is monotonically increasing in b, and moreover when $a = b$ the righthand side is strictly less than the lefthand side. Hence $a < b$ must hold. Combined with strict logconcavity implies from (16) that $p^*_{r,t} < p^*_{f,t}$. ■

Proof of Lemma 5 Following the same steps as in the proof of Lemma 4 it is straightforward to prove that a is strictly higher than in the standard model, and so since $p^*_{f,t}$ has the same formula it follows that $p^*_{f,t}$ is lower than in the standard model. Then use the fact that $p^*_{r,t} < p^*_{f,t}$. ■

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